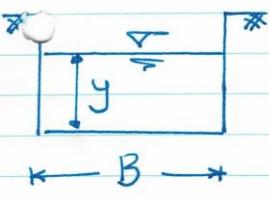
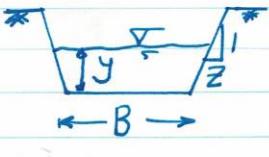
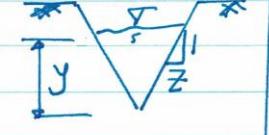


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٤ Area of Prismatic channel :

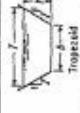
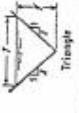
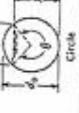
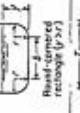
توجد عدة اشكال من القنوات المفتوحة ذات المقطع
الثابت من حيث المساحة لكن المتغير هو الارتفاع كالتالي:

Section	Area A	المحيط المماس P	نصف القطر الحراري hydraulic radius R_h
	$B \cdot y$	$2y + B$	$\frac{B \cdot y}{2y + B}$
		Rectangular Section	
	$By + Zy^2$	$B + 2y\sqrt{1+z^2}$	$\frac{By + Zy^2}{B + 2y\sqrt{1+z^2}}$
		Trapezoidal Section	
	Zy^2	$2y\sqrt{1+z^2}$	$\frac{Zy^2}{2y\sqrt{1+z^2}}$
		Triangle Section	

ويعين حساب معامل يسمى معامل المقطع لبيان انحراف
مائل الماء (الارتفاع) كالتالي:

$$Z = A \sqrt{\frac{A}{P}}$$

TABLE 2-1. GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

Section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
	b_y	$b + 2y$	$\frac{by}{b + 2y}$	b	y	$b^{3/4}$
	$(b + 2y)y$	$b + 2y\sqrt{1+z^2}$	$\frac{(b + 2y)y}{b + 2y\sqrt{1+z^2}}$	$b + 2y$	$\frac{(b + 2y)y}{b + 2y}$	$\frac{[(b + 2y)y]^{1/4}}{\sqrt{b + 2y}}$
	yz^2	$2y\sqrt{1+z^2}$	$\frac{yz}{2\sqrt{1+z^2}}$	$2y$	yz	$\frac{\sqrt{2}}{2}yz^{3/4}$
	$\frac{1}{2}(b - \sin \theta)bz$	$\frac{1}{2}(b - \sin \theta)bz$	$\frac{(\sin \frac{1}{2}\theta)bz}{2\sqrt{bz - y}}$	$\frac{1}{2}(\sin \frac{1}{2}\theta)bz$	$\frac{\sqrt{2}(\theta - \sin \theta)^{1/4}}{32}(\sin \frac{1}{2}\theta)^{1/4}$	$\frac{\sqrt{2}}{32}(\sin \frac{1}{2}\theta)^{1/4}$
	$\frac{1}{2}\pi Ty^2$	$T + \frac{8}{3}\frac{y^3}{T}$	$\frac{2Ty}{3T^2 + 6y^2}$	$\frac{3A}{2y}$	$\frac{3y}{2}$	$\frac{3\sqrt{6}Ty^{5/4}}{32}$
	$(\frac{\pi}{2} - 2)r^2 + (b + 2y)y$	$(\pi/2)r + b + 2y$	$\frac{(\pi/2)r^2 + (b + 2y)y}{(\pi/2)r + b + 2y}$	$b + 2r$	$\frac{(r/2 - 2)r^2}{b + 2r} + y$	$\frac{[(\pi/2 - 2)r^2 + (b + 2y)y]^{1/4}}{\sqrt{b + 2r}}$
	$\frac{r^2}{4} - \frac{r^4}{2}$	$(1 - z \cos^{-2} z)$	$\frac{T}{z}\sqrt{1+z^2} - \frac{2r}{z}(1 - z \cos^{-2} z)$	$\frac{A}{T}$	$\frac{4Ty - r}{T} + r\sqrt{1+z^2}$	$A/\sqrt{\frac{A}{P}}$

* Satisfactory approximation for the interval $0 < z \leq 1$, where $z = 4y/R$. When $z > 1$, use the exact expression $P = (T/2)\sqrt{1+z^2} + (z + \sqrt{1+z^2})$.

5 uniform flow (Basic Equation) 12

Discharge estimation
in open prismatic channel

هناك عدد معادلات لحساب التدفق في المجرى المفتوح المستقيم وكاردي:

A CHEZY'S Eq. For Discharge:-

This eq. Published in 1775 is known as chezy's eq. and the constant C is known chezy's coefficient and noted that chezy coefficient is not dimensionless

$$Q = C A \sqrt{R_h S^l} \quad \dots \quad (1)$$

where :-

S^l = bed slope

The dimension of C is $(\frac{L}{T})^{1/2}$.

and, can be written eq. above as:

$$V = C \sqrt{R_h S^l} \quad \dots \quad (2)$$

ان المعامل (C) اسبة عامل خسائر (الاحتياط) (f) ويعني

كتابه (المعارف) اورته :-

$$C = \sqrt{\frac{8g}{f}} \quad \dots \quad (3)$$

Ex1 Determine the flow rate of water through a rectangular channel (3m wide) with a flow depth of (1m). The bed slope is (1 in 2500) and $f = 0.038$?

Soln

$$C = \sqrt{\frac{8g}{f}} = \sqrt{\frac{8 \times 9.81}{0.038}} = 45.46 \frac{m^{1/2}}{sec}$$

$$P = B + 2y = 3 + (2 \times 1) = 5m$$

$$A = By = 3 \times 1 = 3 m^2$$

$$R_h = \frac{A}{P} = \frac{3}{5} = 0.6 m$$

$$\therefore Q = CA \sqrt{R_h S} = 45.46 \times 3 \sqrt{0.6 \times \frac{1}{2500}}$$

$$Q = 2.112 m^3/sec \quad \underline{ANS}$$

Ex2 A rectangular open channel has (5m) width and (1.5m) depth. The bed slope is 1:1000, $C = 50$, determine the flow rate ?

Soln.

$$A = B \times y = 5 \times 1.5 = 7.5 m^2$$

$$P = B + 2xy = 5 + 2(1.5) = 8 m$$

$$R_h = \frac{A}{P} = \frac{7.5}{8} = 0.938 m$$

$$\therefore Q = CA \sqrt{R_h S}$$

$$Q = 50 \times 7.5 \sqrt{0.938 \times \frac{1}{1000}} = 11.48 \text{ m}^3/\text{sec}$$

ANS

H.W A triangular open channel with (0.25m) depth and 60° angle conveys water. If the bed slope is (1:137) and Chezy constant C = 52, determine the flow rate? (ANS (40 liters/sec))

• Determination of chezy's constant

↓ Bazin's Eq. for chezy's constant..

The Bazin eq. is

$$C = \frac{86.9}{1 + \frac{K}{\sqrt{R_h}}}$$

where K is Bazin constant. The value varies from 0.11 for smooth surface to 3.17 for earthen channel in rough condition, for Brick lined channel is 0.5.

Ex3 calculate the value of chezy's constant using Bazin equation in the case of rectangular channel (3m wide) and (1m deep), bed slope of (1:2500), Find the value of flow rate?

Soln. $A = By = 3 \times 1 = 3 \text{ m}^2$ $P = B + 2y = 3 + 2(1) = 5 \text{ m}$
 $\therefore R_h = \frac{A}{P} = \frac{3}{5} = 0.6 \text{ m}$

Bazin eq. $\Rightarrow C = \frac{86.9}{1 + \frac{K}{\sqrt{R_h}}} = \frac{86.9}{1 + \frac{K}{\sqrt{0.6}}}$

and, chezy eq. discharge $\Rightarrow Q = CA \sqrt{R_h S}$

حسب معادله بازن تغير سبع C كلما تغيرت فيه K ولأن الوال لم يحد فيه K ستأخذ جميع الاتار ونقاري :

Case No.	Nature of surface	Bazin constant	C	Q m^3/sec
1	Smooth cement lining	0.06	80.65	3.75
2	Smooth brick	0.16	72.02	3.35
3	Rubble masonry	0.46	54.52	2.53
4	Earthen channel in ordinary condition	1.303	32.4	1.51
5	Earthen channel in rough condition	1.75	26.66	1.24

2 Kutter's Eq. for chezy's constant C

The Kutter's Eq. is

$$C = \frac{23 + \left(\frac{0.00155}{S} \right) + \left(\frac{1}{N} \right)}{1 + \left[23 + \left(\frac{0.00155}{S} \right) \right] \left(N + R_h^{0.5} \right)}$$

Where N is Kutter constant.

EX4 Determine the flow rate for rectangular channel (3m wide) and (1m deep) with slope (1: 2500), using Kutter constant ?

$$\text{Soln. } A = By = 3 \times 1 = 3 \text{ m}^2 \quad P = B + 2y = 3 + 1(2) = 5 \text{ m}$$

$$R_h = \frac{A}{P} = \frac{3}{5} = 0.6 \text{ m}$$

$$\text{Kutter eq. } \Rightarrow C = \frac{23 + \left(\frac{0.00155}{S}\right) + \left(\frac{1}{N}\right)}{1 + \left[23 + \left(\frac{0.00155}{S}\right)\right] \left(N + R_h^{0.5}\right)}$$

$$\text{and, chezy eq. } \Rightarrow Q = CA \sqrt{R_s S}$$

حسب معادله کاتر تغیرت قیم C که تغیرت قیم N و باعث ایجاد می شود
نماید که در کارهای ایجاد نیز نیز می شود.

Case No.	Type of surface	Kutter constant N	C	Q m³/sec
1	Smooth cement lining	0.11	85.25	3.96
2	Smooth concrete	0.013	71.53	3.32
3	Rough brick	0.015	61.52	2.86
4	Clean earthen channel	0.018	50.74	2.36
5	Rubble masonry	0.017	53.9	2.51

B Manning's Eq.

In 1890 manning Proposed in place of relation given as:

$$Q = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

$$\text{and, } V = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

where n is the manning's discharge coefficient.

ان قيمة معامل ماننگ متغيرة وذلك بسبب اعتماده على العوامل الآتية :

- ١- خصونه السطح المائي
- ٢- عمق الماء وانتظام المقطع
- ٣- امتداد مجاري الماء وقدر الرسارات
- ٤- العورض وحجم رسكل القناة.
- ٥- التغير في التصرف.

nd, the manning's coefficient is not dimensionless, The dimensions is $T L^{1/3}$ ($\text{sec. m}^{1/3}$)

كم اقترح العالم الايرلندي ماننگ عالمة بين معامل سيري C

معامل ماننگ ركاري :

$$C = \frac{R_h^{1/6}}{n}$$

Ex5 For rectangular channel (3m wide) and (1m deep) with slope (1:2500) determine the flow rate ? using manning Eq. 19

$$\text{Soln: } A = B \cdot y = 3 \times 1 = 3 \text{ m}^2 \quad P = B + 2y = 3 + 2(1) = 5 \text{ m}$$

$$R_h = \frac{A}{P} = \frac{3}{5} = 0.6 \text{ m}$$

$$\text{manning eq.} \Rightarrow Q = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

$$Q = \frac{1}{n} (0.6)^{2/3} \left(\frac{1}{2500}\right)^{1/2} (3)$$

حسب معادله ماننینج (اعلاه تقدیم حساب شده φ اولیه عاید مقدار
معامل الخسونه n در کنون السؤال لم يرد شده و سنتاً مذکور مجموعه
من اطارات رکابزی :

case No.	surface type	n	Q m³/sec
1	Smooth cement lined channel	0.011	3.88
2	Rough brick	0.015	2.85
3	Rubble masonry	0.017	2.51

Ex 6 Find the discharge in trapezoidal channel with bed width of (10m), side slope (1:1) and depth of flow (2m), bed slope = 10^{-4} , $n=0.02$ and find chezy constant ? 20

$$\text{Soln} \quad A = By + \frac{Zy^2}{2} = 10(2) + \frac{1(2)^2}{2} = 24 \text{ m}^2$$

$$P = B + 2y\sqrt{1+Z^2} = 10 + 2(2)\sqrt{1+1^2} = 15.66 \text{ m}$$

$$\therefore R_h = \frac{A}{P} = \frac{24}{15.66} = 1.53 \text{ m}$$

$$\text{From manning Eq. } \Rightarrow Q = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

$$Q = \frac{1}{0.02} (1.53)^{2/3} (10^{-4})^{1/2} (24) = 16 \text{ m}^3/\text{sec}$$

$$\text{and, } C = \frac{R_h^{1/6}}{n} = \frac{(1.53)^{1/6}}{0.02} = 52.4 \frac{\text{m}^{1/2}}{\text{sec}}$$

Ex 7 Water flow at rate of ($5 \text{ m}^3/\text{sec}$) in a rectangular open channel of (3m width). Assuming manning constant $n = \frac{1}{50}$, calculate the slope required to maintain a depth of (2m) ? 21

$$\text{Soln. } A = B y = 3 \times 2 = 6 \text{ m}^2$$

$$P = B + 2y = 3 + 2(2) = 7 \text{ m}$$

$$R_h = \frac{A}{P} = \frac{6}{7} = 0.86 \text{ m}$$

$$Q = \frac{1}{n} R_h^{2/3} S^{1/2} A$$

$$5 = \frac{1}{\frac{1}{50}} (0.86)^{2/3} (S)^{1/2} (6)$$

$$\therefore S = 0.00034 \text{ ANS}$$

TABLE 15.1 Typical Values of Roughness Coefficient, Manning's n

Lined Canals	n
Cement plaster	0.011
Untreated gunite	0.016
Wood, planed	0.012
Wood, unplaned	0.013
Concrete, troweled	0.012
Concrete, wood forms, unfinished	0.015
Rubble in cement	0.020
Asphalt, smooth	0.013
Asphalt, rough	0.016
Corrugated metal	0.024
Unlined Canals	
Earth, straight and uniform	0.023
Earth, winding and weedy banks	0.035
Cut in rock, straight and uniform	0.030
Cut in rock, jagged and irregular	0.045
Natural Channels	
Gravel beds, straight	0.025
Gravel beds plus large boulders	0.040
Earth, straight, with some grass	0.026
Earth, winding, no vegetation	0.030
Earth, winding, weedy banks	0.050
Earth, very weedy and overgrown	0.080

Manning Equation: Traditional System of Units

The form of the Manning equation depends on the system of units because Manning's equation is not dimensionally homogeneous. In Eq. (15.15), notice that the primary dimensions on the left side of the equation are L^3/T and the primary dimensions on the right side are $L^{8/3}$.

To convert the Manning equation from SI to traditional units, one must apply a factor equal to 1.49 if the same value of n is used in the two systems. Thus, in the traditional system the discharge equation using Manning's n is

$$Q = \frac{1.49}{n} AR_h^{2/3} S_0^{1/2} \quad (15.16)$$

In Example 15.4, a value for Manning's n is calculated from known information about a channel and compared to tabulated values for n in Table 15.1.

EXAMPLE 15.4

Apply the Chezy Equation to find Manning's Value of n for Flow in a Channel

Problem Statement

If a channel with boulders has a slope of 0.0030, is 100 ft wide, has an average depth of 4.3 ft, and is known to have a friction factor of 0.130, what is the discharge in the channel, and what is the numerical value of Manning's n for this channel?

Define the Situation

Water flows in an channel with boulders:

$$S_0 = 0.003, B = 100 \text{ ft}, y = 4.3 \text{ ft}, f = 0.13$$

Assumptions: $R_h \approx y = 4.3 \text{ ft}$ (because the channel is wide).

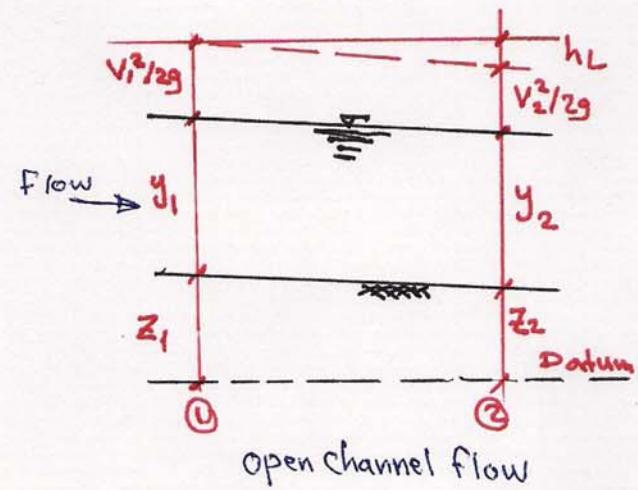
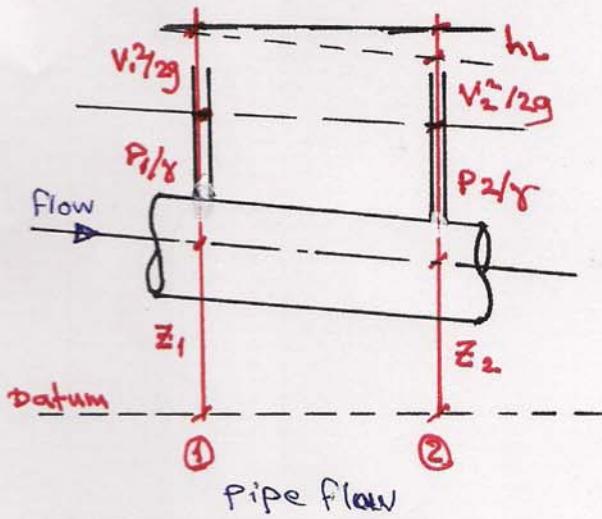
State the Goal

1. $Q(\text{cfs}) \Rightarrow$ discharge in the channel
2. $n \Rightarrow$ Manning's n

open channel flow is an important area of fluid mechanics for civil engineers. It describes the flow in rivers, man made channels and partially full pipes as well as the behaviour of hydraulic structures such as weirs, spillways and sluices. Open channel flow must have a free surface subjected to atmospheric pressure. The flow is gravity driven, with the discharge and flow depth dependent on the balance between the downslope component of gravity and bed friction.

Pipe flow & open channel flow

See the illustration figures for demonstration the flow features of closed conduit and open channel flow :-



Despite the similarity between the two kinds of flow, it must be pointing out the following notes :-

- * The difficulties to solve problems of flow in open channels than in pipe because the flow conditions in open channel are complicated due to:
 - Variability of position of free surface which will change with time and space.
 - The depth of flow, the discharge, and the slope of channel bottom and the free surface are all inter dependent

- * Physical conditions in open channel vary much more than in pipe where, the cross section of pipe is usually regular but for open channel may be irregular.
- * The roughness in open channel is considered the main variable that may be differs in flow manner from section to another (e.g.: the roughness may depend on the depth of flow).

Types of flow

The following classifications are made according to change in flow depth with respect to time and distance:-

* Steady & Unsteady : Time is the criterion

[flow is said to be steady if the depth of flow at a section does not change for the time under consideration. Otherwise is said to be un-steady.]

* Uniform Flow : Distance is the criterion

[The flow is said to be uniform if the depth of flow and hence Velocity are the same at every section of channel (e.g.: the flow in prismatic channels)]

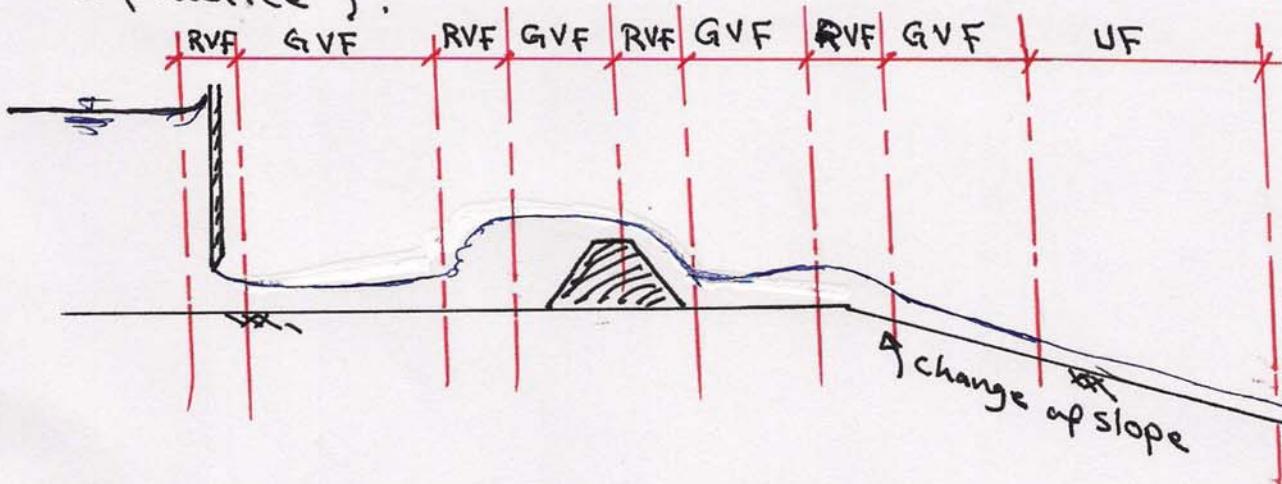
from the above definition " Steady-uniform" flow the depth & velocity will be constant with both Time & distance. This fundamental type of flow in an open channel occurs when gravity forces are in equilibrium with resistance forces.

* Steady non uniform flow (Varied flow)

[The depth varies with distance but not with time. This type of flow may be either Gradually varied or Rapidly varied.]

The gradually varied flow requires to apply the energy and frictional resistance equations, while the rapidly varied flow requires to apply the energy & momentum Eq.

(see fig. below for demonstration the feature type of flow in practice).



Geometric properties necessary for analysis

The commonly needed geometric properties are shown as follows:-

- **Depth (y)** : The vertical distance from lowest point of channel section to the free surface.
- **Stage (z)** : The vertical distance from the free surface to an arbitrary datum.
- **Area (A)** : The cross-sectional area of flow, normal to the direction of flow.
- **wetted perimeter (P)** : The length of the wetted surface measured normal to the direction of flow.
- **Top width (T)** : Width of the channel section at the free surface.
- **Hydraulic Radius (R)** : The ratio of area to wetted perimeter where :- $R = \frac{A}{P}$
- **Hydraulic depth (y_h)** : The ratio of area to top width where :-

Fundamental Equations

The equations which describe the flow of water are derived from three fundamental laws of Physics:

- * Conservation of Mass.
- * Conservation of energy.
- * Conservation of momentum.

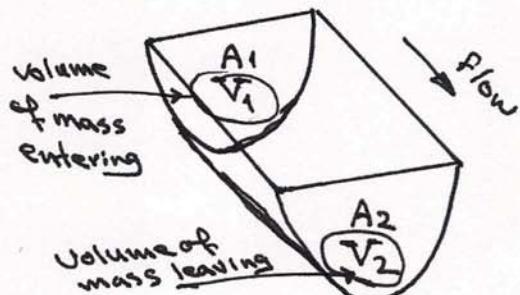
① The Continuity Equation (Conservation of mass)

for any control volume during the small time interval according to this law "The mass entering the control volume - the mass leaving the control volume = the change of mass within the control volume".

If the flow is steady & incompressible the mass of flow entering is equal to the mass leaving, where:-

$$\text{mass entering} = \text{mass leaving} \quad \text{--- (1)}$$

for control volume of flow in open channel show in fig. below



The eq.1 Can be written as:-

$$\rho Q_{\text{ent.}} = \rho Q_{\text{leav.}} \quad \text{--- (1a)}$$

also: the discharge known as: $Q = V \cdot A$

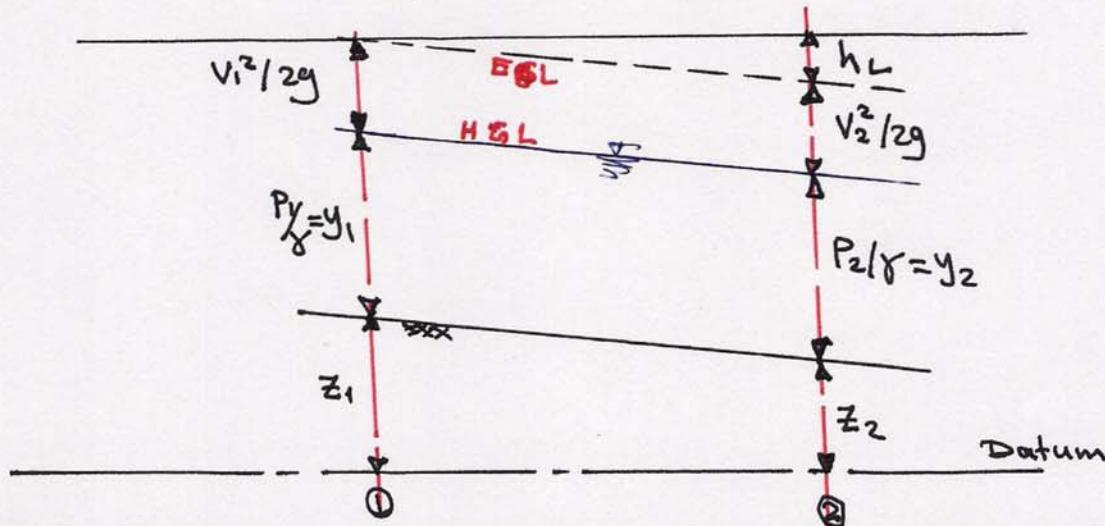
Therefore the continuity equation can be written as:-

$$V_1 \cdot A_1 = V_2 \cdot A_2 \quad \text{--- (2)}$$

② The Energy Equation (Conservation of energy)

for any given system, the change in energy is equal to the difference between the heat transferred to the system and the work done by the system. The energy in open channel flow represent the total energy of the system, which is the sum of the potential energy, Kinetic energy, and pressure energy.

In hydraulic application the "energy" converted to "head" in order to get a better feel for the resulting behavior of the system (see fig. below)



The energy at any point within the cross-section of flow is as can be seen in figure often expressed in three parts

- Elevation head, Z
- Pressure head (depth of flow), P/γ or y
- Velocity head (kinetic energy), $V^2/2g$

Because energy is conserved, it across any two points in the flow control volume must balance, where the energy between any two sections of control volume take the following form:-

$$Z_1 + y_1 + V_1^2/2g = Z_2 + y_2 + V_2^2/2g + h_L$$

③

③ The momentum Equation (conservation of momentum)

Again consider the control volume, during the time "t" :-

$$\text{momentum Entering} = \rho Q_1 t V_1$$

$$\text{Momentum leaving} = \rho Q_2 t V_2$$

$$\text{By the continuity } Q_1 = Q_2 = Q$$

and by Newton's second law $F = \text{rate of change of momentum}$

$$\text{whereas } F = \frac{M_{\text{leaving}} - M_{\text{entering}}}{t}$$

$$\text{then } F = \rho Q (V_2 - V_1) \quad \text{--- (4)}$$

Eq.4 is the momentum equation for steady flow for a region of uniform velocity.

Laminar & Turbulent flow

As in pipes and all flow, the flow in an open channel may be either laminar or turbulent. The criterion for determining the type of flow is the **Reynold's number, Re** .

In practice the limit for turbulent flow is not so well defined in channel as it is in pipes so $Re = 2000$ is often taken as the threshold for turbulent flow. Also in practice, flow in open channels is usually in the **rough turbulent zone** and consequently simpler friction formula may be applied to relate frictional losses to velocity and channel shape.

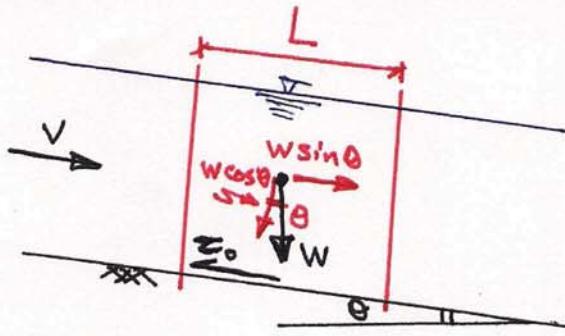
Uniform flow and the development of friction formula

When uniform flow occurs gravitational forces exactly balance with the **frictional resistance forces**, which apply as a shear force along the boundary (channel bed and walls).

Considering the below diagram :-

- The gravity force in the direction of flow is;

$$G_F = \gamma A L \sin \theta \quad \text{--- (a)}$$



- The boundary shear force is

$$S.F = T_0 P L \quad \text{--- (b)}$$

Then in uniform flow $G.F = S.F$, that is;

$$\gamma A L \sin \theta = T_0 P L \quad \text{--- (c)}$$

for Small Slope channel that is refer to $\sin \theta \approx \tan \theta \approx S_0$

where, S_0 , the bed slope, then

$$T_0 = \frac{\gamma A S_0}{P} = \gamma R S_0 \quad \text{--- (d)}$$

The Chezy Equation

In hydraulics and for "rough turbulent" flow by definition of the skin friction coefficient C_f , the bed shear stress can be related to average velocity V :-

$$\begin{aligned} T_0 &= C_f (\frac{1}{2} \rho V^2) \\ \text{i.e. } T_0 &= K V^2 \quad \text{--- (e)} \end{aligned}$$

When substituting into Eq. d gives;

$$V = \sqrt{\frac{\gamma}{K} R S_0} \quad \text{--- (f)}$$

If grouping the constants together as denoted "C" Eq.f will be;

$$V = C \sqrt{R S_0} \quad \text{--- (5)}$$

This is the "Chezy" equation and the "Chezy Coefficient C" as in C_f depending on fluid properties and shear resistance.

The Manning Equation

many studies have been made of the evaluation of "C" for different open channels. Robert Manning (1891 - 1895) derived the following empirical relation for "C" based upon experiments:

$$C = \frac{R^{1/6}}{n} \quad \text{--- --- --- --- ---} \quad (6)$$

Where "n" is the Manning's roughness coefficient. This coefficient is relates the effects of State, kind, shape, configuration of roughness of bed & sides of channel (Tables 802A - 802C) Illustrate a wide range of this coefficient.

When substituting Eq. 6 into Eq. 5 gives the well known Manning's Equation :-

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad \text{--- --- --- --- ---} \quad (7)$$

[It should be noted that the slope referred in Eq. 7 considered energy grade line slope, in uniform flow the lines EGL, HGL & bed slope line are all in parallel, so that $S_E = S_0$, where S_E is the slope of Energy line.]

Conveyance

If re-arranged eq. 7 as the following form

$$Q = V * A = \frac{1}{n} A \left(\frac{A}{P} \right)^{2/3} S_0^{1/2}$$

then $Q = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \cdot S_0^{1/2}$

For the same flow condition, shape and roughness the discharge in open channel can be expressed as:-

$$Q = K S_0^{1/2} \quad \text{--- --- --- --- ---} \quad (8)$$

"K" is called Conveyance Coefficient it is a measure of the

carrying capacity of a channel. For a specified "K" the only influencing parameter on discharge is the slope "S"

$$K = \frac{1}{n} \cdot \frac{A^{5/3}}{P^{2/3}} \quad \text{--- --- --- --- ---} \quad (9)$$

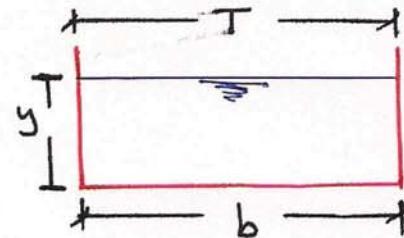
* The application of conveyance may be a solution for calculating the discharge and stage of compound channels.

Particular Channel Shapes

In each case "y" is the depth of flow.

* Rectangular channel

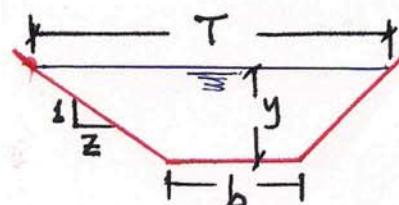
$$R = \frac{A}{P} = \frac{by}{b+2y}$$



* Wide channel

The channel is considered as wide if the limit of $y/b \ll 1$

then $R \approx y$



* Trapezoidal channel

$$R = \frac{y(b+zy)}{b+2y\sqrt{1+z^2}}$$

The geometric elements of different channel section can be shown in Table 801.

Normal Depth

For any given discharge, there will be a particular normal depth " y_n ". To analyze open channel flow, it is usually necessary to know the normal depth, referring to conveyance Eq.9, it can be re-writing (In SI-units)

$$K = \frac{1}{n} \cdot R^{2/3} \cdot A$$

Note that, K , is a function of normal depth, properties of channel section and Manning's coefficient. Eq. 7 will be after multiplying by area of cross section;

$$AR^{2/3} = \frac{nQ}{\sqrt{S}} = nK \quad \text{--- (10)}$$

Eq. 10, illustrate that for a specified values of n , Q , and S it can be solve this equation to determine the normal depth

Computation of Normal depth

- ① Using Chow (1959) design Curve (see Fig. 6-1).
- ② Trial & Error procedure
- ③ For a Specified cross-section of channel, use the following Equations:-

* For wide rectangular channel:

$$- q = \frac{Q}{b} \quad (\text{discharge per unit width})$$

$$- q = y_n V = \frac{1}{n} y_n R^{2/3} S^{1/2} \quad (\text{Mannig's Eq.})$$

$$- R \approx y_n \quad (\text{for wide rectangular channel})$$

$$\text{then :- } q = \frac{y_n^{5/3} S^{1/2}}{n} \quad \text{or} \quad y_n = \left(\frac{nq}{\sqrt{S}} \right)^{3/5}$$

then is refere Eq. 10 will become:-

$$y_n = \left(\frac{nq}{\sqrt{S}} \right)^{3/5}$$

--- (11)

Note that Eq. 11 Used to calculate the normal depth for wide rectangular channel.

* For Rectangular channel:

referring to Table 801 the hydraulic radius of rectangular channel were calculated using the following expression:-

$$R = \frac{b y}{b + 2y} = \frac{y_n}{1 + \frac{2y_n}{b}}$$

then ; $q = \frac{y_n^{5/3} \sqrt{S}}{n \left(1 + \frac{2y_n}{b}\right)^{2/3}}$

If need to calculate the normal depth rearrange the above formula to become as :

$y_n = \left(\frac{nq}{\sqrt{S}}\right)^{3/5} \left(1 + \frac{2y_n}{b}\right)^{2/5}$

----- ⑫

STORMWATER MANAGEMENT MANUAL

TYPICAL ROUGHNESS COEFFICIENTS FOR OPEN CHANNELS

<u>TYPE OF CHANNEL AND DESCRIPTION</u>	<u>MINIMUM</u>	<u>NORMAL</u>	<u>MAXIMUM</u>
EXCAVATED OR DREDGED			
a. Earth, straight and uniform			
1. Clean, recently completed	0.016	0.018	0.020
2. Clean, after weathering	0.018	0.022	0.025
3. Gravel, uniform section, clean	0.022	0.025	0.030
4. With short grass, few weeds	0.022	0.027	0.033
b. Earth, winding and sluggish			
1. No vegetation	0.023	0.025	0.030
2. Grass, some weeds	0.025	0.030	0.033
3. Dense weeds or aquatic plants in deep channels	0.030	0.035	0.040
4. Earth bottom and rubble sides	0.028	0.030	0.035
5. Stony bottom and weedy banks	0.025	0.035	0.040
6. Cobble bottom and clean sides	0.030	0.040	0.050
c. Dragline-excavated or dredged			
1. No vegetation	0.025	0.028	0.033
2. Light brush on banks	0.035	0.050	0.060
d. Rock cuts			
1. Smooth and uniform	0.025	0.035	0.040
2. Jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and brush			
1. Dense weeds, high as flow depth	0.050	0.080	0.120
2. Clean bottom, brush on sides	0.040	0.050	0.080
3. Same as above, but highest state of flow	0.045	0.070	0.110
4. Dense brush, high state	0.080	0.100	0.140

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STORMWATER MANAGEMENT MANUAL

TYPICAL ROUGHNESS COEFFICIENTS FOR OPEN CHANNELS

<u>TYPE OF CHANNEL & DESCRIPTION</u>	<u>MINIMUM</u>	<u>NORMAL</u>	<u>MAXIMUM</u>
Brass, smooth	0.009	0.010	0.013
Steel:			
Lockbar and welded	0.010	0.012	0.014
Riveted and spiral	0.013	0.016	0.017
Cast Iron:			
Coated	0.010	0.013	0.014
Uncoated	0.011	0.014	0.016
Wrought Iron:			
Black	0.012	0.014	0.015
Galvanized	0.013	0.016	0.017
Corrugated Metal:			
Sub-drain	0.017	0.019	0.021
Storm Drain	0.021	0.024	0.030
Lucite	0.008	0.009	0.010
Glass	0.009	0.010	0.013
Cement:			
Neat, surface	0.010	0.011	0.013
Mortar	0.011	0.013	0.015
Concrete:			
Culvert, straight and free of debris	0.010	0.011	0.013
Culvert with bends, connections, and some debris	0.011	0.013	0.014
Finished	0.011	0.012	0.014
Sewer with manholes, inlet, etc., straight	0.013	0.015	0.017
Unfinished, steel form	0.012	0.013	0.014
Unfinished, smooth wood form	0.012	0.014	0.016
Unfinished, rough wood form	0.015	0.017	0.020
Wood:			
Stave	0.010	0.012	0.014
Laminated, treated	0.015	0.017	0.020
Clay:			
Common drainage tile	0.011	0.013	0.017
Vitrified sewer	0.011	0.014	0.017
Vitrified sewer with manholes, inlet, etc.	0.013	0.015	0.017
Vitrified subdrain with open joint	0.014	0.016	0.018
Brickwork:			
Glazed	0.011	0.013	0.015
Lined with cement mortar	0.012	0.015	0.017
Sanitary sewers coated with sewage slime with bends and connections	0.012	0.013	0.016
Paved invert, sewer, smooth bottom	0.016	0.019	0.020
Rubble masonry, cemented	0.018	0.025	0.030

Revision	Date
ORIGINAL ISSUE	3/27/06

STORMWATER MANAGEMENT MANUAL

TYPICAL ROUGHNESS COEFFICIENTS FOR OPEN CHANNELS

<u>TYPE OF CHANNEL AND DESCRIPTION</u>	<u>MINIMUM</u>	<u>NORMAL</u>	<u>MAXIMUM</u>
LINED OR BUILT-UP CHANNELS			
a. CONCRETE			
1. TROWEL FINISH	0.011	0.013	0.015
2. FLOAT FINISH	0.013	0.015	0.016
3. GUNITE, GOOD SECTION	0.016	0.019	0.023
4. GUNITE, WAVY SECTION	0.018	0.022	0.023
b. CONCRETE BOTTOM FLOAT FINISHED WITH SIDE OF			
1. DRESSED STONE IN MORTAR	0.015	0.017	0.020
2. RANSOM STONE IN MORTAR	0.017	0.020	0.024
3. DRY RUBBLE OR RIPRAP	0.020	0.030	0.035
c. GRAVEL BOTTOM WITH SIDES OF			
1. FORMED CONCRETE	0.017	0.020	0.025
2. RANDOM STONE IN MORTAR	0.020	0.023	0.026
3. DRY RUBBLE OR RIPRAP	0.023	0.033	0.036
d. ASPHALT			
1. SMOOTH	0.013	0.013	--
2. ROUGH	0.016	0.016	--
e. GRASSED	0.030	0.040	0.050

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STORMWATER MANAGEMENT MANUAL

GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

SECTION	AREA, A	WETTED PERIMETER, P	HYDRAULIC RADIUS, R	TOP WIDTH, T	HYDRAULIC DEPTH, D	SECTION FACTOR, Z
Rectangle	bT	$b+2y$	$\frac{by}{b+2y}$	b	y	$by^{1.5}$
Trapezoid	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b+2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$
Triangle	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$\frac{y}{2}$	$\frac{\sqrt{2}}{2}zy^{2.5}$
Circle	$\frac{1}{8}(\theta - \sin \theta)d_0^2$	$\frac{1}{4}\theta - \frac{\sin \theta}{\theta}d_0$	$(\sin \frac{1}{4}\theta)d_0$ or $2\sqrt{y(d_0-y)}$	$\frac{3A}{2\sqrt{y}}$	$\frac{y}{2}\left(\frac{\theta - \sin \theta}{\sin \frac{1}{4}\theta}\right)d_0$	$\frac{\sqrt{2}}{32} \frac{(\theta - \sin \theta)^{1.5}}{(\sin \frac{1}{4}\theta)^{0.5}} d_0^{2.5}$
Parabola	$\frac{2}{3}T^2y$	$T + \frac{8}{3}\frac{y^2}{T}$	$\frac{2T^2y}{3T^2+8y^2}$	$\frac{3A}{2y}$	$\frac{2y}{3}$	$\frac{2}{3}\sqrt{6}Ty^{1.5}$
Round-cornered rectangle ($y > r$)	$(\frac{\pi}{2}-2)r^2+(b+2r)y$	$(\pi-2)r+b+2y$	$\frac{(\pi/2-2)r^2+(b+2r)y}{(\pi-2)r+b+2y}$	$b+2r$	$\frac{(\pi/2-2)r^2}{b+2r}+y$	$\frac{[(\pi/2-2)r^2+(b+2r)y]^{1.5}}{\sqrt{b+2r}}$
Round-bottom triangle	$\frac{T^2 - r^2}{4z} (1-z \cot^{-1} z)$	$\frac{T}{z} \sqrt{1+z^2} - \frac{2r}{z} (1-z \cot^{-1} z)$	$\frac{A}{P}$	$2[z(y-r)+r\sqrt{1+z^2}]$	$\frac{A}{T}$	$A \sqrt{\frac{4}{7}}$

* Satisfactory approximation for the interval $0 < x \leq 1$, where $x = 4y/T$. When $x > 1$, use the exact expression $P = (T/2)[\sqrt{1+x^2} + 1/x \ln(x + \sqrt{1+x^2})]$

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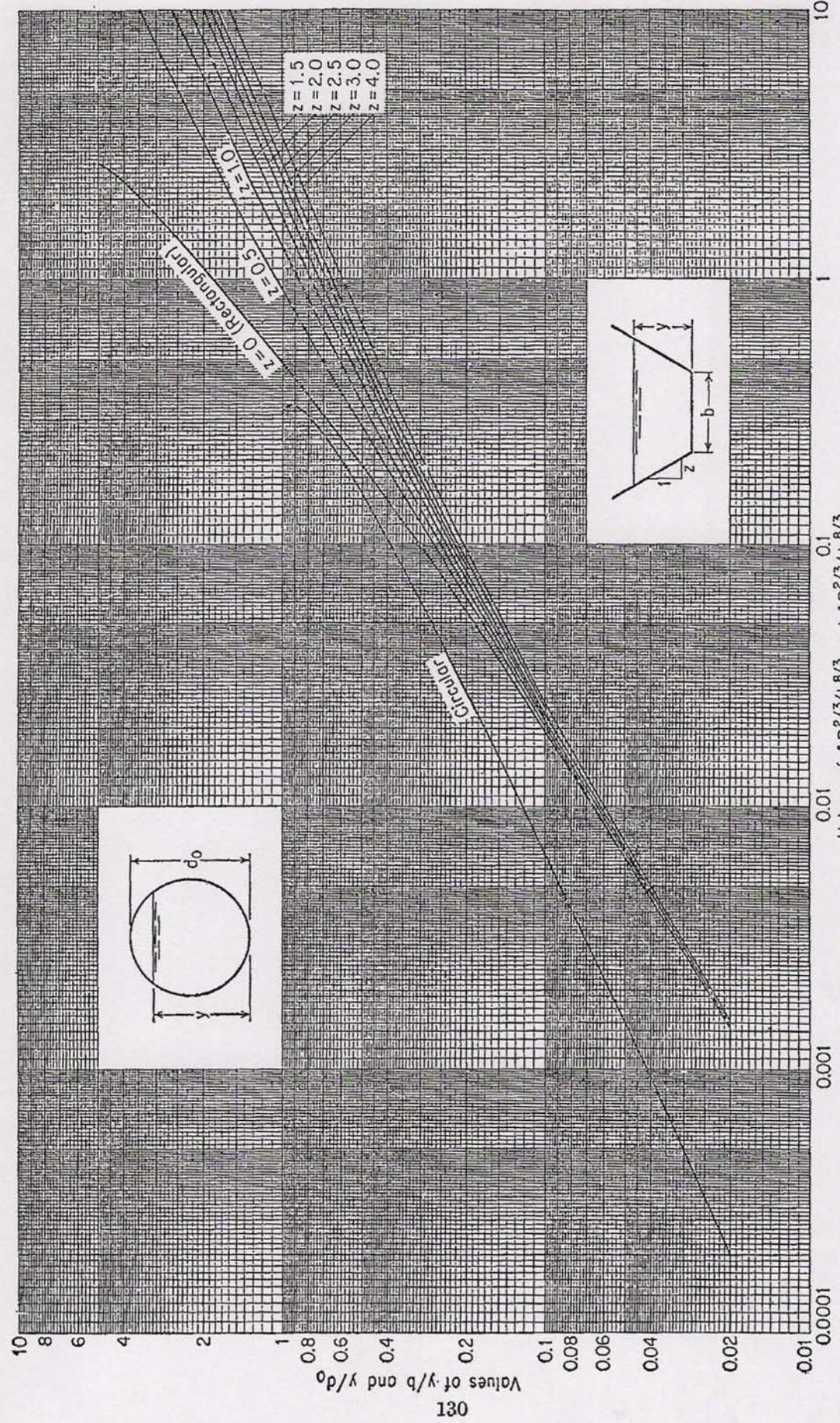


FIG. 6-1. Curves for determining the normal depth.

6. Economical cross-section (Most Efficient section) المقطع الأفضل

The hydraulic engineer upon to determine the shape and size of cross section of channel which transport the Maximum discharge when (P) is minimum.

This illustrated in the Problem by atrial Process assuming depth and width.

Ex8 A smooth cement rectangular channel with a slope (1 : 2500), Manning's coefficient is (0.011), total area (4 m^2), assuming different ratios of depth and width, determine the flow rate?

$$\underline{\text{Sol:}} \quad Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$A = B \times y = 4 \text{ m}^2 \quad \text{--- (1)}$$

$$P = B + 2y$$

$$R = \frac{A}{P} = \frac{4}{B + 2y} \quad \text{--- (2)}$$

هنا لـ y واحد المعرف يجب معرفة قيمة الارتفاع y والعرض B كالتالي:

سٰ

$y:B$	y	B	P	R_h	$Q = \frac{1}{n} R_h^{2/3} S^{1/2} (4)$
1:1.5	1.633	2.45	5.716	0.699	5.73 $y/B = 1/1.5 \quad B=1.5y$
1:1.75	1.512	2.645	5.67	0.706	$A=4 = B.y = (1.5y)y = 1.5y^2$ $y = 1.633 \checkmark$
1:2	1.414	2.828	5.66 Min.	0.707 $y/2$	$B=1.5y \quad B=2.45$
1:2.25	1.33	3	5.67	0.706	5.772 → Max.
1:2.5	1.265	3.16	5.69	0.703	5.766
					5.748

نقطة اعلى عند نسبة (1:2) (الارتفاع : (عرض)) يحصل على اقصى الامثل
لأفضل سطح مماس (أكبر تصرف لائل محلي مثيل)

then, for rectangular section :- $\frac{y}{B} = \frac{1}{2}$

$$B = 2y$$

$$\text{and, } R_h = \frac{y}{2}$$

ونفس الطريقة يمكن لنا إيجاد العوامل التي يجب بالطبع
الامثل لقاطع به المتر و الثالث وكلابين بالجدول
الأخير :

Section	Optimum geometry	normal depth y	R_h	A
rectangular	$B=2y$	$0.917 \left[\frac{Qn}{S^{1/2}} \right]^{3/8}$	$\frac{y}{2}$	$1.68 \left[\frac{Qn}{S^{1/2}} \right]^{3/4}$
Trapezoidal	$\theta = 60^\circ$ $Z = \frac{1}{\sqrt{3}}$ $B = \frac{2y}{\sqrt{3}}$	$0.968 \left[\frac{Qn}{S^{1/2}} \right]^{3/8}$	$\frac{y}{2}$	$1.622 \left[\frac{Qn}{S^{1/2}} \right]^{3/4}$
Triangular	$Z = 1$ $\theta = 45^\circ$	$1.297 \left[\frac{Qn}{S^{1/2}} \right]^{3/8}$	$\frac{y}{2\sqrt{2}}$	$Zy^2 = y^2$

يتم تطبيق الجدول اعلاه ونخال امثلة حسب نوع المقطع اما

حساب التصريف نعم تطبيق معادله مانلث بعد حساب

امثليات في الجدول اعلاه

Ex9 For given slope of $1:2500$ and flow rate of ($4 \text{ m}^3/\text{sec}$), determine the optimum cross section (Most Efficient Section) for ① rectangular
 ② trapezoidal ③ triangular, ($n=0.011$) ?

Section	y	R_h	A
Rectangular	$y = 0.917 \left[\frac{Qn}{S^{1/2}} \right]^{3/8}$ $y = 0.917 \left[\frac{4(0.011)}{\left(\frac{1}{2500}\right)^{1/2}} \right]^{3/8}$ $y = 1.235 \text{ m}$ $B = 2y = 2.465$	$\frac{y}{2}$	$1.68 \left[\frac{Qn}{S^{1/2}} \right]^{3/4}$ $A = 1.68 \left[\frac{4(0.011)}{\left(\frac{1}{2500}\right)^{1/2}} \right]^{3/4}$ $A = 3.04 \text{ m}^2$
Trapezoidal	$Z = \frac{1}{\sqrt{3}}$ $\Theta = 60^\circ$ $B = \frac{2y}{\sqrt{3}}$ $y = 0.968 \left[\frac{Qn}{S^{1/2}} \right]^{3/8}$ $y = 0.968 \left[\frac{4(0.011)}{\left(\frac{1}{2500}\right)^{1/2}} \right]^{3/8}$ $y = 1.302 \text{ m}$ $B = \frac{2(1.302)}{\sqrt{3}}$ $B = 1.502 \text{ m}$	$\frac{y}{2}$	$1.622 \left[\frac{Qn}{S^{1/2}} \right]^{3/4}$ $A = 1.622 \left[\frac{4(0.011)}{\left(\frac{1}{2500}\right)^{1/2}} \right]^{3/4}$ $A = 2.93 \text{ m}^2$

Ex 10 Find the depth of flow in the most efficient triangular section carrying discharge of $(0.2 \text{ m}^3/\text{sec})$ on slope of $(1:2500)$, $n = 0.014$? 25

Solⁿ for triangular section :-

$$Z = 1 \quad \Theta = 45^\circ$$

$$y = 1.297 \left[\frac{\Phi n}{S^{1/2}} \right]^{3/8} = 1.297 \left[\frac{0.2(0.014)}{\left(\frac{1}{2500}\right)^{1/2}} \right]^{3/8}$$

$$\therefore y = 0.62 \text{ m}$$

Ex 11 A rectangular section is to be built of rough unsized timber. If given a drop of $(2 \text{ m in } 1 \text{ Km})$ what will be width and depth for best section to carry $(1.1 \text{ m}^3/\text{sec})$, $n = 0.011$?

$$\underline{\text{Sol¹.}} \quad S = \frac{2 \text{ m}}{1 \text{ Km}} = \frac{2}{1000} = \frac{1}{500}$$

$$y = 0.917 \left[\frac{\Phi n}{S^{1/2}} \right]^{3/8} = 0.917 \left[\frac{1.1(0.011)}{\left(\frac{1}{500}\right)^{1/2}} \right]^{3/8} = 0.56 \text{ m}$$

$$B = 2y = 2(0.56) = 1.12 \text{ m}$$

Ex12 A trapezoidal channel of Best section has
a discharge of ($25 \text{ m}^3/\text{sec}$) with slope ($1:1500$)

Design the section if $n = 0.0135$?

Soln. for trapezoidal section :-

$$\bar{C} = \frac{1}{\sqrt{3}}$$

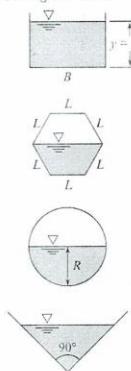
$$Y = 0.968 \left[\frac{Qn}{S^{1/2}} \right]^{3/8} = 0.968 \left[\frac{25(0.0135)}{\left(\frac{1}{1500}\right)^{1/2}} \right]^{3/8}$$

$$\therefore Y = 2.54 \text{ m}$$

$$B = \frac{2Y}{\sqrt{3}} = \frac{2(2.54)}{\sqrt{3}} = 2.93 \text{ m}$$

FIGURE 15.5

Best hydraulic sections for different geometries.



Best Hydraulic Section for Uniform Flow

The **best hydraulic section** is the channel geometry that gives the maximum discharge for a given cross-sectional area. Maximum discharge occurs when a geometry has the minimum wetted perimeter. Therefore, it yields the least viscous energy loss for a given area. Consider the quantity $AR_h^{2/3}$ in Manning's equation given in Eqs. (15.15 and 15.16), which is referred to as the section factor. Because $R_h = A/P$, the section factor relating to uniform flow is given by $A(A/P)^{2/3}$. Thus, for a channel of given resistance and slope, the discharge will increase with increasing cross-sectional area but decrease with increasing wetted perimeter P . For a given area A and a given shape of channel—for example, rectangular cross section—there will be a certain ratio of depth to width (y/B) for which the section factor will be maximum. This ratio is the best hydraulic section.

Example 15.6 shows that the best hydraulic section for a rectangular channel occurs when $y = \frac{1}{2}B$. It can be shown that the best hydraulic section for a trapezoidal channel is half a hexagon as shown; for the circular section, it is the half circle with depth equal to radius; and for the triangular section, it is a triangle with a vertex of 90° (Fig. 15.5). Of all the various shapes, the half circle has the best hydraulic section because it has the smallest perimeter for a given area.

The best hydraulic section can be relevant to the cost of the channel. For example, if a trapezoidal channel were to be excavated and if the water surface were to be at adjacent ground level, the minimum amount of excavation (and excavation cost) would result if the channel of best hydraulic section were used.

EXAMPLE 15.6

Finding the Best Hydraulic Section for a Rectangular Channel

Problem Statement

Determine the best hydraulic section for a rectangular channel with depth y and width B .

Define the Situation

Water flows in a rectangular channel. Depth = y . Width = B .

State the Goal

Find the best hydraulic section (relate B and y).

Generate Ideas and Make a Plan

1. Set $A = By$ and $P = B + 2y$ so that both are a function of y .
2. Let A be constant, and minimize P .
 - Differentiate P with respect to y and set the derivative equal to zero.
 - Express the result of minimizing P as a relation between y and B .

Take Action (Execute the Plan)

1. Relate A and P in terms of y :

$$P = \frac{A}{y} + 2y$$

- 2a. Minimize P :

$$\begin{aligned} \frac{dP}{dy} &= \frac{-A}{y^2} + 2 = 0 \\ \frac{A}{y^2} &= 2 \end{aligned}$$

- 2b. Express result in terms of y and B :

$$A = By, \text{ so}$$

$$\frac{By}{y^2} = 2 \quad \text{or} \quad y = \frac{1}{2}B$$

Review the Solution and the Process

Knowledge. The best hydraulic section for a rectangular channel occurs when the depth is one-half the width of the channel (see Fig. 15.5).

Uniform Flow in Culverts and Sewers

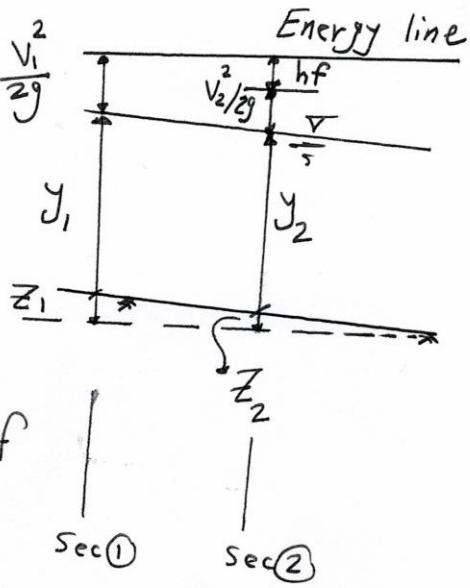
Sewers are conduits that carry sewage (liquid domestic, commercial, or industrial waste) from households, businesses, and factories to sewage disposal sites. These conduits are often circular in cross section, but elliptical and rectangular conduits are also used. The volume rate of sewage varies throughout the day and season, but of course sewers are designed to carry the

Energy Equation for steady Flow and specific Energy (critical flow)

Considering Sections 1 and 2 in the flow as shown in Fig.

Bernoulli eq. is written as:

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + hf$$



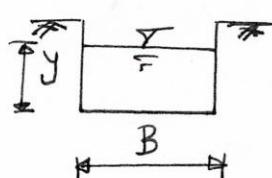
- The term $\frac{V^2}{2g} + y$ is found to be an important

Parameter in open channel flow, This quantity is defined as **Specific Energy** الطاقة المائية OPEN CHANNELS ONLY and, symbol used is E .

$$\therefore \text{Specific energy } E = y + \frac{V^2}{2g} \quad \text{--- (1)}$$

- The case of rectangular section:**

$$E = y + \frac{\phi^2}{2g B^2 y^2} \quad \text{--- (2)}$$



in this Process the value of minimum energy for a given flow is found as:

$$E_{\min} = \frac{y}{c} + \frac{y_c^3}{2y_c^2} = \frac{3}{2} \frac{y}{c} \quad \text{--- (3) Min. Energy with critical flow only}$$

The value of y for minimum energy is called critical depth (y_c).

$$\frac{y}{c} = \sqrt[3]{q^2/g} \quad \text{--- (4)}$$

where, $q = \frac{\Phi}{B}$ m^2/sec (flow rate for unit width)

The flow rate at this condition (Min. Specific energy and critical depth) is:

$$q_{\max} = \sqrt{g y_c^3} \quad \text{--- (5)}$$

at y_c :
the energy is Min.
the discharge is Max.

العوارض من (1) \rightarrow (5) لاملاط المدخلين وعيون

ناتجها مع مدخل المدخلين التاليين وكالثري:

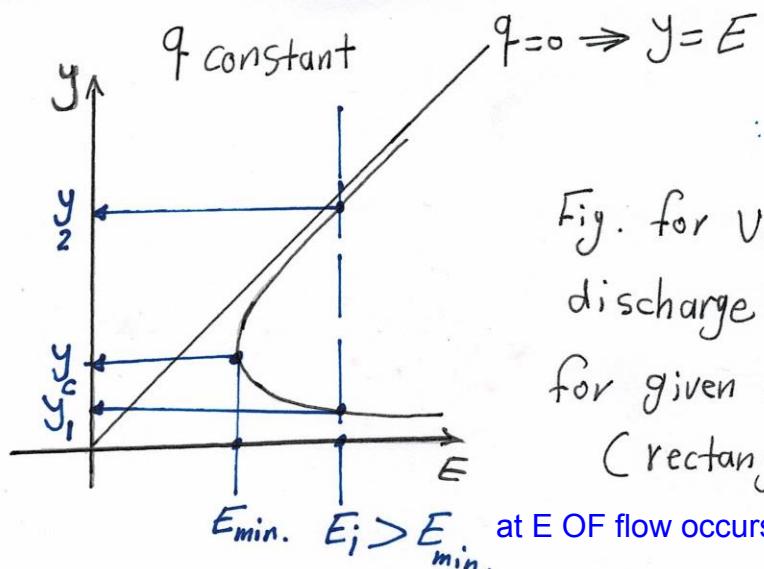


Fig. for Variation of discharge with depth for given Specific Energy.
(rectangular section)

at $E_{\min.}$ occurs y_c only

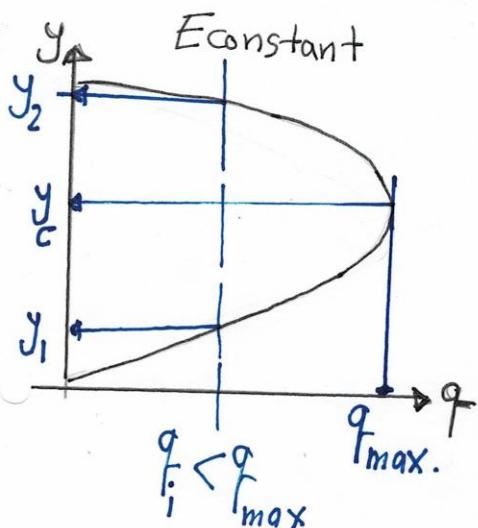


Fig. Variation of depth with Specific Energy for given flow rate (rectangular section)

ملاحظة: $(y_2 > y_1)$ يسمى العمق المترافق في الماء (consequent depth) ، و $(y_1 < y_2)$ يسمى العمق المترافق (alternate depth).

$$\frac{y^3}{3} - E y^2 + \frac{q^2}{2g} = 0 \quad \text{(rectangular sec.)}$$

1 or 2

Ex13 water flow in a rectangular channel at the rate 30 of ($3 \text{ m}^3/\text{sec}$) per m width, the depth being (1.5 m^{y_1})

Determine whether the flow is subcritical or supercritical. Also determine the alternate depth and critical depth?

$$\text{Soln} \quad ① E = \frac{V^2}{2g} + y$$

$$V = \frac{Q}{A} = \frac{3}{1.5 \times 1} = 2 \text{ m/sec}$$

$$\therefore E = \frac{4}{2g} + 1.5 = 1.704 \text{ m} \quad \text{حيث أن العرض ثابت وعمران ثابت} \quad E_i = E_1 = E_2 \\ \text{لذلك السرعة الحدية تتغير عند عمق اتجاه الحركة.}$$

$$Fr = \frac{V}{\sqrt{g y}} = \frac{2}{\sqrt{9.81 \times 1.5}} = 0.52 < 1 \quad \text{The flow is subcritical}$$

$$② y_c = \sqrt[3]{q^2/g} \quad q = \frac{Q}{B} = \frac{3}{1} = 3 \text{ m}^2/\text{sec}$$

$$\therefore y_c = \sqrt[3]{3^2/9.81} = 0.972 \text{ m}$$

$$E_{min.} = \frac{3}{2} y_c = \frac{3}{2} (0.972) = 1.458 \text{ m}$$

③ alternate depth y_2 :

$$\frac{y^3}{2} - E y^2 + \frac{q^2}{2g} = 0 \Rightarrow \frac{y^3}{2} - 1.703 y^2 + \frac{3^2}{2g} = 0$$

٨١

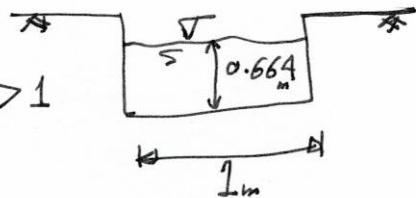
Solving by trial, $y_2 = 0.664 \text{ m}$

وكان المتر دارل (1.5m) حساب: حالة الجريان يمكنه الاردن
حسبما هو الجريان المتر الثاني والذى يقدر بـ (0.664m)

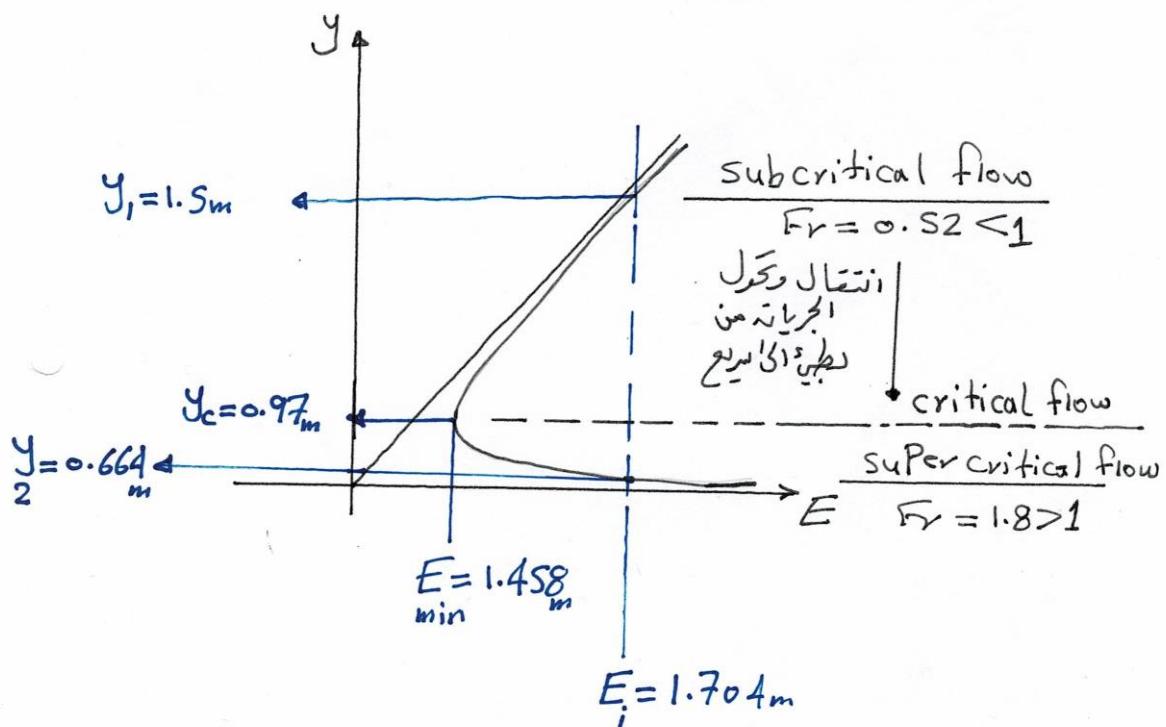
وكان

$$V = \frac{Q}{A} = \frac{J}{1 \times 0.664} = 4.52 \text{ m/sec}$$

$$Fr_2 = \frac{V}{\sqrt{g y_2}} = \frac{4.52}{\sqrt{9.81 \times 0.664}} = 1.8 > 1$$



The flow is super critical.



• Specific Energy and critical flow
of trapezoidal section :-

32

Specific energy is an important parameter in open channel flow symbol is E and writing as:

$$E = y + \frac{V^2}{2g} \quad \text{--- (1)}$$

المعادلة تسمى في الماء
السابقة هي
تحل معادلة عامة للحالة
النوعية بغض النظر
عن نوع وشكل المقطع

. ولأن نبدأ بكتابه المعادلات اقامة بالعاصي السبب المخرج

رموز :-

$$E_{min} = \frac{3}{2} y_c$$

--- (3)

المعادلة (3) this equation for rectangular section only

المستند له سابقاً
العاصي المستطيل لا يكفي لأن استخدناها لات

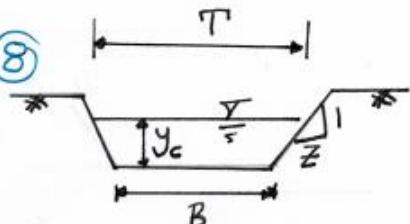
$$y_c = 0.81 \left[\frac{Q^2}{g Z^{0.75} B^{1.25}} \right]^{0.27} - \frac{B}{30 Z} \quad \text{--- (7)}$$

• المعادلة (7) لحساب العمق المخرج في العاصي سبب المخرج.

وكما علمنا سابقاً أن عند حدوث الفرق الحرج τ يصبح
الطاقة النوعية عند أعلى مستوى لها (E_{\max}) ويصيغ
التصريف في أقصى حداته (q_{\max}).

• حساب أكبر تصريف ممكن في القنوات الشبه المفرزة
والذي تتحقق بسبب الفرق الحرج نسبتاً للمعادلة الأساسية:

$$\frac{Q^2}{g A_c^3} \frac{T}{T} = 1 \quad \text{---} \quad (8)$$

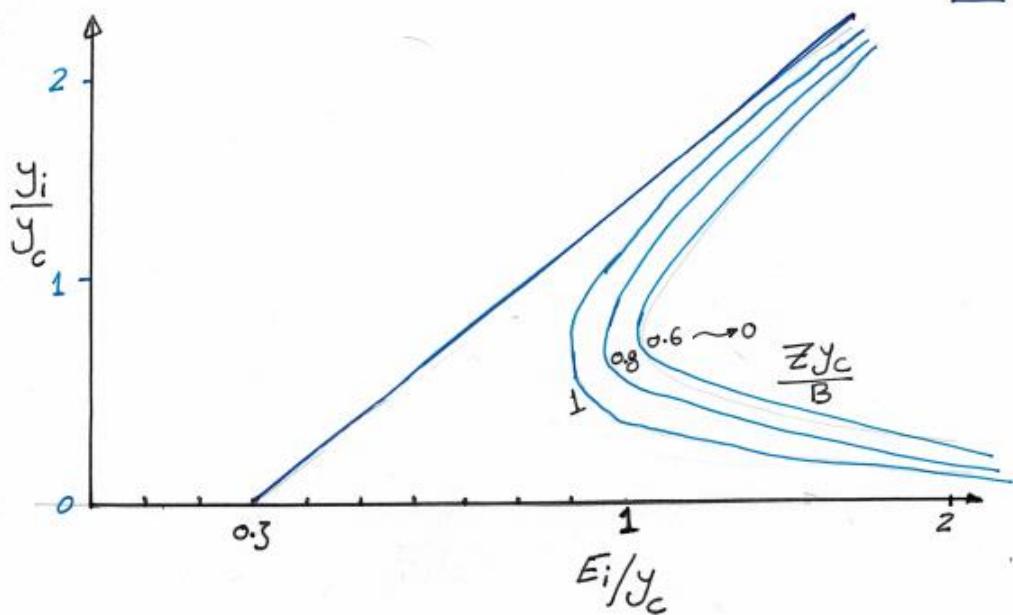


where, $T = B + 2Zy_c$

$$A_c = By_c + Zy_c^2$$

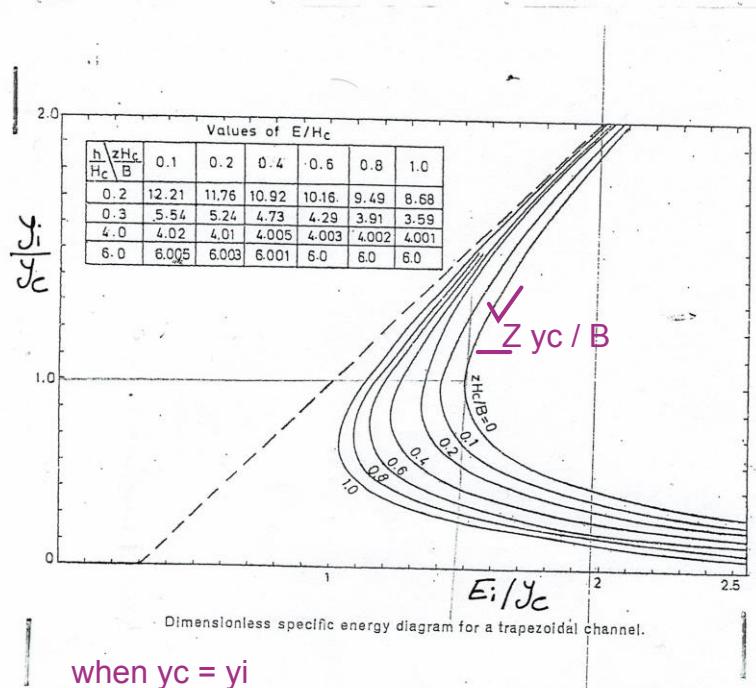
$$y_c = \text{from eq } (7)$$

• المعادلات (7, 8) خاصه بال洽ع مع ثبيه المحرفه يمكن من استعمالها من
طريق الخطط الآتي وكل ما يلي :



ملاحظة هامة

- ① لا يجاد (E_{min}) يجب استخدام المختلط اعلاه من فلار معنده \leq
- ② في السكل اعلاه نلا مختلط تكون عميدين (يلار_1 و يلار_2) والتي تسمى
(Squent depth) وتحب من السكل اعلاه من فلار
معنده يمه (E_i) الطائمه النوعيه عند اي نقطه.



when $y_c = y_i$
then $E_{min} = E_i$

to find E_{min}
choose $y_i/y_c = 1$

Ex 14 Determine the critical depth of channel with trapezoidal section with flow of $(1/3) m^3/\text{sec}$. The base width is (0.6m) and side slope is 45° ?

Soln.

(7) حساب العمق الحراري من y_c

$$y_c = 0.81 \left[\frac{Q^2}{g Z^{0.75} B^{1.25}} \right]^{0.27} - \frac{B}{30 Z}$$

$$B = 0.6\text{m} \quad Z = 1 \quad Q = \frac{1}{3}$$

$$\therefore y_c = 0.81 \left[\frac{(1/3)^2}{9.81 (1)^{0.75} (0.6)^{1.25}} \right]^{0.27} - \frac{0.6}{30 (1)}$$

$$y_c = 0.26\text{m}$$

Ex 15 A trapezoidal channel with side slopes of 45° and bottom width of 8m , The slope is $1/796$, Determine the state of flow and critical depth if the flow rate is $20\text{ m}^3/\text{sec}$? ($n = 0.045$)

Soln.

$$\textcircled{1} Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$20 = \frac{1}{0.03} \left(\frac{A}{P} \right)^{2/3} \left(\frac{1}{796} \right)^{1/2} A$$

$$A = By + Z^2 y^2 = 8y + y^2$$

$$P = B + 2y \sqrt{1 + Z^2}$$

y هي الماء والطاقة، حسب
 $y = 2m$

$$Fr = \frac{V}{\sqrt{g y_h}} \Rightarrow V = \frac{Q}{A} = \frac{20}{8(2) + 4} = \frac{20}{20} = 1 \text{ m/sec}$$

$$y_h = \frac{A}{T} = \frac{20}{B + 2Zy} = \frac{20}{8 + 2(2)} = \frac{20}{12} \checkmark$$

$$y_h = \frac{20}{12} = 1.67m \checkmark$$

$$\therefore Fr = \frac{L}{\sqrt{9.81 \times 1.67}} = \frac{L}{\sqrt{16.35}} = 0.25 < 1$$

\therefore sub critical flow

$$(2) y_c = 0.81 \left[\frac{Q^2}{g Z^{0.75} B^{1.25}} \right]^{0.27} - \frac{B}{30 Z} \quad \text{equation NO. 7}$$

$$y_c = 0.81 \left[\frac{(20)^2}{9.81 (8)^{1.25}} \right]^{0.27} - \frac{8}{30}$$

$$y_c = 0.83m < y \Rightarrow \text{sub critical flow}$$

$$\textcircled{3} \quad E_i = \frac{V^2}{2g} + y = \frac{(1)^2}{2g} + 2 = 2.1 \text{ m}$$

وحساب اند طاقة نستخدم المترى الخاص بالعاصف السبوعى وكارلری:

$$\begin{aligned} \frac{\bar{y}_c}{B} &= \frac{1(0.83)}{8} = 0.1 \\ \frac{y_i}{y_c} &= \frac{0.83}{0.83} = 1 \end{aligned} \quad \Rightarrow \frac{E_{min}}{y_c} = 1.47$$

$\therefore E_{min} = 1.47(y_c)$

$$E_{min} = 1.47(0.83) = 1.22 \text{ m}$$

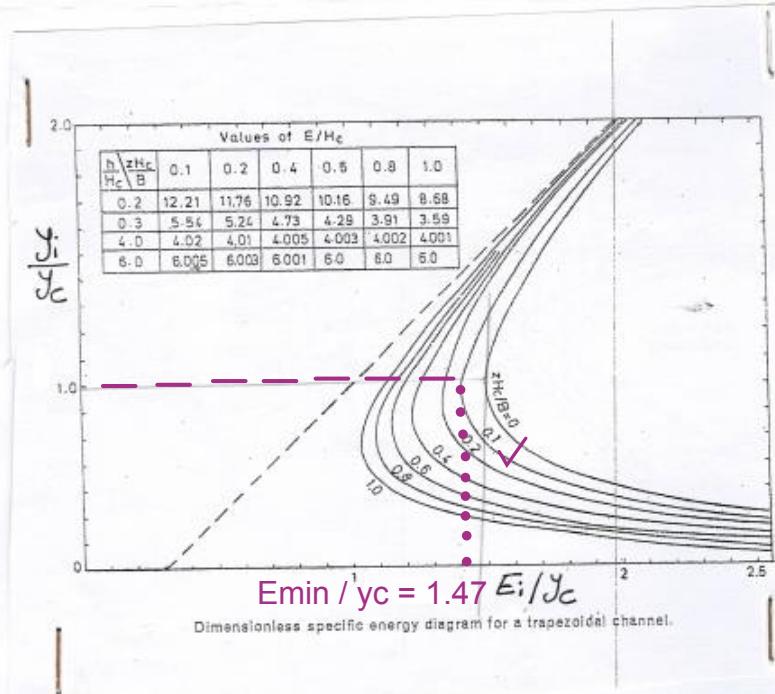
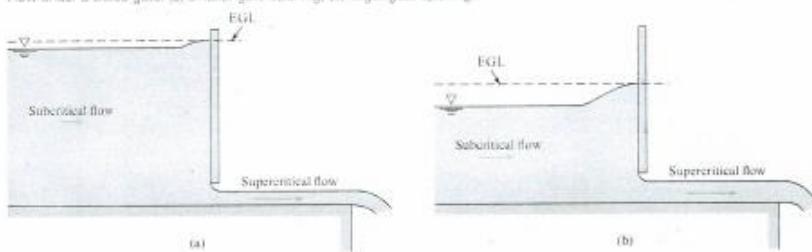


FIGURE 15.9
Flow under a sluice gate: (a) smaller gate opening, (b) larger gate opening.



upstream and supercritical flow occurs downstream of the sluice gate in Fig. 15.9. Subcritical flows corresponds to a Froude number less than one ($Fr < 1$), and supercritical flow corresponds to $Fr > 1$. Some engineers refer to subcritical and supercritical flow as *troughal* and *rapid* flow, respectively. Other aspects of critical flow are shown in the next section.

Characteristics of Critical Flow

Critical flow occurs when the specific energy is minimum for a given discharge. The depth for this condition may be determined by solving for dE/dy from $E = y + Q^2/2gA^2$ and setting dE/dy equal to zero:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{dy} \quad (15.21)$$

However, $dA = T dy$, where T is the width of the channel at the water surface, as shown in Fig. 15.10. Then Eq. (15.21), with $dE/dy = 0$, will reduce to

$$\frac{Q^2 T_c}{g A_c^3} = 1 \quad (15.22)$$

or

$$\frac{A_c}{T_c} = \frac{Q^2}{g A_c^2} \quad (15.23)$$

If the hydraulic depth, D , is defined as

$$D = \frac{A}{T} \quad (15.24)$$

then Eq. (15.23) will yield a critical hydraulic depth D_c , given by

$$D_c = \frac{Q^2}{g A_c^2} = \frac{V^2}{g} \quad (15.25)$$

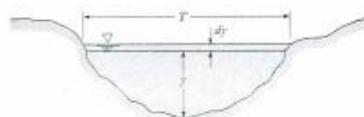


FIGURE 15.10
Open-channel relations.

Rapidly Varied Flow and Hydraulic Jump

The flow in an open channel is called "rapidly varied flow" (RVF) if the flow depth changes markedly (abruptly) over a relatively short distance. Such flow occur in:-

- Flow under sluice gates. ✓
- Flow over Sharp or broad crested weirs. ✓
- Waterfalls (Drops). ✓
- transitions sections of a channel for expansion or contraction. ✓
- change in slope of Channel. ✓

Rapidly Varied flows are typically complicated by the fact that they may involves significant multidimensional, backflows and flow separation also air-entainment can be complicate the flow patterns due to bubbles eddies. Therefore, rapidly varied flows are usually mainly studied experimentally.

The flow in steep channels can be Supercritical and the flow can change to Subcritical if the channel slope reduced or a friction effect of channel increases. Any such change from supercritical to subcritical occur through a hydraulic jump. When hydraulic jump occur a large scale turbulence accompanied lead to dissipation most of the kinetic energy of supercritical flow.

The hydraulic jump occur practically in:-

- Canal below a regulating sluice gate. ✓

- At the bottom of a spillway ✓
- At a place when a steep slope suddenly turns to flat or less steeper.

Consider steady flow through a control volume that encloses the hydraulic jump (shown in fig.). To make a simple analysis possible we make the following assumptions:-

- 1- The Velocity constant across the channel at sections 1 & 2 .
- 2- The distribution of pressure varies hydrostatically .
- 3- The wall shear stress and the associated losses are negligible relative to the losses that occur during the hydraulic jump due to eddies and turbulence .
- 4- The channel is wide and horizontal.
- 5- There are no external or body forces other than gravity.

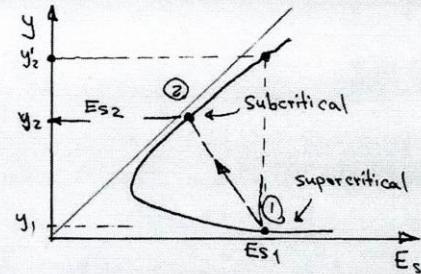
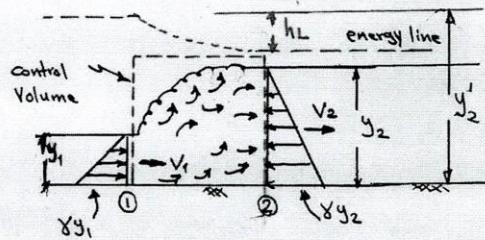
For a channel of width, b , and from Continuity :-

$$y_1 V_1 = y_2 V_2 \quad \text{--- (1)}$$

The only forces acting on control volume in the direction of flow are the pressure forces , thus by Momentum eq:-

$$\sum F = \sum M_{out} - \sum M_{in} \quad \text{--- (2)}$$

In the direction of flow becomes a balance between hydrostatic pressure forces and Momentum transfer .



$$P_{1\text{av}} A_1 - P_{2\text{av}} A_2 = M V_2 - M V_1 \quad \text{--- (2)}$$

where; $P_{1\text{av}} = \frac{\gamma y_1}{2}$, $P_{2\text{av}} = \frac{\gamma y_2}{2}$ for width b , we have

$$A_1 = y_1 b \quad \& \quad A_2 = y_2 b, \quad M = M_1 = M_2 = \rho Q_1 = \rho y_1 b V_1, \text{ then}$$

Substituting in Eq3 and Simplifying the momentum eq. reduces to:-

$$y_1^2 - y_2^2 = \frac{2 y_1 V_1}{g} (V_2 - V_1) \quad \text{--- (4)}$$

Eliminating V_2 by using $V_2 = \left(\frac{y_1}{y_2}\right) V_1$ from the continuity equation give:-

$$y_1^2 - y_2^2 = \frac{2 y_1 V_1^2}{g y_2} (y_1 - y_2) \quad \text{--- (5)}$$

Cancelling the common factor $(y_1 - y_2)$ from both sides and re-arranging give:-

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2 Fr_i^2 = 0 \quad \text{--- (6)}$$

This is a quadratic equation for y_2/y_1 and it has two roots one negative and one positive. Noting that $\frac{y_2}{y_1}$ cannot be negative since both y_2 & y_1 are positive quantities, the depth ratio y_2/y_1 is determined to be:-

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1+8Fr_i^2} - 1 \right)}$$

⑦ for rectangular section only

The depth of flow before a hydraulic jump, y_1 , (at supercritical) state is called the Initial Depth, and the depth of flow after the jump, y_2 (at subcritical) state is called Seguent depth. Note that the specific energy at an initial depth, E_{s1} , don't equal the

specific energy at y_2 (E_{s2}), that is due to a consideration of a significant dissipation of energy through the jump (control volume) (See Fig. in the page 2). Equation 7 demonstrate that the sequent depth, y_2 , should be less than the alternate depth (y'_2) because the consideration of energy dissipation (see Fig. Page 2).

The energy eq. for the two section of control volume can be expressed as:-

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \quad \text{--- (8)}$$

noting that $V_2 = (\frac{y_1}{y_2})V_1$ and $Fri = \frac{V_1}{\sqrt{gy_1}}$, the head loss associated with the hydraulic jump is expressed as:-

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} \quad \text{or ;}$$

$$\boxed{h_L = (y_1 - y_2) + \frac{y_1 Fri^2}{2} \left(1 - \frac{y_1^2}{y_2^2}\right)} \quad \text{--- (9)}$$

If substituting Eq. 7 in Eq. 9 and Simplifying it can conclude to :-

$$\boxed{h_L = \frac{(y_2 - y_1)^3}{4y_1 y_2}} \quad \text{--- (10) head losses for rec. section}$$

If it need to the dissipation ratio , then:-

$$\frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + \frac{V_1^2}{2g}} = \frac{h_L}{y_1 \left(1 + \frac{Fri^2}{2}\right)}, \text{ where : -}$$

$$\boxed{\text{dissipation ratio \%} = \left[\frac{h_L}{y_1 \left(1 + \frac{Fri^2}{2}\right)} * 100 \right]} \quad \text{--- (11) rec.section}$$

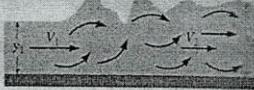
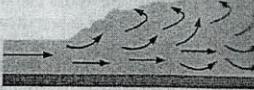
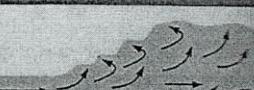
The percent dissipation of kinetic energy ranges from just few percent for weak jump ($F_r < 2$) to approximately 85% for strong jump ($F_r > 9$). However the hydraulic jump occur over a considerable Channel length, in the Froude number range of practical interest the length of the jump is observed to be 4 to 7 times the downstream (sequent depth) y_2 . Over this length the channel will be under a risk of degradation and collapse if it constructed without protections consideration.

Experimental studies indicate that the hydraulic jumps can be considered in five categories as shown in table (13-4) depending on the value of incoming Froude number (F_r). The section shows in table were limit to wide rectangular horizontal channel, so the hydraulic jump in non-rectangular and sloped channels behave similarly, but the flow characteristics and thus the relations for depth ratio, head losses, jump length and dissipation ratio are different.

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FLUID MECHANICS

TABLE 13-4
Classification of hydraulic jumps

Source: U.S. Bureau of Reclamation (1955).

Upstream Fr_1	Depth Ratio y_2/y_1	Fraction of Energy Dissipation	Description	Surface Profile
<1	1	0	<i>Impossible jump.</i> Would violate the second law of thermodynamics.	
1-1.7	1-2	<5%	<i>Undular jump</i> (or <i>standing wave</i>). Small rise in surface level. Low energy dissipation. Surface rollers develop near $Fr = 1.7$.	
1.7-2.5	2-3.1	5-15%	<i>Weak jump.</i> Surface rising smoothly, with small rollers. Low energy dissipation.	
2.5-4.5	3.1-5.9	15-45%	<i>Oscillating jump.</i> Pulsations caused by entering jets at the bottom generate large waves that can travel for miles and damage earth banks. Should be avoided in the design of stilling basins.	
4.5-9	5.9-12	45-70%	<i>Steady jump.</i> Stable, well-balanced, and insensitive to downstream conditions. Intense eddy motion and high level of energy dissipation within the jump. Recommended range for design.	
>9	>12	70-85%	<i>Strong jump.</i> Rough and intermittent. Very effective energy dissipation, but may be uneconomical compared to other designs.	

compare with EQ. 11

المعادلات التي تهـا بالتفصـل الـهـيدرولـيـكـيـه في المـعـاـصـعـ المـسـطـيلـيهـ:-

المعادله العامه لحساب النسبة بين العمقين y_1 و y_2 هي
حالات :-

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 F_r^2} - 1 \right] \quad \dots \quad (1)$$

and, head loss can be determined as :-

$$\frac{h_L}{y_1} = \frac{y_1}{4y_2} \left[\frac{y_2}{y_1} - 1 \right]^3 \quad \dots \quad (2)$$

Ex16 in the flow through asluic in a large reservoir the Velocity is (5.33 m/sec) while the depth is (0.056 m), determine the downstream conditions if a hydraulic jump takes place downstream?

Soln.

$$F_r = \frac{V}{\sqrt{gy_1}} = \frac{5.33}{\sqrt{9.81 \times 0.056}} = 7.172$$

$$\therefore \frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 F_r^2} - 1 \right]$$

$$\frac{y_2}{0.056} = \frac{1}{2} \left[\sqrt{1 + 8 \times (7.172)^2} - 1 \right] \Rightarrow y_2 = 0.544 \text{ m}$$

$$\frac{h_L}{y_1} = \frac{y_1}{4y_2} \left[\frac{y_2}{y_1} - 1 \right]^3$$

$$\frac{h_L}{0.0563} = \frac{0.0563}{4 \times 0.544} \left[\frac{0.544}{0.0563} - 1 \right]^3 \Rightarrow h_L = 0.945 \text{ m}$$

و لا يجد السبب المسوبي للخسارة بسبب حدوث الفجوة فعلى

ما يلي :

$$E_1 = \frac{V_1^2}{2g} + y_1 = \frac{(5.33)^2}{2g} + 0.0563 = 1.504 \text{ m}$$

$$\% \text{ dissipation} = \frac{h_L}{E_1} \times 100 = \frac{0.945}{1.504} \times 100 = 63\%$$

about 63% of mechanical energy is dissipated by hyd. jump.

Ex 17 A rectangular channel of 5m width discharge water at the rate of $(1.5 \text{ m}^3/\text{sec})$ into a (5m) wide with $(1/3000)$ slope at velocity of (5 m/sec) , Determine the height of hyd. jump and energy loss?

$$\underline{\text{Soln.}} \quad Q = V_1 A_2$$

$$\therefore 1.5 = 5 \times (5 \times y_1) \Rightarrow y_1 = 0.06 \text{ m}$$

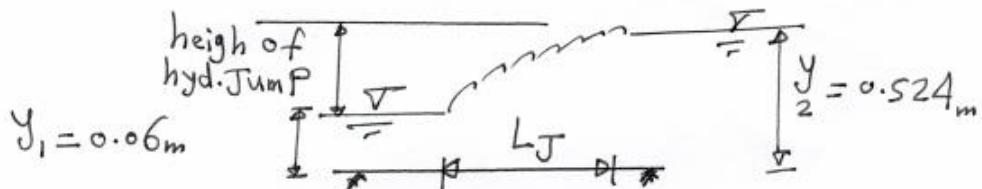
$$F_r = \frac{V_1}{\sqrt{g y_1}} = \frac{5}{\sqrt{9.81 \times 0.06}} = 6.52$$

- ↓
- ① Hence flow is supercritical
 - ② Hence hyd. Jump is possible.

$$\frac{y_2}{y_1} = \frac{1}{2} \left[\sqrt{1 + 8 F_r^2} - 1 \right]$$

$$\frac{y_2}{0.06} = \frac{1}{2} \left[\sqrt{1 + 8(6.52)^2} - 1 \right] \Rightarrow y_2 = 0.524 \text{ m}$$

$$\therefore \text{Height of hyd. Jump} = y_2 - y_1 = 0.524 - 0.06 = 0.464 \text{ m}$$



$$\frac{h_L}{y_1} = \frac{y_1}{4y_2} \left[\frac{y_2}{y_1} - 1 \right]^3$$

$$\frac{h_L}{0.06} = \frac{0.06}{4 \times 0.524} \left[\frac{0.524}{0.06} - 1 \right]^3 \Rightarrow h_L = 0.794 \text{ m}$$

$$\text{and, } L_J = 6.9(y_2 - y_1) = 6.9(0.524 - 0.06) = 3.2 \text{ m}$$

hydraulic Jump in non-rectangular channel 42

لديك ادعيه ولابد للتفريغه ما يعاد تعداد الخسائر سلسله لاستوان
تبه المخزنه نطبق المنهج الوابي هلمونق:

Ex18 Find the sequent depth corresponding to
depth of (0.5m) in (3m) wide trapezoidal
channel ($Z=1.5$) at discharge of $20 \text{ m}^3/\text{sec}$?

$$\text{Soln. } \frac{Z y_1}{B} = \frac{1.5 \times 0.5}{3} = 0.25$$

$$\frac{Z^{3/2}}{\sqrt{g}} \frac{Q}{B^{5/2}} = \frac{(1.5)^{3/2} (20)}{\sqrt{9.81} (3)^{5/2}} = 0.75$$

$$\therefore \text{from diagram} \Rightarrow \frac{Z h_L}{B} = 1.8 \Rightarrow h_L = 3.6 \text{ m}$$

$$\text{and, from } \frac{Z h_L}{B} = 1.8 \text{ and } \frac{Z^{3/2}}{\sqrt{g}} \frac{Q}{B^{5/2}} = 0.75 \text{ find:..}$$

$$\frac{Z y_2}{B} = 1.34 \Rightarrow y_2 = 2.68 \text{ m}$$

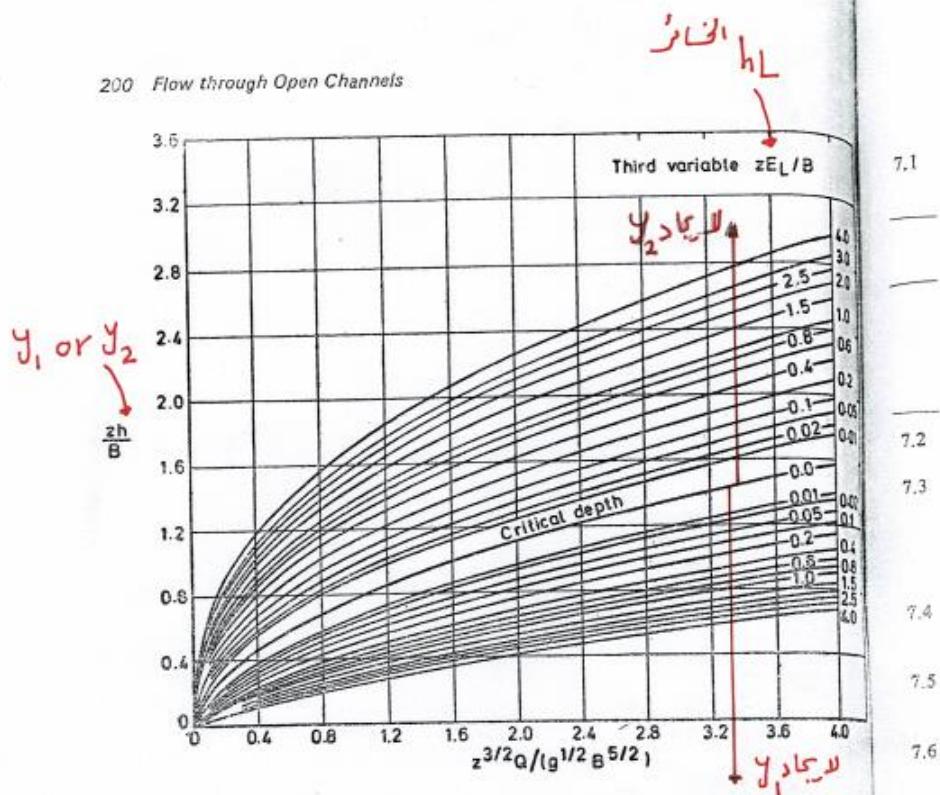


Fig. 7.29 Pre-jump and post-jump depths in trapezoidal channels.

Solution

$$zh_1/B = \frac{1.5 \times 0.5}{3.0} = 0.25$$

$$z^{3/2}Q/g^{1/2}B^{5/2} = \frac{1.5^{3/2} \times 20}{(9.8)^{1/2} 3^{5/2}} = 0.75$$

The corresponding value of $zE_L/B = 1.8$

$$\therefore E_L = \frac{1.8 \times 3}{1.5} = 3.6 \text{ m}$$

Corresponding to $zE_L/B = 1.8$ and $z^{3/2}Q/g^{1/2}B^{5/2} = 0.75$

$$zh_2/B = 1.34$$

$$\therefore h_2 = \frac{3 \times 1.34}{1.5} = 2.68 \text{ m}$$

Solving Eq. (15.35) for c after discarding terms with $(\Delta y)^2$, assuming an infinitesimally small wave, yields the

$$c = \sqrt{gy} \quad (15.36)$$

It has thus been shown that the speed of a small solitary wave is equal to the square root of the product of the depth and g .

15.6 Hydraulic Jump

Occurrence of the Hydraulic Jump

An interesting and important case of rapidly varied flow is the hydraulic jump. A *hydraulic jump* occurs when the flow is supercritical in an upstream section of a channel and is then forced to become subcritical in a downstream section (the change in depth can be forced by a sill in the downstream part of the channel or just by the prevailing depth in the stream further downstream), resulting in an abrupt increase in depth and considerable energy loss. Hydraulic jumps (Fig. 15.23) are often considered in the design of open channels and spillways of dams. If a channel is designed to carry water at supercritical velocities, the designer must be certain that the flow will not become subcritical prematurely. If it did, overtopping of the channel walls would undoubtedly occur, with consequent failure of the structure. Because the energy loss in the hydraulic jump is initially not known, the energy equation is not a suitable tool for analysis of the velocity-depth relationships. Because there is a significant difference in hydrostatic head on both sides of the equation causing opposing pressure forces, the momentum equation can be applied to the problem, as developed in the following sections.

Derivation of Depth Relationships in Hydraulic Jumps

Consider flow as shown in Fig. 15.23. Here, it is assumed that uniform flow occurs both upstream and downstream of the jump and that the resistance of the channel bottom over the relatively short distance L is negligible. The derivation is for a horizontal channel, but experiments show that the results of the derivation will apply to all channels of moderate slope ($S_0 < 0.02$). The derivation is started by applying the momentum equation in the x direction to the control volume shown in Fig. 15.24.

$$\sum F_x = m_x V_2 - m_x V_1$$

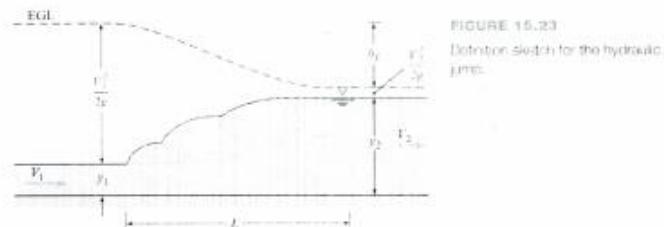


FIGURE 15.23
Diagram sketch for the hydraulic jump.

or y_1/y_2 (both terms are in common use) to each other, in contrast to the alternate depths obtained from the energy equation. Numerous experiments show that the relation represented by Eqs. (15.41) and (15.42) is valid over a wide range of Froude numbers.

Although no theory has been developed to predict the length of a hydraulic jump, experiments [see Chow (5)] show that the relative length of the jump, L/y_2 , is approximately 6 for Fr_1 ranging from 4 to 18.

Head Loss in a Hydraulic Jump

In addition to determining the geometric characteristics of the hydraulic jump, it is often desirable to determine the head loss produced by it. This is obtained by comparing the specific energy before the jump to that after the jump, the head loss being the difference between the two specific energies. It can be shown that the head loss for a jump in a rectangular channel is

$$h_L = \frac{(y_1 - y_2)^2}{4y_1 y_2} \quad (15.45)$$

For more information on the hydraulic jump, see Chow (5). The following example shows that Eq. (15.43) yields a magnitude that equals the difference between the specific energies at the two ends of the hydraulic jump.

EXAMPLE 15.11

Calculating Head Loss in a Hydraulic Jump

Problem Statement

Water flows in a rectangular channel at a depth of 30 cm with a velocity of 16 m/s, as shown in the following sketch. If a downstream sill (not shown) forces a hydraulic jump, what will be the depth and velocity downstream of the jump? What head loss is produced by the jump?



Define the Situation

A hydraulic jump is occurring in a rectangular channel.

State the Goal

- Calculate downstream depth and velocity.
- Calculate head loss produced by the jump.

Develop the Plan

1. To calculate h_L using Eq. (15.43), calculate y_2 from the depth ratio equation (Eq. 15.42). This requires Fr_1 .
2. Check validity of head loss by comparing to $E_1 - E_2$.

Take Action (Execute the Plan)

1. Calculate Fr_1 , y_2 , V_2 , and h_L from Eqs. (Eq. 15.42) and (15.43):

$$Fr_1 = \frac{V}{\sqrt{g y_1}} = \frac{16}{\sqrt{9.81(0.30)}} = 9.33$$

$$y_2 = \frac{0.30}{2} \left[\sqrt{1 + 8(9.33)^2} - 1 \right] = 3.81 \text{ m}$$

$$V_2 = \frac{q}{y_2} = \frac{(16 \text{ m/s})(0.30 \text{ m})}{3.81 \text{ m}} = 1.26 \text{ m/s}$$

$$h_L = \frac{(3.81 - 0.30)^2}{4(0.30)(3.81)} = 9.46 \text{ m}$$

2. Compare the head loss to $E_1 - E_2$:

$$h_L = \left(0.30 + \frac{16^2}{2 \times 9.81} \right) - \left(3.81 + \frac{1.26^2}{2 \times 9.81} \right) = 9.46 \text{ m}$$

The value is the same, so validity of h_L equation is verified.

Use of Hydraulic Jump on Downstream End of Dam Spillway

Previously it was shown that the transition from supercritical to subcritical flow produces a hydraulic jump and that the relative height of the jump (y_2/y_1) is a function of Fr_1 . Because

hydraulic Jump in non-rectangular channel 42

لديك ادعيه ولابد للتفريغه ما يعاد تعداد الخسائر سلسله لاستوان
تبه المخزنه نطبق المنهج الوابي هلمونق:

Ex18 Find the sequent depth corresponding to
depth of (0.5m) in (3m) wide trapezoidal
channel ($Z=1.5$) at discharge of $20 \text{ m}^3/\text{sec}$?

$$\text{Soln. } \frac{Z y_1}{B} = \frac{1.5 \times 0.5}{3} = 0.25$$

$$\frac{Z^{3/2}}{\sqrt{g}} \frac{Q}{B^{5/2}} = \frac{(1.5)^{3/2} (20)}{\sqrt{9.81} (3)^{5/2}} = 0.75$$

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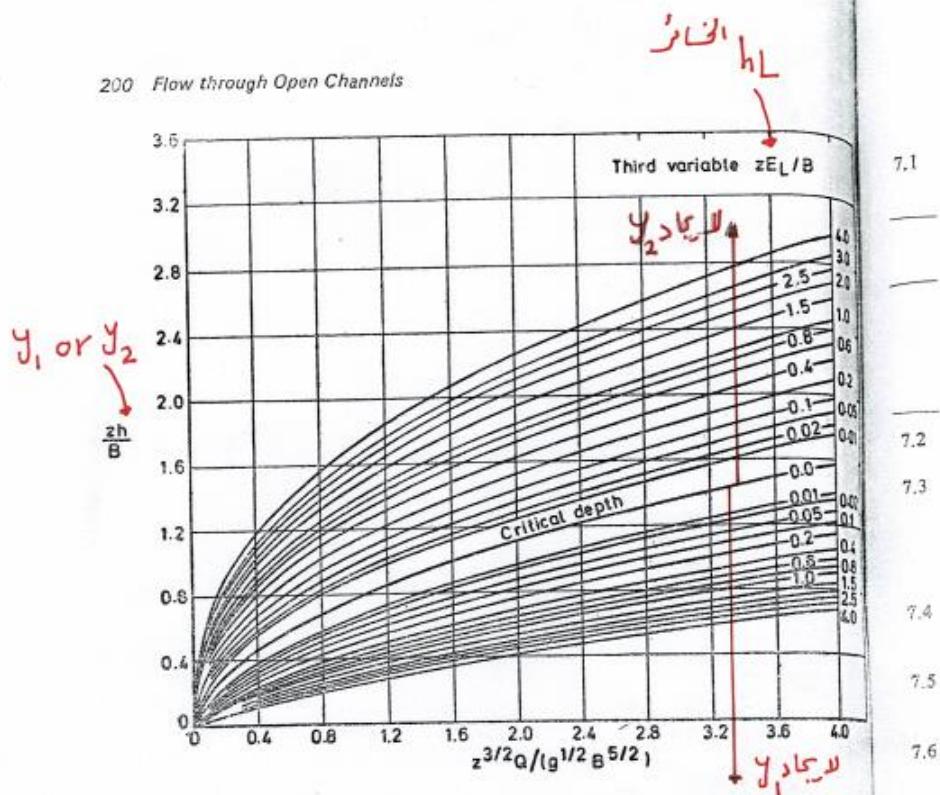


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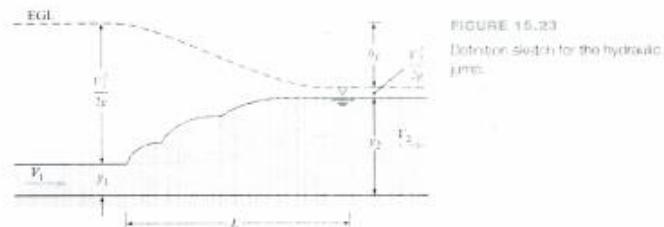


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Calculating Head Loss in a Hydraulic Jump

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Water flows in a rectangular channel at a depth of 30 cm with a velocity of 16 m/s, as shown in the following sketch. If a downstream sill (not shown) forces a hydraulic jump, what will be the depth and velocity downstream of the jump? What head loss is produced by the jump?



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Use of Hydraulic Jump on Downstream End of Dam Spillway

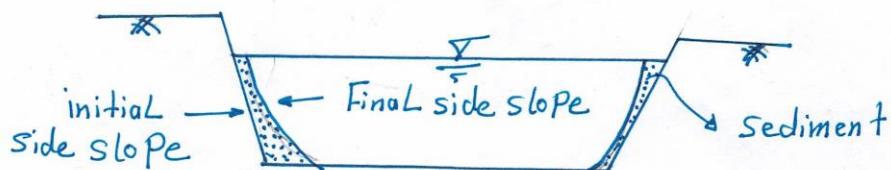
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Alluvial Channels

القنوات اللسوبيه

47

- An alluvial channel was defined as transporting water as well as sediment.
- The shape, Longitudinal slope and cross-sectional dimensions of such a stable channel depend on discharge, size of sediment and sediment load.
- The two methods commonly for design of alluvial channels are :
 - (a) Kennedy's Equation
 - (b) Lacey's Equation.



(a) Kennedy's Equation

Kennedy analysed data from stable canals of the upper Bari Doab system and found that non-scouring and non-silting Velocity (V_o) (initial velocity) is:

$$V_o = 0.55 y^{0.64} \quad \text{--- (1)}$$

السرعة ابتداء
 تكون المتر ميا

Kennedy also found that critical velocity ratio (m)⁴⁸
as: $V = 0.55 m y^{0.64}$ --- ②

or, $m = \text{critical Velocity ratio} = \frac{V}{V_0}$

$m > 1$ for sands coarser] مسحوق من كثيف

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الرسبات

and, $V = \text{Velocity of flow of alluvial channel}$ can
be combined with flow equation of
manning equation:

$$V = 0.55 m y^{0.64} = \frac{1}{n} R^{2/3} S^{1/2} \quad \text{--- ③}$$

سرعه اجراءه المائي بعد تكون
الرسبات

- The commonly ranges for n (0.02 - 0.025) and
 m (0.9 - 1.1).

- The cross-section to assume trapezoidal with
side slope ($Z = 0.5$).

in which $P = \frac{B}{y}$ --- ④

and, $S = \frac{Q^2 n^2 (P+2.236)^{4/3}}{y^{16/3} (P+0.5)^{10/3}} \quad \text{--- ⑤}$

$$y = \left[\frac{1.818 Q}{(P+0.5)m} \right]^{0.378} \quad \text{--- (6)}$$

and, can Eliminate P from following Eq. :

$$\frac{s^{\frac{0.0202}{2}} Q}{n^2 m^{\frac{2.02}{2}}} = 0.299 \frac{(P+2.236)^{1.33}}{(P+0.5)^{1.313}} \quad \text{--- (7)}$$

Hint but not all of these channels will be stable, the resulting solution is compared with the following recommended value of (B/y), if the two values differ significantly, suitable modification in the slope would be necessary.

Q	5	10	15	50	100	200	300
B/y	4.5	5	6.5	9	12	15	18

Ex21 Design a stable canal to carry a discharge of ($10 \text{ m}^3/\text{sec}$) at slope of (2×10^{-4}), $n=0.023$, $m=1$?

Soln. المطلوب من التصميم حساب كثافة y و B على نرخ s
ان المطلوب معرف بعل جابي سنه ($Z=0.5$)

$$\frac{S^2 Q^{0.02}}{n^2 m^2} = 0.299 \frac{(P + 2.236)^{1.33}}{(P + 0.5)^{1.3}}$$

$$\frac{(2 \times 10)^{-4} (10)^{0.02}}{(0.023)^2 (1)^2} = 0.299 \frac{(P + 2.236)^{1.33}}{(P + 0.5)^{1.3}} \Rightarrow P = 7.5$$

$$P = 7.5 = \frac{B}{y}$$

$$y = \left[\frac{1.818 \Phi}{(P + 0.5)m} \right]^{0.378} = \left[\frac{1.818 (10)}{(7.5 + 0.5)(1)} \right]^{0.378} = 1.364_m$$

$$\therefore B = P \cdot y = 7.5 (1.364) = 10.2_m$$

ملاحظة / إن فيه P المحسوب هنا ($P = 7.5$) يجب أن تقارب مع

العينة الصناعية المعروفة في الجدول المرفق مع هذا الموضع.

وبالعوره إلى الجدول السابق نلقي نظرة ملحوظة :

$$\text{when } Q = 10 \quad \xrightarrow[\text{table}]{\text{From}} \frac{B}{y} = 5$$

$\therefore P_{\text{cal.}}(7.5) > 5 \Rightarrow 7.5$ عليه يكن اعتماد العينة
في الحال رقميه يكروه فيه

$$B = 10.2_m \quad \text{and,} \quad y = 1.364_m$$

Alluvial channels
(stable channel)

Kennedy's Eq. المعادلة كينيدي

$$m, n \rightarrow Eq. 7 \rightarrow Eq. 6 \rightarrow Eq. 4$$

$P_{cal.}$ y B

بيانه بالشكل
if $P_{cal.} \geq P_{table}$ \rightarrow تم اعتماد تم B
وذلك الموجب بتنازل
ويتحقق الحال

if $P_{cal.} < P_{table}$ \rightarrow هناك اسلوبين
لكل

ارسلب الزوال في حال

تغير فيه S وبالطريق الائمه:

ادرر : تجسس العادم رم (7) طابعه د ناخدا

$$\frac{S^l \times Q^{0.02}}{\frac{n^2 m^2}{y}} = 0.299 \quad \frac{(P_t + 2.236)^{1.33}}{(P_t + 0.5)^{1.313}} \quad (7)$$

جاتر : م نھضه د لایا : سی العادم رم (5) لایادیه د

$$S^l_{\text{New}} = \frac{Q^2 n^2 (P_t + 2.236)^{4/3}}{y^{16/3} (P_t + 0.5)^{10/3}} \quad (5)$$

جاتر : نصیق العادم رم (4) لایاد

$$P_{\text{table}} = \frac{B}{y} \quad (4)$$

$$\frac{P_{\text{cal.}} < P_{\text{table}}}{}$$

لایادوب ایتی دی مار

Lacey's Eq. صورتیجی

b) Lacey's Equation

The main limitation of Kennedy's equation is that it does not specify a stable width, that any $(B/y)_{cal.}$ is satisfactory long as $(B/y)_{table}$.

i.e if $P_{cal.} < (B/y)_{table}$ then,

Lacey Proposed the following equations for channel design:

$$P = 4.75 \sqrt{Q} \quad \text{--- (1)}$$

$$R = 0.47 \sqrt[3]{Q/f_s} \quad \text{--- (2)}$$

$$S = 3 \times 10^{-4} f_s^{5/3} Q^{-1/6} \quad \text{--- (3)}$$

$$f_s = \text{silt factor} = 1.76 d^{1/2} \quad \text{--- (4)}$$

d = sediment size (mm)

and, Lacey flow equation is:

$$V = 10.8 R^{2/3} S^{1/3} \quad \text{--- (5)}$$

Equation (5) has been found to be applicable to alluvial rivers in floods at their full supply discharge.

Ex 22

52

Design a Lacey channel to carry ($5 \text{ m}^3/\text{sec}$) through (0.5 mm) sand?

Soln.

$$f_s = 1.76 d^{1/2} = 1.76 \sqrt{0.5} = 1.24$$

$$S = \frac{3 \times 10^{-4} f_s^{5/3}}{Q^{1/6}} = \frac{3 \times 10^{-4} \times (1.24)^{5/3}}{(5)^{1/6}} = 3.27 \times 10^{-4}$$

$$R = 0.47 \sqrt[3]{\frac{Q}{f_s}} = 0.47 \sqrt[3]{\frac{5}{1.24}} = 0.746 \text{ m}$$

$$P = 4.75 \sqrt{Q} = 4.75 \sqrt{5} = 10.6 \text{ m}$$

assume $Z = 0.5$

$$\therefore P = B + 2 \sqrt{1 + Z^2} y = B + 2.24 y = 10.6 \quad \textcircled{1}$$

$$A = PR = 10.6 \times 0.746 = 7.9 \text{ m}^2$$

$$A = By + Zy^2 = By + 0.5y^2 = 7.9 \quad \textcircled{2}$$

Solving eq $\textcircled{1}$ and eq $\textcircled{2}$ for y :

$$y = 0.877 \text{ m} \quad B = 8.63 \text{ m}$$

H.W ① Use Kennedy equation to design canal for following:

$$S = 2.5 \times 10^{-4} \quad n = 0.0225 \quad m = 0.9 \quad Q = 30 \text{ m}^3/\text{sec}$$

② Using Lacey's equation design channel given:

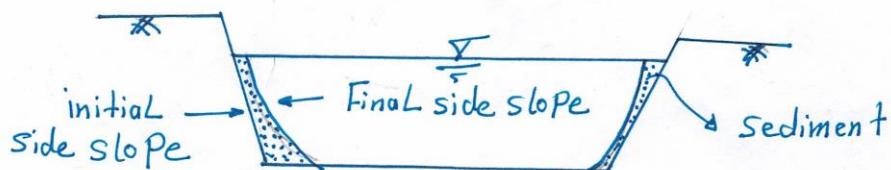
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$\therefore P_{\text{cal.}}(7.5) > 5 \Rightarrow 7.5$ عليه يكن اعتماد العينة
في الحال رقميه يكروه فيه

$$B = 10.2_m \quad \text{and,} \quad y = 1.364_m$$

Alluvial channels
(stable channel)

Kennedy's Eq. المعادلة كينيدي

$$m, n \rightarrow Eq. 7 \rightarrow Eq. 6 \rightarrow Eq. 4$$

$P_{cal.}$ y B

متاردة بالدريل
if $P_{cal.} \geq P_{table}$ \rightarrow تم اعتماد تم
دل المجرى بتاتاً
وينصي على

if $P_{cal.} < P_{table}$ \rightarrow هناك اسلوبين
لكل

$P_{cal.} < P_{table}$ في حال

تغير فيه S وبالطريق الائمه:

ادرر : تجسس العادم رم (7) طابعه د ناخدا

$$\frac{S^l \times Q^{0.02}}{\frac{n^2 m^2}{y}} = 0.299 \quad \frac{(P_t + 2.236)^{1.33}}{(P_t + 0.5)^{1.313}} \quad (7)$$

جاتر : م نھضه د لایتیا : م نھضه د لایتیا

$$S^l_{\text{New}} = \frac{Q^2 n^2 (P_t + 2.236)^{4/3}}{y^{16/3} (P_t + 0.5)^{10/3}} \quad (5)$$

جاتر : نطبق العادم رم (4) لایجاد B

$$P_{\text{table}} = \frac{B}{y} \quad (4)$$

$$\frac{P_{\text{cal.}} < P_{\text{table}}}{}$$

لایکوب ایتی دی مار

Lacey's Eq. صورتیق

b) Lacey's Equation

The main limitation of Kennedy's equation is that it does not specify a stable width, that any $(B/y)_{cal.}$ is satisfactory long as $(B/y)_{table}$.

i.e if $P_{cal.} < (B/y)_{table}$ then,

Lacey Proposed the following equations for channel design:

$$P = 4.75 \sqrt{Q} \quad \text{--- (1)}$$

$$R = 0.47 \sqrt[3]{Q/f_s} \quad \text{--- (2)}$$

$$S = 3 \times 10^{-4} f_s^{5/3} Q^{-1/6} \quad \text{--- (3)}$$

$$f_s = \text{silt factor} = 1.76 d^{1/2} \quad \text{--- (4)}$$

d = sediment size (mm)

and, Lacey flow equation is:

$$V = 10.8 R^{2/3} S^{1/3} \quad \text{--- (5)}$$

Equation (5) has been found to be applicable to alluvial rivers in floods at their full supply discharge.

Ex 22

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Design a Lacey channel to carry ($5 \text{ m}^3/\text{sec}$) through (0.5 mm) sand?

Soln.

$$f_s = 1.76 d^{1/2} = 1.76 \sqrt{0.5} = 1.24$$

$$S = \frac{3 \times 10^{-4} f_s^{5/3}}{Q^{1/6}} = \frac{3 \times 10^{-4} \times (1.24)^{5/3}}{(5)^{1/6}} = 3.27 \times 10^{-4}$$

$$R = 0.47 \sqrt[3]{\frac{Q}{f_s}} = 0.47 \sqrt[3]{\frac{5}{1.24}} = 0.746 \text{ m}$$

$$P = 4.75 \sqrt{Q} = 4.75 \sqrt{5} = 10.6 \text{ m}$$

assume $Z = 0.5$

$$\therefore P = B + 2 \sqrt{1 + Z^2} y = B + 2.24 y = 10.6 \quad \textcircled{1}$$

$$A = PR = 10.6 \times 0.746 = 7.9 \text{ m}^2$$

$$A = By + Zy^2 = By + 0.5y^2 = 7.9 \quad \textcircled{2}$$

Solving eq $\textcircled{1}$ and eq $\textcircled{2}$ for y :

$$y = 0.877 \text{ m} \quad B = 8.63 \text{ m}$$

H.W ① Use Kennedy equation to design canal for following:

$$S = 2.5 \times 10^{-4} \quad n = 0.0225 \quad m = 0.9 \quad Q = 30 \text{ m}^3/\text{sec}$$

② Using Lacey's equation design channel given:

$$f_s = 1 \quad Q = 4.5 \text{ m}^3/\text{sec}$$