CHAPTER 6 MOMENT OF INERTIA

1. MOMENTS OF INERTIA FOR AREAS

In previous sections we learnt to determine the centroid for an area by calculating the first moment of the area about an axis; this is, we evaluated the integral of the form

$$\int_A x dA$$

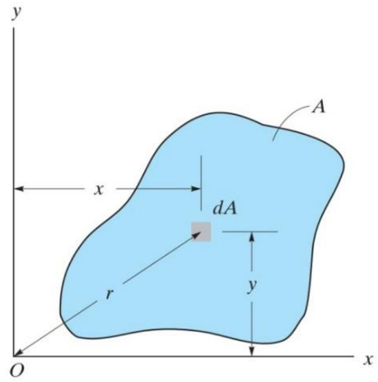
An integral of the second moment of an area, such as

$$\int_A x^2 dA$$

is known as *moment of inertia* for the area

Thus, the moments of inertia for the area are determined by

$$I_x = \int_A y^2 \, dA$$
$$I_y = \int_A x^2 \, dA$$



Polar moment of inertia is defined as

$$J_O = \int_A r^2 \, dA = I_x + I_y$$

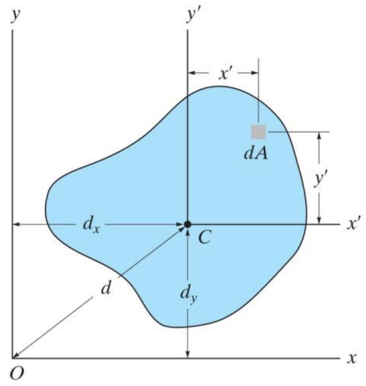
2. RADIUS OF GYRATION OF AN AREA

The radius of gyration of an area about an axis has units of length and is a quantity that is often used for the design of columns in structural mechanics. Provided the areas and moments of inertia are known, the radii of gyration are determined from

$$k_x = \sqrt{\frac{I_x}{A}} \quad k_y = \sqrt{\frac{I_y}{A}} \quad k_O = \sqrt{\frac{J_O}{A}}$$

3. PARALLEL-AXIS THEOREM FOR AN AREA

If the moment of inertia for an area is known about an axis passing through its centroid, which is often the case, it is convenient to determine the moment of inertia of the area about a corresponding parallel axis using the ***parallel-axis theorem***.



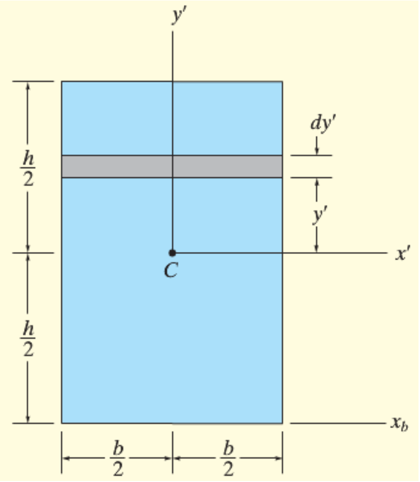
$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$I_y = \bar{I}_{y'} + Ad_x^2$$

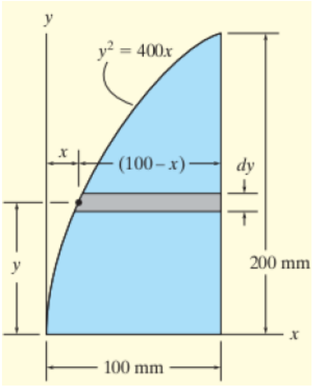
$$J_O = \bar{J}_C + Ad^2$$

The form of each of these three equations states that ***the moment of inertia for an area about an axis is equal to its moment of inertia about a parallel axis passing through the area's centroid plus the product of the area and the square of the perpendicular distance between the axes***.

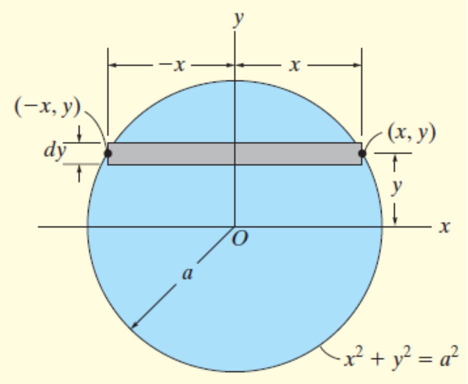
Example 1: Determine the moment of inertia for the rectangular area shown in Figure with respect to (a) the centroidal *x*' axis. (b) the axis x_b passing through the base of the rectangle, and (c) the pole or z' axis perpendicular to the x'-y' plane and passing through the centroid C.



Example 2: Determine the moment of inertia for the shaded area shown in Fig about the *x* axis.



Example 3: Determine the moment of inertia with respect to the x axis for the circular area shown in Fig.



4. MOMENTS OF INERTIA FOR COMPOSITE AREAS

A composite area consists of a series connected "simpler" parts or shapes such as rectangles, triangles, and circles. Provided the moment of inertia of each of these parts is known or can be determined about a common axis, then the moment of inertia for the composite area about this axis equals ***the algebraic sum*** of the moments of inertia of all its parts.

The moment of inertia for a composite area about a reference axis can be determined using the following procedure:

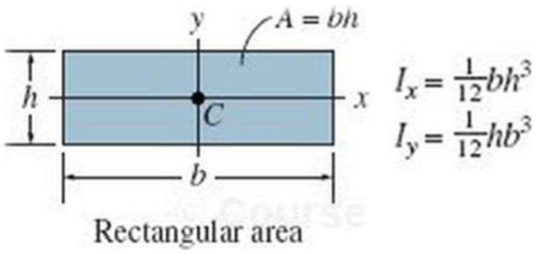
• Using a sketch, divide the area into its composite parts and indicate the perpendicular distance from the centroid of each part to the reference axis.

• **If the centroidal axis for each part does not coincide with the reference axis, the parallel-axis theorem, $I = \bar{I} + Ad^2$, should be used to determine the moment of inertia of the part about the reference axis.** For the calculation of $\bar{I}$, use the table on the inside back cover.

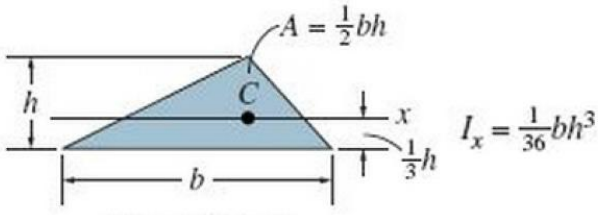
• The moment of inertia of the entire area about the reference axis is determined by summing the results of its composite parts about this axis.

• If a composite part has a "hole", its moment of inertia is found by "subtracting" the moment of inertia of the hole from the moment of inertia of the entire part including the hole.

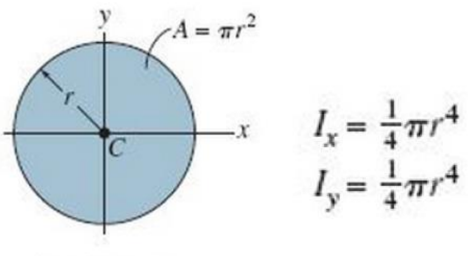
The geometric properties of the following areas are very useful in finding the moment of inertia for composite areas.



Rectangular area

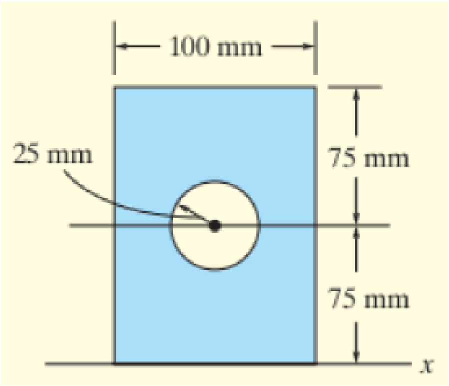


Triangular area

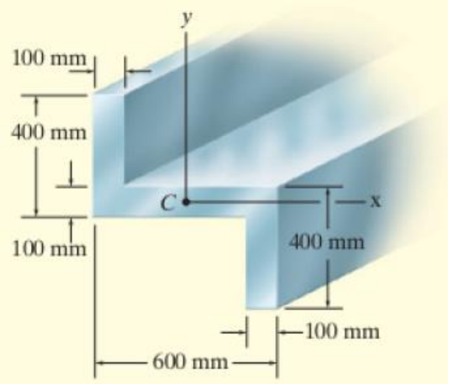


Circular area

Example 4: Determine the moment of inertia of the area shown in Figure about the x axis.

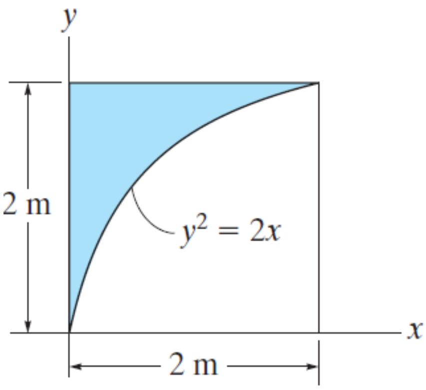


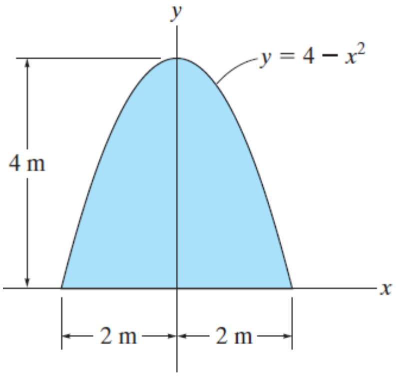
Example 5: Determine the moments of inertia for the cross-sectional area of the member shown in Figure about the x and y centroidal axes.



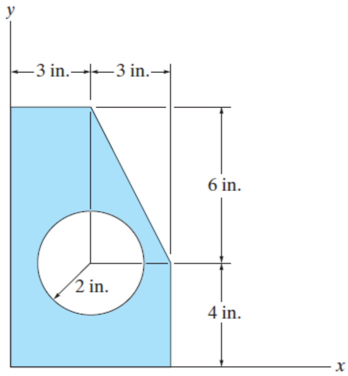
Sheet No. 1

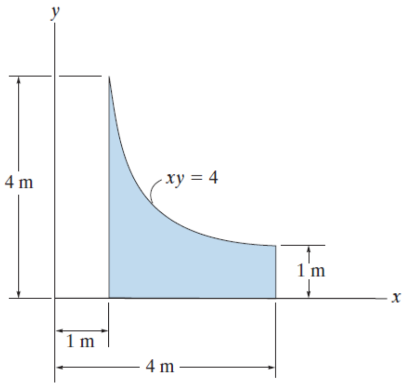
Q 1: Determine the moment of inertia of the area about the x and y axes.



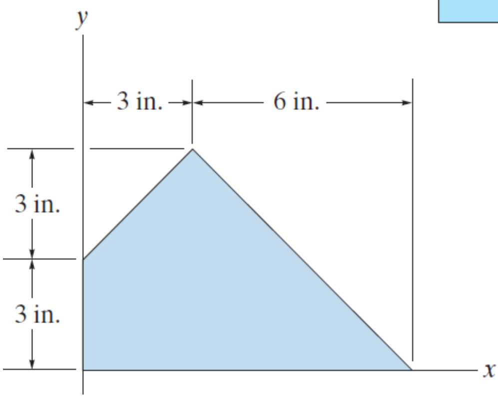


Q 2: Determine the moment of inertia for the shaded area about the x axis.



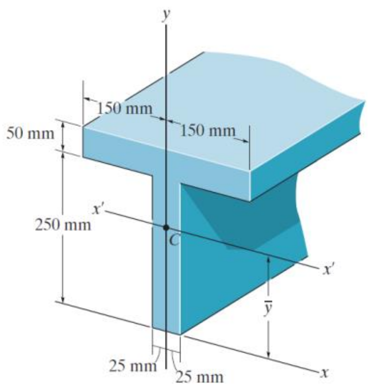


Q 3: Determine the moment of inertia of the shaded area about the y axis.



Q 4: Determine the moment of inertia of the composite area about the x and y axis.

Q 5: Determine the moment of inertia of the beam's cross-sectional area about the y axis. Determine $\bar{y}$ which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moment of inertia about the x' axis.



CHAPTER 7 EQUILIBRIUM OF A RIGID BODY

5. CONDITIONS FOR RIGID-BODY EQUILIBRIUM

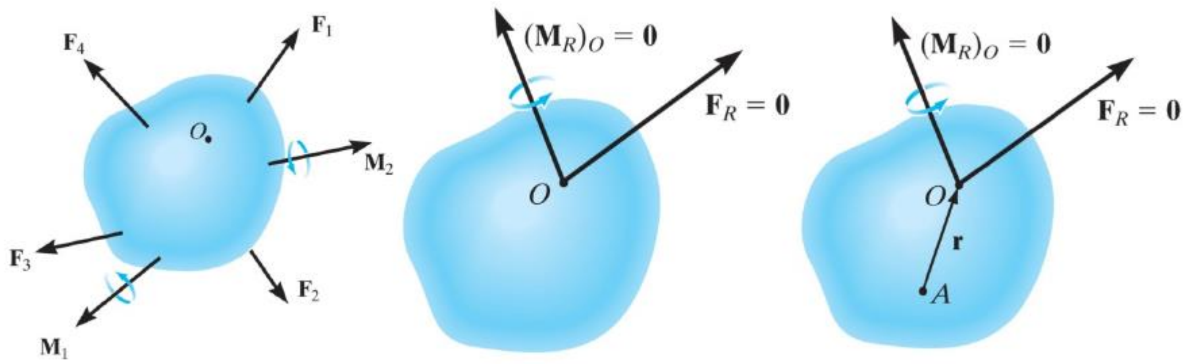
• A rigid body (RB) is formed by an infinite number of particles.

• Individual particles of the RB are subjected to internal and external forces.

• Internal forces are caused by the interaction of the particle with the particles adjacent to it.

• External forces are those caused by external effects such as gravitational, electrical, magnetic, or contact forces.

• For the rigid body, the internal forces will cancel out because they occur in equal but opposite collinear pairs.



The force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point *O*. If the body is in equilibrium, we have

$$\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{0}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O = \mathbf{0}$$

If we sum moments about another point *A*, we require,

$\Sigma\mathbf{M}_A = \mathbf{r} \times \mathbf{F}_R + (\mathbf{M}_R)_O = \mathbf{0}$, with, $\mathbf{F}_R = \mathbf{0}$ and $(\mathbf{M}_R)_O = \mathbf{0}$, the equation is satisfied.

6. FREE-BODY DIAGRAMS

A FBD is a sketch of an element of a system where all the remaining components of the system are removed and all of the forces acting on the element of interest are shown.

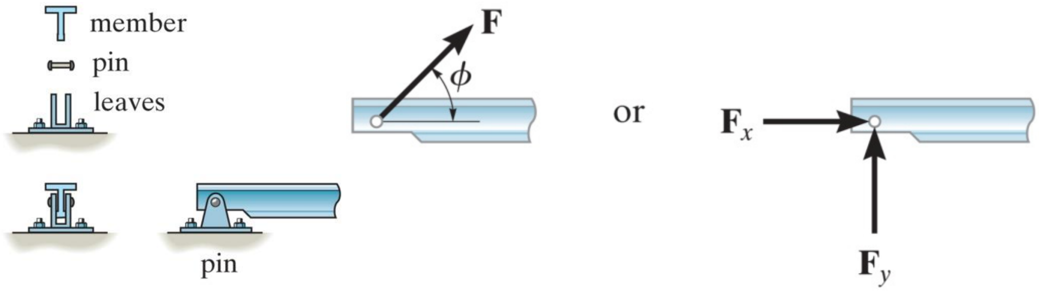
The correct computation of the forces and moments acting on a RB highly depends on the correct presentation of the FBD.

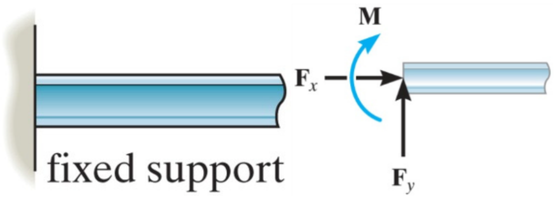
Before presenting FBD, we first consider a number of different ***structural supports***, which impose different sorts of forces on a rigid body:

• If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.

• If a support prevents rotation, then a couple moment is exerted on the body.

Three of the most common supports are: ***Roller Support, Pin Support, Fixed Support***.





Other support types can refer to the textbook.

When preparing a free–body diagram the internal forces are not presented in the diagram since these forces always occur in opposite collinear pairs having a net effect of zero on the body.

When a body is subjected to a gravitational field, each of its particles has a specified weight. The sum of all the individual weights represents the weight of the body **W** and the location where this weight **W** acts corresponds to the center of gravity.

The procedure to draw a FBD is:

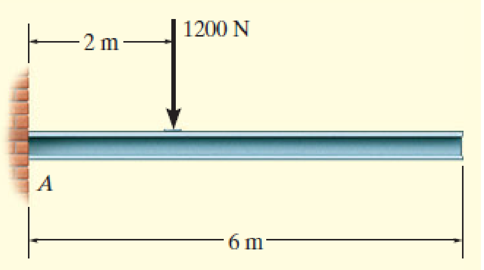
(1) Draw outline shape. Imaging the body to be isolated from its constraints and connections and sketch its outlined shape.

(2) Show all external forces and couple moments that act on the body, usually including (a) applied loadings, (b) reactions at the supports, (c) weight of the body.

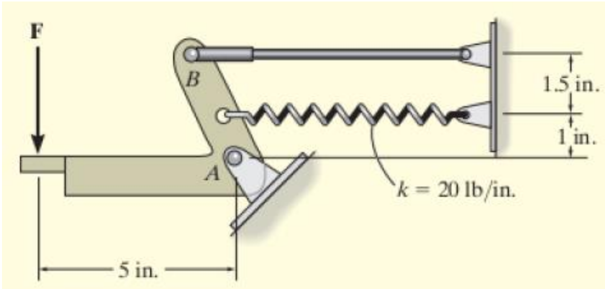
(3) Label the known forces and couple moments with their magnitudes and directions.

Indicate the dimensions of the body necessary for calculating the moments of forces.

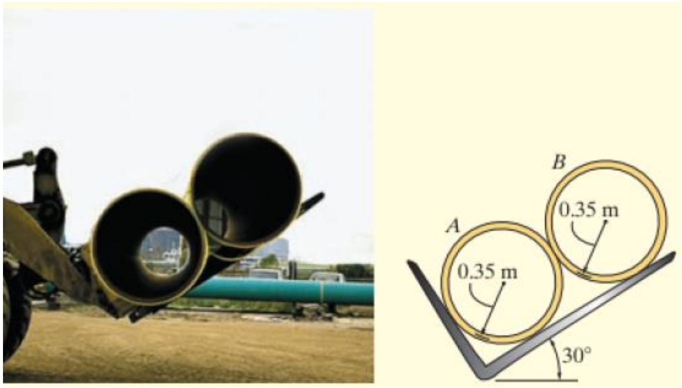
Example 1: Draw the free-body diagram of the uniform beam shown in Fig. The beam has a mass of 100 kg.



Example 2: Draw the free-body diagram of the foot lever shown in Fig. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb.



Example 3: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. Draw the free-body diagrams for each pipe and both pipes together.



7. EQUATIONS OF EQUILIBRIUM IN TWO DIMENSIONS

The conditions sufficient and necessary to ensure equilibrium in a rigid body are:

• Summation of forces equal to zero.

• Summation of moments equal to zero.

In a coplanar system, these equations are reduced to:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

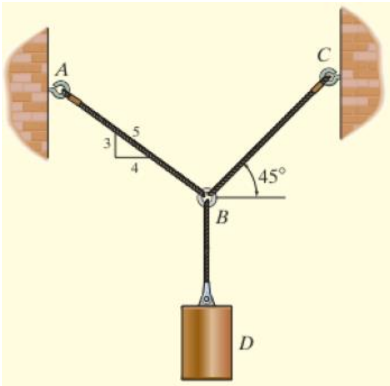
External and Internal Forces

Since a rigid body is formed by an infinite number of particles, both external and internal loadings may act on it. However, when preparing a free–body diagram the internal forces are not presented in the diagram since these forces always occur in opposite collinear pairs having a net effect of zero on the body.

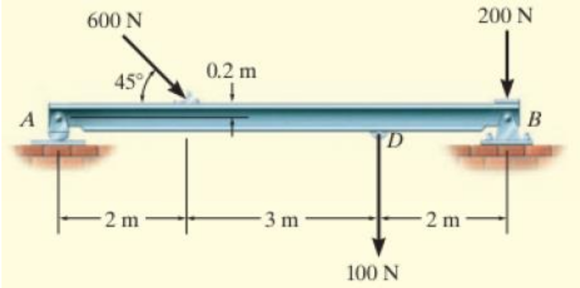
Weight and Center of Gravity

When a body is subjected to a gravitational field, each of its particles has a specified weight. The sum of all the individual weights represents the weight of the body **W** and the location where this weight **W** acts corresponds to the center of gravity.

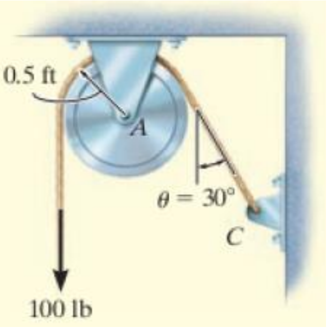
Example 4: Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig.



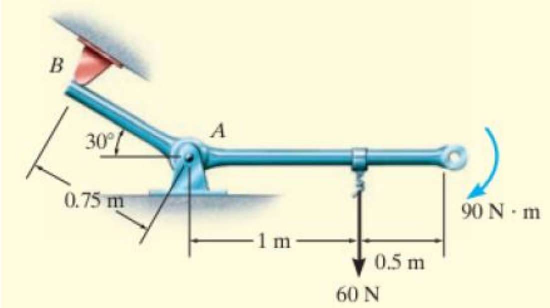
Example 5: Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. Neglect the weight of the beam.



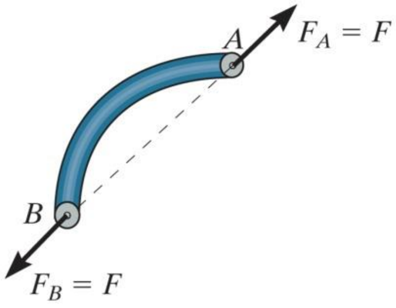
Example 6: The cord shown in Figure supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



Example 7: The member shown in Figure is pin connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.



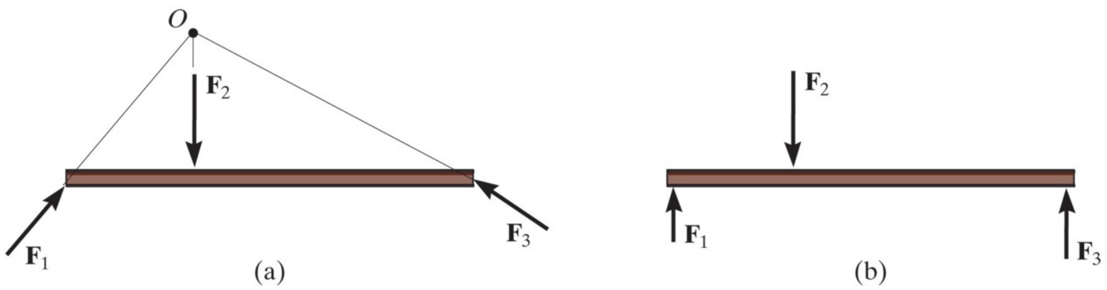
8. TWO- AND THREE-FORCE MEMBERS



When a member is subjected to ***no couple moments*** and forces are applied only at two points on the member, the element is called ***two–force member***.

For any two-force member to be in equilibrium, the two forces must have the ***same magnitude***, act in ***opposite directions***, and have ***the same line of action, directed along the line joining the two points*** where these forces act.

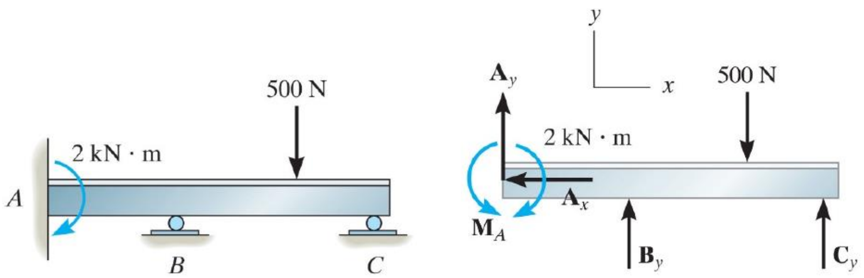
If a member is subjected to **only** three forces, it is called a ***three-force member***. It is necessary that the forces be *concurrent* or *parallel* for the member to be in equilibrium.

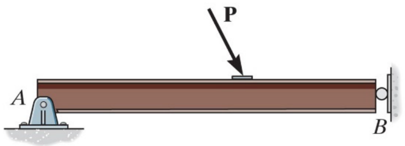


9. CONSTRAINTS AND STATICAL DETERMINACY

A body is considered to have ***sufficient supports*** if it has just enough supports to maintain its equilibrium. That is, it has just enough supports to keep it from translating in any direction and to keep it from rotating about any axis. In this case, the system is ***statically determinate***, meaning that we can find the support forces using the equations of equilibrium alone.

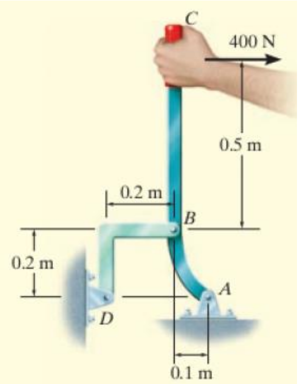
A body has ***redundant supports*** if it has ***more than enough supports*** to maintain its equilibrium. In this case, the system is ***statically indeterminate***, meaning that we cannot find the support forces using the equations of equilibrium alone. We need to include ***additional equations*** associated with the deformation/displacement in the body.





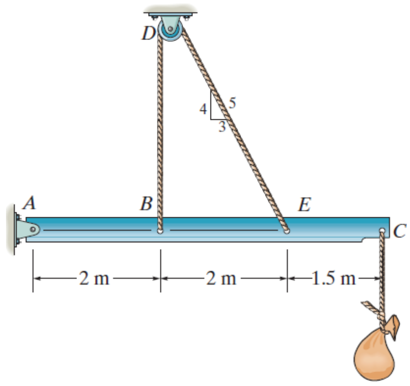
Having the same number of unknown reactive forces as available equations of equilibrium does not always guarantee that a body will be stable when subjected to a particular loading.

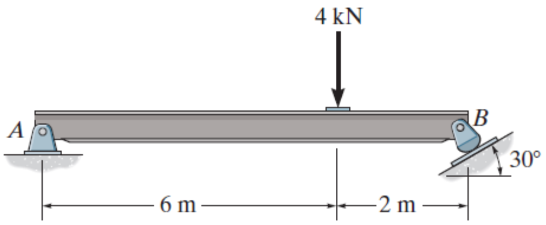
Example 8: The lever ABC is pin supported at A and connected to a short link BD as shown in Fig. If the weight of the members is negligible, determine the force of the pin on the lever at A.



Sheet No. 1

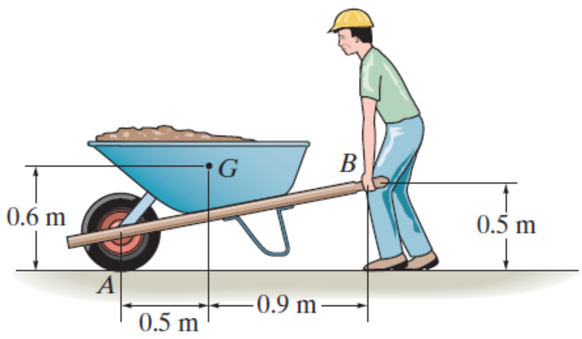
Q 1: Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram.

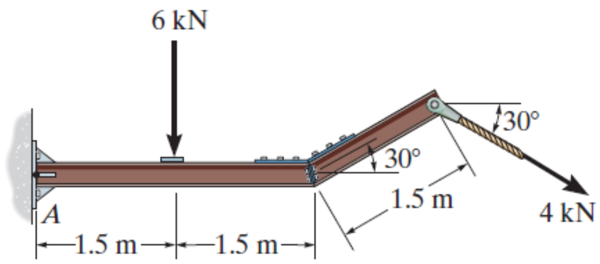




Q 2: Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.

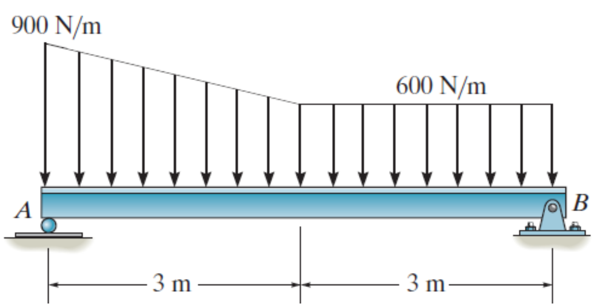
Q 3: If the wheelbarrow and its contents have a mass of 60 kg and center of mass at G, determine the magnitude of the resultant force which the man must exert on each of the two handles in order to hold the wheelbarrow in equilibrium.

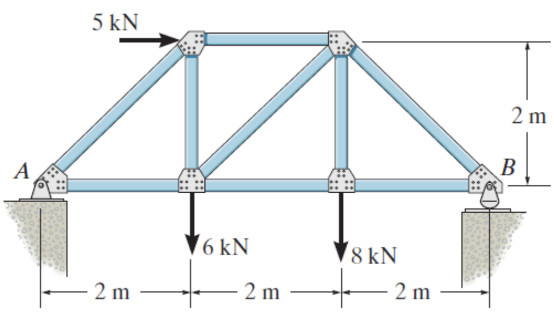




Q 4: Determine the components of the support reactions at the fixed support A on the cantilevered beam.

Q 5: Determine the reactions at the supports.





10. EQUILIBRIUM IN THREE DIMENSIONS

The first step when analyzing a three dimensional problem is to draw a FBD. This must include:

• The proper reactions at the supports. As in the two dimensional case, *a force is developed by a support that restricts the translation of the attached member, whereas a couple moment is developed when the rotation of the attached member is prevented*. Refer to Table 5-2 in the textbook for typical supports.

• The reactive forces and couple moments acting on the body analyzed.

The conditions for equilibrium if a rigid body subjected to a three dimensional force system require that both the *resultant* force and the *resultant* couple moment acting on the body be equal to *zero*. The equations of equilibrium are:

Vector form

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M}_O = \mathbf{0}$$

or ***scalar form***

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

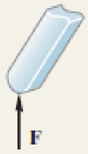
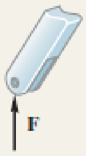
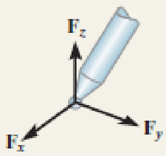
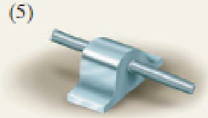
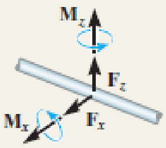
Typical examples of actual supports that are referenced to Table 4–2 are shown in the following sequence of photos.

This ball-and-socket joint provides a connection for the housing of an earth grader to its frame. (4)

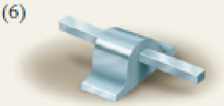
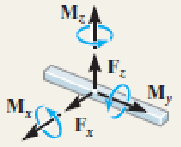
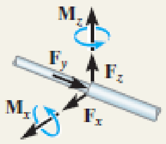
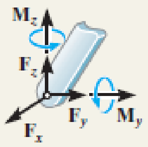
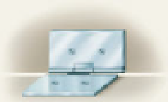
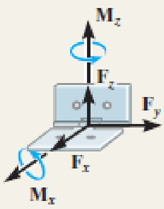
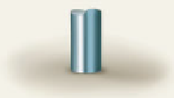
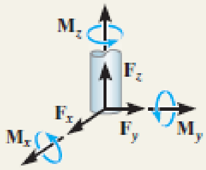
This journal bearing supports the end of the shaft. (5)

This thrust bearing is used to support the drive shaft on a machine. (7)

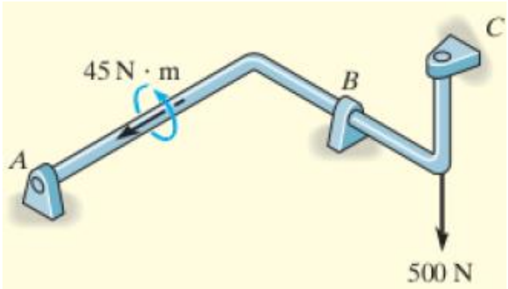
This pin is used to support the end of the strut used on a tractor. (8)

Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems		
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

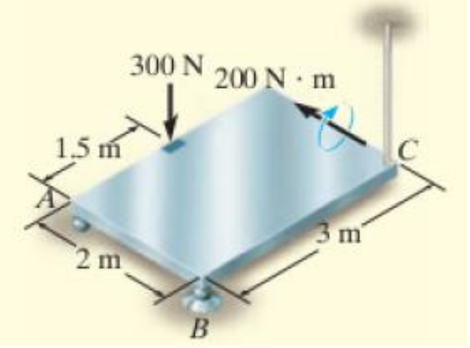
continued

Continued		
Types of Connection	**Reaction**	**Number of Unknowns**
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

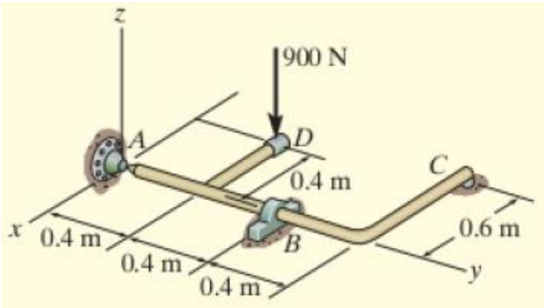
Example 9: Consider the two rods and plate, along with their associated free-body diagrams shown in Fig. The x, y, z axes arc established on the diagram and the unknown reaction components are indicated in the *positive sense*. The weight is neglected.



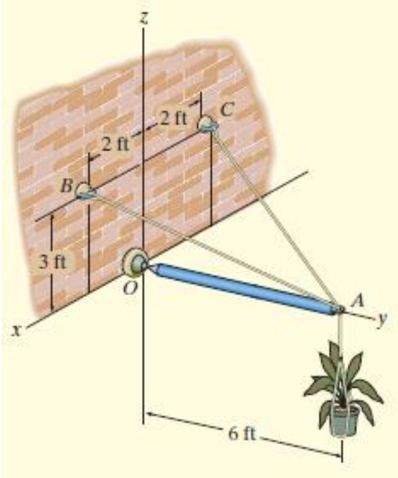
Example 10: The homogeneous plate shown in Figure has a mass of 100 kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at B, and a cord at C. determine the components of reaction at these supports.



Example 11: Determine the components of reaction that the ball-and-socket joint at A. the smooth journal bearing at B, and the roller support at C exert on the rod assembly in Fig.

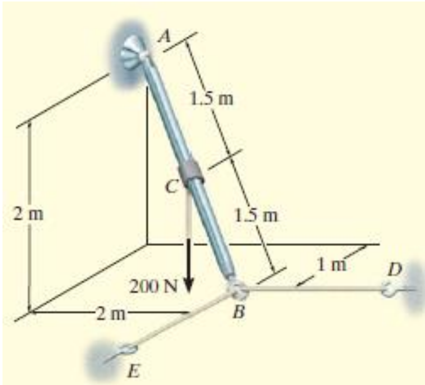


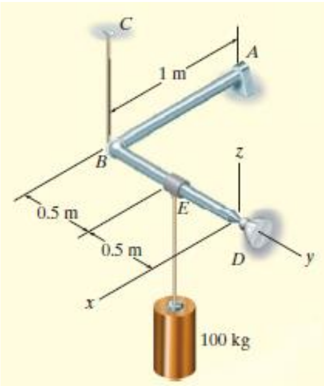
Example 12: The boom is used to support the 75-lb flowerpot in Figure. Determine the tension developed in wires AB and AC.



Sheet No. 2

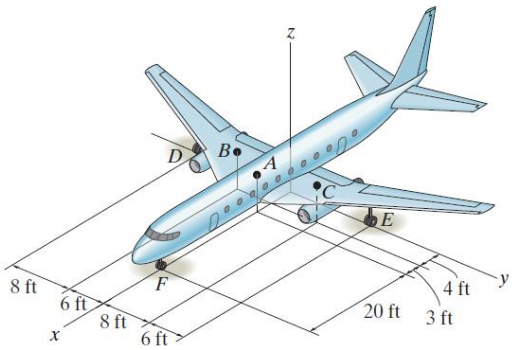
Q 1: Rod AB shown in Figure is subjected to the 200-N force. Determine the reactions at the ball-and-socket joint A and the tension in the cables BD and BE. The collar at C is fixed to the rod.

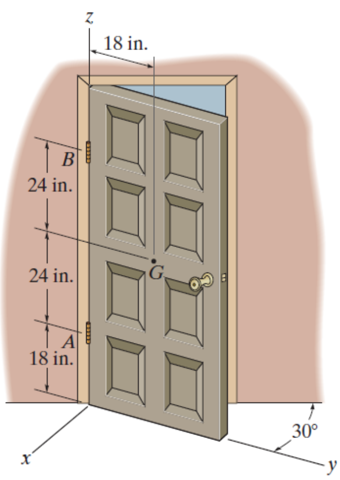




Q 2: The bent rod in Figure is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by means of cable BC. Using only one equilibrium equation, obtain a direct solution for the tension in cable BC. The bearing at A is capable of exerting force components only in the z and y directions since it is properly aligned on the shaft. In other words, no couple moments are required at this support.

Q 3: Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights W_A= 45000 Ib, W_B= 8000 Ib and W_C= 6000 Ib, determine the normal reactions of the wheels D, E, and F on the ground.





Q 4: The 100-lb door has its center of gravity at G. Determine the components of reaction at hinges A and B if hinge B resists only forces in the x and y directions and A resists forces in the x, y, z directions.

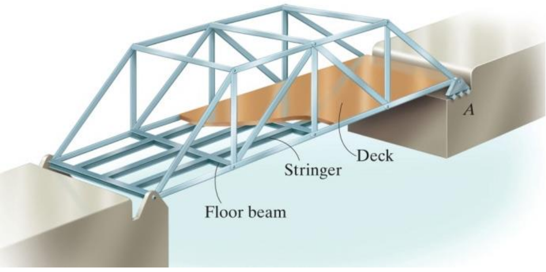
CHAPTER 8 STRUCTURAL ANALYSIS

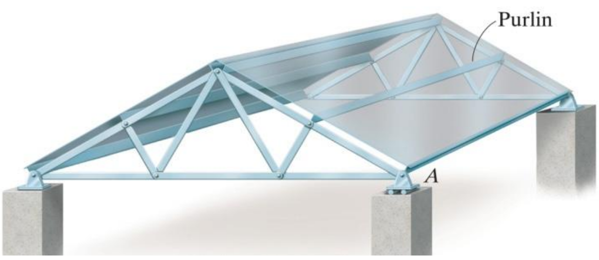
11. SIMPLE TRUSSES

A truss is a structure composed of slender members joined together at their end points.

The members commonly used in construction consist of wooden struts or metal bars.

The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*.





Planar **trusses** lie in a single plane and are often used to support roofs and bridges.

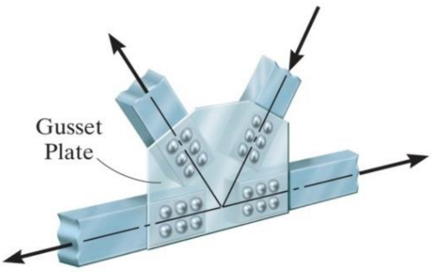
For example, in the roof truss presented above, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins*. Since the imposed loading acts in the same plane as the truss, the analysis of the forces developed in the truss members is two-dimensional.

To design the members and the connections of a truss, the *force* developed in each member under working conditions must be calculated.

Two important assumptions are made:

1. ***All loadings are applied at the joints***. In most situations, such as for bridge and roof trusses, this assumption is true. ***Frequently in the truss analysis the weight of the members is neglected*** since the forces supported by the members are usually large in comparison with their weight.

If the member's weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.



2. ***The members are joined together by smooth pins***. In cases where bolted or welded joint connections are used, this assumption is satisfactory provided ***the center lines of the joining members are concurrent***.

Thus, each truss member acts as a ***two-force member***, and the forces at the ends of the member must be directed along the axis of the member.

If the force tends to *elongate* the member, it is a *tensile force* (T). If the force tends to *shorten* the member, it is a *compressive force* (C).

12. THE METHOD OF JOINTS

The analysis or design of a truss requires the calculation of the force in each of its members.

If a free–body diagram of the entire truss is sketched, the forces in the members are *internal forces* and cannot be obtained from an equilibrium analysis.

Considering the ***equilibrium of a joint*** of the truss then a member force becomes an *external force* on the joint's free-body diagram, and the equations of equilibrium can be applied to obtain its magnitude.

This forms the basis for the ***method of joints***.

Since the truss members are all straight two-force members lying in the same plane, the force system acting at each joint is *coplanar and concurrent.*

Rotational or moment equilibrium is automatically satisfied at the joint (or pin), thus equilibrium is ensured if $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

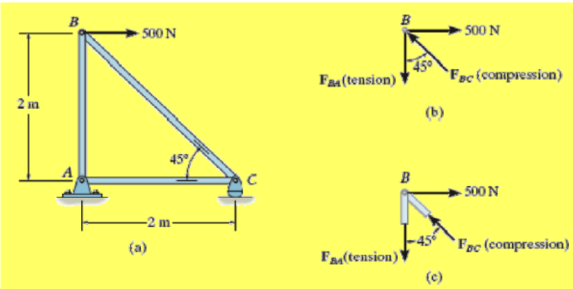
The main steps to follow when using the method of joints are:

• Draw a FBD of the entire truss to be analyzed

• Determine the forces (reactions) produced by the supports on the structure by applying the equilibrium equations. This step sometimes is not necessary.

• Draw the FBD of the joint to be analyzed. Recall that the *line of action* of a force has the same direction as the corresponding member.

• Determine the forces on each of the members on the joint by applying the equilibrium conditions on the joint.



In the analysis of a joint we must start at a point that has **at least one known force and at most two unknown** forces.

We can determine the correct sense of an unknown member force by always assuming the unknown member forces acting on the joint's FBD to be in tension, i.e., "pulling" on the pin. If this is done, then numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression. Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint free body diagrams.

When drawing FBD of a joint, keep in mind that **tension forces point outward from the joint** and **compression forces point toward the joint**.

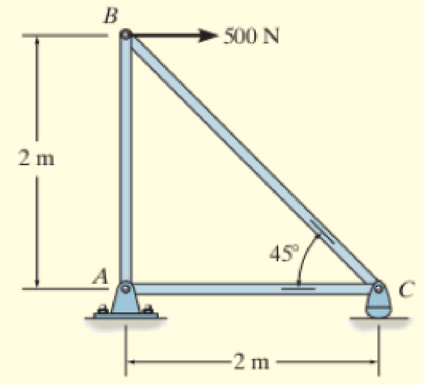
Sense of an Unknown Member Force

The correct sense of an unknown member force can be determined using one of two possible methods:

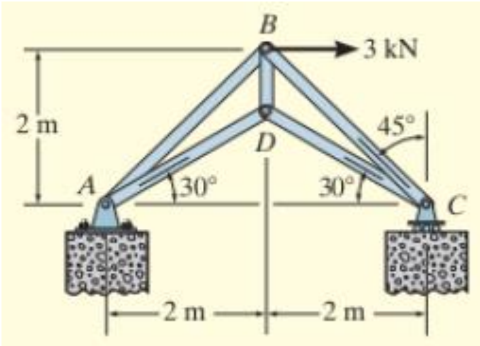
1. *Always assume* the *unknown member forces* acting on the joint's free-body diagram to be in *tension,* i.e., "pulling" on the pin. If this is done, then numerical solution of the equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.* Once an unknown member force is found, use its *correct* magnitude and sense (T or C) on subsequent joint free-body diagrams.

2. The *correct* sense of direction of an unknown member force sometimes can be determined "by inspection". In complicated cases, *assume* the sense of an unknown member force. After applying the equilibrium equations, a *positive* force indicates that the sense is *correct,* a *negative* force indicates that the sense shown on the free-body diagram must be *reversed.*

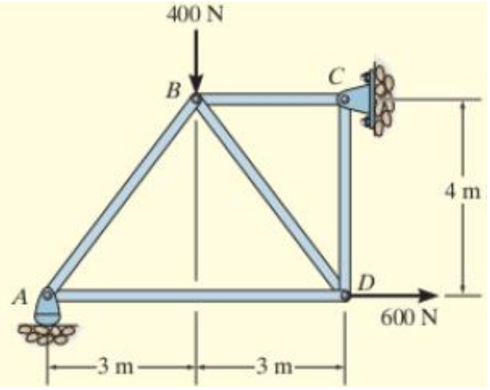
Example 1: Determine the force in each member of the truss shown in Figure and indicate whether the members are in tension or compression.



Example 2: Determine the forces acting in all the members of the truss shown in Fig.



Example 3: Determine the force in each member of the truss shown in Fig. Indicate whether the members are in tension or compression.



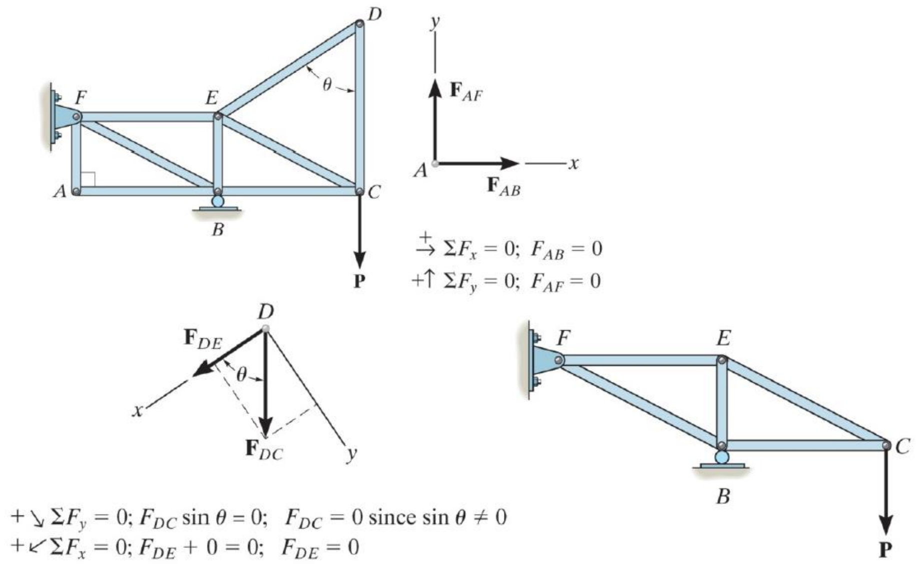
13. ZERO-FORCE MEMBERS

Simplification on the analysis of a truss is achieved by first identifying the ***zero force members*** (support *no loading*).

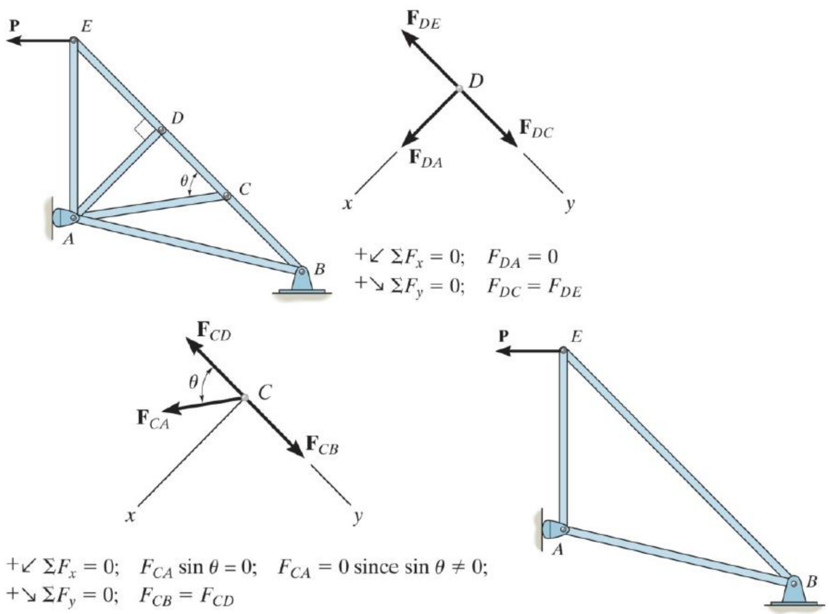
Zero-force members are used to increase the stability of the truss during construction and to provide support if the applied loading is changed.

Zero-force members of a truss can generally be found *by inspection* of each of its joints.

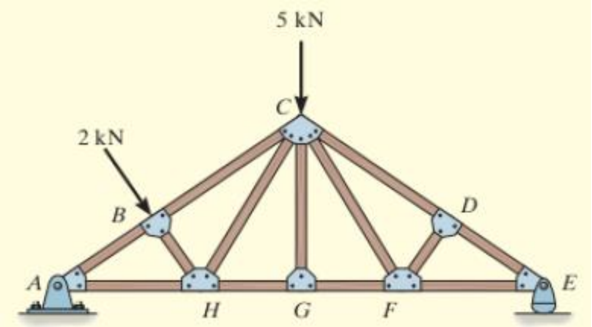
• If only two members form a truss joint and no external load or support reaction is applied to the joint, the members must be zero-force members.



• If three members form a truss joint for which two of them are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.

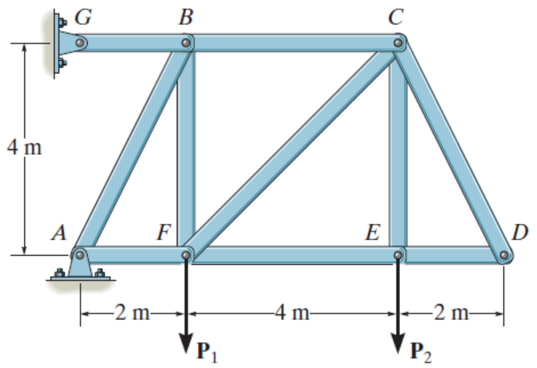


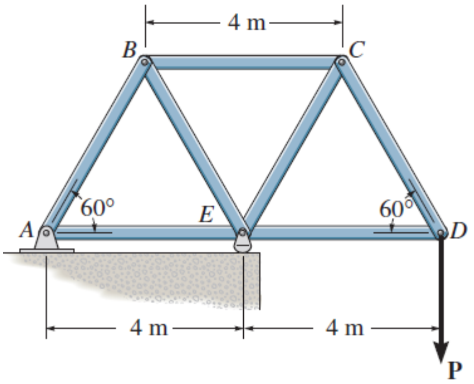
Example 4: Using the method of joints, determine all the zero-force members of the Fink roof MISS shown in Fig. Assume all joints are pin connected.



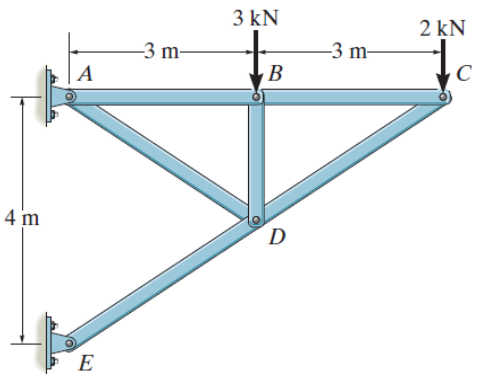
Sheet No. 1

Q 1: Determine the force in each member of the truss and indicate whether the members are in tension or compression. Set P1 = 10 kN, P2 = 15 kN.

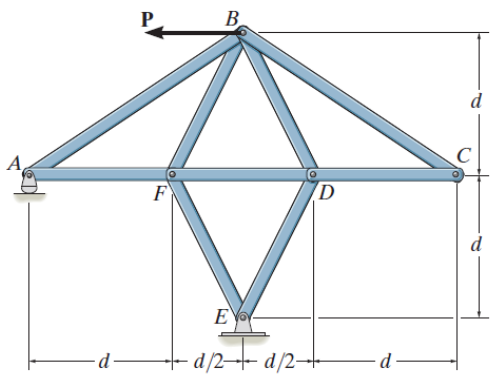




Q 2: Determine the force in each member of the truss. State whether the members are in tension or compression. Set P = 8 kN.

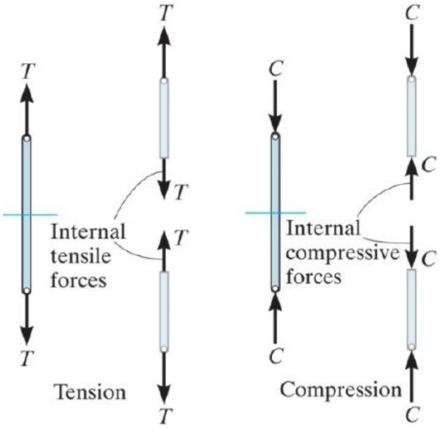


Q 3: Determine the force in each member of the truss and state if the members are in tension or compression. Hint: The resultant force at the pin E acts along member ED. Why?



Q 4: Determine the force in each member of the truss in terms of the load P, and indicate whether the members are in tension or compression.

14. THE METHOD OF SECTIONS



If it is necessary to find the force in only a few members of a truss, the ***method of sections*** is an adequate process of analysis. This method is based on the principle that if a body is in equilibrium then any part of the body is also in equilibrium.

Method of Sections Applied to a Structure

Based on the concept that if a structure is in equilibrium, every element of the structure must be in equilibrium, the method of sections can also be used to analyze a section the members of an entire truss.

If the section passes through the truss and the free-body diagram of either of its two parts is drawn, then the equations of equilibrium can be applied to that part to determine the member forces at the "cut section."

Since only three independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the isolated part of the truss.

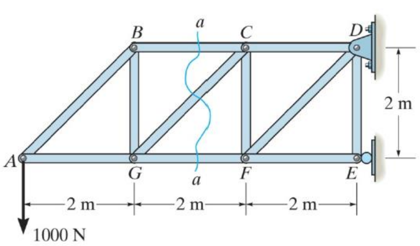
Try to select a section that, in general, passes through ***not more than three members*** in which the forces are unknown.

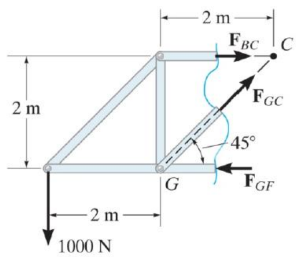
Consider the truss shown next. To determine $\mathbf{F}_{GC}$ a cut through *aa* is appropriate.

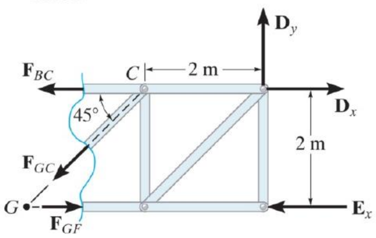
Note that the line of action of each member force is specified from the *geometry* of the truss, since the force in a member passes along its axis. The member forces acting on one part of the truss are equal but opposite of those acting on the other part—Newton's third law.

The members assumed to be in *tension* (*BC* and *GC*) are subjected to a "pull", whereas the member in *compression* (*GF*) is subjected to a "push".

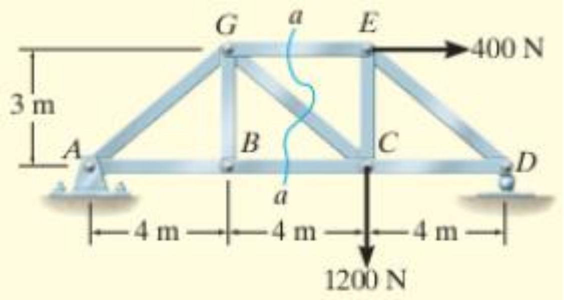
The three unknown member forces $\mathbf{F}_{BC}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{GF}$ can be obtained by applying the three equilibrium equations to the FBD of the sectioned figure.



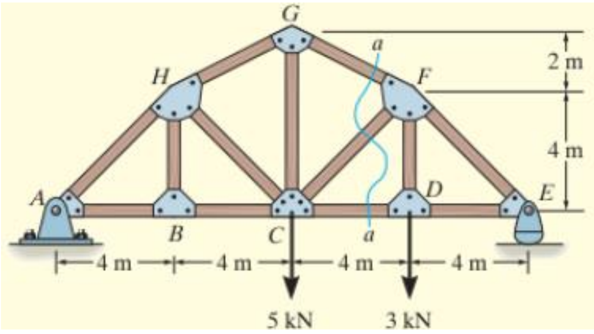




Example 5: Determine the force in members GE, GC, and BC of the truss shown in Fig. Indicate whether the members are in tension or compression.



Example 6: Determine the force in member CF of the truss shown in Fig. Indicate whether the member is in tension or compression. Assume each member is pin connected.



15. FRAMES AND MACHINES

Frames and machines are two common types of structures which are often composed of pin-connected *multi-force members*, i.e., members that are subjected to more than two forces.

• *Frames* are generally stationary and are used to support loads.

• *Machines* contain moving parts and are designed to transmit and alter the effect of forces.

To determine the forces acting at the joints and supports of a frame or machine, the structure must be *disassembled* and the free-body diagrams of its parts must be drawn. The following points must be observed:

• Isolate each part by drawing its *outlined shape*. Then show all the forces and/or couple moments that act on the part.

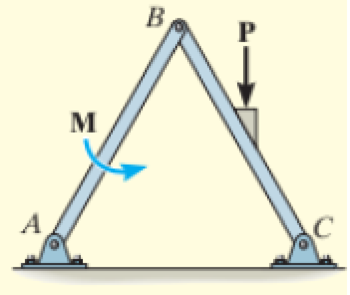
• Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. Finding two force members will avoid solving extra equations.

• Forces common to any two *contacting* members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a *"system" of connected members*, then these forces are *"internal"* and are *not shown* on the *free-body diagram of the system*; however, if the free-body diagram of *each member* is drawn, the forces are *"external"* and *must* be shown on each of the free-body diagrams.

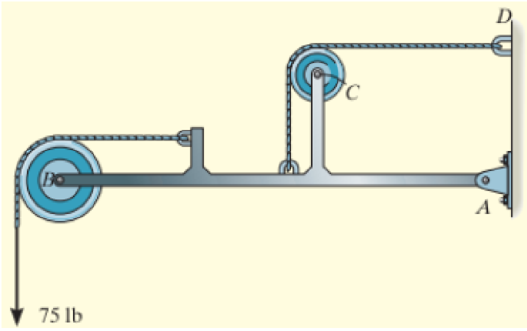
Equations of Equilibrium

If the frame or machine is properly supported and statically determinate, then the unknown forces at the supports and connections can be determined from the equations of equilibrium. If the structure lies in the *x–y* plane, then for *each* free-body diagram drawn the loading must satisfy $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_O = 0$.

Example 7: For the frame shown in Figure draw the free-body diagram of (a) each member, (b) the pins at B and A, and (c) the two members connected together.

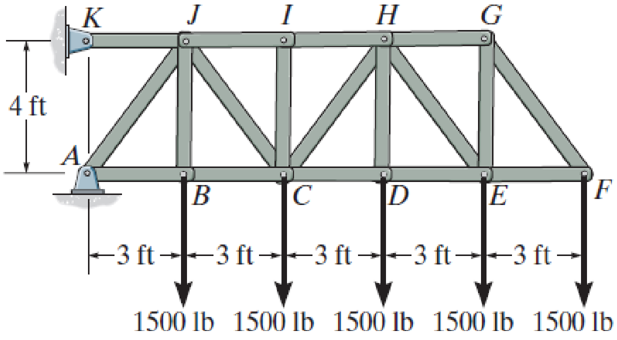


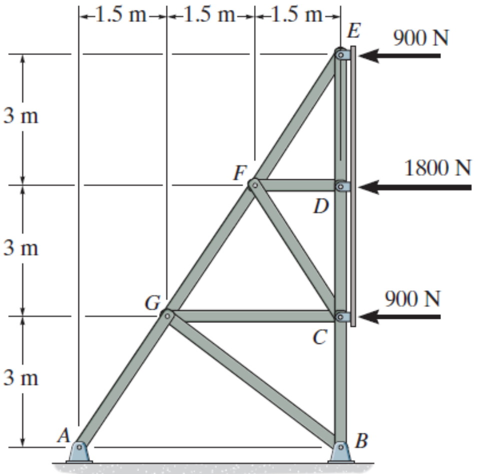
Example 8: For the frame shown in Figure draw the free-body diagrams of (a) the entire frame including the pulleys and cords, (b) the frame without the pulleys and cords, and (c) each of the pulleys.



Sheet No. 2

Q 1: Determine the force in members CD, HI, and CJ of the truss, and state if the members are in tension or compression.



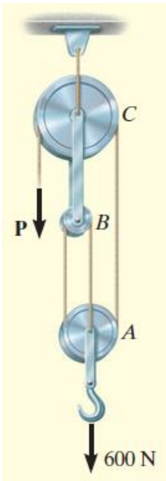


Q 2: Determine the force in members FG, GC and CB of the truss used to support the sign, and state if the members are in tension or compression.

Q 3: A constant tension in the conveyor belt is maintained by using the device shown in Fig. Draw the free-body diagrams of the frame and the cylinder (or pulley) that the belt surrounds. The suspended block has a weight of W.

Q 4: Draw the free-body diagrams of the members of the backhoe, shown in the photo, Fig. The bucket and its contents have a weight W.

Q 5: Determine the tension in the cables and also the force P required to support the 600-N force using the frictionless pulley system shown in Fig.

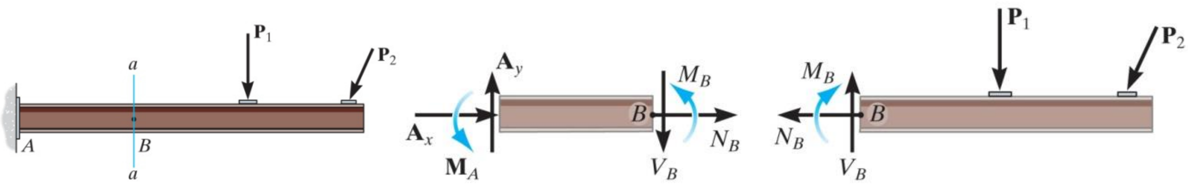


CHAPTER 9 INTERNAL FORCES

16. INTERNAL LOADINGS DEVELOPED IN STRUCTURAL MEMBERS

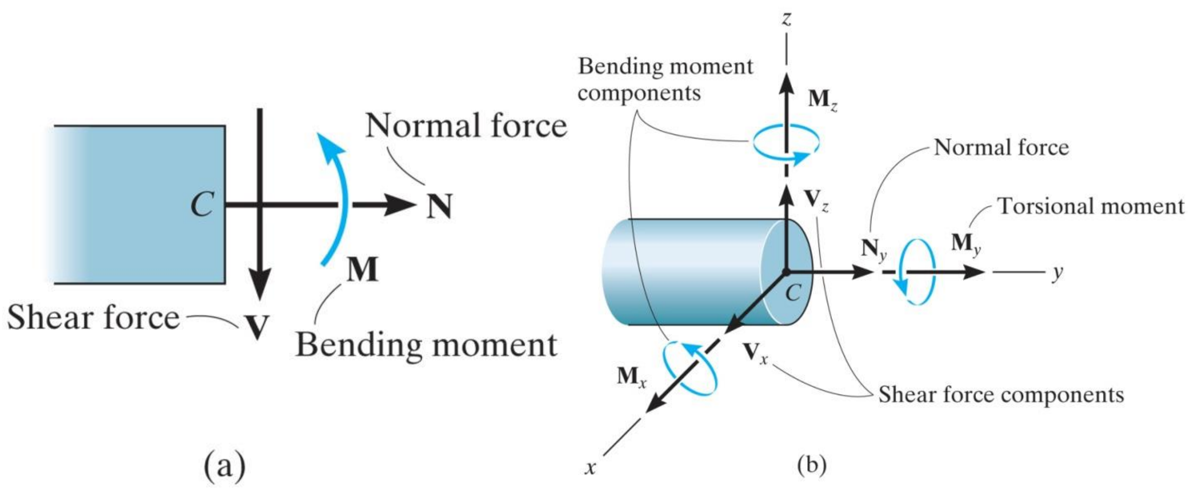
Designing any structural member requires an investigation of the loads acting within the element to ensure that the material will be able to support such loads.

The determination of these internal loads can be achieved by using the ***method of sections***.



Segments *AB* and *BC* were in equilibrium *before* the beam was sectioned. Thus, the equilibrium of each segment is maintained provided the force components N_B and V_B and the resultant couple moment M_B are developed at the section. These loadings must be equal in magnitude and opposite in direction on each of the segments. The magnitude of each of these loadings can be calculated by applying the three equations of equilibrium to either segment *AB* or *BC*.

In 2-D, the force components **N**, acting normal to the beam at the cut section, and **V**, acting tangent to the section, are the ***normal or axial force*** and the ***shear force***. The couple moment **M** is the ***bending moment***.



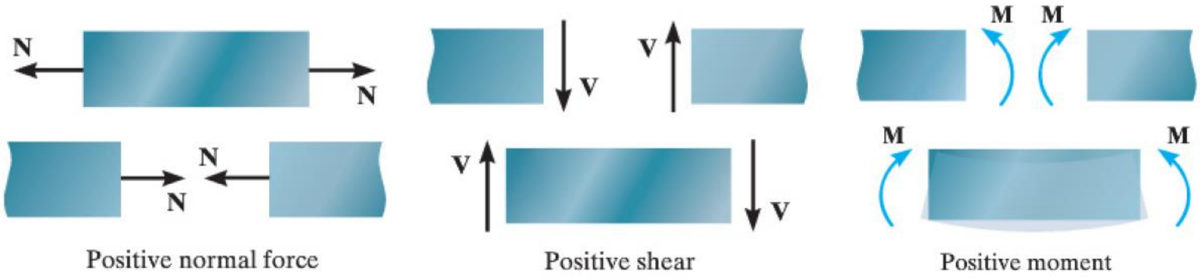
In 3-D, a general internal force and couple moment resultant will act at the section. The *x*, *y*, *z* components of these loadings are shown above. $\mathbf{N}_y$ is the *normal force*, and $\mathbf{V}_x$ and $\mathbf{V}_z$ are *shear force components*. $\mathbf{M}_y$ is a *torsional or twisting moment*, and $\mathbf{M}_x$ and $\mathbf{M}_z$ are *bending moment components*. For most applications, these resultant loadings will act at the geometric center or centroid (*C*) of the section's cross-sectional area.

Free–Body Diagrams

• Trusses are composed of two-force members that only support normal loads.

• Frames and machines are composed of multi-force members, and so each of these members will generally be subjected to internal normal, shear, and bending loadings.

Sign Convention



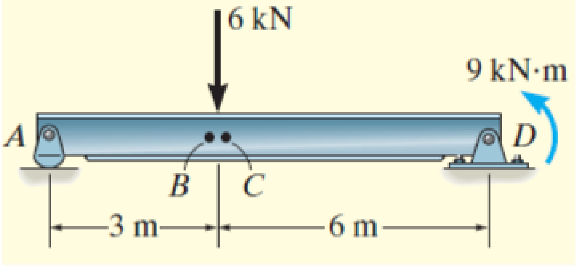
The choice of a sign convention **is arbitrary**. However, we will adopt the following convention:

• ***Normal Force*** (**N**) will be considered positive when generates tension.

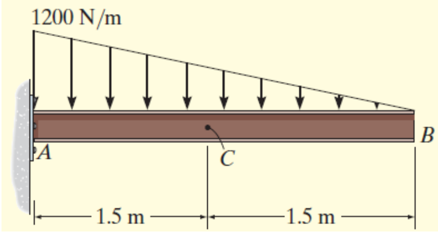
• ***Shear Force*** (**V**) will be considered positive when causes the beam segment where it acts to rotate clockwise.

• ***Bending Moment*** (**M**) will be considered positive when the segment on which it acts tends to bend in a concave upward manner.

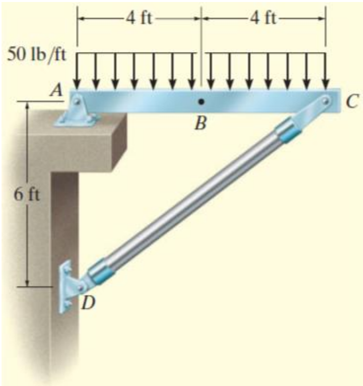
Example 1: Determine the normal force, shear force, and bending moment acting just to the left, point B , and just to the right, point C , of the 6-kN force on the beam in Fig.



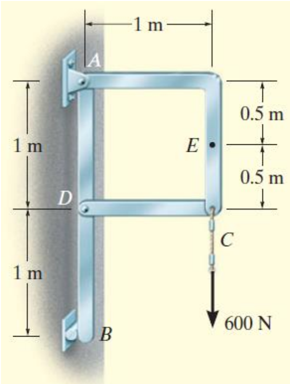
Example 2: Determine the normal force, shear force, and bending moment at C of the beam in Fig.



Example 3: Determine the normal force, shear force, and bending moment acting at point B of the two-member frame shown in Fig.



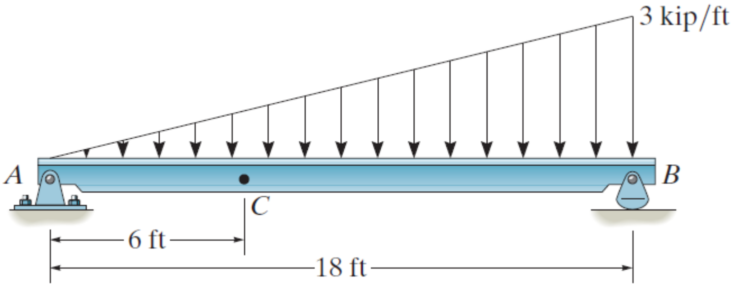
Example 4: Determine the normal force, shear force, and bending moment acting at point E of the frame loaded as shown in Fig.



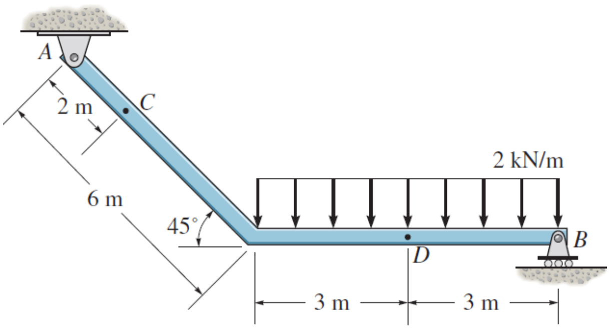
Example 5: The uniform sign shown in Figure has a mass of 650 kg and is supported on the fixed column. Design codes indicate that the expected maximum uniform wind loading that will occur in the area where it is located is 900 Pa. Determine the internal loadings at A.

Sheet No. 1

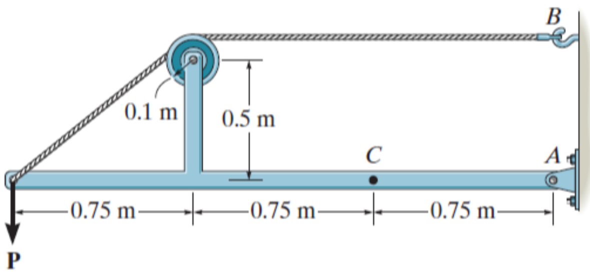
Q 1: Determine the shear force and moment acting at a section passing through point C in the beam.



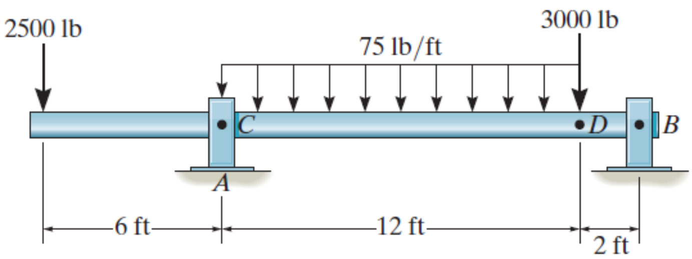
Q 2: Determine the internal normal force, shear force, and the moment at points C and D.



Q 3: Determine the normal force, shear force, and moment at a section passing through point C. Take P = 8 kN.



Q 4: The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the normal force, shear force, and moment at a section passing through (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the 3000-lb force.



17. SHEAR AND MOMENT EQUATIONS AND DIAGRAMS

Beams are structural members designed to support loadings applied perpendicular to their axes.

Beams are usually long, straight bars with constant cross-sectional area. Beams are usually classified as to how they are supported:

• *Simply supported beam.* Pinned at one end and roller-supported at the other.

• *Cantilevered beam* is fixed at one end and free at the other.

Why we need to know shear force and bending moment in a beam?

• The design of a beam requires a detailed knowledge of the variation of the internal shear force *V* and bending moment *M* acting at each point along the axis of the beam.

• After the force and bending-moment analysis is done, the theory of mechanics of materials and an appropriate engineering design code can be used to determine the beam's required cross-sectional area.

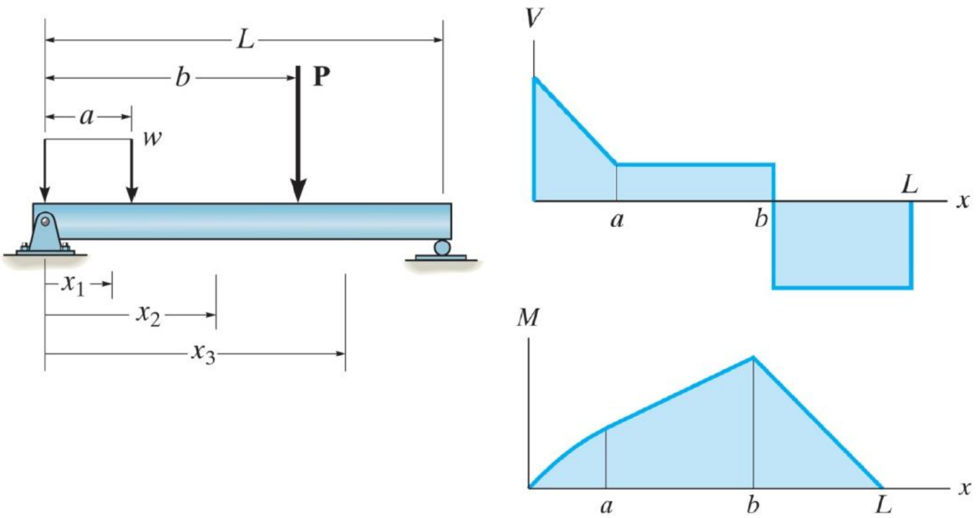
How to obtain a relation between *V* and *M* in terms of *x*?

• We section the beam at an arbitrary distance *x* from one end.

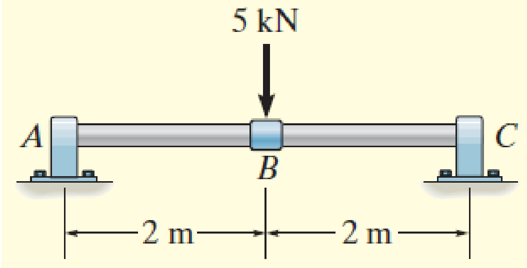
• Using the method of sections, we obtain *V* and *M* in terms of the position *x.*

• In general, the internal *V* and *M* functions will be discontinuous, or their slopes will be discontinuous, at points where a distributed load changes or where concentrated forces or couple moments are applied. Thus these functions must be determined for ***each segment*** of the beam located between any two discontinuities of loadings.

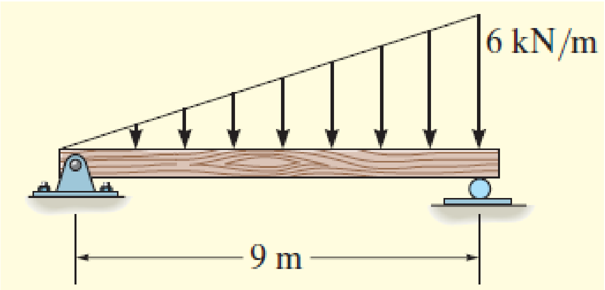
• If the resulting functions of *x* are plotted, the graphs are named ***shear diagram*** and ***bending-moment diagram***.



Example 6: Draw the shear and moment diagrams for the shaft shown in Fig. The support at A is a thrust bearing and the support at C is a journal bearing.

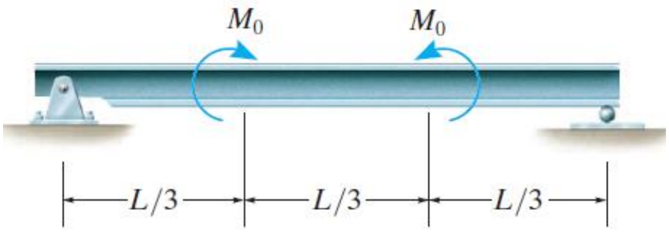


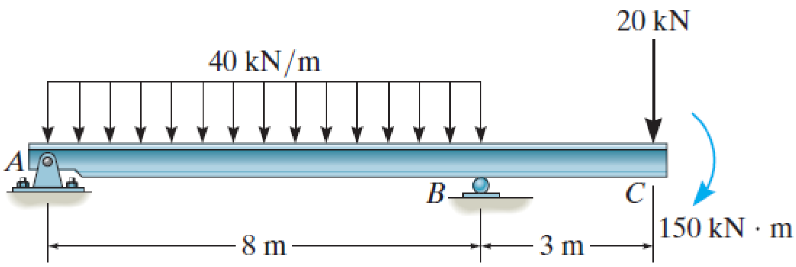
Example 7: Draw the shear and moment diagrams for the beam shown in Fig.



Sheet No. 2

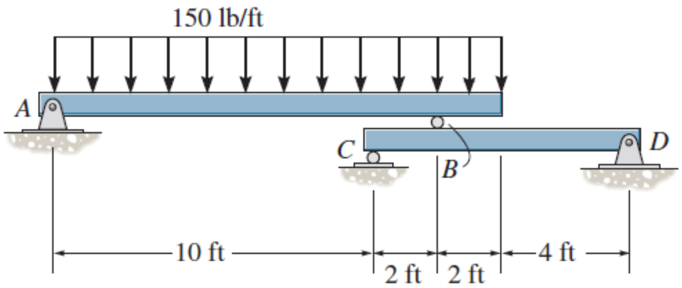
Q 1: Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set Mo = 500 N.m, L = 8 m.

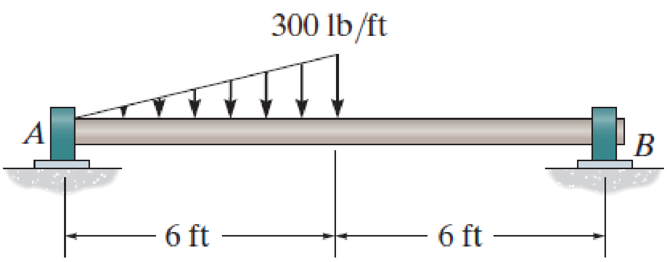




Q 2: Draw the shear and moment diagrams for the beam.

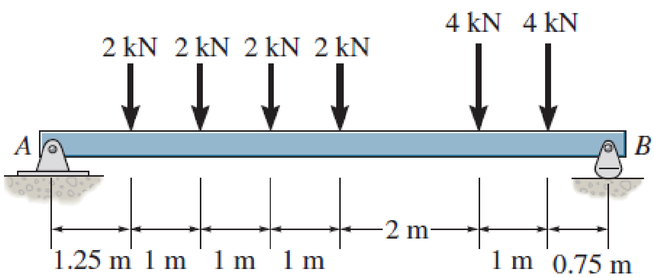
Q 3: Draw the shear and bending-moment diagrams for each of the two segments of the compound beam.





Q 4: The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.

Q 5: Draw the shear and moment diagrams for the beam.



CHAPTER 10 FRICTION

CHARACTERISTICS OF DRY FRICTION

Friction can be defined as a force of resistance acting on a body that prevents or retards slipping of the body relative to a second body or surface with which it is in contact.

This force always acts *tangent* to the surface at points of contact with other bodies and is directed so as to oppose the possible or existing motion of the body relative to these points.

Two types of friction can occur between surfaces:

• *Fluid friction* exists when the contacting surfaces are separated by a film of fluid (gas or liquid).

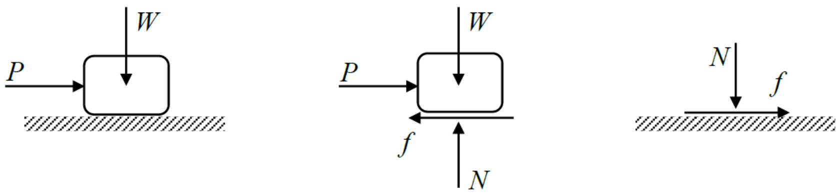
• *Dry friction*. This type of friction is often called *Coulomb friction* since its characteristics were studied extensively by C. A. Coulomb in 1781.

Dry friction occurs between the contacting surfaces of bodies when there is no lubricating fluid.

When two bodies are in ***contact***, the ***reactive force*** can be broken into ***two components***, one ***normal*** to the plane of contact and one ***tangent*** to the plane.

The component of the contact force which is ***tangent*** to the plane of contact is called the ***friction force***, and it is the result of the ***relative roughness*** of the contacting surfaces.

Consider a ***block*** resting on a ***horizontal plane*** with an ***applied force*** *P*.



Depending on the ***magnitude*** of *P* and the ***roughness*** of the contacting surfaces, the block ***may*** or ***may not*** move.

Actually, there are ***three*** distinct ***possibilities*** in this case.

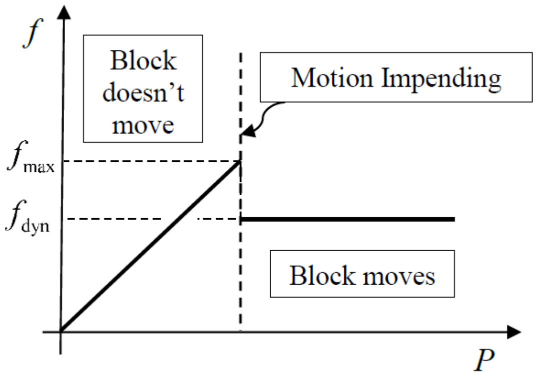
 the block ***slides***

 the block ***does not slide***, even when *P* is increased slightly

 the block ***does not slide***, but it does slide when *P* is increased slightly

In the last case, we say ***motion*** is ***impending***, because if P is increased slightly, the block will move.

One ***model*** of ***friction*** is described by the plot below. As P is ***increased*** from zero, the friction force ***matches it*** up to a limiting value f_{max}. As P is ***increased beyond this value***, the ***friction force drops*** to its dynamic value f_{dyn} which is ***approximately constant***.



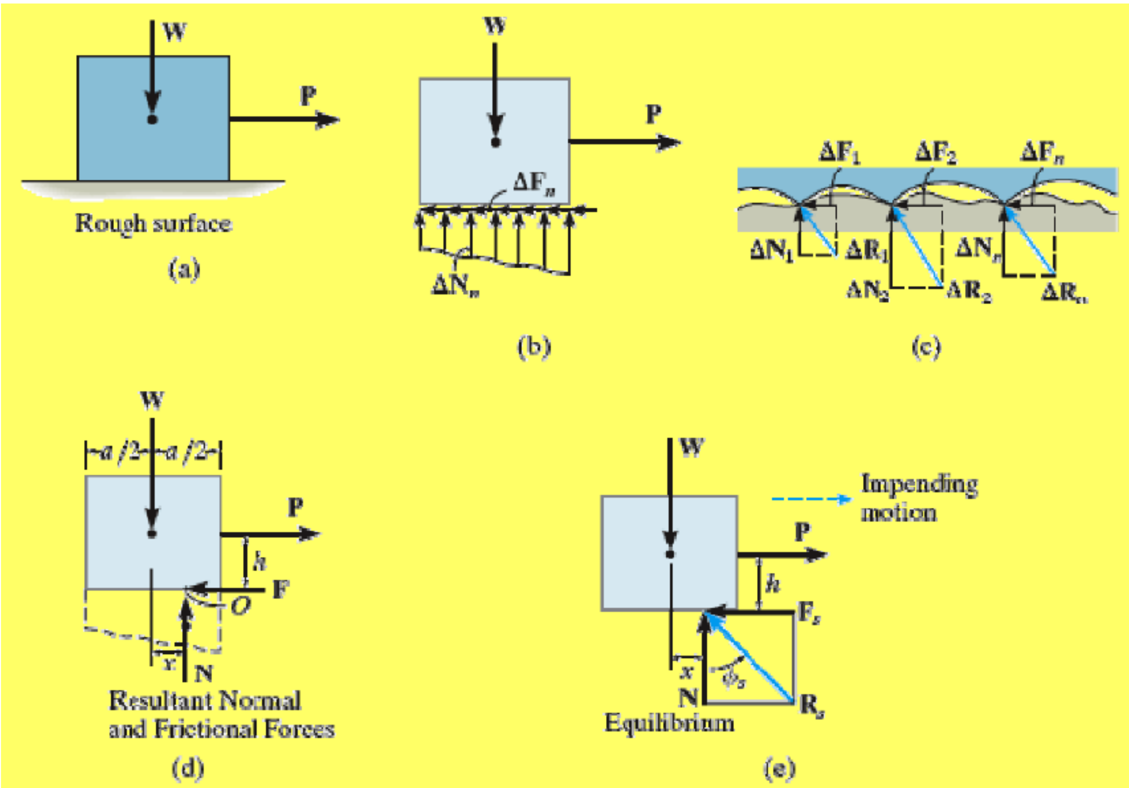
The values of f_{max} and f_{dyn} are given in terms of the ***normal force*** and the ***coefficients*** of ***static*** and ***kinetic friction***.

$$f_{\max} = \mu_s N$$ μ_s is the ***coefficient*** of ***static friction***

$$f_{\text{dyn}} = \mu_k N$$ μ_k is the ***coefficient*** of ***kinetic friction***

The ***coefficients*** of ***friction*** (μ_s and μ_k) depend on the ***relative roughness*** of the two surfaces. See your textbook for some ***typical values***.

Considering the effects caused by pulling horizontally on a block of uniform weight **W** which is resting on a rough horizontal surface.



Characteristics of Dry Friction

•Friction force acts *tangent* to the contacting surfaces and *opposed* to the *relative motion* or tendency for motion of one surface against another.

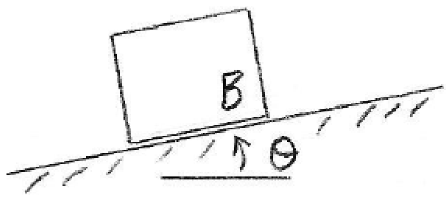
•The maximum static friction force f_s that can be developed is independent of the area of contact, provided the normal pressure is not very low nor great enough to severely deform or crush the contacting surfaces of the bodies.

•The maximum static frictional force is generally greater than the kinetic frictional force for any two surfaces of contact.

•When *slipping* at the surface of contact is *about to occur,* the maximum static frictional force is proportional to the normal force: $f_s = \mu_s N$.

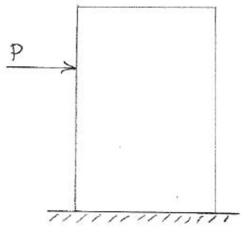
•*W*hen *slipping* at the surface of contact is *occurring,* the kinetic frictional force is proportional to the normal force: $f_k = \mu_k N$.

Example 1: Weight, W and coefficient of static friction μ_s. Find the maximum angle θ for which the block will remain in equilibrium.

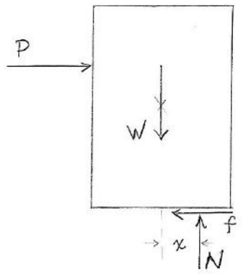


Tipping Problems

Consider the box resting on a plane with ***horizontal force*** *P* = 0. If the ***weight*** within the box is ***evenly distributed***, the ***normal force*** exerted by the ground on the box is ***uniformly distributed along the bottom*** of the box.



If we begin to ***push*** on the box to the ***right*** (*P* ≠ 0), then the ***normal distributed force*** exerted by the ground on the box is ***skewed*** either ***to the right*** or ***to the left*** of center.



If the box is in ***static equilibrium*** (*P* = *f*), the ***normal force*** will always be ***skewed*** to the ***right of center***.

. If *P* is ***above*** the ***mass center*** *G*, both *P* and *f* will have ***clockwise moments*** about *G* which will be ***balanced*** by a ***counter-clockwise moment*** from *N*.

. If *P* is ***below*** the ***mass center***, *P* will have a ***counterclockwise moment*** about *G*, and *f* will have a ***larger clockwise moment*** about *G*. This ***net clockwise moment*** will be balanced by a ***counter-clockwise moment*** of *N*. (Note: The friction force *f* has a ***larger moment arm*** about *G* than does *P*.)

If the box is ***not*** in ***static equilibrium*** (*P* > *f*), the ***normal force*** will be ***skewed to the right of center*** if *P* is ***above*** *G*, and it ***can*** be ***skewed left*** or ***right of center*** if *P* is ***below*** *G* , depending on the relative magnitudes of *P* and *f* .

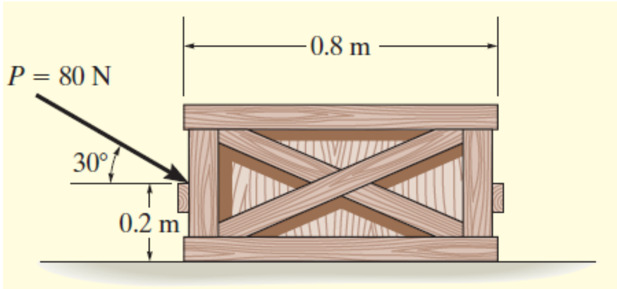
Consider the case of static equilibrium where the ***applied force*** *P* is ***above*** the center of gravity, and the ***resultant normal force*** (*N*) is to the ***right*** of ***center*** as shown in the free-body diagram.

In this case, the moments of the ***applied force*** *P* and the ***friction force*** *f* will both have a ***tendency*** to ***tip*** the box to the right, and the moment of ***normal force*** *N* will ***counter*** that tendency.

If *N* acts at the ***edge of the box*** to counter the effects of *P* and *f*, the box will be on the ***verge*** of ***tipping***.

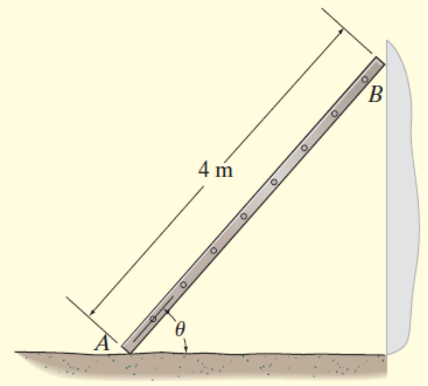
If *N* needs to act beyond the ***edge of the box*** to counter the effects of *P* and *f*, the box will ***tip***.

Example 2: The uniform crate shown in Figure has a mass of 20 kg. If a force P = 80 N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.

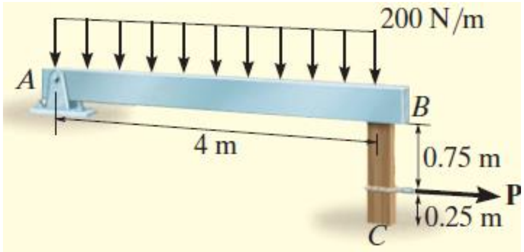


Example 3: It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25^o$ the vending machines will begin to slide off the bed, Fig. Determine the static coefficient of friction between a vending machine and the surface of the truckbed.

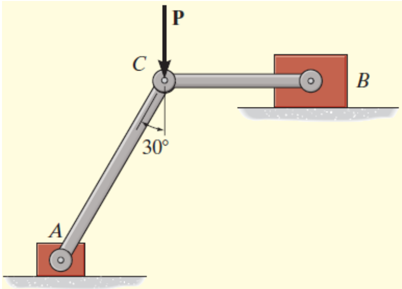
Example 4: The uniform 10-kg ladder in Figure rests against the smooth wall at B , and the end A rests on the rough horizontal plane for which the coefficient of static friction is $\mu_s = 0.3$. Determine the angle of inclination θ of the ladder and the normal reaction at B if the ladder is on the verge of slipping.



Example 5: Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC, Fig. If the coefficients of static friction at B and C are μ_B = 0.2 and μ_C = 0.5, determine the force P needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.

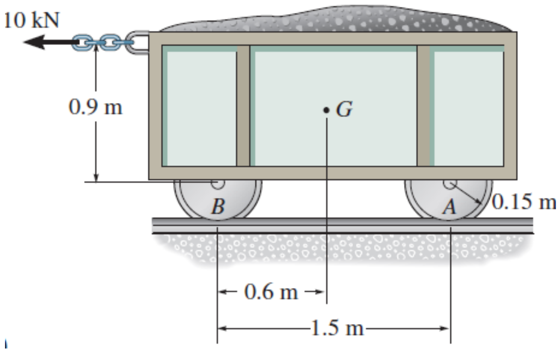


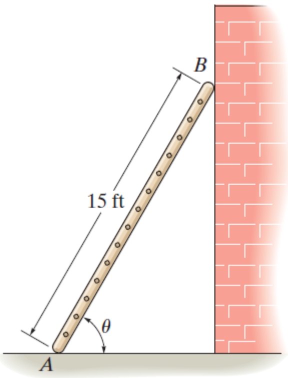
Example 6: Blocks A and B have a mass of 3 kg and 9 kg, respectively, and are connected to the weightless links shown in Fig. Determine the largest vertical force P that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is $\mu_s = 0.3$.



Sheet No. 1

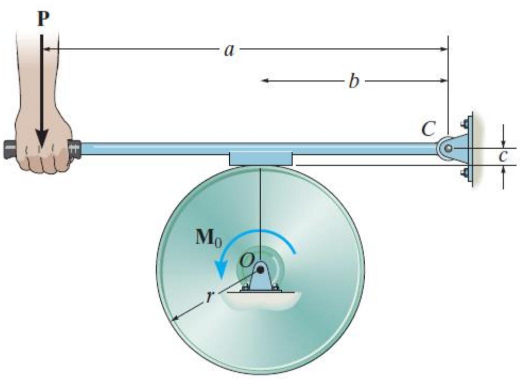
Q 1: The mine car and its contents have a total mass of 6 Mg and a center of gravity at G. If the coefficient of static friction between the wheels and the tracks is μ_s = 0.4 when the wheels are locked, find the normal force acting on the front wheels at B and the rear wheels at A when the brakes at both A and B are locked. Does the car move?

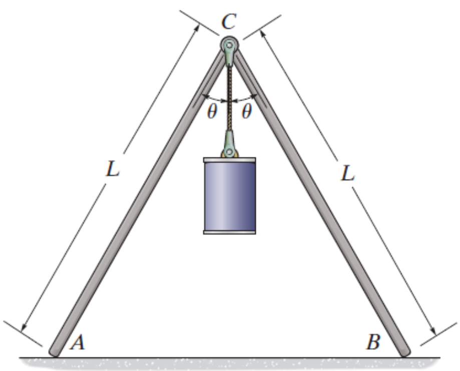




Q 2: The ladder has a uniform weight of 80 lb and rests against the wall at B. If the coefficient of static friction at A and B is μ = 0.4, determine the smallest angle at which the ladder will not slip.

Q 3: The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment M_o. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.





Q 4: If the coefficient of static friction at A and B is μ_s =0.6, determine the maximum angle so that the frame remains in equilibrium, regardless of the mass of the cylinder. Neglect the mass of the rods.