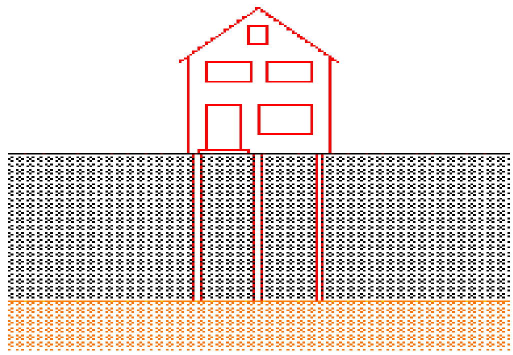
Soil Mechanic

Introduction:

Soil problems in civil engineering

Soil → to support loads from the foundation of buildings in embankments.
→ As a construction materials : Fill materials: dams , highways.



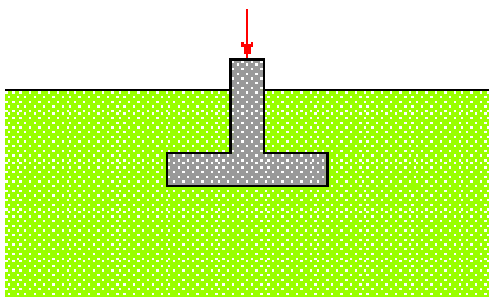
Criteria of Foundation Design:-

- bearing Capacity (B.C)
- Settlement
- Overall Stability

B.C:- actual foundation Pressure or contact pressure = $\frac{\text{Load}}{\text{footing Area}}$ > B.C ↔ B.C failure

Hence, Actual foundation Pressure ≥ B.C

F.S against B.C failure = $\frac{\text{Unit B.C}}{\text{Contact pressure}}$ (< 2-3)



Settlement: computed settlement or actual settlement< Acceptable values that depend on:

- Structure
- Soil

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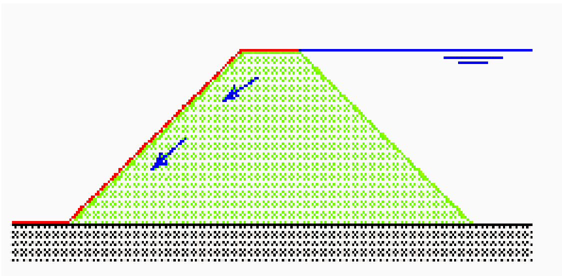
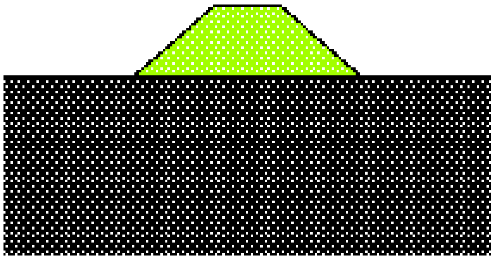
- Foundation

Soil as a construction material

1 Earth Dams



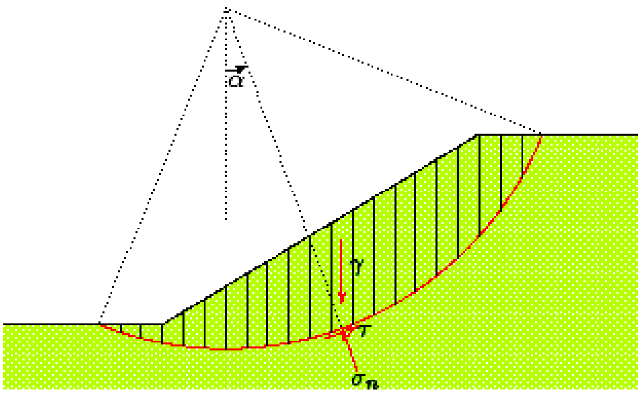


2 Highways

3 Slopes and Excavations

T driving force

if T > resisting force ↔ slope failure

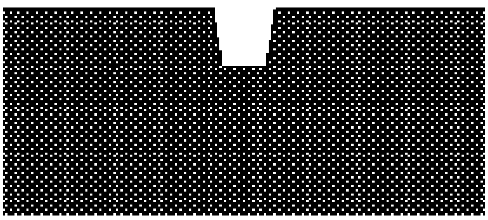


4 Excavations

For deep Excavation:

Lateral pressure ↔ Side failure

Hence bracing is to be used



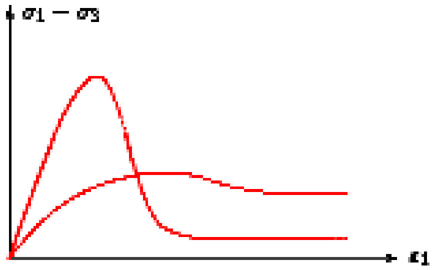
The solution of soil engineering problems:

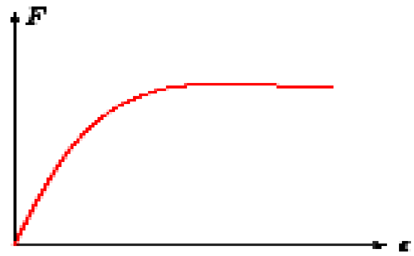
1 Soil Mech.: stress-strain properties analysis of soil mass
2 Geology Experience
3 Economic
4 Experience

+ Eng. Judgment → Solution to soil Eng. problems

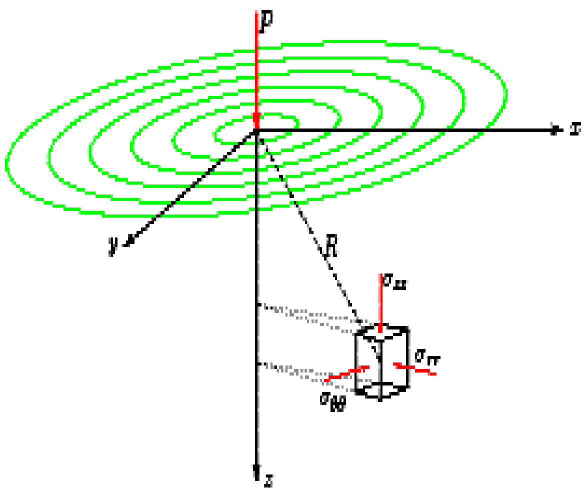
Soil problems are statically indeterminate, since:

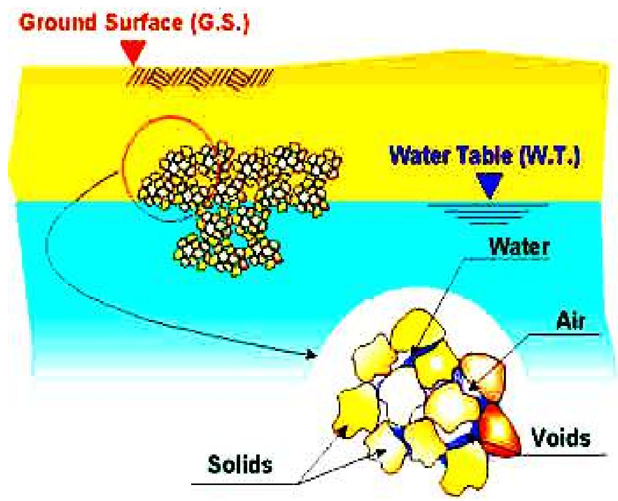
1-Stress-strain relationship of soils is not linear.





2-Soil behavior depends on pressure, time and environments(بيئة)





3-Soils are not homogeneous(غير متجانس)

4-Soil mass can not be seen entirely and its properties evaluated on the basis of small samples.

5-Most soils are very sensitive(حساس) to disturbance (تشوش)and the behavior measured by a laboratory test may be unlike that of the in situ soil.

Definitions:

Soil mechanic: is application of lows of mechanics and hydraulics to engineering problems relating to soils.

Geotechnology : is overall study of geology ,soil mechanics and rock mechanics

Geotechnical Engineering: is the application of civil engineering technology to some aspects of the earth.

Soil Formation:

Soil

Geological origin (By Nature) ← Soil → Fill (Man made)

Geological origin : weathering process → Disintegration(تحلل) of Rock → Soil formation

Type of Geological origin

Mechanical weathering of Rocks (physical forces)
↓
Impact of Sand grains carried by High winds or water or glaciers
↓
Erosion (تعرية)of rock surface
↓
Silt, Sand , Gravel

Chemical weathering of Rocks (due to water or alkaline (قلوي)or O_2)
↓
Clay size particles

Definitions

Residual Soils(التربة المتبقية): is the products of weathering remain at their original location.

Transported Soils(التربة المنقولة): is the products of weathering transported by water (alluvium soilsترسبات نهرية), wind (Aeolian soils"غبار ناعم جدا), glaciers (glacial soils ترسبات جليدية) or gravity.

Marine Soils(التربة البحرية) ; Soils formed by deposition in the sea.

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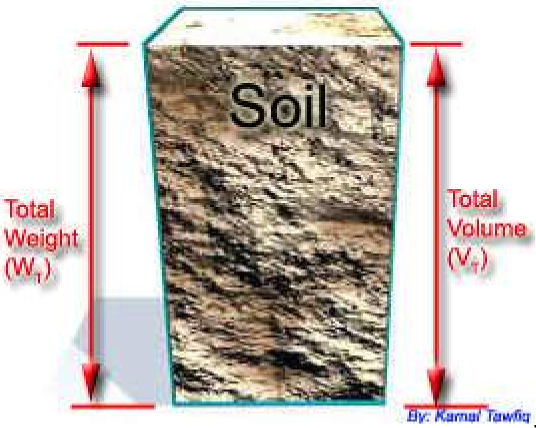
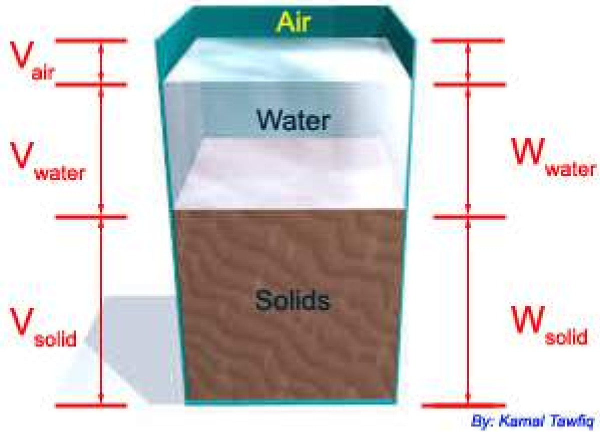
The physical state of a soil sample





Total Volume = $V_T = V_s + V_w + V_a$
Total Weight = $W_T = W_s + W_w$

Porosity (n): is the ratio of void volume.

$$\mathbf{n = V_v/V_T}$$

Void Ratio (e):is the ratio of void volume to solid volume. $\mathbf{e = V_v/V_s}$

now $\mathbf{n = V_v/V_T = V_v/ V_v + V_s} = \dfrac{V_v/V_s}{V_v/V_s + 1} = \dfrac{e}{e+1}$

Note also that:

$$\mathbf{e = n / (1 - n)}$$
$$\mathbf{v = 1 / (1 - n)}$$

Note:

- ***n < 1 and is expressed as %***
- ***e may be any value greater or smaller than unity.***

Example: A soil has a total volume of 250ml and a void ratio of 0.872. What is the **volume of solids** of the sample?

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s}$$

$$250 - V_s = 0.872 V_s$$

$$250 = 1.872\ V_s$$

$$V_s = 133.55\ \text{ml}$$

Example: A soil has a porosity of 0.45. What is the value of its **void ratio**?

$$n = \frac{e}{1+e}$$

$$e = n(1+e) = n + ne \qquad \{e - ne = n\}$$

$$e(1 - 0.45) = 0.45$$

$$e = \underline{\underline{0.818}}$$

Degree of saturation (s):is the ratio of water volume to void volume.

$$S_r = V_w/V_v$$

if ***S = 0*** ***dry soil (Vw = 0)***

S=100 ***saturated soil (V_w =V_v)***

0 < S < 1→the soil is partially sat.

Water Content (w):is the ratio of water weight in a soil sample to the solids weight.

$$w_c = W_w/ W_s$$

Specific gravity (GS): specific gravity of *soil solids* of a soil is defined as the ratio of the density of a given volume of the solids to the density of any equal volume of water at 4ºC.

$$G_S = \frac{\text{mass of a soil grain}}{\text{mass of an equal volume of water}}$$

$$G_s = \frac{\gamma_s}{\rho_w} = \frac{M_s / V_s}{\rho_w} = \frac{M_s}{V_s\ \rho_w}$$

Soil type	G
Gravel	2.65-2.68
Sand	2.65-2.68

Silt	2.66-2.7
Clay	2.68-2.8
Organic soils	may be less than 2.0

Example: A sample of oven dried soil had a mass of 306g. The soil was broken down and placed into a jar of internal volume 1000 ml. Water at 20C was then poured into the jar and a rubber stopper placed to seal the jar. The soil and water were thoroughly mixed using an end over end shaker until all air had been removed. The jar was then topped up with water to the 1000ml mark and the total mass of the jar and its contents was found to be 1440.5g. The mass of the empty jar was 250g.

Determine the **particle specific gravity** of the soil.

Mass of soil + water = 1440.5 - 250 = 1190.5g
Mass of dry soil = 306g
Mass of water present with soil = 1190.5 - 306 = 884.5g

$$\text{Mass of water present without soil} = \rho_w \text{ x vol} = \underset{(g/m^3)}{1 \text{ x}10^6} \text{ x } \underset{(m^3)}{1000 \text{ x}10^{-6}} = 1000\text{g}$$

$\Rightarrow$ Mass of water of same volume as soil = 1000 - 884.5 = $\underline{\underline{115.5\text{g}}}$

$$G_s = \frac{\text{Mass of soil}}{\text{Mass of water of same volume as soil}}$$

$$= \frac{306}{115.5}$$

$$G_s = \underline{\underline{2.65}}$$

or...

$$G_s = \frac{\text{Mass of dry soil}}{\{(\text{jar+ water}) - (\text{jar+ water+ soil})\} + \text{dry soil}}$$

$$(\text{jar} + \text{water}) = 1000 + 250 = 1250\text{g}$$

$$\Rightarrow G_s = \frac{306}{(1250 - 1440.5) + 306}$$

$$\Rightarrow G_s = \underline{\underline{2.65}}$$

Air Content (A):is the ratio of air volume to total volume.

The **air-voids volume, V_a** , is that part of the void space not occupied by water(is the ratio of air volume to total volume).

$$\boldsymbol{A_v = V_a / V}$$
$$\boldsymbol{V_a = V_v - V_w}$$
$$\boldsymbol{= e - e.S_r}$$
$$\boldsymbol{= e.(1 - S_r)}$$

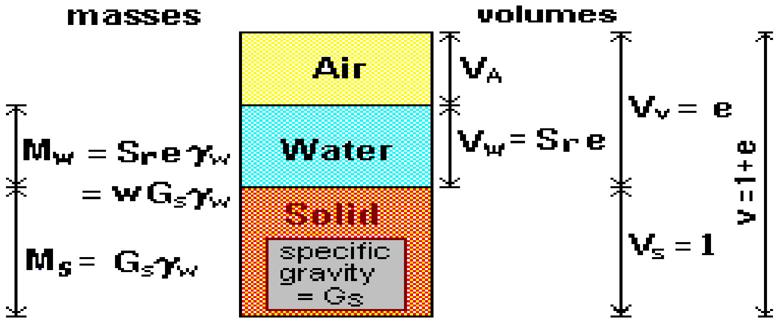
Air-voids content, A_v

$$A_v = (\text{air-voids volume}) / (\text{total volume})$$
$$\boldsymbol{= Va / V}$$
$$\boldsymbol{= e.(1 - S_r) / (1+e)}$$
$$\boldsymbol{= n.(1 - S_r)}$$

For a perfectly *dry* soil: $A_v = n$
For a *saturated* soil: $A_v = 0$

e in term of V, Ws, Gs

$$e=\frac{V_v}{V_s}=\frac{V-V_s}{V_s}=\frac{V}{V_s}-1=\frac{V}{\frac{W_s}{G_s\gamma_w}}-1=\frac{G_s\gamma_w V}{W_s}-1$$



<u>Bulk (Total)density (ρt)and</u> Dry density (ρ_d):

$$\text{Dry density, } \rho_d = \frac{\text{Mass of solids}}{\text{Total volume}} = \frac{M_S}{V} = \frac{G_s\rho_w}{1+e}$$

$$\text{Bulk density, } \rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{M_S + M_w}{V} = \frac{G_s\rho_w + S_r e\rho_w}{1+e}$$

<u>Bulk (Total)unit weight (γt)and</u> Dry Unit weight (γ_d):

$$\text{Dry unit weight, } \gamma_d = \frac{\text{Dry weight}}{\text{Total volume}} = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e} = 9.81\ \rho_d$$

$$\text{Unit weight, } \gamma = \frac{\text{Total weight}}{\text{Total volume}} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + S_r e\gamma_w}{1+e} = 9.81\ \rho$$

Saturated Unit weight (γ_s):for sat .soil S= 100% =1

$$\gamma_s=\frac{e+G_s}{1+e}\gamma_w$$

submerged unit weight γ' (or **buoyant unit**

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s+Se}{1+e}\gamma_w - \gamma_w = \frac{G_s-1-e(1-S)}{1+e}\gamma_w \text{ (for \textbf{partially saturated})} \quad (2.11)$$

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s+e}{1+e}\gamma_w - \gamma_w = \frac{G_s-1}{1+e}\gamma_w \text{ (for \textbf{fully saturated})} \quad (2.12)$$

Ex1:

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For a given soil, $w = 25\%$ and $\gamma_t = 18.5$ kN/m³ are measured. Determine void ratio e and degree of saturation S. Assume that G_s is 2.70.

Solution (a):

First assume $W_s = 100$ kN as shown in Figure 2.7a. Then, $W_w = 100 \times 0.25 = 25$ kN.

Calculate $V_s = W_s/G_s\gamma_w = 100/(2.7 \times 9.81) = 3.775$ m³.

Calculate $V_w = W_w/\gamma_w = 25/9.81 = 2.548$ m³.

Since $\gamma_t = 18.5$ kN/m³ $= (W_s + W_a)/(V_s + V_w + V_a) = (100 + 25)/(3.775 + 2.548 + V_a)$, thus, $V_a = 0.434$ m³.

Now, all components in the three phases are obtained as shown in Figure 2.7a and,

$e = (V_w + V_a)/V_s = (2.548 + 0.434)/3.775 = \mathbf{0.790} \leftarrow$

$S = V_w/(V_w + V_a) = 2.548/(2.548 + 0.434) = 0.854 = \mathbf{85.4\%}. \leftarrow$

First assume $V = 10$ m³ as seen in Figure 2.7b.

From $W_s + W_w = W_s + wW_s = (1 + w)W_s = V\gamma_t = 10 \times 18.5 = 1\ 85$ kN,

$W_s = 185/(1 + 0.25) = 148$ kN, and $W_w = 185 - 148 = 37$ kN.

Using Gs as a bridge value, $V_s = W_s/G_s\gamma_w = 148/(2.7 \times 9.81) = 5.588$ m³.

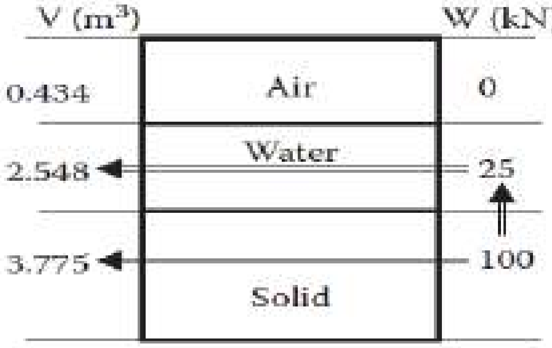
Using γ_w as a bridge value, $V_w = W_w/\gamma_w = 37/9.81 = 3.772$ m³.

Thus $V_a = V - (V_s + V_w) = 10 - (5.588 - 3.772) = 0.641$ m³.

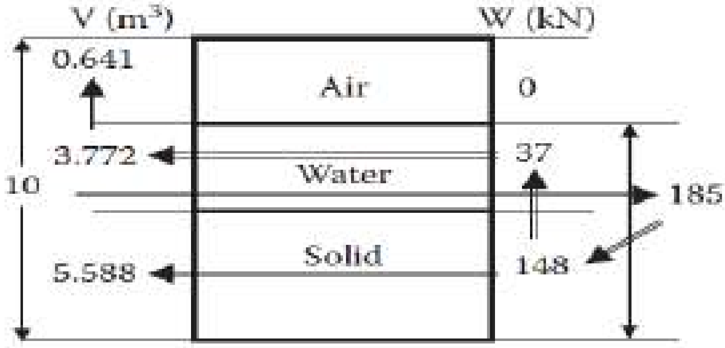
Now, all components in the three phase are obtained as shown in Figure 2.7b and,

$e = (V_w + V_a)/V_s = (3.772 + 0.641)/5.588 = \mathbf{0.789}. \leftarrow$

$S = V_w/(V_w + V_a) = 3.772/(3.772 + 0.641) = 0.855 = \mathbf{85.5\%}. \leftarrow$



Solution (a)



Solution (b)

EX2.

In a fill section of a construction site, 1500 m^3 of moist compacted soils is required. The design water content of the fill is 15%, and the design unit weight of the compacted soil is 18.5 kN/m^3. Necessary soil is brought from a borrow site, with the soil having 12% natural water content, 17.5 kN/m^3 wet unit weight of the soil, and $G_s = 2.65$. How much (in cubic meters) of the borrow material is required to fill the construction fill section? And how heavy is it?

Solution:

Draw three-phase diagrams of the fill site and the borrow site in Figure 2.8a and b, respectively.

First for the fill site in Figure 2.8a, $V = 1500\ m^3$ so that $W_s + W_w = V\gamma_t = 1500 \times 18.5 = 27750$ kN.

$$W_s + W_w = (1 + w)W_s = 27750 \text{ kN, so that } W_s = 27750/(1 + 0.15) = 24130 \text{ kN.}$$

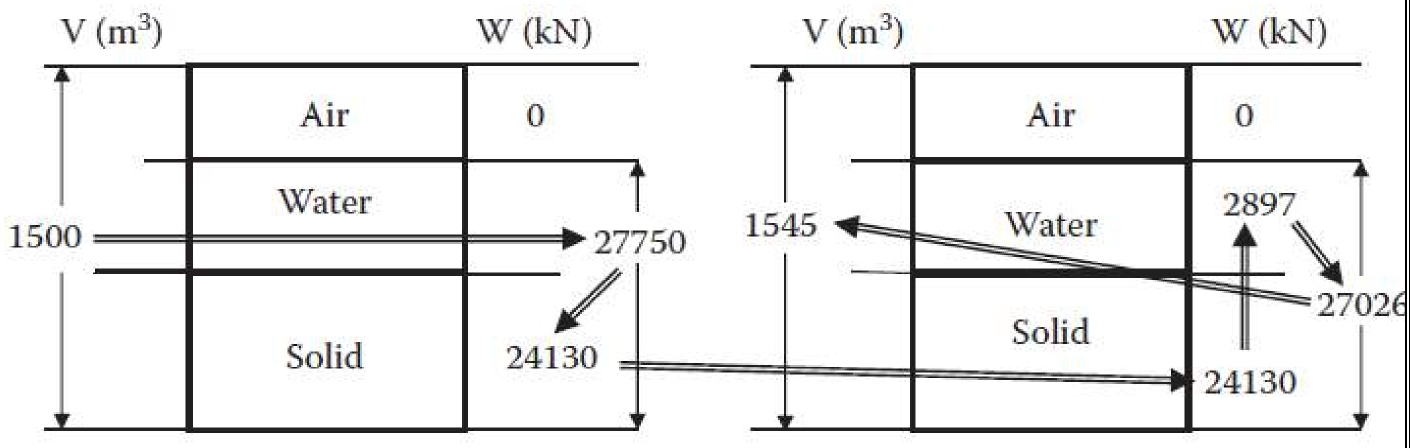
This much solid weight of the soil is required at the fill site.

At the borrow site, the same solid weight 24130 kN is needed as shown in Figure 2.8b.

Thus, $W_w = wW_s = 0.12 \times 24130 = 2897$ kN, and $W_s + W_w = 24130 + 2897 =$ **27026 kN**. ←

Since $\gamma_t = (W_s + W_w)/V = 17.5\ kN/m^3$, $V = 27026/17.5 =$ **1545 m^3**. ←

Thus, 1545 m^3 of the borrow material is needed for the project carrying a total weight of 27026 kN.



(a) Fill site (b) Borrow site

Problem 3.1 A soil sample was collected for laboratory examination. It has a wet mass of 5.2 kg, bulk density of 1.65 g cm^{-3}, dry density of 1223 kg m^{-3} and degree of saturation of 82%. Determine the density of solids.

Solution

There are different ways to solve this problem; we will use the definitions of soil constituents. We will first find the mass of solids (i.e., the mass of dry soil), then the volume of solids and finally its density.

Volume of soil sample, $V = \frac{M}{\rho} = \frac{5.2}{1650} = 0.00315\,\mathrm{m}^3$

Mass of dry sample, $M_d = \rho_d \cdot V = 1223 \cdot 0.00315 = 3.85\,\mathrm{kg}$

Mass of water, $M_w = M_{soil} - M_d = 5.2 - 3.85 = 1.35\,\mathrm{kg}$

Volume of water, $V_w = \frac{M_w}{\rho_w} = \frac{1.35}{1000} \approx 0.00135\,\mathrm{m}^3$

From $S = \frac{V_w}{V_v} = 0.82$

We will obtain the volume of voids as

$$V_v = \frac{V_w}{S} = \frac{0.00135}{0.82} = 0.00164\,\mathrm{m}^3$$

Then, the volume of solids equals

$$V_s = V - V_v = 0.00315 - 0.00164 \approx 0.0015\,\mathrm{m}^3$$

Therefore, the density of solids is

$$\rho_s = \frac{M_d}{V_s} = \frac{3.85}{0.0015} = 2{,}566\,\mathrm{kg\ m}^{-3}$$

Problem 3.2 Site investigation was performed to study soil conditions at a construction site in a new development area. A cylindrical soil sample (height = 100 mm, diameter = 50 mm) was collected at a depth of 1.5 m below the ground. The following soil characteristics were obtained: soil density was 1.52 t m^{-3}, moisture content was 68.2% and density of solid particles was 2.53 g cm^{-3}. Determine:

a) Weight of solids (in *N*)
b) Volume of air (in m^3).

Solution

Similar to Problem 3.1, there are different ways to solve it; we will use the definitions of soil constituents.

Volume of soil sample, $V = \pi r^2 h = 0.000196\,\text{m}^3$

Unit weight of soil, $\gamma = 1.52 \cdot 9.81 = 14.9\,\text{kN m}^{-3}$

Weight of soil, $W = \gamma \cdot V = 14.9 \cdot 1000 \cdot 0.000196 = 2.92\,\text{N}$

Weight of solids, $W_s = \dfrac{W}{1+w} = \dfrac{2.92}{1+0.682} \approx 1.74\,\text{N}$

Specific gravity, $G_s = \dfrac{\rho_s}{\rho_w} = \dfrac{2.53}{1} = 2.53$

Volume of solids, $V_s = \dfrac{W_s}{\gamma_s} = \dfrac{W_s}{G_s \cdot \gamma_w} = \dfrac{1.74}{2.53 \cdot 9.81 \cdot 1000} \approx 7 \cdot 10^{-5}\,\text{m}^3$

Weight of water, $W_w = W_s \cdot w = 1.74 \cdot 0.682 = 1.19\,\text{N}$

Volume of water, $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{1.19}{9.81 \cdot 1000} \approx 0.00012\,\text{m}^3$

Volume of air, $V_a = V - V_s - V_w \approx 5.2 \cdot 10^{-6}\,\text{m}^3$

Problem 3.3 Soil excavated from a borrow pit is being used to construct an embankment (Fig. 3.3). The soil sample from the borrow pit has a specific gravity of 2.7 and unit weight of 17.8 kN m^{-3}. The weight of the sample was 3.5 N. The sample was then placed in an oven for 24 h at 105°C and its weight reduced to a constant value of 2.9 N.

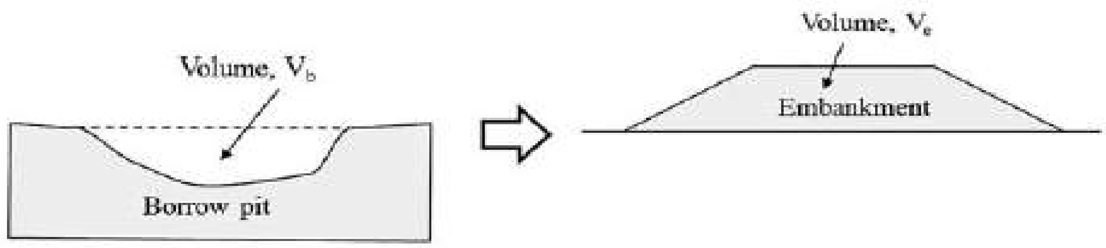


Figure 3.3 Borrow pit and embankment for Problem 3.3.

The soil at the embankment is required to be compacted to a void ratio of 0.71. If the finished volume of the embankment (V_e) is 80,000 m^3, what would be the volume of the soil (V_b) excavated at the borrow area?

Solution

Please note that there are different ways to solve this problem. This solution will deal with the volume of soil in the embankment (V), in the borrow pit (V_p) and soil void ratios (e_e and e_p, respectively). From the three phase diagram (Fig. 3.1), we can derive that the total volume of soil can be written as V = 1 + e.

Question: *How is the total volume (V) related to the void ratio (e)?*
Answer: For many problems related to soil constituents, it can be assumed that the volume of solids (V_s) is equal to 1 m^3 as it makes the solution work-out much easier. Then, from the definition of void ratio (Equation 3.1), the volume of voids (V_v) will be equal to e and thus the total volume of soil will be V = 1 + e.

It can be stated that

$$\frac{V_p}{V_e} = \frac{1+e_p}{1+e_e}$$

To find e_p, the following calculations involving soil water content and unit weight should be done:

$$\text{Weight of water, } W_w = W - W_s = 3.5 - 2.9 = 0.6\,\text{N}$$

$$\text{Water content, } w = \frac{W_w}{W_s} = \frac{0.6}{2.9} = 0.21$$

$$\text{Dry unit weight, } \gamma_d = \frac{\gamma}{1+w} = \frac{17.8}{1+0.21} = 14.7\,\text{kN m}^{-3}$$

$$\text{From } \gamma_{dry} = \frac{G_s \cdot \gamma_{water}}{1+e}$$

We will find that the void ratio of soil from the borrow pit equals

$$e = e_p \approx 0.796$$

Finally, we will have

$$\frac{V_p}{80,000} = \frac{1+0.796}{1+0.71}$$

Giving the volume of soil from the borrow pit

$$V_p \approx 84,019\,\mathrm{m}^3$$

Problem 3.4 A 1 m thick soil with the initial void ratio of 0.94 was compacted by a roller and its thickness reduced by 0.09 m (Fig. 3.4). The specific gravity of this soil was 2.65. A 178 g soil sample was collected from the compacted soil mass to examine the degree of compaction; it was dried in an oven for 24 h and it had a dry mass of 142.4 g. Determine the degree of saturation after the compaction.

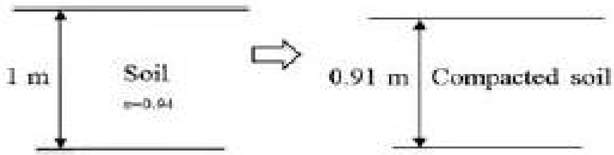


Figure 3.4 Changes in the soil layer thickness in Problem 3.4

Solution

Assume that a width of the soil mass before compaction is 1 m, then its volume is V = 1 m^3

From the definition of void ratio

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s}$$

We will find the volume of solids as

$$V_s = \frac{V}{1+e} = \frac{1}{1+0.94} = 0.515\,\mathrm{m}^3$$

It is logical to assume that the volume of solids remains the same after the compaction; however, the volume of voids would likely decrease.

The new volume of the compacted soil mass equals

$$V_{new} = 0.91 \cdot 1 = 0.91\mathrm{m}^3$$

Then, the volume of voids will become

$$V_v = V_{new} - V_s = 0.91 - 0.515 = 0.395\,\mathrm{m}^3$$

The new void ratio of compacted soil equals

$$e_{new} = \frac{0.395}{0.515} = 0.765$$

The water content will change to

$$w=\frac{178-142.4}{142.4}=0.25$$

And the degree of saturation will become

$$S=\frac{w\cdot G_s}{e}=\frac{0.25\cdot 2.65}{0.765}\approx 0.866\ or\ 86.6\ \%$$

Problem 3.5 A laboratory specimen of soil has a volume of 2.3 m^3. The void ratio of the sample is 0.712 and water content is 16.1%. The specific gravity of the solid particles is 2.7. Determine:

a) Volume of water
b) Mass of solids
c) Dry density
d) Bulk density

Solution

Volume of soil (V) consists of the volume of voids (V_v) and volume of solids (V_s), i.e.,

$$V=V_v+V_s=2.3\,\text{m}^3 \qquad \text{(a)}$$

We also know (Equation 3.1) that

$$e=\frac{V_v}{V_s}=0.712 \qquad \text{(b)}$$

Substituting V_s from Equation (b) to Equation (a), we get

$$1.712V_v=1.64$$

Therefore,

$V_v=0.96\,\text{m}^3$, and $V_s=1.34\,\text{m}^3$

From $w=\dfrac{S\cdot e}{G_s}$

We will get the degree of saturation (S) as

$$S=\frac{wG_s}{e}=\frac{0.161\cdot 2.7}{0.712}=0.61$$

From $S=\dfrac{V_w}{V_v}$

We will find the volume of water

$$V_w=S\cdot V_v=0.61\cdot 0.96\approx 0.59\,\text{m}^3$$

Mass of solids equals

$$M_s = \rho_s \cdot V_s = G_s \cdot \rho_w \cdot V_s = 2.7 \cdot 1000 \cdot 1.34 \approx 3618\,\text{kg}$$

Dry density (ρ_d) of soil will be

$$\rho_d = \frac{M_s}{V} = \frac{3618}{2.3} = 1573\,\text{kg/m}^3 = 1.57\,\text{g/cm}^3$$

Mass of water equals

$$M_w = \rho_w \cdot V_w = 1000 \cdot 0.59 \approx 590\,\text{kg}$$

From the definition of soil density, we have

$$\rho = \frac{M_s + M_w}{V} = \frac{3618 + 590}{2.3} \approx 1829.6\,\text{kg/m}^3$$

Problem 3.6 A cylindrical sample of clay, 50 mm (diameter) × 100 mm long, had weight of 3.5 N. It was placed in an oven for 24 h at 105°C. The sample weight reduced to a constant value of 2.9 N. If the specific gravity is 2.7, determine:

a) Void ratio
b) Dry unit weight
c) Degree of saturation

Solution

This problem will be solved using the aforementioned equations/relationships between the soil constituents.

Weight of water, $W_w = 3.5 - 2.9 = 0.6\,\text{N}$

Bulk unit weight, $\gamma_{bulk} = \frac{W}{V} = \frac{3.5 \cdot 10^{-3}}{196.4 \cdot 10^{-6}} \approx 17.8\,\text{kN m}^{-3}$

Water content, $w = \frac{W_w}{W_s} = \frac{0.6}{2.9} \approx 0.207 \text{ or } 20.7\ \%$

Dry unit weight, $\gamma_d = \frac{\gamma}{1+w} = \frac{17.8}{1+0.207} \approx 14.7\,\text{kN m}^{-3}$

From $\gamma_d = \frac{G_s}{1+e} \cdot \gamma_w$

We will get that

$$e \approx 0.8$$

Finally, the degree of saturation equals

$$S = \frac{w \cdot G_s}{e} = \frac{0.207 \cdot 2.7}{0.8} \approx 0.7 \text{ or } 70\ \%$$

Tutorial

1. For a given soil , show that $Gs = \dfrac{\gamma_{sat}}{\gamma_w - \omega_{sat}(\gamma_{sat} - \gamma_w)}$

Solution $Gs = \dfrac{\gamma_{sat}}{\gamma_w - \omega_{sat}(\gamma_{sat} - \gamma_w)}$

$\because \ \gamma_b = \gamma_{sat} - \gamma_w$

$$\therefore \ Gs = \frac{\gamma_{sat}}{\gamma_w - \omega_{sat}\gamma_b} = \frac{\frac{e+Gs}{1+e}\gamma_w}{\gamma_w - \omega_{sat}(\frac{Gs-1}{1+e}\gamma_w)}$$

$$Gs = \frac{\gamma_w \ \frac{e+Gs}{1+e}}{\gamma_w(1-\omega_{sat}(\frac{Gs-1}{1+e})} = \frac{e+Gs}{1+e-\omega_{sat}.Gs+\omega_{sat}}$$

For γ_{sat} $\rightarrow$ S = 100%

$\because \ Se = G_s\omega \rightarrow$ e = ω_{sat} Gs

$$\therefore \ Gs = \frac{e+Gs}{1+e-e+\omega_{sat}} = \frac{e+Gs}{1+\omega_{sat}} = \frac{\omega_{sat}Gs+Gs}{1+\omega_{sat}}$$

$$\therefore Gs = \frac{Gs(\omega_{sat}+1)}{(1+\omega_{sat})}$$

$\therefore$ Gs = Gs o.k.

2. For a given soil , show that $\omega_{sat} = \dfrac{n\gamma_w}{\gamma_{sat} - n\gamma_w}$

Solution $\omega_{sat} = \dfrac{n\gamma_w}{\frac{e+Gs}{1+e}\gamma_w - n\gamma_w}$

$\because n = \dfrac{e}{1+e}$

$$\omega_{sat} = \frac{\frac{e.\gamma w}{1+e}}{\frac{e+Gs}{1+e}\gamma w - \frac{e.\gamma w}{1+e}} = \frac{\frac{e.\gamma w}{1+e}}{\frac{\gamma w.e+Gs\gamma w-e.\gamma w}{1+e}}$$

$\omega_{sat} = \dfrac{e.\gamma_w}{Gs.\gamma_w}$

At $\omega_{sat} \rightarrow \omega_{sat}.Gs =$

$\therefore \omega_{sat} = \dfrac{e}{Gs}$

$\therefore \ \omega_{sat} = \omega_{sat}$ o.k.

3. For a given soil , the following are given : Gs= 2.67 ; moist. Unit weight $\gamma = 112Ib/ft^3$; moisture content $\omega = 10.8\%$. Determine :

a-Dry unit weight
b-Void ratio
c-Porosity
d-Degree of saturation

Solution

$$\gamma_t = \frac{w_c + 1}{1+e}\gamma_w Gs$$

$$112 = \frac{0.108+1}{1+e}(62.4)(2.67)$$

$$\therefore \quad e = \frac{184.6}{112} - 1 = 0.6482$$

$$n = \frac{e}{1+e} = \frac{0.6482}{1+0.6482} = 0.393$$

$$\gamma_d = \frac{Gs.\gamma_w}{1+e} = \frac{(2.67)(62.4)}{1+0.6482} = 101.08 lb/ft^3$$

$$\because Se = G_s\omega$$

$$\therefore \; S = \frac{\omega_c.Gs}{e} = \frac{(0.108)(2.67)}{0.6482} = 44.48\%$$

4. For the soil describe in problem 2.6 , determine the weight of water , in pounds to be added per ft^3 of soil for saturation ?

Solution

1 ft^3 of soil weight 112 lb

Before saturation ω_c =0.108

$\because \; \omega_c = \frac{W_w}{W_s} \Rightarrow W_w = (\omega_c)(W_s)$ = 0.108 x 112 = 12.096 lb

After saturation S = 100% → $\because \; Se = G_s\omega$

$$\therefore \; e = G_s\omega \Rightarrow \omega_{c for saturation} = \frac{e}{Gs} = \frac{0.6482}{2.67} = 0.2427$$

After saturation Ww = ω_c x Ws = 0.2427 x 112 = 27.1824 lb

$\therefore$ Ww_{after} – Ww_{before} = 27.1824 – 12.096 = 15.08 lb added .

5. For a moist soil , given the following : V = 0.25 ft^3 ; W = 30.75 lb ; ω = 9.8% ; Gs = 2.66 . determine the following :

a. γ (lb/f^3)
b. γ_d(lb/f^3)
c. e
d. n
e. S
f. Volume occupied by water

Solution

a. $\gamma_t = \frac{W}{V} = \frac{30.75}{0.25} = 123 lb/ft^3$

c. $\because \gamma_t = \frac{\omega_c + 1}{1+e}\gamma_w Gs \Rightarrow 123 = \frac{0.098+1}{1+e}(62.4)(2.66)$

$\therefore$ e =0.4817

b. $\gamma_d = \frac{Gs}{1+e}\gamma_w = \frac{2.66}{1+0.4817}(62.4) = 112.02 lb/ft^3$

d. $n = \frac{e}{1+e}$ $\therefore n = \frac{0.4817}{1+0.4817} = 0.325 = 32.5\%$

e. $Se = G_s\omega \rightarrow$ S x 0.4817 =2.66 x 0.098 S= 0.5411 =54.11%

f. $n = \frac{V_v}{V} \rightarrow 0.325 = \frac{V_v}{0.25} \Rightarrow V_v = 0.08125 ft^3$ $\because$ $S = \frac{V_w}{V_v} \Rightarrow 0.5411 = \frac{V_w}{0.08125}$

$\therefore V_w = 0.0439 ft^3$

6. For a soil , given ρ_d = 1712 Kg/m^3 ; e = 0.51 determine
   a. n
   b. Gs

Solution

$$n = \frac{e}{1+e} = \frac{0.51}{1+0.51} = 0.3377 = 33.77\%$$

$$\gamma_d = \rho_d(9.81) = (1.712)(9.81) = 16.794 KN/m^3$$

$$\gamma_d = \frac{Gs\gamma_w}{1+e} \Rightarrow 16.794 = \frac{(Gs)(9.81)}{1+0.51}$$

$$Gs = 2.585$$

7. A soil has unit weight of 126.8 lb/ft^3. Given Gs = 2.67 and ω= 12.6% determine
a. dry unit weight (lb/ft^3)
b. void ratio
c. porosity
d. The weight of water in (lb/ft^3) of soil needed for full saturation .

Solution

$$\gamma_t = \frac{\omega_c + 1}{1+e}(Gs)(\gamma_w) \Rightarrow 126.8 = \frac{0.126+1}{1+e}(2.6)(62.4)$$

$$\therefore e = 0.4407$$

$$\gamma_d = \frac{Gs}{1+e}\gamma_w = \frac{2.6}{1+0.4407}(62.4) = 112.61 lb/ft^3$$

$$n=\frac{e}{1+e}=\frac{0.4407}{1+0.4407}=0.3058=30.58\%$$

for S = 100%

$$Se=\omega_c Gs \Rightarrow (1)(0.4407)=\omega_c(2.6)$$

$$\omega_c=0.1695=16.95\%$$

1 lb/ft3 of dry soil has

$$\omega_c=\frac{W_w}{W_s}\Rightarrow \because Ws=1lb$$

$$\therefore 0.1695=\frac{W_w}{1}\Rightarrow W_w=0.1695lb$$

8. The saturated unit weight of soil is 20.12 kN/m^3 . Given Gs = 2.74 , determine
   a. γ_{dry}
   b. e
   c. n
   d. ω_c

   Solution

$$\gamma_{sat}=\frac{e+Gs}{1+e}\gamma_w \Rightarrow 20.12=\frac{e+2.74}{1+e}(9.81)$$

$\therefore$ e = 0.657

$$\gamma_d=\frac{Gs}{1+e}\gamma_w=\frac{0.657+2.74}{1+0.657}(9.81)=16.22kN/m^3$$

$$n=\frac{e}{1+e}=\frac{0.657}{1+0.657}=0.3964=39.64\%$$

$$\because Se=\omega_c Gs$$

$\because$ S = 100%

$$\therefore (1)(0.6574)=\omega_c(2.74)\Rightarrow \omega_c=0.24=24\%$$

9. For a soil given e = 0.86 , ω_c = 28% and Gs = 2.72 determine
   a. moist unit weight (lb/ft3)
   b. degree of saturation (%)

Solution

$$\gamma_t=\frac{\omega_c+1}{1+e}(Gs)(\gamma_w)=\frac{0.28+1}{1+0.86}(2.72)(9.81)=18.362kN/m^3$$

$$\because Se=\omega_c Gs$$

S x 0.86 = 0.28 x 2.72

$\therefore$ S = 0.8855 = 88.55%

10. For a saturated soil ; given γ_d = 15.29 kN/m^3 ; and ω_c =21% ; determine
    a. $\gamma_{sat.}$
    b. e
    c. Gs
    d. γ_{moist} when the degree of saturation is 50%.

Solution

$$\gamma_{sat} = \frac{e + Gs}{1+e}(\gamma_w)$$

$\because Se = \omega_c Gs$ for S = 100%

1 x e = 0.21 x Gs

$$\therefore G_s = \frac{e}{0.21}$$

$$\gamma_d = \frac{Gs}{1+e}\gamma_w \Rightarrow \gamma_d = \frac{\frac{e}{0.21}}{1+e}(9.81) \Rightarrow 15.29 = \frac{\frac{e}{0.21}}{1+e}(9.81)$$

$\therefore$ e = 0.4865

$$\because G_s = \frac{e}{0.21} = \frac{0.4865}{0.21} = 2.316$$

$$\gamma_{sat} = \frac{e + Gs}{1+e}(\gamma_w)$$

$$\gamma_{sat} = \frac{0.4865 + 2.316}{1 + 0.4865}(9.81) = 18.49 kN/m^3$$

For 50% = S

$\because Se = \omega_c Gs$

$\therefore$ 0.5 x 0.4865 = ω_c x 2.316

$\therefore$ ω_c = 0.105 = 10.5%

$$\gamma_t = \frac{\omega_c + 1}{1+e} Gs \gamma_w = \frac{0.105 + 1}{1 + 0.4865}(2.316)(9.81) = 16.889 kN/m^3$$

Or:

$$\gamma_t = \frac{Gs + se}{1+e}\gamma_w = \frac{2.316 + (0.5)(0.4865)}{1 + 0.4865}(9.81) = 16.889 kN/m^3$$

Measurement of Soil Properties

1. The in-situ density of a soil is 1.85 Mg/m^3. A moisture content determination test on a sample of the soil gave the following results.

Test No.	Mass of tin (g)	Tin + wet soil (g)	Tin + dry soil (g)
1	20.24	30.61	28.73
2	20.36	32.44	30.28

Determine the **moisture content** and **dry density** of the soil.

$$w = \frac{(wet + tin) - (dry + tin)}{(dry + tin) - (tin)} = \frac{30.61 - 28.73}{28.73 - 20.24} = 22.1\%$$

$$w = \frac{(wet + tin) - (dry + tin)}{(dry + tin) - (tin)} = \frac{32.44 - 30.28}{30.28 - 20.36} = 21.8\%$$

Average, $w = \underline{\underline{22\%}}$

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{185 \times 100}{122} = \underline{\underline{1.52 \text{ Mg/m}^3}}$$

2. The bulk density of a soil sample was found to be 1.90 g/ml and the moisture content 12%.

Determine the **dry density**, **void ratio** and **degree of saturation** if the particle specific gravity was 2.68.

What would the **moisture content** be if the soil were completely saturated at the same void ratio?

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{1.9 \times 100}{112} = 1.7\ \text{g/ml} = \underline{1.7\ \text{Mg/m}^3}$$

$$\rho_d = \frac{\rho_w G_s}{1+e}$$

$$1 + e = \frac{\rho_w G_s}{\rho_d}$$

$$e = \frac{2.68}{1.7} - 1$$

$$e = \underline{0.58}$$

$$\rho_b = \frac{G_s + eS_r}{1+e} \qquad (\text{N.B. } \rho_w = \text{ density of water } = 1\ \text{Mg/m}^3)$$

$$1.9 = \frac{(2.68 + 0.58S_r)}{1.58}$$

$$S_r = \frac{3 - 2.68}{0.58} = \underline{56\%}$$

$$\text{Saturated}, \Rightarrow e = wG_s$$

$$w = \frac{e}{G_s} = \frac{0.58}{2.68} = \underline{21.6\%}$$

3. A sample of saturated clay has a volume of 245ml and, after oven drying, has a mass of 453g.

If the particle specific gravity of the soil is 2.75, determine the **dry** and **saturated unit weights** of the soil in its natural state.

$$G_s = \frac{M_s}{V_s \rho_w}$$

$$2.75 = \frac{453 \times 10^{-6}}{V_s \times 1}$$

$$V_s = \frac{453 \times 10^{-6}}{2.75} = 164.7 \text{ x } 10^{-6} \text{ m}^3 = \underline{\underline{164.7 \text{ ml}}}$$

$$\text{Now, } V_v = V - V_s = 245 - 164.7 = \underline{\underline{80.3 \text{ ml}}}$$

$$e = \frac{V_v}{V_s} = \frac{80.3}{164.7} = 0.488$$

$$\gamma_d = \frac{\gamma_w G_s}{1+e} = \frac{9.81 \times 2.75}{1.488} = 18.1 \ \underline{\underline{\text{kN/m}^3}}$$

$$\gamma_{sat} = \gamma_w \left(\frac{G_s + e}{1+e}\right) = 9.81\left(\frac{2.75 + 0.488}{1.488}\right) = 21.4 \ \underline{\underline{\text{kN/m}^3}}$$

4. During a particle specific gravity test on a soil sample the following masses were recorded:

Mass of dry soil sample = 450g

Mass of pycnometer when full of water = 1875g

Mass of pycnometer + soil sample and full of water = 2160g

Determine the **particle specific gravity** of the soil.

$$G_s = \frac{\text{Mass of dry soil}}{\{(\text{jar + water}) - (\text{jar + water + soil})\} + \text{dry soil}}$$

$$\therefore G_s = \frac{450}{(1875 - 2160) + 450}$$

$$\therefore G_s = \underline{2.73}$$

5. A sand deposit was found to have a bulk density of 1.93 Mg/m^3 and a moisture content of 16%. Laboratory tests established that the maximum and minimum void ratio values were 0.75 and 0.48 respectively. If the particle specific gravity was 2.65, determine the **void ratio**, the **degree of saturation** and the **relative density** of the deposit.

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{1.93 \times 100}{116} = 1.66 \ Mg/m^3$$

$$\& \ \rho_d = \frac{\rho_w G_s}{1 + e}$$

$$\Rightarrow e = \frac{2.65}{1.66} - 1 = \underline{0.596}$$

$$\rho_b = \rho_w \left(\frac{G_s + eS_r}{1 + e} \right)$$

$$1.93 = \frac{2.65 + 0.596 S_r}{1.596}$$

$$\Rightarrow S_r = \underline{72.2\%}$$

$$R.D. = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.75 - .596}{0.75 - 0.48} = \underline{0.57}$$

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Soil Texture (نسيج التربة)

Composition and description of an Individual Soil Particle :

Soil Particle

Inorganic (Minerals and rocks) Organic (peat: decomposition of plants and animals)

in general: Soil may be divided into three main classes

1- Coarse – grained or non- cohesive soils
2- fine grained or cohesive soils
3- organic soil

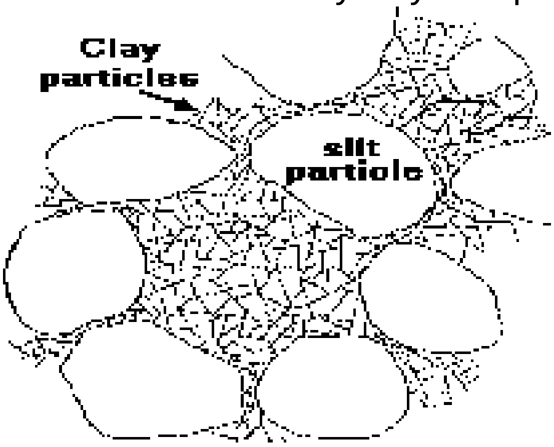
(1 and 2: Inorganic due to weathering process)

Particle Size : Vary from 1x 10^{-6} to rocks of several meters in thickness.

According to MIT:

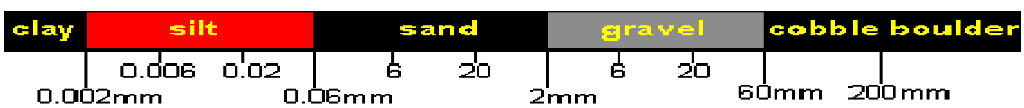
Coarse grained soils:	Boulders	> 300mm
	Cobble	150-300mm
	Gravel	2 - 150mm
	Sand	0.06- 2 mm
Fine –grained soil	Silt	0.002-0.06 mm
	Clay	< 0.002

Note: particles of size < 0.002 mm is denoted by clay size particles



Clay size particles:

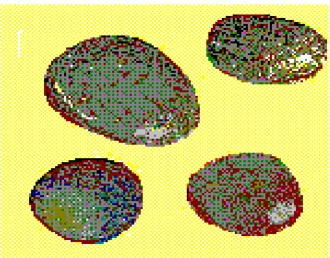
- clay minerals (silicate of Mg, Al, Fe)
- Fine particles of quartz or feldspar



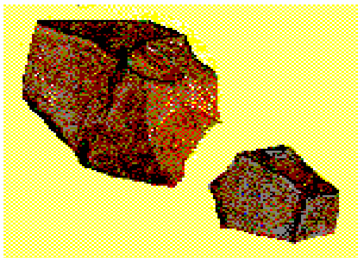
Particle shape

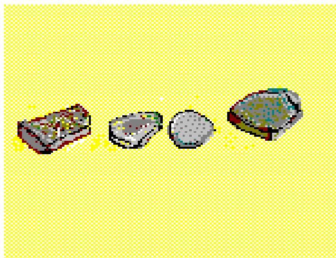
For *sand and silt* : equidimensional , cube or sphere.
For *clay* : platy shape **.**

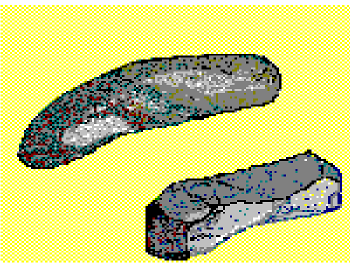
Rounded: Water- or air-worn; transported sediments
Irregular: Irregular shape with round edges; glacial sediments (sometimes sub-divided into 'sub-rounded' and 'sub-angular')
Angular: Flat faces and sharp edges; residual soils, grits
Flaky: Thickness small compared to length/breadth; clays
Elongated: Length larger than breadth/thickness; screen, broken flagstone
Flaky & Elongated: Length>Breadth>Thickness; broken schists and slates
Sieve analysis example


Rounded


Irregular


Angular


Flaky


Elongated


Flaky & Elongated

Clays Platy Shape

Specific surface: is the surface area per unit mass.

Kaolinite	10 - 20 m^2 /g
Illite	80 -100 m^2 /g
Montmorillonite	800 m^2 /g

Forces on soil particle:

Forces:

- Surface derived forces (*fine – grained*) → Colloid
- Mass derived forces (*Coarse – grained*)

Colloid: this term is used to describe a particle whose behavior is controlled by the surface derived surface.

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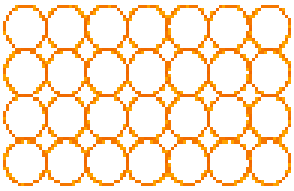
- Colloid range: 1ηm→1μm ,lower limit has a sp. Surface (10^{-9}m) → (10^{-6}m) from 25 m^2/g. (<1ηm lie the diameter of atoms and molecules)
- Clay particle is a colloid because of its small size (< 0.002mm = 2 μm) and irregular

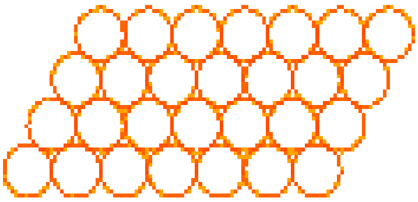
shape (plately Shape).

Soil Structure (Fabric): refers to orientation and distribution of particles in a soil mass.

1-for *coarse-grained soils*



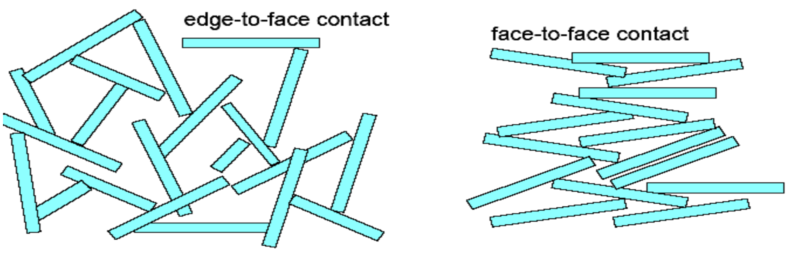
Loose State



dense State

2-for *clay*

Clay Fabric



Flocculated Dispersed

Dispersed structure: has parallel particles which tend to repel each other.

Flocculated Structure: in which the soil particles are edge to face and attract each other.

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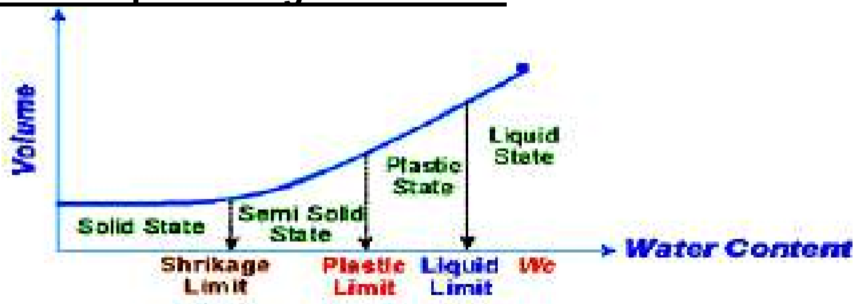
Consistency and Atterberg Limits:

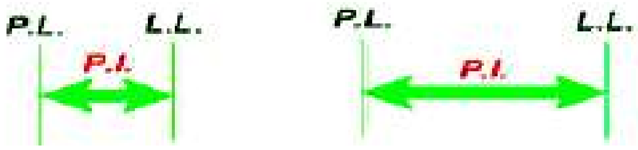
When *clay minerals* are present in fine grained soil, that soil can be remolded in the presence of some *moisture content* without crumbling. This *cohesive nature* is due to the *adsorbed water* surrounding the clay particles.

Consistency: the relative case with which a soil mass can be deformed.

Atterberg Limits: these limits are based on the concept that a fine grained soil can exist in any *four states depending on its water content.*

W_c - Volume relationship of fine grained soils:





Shrinkage Limit S.L : min. water content at which further loose in water does not cause a decrease in soil volume.

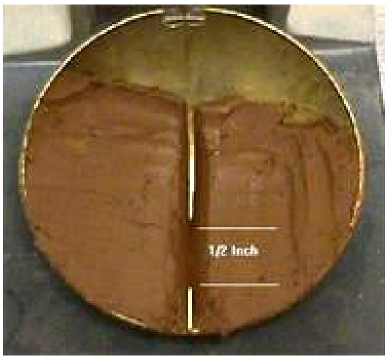
Plastic Limit P.L: is the water content of the soil between the plastic and semi-solid states at which threads of 3mm of the soil can be rolled without breaking.

Liquid Limit L.L; Min water content at which soil flow under its own weight.

Plasticity Index P.I: is the numerical difference between the liquid limit and plastic limit.

$$PI = LL - PL$$





- The classification system uses the term “fines” to describe everything that passes through a # 200 sieve (<0.075mm)
- No attempt to distinguish between silts and clays in terms of particles sizes since the biggest difference between silt and clay is not their particle sizes, but their physical and chemical structures
- The soil consistency is used as a practical and an inexpensive way to distinguish between silts and clays
- Plasticity property is important because it describes the response of a soil to change in moisture content
- Water Content Significantly affects properties of Silty and Clayey soils (unlike sand and gravel)
  - ***Strength decreases as water content increases***
  - ***Soils swell-up when water content increases***
  - ***Fine-grained soils at very high water content possess properties similar to liquids***
  - ***As the water content is reduced, the volume of the soil decreases and the soils become plastic***
  - ***If the water content is further reduced, the soil becomes semi-solid when the volume does not change***

- Atterberg limits are important to describe the consistency of fine-grained soils
- The knowledge of the soil consistency is important in defining or classifying a soil type or predicting soil performance when used a construction material
- A fine-grained soil usually exists with its particles surrounded by water.

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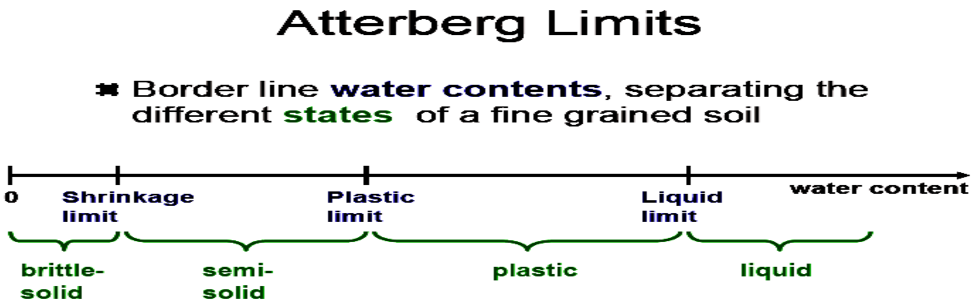
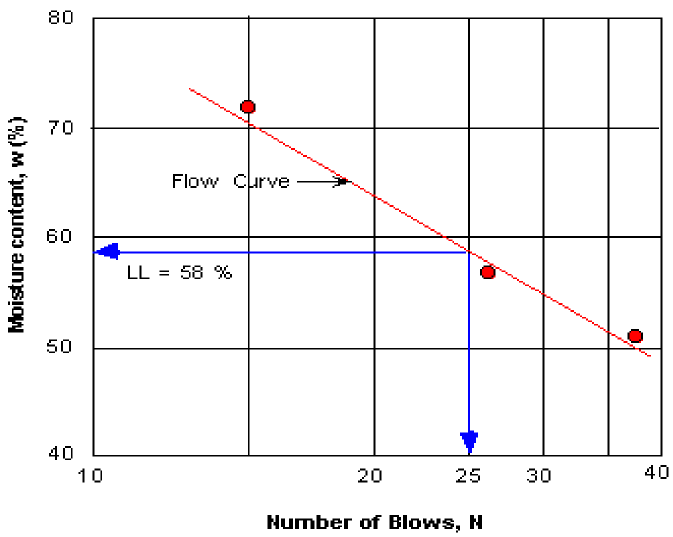
- The amount of water in the soil determines its state or consistency
- Four states are used to describe the soil consistency;
  solid, semi-solid, plastic and liquid





<u>Toughness Index</u> I_t: $I_t = \dfrac{P.I}{I_f}$

<u>Liquidity Index L.I</u>: is the ratio expressed as a percentage of the natural water content of a given soil sample minus its plastic limit to its plasticity index.

$$L.I = \frac{Wc - P.L}{L.L - P.L} = \frac{Wc - P.L}{P.I}$$

<u>Now if</u> L.I < 0 : Wc < P.L → Soil in semi or solid State
L.I = 0; Wc = P.L → Soil at P.L
0 < L.I < 1: Wc < L.L → Soil at plastic State
L.I =1: Wc = L.L → Soil at L.L
L.I > 1: Wc > L.L → Soil at liquid State.

Notes on Atterberg Limits:

1-The limits are used in classification and specification (ex: for controlling soil for use in fill).
2-The limits depend on a mount and type of *clay minerals* and the nature of (+ ve) ions in pore water, a soil of greater tendency to attach water to the particle surface will have larger L.L.
3- Soil of higher L.L has higher P.L and higher compressibility.

Activity of clay: is the ratio of plasticity index of a soil sample to percent by weight of the particles finer than 0.002 mm in size.

*So-called 'clay' soils are not 100% clay. The proportion of clay mineral flakes (< 2 μm size) in a fine soil affects its current state, particularly its tendency to swell and shrink with changes in water content. The degree of plasticity related to the clay content is called the **activity** of the soil.*

Activity = I_P / (% clay particles)

Activity depends on:

- specific surface.
- amount of clay particles.
- type of clay minerals.

Atterberg limits for clay minerals.

Mineral	*LL*	*PL*	*SL*	*I_P*	*Activity, A*
Kaolinite	30 - 110	25 - 40	25 - 29	5 – 70	0.5
Illite	60 - 120	35 - 60	15 - 17	25 – 60	0.5 – 1
Montmorillonite	100 - 900	50 - 100	8.5 - 15	50 - 800	1 – 7

Void Ratio For Granular Soils and Cohesive Soils:

- for cohesive soils, values of (e) mainly depend on pressure.
- for granular soils, (e) depends on :
  - vibration,
  - Range of particle sizes

Relative Density D_r: its use to describe density of natural granular soils.

$$Dr = \frac{e_{max} - e}{e_{max} - e_{min}} x100\%$$

e_{min}, e_{max} =Void ratio of soil in densest and loosest condition
e = natural or in situ void ratio

Dr%	*Description of soil*
0 - 15	***Very loose***
15 - 35	***loose***
35- 65	***medium***
65 - 85	***dense***
85 - 100	***Very dense***

The expression for relative density can also be written in terms of the dry unit weights associated with the various voids ratios. From the definitions we have

$$e = \frac{G_s \gamma_w}{\gamma_{dry}} - 1$$

and hence

$$I_d = \frac{\dfrac{1}{\gamma_{dry_{\min}}} - \dfrac{1}{\gamma_{dry}}}{\dfrac{1}{\gamma_{dry_{\min}}} - \dfrac{1}{\gamma_{dry_{\max}}}} = \frac{\gamma_{dry_{\max}}(\gamma_{dry} - \gamma_{dry_{\min}})}{\gamma_{dry}(\gamma_{dry_{\max}} - \gamma_{dry_{\min}})}$$

<u>Particle size Distribution:</u>

steps:

- sieve analysis (dry mechanical analysis).
- hydrometer analysis (wet analysis).
- combined analysis.

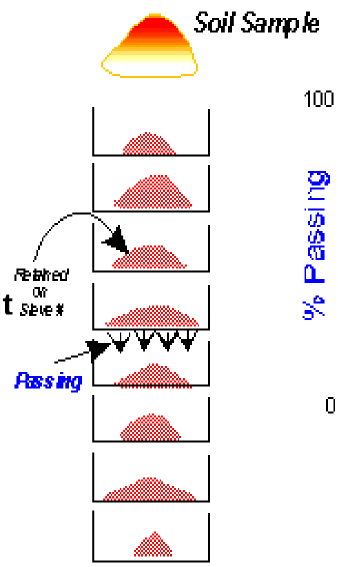
<u>Sieves</u> (U.S standards)

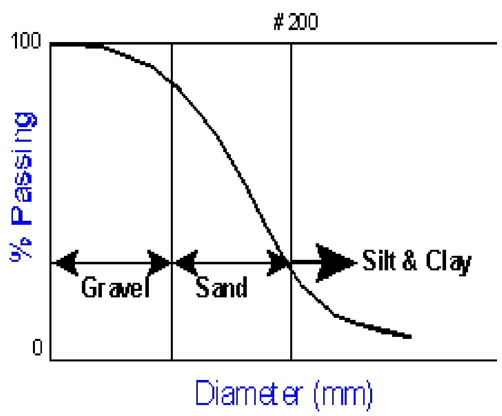
No.	*Penning size*
4	***4.76***
10	***2.00***
20	***0.84***
30	***0.59***
40	***0.42***
60	***0.25***
100	***0.147***
200	***0.075***

Atterebege Limit

Sieve

Hydrometer



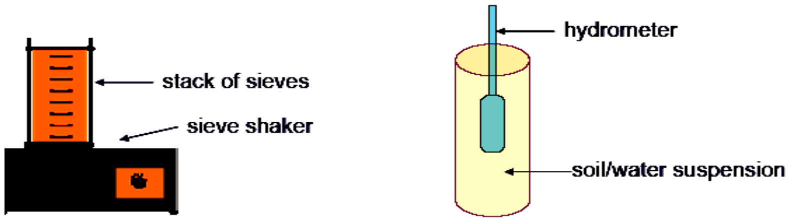


3/8", ¼" sieves is the size of the opening

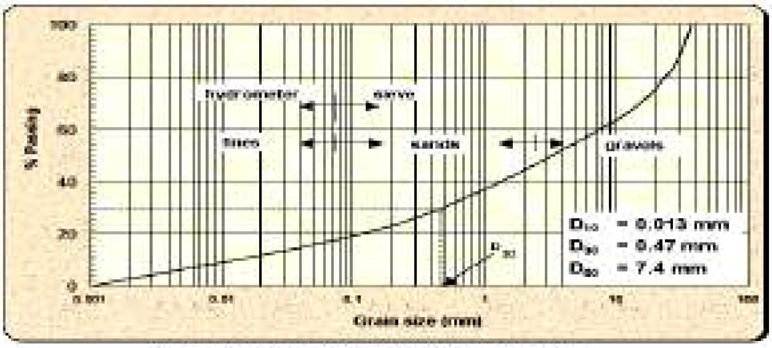
- ***No.10 sieve …. has 10 apertures per linear inch***
- ***Use sieves No.3/8", No.4, No.10, No.40, No.140 & No.200***

Determination of GSD:

- In coarse grain soils …… By sieve analysis
- In fine grain soils …… By hydrometer analysis



Sieve Analysis　　Hydrometer Analysis



Grain Size Distribution Curve

- can find % of gravels, sands, fines
- define D_{10}, D_{30}, D_{60}.. as above.

Uniformity Coefficient (Cu)

$$Cu = \frac{D_{60}}{D_{10}}$$

D_{60},D_{10}=soil diameter at which 60% and 10% of the soil weight is finer.

- smaller Cu means smaller range of particle size.
- A soil of Cu < 4 is considered uniform for gravel
- Cu < 6 is considered uniform for sand

Coefficient of Graduation (Cc) or Curvature:

$$C_c = \frac{(D_{30})^2}{D_{60} x D_{10}}$$

If 1 < **Cc** < 3 the soil is well graded.

Sorting Coefficient

$$S_c = \sqrt{\frac{D_{75}}{D_{25}}}$$

Well or Poorly Graded Soils

Well Graded Soils

Wide range of grain sizes present

Gravels: C_c = 1-3 & C_u >4

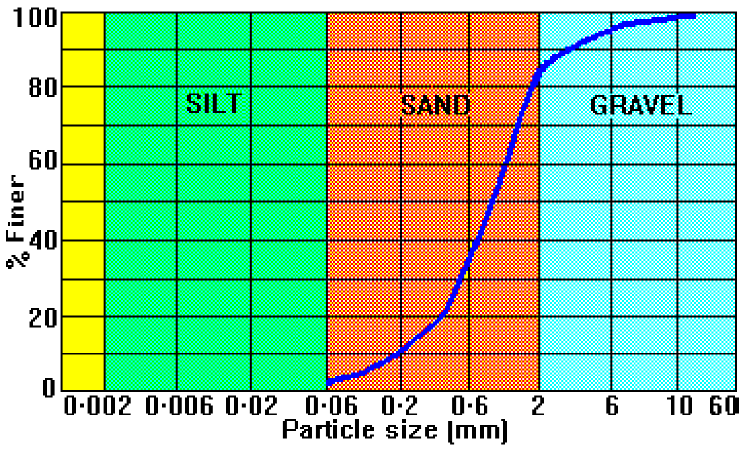
Sands: C_c = 1-3 & C_u >6

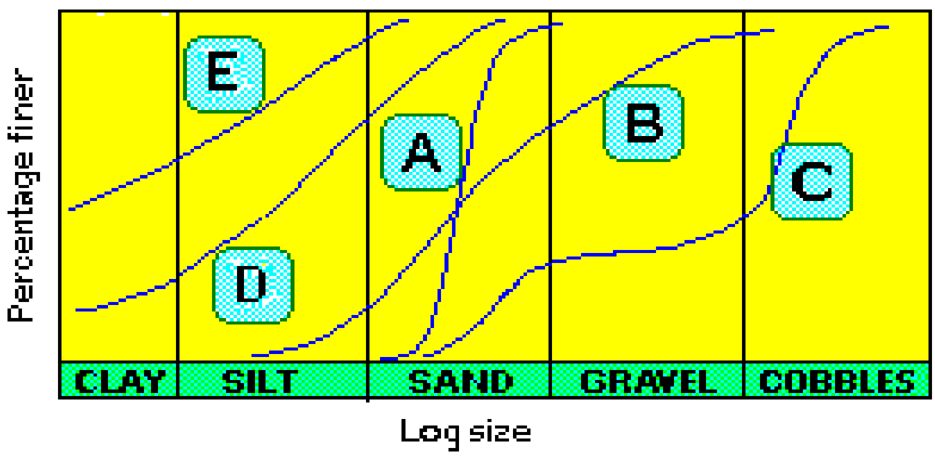
Poorly Graded Soils

Others, including two special cases:

(a) **Uniform** soils – grains of same size

(b) **Gap** graded soils – no grains in a specific size range





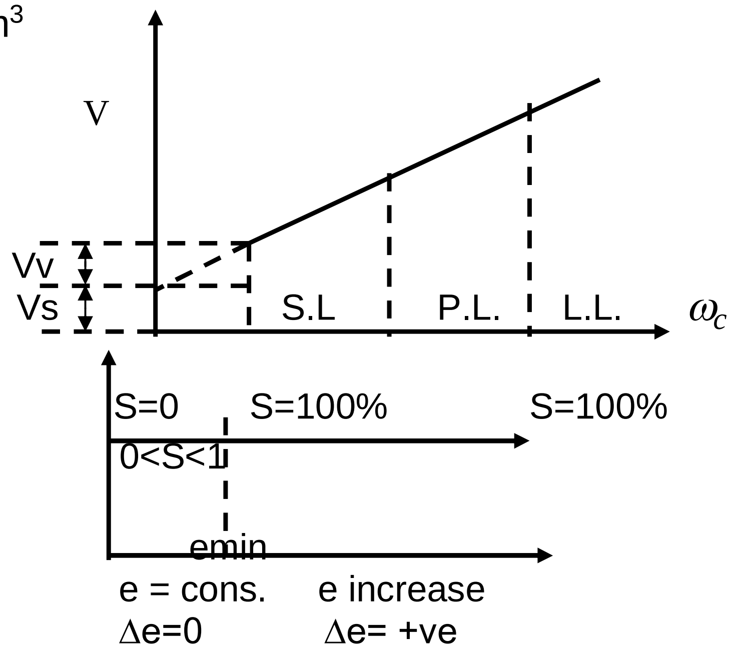
Tutorial

Problem 4: A dry sample of soil having the following properties: L.L=52.1, PL=30% , Gs=2.7 and e = o.53 .Find shrinkage limit , dry density , dry unit weight and air content of dry state .

$$\gamma_d = \frac{Gs}{1+e}\gamma_w = \frac{(2.7)(9.81)}{1+0.53} = 17.311 kN/m^3$$

$$\rho_d = \frac{17.311}{9.81} = 1.764 Ton/m^3$$

$$n = \frac{e}{1+e} = \frac{0.53}{1+0.53} = 0.3464$$



For the dry state S = 0%

$$A = n(1-S) = 0.3464(1-0) = 0.3464$$

At S = 100% ω_c find S.L.

$$\because Se = \omega_c Gs$$

1 x 0.53 = S.L. x 2.7

$\therefore$ S.L = 0.1962 = 19.62 %

Problem 5

A saturated soil sample has a volume of 20 cm^3 at L.L. Given L.L=42% ,P.L=30 % , S.L=17% and Gs=2.74. Find minimum volume which the soil can attain.

Solution

At L.L $\because Se = \omega_c Gs$ $\therefore$ S = 1

1 x e = 0.42 x 2.74 $\therefore$ $e_{at\ L.L}$= 1.1508

$$n = \frac{e}{1+e} = \frac{1.1508}{1+1.1508} = 0.535$$

$$n_{atL.L} = \frac{Vv}{V} \Rightarrow 0.535 = \frac{Vv}{20} \Rightarrow Vv = 10.701 cm^3$$

$$Vs = V - Vv = 20 - 10.701 = 9.3 cm^3$$

$e_{atS.L.} = \omega_c G_s$ because S = 1

$\therefore$ e = 0.17 x 2.74 = 0.4658

$$n_{atS.L} = \frac{e}{e+1} = \frac{0.4658}{1+0.4658} = 0.3177$$

$$e_{S.L} = \frac{Vv}{Vs} \Rightarrow 0.4658 = \frac{Vv}{9.3} \Rightarrow Vv = 4.3319cm^3$$

$$\therefore V = Vs + Vv = 9.3 + 4.3319 = 13.63cm^3$$

Problem 6-A sample of saturated clay had a volume of 97cm^3 and a mass of 0.202 kg. When completely dried out the volume of the sample was 87 cm^3 and its mass 0.167 kg. Find initial water content, shrinkage limit and specific gravity of the solid particles.

Solution

Ww = 0.202 – 0.167 = 0.035 kg = 35g

$$\omega_c = \frac{W_w}{W_s} = \frac{35}{167} = 0.21 = 21\%$$

$\because$ $W_{water} = V_{water} = 35cm^3$

$\therefore$ $V_{solid} = 97 - 35 = 62\ cm^3$

For shrinkage limit $Vv = V_{dry} - V_{solid} = 87 - 62 = 25\ cm^3$

$\because$ at S.L. → S = 100%

$\therefore$ Vv = Vw = 25 cm^3

$\because$ Vw = Ww = 25g

$$\therefore \quad S.L = \frac{W_w}{W_s} = \frac{25}{167} = 0.15 = 15\%$$

$$e = \frac{V_v}{V_s} = \frac{25}{62} = 0.4032$$

$$Se = \omega_c Gs$$

1 x 0.4032 = 0.15 x Gs

$\therefore$ Gs = 2.688

Problem 7- The Atterberg Limits of a clays soil are : LL= 52%, P.L =30% and SL= 18%. If a Specimen of this soil Shrinks from a volume of 39.5cm^3 at the L.L to a volume of 24.2 cm^3 at the S.L . Calculate the specific gravity.

Solution

$\because$ $V_{solid\ at\ S.L} = V_{solid\ at\ L.L.}$

$(V - Vw)_{at\ S.L} = (V - Vw)_{at\ L.L}$

$\therefore$ $24.2 - (Vw_{at\ S.L}) = 39.5 - (Vw_{at\ L.L})$

$$\omega_c = \frac{W_w}{W_s} \qquad \therefore W_w = \omega_c W_s$$

$$Ww_{atS.L} = 0.18(Ws)$$

$\because$ Ww = Vw (because γ_w =1)

$\therefore$ 24.2 - 0.18 Ws =39.5 – 0.52Ws → Ws = 45g.

$\therefore$Ww $_{\text{at S.L}}$ =0.18 x 45 = 8.1g =Vw $_{\text{at S.L.}}$

$$\therefore e_{\text{at S.L.}} = \frac{V_v}{V_s} = \frac{8.1}{16.1} = 0.53$$

$\because Se = \omega_c Gs$

$$1x0.503 = 0.18Gs \Rightarrow Gs = 2.79$$

Another Solution

$$\rho_{soild} = \frac{Ws}{Vs} = \frac{45}{16.1} = 2.79g/cm3$$

$$Gs = \frac{\rho_s}{\rho_w} = \frac{2.79}{1} = 2.79$$

Problem 8- A saturated Sample of clay with an SL of 22% has a natural water content of 35%. What would its dry volume be as a percentage of its original (natural) volume if Gs = 2.70

Solution:

At S.L S = 100%

$\therefore Se = \omega_c Gs$

1 x e = 0.22 x 2.7

$\therefore$ e = 0.594

At natural water content S = 100%

$$Se = \omega_c Gs$$

1 x e = 0.35 x 2.7

$\therefore$ e = 0.945

$$\frac{V_{dry}}{V_{sat.}} = \frac{1+e_{dry}}{1+e_{sat}} = \frac{1+0.594}{1+0.945} = 0.82 = 82\%$$

Problem 9-The Shrinkage limit of a 0.1m^3 sample of a clay is 15% and its natural water content is 34%. Assume Gs is 2.68 , estimate the volume of the sample when the water content.

Solution

At S.L. S = 100%

$$Se = \omega_c Gs$$

$\therefore$ 1 x e = 0.15 x 2.68

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$\therefore e_{S.L} = 0.402$

At natural water content ω_n

$$Se = \omega_c G s \rightarrow 1 \times e = 0.34 \times 2.68$$

$\therefore e_{n} = 0.9112$

$$n = \frac{e}{1+e} = \frac{0.9112}{1+0.9112} = 0.4767$$

$$\therefore\ n = \frac{V_v}{V}$$

1t ω_n

$$0.4767 = \frac{V_v}{0.1}$$

$\therefore Vv = 0.04767\ m^3$

$\therefore Vs = V – Vv = 0.1 – 0.04767 = 0.05233 m^3$

$$e_{atS.L.} = \frac{Vv}{Vs} \Rightarrow 0.402 = \frac{Vv}{0.05233}$$

$\therefore Vv = 0.021\ m^3{}_{at\ S.L.}$

$\therefore V_{at\ S.L} = Vs + Vv = 0.05233 + 0.021 = 0.0733 m^3$

Problem 10- The L.L of a medium sensitive clay is 56% and P.I 28%.At it natural water content , the void ratio is 1.03 while after shrinkage the minimum void ratio is 0.72. Assuming Gs = 2.72 , calculate the shrinkage limit of the clay.

Solution :

At shrinkage limit S = 100%

$$Se = \omega_c G s$$

$1 \times 0.72 = \omega_c \times 2.72$

$\therefore \omega_c = S.L = 0.2647 = 26.47\%$

2-24 For a given sandy soil $e_{max} = 0.86$ and $e_{min} = 0.43$ what is the void ratio at $D_r = 56\%$

Solution

$$Dr = \frac{e_{max} - e}{e_{max} - e_{min}}(100) \Rightarrow 0.56 = \frac{0.86 - e}{0.86 - 0.43}$$

$\therefore\ e = 0.6192$

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2-26 For a given sandy soil; e_{max} = 0.72 ; e_{min}= 0.46 ; Gs = 2.68 what will be the moist. unit weight of compaction(kN/m3)in the field if D_r =78% and ω_c =9%?

Solution

$$Dr = \frac{e_{max} - e}{e_{max} - e_{min}}(100) \Rightarrow 0.78 = \frac{0.72 - e}{0.72 - 0.46}$$

$\therefore$ e = 0.5172

$$\gamma_t = \frac{1+\omega_c}{1+e} Gs\gamma_w = \frac{1+0.09}{1+0.5172}(2.68)(9.81) = 18.88\ \text{kN/m}^3$$

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Soil classification Systems

Are used to group soils in accordance with their general behavior under given physical conditions.

Unified Soil Classification System (USCS):

1- Is the most popular soil classification system among soil and foundation engineers.

USCS (Unified Soil Classification System)

soils are classified on basis of parameters which influence their engineering properties .

Coarse – grained soils (gravels and sands) classified on basis of **grain size characteristics**

Fine-grained soils (silts and clays) classified on basis of **plasticity characteristics**.

Symbols:

G	Gravel
S	Sand
M	silt
C	Clay
O	Organic

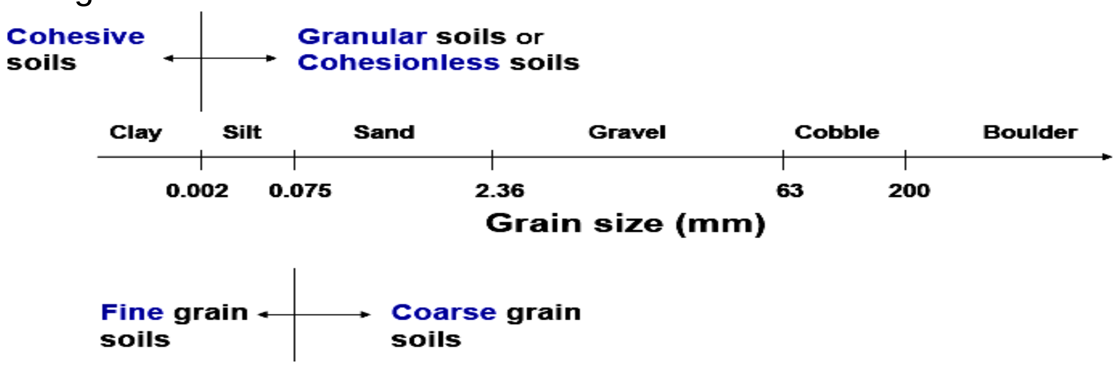
Modifiers:

W	Well Graded
P	Poorly Graded
H	High Plasticity
L	Low Plasticity

Examples:

GW	Well-graded gravel
SP	Poorly-graded (uniform) sand
MH	Highly plastic silt
CL	Low plasticity clay
GM	Silty gravel

2-According to USCS:



3-according to U.S standards

Sieve No.	*Opening Size*	
4	*4.76*	*G*
10	*2.00*	*S, M, C*
40	*0.42*	↑ *Coarse*
200	*0.075*	↓ *fine*

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This system divides soils into three main groups:

1-Coarse-grained Soils: more than 50% of the soil *retained* on the No.200 sieve (0.075mm).

Soil group		Symbol		Recommended name
Coarse soils			Fines %	
GRAVEL	G	GW	0 - 5	Well-graded GRAVEL
		GPu/GPg	0 - 5	Uniform/poorly-graded GRAVEL
	G-F	GWM/GWC	5 - 15	Well-graded silty/clayey GRAVEL
		GPM/GPC	5 - 15	Poorly graded silty/clayey GRAVEL
	GF	GML, GMI...	15 - 35	Very silty GRAVEL [plasticity sub-group...]
		GCL, GCI...	15 - 35	Very clayey GRAVEL [..symbols as below]
SAND	S	SW	0 - 5	Well-graded SAND
		SPu / SPg	0 - 5	Uniform/poorly-graded SAND
	S-F	SWM/SWC	5 - 15	Well-graded silty/clayey SAND
		GPM/GPC	5 - 15	Poorly graded silty/clayey SAND
	SF	SML, SMI...	15 - 35	Very silty SAND [plasticity sub-group...]
		SCL, SCI...	15 - 35	Very clayey SAND [..symbols as below]

2-Fine-grained Soils: more than 50% of the soil passing the No.200 sieve.

Fine soils		>35% fines	Liquid limit%	
SILT	M	MG		Gravelly SILT
		MS		Sandy SILT
		ML, MI...		[Plasticity subdivisions as for CLAY]
CLAY	C	CG		Gravelly CLAY
		CS		Sandy CLAY
		CL	<35	CLAY of low plasticity
		CI	35 - 50	CLAY of intermediate plasticity
		CH	50 - 70	CLAY of high plasticity
		CV	70 - 90	CLAY of very high plasticity
		CE	>90	CLAY of extremely high plasticity
Organic soils	O			[Add letter 'O' to group symbol]
Peat	Pt			[Soil predominantly fibrous and organic]

3-High organic soil , peat (pt).

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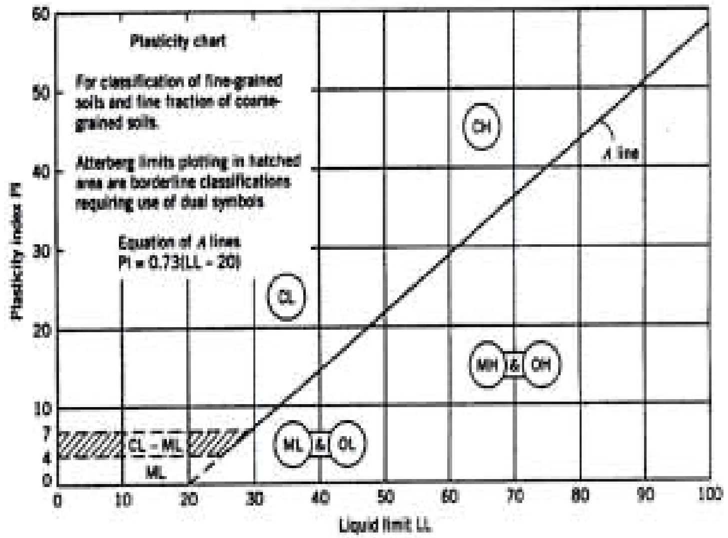
Major Divisions			Group Symbols	Typical Names	Classification Criteria		
Coarse-Grained Soils More than 50% retained on No. 200 sieve*	Gravels 50% or more of coarse fraction retained on No. 4 sieve	Clean gravels −200 <5%	GW	Well-graded gravels and gravel–sand mixtures, little or no fines	Classification on Basis of Percentage of Fines: Less than 5% pass No. 200 sieve — GW, GP, SW, SP; More than 12% pass No. 200 sieve — GM, GC, SM, SC; 5–12% pass No. 200 sieve — Borderline classification requiring use of dual symbols	$C_u = D_{60}/D_{10} > 4$ $C_c = \frac{D_{30}^2}{D_{10}D_{60}}$ between 1 and 3	
			GP	Poorly graded gravels and gravel–sand mixtures, little or no fines		Not meeting both criteria for GW	
		Gravels with fines −200 >12%	GM	Silty gravels, gravel–sand–silt mixtures	More than 12%: GM, GC	Atterberg limits plot below A line or plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			GC	Clayey gravels, gravel–sand–clay mixtures	5–12% Borderline	Atterberg limits plot above A line and plasticity index greater than 7	
	Sands More than 50% of coarse fraction passes No. 4 sieve	Clean sands −200 <5%	SW	Well-graded sands and gravelly sands, little or no fines		$C_u = D_{60}/D_{10} > 6$ $C_c = \frac{D_{30}^2}{D_{10}D_{60}}$ between 1 and 3	
			SP	Poorly graded sands and gravelly sands, little or no fines		Not meeting both criteria for SW	
		Sands with fines −200 >12%	SM	Silty sands, sand–silt mixtures		Atterberg limits plot below A line or plasticity index less than 4	Atterberg limits plotting in hatched area are borderline classifications requiring use of dual symbols
			SC	Clayey sands, sand–clay mixtures		Atterberg limits plot above A line and plasticity index greater than 7	
Fine-Grained Soils 50% or more passes No. 200 sieve*	Silts and Clays Liquid limit 50% or less		ML	Inorganic silts, very fine sands, rock flour, silty or clayey fine sands	Plasticity chart (below)		
			CL	Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays			
			OL	Organic silts and organic silty clays of low plasticity			
	Silts and Clays Liquid limit greater than 50%		MH	Inorganic silts, micaceous or diatomaceous fine sands or silts, elastic silts			
			CH	Inorganic clays of high plasticity, fat clays			
			OH	Organic clays of medium to high plasticity			
Highly Organic Soils			PT	Peat, muck, and other highly organic soils	Visual-manual identification, see ASTM D-2488.		



* 0.075-mm sieve.

Figure 4.9 Unified Classification System, ASTM D-2487-69.

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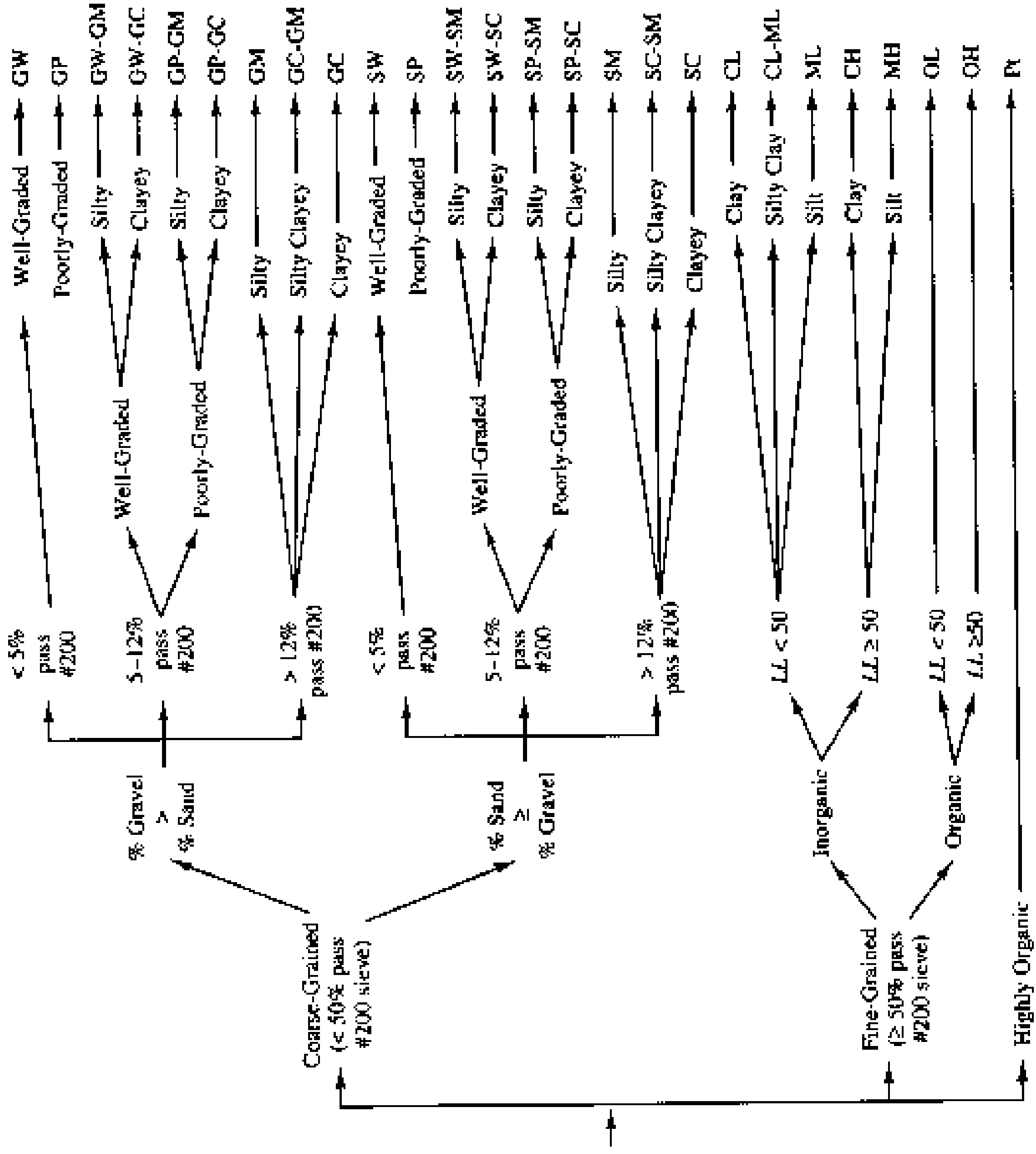
GW
GP
GW-GM
GW-GC
GP-GM
GP-GC
GM
GC-GM
GC
SW
SP
SW-SM
SW-SC
SP-SM
SP-SC
SM
SC-SM
SC
CL
CL-ML
ML
CH
MH
OL
OH
Pt
Well-Graded
Poorly-Graded
Silty
Clayey
Silty
Clayey
Silty
Silty Clayey
Clayey
Well-Graded
Poorly-Graded
Silty
Clayey
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Clayey
Silty
Silty Clayey
Clayey
Clay
Silty Clay
Silt
Clay
Silt
Well-Graded
Poorly-Graded
Well-Graded
Poorly-Graded
< 5% pass #200
5–12% pass #200
> 12% pass #200
< 5% pass #200
5–12% pass #200
> 12% pass #200
LL < 50
LL ≥ 50
LL < 50
LL ≥50
% Gravel > % Sand
% Sand ≥ % Gravel
Inorganic
Organic
Coarse-Grained (< 50% pass #200 sieve)
Fine-Grained (≥ 50% pass #200 sieve)
Highly Organic

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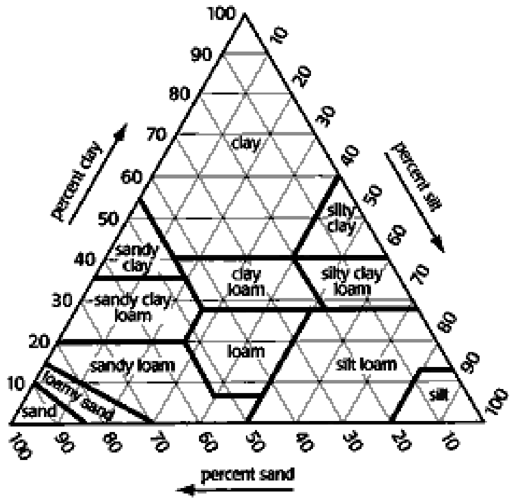
4- Soil texture is the percent sand, silt and clay in any given sample. It describes how gritty, smooth or sticky the soil is...

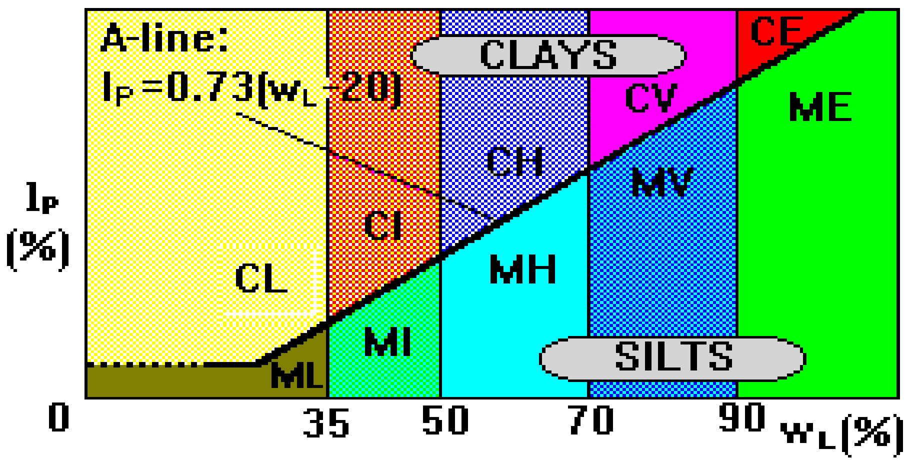
We will practice this a lot in the field, but on the exam you will need to know how to describe each of the textural classes. Below is the most common terminology:



SAND: Will not form a ball or a ribbon; very gritty.
LOAMY SAND: Will form a weak ball, but no ribbon; very gritty.
SANDY LOAM: Will form a weak ribbon (< 1"); very gritty.
LOAM: Will form a weak (<1") ribbon; intermediate (some) grittiness.
SILT LOAM: Will form a weak (<1") ribbon; very smooth (little grit).
SANDY CLAY LOAM: Will form a moderate (1-2") ribbon; very gritty.
CLAY LOAM: Will form a moderate (1-2") ribbon; intermediate (some) grittiness.
SILTY CLAY LOAM: Will form a moderate ribbon (1-2"); very smooth (little grit).
CLAY: Will form a strong ribbon (>2"); very plastic; can make little animals out of it, etc.
*Technically SANDY CLAY and SILTY CLAY also exist but we will not cover them in this class.

	VERY GRITTY	INTERMEDIATE	VERY SMOOTH
NO RIBBON	sand / loamy sand	X	X
<1" RIBBON	sandy loam	loam	silt loam
1-2" RIBBON	sandy clay loam	clay loam	silty clay loam
>2" RIBBON	[sandy] clay	clay	[silty] clay

Plasticity Chart



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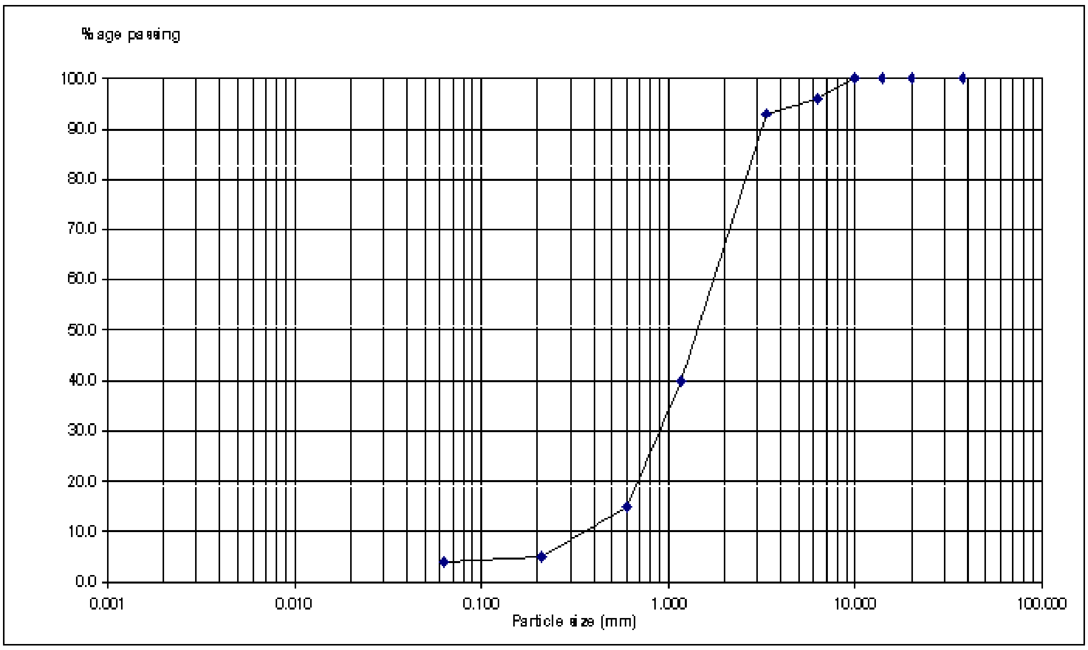
Example1. Plot the **particle size distributions** for each of the soils whose sieve analyses are given below. Use the PSD chart provided.

Sieve size (mm)	Mass retained on sieve (g) (1)	(2)
37.5	0.0	15.5
20.0	0.0	17.0
14.0	0.0	10.0
10.0	0.0	11.0
6.30	4.2	33.0
3.35	3.1	114.5
1.18	55.1	63.3
0.60	26.0	18.2
0.20	10.4	17.0
0.063	1.0	10.5
pan	4.2	2.5

Sieve (mm)	Mass (g)	% retained	% passing
37.5	0	0.0	100.0
20.0	0	0.0	100.0
14.0	0	0.0	100.0
10.0	0	0.0	100.0
6.3	4.2	4.0	96.0
3.35	3.1	3.0	93.0
1.18	55.1	53.0	40.0
0.60	26	25.0	15.0
0.212	10.4	10.0	5.0
0.063	1	1.0	4.0
pan	4.2	4.0	

Determine the **uniformity coefficient f**or each sample.

(a)



Cu = 1.8/0.36 = <u>5.0</u>

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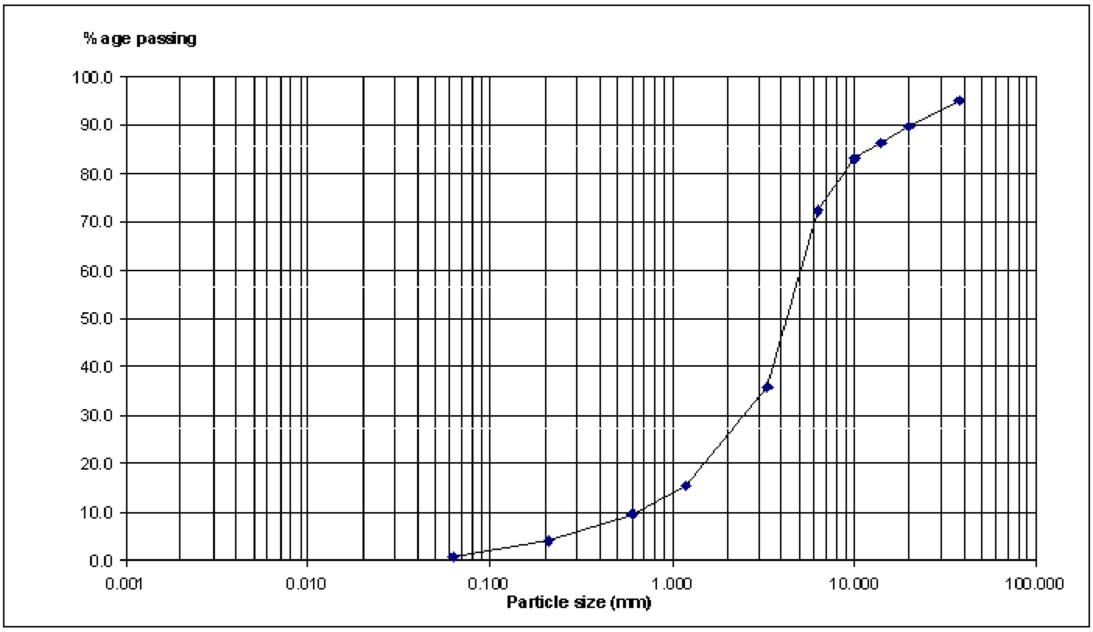
(b)

Sieve (mm)	Mass (g)	% retained	% passing
37.5	15.5	5.0	95.0
20.0	17	5.4	89.6
14.0	10	3.2	86.4
10.0	11	3.5	82.9
6.3	33	10.6	72.3
3.35	114.5	36.6	35.7
1.18	63.3	20.3	15.4
0.60	18.2	5.8	9.6
0.212	17	5.4	4.2
0.063	10.5	3.4	0.8
pan	2.5	0.8	



Cu = 5.1/0.62 = <u>8.3</u>

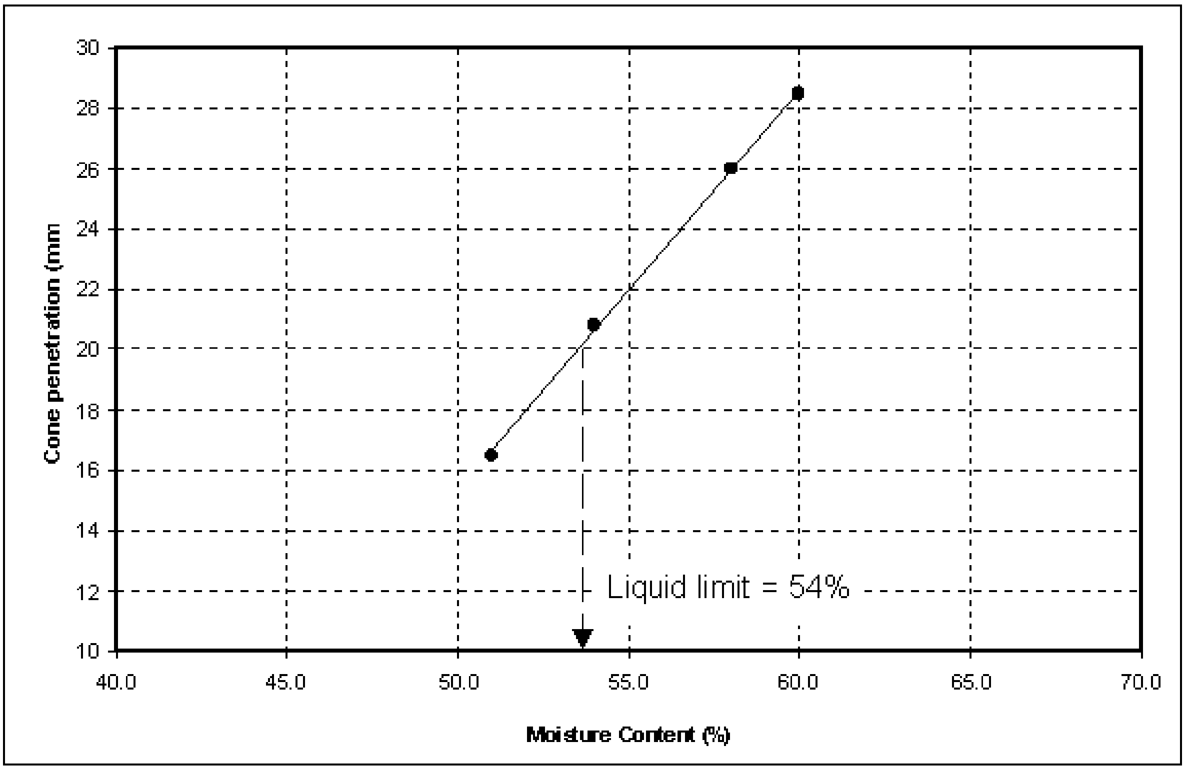
Example2. A liquid limit test gave the following results:

Mass of wet soil (g)	**Mass of dry soil (g)**	Penetration (mm)
39.73	26.31	16.5
50.51	32.80	20.8
47.07	29.79	26.0
61.28	38.30	28.5

The plastic limit of the soil was found to be 25%. Determine the **liquid limit**, the **plasticity index** and, hence, **classify** the soil.

If the natural moisture content was 40%, what would be the **liquidity index** in the field?

The calculated moisture contents can be seen on the liquid limit plot.



From Plot, LL = <u>54%</u>

PI = LL - PL = 54 - 25 = <u>29%</u>

From A-line, soil is classified ***CH***

IL = (w-PL)/PI = (40-25)/29 = <u>0.52</u>

Examples:

Ex1

Classification tests were performed on a light-brown sandy soil which visually has several pieces of gravel larger than 6 mm. The following laboratory data were obtained:

Sieve No.	*% Passing*
4	98.0
40	36.5
200	28.5

$w_{LL} = 33.2\ \%$

$w_{PL} = 22.6\ \%$

Classify this soil.

Solution:

Note that the 28.5 % that passed the # 200 sieve are fine grained soils and the remaining 71.5 % are sands and gravels. Furthermore, these are mostly sand, since there are only 2 % gravels.

The $I_P = w_{LL} - w_{PL} = 33.2 - 22.6 = 10.6\ \%$

The A-chart indicates that this portion of the soil is a CL.

Therefore, this soil can be described as a light-brown clayey sand, with a trace of gravel.

Ex2

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Classify soils # 4 and # 5 as listed in the attached page according to the Unified Soil Classification System. Explain your steps, and compare with the AASHTO Table.

Soil # 4

1) 70% is smaller than # 200 sieve, therefore M or C.
2) LL < 50 therefore, ML, CL, or OL
3) On plasticity chart (Activity chart) for PI = 25 and LL = 49….CL

Answer: CL (low plasticity clay).

However,

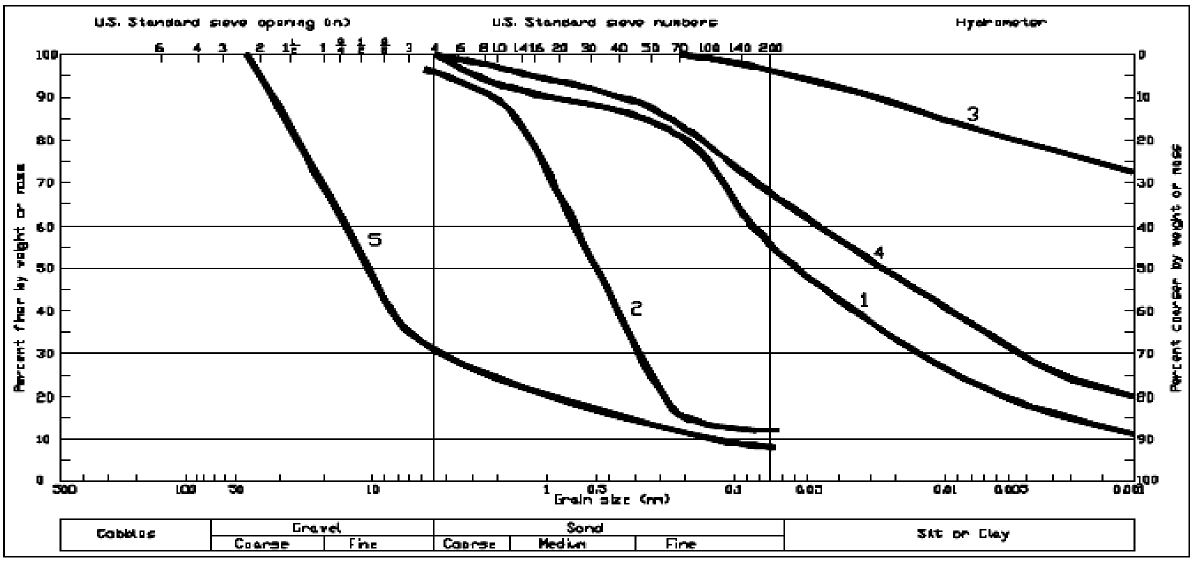
4) Since LL = 49 ~ 50 use CL – CH (AASHTO silt-clay)

Soil # 5

1) $H_U = \frac{D_{60}}{D_{10}} = \frac{28mm}{0.5mm} = 56$ *Well graded*?
2) $C_C = \frac{D_{30}{}^2}{D_{10}D_{60}} = \frac{9.5^2}{0.5(28)} = 6.4 > 3 \therefore$ *gap graded* $\longrightarrow P$
3) 77% gravel, 29% sand, 4% fines.

Answer: GP (a poorly graded or gap-graded gravel); AASHTO gravel (see note below).

Note: The USCS classifies fine soils (M,C) according to their plasticity (LL, PL and PI) versus their grain size, as done by AASHTO. For example, a soil may contain large amounts of fine silica sand, which has no plasticity and yet be deemed unsuitable as a road base by AASHTO. On the other hand, a 95% G and S, with 5% montmorrilonite clay will be classified as an A-2 by AASHTO (which is considered a good road material) and yet it could cause severe rutting.



Ex3

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Using the attached grain size distribution curves, classify the following soils:

Sample # 1: LL = 19, PI =0 35% < # 200 sieve.
Classification : S M ←

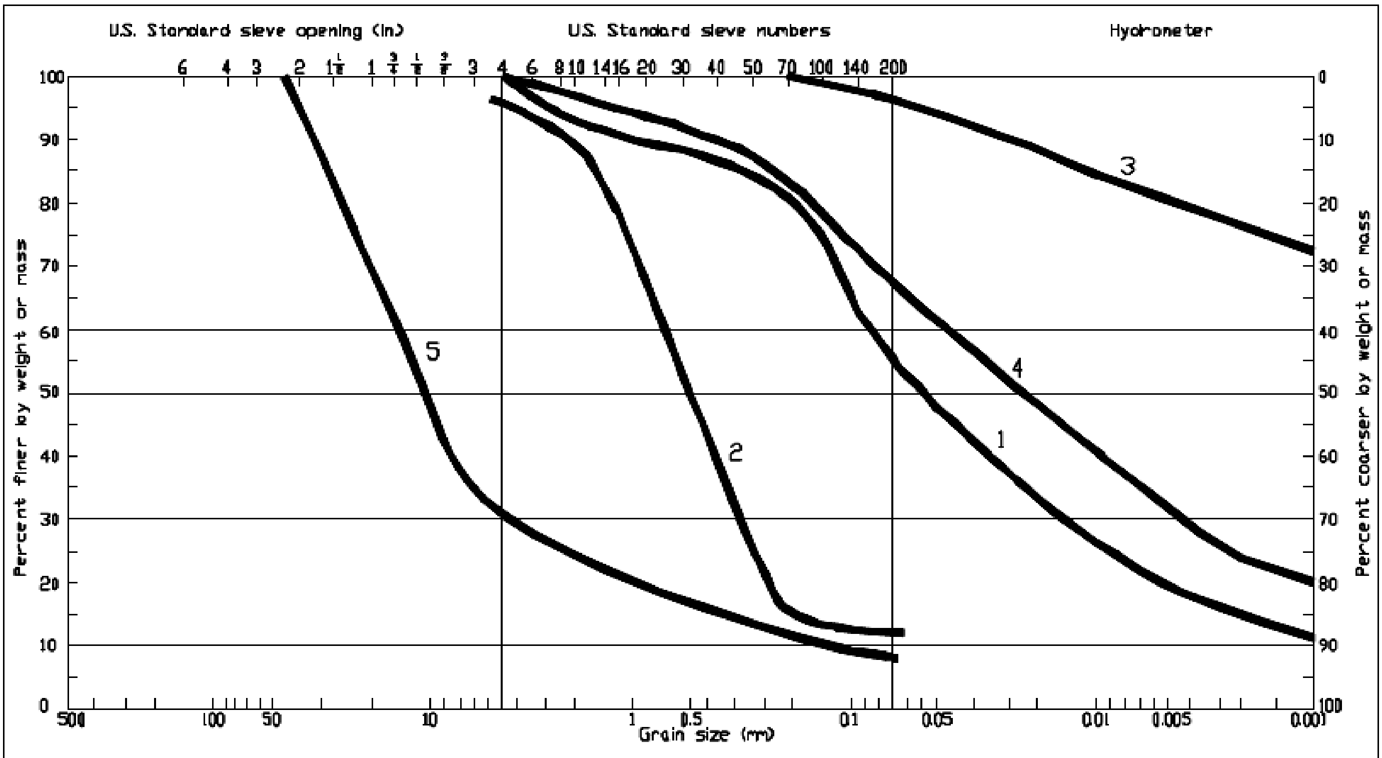
Sample # 2: LL = 44, PI = 0 42% < # 200 sieve
Classification : S M ←

Sample # 3: LL = 30, PI = 0 42% < # 200 sieve
Classification : S M ←

Sample # 4: LL = 40, PI = 12 76% < # 200 sieve
Classification : M L ←

Sample # 5: LL = 67, PI = 27
Classification : M H ←

Sample # 6: LL = 0, PI = 0 D_{60} = 0.2 mm D_{10} = 0.7 mm 100% < # 4 sieve
$D_{60} / D_{10} \sim 3$
Classification : S P ←



Cobbles	Gravel		Sand			Silt or Clay
	Coarse	Fine	Coarse	Medium	Fine	

Ex4

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A sample of soil weights 1.5 N. Its clay fraction is 0.34 N by weight. If its liquid limit is 60% and its plastic limit is 26%, what type of clay are you probably studying?

W = 1.5 N
W_c = 0.34 N (or 23% of W)
I_p = LL – PL = 60 – 26 = 34 %

$$A = \frac{I_P}{\%of\ clay\ fraction} = \frac{34\%}{23\%} \approx 1.5 \quad \therefore\ \textit{this clay is probably a member of the Kaolinite family.}$$

Ex5
During a hydrometer analysis, a soil with a $G_S = 2.60$ is immersed in a water suspension with a temperature of 24 °C. An R = 43 cm is obtained after 60 minutes of sedimentation. What is the diameter D of the smallest size particles that have settled during that time?

$G_S = 2.60$, at T =24 °C $K = 0.01321$

$R = 43$ cm corresponds to L = 9.2 cm.

$$\therefore \quad D = K\sqrt{\frac{L}{t}} = 0.01321\sqrt{\frac{9.2cm}{60\min}} = 0.00517\ mm$$

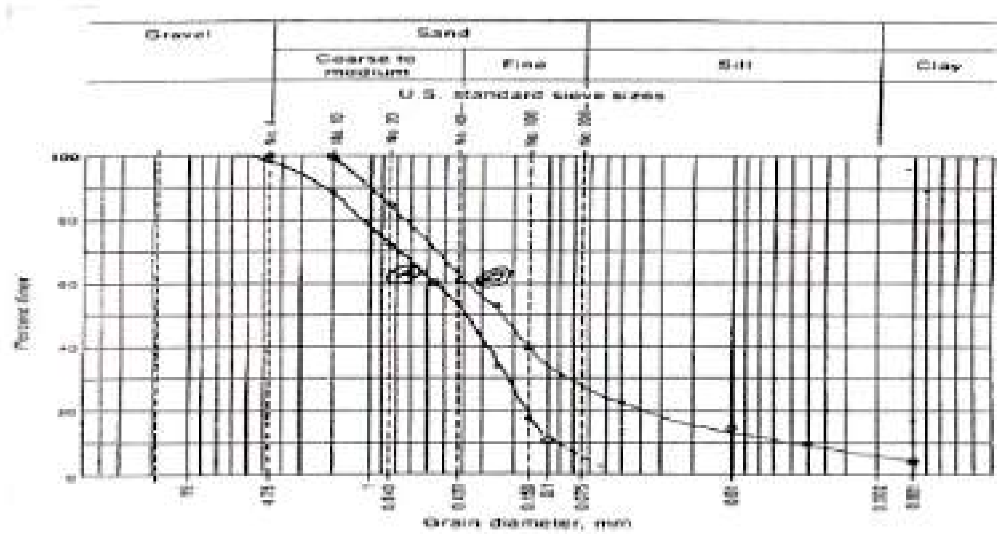
$$\therefore \rightarrow \quad L = 16.29 - (0.164R) = 16.29 - (0.164(43)) = 9.2cm$$

Ex6

The grain size distribution of two soil samples A and B are listed below.
a) Plot the grain size distribution

Sieve No.	Percent Passing Soil A	Percent Passing Soil B
4	98.0	100.0
10	88.2	100.0
20	72.3	84.5
40	54.1	61.3
60 (0.25 mm)	32.7	53.5
100	17.1	39.2
	.05 mm	23.2
	.01 mm	15.8
	.005 mm	9.7
	.001 mm	3.4

b) Find H_u for soil A. How would you classify this soil?
c) Estimate the clay fraction of soil B. If LL = 55%, and the PL = 51%, what is the activity of this clay? Suggest to what clay family it probably belongs.



Solution:

b) For soil A, uniformity coefficient $H_U = \dfrac{D_{60}}{D10} = \dfrac{0.56mm}{0.10mm} = 5.6$ Thus, soil A is a well graded sand.

c) Estimate the clay fraction as 5%;

$$LL = 55\%,\quad PL = 51\%,\quad \text{thus } I_P = 4\% \quad \text{Activity } A = \frac{I_P}{\%CLAY} = \frac{4}{5} = 0.8$$

Thus clay probably belongs to the Kaolinite family.

Ex7

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Data were obtained from a relative density test using information from six laboratory tests.

	Limiting ?	average ? in kN/m³
? max	18.07	17.52
? min	14.77	15.56
? field		16.97

Compute the range of D_r (relative density)

$$\therefore \quad D = K\sqrt{\frac{L}{t}} = 0.01321\sqrt{\frac{9.2cm}{60\min}} = 0.00517\ mm$$

$$\therefore \quad D_r = \left(\frac{\gamma_n - \gamma_{\min}}{\gamma_{\max} - \gamma_{\min}}\right)\left(\frac{\gamma_{\max}}{\gamma_n}\right)$$

$$range\ 1\ (low\ \gamma_{\min})(high\ \gamma_{\max}) \qquad D_r = \left(\frac{16.97-14.77}{18.07-14.77}\right)\left(\frac{18.07}{14.97}\right) = 0.71$$

$$range\ 2\ (avg\ \gamma_{\min})(high\ \gamma_{\max}) \qquad D_r = \left(\frac{16.97-15.56}{18.07-15.56}\right)\left(\frac{18.07}{14.97}\right) = 0.60$$

$$range\ 3\ (low\ \gamma_{\min})(avg\ \gamma_{\max}) \qquad D_r = \left(\frac{16.97-14.77}{17.52-14.77}\right)\left(\frac{18.07}{16.97}\right) = 0.83$$

$$range\ 4\ (avg\ \gamma_{\min})(avg\ \gamma_{\max}) \qquad D_r = \left(\frac{16.97-15.56}{17.52-15.56}\right)\left(\frac{17.52}{16.97}\right) = 0.74$$

$$\therefore \quad 60\% \le D_r \le 83\%$$

Ex8

Prove that $e_{min} = 0.35$.

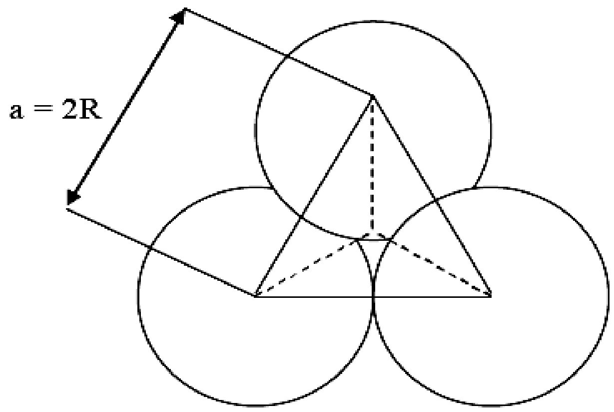
$$V_{tetr} = 0.1179a^3 = 0.1179(2R)^3 = 0.943R^3$$

$$V_{sphere} = \frac{4}{3}\pi R^3$$

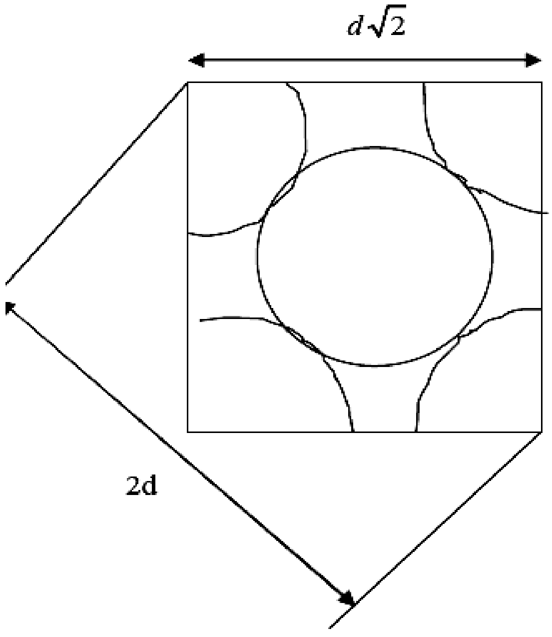
the volume of the sphere occupied by the tetrahedron is:

$$\therefore \quad V_{sphere(tetr)} \equiv \frac{60}{360} = 0.167 = 16.7\%$$

$$\therefore \quad e = \frac{V_V}{V_S} = \frac{V - V_{sphere}}{V_{sphere}} = \frac{0.943R^3}{0.167\left(\frac{4}{3}\pi R^3\right)} = 0.35$$



ALTERNATE METHOD:



$$Volume\ of\ \ cube = \left(d\sqrt{2}\right)^3 = d^3 2\sqrt{2}$$

$$Volume\ of\ sphere = 4\left(\frac{\pi d^3}{6}\right) = \frac{2}{3}\pi d^3$$

$$e_{min} = \frac{V_{cube} - V_{sphere}}{V_{sphere}} = \frac{2d\sqrt{2} - \left(\frac{2}{3}\pi d^3\right)}{\frac{2}{3}\pi d^3} = 0.35$$

$$e_{min} = 0.35 \qquad OK\ \ !!!$$

Ex9

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The fines fraction of a soil to be used for a highway fill was subjected to a hydrometer analysis, by placing 20 grams of dry fines in a 1 liter solution of water (dynamic viscosity 0.01 poise at 20 degrees centigrade). The specific gravity of the solids was 2.65.

a) Estimate the maximum diameter d of the particles found at a depth of 5 cm after a sedimentation time of 4 hours has elapsed;
b) What type of soil is this?

Solution:

Using Stoke's relation:

a)

$$v = \frac{\gamma_S - \gamma_w}{18\eta} D^2$$

or

$$D(mm) = \sqrt{\frac{18(\eta)}{\gamma_S - \gamma_W}\left(\frac{L(cm)}{t(\min)}\right)}$$

where: $t = 4$ hours $= 14{,}400$ sec. , $L = 5$ cm

$$\eta = 10^{-2}\, poise = 10^{-2}\ \frac{g.\sec}{cm^2}$$

$$G_S = 2.65 = \frac{\gamma_S}{\gamma_W} = 2.65(9.81)\left(\frac{g.\sec}{cm^2}\right)$$

$$d = \sqrt{\frac{18x10^{-2}\left(\frac{g.\sec}{cm^2}\right)(5cm)}{9.81\left(\frac{g.\sec}{cm^2}\right)(2.65-1.00)(14{,}400\sec)}} = 0.020\ mm$$

This diameter corresponds to a silt. (M).

Ex10

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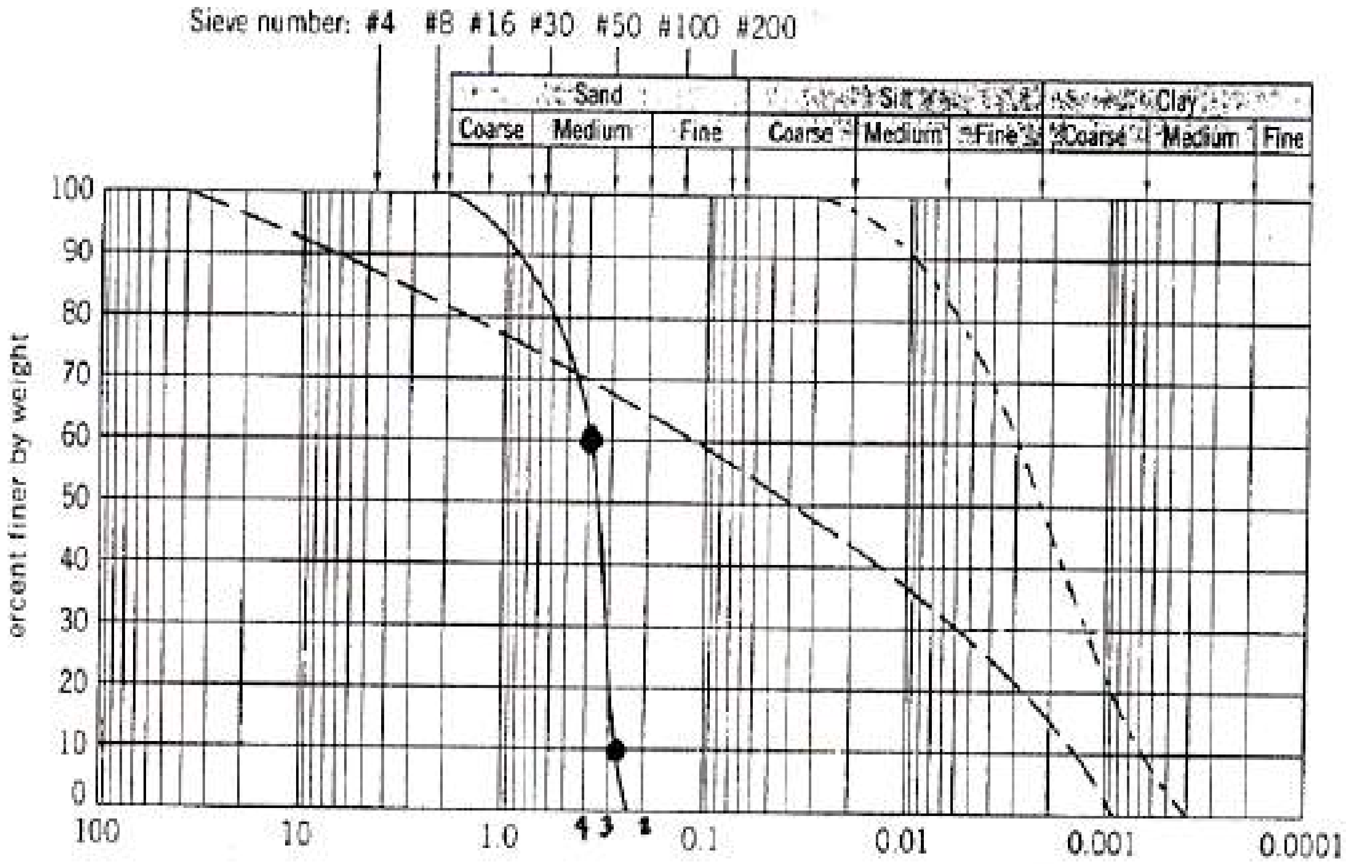
Using the grain-size distribution curve shown below, determine the coefficient of uniformity for the soil A (the solid curve).



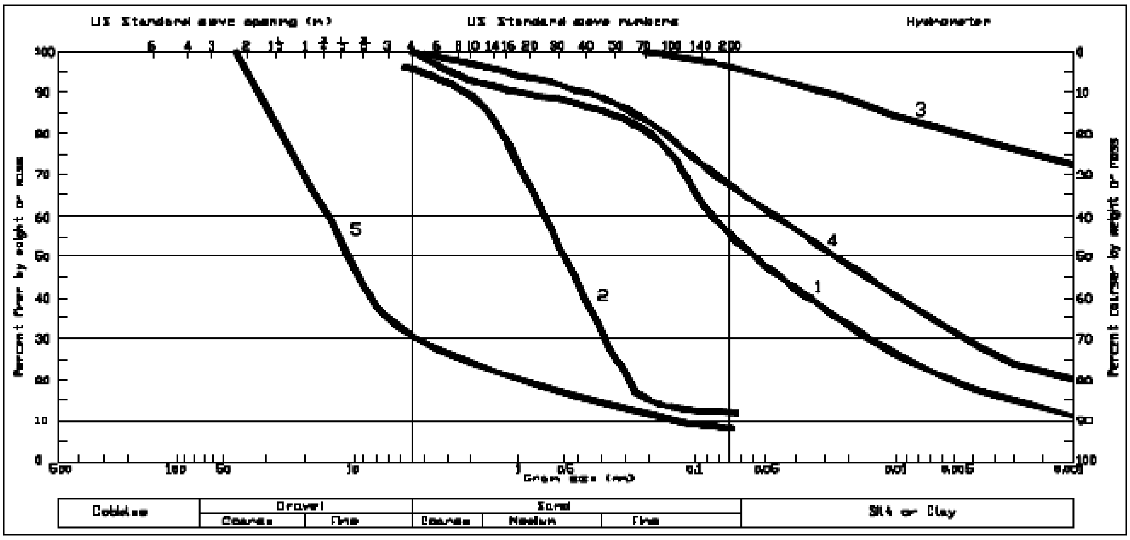
Solution:

$D_{60} = 0.40$
$D_{10} = 0.28$

$$C_U = \frac{D_{60}}{D_{10}} = \frac{0.40}{0.28} = 1.43$$

Ex11

1. Using the grain size distribution plot of soils 1, 2 and 3 shown below, find their H_u.



AASHTO Definitions of Gravel, Sand, and Silt-Clay

Soil Fraction	Size Range
Boulders	Above 75 mm
Gravel	75 mm to No. 10 sieve (2.0 mm)
Coarse sand	No.10 (2.0 mm) to No. 40 (0.425 mm)
Fine sand	No. 40 (0.425 mm) to No. 200 (0.075 mm)
Silt-clay (combined silt and clay)	Material passing the 0.075 mm (No. 200) sieve

2. Find H_u for all three soils, and classify as to grading.

A) $H_u = D_{60} / D_{10} = 0.075$ / ? cannot classify
B) $H_u = 0.60 / 0.20 = 3$ uniform
C) $H_u =$? / ? cannot classify

Ex12

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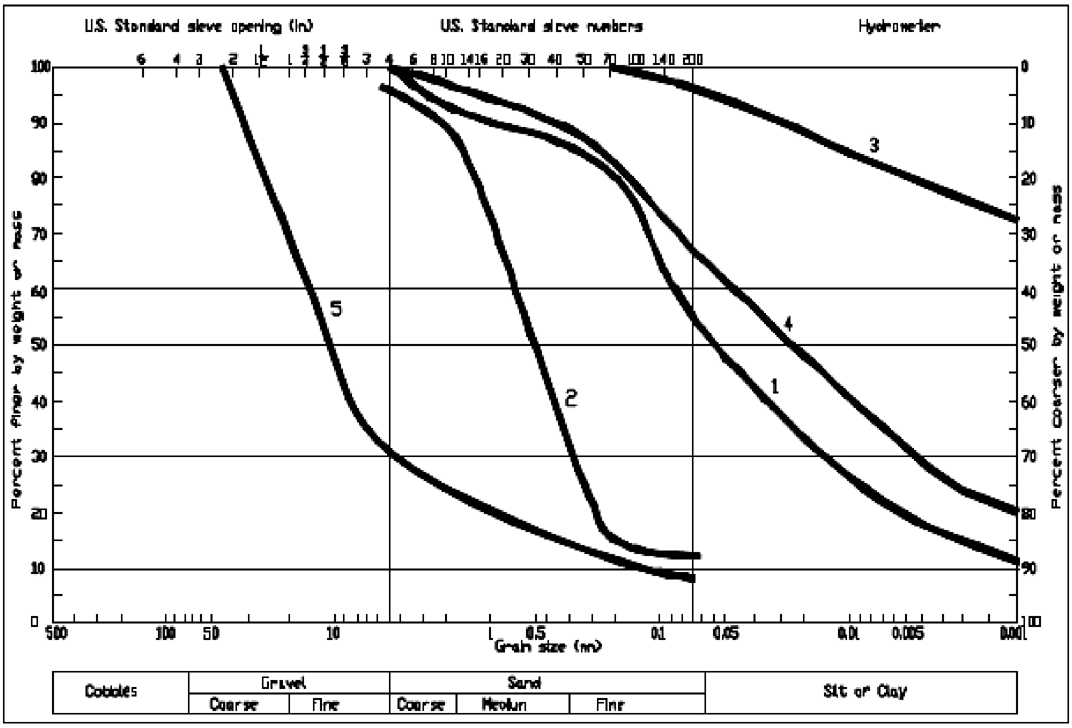
Classify soils #4 and #5 as listed in the attached page according to the Unified Soil Classification System. Explain your steps, and compare with the AASHTO system.

Soil #4

1) 70% is smaller than #200 sieve therefore a M or a C
2) LL < 50 therefore ML, CL or OL
3) On Plasticity Chart (Activity Chart) for PI = 25 and LL = 49 $\longrightarrow$ CL
   Ans. CL (low plasticity clay)
4) However, since LL = 49=50 use CL-CH (AASHTO silty-clay)

Soil #5

1) Hu =D60 / D10 = 28 mm / 0.5mm = 56 well graded?
2) Cc = D30 / D10*D60 = 9.5^2/0.5*28 = 6.4 > 3 therefore gap graded $\longrightarrow$ GP
3) 77% gravel, 29% sand, 4% fines
   Ans. GP (poorly or gap-graded gravel) (AASHTO gravel)



Ex13

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If a hydrometer's mark has descended a distance L = 4.5cm after 1hr = 60 minutes, and the suspension temperature = 25°C, for a G_s = 2.80 what diameter is precipitated?

(For 25° C, the constant K = 0.01232)

Solution:

$$D\ (mm) = K\left(\sqrt{\frac{4.5cm}{60min}}\right) = (0.01232)\left(\sqrt{\frac{4.5cm}{60min}}\right) = 0.0033\ mm$$

This soil is a silt (M)

Ex14

During a hydrometer analysis, a soil with a G_s = 2.60 is immersed in a water suspension with a temperature of 24°C. An R = 43 cm is obtained after 60 minutes of sedimentation. What is the diameter D of the smallest-size particles that have settled during that time?
(K = 0.01321)
(L = 9.2cm)

Solution:

$$D = K\left(\sqrt{\frac{L}{t}}\right) = 0.01321\left(\sqrt{\frac{9.2cm}{60min}}\right) = 0.00517mm = 5.2 \times 10^{-3}\ mm\ (a\ silt)$$

Ex15

The fines fraction of a soil to be used for a highway fill was subjected to a hydrometer analysis by placing 20 grams of dry fines in a 1-liter solution of water (dynamic viscosity 0.01 Poise at 20°C). The specific gravity of the solids was 2.65.

a) Estimate the maximum diameter d of the particles found at a depth of 5cm after a sedimentation time of 4 hours has elapsed.

b) What type of soil is this?

Solution:

a) Using *Stoke's* relation $\quad d = \left(\sqrt{\frac{18\eta}{(\gamma s - \gamma w)} \left(\frac{L}{t}\right)} \right)$

T = 4 hours = 14,400 sec.
L = 5 cm

$\eta = 10^{-2}$ Poise $\left(\text{dyne} \left(\frac{\text{dynes}}{\text{cm}^3} \right) \right)$

$G_s = 2.65 = \gamma s / \gamma w$ therefore $\gamma s = 2.65 \text{ x } 9.81 \left(\frac{\text{dynes}}{\text{cm}^3} \right) = 26 \left(\frac{\text{dynes}}{\text{cm}^3} \right)$

$$d = \left[\sqrt{\frac{\left(18 \cdot 10^{-2} \frac{dynes\ x\ \sec}{cm^2\ x\ 5cm} \right)}{\left(9.81 \cdot \frac{dynes}{cm^3} (2.65 - 1.00)\ 14{,}400\sec \right)}} \right] = 0.020 \text{ mm}$$

b) This soil diameter corresponds to a silt.

Ex16

The formula for the relative compaction D_r is:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

From this expression, derive the equivalent equation,

$$D_r = \frac{(\gamma_{d(field)} - \gamma_{d\,min})}{(\gamma_{d\,max} - \gamma_{d\,min})} \frac{(\gamma_{d\,max})}{(\gamma_{d(field)})}$$

Solution:

g$_{d\,(min)}$ = dry unit weight loosest condition (void ratio e $_{max}$)

g$_d$ = in-situ dry unit weight

g$_{d\,(max)}$ = dry unit in densest condition (void ratio e is min)

where

$$\gamma_d = \frac{WS}{V} = \frac{G_S \gamma W}{1+e}$$

D_r = 0 = loose; 1= very dense

$$D_r = \frac{\dfrac{1}{\gamma_{d\,min}} - \dfrac{1}{\gamma_d}}{\dfrac{1}{\gamma_{d\,min}} - \dfrac{1}{\gamma_{d\,max}}} = \left[\frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_d}\right] = \left[\frac{\gamma_{d(field)} - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_{d(field)}}\right]$$

For example, what is the RC (relative density) of a sand in the field, if it was tested to be at 98% Standard Proctor, its maximum unit weight was 18.8 KN/m^3 and the minimum unit weight was 14.0 kN/m^3.

$$RC = 98\% = \frac{\gamma_{d(field)}}{\gamma_{d\,(Std.Proct.)}} = \frac{\gamma_{d(field)}}{18.8} \therefore \gamma_{d(field)} = 18.4 kN/m^3$$

$$D_r = \left[\frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_d}\right] = \frac{(18.4 - 14.0)(18.8)}{(18.8 - 14.0)(18.4)} = 94\%$$

<u>Additional Problems</u>

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Question 1: The volume of an odd shaped object can be calculated by measuring the mass of the object in air and water. According to Archimedes Principle, the weight difference can be used to calculate the density of the object (the mass of displaced water is related to the volume of the object). This principle can be used to determine the volume of a soil sample with an irregular shape. A coating of wax around a soil sample is used to prevent water from entering or leaving a soil sample during storage. Using a phase diagram, calculate the bulk density (ρ_T), dry density(ρ_d), void ratio (e), degree of saturation (S_r) and unit weight (γ) of the soil.

Given: Mass of soil sample (in air): 191.6 g; Mass of soil + wax (in air): 234.2 g
Mass of soil + wax (in water): 66.4 g; Mass of dry soil (in air): 157.3 g
G_S: 2.70; G_{WAX}: 0.95

Calculate the volume of water required to raise the degree of saturation to 95% (Assuming the total volume of the sample remains constant).

Question 2: Plot the grain size distribution curve and determine the coefficients of uniformity C_U and curvature C_C for each soil. Comment on the shape of each curve and estimate permeability values.

Metric Sieve Size	US Sieve Size	Soil A (g)	Soil B (g)	Soil C (g)
25 mm	1 in	0	0	0
19 mm	0.75 in	5.90	0	0
9.5 mm	0.375 in	5.02	134.25	0
4.76 mm	No. 4	10.98	90.3	0
2.38 mm	No. 8	47.25	62.34	0
0.84 mm	No. 20	465.32	68.23	0
420 μm	No. 40	34.66	65.75	162.32
250 μm	No. 60	17.35	53.78	148.23
150 μm	No. 100	0	42.50	96.56
75 μm	No. 200	0	41.59	195.68
Pan	Pan	0	2.13	0

Question 3: Compute the specific surface (S_O) [m^2/g] for the following particulate arrangements:

a) equal spheres of diameter d (m)
b) cylinders of length L and diameter αL
c) rectangular prismatic particles L*L* αL

Question 4: Soil having a void ratio of 0.68 as it exists in a gravel pit is to be excavated and transported to a fill site where it will be compacted to a void ratio of 0.45. The volume of fill required at the construction site (at a void ratio of 0.45) is 2,500m^3. Find the volume of soil that must be excavated from the gravel pit to furnish the required the volume of fill.

#1 Mass of soil sample (in air) = 191.6 g
Mass of soil + wax (in air) = 234.2 g
Mass of soil + wax (in water) = 66.4 g
Mass of dry soil (in air) = 157.3 g
$G_s = 2.70$ $\quad G_{wax} = 0.95$

$$w = \frac{m_w}{m_s} = \frac{191.6 - 157.3\,g}{157.3\,g} = 0.218$$

$$M_{wax} = 234.2 - 191.6 = 42.6\,g$$

$$V_{wax} = \frac{M_{wax}}{G_{wax}\,\rho_w} = \frac{42.6\,g}{0.95\,(1.0\,g/cm^3)} = 44.84\,cm^3$$

$$V_s = \frac{M_s}{G_s\,\rho_w} = \frac{157.3\,g}{2.7\,(1.0\,g/cm^3)} = 58.26\,cm^3$$

$$M_{water} = 191.6 - 157.3 = 34.3\,g$$

$$V_w = \frac{M_w}{\rho_w} = \frac{34.3\,g}{1.0\,g/cm^3} = 34.3\,cm^3$$

Mass of displaced water = 234.2 − 66.4 = 167.8 g

Volume " " " $= \dfrac{167.8\,g}{1.0\,g/cm^3} = 167.8\,cm^3$

Since $V_{wax} = 44.84\,cm^3$, Volume of displaced soil = 167.8 − 44.84 $= 122.96\,cm^3$

Volume of displaced soil sample = V_T

$$\therefore V_{air} = V_a = V_T - V_w - V_s = 122.96 - 34.3 - 58.26 = 30.4\,cm^3$$

Ps 1 (con't).

$$\rho_T = \frac{M}{V} = \frac{191.6\,g}{122.96\,cm^3} = 1.558\,g/cm^3 \quad or \quad 1558\,kg/m^3$$

$$\rho_d = \frac{\rho_T}{1+w} = \frac{1.558\,g/cm^3}{1+0.218} = 1.279\,g/cm^3 \quad or \quad 1279\,kg/m^3$$

$$e = \frac{V_v}{V_s} = \frac{V_a + V_w}{V_s} = \frac{34.3+30.4}{58.26} = 1.11$$

$$S_r = \frac{V_w}{V_v} \times 100\% = \frac{34.3}{34.3+30.4} \times 100\% = 53.0\%$$

$$\gamma = \frac{Mg}{V} = \rho g = (1558\,kg/m^3)(9.8\,m/s^2) = 15268.4\,\frac{N}{m^3} \quad or \quad 15.3\,\frac{kN}{m^3}$$

Volume of water to be added to raise S_r to 95% ?

$$S_r = 0.95 = \frac{V_w}{V_v} \qquad \therefore V_w = 0.95\,V_v = 0.95 \times 64.7 = 61.47\,cm^3$$

Question 3

$$\text{Specific Surface} = \frac{\text{Total Area}}{\text{Total Mass}} \qquad \rho_s = \rho_w G_s$$

a) spheres of diameter $= 2r$

$$S_o = \frac{4\pi r^2}{\frac{4}{3}\pi r^3 \rho_s} = \frac{3}{\rho_w G_s r} = \frac{6}{\rho_w G_s d}$$

b) cylinders Length $= L$ diameter αL.

$$S_o = \frac{2\pi r L + 2\pi r^2}{\pi r^2 L \rho_s} = \frac{2}{\rho_w G_s r} + \frac{2}{\rho_w G_s L} = \frac{4}{\rho_w G_s L}\left(\frac{1}{\alpha} + \frac{1}{2}\right)$$

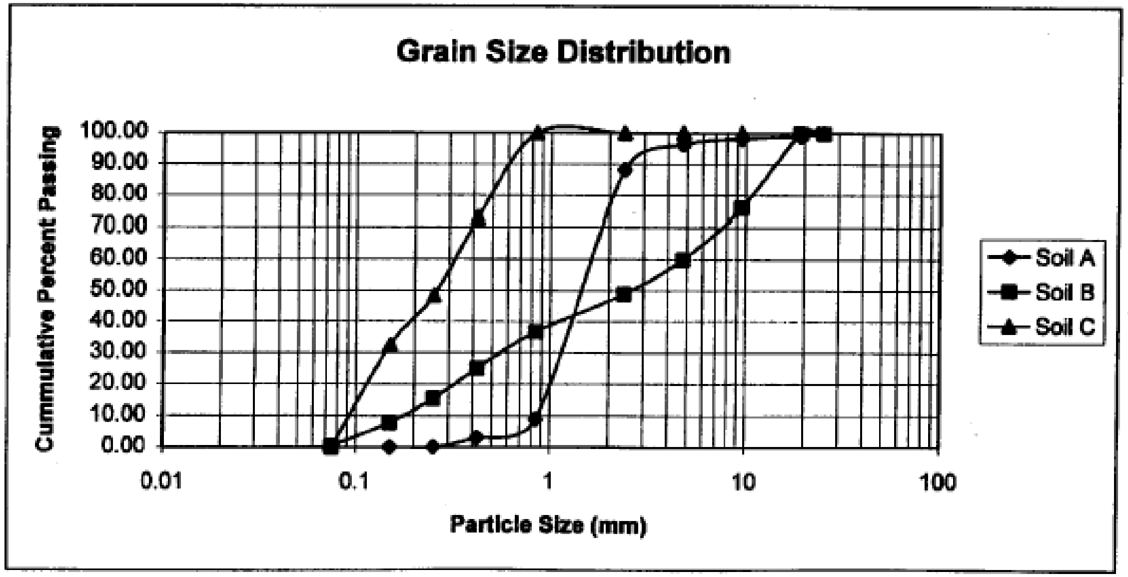
c) rectangular prism $(L . L . \alpha L)$

$$S_o = \frac{4\alpha L^2 + 2L^2}{\alpha L^3 \rho_s} = \frac{2}{\rho_w G_s L}\left(2 + \frac{1}{\alpha}\right)$$

Particle Size (mm)	Soil A Weight Retained (g)	Soil B Weight Retained (g)	Soil C Weight Retained (g)	Soil A Percent Retained (%)	Soil B Percent Retained (%)	Soil C Percent Retained (%)	Soil A Cummulative Percent Passing (%)	Soil B Cummulative Percent Passing (%)	Soil C Cummulative Percent Passing (%)
25	0	0	0	0.00	0.00	0.00	100.00	100.00	100.00
19	5.9	0	0	1.01	0.00	0.00	98.99	100.00	100.00
9.5	5.02	134.25	0	0.86	23.94	0.00	98.14	76.06	100.00
4.76	10.98	90.3	0	1.87	16.10	0.00	96.27	59.96	100.00
2.38	47.25	62.34	0	8.06	11.11	0.00	88.21	48.85	100.00
0.84	465.32	68.23	0	79.34	12.17	0.00	8.87	36.68	100.00
0.42	34.66	65.75	162.32	5.91	11.72	26.97	2.96	24.96	73.03
0.25	17.35	53.78	148.23	2.96	9.59	24.63	0.00	15.37	48.40
0.15	0	42.5	95.56	0.00	7.58	15.88	0.00	7.80	32.52
0.075	0	41.59	195.68	0.00	7.42	32.52	0.00	0.38	0.00
Pan	0	2.13	0	0.00	0.38	0.00	0.00	0.00	0.00
Total Mass	586.48	560.87	601.79						

$$C_u = \frac{D_{60}}{D_{10}} \qquad C_c = \frac{D_{30}^2}{D_{10} D_{60}}$$

	D_{10}	D_{30}	D_{60}	C_u	C_c
Soil A	0.85	1.1	1.75	2.06	0.81
Soil B	0.18	0.54	1.60	8.89	1.01
Soil C	0.09	0.14	0.31	3.44	0.70



4.1 Void Ratio = 0.68

Compacted Void Ratio = 0.45

Volume of compacted fill = 2500 m^3

$e = \frac{V_v}{V_s}$ $\quad V_T = V_s + V_v \quad V_{T_{0.45}} = 2500 m^3$

$0.45 = \frac{V_T - V_s}{V_s}$ where $V_T = 2500 m^3$

$\therefore 0.45 = \frac{2500 - V_s}{V_s}$

$0.45 V_s = 2500 - V_s$

$1.45 V_s = 2500 \ m^3$

$\therefore V_s = 1724.14 \ m^3$

$e = 0.68 \quad V_T = ?$

$0.68 = \frac{V_v}{V_s}$ using $V_s = 1724.14 m^3$ (V_s remain the same for both soils), we need to determine V_v for the 0.68 void ratio soil

$0.68 (1724.14) = V_v$

$V_v = 1172.42 \ m^3$

$\therefore V_T = V_s + V_v$

$= 1724.14 + 1172.42$

$= 2896.56 \ m^3$

problems

1-A laboratory sample of silty clay has a volume of 14,.88 cm^3 , a total mass of 28.81g, a dry mass 24.83g and a specific gravity 2.7.Determine void ratio and degree of saturation.

2-A sample of saturated soil has a moisture content of 29% and bulk density of 1930 kg/cm^3. Determine the dry density and the void ratio of the soil and specific gravity of the particles.
What would be the bulk density if S=90% .

3-The natural water content of a sample taken from a soil deposit was found to be 11.5% .It has been calculated that the maximum density for the soil will be obtained when the water content reaches 21.5% .Compute how many grams of water must be added to each 1000g of soil in its natural state in order to increase the water content to 21.5% .(Ans.100g).

4-A dry sample of soil having the following properties: L.L=521., PL=30% , Gs=2.7 and e = o.53 .Find shrinkage limit , dry density , dry unit weight and air content of dry state .

5-A saturated soil sample has a volume of 20 cm^3 at L.L. Given L.L=42% ,P.L=30 % , S.L=17% and Gs=2.74. Find minimum volume which the soil can attain.
6-A sample of saturated clay had a volume of 97cm^3 and a mass of o.202 kg. When completely dried out the volume of the sample was 87 cm^3 and its mass o.167 kg. Find initial water content, shrinkage limit and specific gravity of the solid particles. (Ans. 21%,15%,2.69)

7- The Atterberg Limits of a clays soil are : LL= 52%, P.L =30% and SL= 18%.If a Specimen of this soil Shrinks from a volume of 39.5cm^3 at the L.L to a volume of 24.2 cm^3 at the S.L . Calculate the specific gravity. (Ans. 2.79)

8- A saturated Sample of clay with an SL of 22% has a natural water content of 35%. What would its dry volume be as a percentage of its original (natural) volume if Gs = 2.70 (Ans. 82%)

9-The Shrinkage limit of a 0.1m^3 sample of a clay is 15% and its natural water content is 34%. Assume Gs is 2.68 , estimate the volume of the sample when the water content 12.7%.(Ans.0.07)

10- The L.L of a medium sensitive clay is 56% and P.I 28%.At it natural water content , the void ratio is 1.03 while after shrinkage the minimum void ratio is o.72. Assuming Gs=2.72 , calculate the shrinkage limit of the clay.

11-Use the USCS to classify the soil with the following data:

- grain size distribution:
    - A-70% of the material is retained on the No.200 sieve.
    - B-more than 50% of the percent above is retained on the 4.75 mm sieve.
- For the material passing the o.42 mm sieve (No.40):L.L=39% and P.I=19%.

12-Classify a soil having the following informations:

- particle size distribution :material passing the No.200 sieve is 55%
- Atterberg Limits: L.L=56% ,P.I=28%

13- Use the USCS to classify a soil having:

- 61% of the soil passes through the No.200 sieve.
- L.L=26% , P.L=20%

$-\tau_{xy}\cos 2\theta$

$+\tau\ \sin 2\theta$

14- Classify a soil using the USCS:

D_{10} = 0.085 mm   L.L=30%

D_{30} = 0.12 mm

D_{60} = 0.135 mm   P.L=22%

% retained on the No.200 sieve = 90%

% passing the No.4 sieve = 95%

15- Classify a soil using the USCS :

Sieve No:	4	10	40	100	200
Finer:	97	90	40	8	5

P.I :NP

D_{10}=o.18 mm

D_{30}=0.34 mm

D_{60}=0.71 mm

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Origin of Clay Minerals

The contact of rocks and water produces clays, either at or near the surface of the earth" (from Velde, 1995).

Rock + Water → Clay

For example,

The CO_2 gas can dissolve in water and form carbonic acid, which will become hydrogen ions H^+ and bicarbonate ions, and make water slightly acidic.

$$CO_2 + H_2O \rightarrow H_2CO_3 \rightarrow H^+ + HCO_3^-$$

The acidic water will react with the rock surfaces and tend to dissolve the K ion and silica from the feldspar. Finally, the feldspar is transformed into kaolinite.

Feldspar + hydrogen ions+water → clay (kaolinite) + cations, dissolved silica

$$2KAlSi_3O_8 + 2H^+ + H_2O \rightarrow Al_2Si_2O_5(OH)_4 + 2K+ +4SiO_2$$

- Note that the hydrogen ion displaces the cations.
- The alternation of feldspar into kaolinite is very common in the decomposed granite.

- The clay minerals are common in the filling materials of joints and faults (fault gouge, seam) in the rock mass.

Clay Mineral:

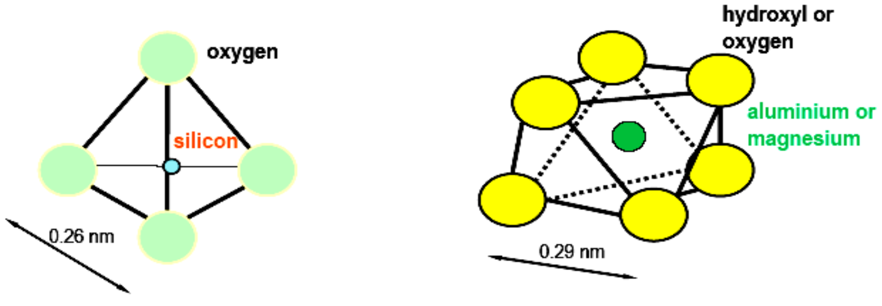
1. Possess the tendency to develop plasticity when mixed with water.
2. More than 90% of soils in the world are silicate minerals.

Two basic minerals:

1. silicon Oxygen tetrahedron (SiO_4)
2. Aluminum Magnesium octahedron $Al_2(OH)_3$,$Mg_2(OH)_3$

Basic Structural Units

Clay minerals are made of two distinct structural units.



Silicon tetrahedron **Aluminium Octahedron**

$(Si_2O_{10})^{-4}$ Replace four Oxygen with hydroxyls or combine with positive union

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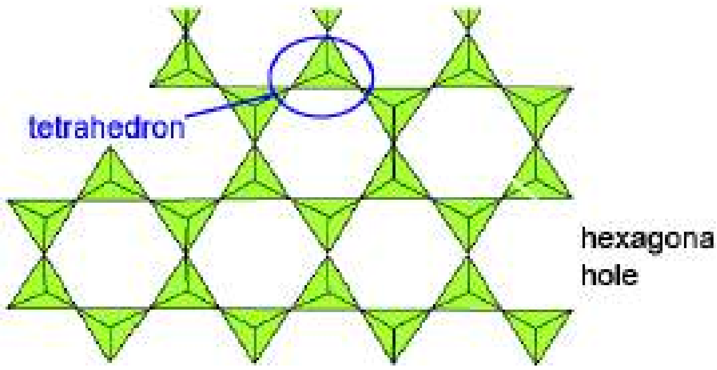
Tetrahedral Sheet

Several tetrahedrons joined together form a tetrahedral sheet.



Tetrahedron
Plural: Tetrahedral

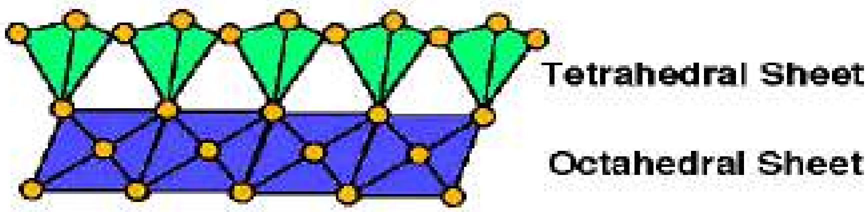
For simplicity, let's represent silica **tetrahedral sheet** by:

and alumina **octahedral sheet** by:

Different Clay Minerals

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

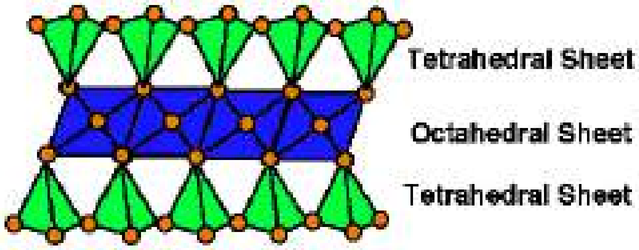
<u>1:1 Clay Mineral</u> (e.g., kaolinite, halloysite):



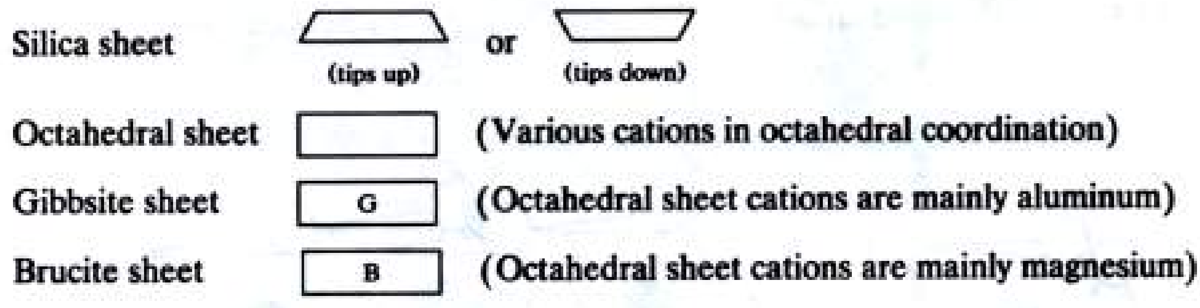
Different Clay Minerals

Different combinations of tetrahedral and octahedral sheets form different clay minerals:

<u>2:1 Clay Mineral</u> **(e.g., montmorillonite, illite)**



Unit-Summary



Kaolinite

- $Si_4Al_4O_{10}(OH)_8$. Platy shape
- The bonding between layers are van der Waals forces and hydrogen bonds (strong bonding).
- There is no interlayer swelling
- Width: 0.1~ 4μm, Thickness: 0.05~2 μm

17 μm

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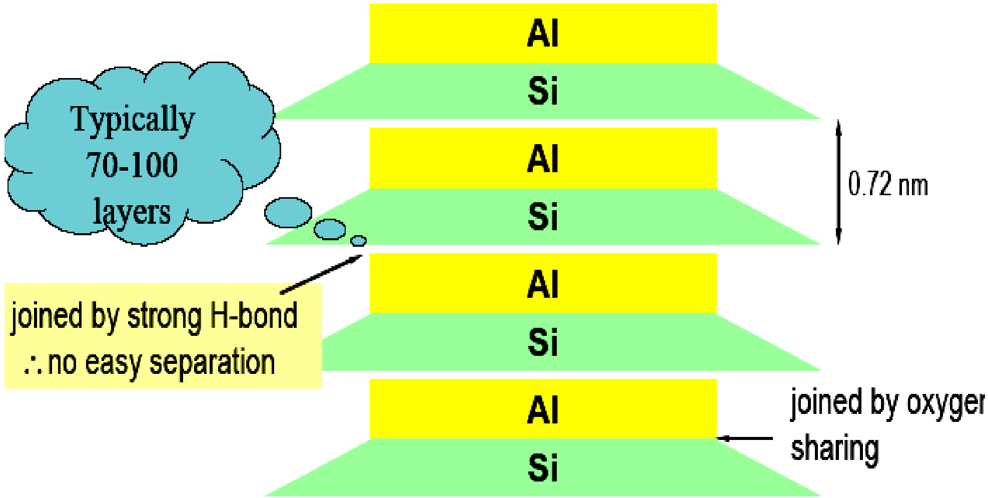
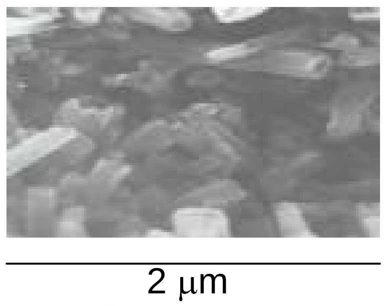
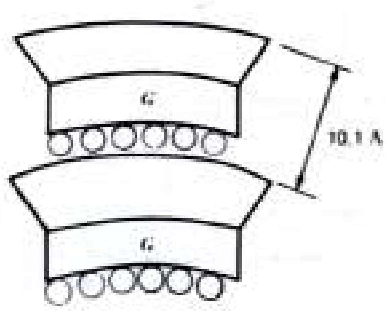
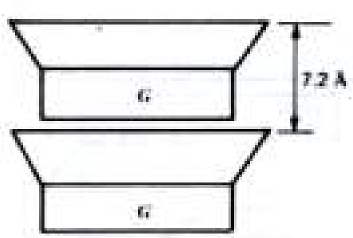


Halloysite

- $Si_4Al_4O_{10}(OH)_8 \cdot 4H_2O$
- A single layer of water between unit layers.
- The basal spacing is 10.1 Å for hydrated halloysite and 7.2 Å for dehydrated halloysite.
- If the temperature is over 50 °C or the relative humidity is lower than 50%, the hydrated halloysite will lose its interlayer water (Irfan, 1966). Note that this process is **irreversible** and will affect the results of soil classifications (GSD and Atterberg limits) and compaction tests.
- There is no interlayer swelling.
- Tubular shape while it is hydrated.







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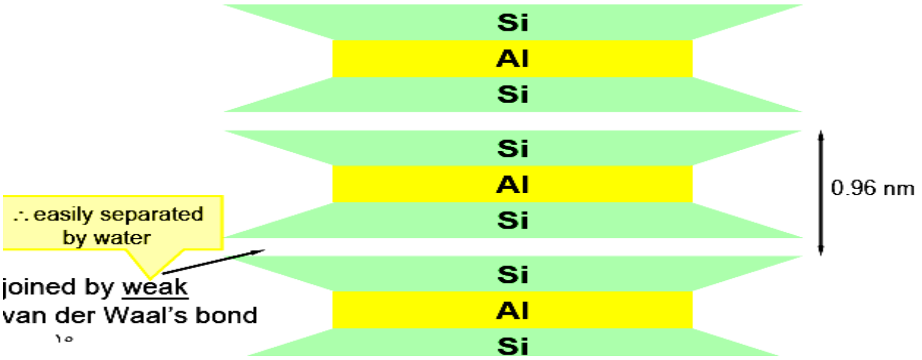
Montmorillonite

- also called **smectite**; expands on contact with water



- A highly reactive (expansive) clay
- $(OH)_4Al_4Si_8O_{20}.nH_2O$

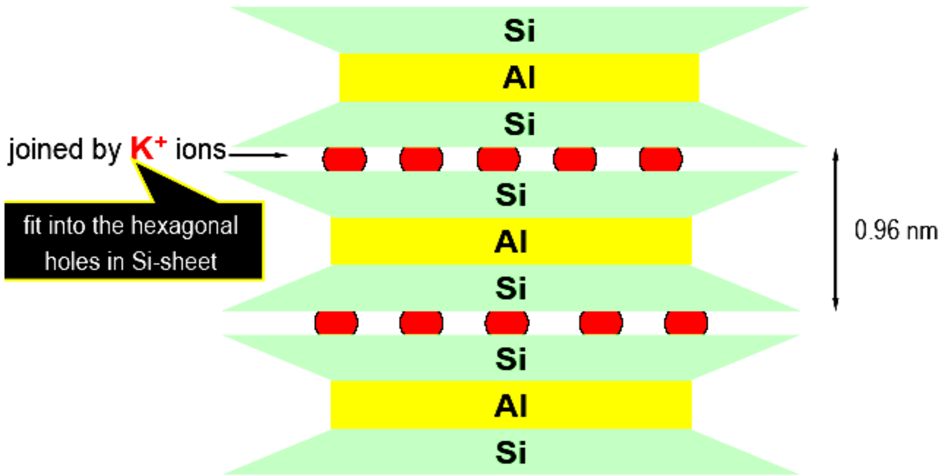
swells on contact with water

high affinity to water

Bentonite

- montmorillonite family
- used as drilling mud, in slurry trench walls, stopping leaks

Illite



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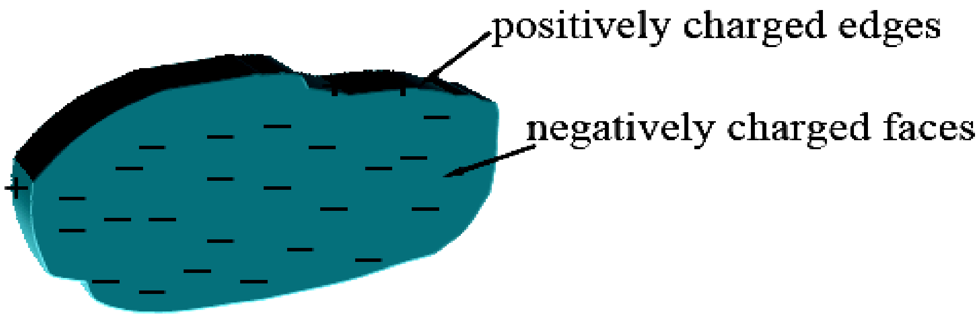
Clay Minerals : are complex aluminum silicates composed of two basic units , silica and alumina ,they possess the tendency to develop plasticity when mixed with water.

Nature of water in clay

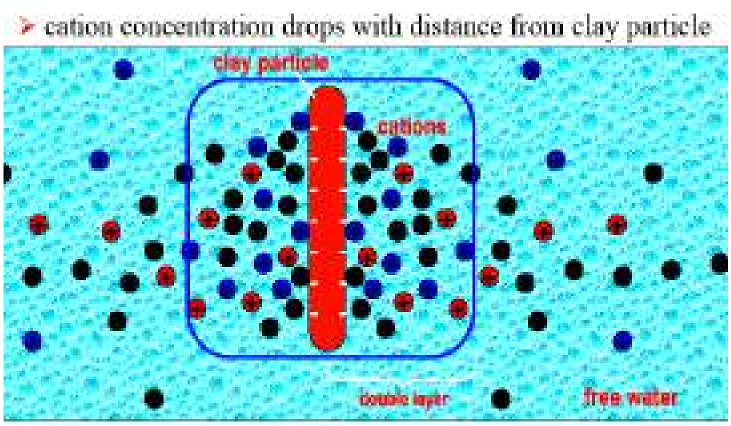
- shape of a clay particle is plately.
- The net charge at the face of clay particle is (-ve).
- There are (+ve) charge at edges of a clay particle.
- There are (+ve) ions (cautions) from salts in water, also the water molecules are dipolar or dipoles.



Clay Particle with Net negative Charge

Exchangeable Ions : a soil particle in nature attracts ions to neutralize its net charge, these ions are weakly held on the particle surface and can be replaced by other ions.
Al^{+3} >ca^{+2} >Mg^{+2} >NH_4^{+} >K^{+}>H^{+} >Na^{+} >Li^{+}

Cation Concentration in Water



Double Layer: describes *all the water* hold to clay particle by attractive force.

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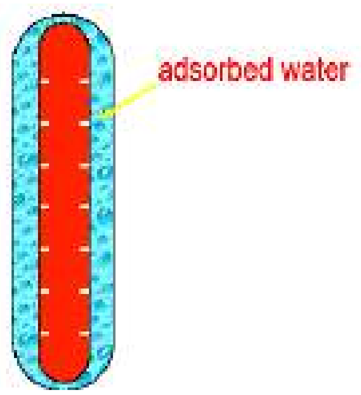
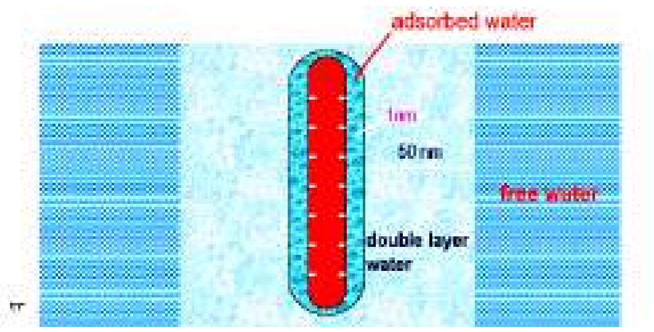
Adsorbed water: the *innermost* layer of double layer water which is held very strongly by clay.



Clay Particle in Water



Thickness of the double layer: the distance from the surface required to neutralize the net charge on the particle or the distance over which there is electrical potential.

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4-1. Calculate the specific surface of a cube (1) 10 mm, (b) 1 mm, (c) 1 μm, and (d) 1 nm on a side. Calculate the specific surface in terms of both areas and m^2/kg. Assume for the latter case that $\rho_s = 2.65\ Mg/m^3$.

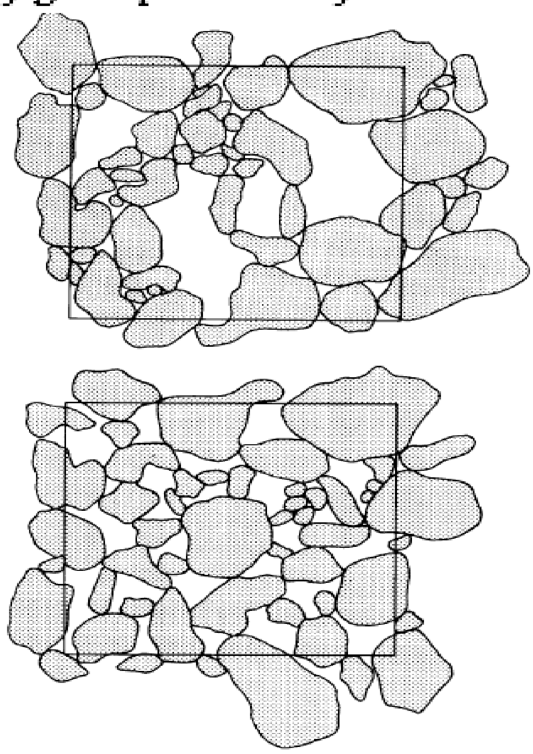
4-2. The values of e_{min} and e_{max} for a pure silica sand ($\rho_s = 2.65\ Mg/m^3$) were found to be 0.46 and 0.66 respectively. (a) what is the corresponding range in dry density? (b) If the in situ void ratio is 0.63, what is the density index?

4-3. Describe briefly the crystalline or atomic structure of the following ten minerals. Also list any important distinguishing characteristics.

(a) Smecite; (b) Brucite; (c) Gibbsite
(d) Attapulgite (e) Bentonite (f) Allophane
(g) Halloysite (h) Ilite (i) Mica
(j) Chlorite

4-4. Which sheet, silica or alumina, would you wear to a fancy dress dance? Why?

4-5. Given the particles in the attached Figure, is it realistic to show that all the particles are in contact with each other for this given plane? Any given plane? Why?



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QUESTION 4-1

GIVEN: Cube; • •= 2.65 Mg/m^3;
Find specific surface = surface area divide by volume
and specific surface = surface area divide by mass

(A) S = 10 MM

$$Specific\ Surface = \frac{6 \times 0.01^2}{0.01^3} = 0.6\ mm^{-1}$$

$$Aternately = \frac{0.6 \times 1000}{2.65 \times 1000} = 0.226\ \frac{m^2}{kg}$$

(B) S = 1 MM

$$Specific\ Surface = \frac{6 \times 0.001^2}{0.001^3} = 6.0\ mm^{-1}$$

$$Aternately = \frac{6.0 \times 1000}{2.65 \times 1000} = 2.26\ \frac{m^2}{kg}$$

(C) S = 1 • M

$$Specific\ Surface = \frac{6 \times 0.000001^2}{0.000001^3} = 6000\ mm^{-1}$$

$$Aternately = \frac{6000 \times 1000}{2.65 \times 1000} = 2260\ \frac{m^2}{kg}$$

(D) S = 1 NM

$$Specific\ Surface = \frac{6 \times 0.000000001^2}{0.000000001^3} = 6 \times 10^6\ mm^{-1}$$

$$Aternately = \frac{6 \times 10^6 \times 1000}{2.65 \times 1000} = 2.26 \times 10^6\ \frac{m^2}{kg}$$

QUESTION 4-2

$$Density = \frac{Mass}{Volume} = \frac{\rho_s}{1+e}$$

$$\therefore \rho_{max} = \frac{2.65}{1+e_{min}} = \frac{2.65}{1+0.46} = 1.82\ g/cm^3$$

$$\therefore \rho_{min} = \frac{2.65}{1+e_{max}} = \frac{2.65}{1+0.66} = 1.60\ g/cm^3$$

$$Relative\ Density = Density\ Index = D_r$$

$$= \frac{e_{max} - e_{field}}{e_{max} - e_{min}} = \frac{0.66 - 0.63}{0.66 - 0.46} = 0.15$$

$$or\ \ 15\ \%$$

QUESTION 4-3

(A) SMECTITE

Smectite also known as Montmorillenite is a 2:1 mineral composed of a repetition of one octahedral alumina (gibbsite) sheet sandwiched between two tetrahedral silica sheets (i.e. TOT). The ideal formula is $(OH)_4Si_8Al_4O_{20}$(interlayer)H_2O and the composition without the interlayer SiO_2, 66.7%; Al_2O_3, 28.3%; H_2O, 5%. The Silicon places in the tetrahedral sheets may be occupied by Aluminum. Similarly the Aluminum places may be occupied by Iron, Magnesium or both.

Between the adjacent (repetitive) terahedral sheets of two TOT units there is a weak bonding where water and exchangeaable ions can easily entre. This results in a high swell potential (or high attraction for water) for the smectite mineral. Another mineral with high swelling potential is Halloysite or Vermiculite.

(B) BRUCITE

Brucite is composed of single octahedral sheets where the anion (oxygen) postions are all occupied by hydroxils and the cation postions are occupied by Magnesium. Its ideal formula is $Mg_3(OH)_6$. Its importance is that of being a single layer mineral which in combinaton with tetrahedral sheets makes up the crystal structure of other minerals.

(C) GIBBSITE

Gibbsite like Brucite is a one layer mineral. This layer is octahedral where 1/3 of the cation positions are empty and the remaining 2/3 positions have Aluminum ions. Its ideal formula is $Al_2(OH)_6$

(D) ATTAPULGITE

Attapulgite has a chain silicate crystal structure. Chain silicates basic unit consists of rows of tetrahedrals each sharing two corners. This makes it look columnar. It is not a clay mineral and it is not a common clay constituent. The composition of an ideal cell is $(OH_2)_4(OH)_2Mg_5Si_8O_{20}.4H_2O$.

(E) BENTONITE

Bentonite is not a mineral but an altered volcanic ash. The dominant clay mineral in Bentonite is Sodium Montmorillenite. Bentonite expands its volume when placed in water (possibly to 1200% or more). It is used as a drilling fluid because of it increases the viscosity of a fluid to several times the viscosity of water.

(F) ALLOPHANE

Allophane is an amorphous or poorly crystallized alumino silicate. Even though it is poorly crytallized it is often classified as a clay mineral. Any amorphous clay is classified as allophane.

(G) HALLOYSITE

Halloysite exists with the crystal structure of Kaolinite (tetrahedral-Octahedral units bonded together by a hydrogen bond between the hydroxil ions). There are two forms of Halloysite. One with Kaolinite composition, $Al_4Si_4O_{10}(OH)_8$, and the other with the composition $Al_4Si_4O_{10}(OH)_8.4H_2O$. The second dehydrates to the first with the loss of interlayer water molecules. This all adds up to Hallosite being a clay mineral with swelling potential.

(H) ILLITE

Illite has the same TOT crystal structure as Montmorillite (i.e. a 2:1 mineral). The difference is that the hexagonal holes in the terahedral sheets are occupied with a Potasium ion bonding the layers together and preventing the formation of an interrlayer of water. Illite is a general term for the mica like clay minerals. The illites differ from the micas in having less substitution of Aluminum for Silicon. They contain more water and they have Potasium partly replaced by Calcium and Magnesium. Even though is a non-swelling clay Illites are chemically more active tham micas. Their ideal formula is $(OH)_4K_2(Al_2Si_6)AL_4O_{20}$.

(I) MICA

Mica is not a clay mineral but rather a clay soil constituent. Micas are a group of 2:1 minerals with interlayer cations and little or no exchangeable water in between. They consist of 2 tetradehal sheets with one octrahedral sheet sandwiched in between (i.e. TOT). Due to the strong bonding by ions Mica has no swell potential.

(J) CHLORITE

Chlorite like mica is a group of minerals. It is a 2:1:1 mineral and consists of a sequence of: a Silica sheet, an Alumina sheet, another Silica sheet, and either a Gibbsite sheet or a Brucite sheet making it sensitive to hydration. It has swell potential, but is much less active than montmorillonite.

QUESTION 4-4

Based on usage, Sillica in glass or Aluminum in foil the choice is ALUMINA.

Based on the drawings of the crystal lattices, a Sillica sheet has gaping hexagonal holes while a Alumina sheet has a densely packed structure providing better coverage. Again the choice is ALUMINA

QUESTION 4-5

Soil particles are highly irregular and three-dimensional in nature, it is unrealistic to show them all in contact in any one plane. The particles would need to be a bundle of rods, in which case they would be regular in a two-dimensional plane.

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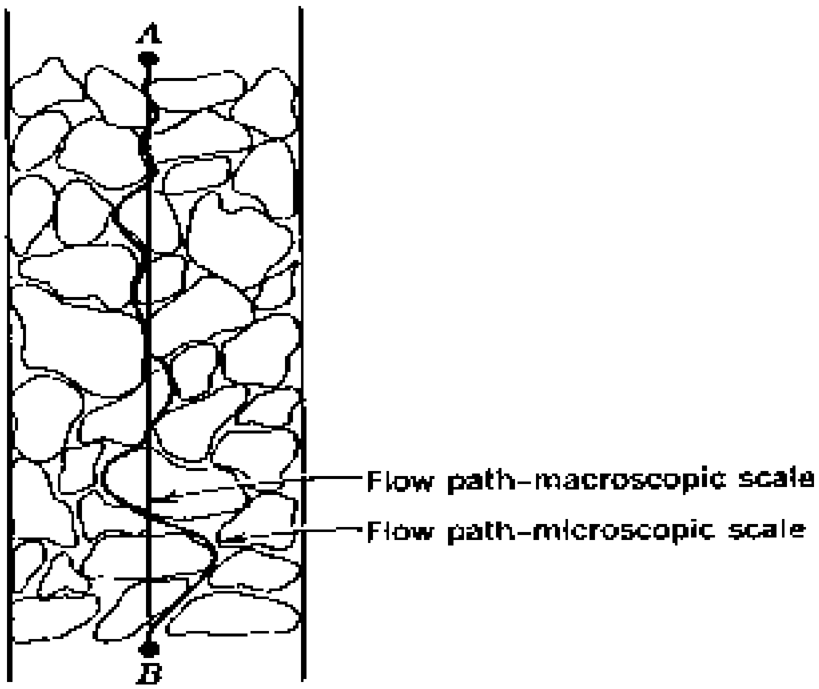
Fluid Flow in Soils

Problems of fluid flow in Soils
1. rate of flow of fluid through an earth dam (e.g. determination of rate of leakage through an earth dam).
2. problems involving compression (e.g. determination of the rate of settlement of a foundation).
3. problems involving strength (e.g. the evaluation of factor of safety of a given soil under a given loading).

One dimensional flow

Flow Path in Soils

A measure of how easily a fluid (e.g., water) can pass through a porous medium (e.g., soils)



Macroscopic scale : Strait path (in soil engineering problems).
Microscopic scale : winding path (actual path)

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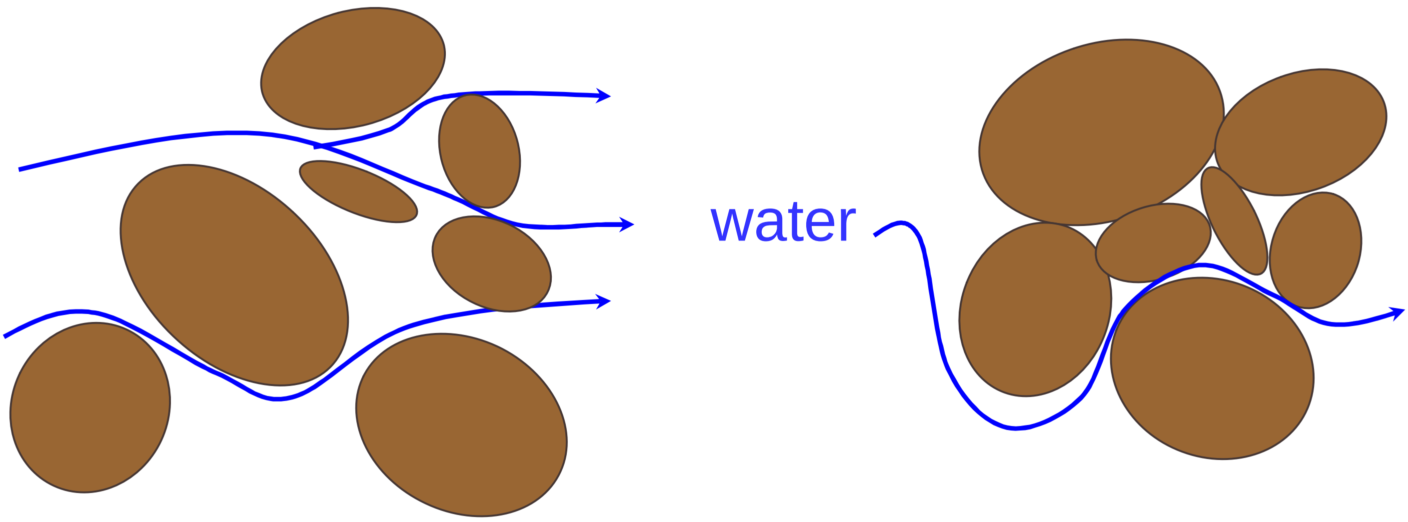


Loose soil
- easy to flow
- **high** permeability

Dense Soil
- difficult to flow
- **Low** permeability

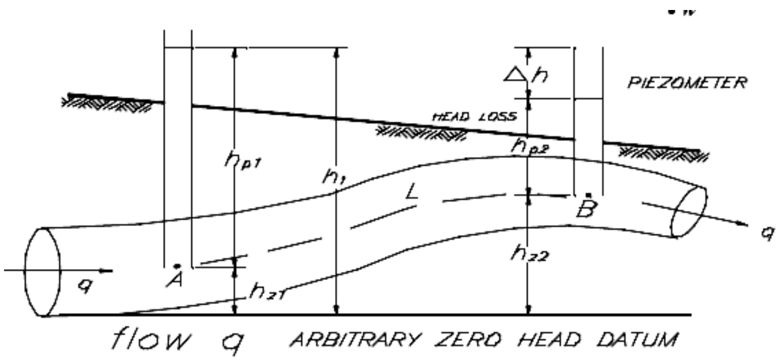
Hydraulic Gradient

According to Bernoulli's equation, the total head of a point under motion can be given by :

$$H = h_e + \frac{P}{g_w} + \frac{v^2}{2g}$$

total head = position head + pressure head + velocity head

where h = total head,
P = Pressure ,
V = Velocity , and g = Acceleration due to gravity.



Note: Velocity head $\frac{v^2}{2g}$ in soils is too small and can be neglected.

Hence:

$H = h_e + \frac{P}{g_w}$ and this is defined as the <u>piezometric head</u>

Now A→B $\Delta h = h_A - h_B$

And $\frac{\Delta h}{L}$ = i = hydraulic gradient

L = length of flow over which loss of head (Δh) is measured.

Therefore : *Flow of Water in Soils*

1- Hydraulic Head in Soil

Total Head = Pressure head + Elevation Head

$$h_t = h_p + h_e$$

- Elevation head at a point = Extent of that point from the datum

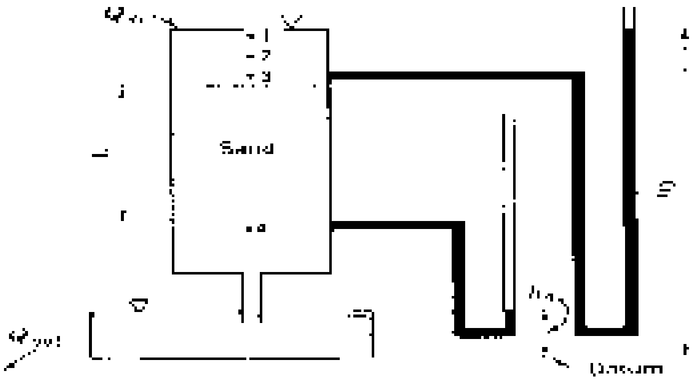
- Pressure head at a point = Height of which the water rises in the piezometer above the point.

- Pore Water pressure at a point = P.W.P. = $g_{water} \cdot h_p$

<u>Darcy's Law:</u> the velocity or discharge through a soil :

V ¥ i V= k i $\frac{Q}{A} = q = k\,i$

Where k = Coefficient of permeability



$i = \frac{\Delta h}{L} = \frac{h_3 - h_4}{L}$ = Hydraulic gradient between pts 3→ 4

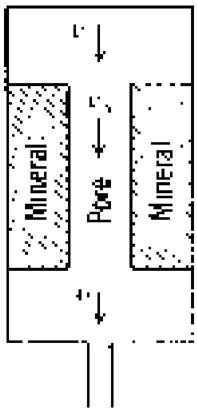
V = Velocity of water between pts.1→2→3

Now $Q_{in} = Q_{out}$

$VA = V_s A_v$

V_s = Velocity of water through the soil between pts. 3→ 4 = Seepage velocity

Then



$$V_s = V\frac{A}{A_v} = V\frac{AL}{A_v L} = V\frac{\text{total vol.}}{\text{voids vol.}}$$

$$\therefore V_s = \frac{V}{n}$$

(since n < 1 →Vs >V)

***How to measure the Pressure Head or the Piezometric Head**

1- Assume that you do not have seepage in the system (Before Seepage)

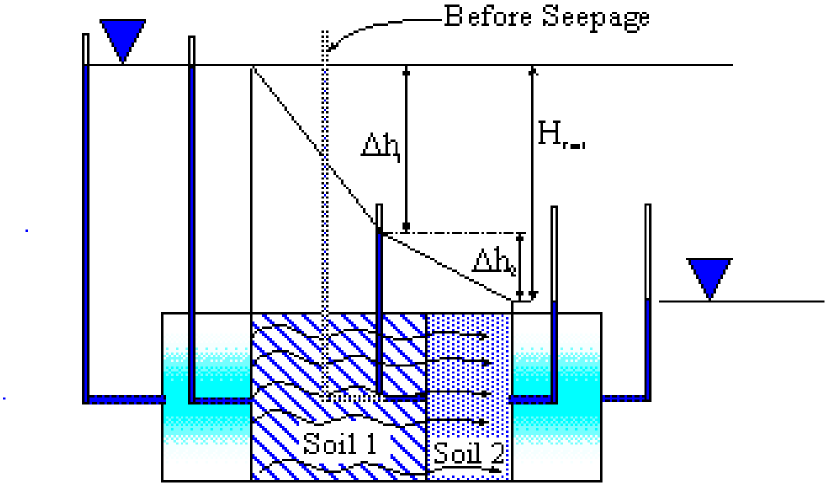
2- Assume that you have piezometer at the point under consideration

3- Get the measurement of the piezometric head (Water column in the Piezometer before seepage) = $h_{p(\text{Before Seepage})}$

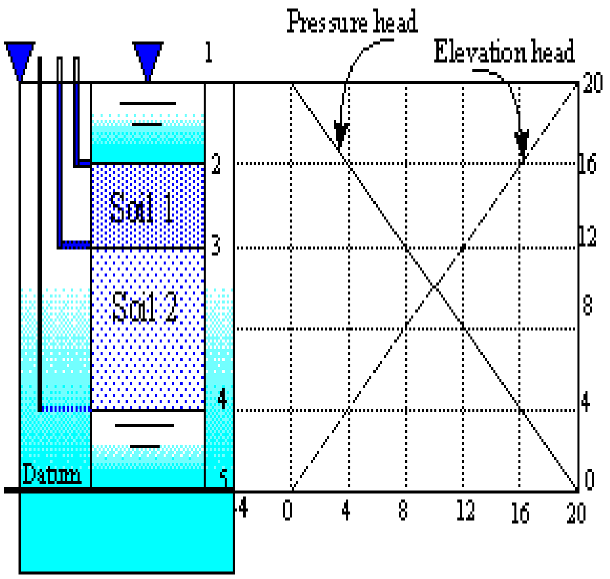
4- Now consider the problem during seepage

5- Measure the amount of the head loss in the piezometer (D_h) or the drop in the piezometric head.

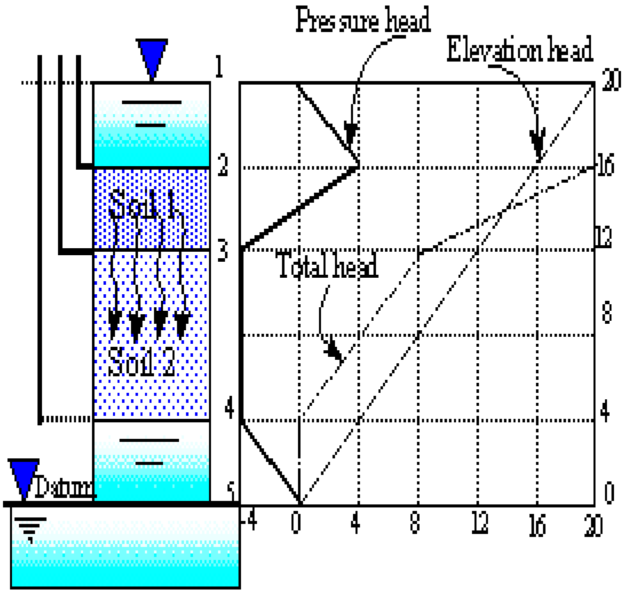
6- The piezometric head during seepage = $h_{p(\text{during seepage})} = h_{p(\text{Before Seepage})} - D_h$



$$H_{total} = \Delta h_1 + \Delta h_2$$
$$q_1 = q_2 = v_1 A_1 = v_2 A_2 = k_1 \Delta h_1 / L_1 = k_2 \Delta h_2 / L_2$$



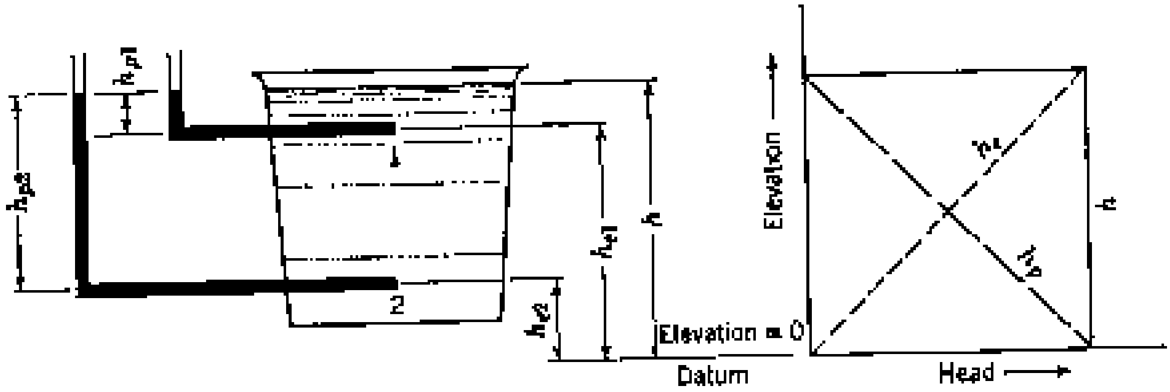
Before Seepage



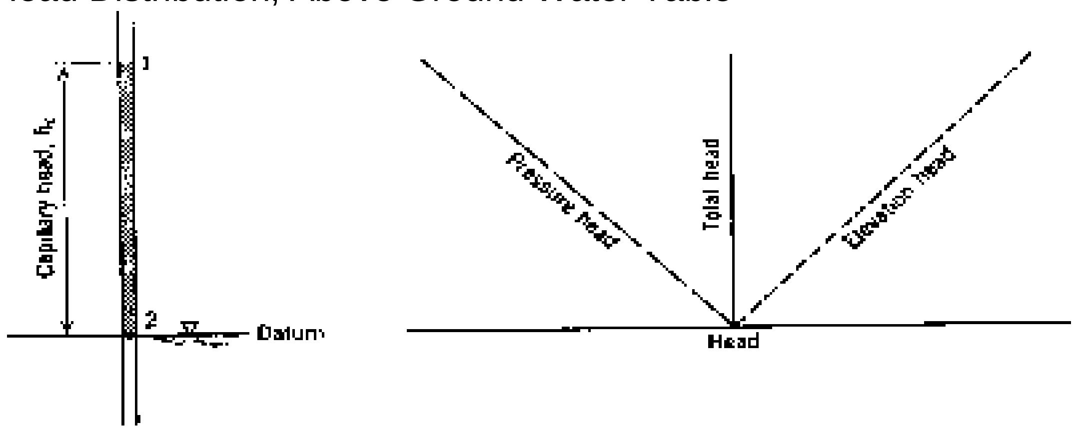
During Seepage

Heads in Static Water

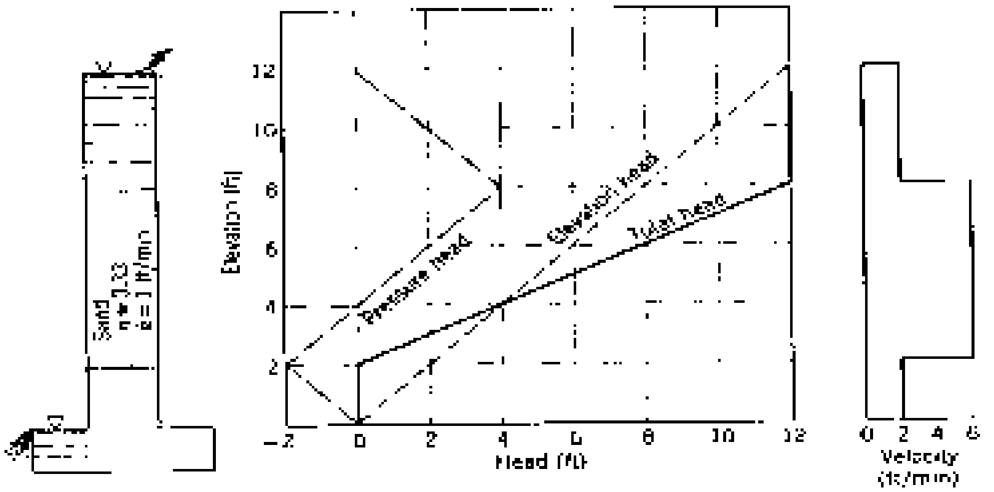
Head Distribution; Below Ground Water Table



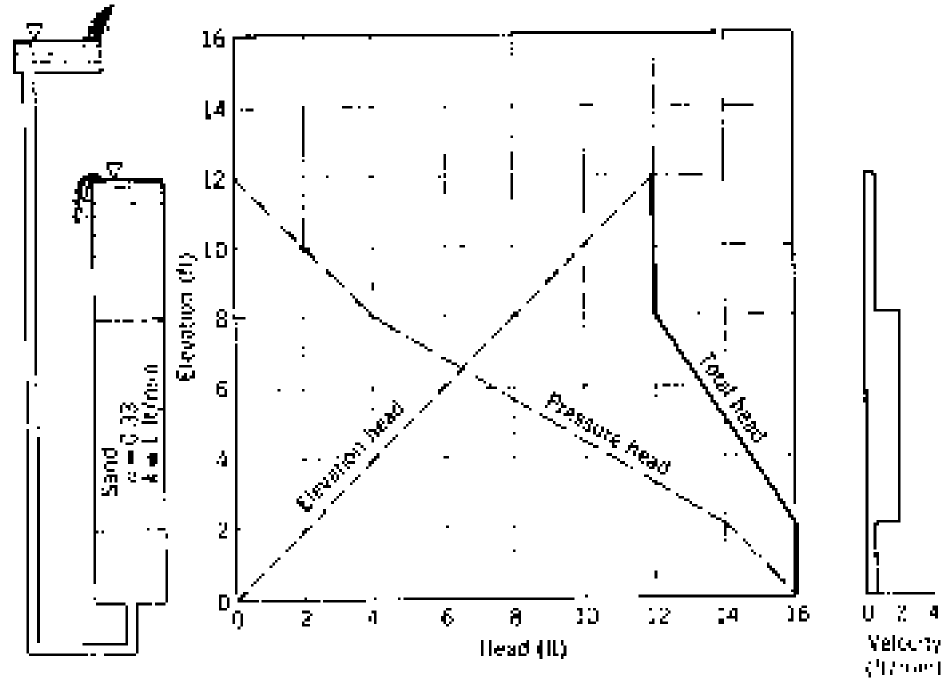
Head Distribution; Above Ground Water Table



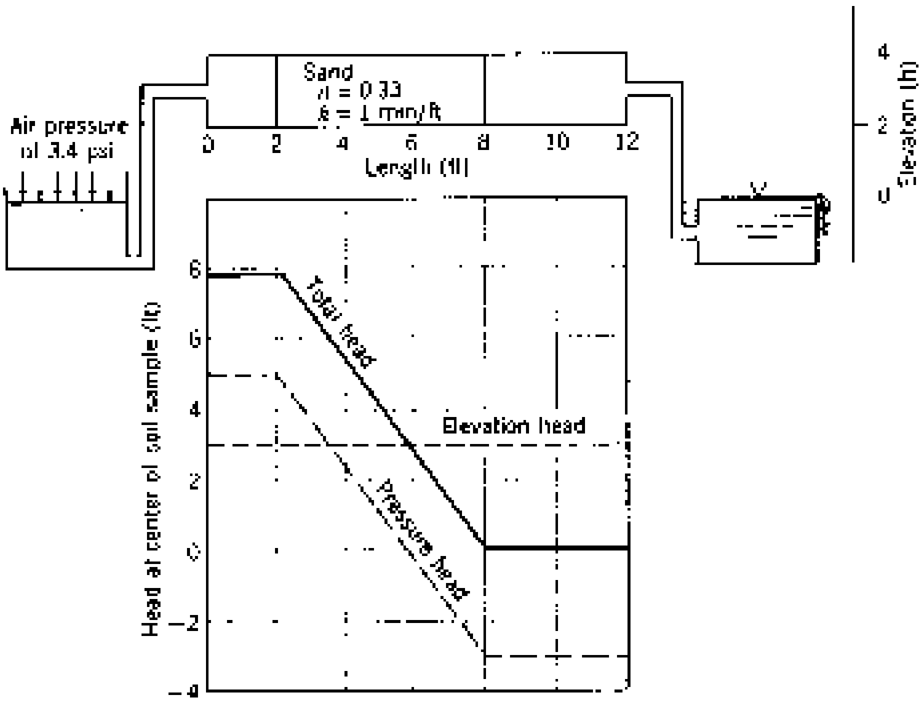
Calculation of Pressure Head in Seepage Conditions (Downward Flow)



Calculation of Pressure Head in Seepage Conditions (Upward Flow)

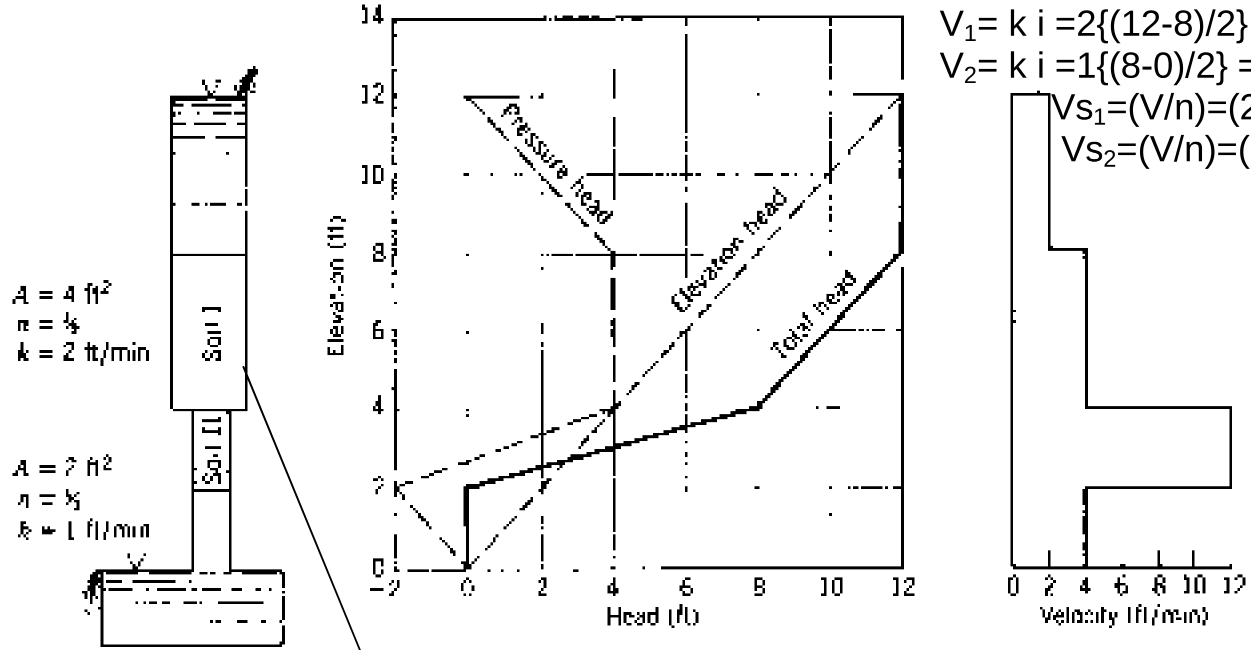


Calculation of Pressure Head in Seepage Conditions (Horizontal flow)



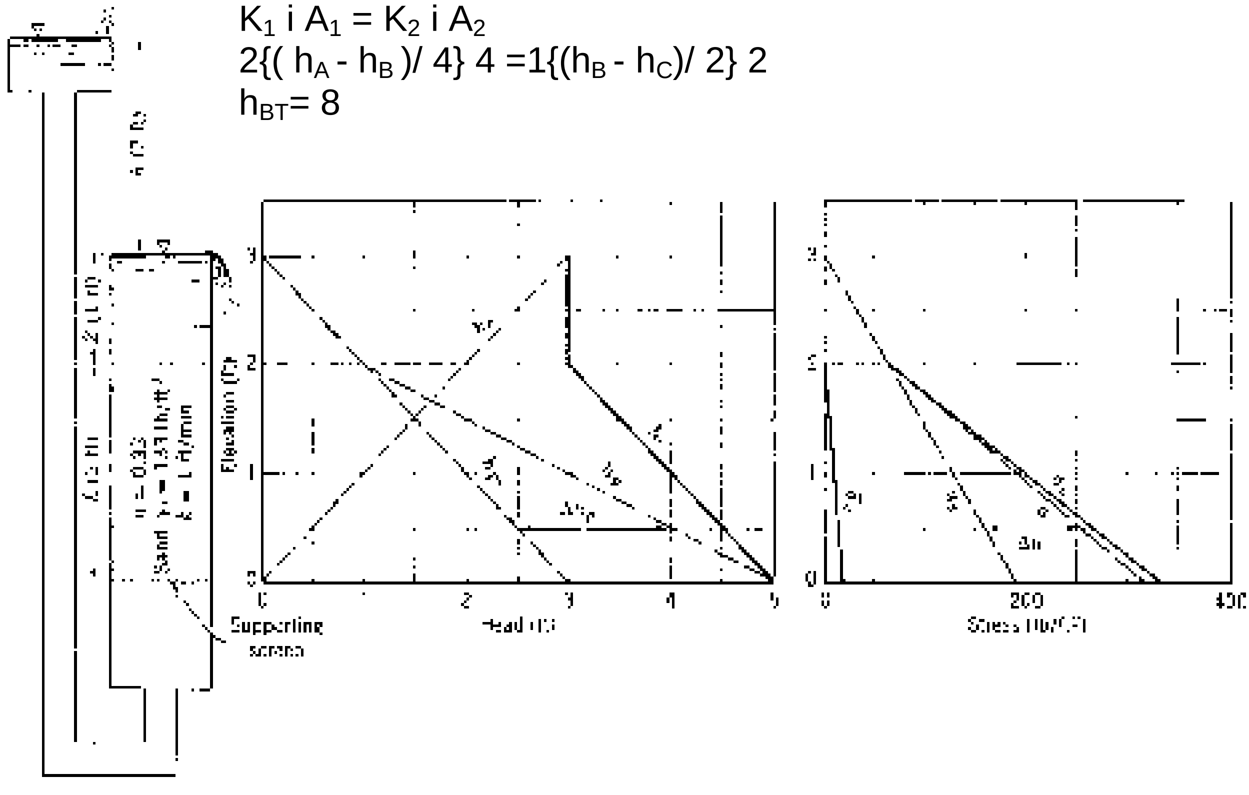
Calculation of Pressure Head in Seepage Conditions (two soils)

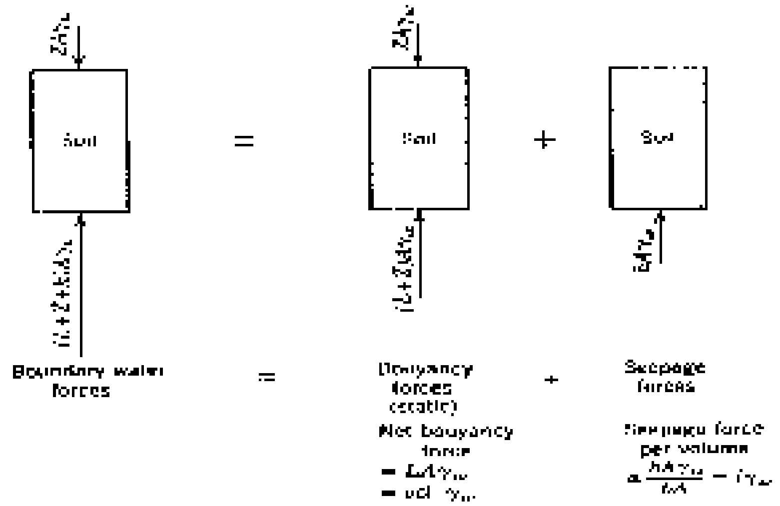
V_1= k i =2{(12-8)/2} = 2
V_2= k i =1{(8-0)/2} = 4
Vs_1=(V/n)=(2/1/2)=4
Vs_2=(V/n)=(2/1/3)=12



Seepage Force : is applied by moving water to the soil Skelton and acts in the flow direction

Q_1 =Q_2
K_1 i A_1 = K_2 i A_2
2{(h_A - h_B)/ 4} 4 =1{(h_B - h_C)/ 2} 2
h_{BT}= 8





Hence :we can work either with :

- Boundary water forces + Total weights
- <u>OR</u> seepage forces + submerged weights

<u>Seepage force/volume = $i\gamma_w$</u>

$$j = \frac{\text{Seepage Force}}{\text{Vol. of Soil}} = \frac{hAg_w}{LA} = \frac{h}{L} g_w = i g_w$$

Note:

- seepage force exerted by the flowing water acts as an external force on the soil Skelton
- J is a constant value for a given soil mass.

Ex.: For the figure above find J if h=0.6 m and L=0.6 m.

$J = i\, g_w = \frac{h}{L} g_w = \frac{0.6}{0.6} x9.81 = 9.81 \text{ kN/m}^3$

Force Equilibrium :for the setup shown in page above:

a-Total weight plus boundary water force :

$\sigma_A - u_A = \sigma`_A$

$F = Z\, g_w A + L\, A\, g_t - (h + Z + L)\, g_w A$

$= L\, A(g_t - g_w) - h\, g_w A$

$= L\, A\, g_b - h\, g_w A = A\,(L\, g_b - h\, g_w)$

$F = \sigma` A$

b-Submerged weight +seepage force

$F = L\, A\, g_b - h\, g_w A$

$F = \sigma` A$

Hence :total wt. + boundary water force =submerged weight + seepage force

Quick Condition (Boiling)

$$S = C + \sigma` \tan \phi \qquad \text{(Coulomb eq.)}$$

Where S = Shear Strength
C = soil cohesion
Φ = internal friction

For cohesionless soils : C = 0 → $S = \sigma` \tan \phi$ Then S = 0 (boiling ) when $\sigma` = 0$

Now $\sigma` = L\,A\,\gamma_b - h\,\gamma_w\,A$ and for $\sigma` = 0$ → $\frac{h}{L} = \frac{\gamma_b}{\gamma_w} = i$

Hence: the gradient required to cause quick condition = critical gradient $= i_c$:

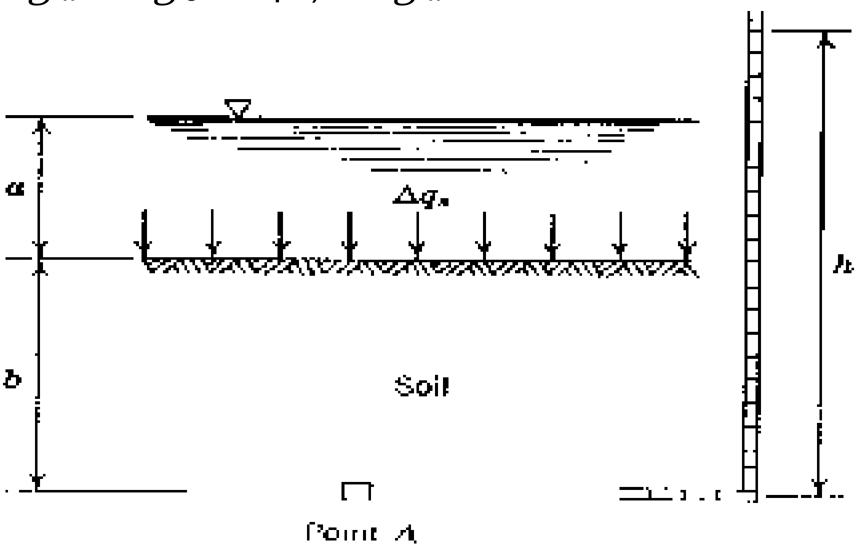
$$i_c = \frac{\gamma_b}{\gamma_w}$$

Show that $i_c = \frac{G_s - 1}{1 + e}$

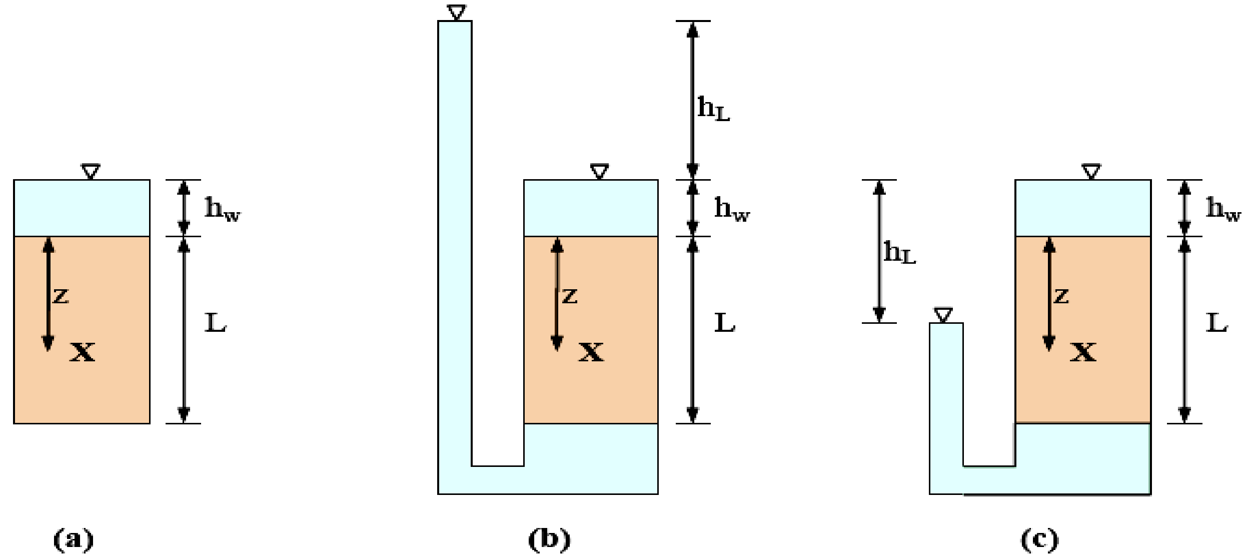
Note : if γ_b not given ,take $\gamma_b \approx 1$

The effective stress at point A is

$$\sigma_v` = \sigma_v - u$$
$$= (a\,\gamma_w + b\,\gamma_s + \Delta q_s) - h\,\gamma_w$$



STRESSES IN SOILS DUE TO FLOW



Three different scenarios (a) Static (b) Flow-up (c) Flow-down

For all three situations, the total vertical stress is the same. The pore water pressures and effective stresses are summarized below.

$u = (h_L + h_w + z)\gamma_w$ $\qquad$ $u = (+h_w + z - h_L)\gamma_w$

(a) <u>Static situation</u>:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z)$
$\sigma_v' = \gamma' z$

(b) <u>Flow-Up Situation</u>:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) + \mathbf{i\, z\, \gamma_w}$
$\sigma_v' = \gamma' z - \mathbf{i\, z\, \gamma_w}$

$i = h_L/z$, $h_L = i\, z$

(c) <u>Flow-Down Situation</u>:
$\sigma_v = \gamma_w h_w + \gamma_{sat} z$
$u = \gamma_w (h_w + z) - \mathbf{i\, z\, \gamma_w}$
$\sigma_v' = \gamma' z + \mathbf{i\, z\, \gamma_w}$

When the flow is upwards in the soil, pore water pressure increases and effective stress decreases. When the flow is downward, the pore water pressure decreases and the effective stress increases. Higher the hydraulic gradient, higher the increase or decrease in the values of pore pressure and effective stress.

Now let's have a closer look at the flow-up situation, in a *granular soil*. The effective stress is positive as long as $\gamma' z$ is greater than $iz\gamma_w$. If the hydraulic gradient is too large, $iz\gamma_w$ can exceed $\gamma' z$, and the effective vertical stress can become negative. This implies that there is no inter-particle contact stress, and the grains are no longer in contact. When this occurs, the granular soil is said to have reached *quick condition*.

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Determination of k in the Laboratory

Permeability in Soils

- Permeability is the measure of the soil's ability to permit water to flow through its pores or voids
- It is one of the most important soil properties of interest to geotechnical engineers

The Constant head test

- The constant head test is used primarily for coarse-grained soils
- This test is based on the assumption of laminar flow where *k* is independent of *i* *(low* values of i)
- ASTM D 2434
- This test applies a constant head of water to each end of a soil in a "permeameter"

Procedure (Constant head)

1. Setup screens on the permeameter
2. Measurements for permeameter, (D), (L), H_1
3. Take 1000 g passing No.4 soil (M_1)
4. Take a sample for M.C.
5. Assemble the permeameter – *make sure seals are air-tight*
6. Fill the mold in several layers and compact it as prescribed.
7. Put top porous stone and measure H_2
8. Weigh remainder of soil (M_2)
9. Complete assembling the permeameter. (keep outlet valve closed)
10. Connect Manometer tubes, but keep the valves closed.
11. Apply vacuum to remove air for 15 minutes (through inlet tube at top)
12. Run the Test (follow instructions in the lab manual) …..
13. Take readings
    - Manometer heads h_1 & h_2
    - Collect water at the outlet, Q ml at time t $\approx$ 60 sec.

- Determine the unit weight
- Calculate the void ratio of the compacted specimen

Calculate *k* as

- $$k = \frac{QL}{Aht}$$
- Calculate $$k_{20^0C} = k_{T^0C}\frac{h_{T^0C}}{h_{20^0C}}$$

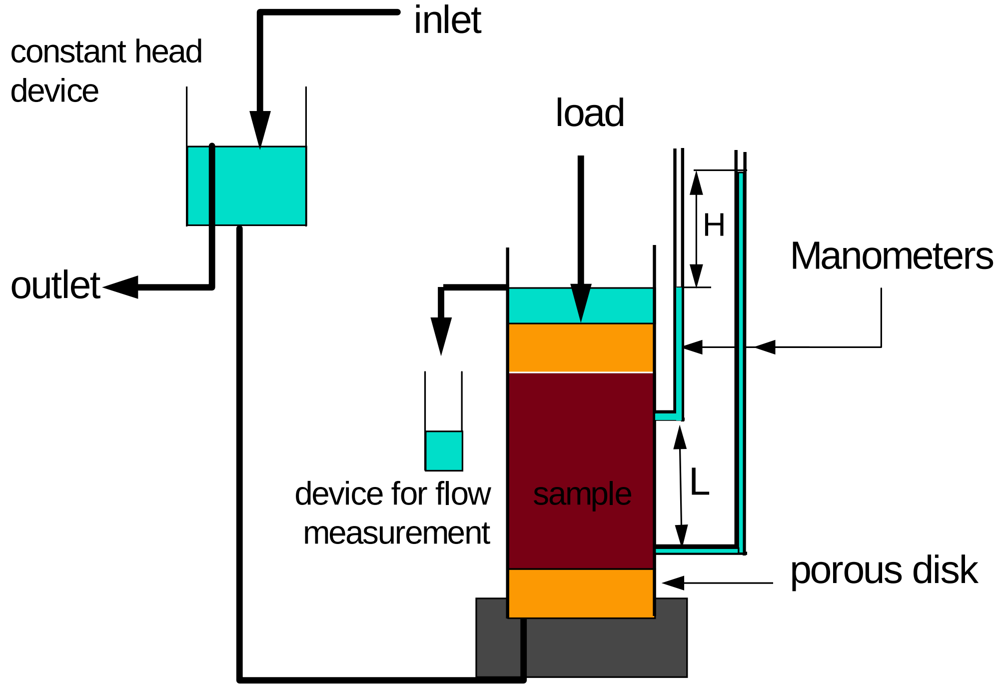


Fig. 4 Constant Head Permeameter

2-The Falling head test

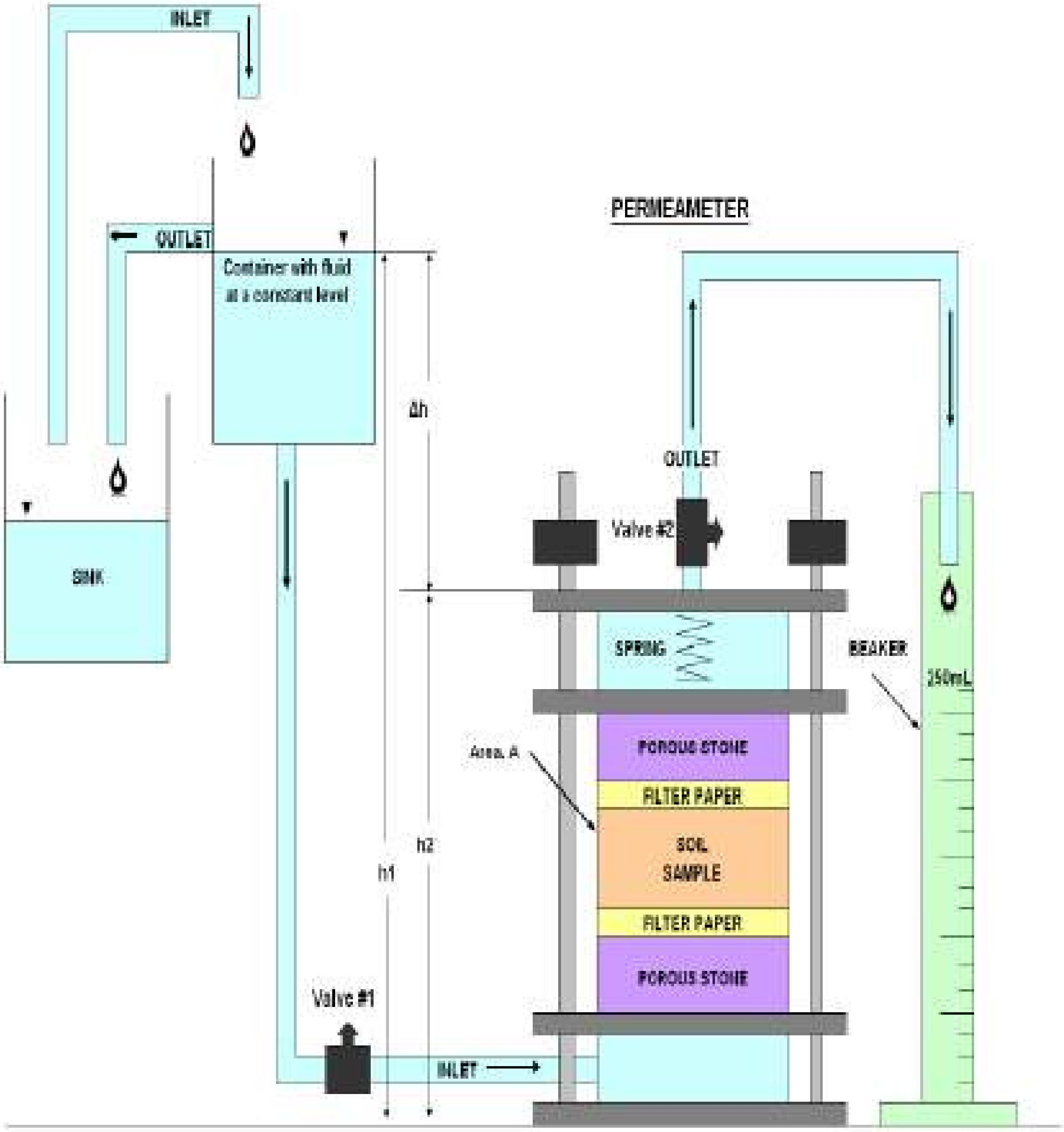
- The falling head test is used both for coarse-grained soils as well as fine-grained soils
- Same procedure in constant head test except:
  - Record initial head difference, h_1 at t = 0
  - Allow water to flow through the soil specimen
  - Record the final head difference, h_2 at time
    t = t_2
  - Collect water at the outlet, Q ml at time t ≈ 60 sec
- Calculate *k* as
- Where;

$$k = \frac{aL}{At} \ln \frac{h_1}{h_2}$$

a = inside cross sectional area of the water tank
h_1 = distance to bottom of the beaker before the test
h_2 = distance to bottom of the beaker after the test

- Calculate

$$k_{20^0C} = k_{T^0C} \frac{h_{T^0C}}{h_{20^0C}}$$



Typical permeability ranges

Soils exhibit a very wide range of permabilities and while particle size may vary by about 3-4 orders of magnitude, permeability may vary by about 10 orders of magnitude.

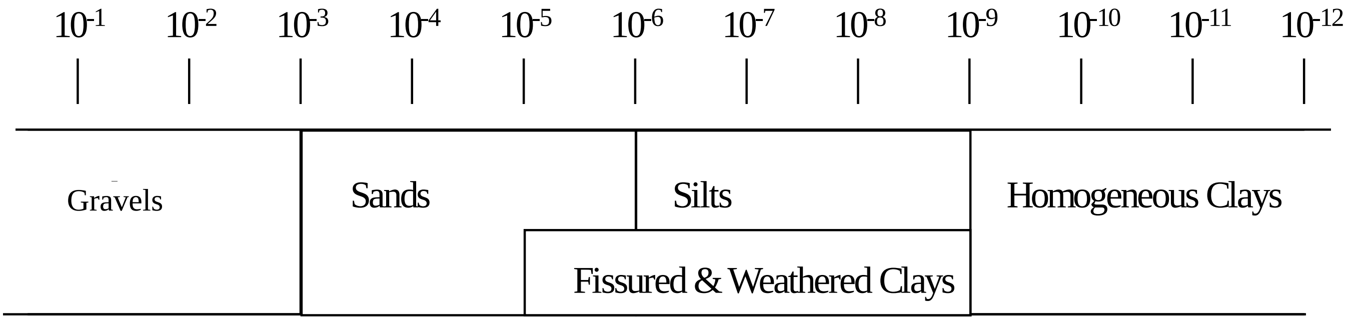
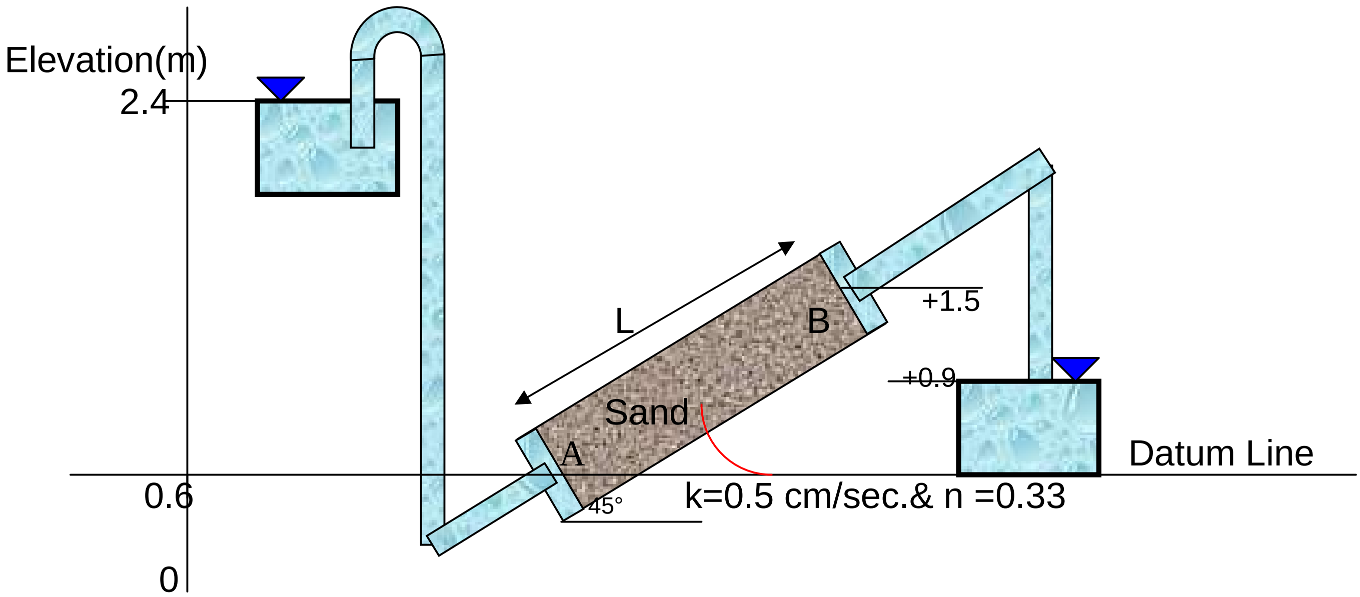


Fig 6 Typical Permeability Ranges

Q. For the setup shown below; plot to scale elevation head, pressure head, total head , and seepage velocity versus distance along the sample axis.



Solution:

$$\sin 45 = \frac{0.9}{L} \Rightarrow L = 1.272\text{m}$$

Point	he	hp	ht	v
A	0	1.8	1.8	0.589
B	0.9	- 0.6	0.3	0.589

$$i = \frac{\Delta h_t}{L} = \frac{1.8-0.3}{1.272} = 1.18$$

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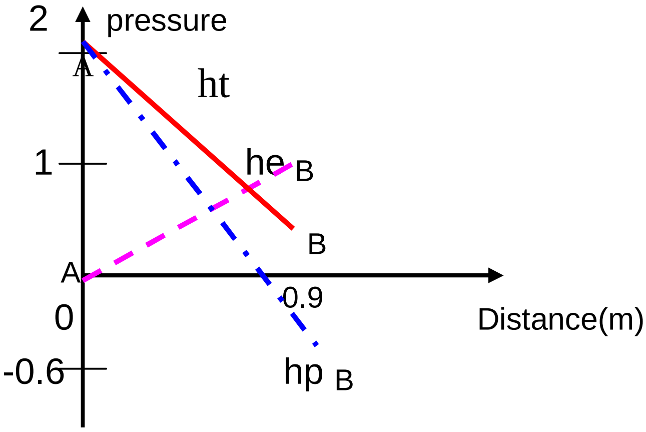
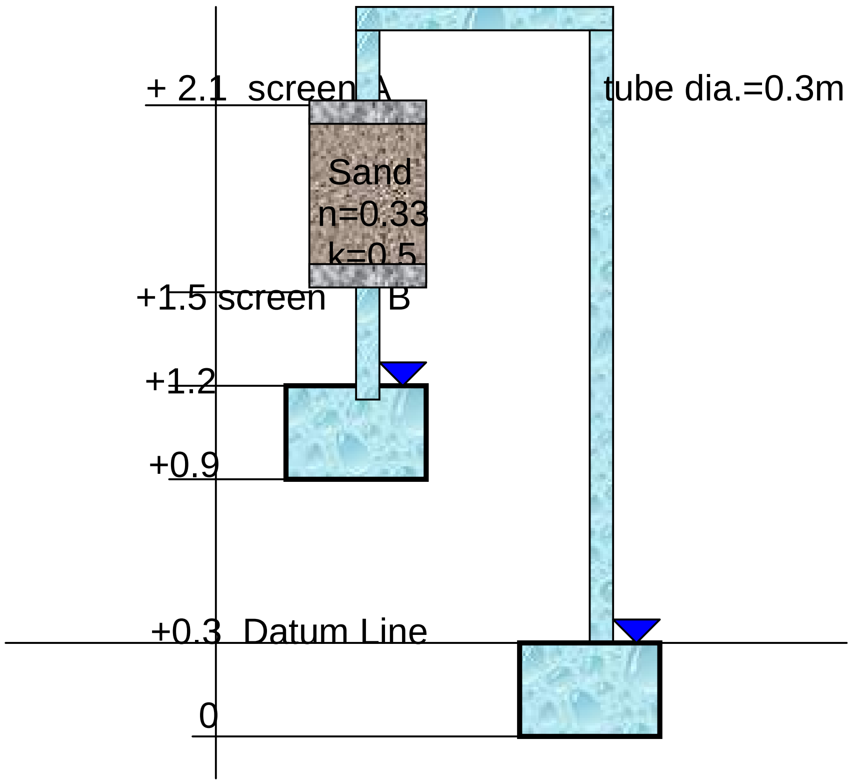
V= ki= 1.18 x 0.5 = 0.589 cm/sec.

$$V_s = \frac{V}{n} = \frac{0.589}{0.33} = 1.77 \text{cm/sec.}$$



Q. For the setup shown below; compute the vertical force exerted by the soil on screen A and that on screen B. Neglect friction between the soil and tube. G= 2.75



Solution:

$$Area = \frac{\pi}{4}(0.3)^2 = 0.07 m^2$$

$$e = \frac{n}{1-n} = \frac{0.33}{1-0.33} = 0.5$$

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$$\gamma_{sand}=\frac{G+e}{1+e}\gamma_w=\frac{2.75+0.5}{1+0.5}(9.81)=21.25\,kN/m^2$$

Point	***he***	***hp***	***ht***
A	***1.8***	***-1.8***	***0***
B	***1.2***	***- 0.3***	***0.9***

$$i_c=\frac{G-1}{1+e}=\frac{2.75-1}{1+0.5}=1.1666$$

$$i=\frac{\Delta h_t}{L}=\frac{1.2-0.3}{0.6}=1.5$$

$$i=\frac{\Delta h_t}{L}=\frac{1.2-0.3}{0.6}=1.5 > i_c=1.1666 \Rightarrow \therefore \text{Boiling}$$

∴ the soil stratum don't effect on the screen B.

For Screen A the force effected by the soil on it is Seepage force – weight of the soil

$$\frac{(\text{Seepage Force})}{(\text{Soil Volume})}=i\gamma_w$$

∴

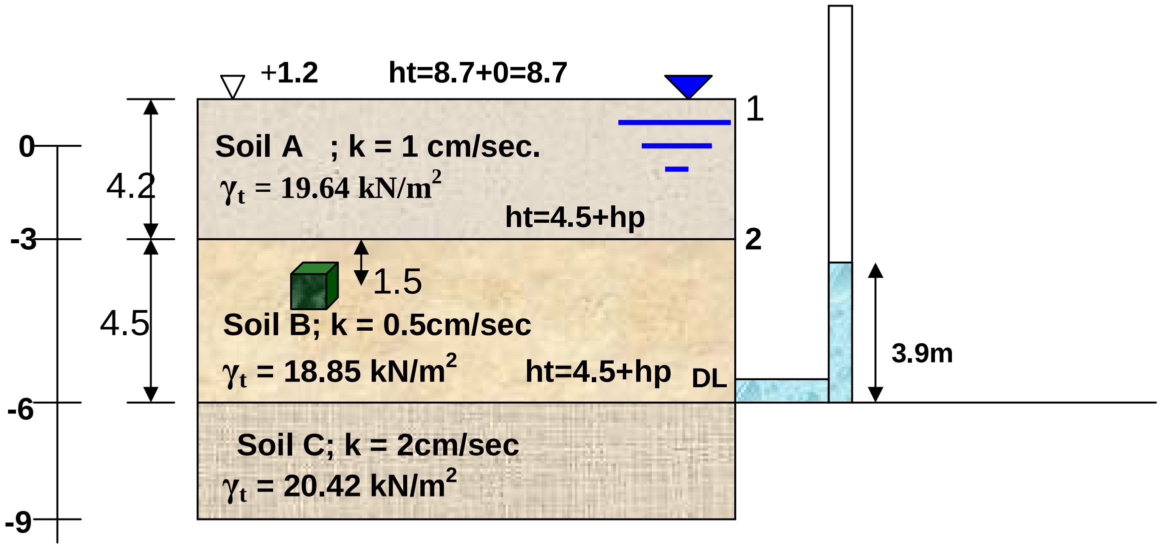
$$(\text{Seepage Force})=i\gamma_w(\text{Soil Volume})=\frac{0.9}{0.6}(9.81)\left(0.6x\frac{\pi}{4}(0.3)^2\right)=8.829\left(\frac{\pi}{4}(0.3)^2\right)$$

$$(\text{Soil Weight})=(\gamma_{sat}-\gamma_w)(\text{Soil Volume})$$

$$(\text{Soil Weight})=(21.25-9.81)\left(0.6x\frac{\pi}{4}(0.3)^2\right)=6.864\left(\frac{\pi}{4}(0.3)^2\right)$$

∴ Force on the screen A $=8.829\left(\frac{\pi}{4}(0.3)^2\right)-6.864\left(\frac{\pi}{4}(0.3)^2\right)=0.14137$ kN.

Q. In the profile shown below , steady vertical seepage is occurring. Make a scaled plot of elevation versus pressure head pore pressure, seepage velocity, and vertical effective stress. Determine the seepage force on a 0.3m cube whose center is at elevation -4.5m. G for all soil = 2.75.



Datum line at point 3

point	he	hp	ht	u	σ_v	σ'_v	v
1	8.7	0	8.7	0	0	0	0.375
2	4.5	2.7	7.2*	27	82.48	55.48	0.375
3	0	3.9	3.9	39	167.3	128.3	0.366

$$* \ v = ki = k\frac{\Delta h_t}{L}$$

$$q_A = q_B$$

$$A_A i_A k_A = A_B i_B k_B$$

$$A_A = A_B \Rightarrow i_A k_A = i_B k$$

$$\therefore (1)\left(\frac{8.7-(4.5+hp)}{4.2}\right) = (0.5)\left(\frac{(4.5+hp)-3.9}{4.5}\right) \Rightarrow hp = 2.7m$$

Or

$$\therefore (1)\left(\frac{8.7-h_{t2}}{4.2}\right) = (0.5)\left(\frac{h_{t2}-3.9}{4.5}\right) \Rightarrow h_{t2} = 7.2m$$

$$J = i\gamma_w = \frac{7.2-3.9}{4.5}(1) = 7.2 kN/m^3$$

J for the cube = 7.2 x 0.3 x 0.3 x 0.3 = 0.2 kN

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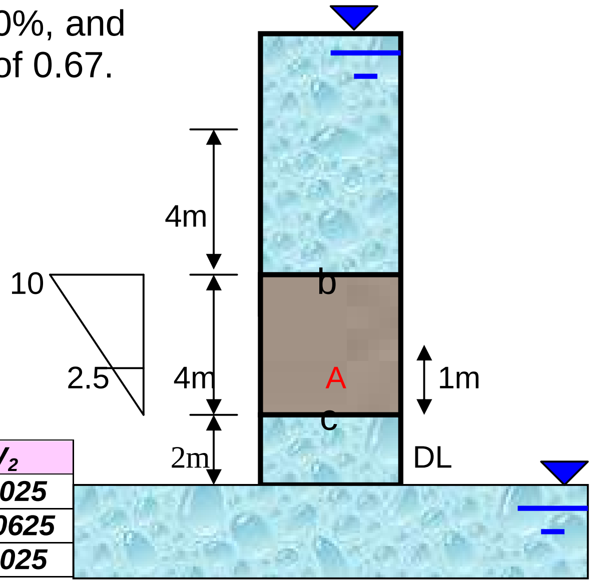
$v = ki$

$$v_2 = (1)\frac{8.7-7.2}{4.2} = 0.357 \text{cm/sec.}$$

$$v_3 = (0.5)\frac{7.2-3.9}{4.5} = 0.366 \text{cm/sec.}$$

Q. for each of cases shown below ; determine the discharge velocity, the seepage velocity, and the seepage force per unit volume for
a. a permeability of 0.1 cm/sec and a porosity of 50%, and
b. a permeability of 0.001 cm/sec and a void ratio of 0.67.
solution:
For a k= 0.1 cm/sec & n=0.5

For b k =0.001 cm/sec & $n = \frac{e}{1+e} = \frac{0.67}{1+0.67} = 0.4$



Case 1
Let $\gamma_{sat.} = 20$ kN/m^2
Let $\gamma_w = 10$ kN/m^2
c

point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	6	4	10	40	40	0	0.25	0.0025
A	3	-0.5	2.5	-5	100	+105	0.5	0.00625
C	2	-2	0	-20	120	+140	0.25	0.0025

$$i = \frac{\Delta h_t}{L} = \frac{10-0}{4} = \frac{10}{4} = 2.5$$

For K = 0.1cm / sec
V = K x i = 0.1 x 2.5 = 0.25 cm / sec

$$V = \frac{v}{n} = \frac{0.25}{0.5} = 0.5 \text{cm/sec}$$

For K = 0.001 cm /sec
V = K x I = 0.001 x 2.5 = 0.0025 cm / sec

$$Vs = \frac{v}{n} = \frac{0.0025}{0.4} = 0.00625 \text{cm/sec}$$

Seepage force / unit volume = i x γ_w = 2.5 x 10 = 25 kN/m^3

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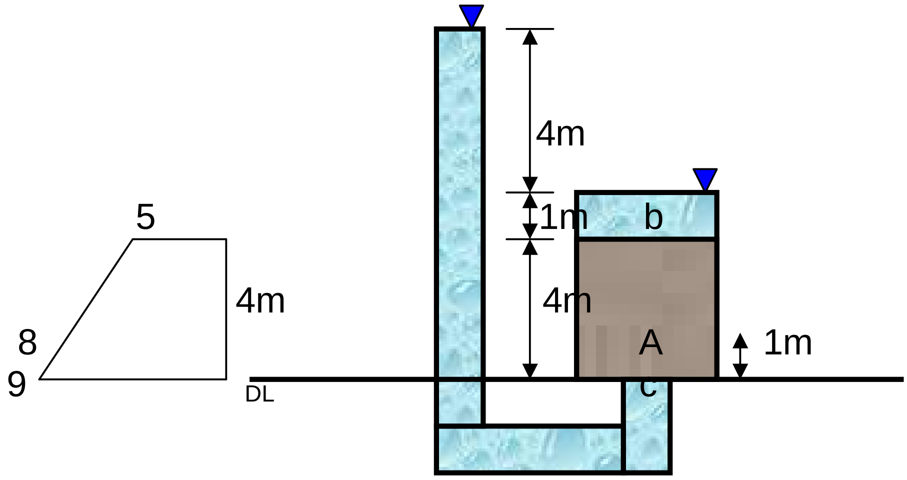
Case 2



point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	4	1	5	10	10	0	0.1	0.001
A	1	7	8	70	70	0	0.2	0.0025
C	0	9	9	90	90	0	0.1	0.001

$$i=\frac{\Delta h_t}{L}=\frac{9-5}{4}=\frac{4}{4}=1 \ \backslash \quad \text{Boiling}$$

$$V_1=Ki=0.1(1)=0.1cm/sec$$

$$Vs=\frac{V}{n}=\frac{0.1}{0.5}0.2cm/sec$$

$$V_2=K_2(i)=0.001(1)=0.001cm/sec$$

$$Vs_2=\frac{0.001}{0.4}=0.0025cm/sec$$

Seepage force / unit volume = I x γ_w = 1 x10 = 10 kN/m^3

Case 3

point	he	hp	ht	u	σ_v	σ'_v	V_1	V_2
b	0	9	9	90	90	0	0.1	0.001
A	0	6	6	60	60	0	0.2	0.0025
c	0	5	5	50	50	0	0.1	0.001

$$i=\frac{\Delta h_t}{L}=\frac{9-5}{4}=\frac{4}{4}=1 \ \backslash \quad \text{Boiling}$$

$$V_1=Ki=0.1(1)=0.1cm/sec$$

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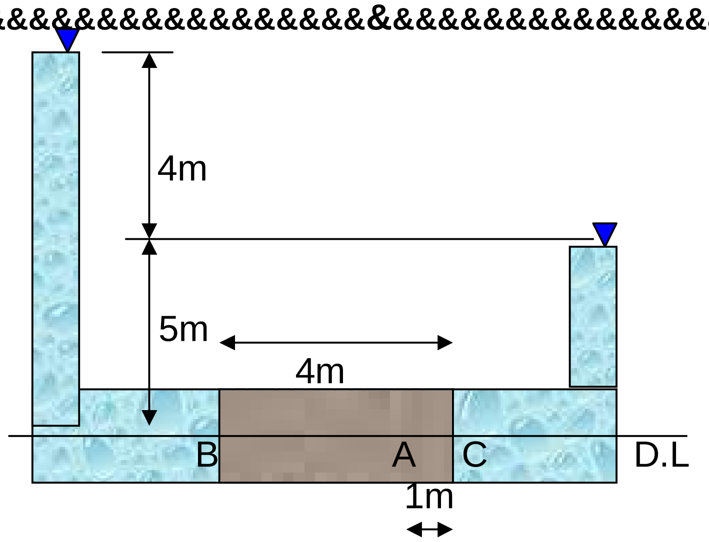
$$Vs_1 = \frac{V_1}{n} = \frac{0.1}{0.5} = 0.2\text{cm/sec}$$

$$V_2 = K_2(i) = 0.001(1) = 0.001\text{cm/sec}$$

$$Vs_2 = \frac{V_2}{n} = \frac{0.001}{0.4} = 0.0025\text{cm/sec}$$



Q. For the soil profile shown ; find (H) that make the soil in Boiling condition

$\sigma'_v = \sigma_v - 4$

= (3 x 18 + 3 x 20 + 6 x 18) – 16 x 10 = 62 kN/m^2

$\sigma_v = 4$

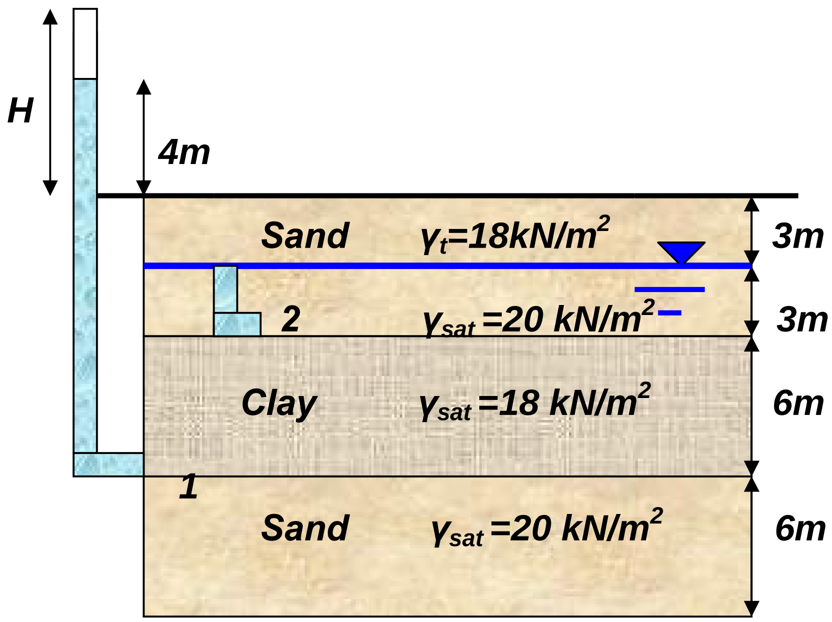
222 = (6 + 6 + H) x 10

\ H = 10.2 m

Or

$$i_c = \frac{g_{sat} - g_w}{g_w} = \frac{18-10}{10} = 0.8 \Rightarrow i_c = \frac{\Delta h_t}{L} = \frac{12+H-9}{6}$$

\ H = 1.8m



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Example**1.** A sample of sand was tested in a constant head permeameter. The results were:

Diameter of sample = 100mm

Length between manometer tappings = 120mm

Head difference measured by manometer = 80mm

Quantity of water passing through sample in 10 minutes = 150ml

Determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 100^2}{4} = 7.85 \text{ x } 10^3 \text{ mm}^2$$

$$Q = 150 \text{ ml} = 150 \text{ cc} = 150 \text{ x } 10^3 \text{ mm}^3$$

$$k = \frac{Ql}{At\Delta h} = \frac{150 \times 10^3 \times 120}{7.85 \times 10^3 \times (10 \times 60) \times 80} = \underline{\underline{4.78 \text{ x } 10^{-2} \text{ mm/s}}}$$

Example**2.** A 100mm diameter sample of fine sand was tested in a falling head permeameter. The length of the sample was 150mm. Water in the standpipe fell from 1000 to 400mm in 44 seconds. If the diameter of the standpipe was 10mm, determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 100^2}{4} = 7.85 \text{ x } 10^3 \text{ mm}^2$$

$$a = \frac{\pi d^2}{4} = \frac{\pi \times 10^2}{4} = 78.5 \text{ mm}^2$$

$$k = \frac{al}{At} \ln\left(\frac{h_1}{h_2}\right) = \frac{78.5 \times 150}{7.85 \times 10^3 \times 44} \ln\left(\frac{1000}{400}\right) = 0.0312 \text{ mm/s} = \underline{\underline{3.12 \text{ x } 10^{-2} \text{ mm/s}}}$$

Example**3.** A sample of coarse sand, 55mm in diameter, was tested in a constant head permeameter. Water percolated through the soil and a head loss of 100mm was recorded over a length of sample of 150mm. The discharge water, collected after 6.0 seconds had a mass of 400g.

Determine the **coefficient of permeability** of the soil.

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 55^2}{4} = 2375.8 \text{ mm}^2$$

$$k = \frac{Ql}{At\Delta h} = \frac{400 \times 10^3 \times 150}{2375.8 \times 6 \times 100} = \underline{\underline{42 \text{ mm/s}}}$$

N.B. *400g water has volume 400 ml*

Example**4.** A falling head permeability test is to be performed on a soil whose permeability is estimated at 3.0 x 10-3 mm/s. What **diameter of standpipe** should be used if the head is to drop from 275mm to 200mm in 5 minutes?

Assume that the area of the sample is 1500mm and its length is 85mm.

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$$k = \frac{al}{At}\ln\left(\frac{h_1}{h_2}\right)$$

$$3 \times 10^{-3} = \frac{a \times 85 \times \ln\left(275/200\right)}{1500 \times (5 \times 60)}$$

$$\Rightarrow \underline{a = 49.87\ \text{mm}^2}$$

$$\text{Now,}\quad a = \frac{\pi d^2}{4}$$

$$\Rightarrow \quad d^2 = \frac{a \times 4}{\pi} = \frac{49.87 \times 4}{\pi} = 63.5\ \text{mm}^2$$

$$\Rightarrow d = \sqrt{63.5} = 7.97\ \text{mm},\ \underline{\underline{\text{say } 8\ \text{mm}}}$$

Example**5.** A pumping out test was carried out on a soil stratum which extended to a depth of 20m where an impermeable layer was encountered. Ground water level originally occurred at 0.5m below the ground level. Observation wells were placed at 5m and 10m from the pumping well.

During steady pumping conditions water was discharged at the rate of 250 kg/minute and the drawdowns in the two wells were 1.5 and 0.2m

Determine the **coefficient of permeability** of the soil in metres/hour.

$z_2 = 19.5 - 0.2 = 19.3\text{m}$

$z_1 = 19.5 - 1.5 = 18.0\text{m}$

$q = 250 \text{ kg/min} \equiv 0.25 \text{ m}^3/\text{min} = 0.25 \times 60 \text{ m}^3/\text{hr} = 15 \text{ m}^3/\text{hr}$

$$k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{\pi\left(z_2^2 - z_1^2\right)} = \frac{15 \times \ln\left(\frac{10}{5}\right)}{\pi\left(19.3^2 - 18^2\right)} = \underline{\underline{68.3 \times 10^{-3} \text{ m/hr}}}$$

Tutorial

1. If the flow out is 200 mm^3/sec in figure 1, find K_1 and K_2
2. In figure 2, when $\frac{dh}{dt} = 0.01\, mm/\sec$,
   1) Derive the equation giving K
   2) Calculate K

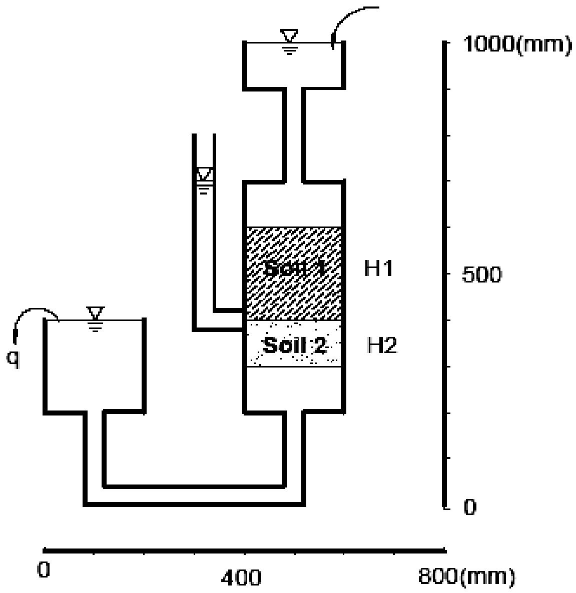

Figure 1.

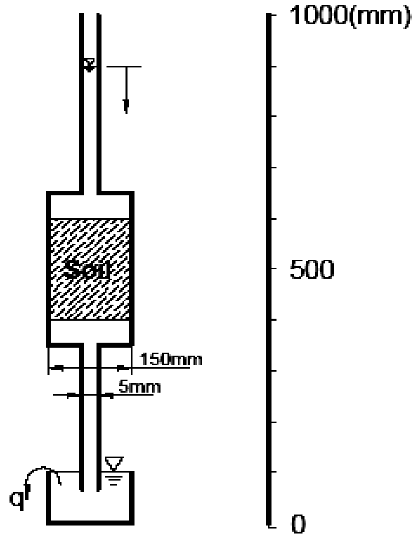

Figure 2.

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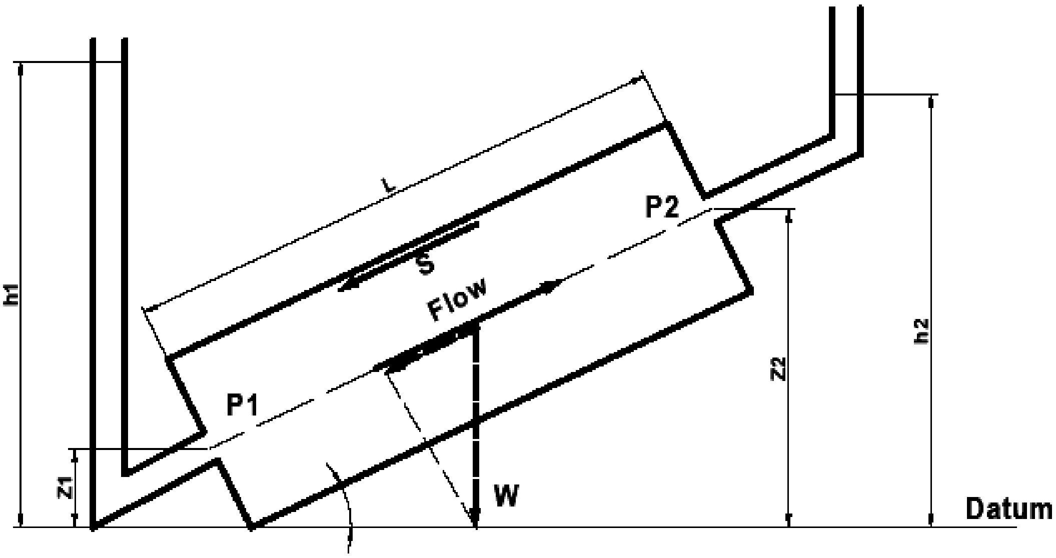
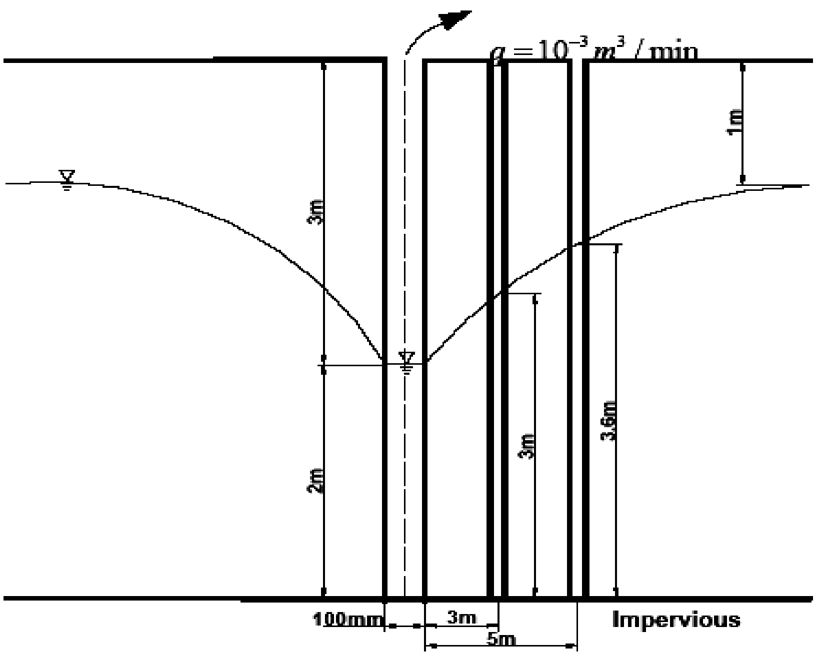
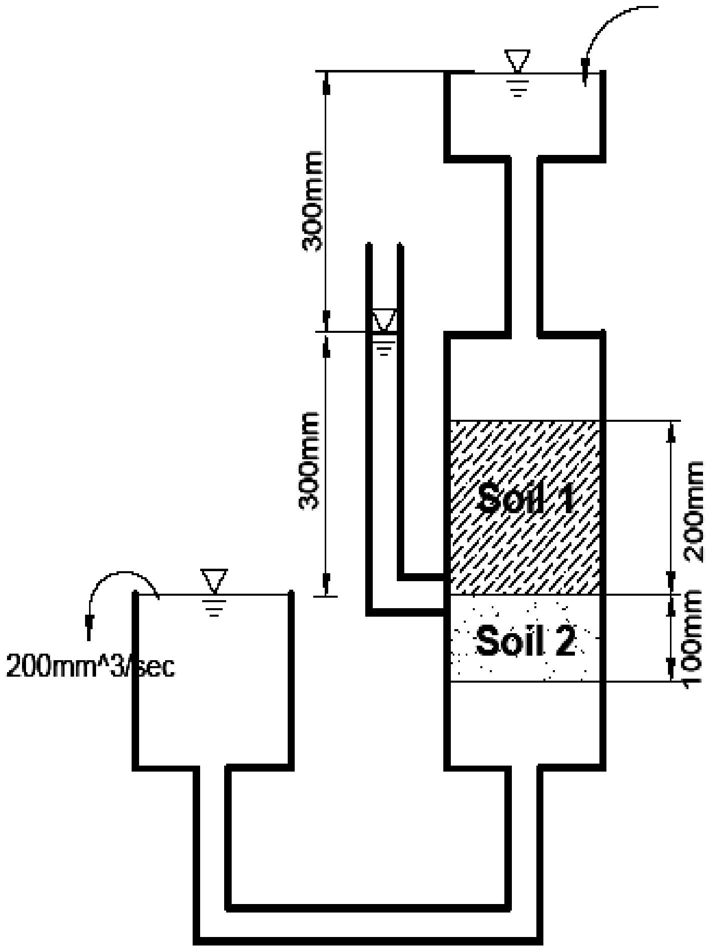
3. Develop the equation for the seepage force per unit soil volume in the Figure below (Hint : Do a free body diagram of the water in the tube)



4. Referring to the Figure below, derive the equation linking q to K and th dimensions involved and calculate K for the numbers given.



Problem 1.



$$Q_{Soil\,1} = Q_{Soil\,2}$$

$$Q = A \times V \qquad A_{Soil\,1} = A_{Soil\,2}$$

$$\therefore V_{Soil\,1} = V_{Soil\,2}$$

$$V = \frac{Q}{A} = \frac{200}{\frac{\pi}{4} \times 200^2}$$

$$= 6.37 \times 10^{-3}\, mm/\sec$$

1) Calculation K_1

$$V_{Soil\,1} = K_{Soil\,1} i = K_{Soil\,1} \times \frac{\Delta h_1}{\ell_1}$$

$$\therefore K_{Soil\,1} = V_{Soil\,1} \times \frac{\ell_1}{\Delta h_1}$$

$$= 6.37 \times 10^{-3} \times \frac{200}{300}$$

$$K_{Soil\,1} = 4.24 \times 10^{-3}\, mm/\sec$$

$$or \;\; 4.24 \times 10^{-6}\, m/\sec$$

2) Calculation K_2

$$V_{Soil\,2} = K_{Soil\,2} i = K_{Soil\,2} \times \frac{\Delta h_2}{\ell_2}$$

$$\therefore K_{Soil\,2} = V_{Soil\,2} \times \frac{\ell_2}{\Delta h_2}$$

$$= 6.37 \times 10^{-3} \times \frac{100}{300}$$

$$K_{Soil\,2} = 2.12 \times 10^{-3}\, mm/\sec \quad or\; 2.12 \times 10^{-6}\, m/\sec$$

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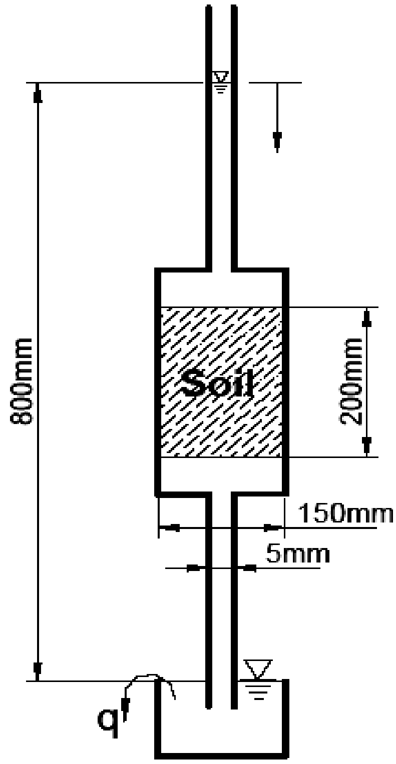
Problem 2.



1) Derivation

$$q = -K\frac{h}{L}A = a\frac{dh}{dt}$$

$$dt = -\frac{aL}{KA}\frac{dh}{h}$$

$$\int_0^t dt = -\frac{aL}{KA}\int_0^t \frac{1}{h}dh$$

$$t = -\frac{aL}{KA}\ln(h)\Big]_0^t$$

Boundary conditions

$$t = 0 \quad h = h_1 \text{ , } t = t \quad h = h_2$$

$$t = -\frac{aL}{KA}\left[\ln(h_2) - \ln(h_1)\right]$$

$$= \frac{aL}{KA}\ln\left(\frac{h_1}{h_2}\right)$$

$$\therefore \ K = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

2) Calculate K

Solution 1)

$$K = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

$$\text{If } t = 100\sec, \quad dh = 0.01\times 100 = 1mm$$

$$\therefore \ h_1 = 800mm, \ h_2 = 799mm$$

$$K = \frac{\frac{\pi}{4}\times 5^2\times 200}{\frac{\pi}{4}\times 150^2\times 100}\ln\left(\frac{800}{799}\right)$$

$$= 2.78\times 10^{-6}mm/\sec \quad or \quad 2.78\times 10^{-9}m/\sec$$

Solution 2)

$$Q_{tube} = Q_{Soil}$$

$$Q \text{ in } d = 5mm \text{ tube}, Q = AV = \frac{\pi}{4}\times 5^2\times 0.01 = 0.196mm^3/\sec$$

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$Q\ in\ d = 150mm\ tube$

$$Q = \frac{\pi}{4} \times 150^2 \times V = 0.196mm^3/\sec$$

$$V = 1.11 \times 10^{-5} mm/\sec$$

$$= Ki = K\frac{\Delta h}{l} = K\frac{800}{200}$$

$$\therefore K = 2.78 \times 10^{-6} mm/\sec,\ or\ 2.78 \times 10^{-9} m/\sec$$

Problem 3.

$$P_1A = P_2A + S + \gamma_w AL\sin\theta$$

$$\gamma_w(h_1 - z_1)A = \gamma_w(h_2 - z_2)A + S + \gamma_w AL\frac{z_2 - z_1}{L}$$

$$h_1 - z_1 = h_2 - z_1 + \frac{S}{\gamma_w A} + z_2 - z_1$$

$$\frac{h_1 - h_2}{L} = \frac{S}{\gamma_w AL} = i$$

The seepage force $= \gamma_w i = \dfrac{S}{AL}$ (per unit soil volume)

Problem 4.

1) Derivation

$$q = K\left(\frac{dh}{dr}\right) \times 2\pi rh$$

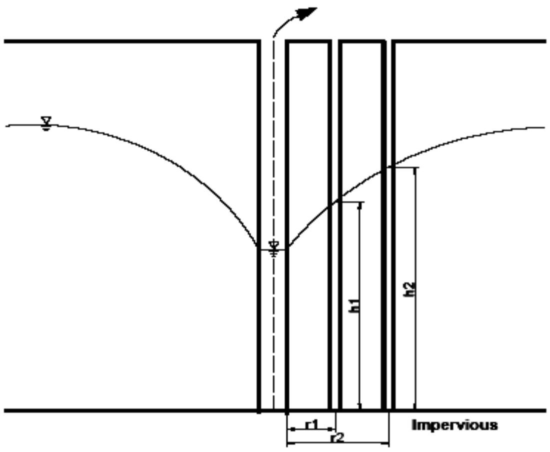
$$\frac{dr}{r} = \frac{2\pi K}{q} hdh$$

$$\int_{r_1}^{r_2} \frac{1}{r}dr = \frac{2\pi K}{q}\int_{h_1}^{h_2} hdh$$

$$\ln R\Big]_{r_1}^{r_2} = \frac{2\pi K}{q} \times \frac{1}{2}H^2\Big]_{h_1}^{h_2}$$

$$\ln\left(\frac{r_2}{r_1}\right) = \frac{\pi K}{q}(h_2^2 - h_1^2)$$

$$\therefore K = \frac{q}{\pi(h_2^2 - h_1^2)}\ln\left(\frac{r_2}{r_1}\right)$$



2) Calculation

$$K = \frac{q}{\pi(h_2^2 - h_1^2)}\ln\left(\frac{r_2}{r_1}\right) = \frac{10^{-3}/60}{\pi(3.6^2 - 3^2)}\ln\left(\frac{5.05}{3.05}\right)$$
$$= 6.76 \times 10^{-7} m/\sec$$

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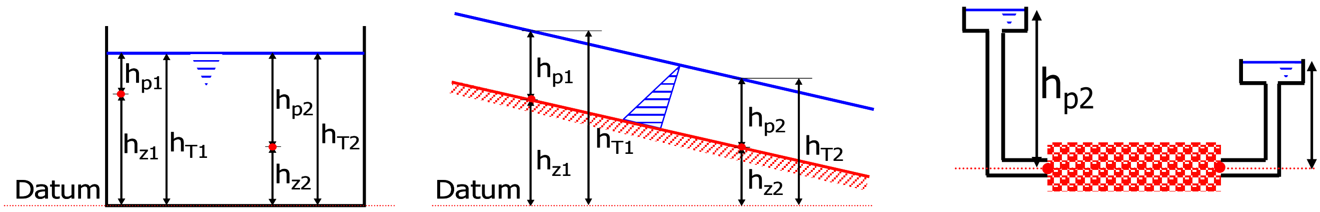
Introduction

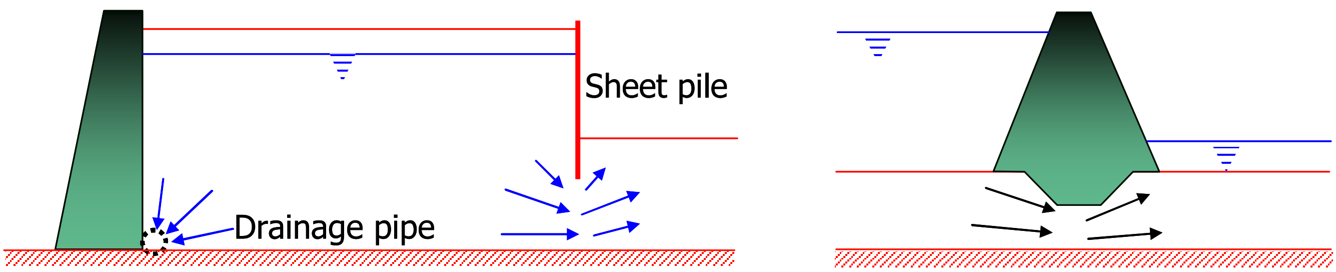
Swimming pool   Open Channel



Objectives

- To obtain pore pressure (stability analysis)
- To calculate flow
- To verify piping conditions

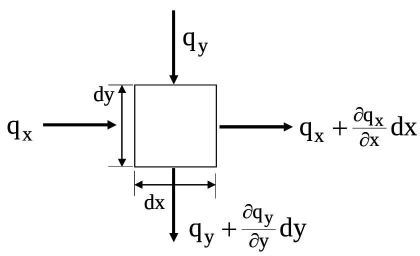
Retaining wall   Cofferdam   Hydraulic dam



Laplace's Equation

Elemental Cube:

- Saturation S=100 %
- Void ratio e= constant
- Laminar flow



$$q_{in} = q_{out}$$

Continuity:

$$q_x + q_y - \left(q_x + \frac{\partial q_x}{\partial x} dx + q_y + \frac{\partial q_y}{\partial y} dy \right) = 0$$

$$\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$$

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$$\frac{\partial q_x}{\partial x} dx + \frac{\partial q_y}{\partial y} dy = 0$$

- Darcy's law:

$$\boxed{q_x = k_x \cdot i \cdot A = k_x \cdot \frac{\partial h_T}{\partial x} \cdot dy \cdot 1}$$

- Replacing:

$$0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} dx \cdot dy \cdot 1 + k_y \cdot \frac{\partial^2 h_T}{\partial y^2} dy \cdot dx \cdot 1$$

$$0 = k_x \cdot \frac{\partial^2 h_T}{\partial x^2} + k_y \cdot \frac{\partial^2 h_T}{\partial y^2}$$

- if $k_x = k_y$

$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$$

Laplace's Equation! (Isotropy):

Typical cases

1 Dimensional:

$$0 = \frac{\partial^2 h_T}{\partial x^2} \quad , \quad constant = \frac{\partial h_T}{\partial x} = i$$

linear variation!!

$$h_T = a + b \cdot x$$

2-Dimensional:

$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2}$$

3-Dimensional:

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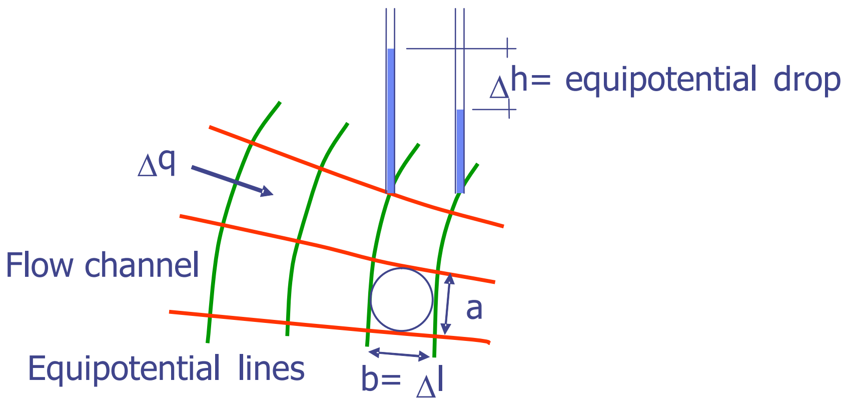
$$0 = \frac{\partial^2 h_T}{\partial x^2} + \frac{\partial^2 h_T}{\partial y^2} + \frac{\partial^2 h_T}{\partial z^2}$$

Laplace's Equation Solutions

- Exact solutions (for simple B.C.'s)
- Physical models (scaling problems)
- Approximate solutions: method of fragments
- Graphical solutions: flow nets
- Analogies: heat flow and electrical flow
- Numerical solutions: finite differences

Flow Nets

- The procedure consists on drawing a set of perpendicular lines: equi-potentials and flow lines.
- These set of lines are the solution to the Laplace's equation.
- It is an iterative (and tedious!) process.
- Identify boundaries:
  - First and last equi-potentials
  - First and last flow lines



- gradient:

$$i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b} = \frac{h/N_e}{b}$$

- flow per channel:

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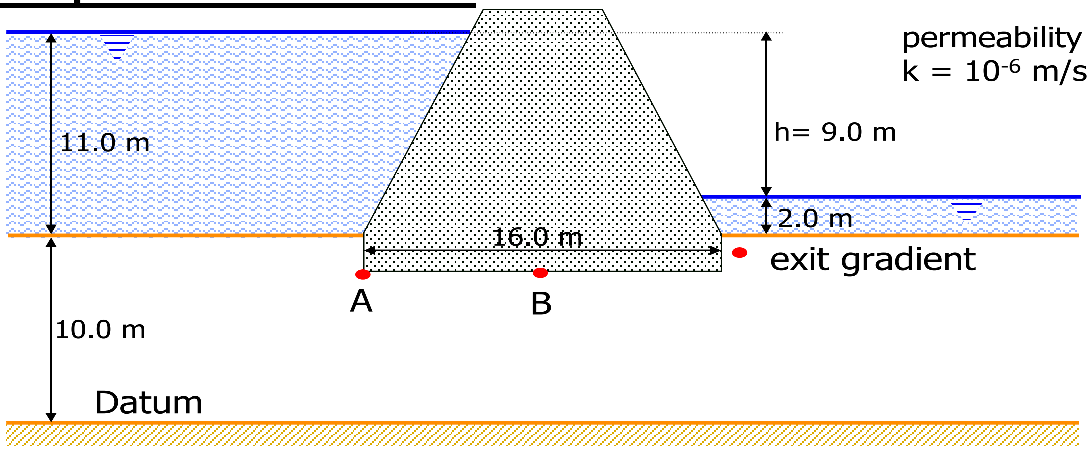
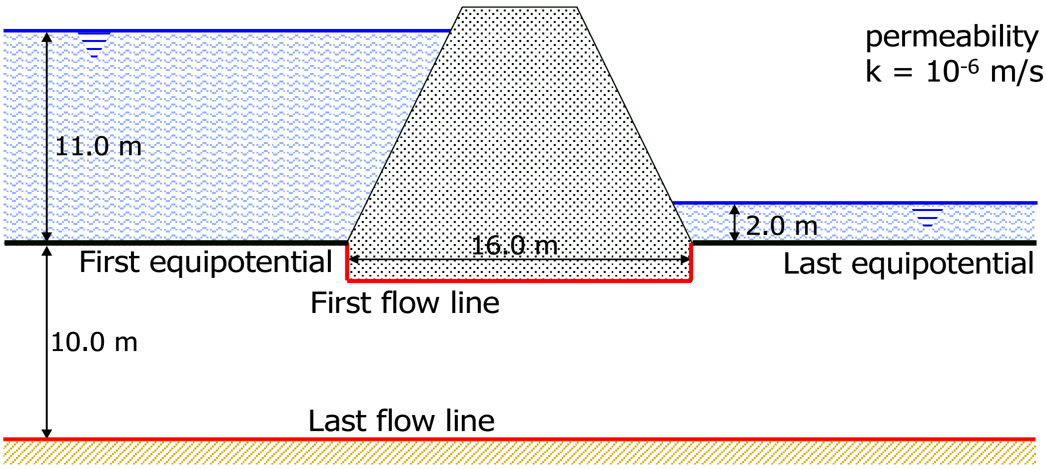
$$\Delta q = k \cdot \frac{\Delta h}{\Delta l} \cdot A = k \cdot \frac{h/N_e}{b} \cdot A$$

- total flow:

$$q = \Delta q \cdot N_f = k \cdot h \cdot \frac{a}{b} \cdot \frac{N_f}{N_e}$$

Example: Confined flow





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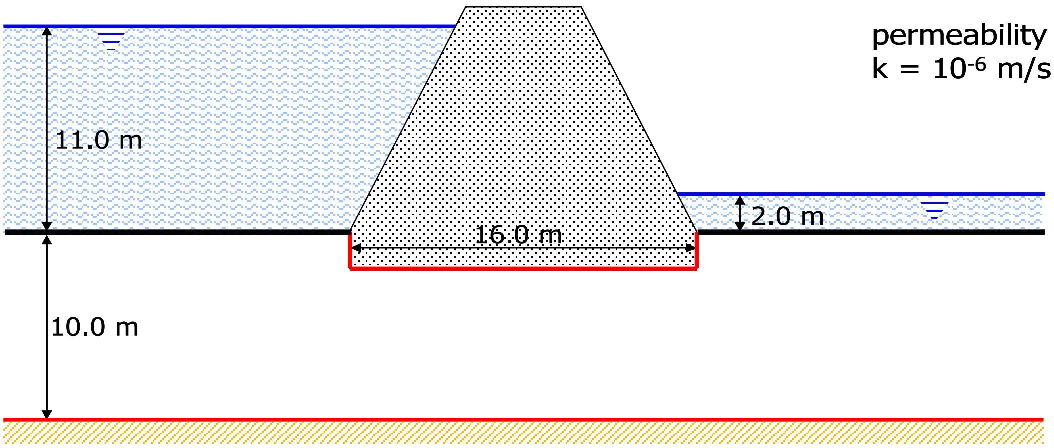
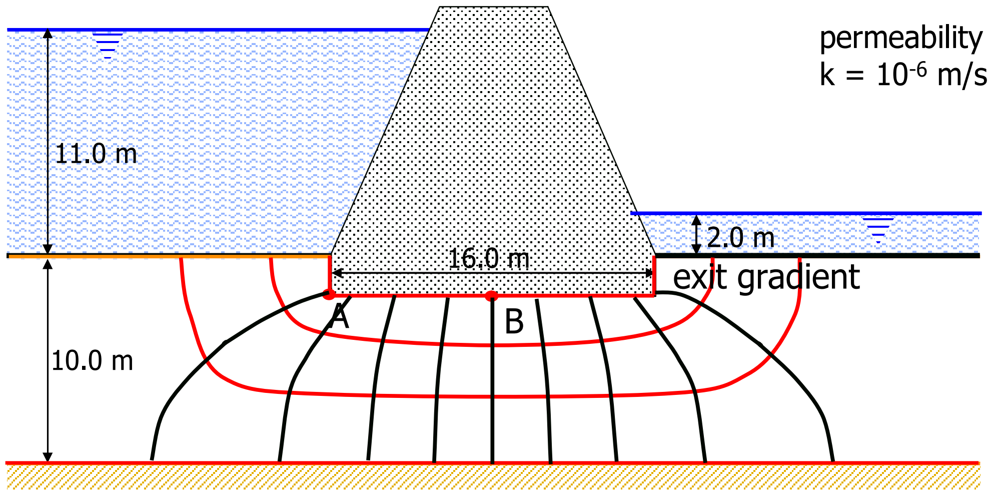


Scale: 1 square = 4 m²



Scale: 1 square = 4 m²

Seepage loss under the dam:

$$q = k \cdot \Delta h \cdot \left(\frac{N_f}{N_e}\right) = 10^{-6}\,\frac{m}{s} \cdot 9m \cdot \frac{3}{10} = 2 \cdot 10^{-4}\,\frac{m^3}{s \cdot m}$$

Exit gradient:

$$i_e = \frac{\Delta h}{\Delta l} = \frac{h/N_e}{\Delta l} = \frac{0.9\ m}{2\ m} = 0.45$$

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$$i_{crit} = 1 \therefore \Delta h \cdot \gamma_w = \Delta l \cdot (\gamma_{sat} - \gamma_w) \Rightarrow \text{piping !!}$$

Total head at points A and B:

$$h_{TA} = h_{To} - h \cdot \frac{N_A}{N_f} = 21\,m - 9\,m \cdot \frac{1}{10} = 20.1\,m$$

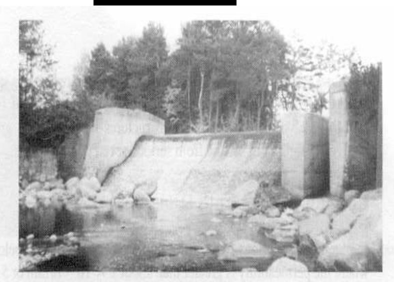
$$h_{TB} = h_{To} - h \cdot \frac{N_B}{N_f} = 21\,m - 9\,m \cdot \frac{5}{10} = 16.5\,m$$

Pressure head at points A and B:

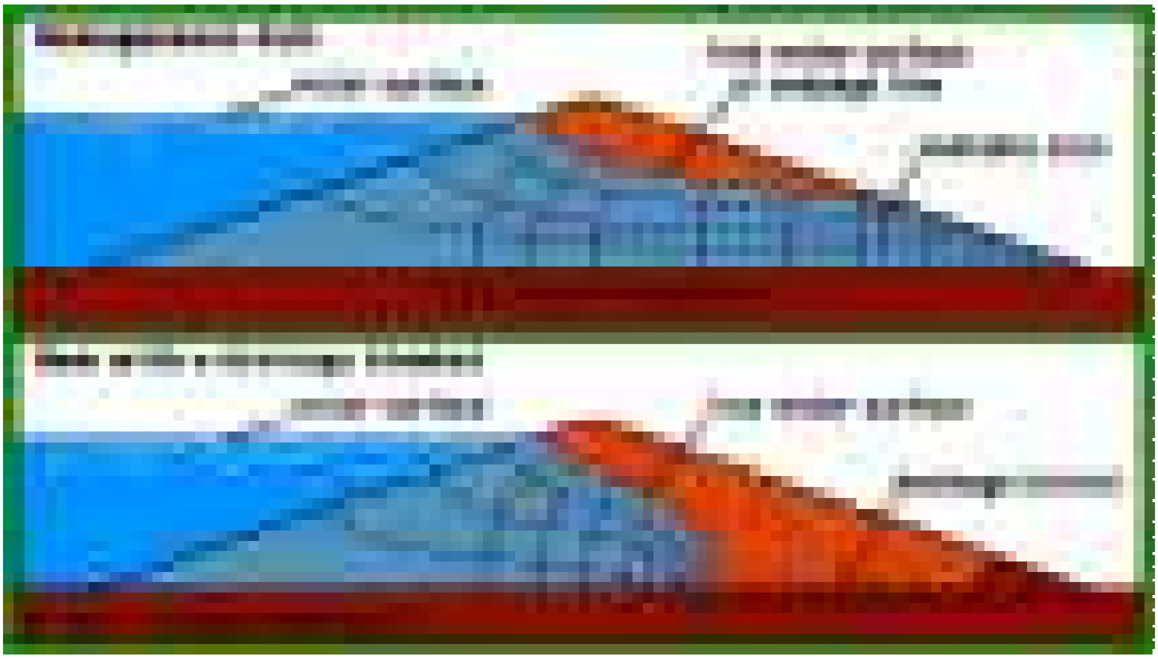
$$h_{PA} = h_{TA} - h_{zA} = 20.1\,m - 8\,m = 12.1\,m \Rightarrow \approx 121\ kPa$$

$$h_{PB} = h_{TB} - h_{zB} = 16.5\,m - 8\,m = 8.5\,m \Rightarrow \approx 85\,kPa$$

Piping



Example: Unconfined flow



Seepage Control - Filters

- Seepage: Cut-off walls
  Impervious blankets
- Erosion and piping: Filters

Requirements

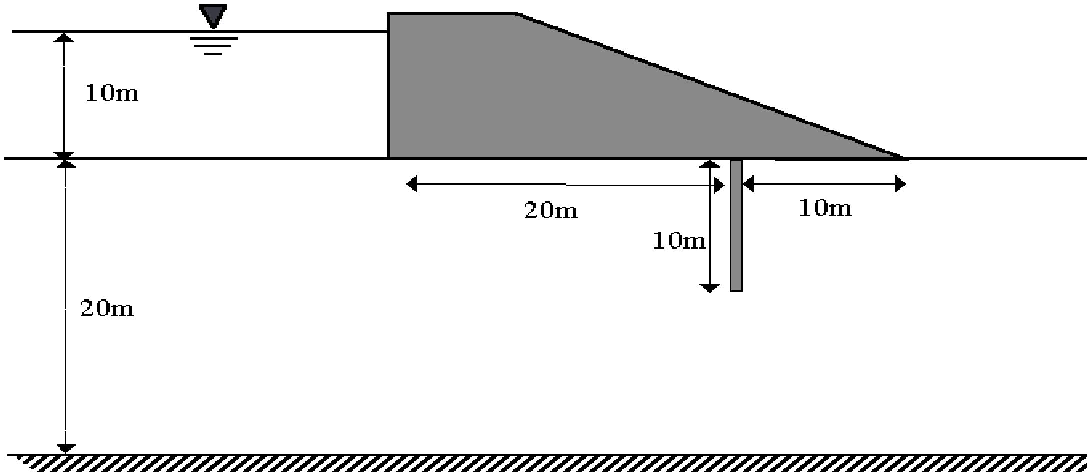
Piping: D15 (filter) =< 5 D85 (soil)

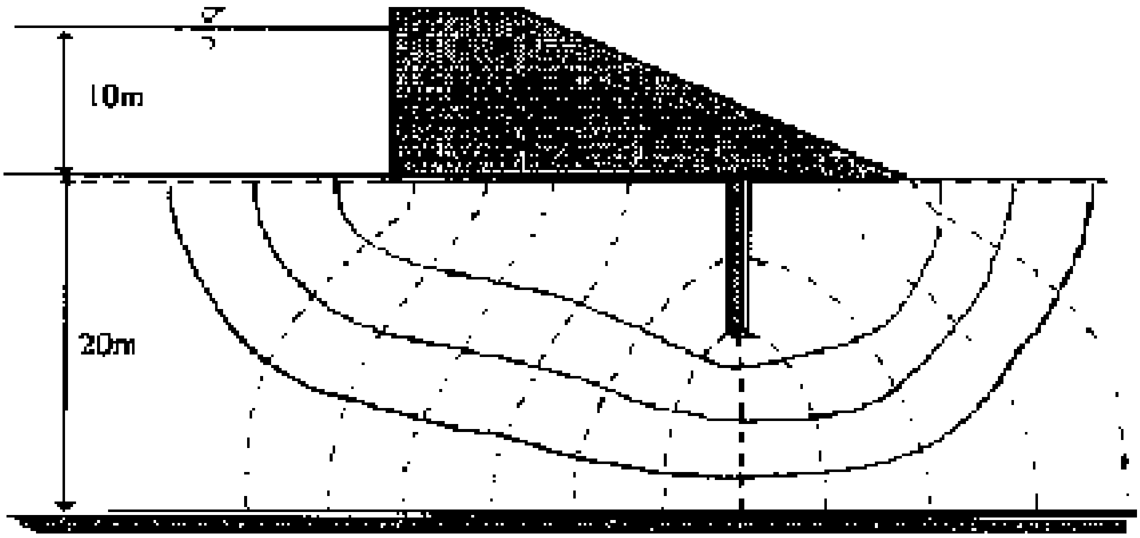
Permeability: D15 (filter) >= 5 D15 (soil)

Uniformity: D50 (filter) =< 25 D50 (soil)

Example**.** Impervious concrete dam with cut-off.

Sketch a flow net for the situation shown below, and hence calculate the seepage quantity per unit width of dam per day. The permeability of the soil, k = 10^{-3} mm/sec.





$$q = kh\frac{N_f}{N_d} = \frac{10^{-3}}{1000} \times 10 \times \frac{3.33}{12} = 2.78 \times 10^{-6}\ m^3/sec$$

(This is q/unit length)

To m^3/day: $2.78 \times 10^{-6} \times 60 \times 60 \times 24 = \underline{\underline{0.24\ m^3/day}}$

Tutorial

Ex1:

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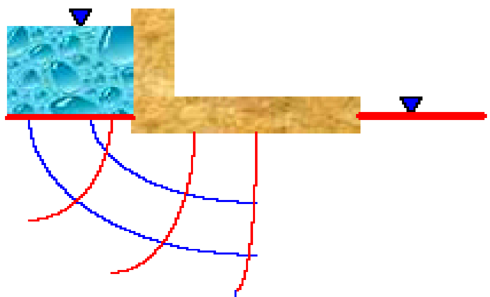
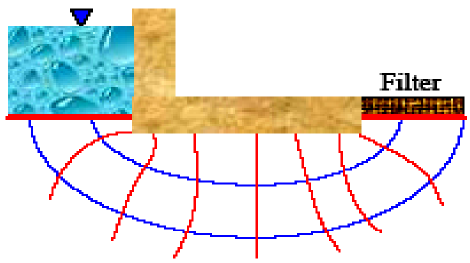
State what is wrong, if anything, with each of the following flow nets.

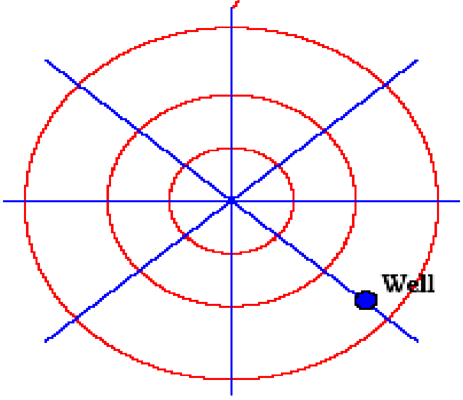
a)



b)



c)



Equipotential Lines
Flow Lines

Solution:

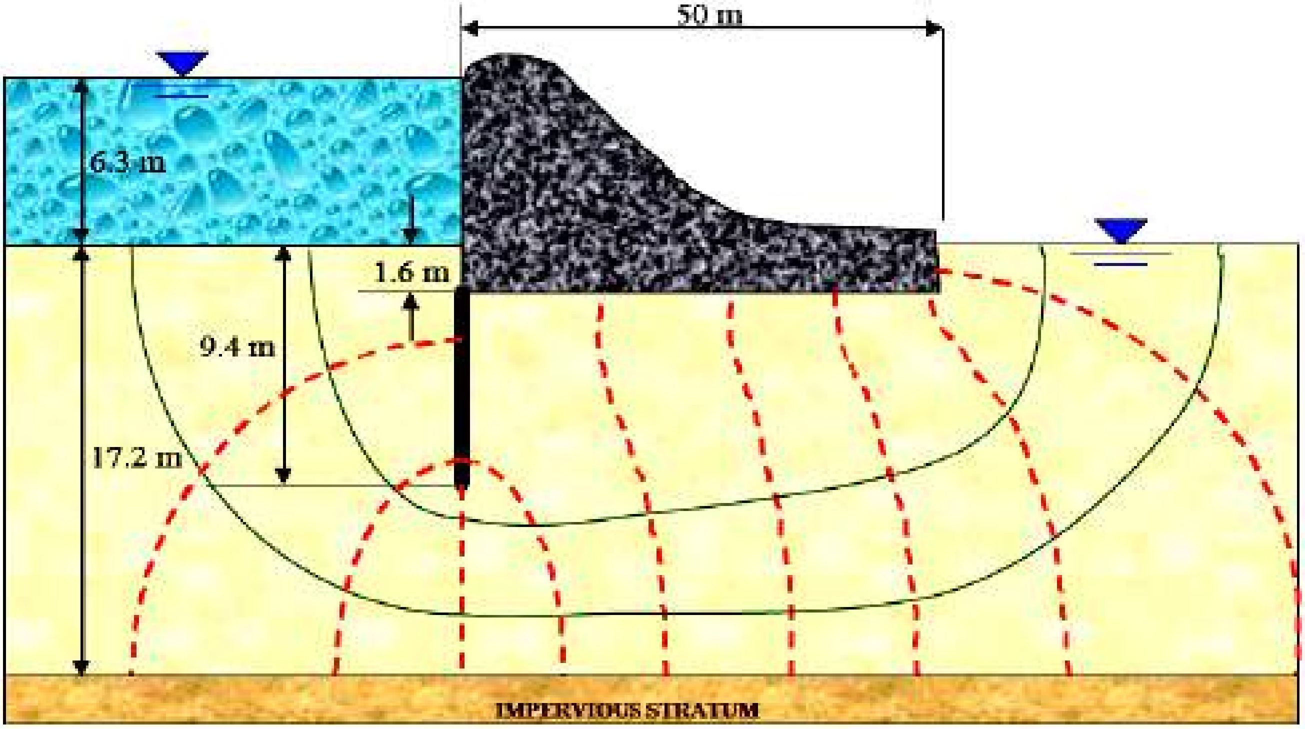
a) Impossible mesh, because two equipotential lines intersect.

b) Impossible mesh, because two flow-lines intersect.

c) The well should be at the center of the net (a sink or a source point).

Ex2:

The completed flow net for the dam shown below includes a steel sheet-pile cutoff wall located at the head (head-water side) of the dam, in order to reduce the seepage loss. The dam is half a kilometer in width (shore to shore), and the permeability of the under-laying silty sand is 3.5×10^{-4} cm/s. Find the total seepage loss under the dam in liter per year. Would the dam be more stable if the cutoff wall was placed under its toe (tail-water side)?



Solution:

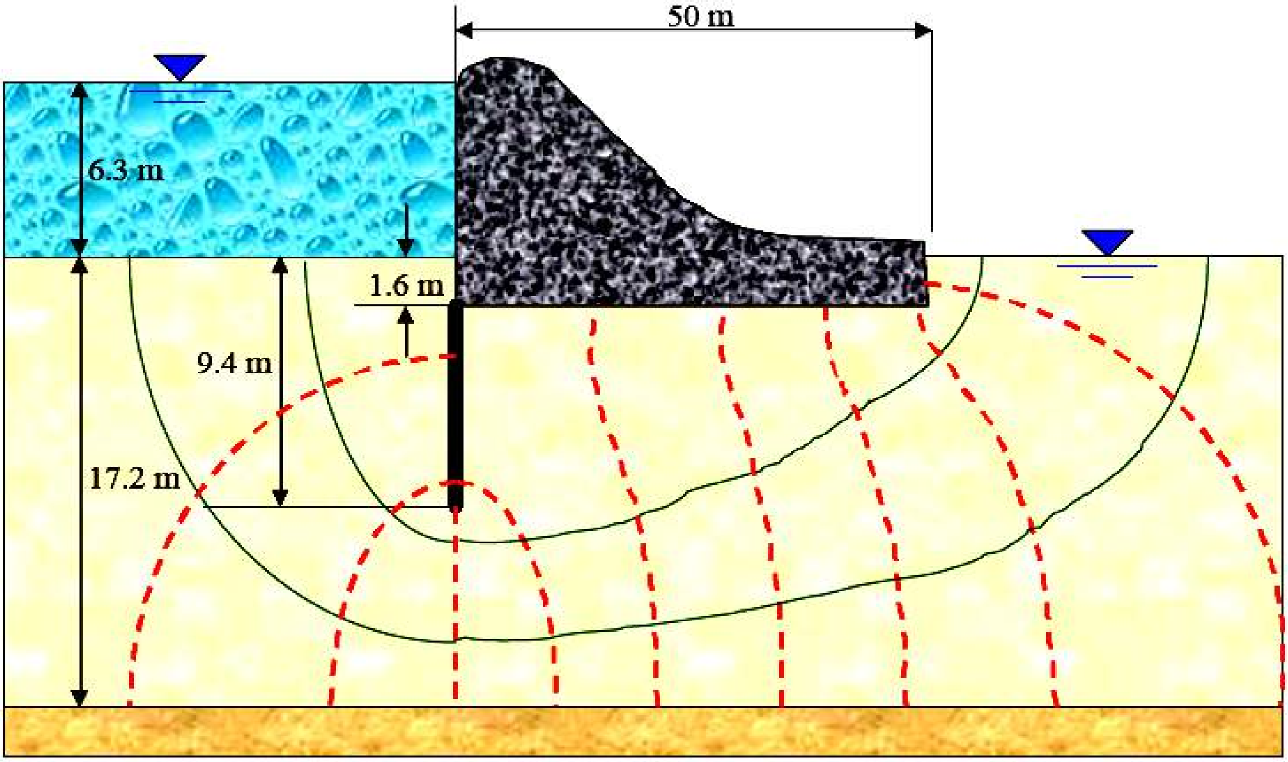
a) Using ***Forcheimer's*** equation:

$$q = k\Delta h \frac{N_f}{N_p} = 3.5 \times 10^{-4} \frac{cm}{\sec}\left(\frac{m}{100\,cm}\right)(6.3m)\left(\frac{3}{10}\right) = 6.6 \times 10^{-6}\ m^2/\sec/m$$

b) $$Q = Lq = 500\,m\left[6.6 \times 10^{-6}\ m^2/\sec/m\right]\left(\frac{lts}{10^{-3}\ m^3}\right)\left(31.5 \times 10^6\ \frac{\sec}{year}\right) = 104\,\frac{million\ liters}{year}$$

c) No. Placing the cutoff wall at the toe would allow high uplift hydrostatic pressures under the dam, thereby decreasing the dam's stability against sliding.

The completed flow net for the dam shown below includes a steel sheet-pile cutoff wall located at the head (head-water side) of the dam, in order to reduce the seepage loss. The dam is half a kilometer in width (shore to shore), and the permeability of the under-laying silty sand is 3.5 x 10^{-4} cm/s. Find the total seepage loss under the dam in liter per year. Would the dam be more stable if the cutoff wall was placed under its toe (tail-water side)?



Solution:

a) Using ***Forcheimer's*** equation:

$$q = k\Delta h\frac{N_f}{N_p} = 3.5\times10^{-4}\,\frac{cm}{\sec}\left(\frac{m}{100\,cm}\right)(6.3m)\left(\frac{3}{10}\right) = 6.6\times10^{-6}\ m^2/\sec/m$$

b) $$Q = Lq = 500\ m\left[6.6\times10^{-6}\ m^2/\sec/m\right]\left(\frac{lts}{10^{-3}\ m^3}\right)\left(31.5\times10^6\ \frac{\sec}{year}\right) = 104\,\frac{million\ liters}{year}$$

c) No. Placing the cutoff wall at the toe would allow high uplift hydrostatic pressures under the dam, thereby decreasing the dam's stability against sliding.

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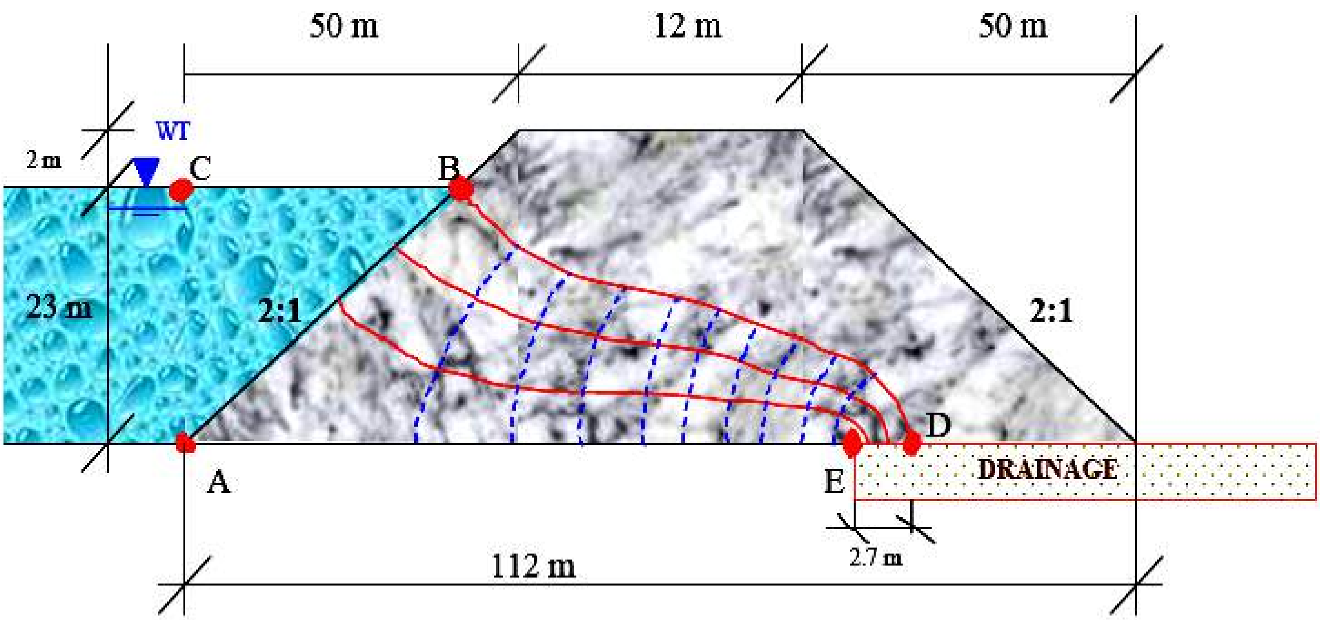
In Miami-Dade County, the Everglades are kept as wetlands by containing their runoff with levees. Levee #111 runs North-South, and is 2 kilometers west of Krome Avenue (its cross section is show below). If your laboratory tests indicate that the permeability of the fill of the 80-year old levee is now 0.30 m/day, what is the volume of water lost through the levee along each kilometer of length, in m^3/day?



Section of levee looking North

$$Q = qL = k\Delta h \frac{N_f}{N_{eq}} \bullet L = \left(0.30\ \frac{m}{day}\right)(23\ m)\left(\frac{3}{9}\right)1000\ m$$

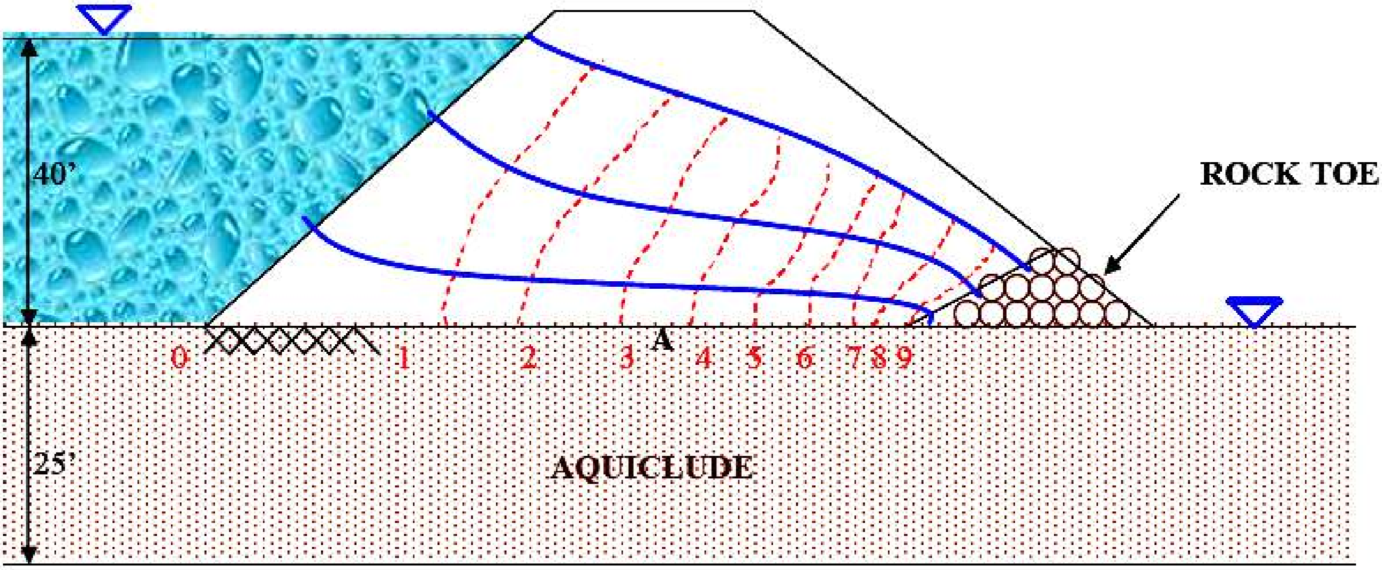
$$\boxed{Q = 2{,}300\ m^3/day}$$

Find the seepage through the earth dam shown below in gallons/day, if the sieve analysis shows the D_{10} to be 0.17 mm, and the dam is 1200 feet wide. What is the pressure head at the top of the aquiclude and at mid-dam (point A)?

$$N_f = 3$$

$$N_p = 9$$

$$k = 15 D_{10}^2 = 15(0.17\,mm)^2 = 0.43\,\frac{mm}{\sec}$$



$$Q = q \times h = k \times i \times A \times h = \frac{\left(0.43\frac{mm}{\sec} \times 1\,in \times 40\,ft \times 86{,}400\,\sec \times 7.5\,gallons \times 1200\,ft \times \frac{3}{9}\right)}{(25.4\,mm \times 12\,in \times day)}$$

$$Q = 14.6 \times 10^6\,\frac{\text{gal}}{day}$$

At point "A" the dynamic pressure head is $\frac{3.4}{9}(40\,ft) = 15\,ft.$

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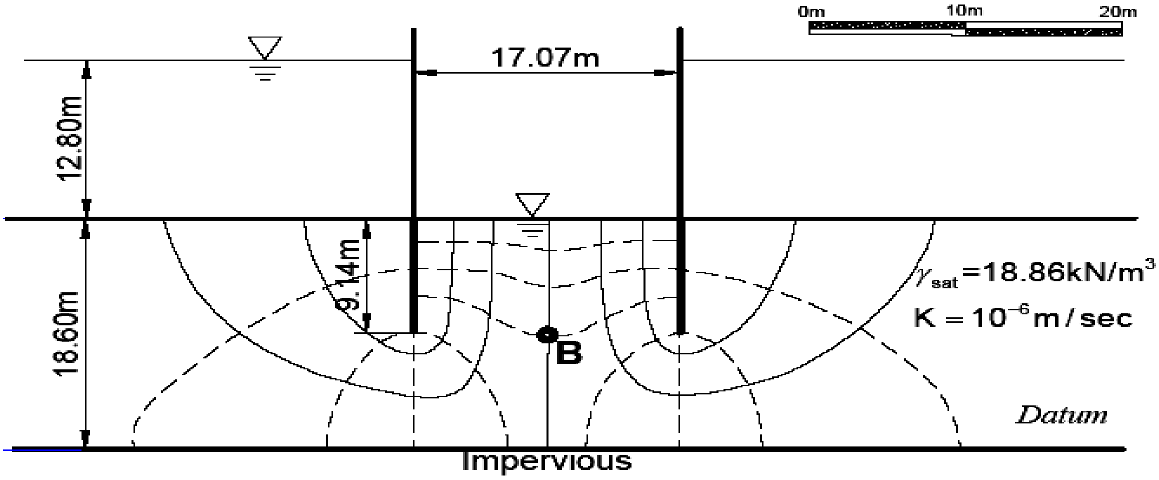
Problem 2. Cofferdam

Solution 1) The datum line is impervious base.



1) The pore water pressure at B (u_b)

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{12.8}{8} = 1.6m$$

- The total head

$$h_t = \Delta H_t - \Delta h \times (N_d)_B = (12.8 + 18.6) - 1.6 \times 5 = 23.4m$$

- The elevation head at B : h_e = 18.6-9.14=9.46m

- The pressure head at B

$$(h_p)_B = h_t - h_e = 23.4 - 9.46 = 13.94m$$

- The pore water pressure at B

$$u_B = h_p \times \gamma_w = 13.94 \times 9.81 = 136.75 kN/m^2 = 136.75 kPa$$

2) The maximum hydraulic gradient(i_e)

$$i_{\max} = \frac{\Delta h}{L_{\min}} = \frac{1.6}{1.75} = 0.91$$

The critical hydraulic gradient(i_c)

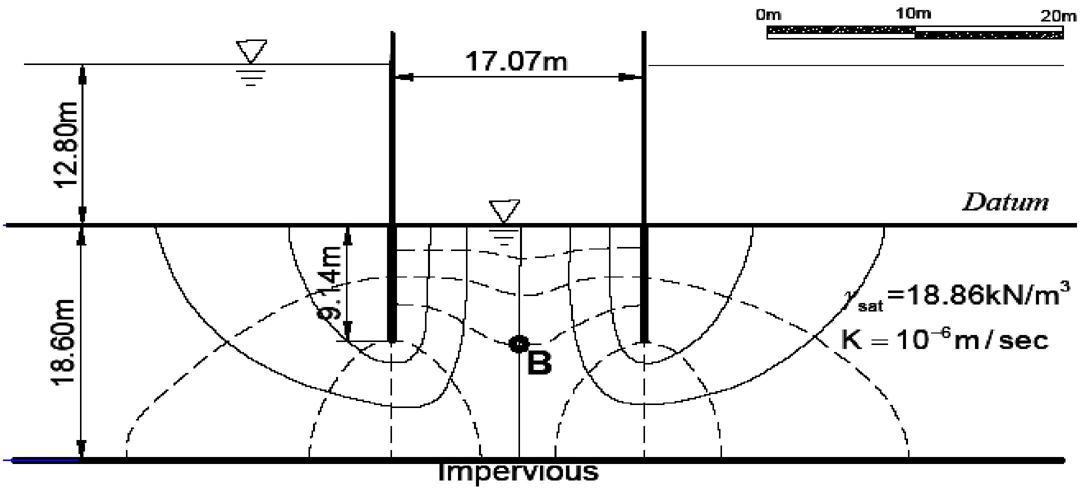
$$i_c = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{18.86 - 9.81}{9.81} = 0.92$$

3) Determine the factor of safety against quick sand and explain it

$$FOS = \frac{i_c}{i_{\max}} = \frac{0.92}{0.91} = 1.01 < 4$$

The piping may occur.

Solution 2) The datum line is the right surface.



1) The pore water pressure at B (u_b)

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{12.8}{8} = 1.6m$$

- The total head

$$h_t = \Delta H - \Delta h \times (N_d)_B = 12.8 - 1.6 \times 5 = 4.8m$$

- The elevation head at B : h_e = -9.14m
- The pressure head at B

$$(h_p)_B = h_t - h_e = 4.8 - (-9.14) = 13.94m$$

- The pore water pressure at B

$$u_B = h_p \times \gamma_w = 13.94 \times 9.81 = 136.75kN/m^2 = 136.75kPa$$

4) The maximum hydraulic gradient(i_e)

$$i_{max} = \frac{\Delta h}{L_{min}} = \frac{1.6}{1.75} = 0.91$$

The critical hydraulic gradient(i_c)

$$i_c = \frac{\gamma_{sat} - \gamma_w}{\gamma_w} = \frac{18.86 - 9.81}{9.81} = 0.92$$

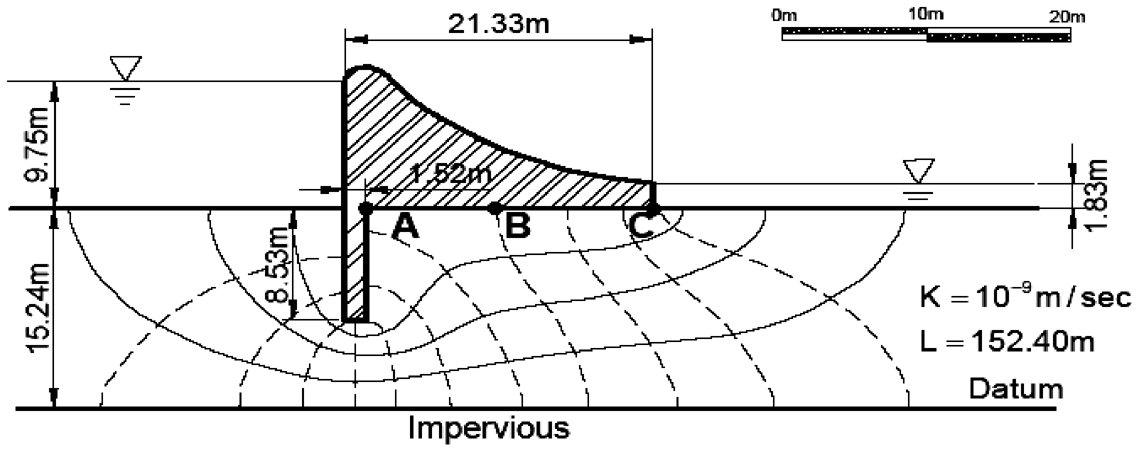
5) Determine the factor of safety against quick sand and explain it

$$FOS = \frac{i_c}{i_{max}} = \frac{0.92}{0.91} = 1.01 < 4$$

The piping may occur.

Problem 3. Spillway

Solution 1) The datum line is impervious base.



1) Determine the pore water pressure distribution at A, B ,C

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{(9.75-1.89)}{12} = 0.66m$$

- The pressure head at each point : $h_p = h_t - h_e = \Delta H - N_d \times \Delta h - h_e$

$$(h_p)_A = (9.75+15.24) - 7.2 \times 0.66 - 15.24 = 5m$$

$$\therefore u_A = 5 \times 9.81 = 49.05kPa$$

$$(h_p)_B = (9.75+15.24) - 8 \times 0.66 - 15.24 = 4.47m$$

$$\therefore u_B = 43.85kPa$$

$$(h_p)_C = (9.75+15.24) - 11 \times 0.66 - 15.24 = 2.49m$$

$$\therefore u_C = 24.43kPa$$

h_e at end of wall =(15.24-8.53)m

$$(h_p)_{end\ of\ wall} = (9.75+15.24) - 4 \times 0.66 - (15.24-8.53) = 15.64m$$

$$\therefore u_{end\ of\ wall} = 15.64 \times 9.81 = 153.43kPa$$

2) The resultant uplift force (Hint : $F_{up} = A_{bottom} * u_{aver}$)

$$F_{up} = (A_{wall} \times u_{wall}) + A_{AB} \times u_{AB} + A_{BC} \times u_{BA}$$

$$= ((1.52 \times 152.4) \times 153.43) + (9 \times 152.4) \times \frac{49.05+43.85}{2} + (11 \times 152.4) \times \frac{43.85+24.43}{2}$$

$$= 156484.9kN = 156.5MN\ (When\ consider\ the\ wall)$$

$$= 120943.1kN = 120.9\ MN\ (When\ ignore\ the\ wall)$$

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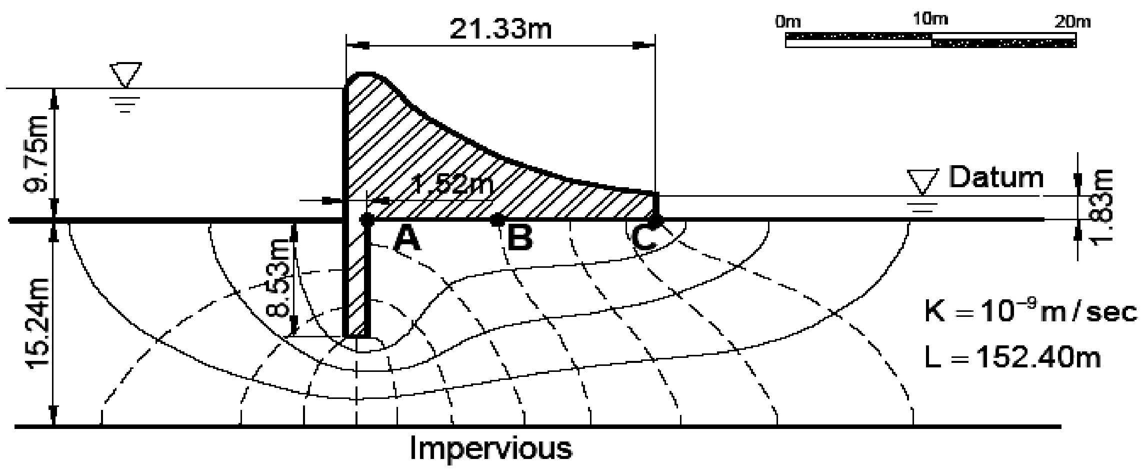
Solution 2) The datum is the surface of downstream.



3) Determine the pore water pressure distribution at A, B ,C

- The head loss between each equipotential line

$$\Delta h = \frac{\Delta H}{N_d} = \frac{(9.75-1.89)}{12} = 0.66m$$

- The pressure head at each point: $h_p = h_t - h_e = \Delta H - N_d \times \Delta h - (-1.83)$

$$(h_p)_A = (9.75-1.83) - 7.2 \times 0.66 + 1.83 = 5m$$

$$\therefore u_A = 5 \times 9.81 = 49.05kPa$$

$$(h_p)_B = (9.75-1.83) - 8 \times 0.66 + 1.83 = 4.47m$$

$$\therefore u_B = 43.85kPa$$

$$(h_p)_C = (9.75-1.83) - 11 \times 0.66 + 1.83 = 2.49m$$

$$\therefore u_C = 24.43kPa$$

h_e at end of wall =-(8.53+1.83)m

$$(h_p)_{end\ of\ wall} = (9.75-1.83) - 4 \times 0.66 + (8.53+1.83) = 15.64m$$

$$\therefore u_{end\ of\ wall} = 15.64 \times 9.81 = 153.43kPa$$

4) The resultant uplift force (Hint : $F_{up} = A_{bottom} * u_{aver}$)

$$F_{up} = (A_{wall} \times u_{wall}) + A_{AB} \times u_{AB} + A_{BC} \times u_{BA}$$

$$= ((1.52 \times 152.4) \times 153.43) + (9 \times 152.4) \times \frac{49.05+43.85}{2} + (11 \times 152.4) \times \frac{43.85+24.43}{2}$$

$$= 156484.9kN = 156.5MN \ (\textit{When consider the wall})$$

$$= 120943.1kN = 120.9\ MN \ (\textit{When ignore the wall})$$

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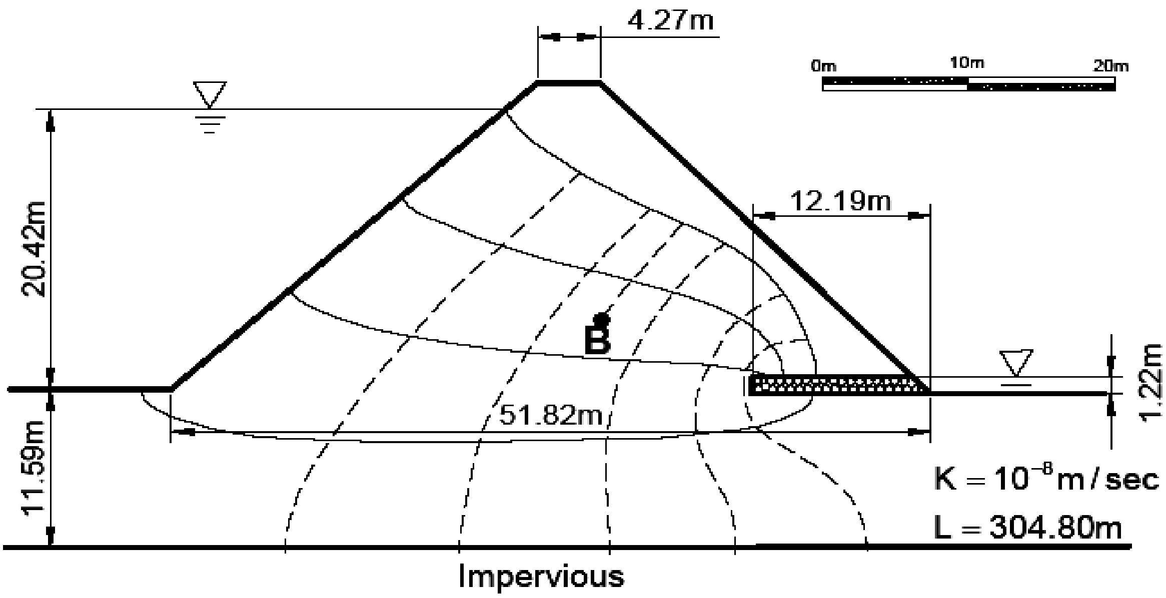
Problem 4. Evaluate the flow rate of the earth dam



$$q = 10^{-8} \times (20.42 - 1.22) \times \frac{4}{6} = 1.28 \times 10^{-7} m^3/\sec/m$$

$$Q = q \times L = k\Delta H \frac{N_f}{N_d} L = 10^{-8} \times (20.42 - 1.22) \times \frac{4}{6} \times 304.8$$

$$= 3.9 \times 10^{-5} m^3/\sec$$

or

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = (1.152)(0.325)(116.67 + 83.33 - 200) + 100 = 100$$

At $\bar{z} = 0.75$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = \frac{\Delta\bar{t}_{(2)}}{(\Delta\bar{z})^2}(\bar{u}_{1,\bar{t}} + \bar{u}_{3,\bar{t}} - 2\bar{u}_{0,\bar{t}}) + \bar{u}_{0,\bar{t}}$$

$$= 0.475[100 + 0 - 2(100)] + 100 = 52.5$$

At $\bar{z} = 1.0$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0$$

For $t = 10$ days,
At $\bar{z} = 0$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0$$

At $\bar{z} = 0.25$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.325[0 + 100 - 2(67.5)] + 67.5 = 56.13$$

At $\bar{z} = 0.5$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = (1.152)(0.325)\left[\frac{2 \times 2.8}{2 + 2.8}(67.5) + \frac{2 \times 2}{2 + 2.8}(52.5) - 2(100)\right] + 100$$

$$= (1.152)(0.325)(78.75 + 43.75 - 200) + 100 = 70.98$$

At $\bar{z} = 0.75$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.475[100 + 0 - 2(52.5)] + 52.5 = 50.12$$

At $\bar{z} = 1.0$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0$$

The variation of the nondimensional excess pore water pressure is shown in Fig. 6.10*b*. Knowing $\bar{u} = (\bar{u})(u_R) = \bar{u}(1.5)$ kN/m², we can plot the variation of u with depth.

EXAMPLE 6.7

For Example 6.6, assume that the surcharge q is applied gradually. The relation between time and q is shown in Fig. 6.11*a*. Using the numerical method, determine the distribution of excess pore water pressure after 15 days from the start of loading.

SOLUTION As before, $z_R = 8$ m, $u_R = 1.5$ kN/m². For $\Delta t = 5$ days,

$$\frac{\Delta\bar{t}_{(1)}}{(\Delta\bar{z})^2} = 0.325 \qquad \frac{\Delta\bar{t}_{(2)}}{(\Delta\bar{z})^2} = 0.475$$

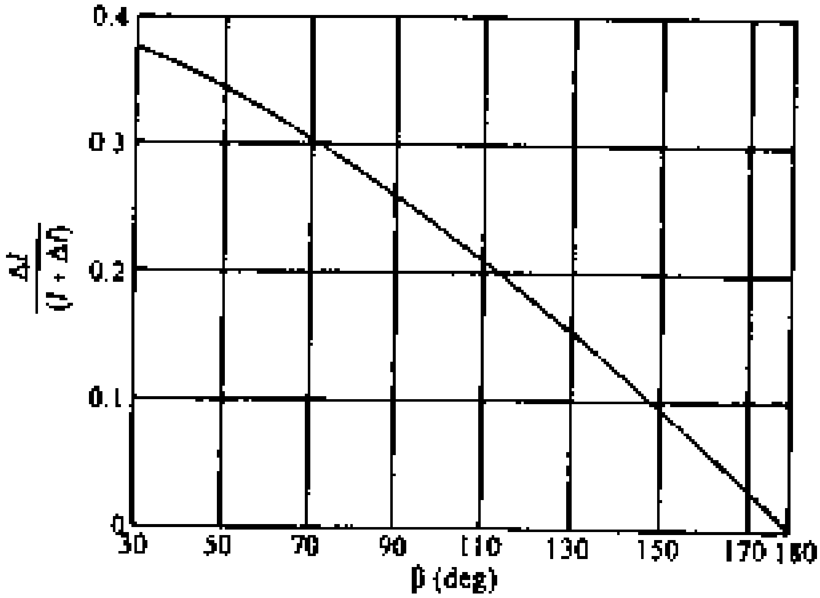


Fig. 5.63 Casagrande's (1937) plot of $\Delta l/(l + \Delta l)$ against downstream slope angle.

SOLUTION

$$\beta = \tan^{-1}(1/1.5) = 33.69^\circ$$

$$\Delta = 70 \cot 45^\circ = 70 \text{ ft}$$

$$aa' = 0.3\Delta = 0.3(70) = 21 \text{ ft}$$

and

$$d = 80 \cot 33.69^\circ + 15 + 10 \cot 45^\circ + 21 = 120 + 15 + 10 + 21 = 166 \text{ ft}$$

From Eq. (5.201),

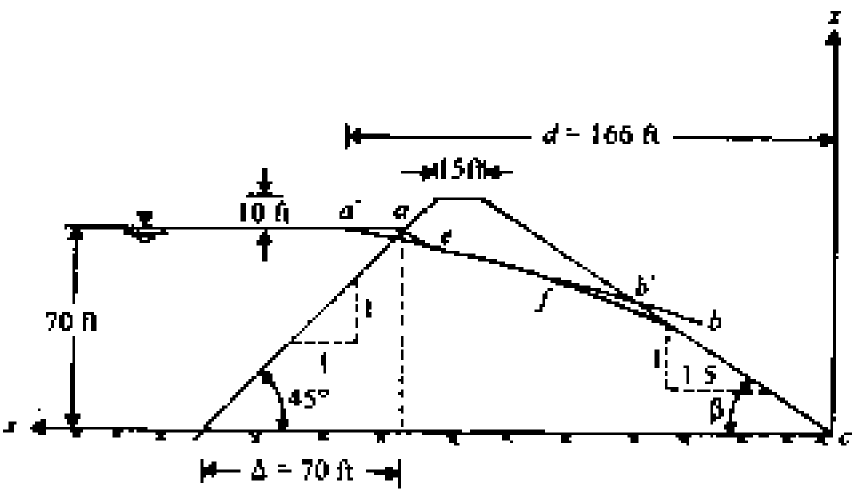


Fig. 5.64 Plot of phreatic line in an earth dam

$$p = \tfrac{1}{2}(\sqrt{d^2 + H^2} - d) = \tfrac{1}{2}(\sqrt{166^2 + 70^2} - 166)$$
$$= \tfrac{1}{2}(180.16 - 166) = 7.08 \text{ ft}$$

Using Eq. (5.202), we can now determine the coordinates of several points of the parabola $a'efb'c'$:

z, ft	x from Eq. (5.202), ft
70	166
65	142.1
60	120.04
55	99.73
50	81.2
45	64.42

Using the values of x and corresponding z calculated in the above table, the basic parabola has been plotted in Fig. 5.64.

We calculate l as follows. The equation of the line cb' can be given by $z = x \tan \beta$, and the equation of the parabola [Eq. (5.202)] is $x = (z^2 - 4p^2)/4p$. The coordinates of point b' can be determined by solving the above two equations:

$$x = \frac{z^2 - 4p^2}{4p} = \frac{(x \tan \beta)^2 - 4p^2}{4p}$$

$$\text{or} \qquad x^2 \tan^2 \beta - 4px - 4p^2 = 0$$

Hence

$$x^2 \tan^2 33.69° - 4(7.08)x - 4(7.08)^2 = 0$$

$$0.444x^2 - 28.32x - 200.5 = 0$$

The solution of the above equation gives $x = 70.22$ ft. So,

$$cb' = \sqrt{70.22^2 + (70.22 \tan 33.69°)^2} = 84.39 \text{ ft} = l + \Delta l$$

From Fig. 5.63, for $\beta = 33.69°$,

$$\frac{\Delta l}{l + \Delta l} = 0.366 \qquad \Delta l = (0.366)(84.39) = 30.9 \text{ ft}$$

$$l = (l + \Delta l) - (\Delta l)$$
$$= 84.39 - 30.9 = 53.49 \text{ ft} \approx 54 \text{ ft}$$

So, $l = cb = 54$ ft.

The curve portions ae and fb can now be approximately drawn by hand, which completes the phreatic line $aefb$ (Fig. 5.64).

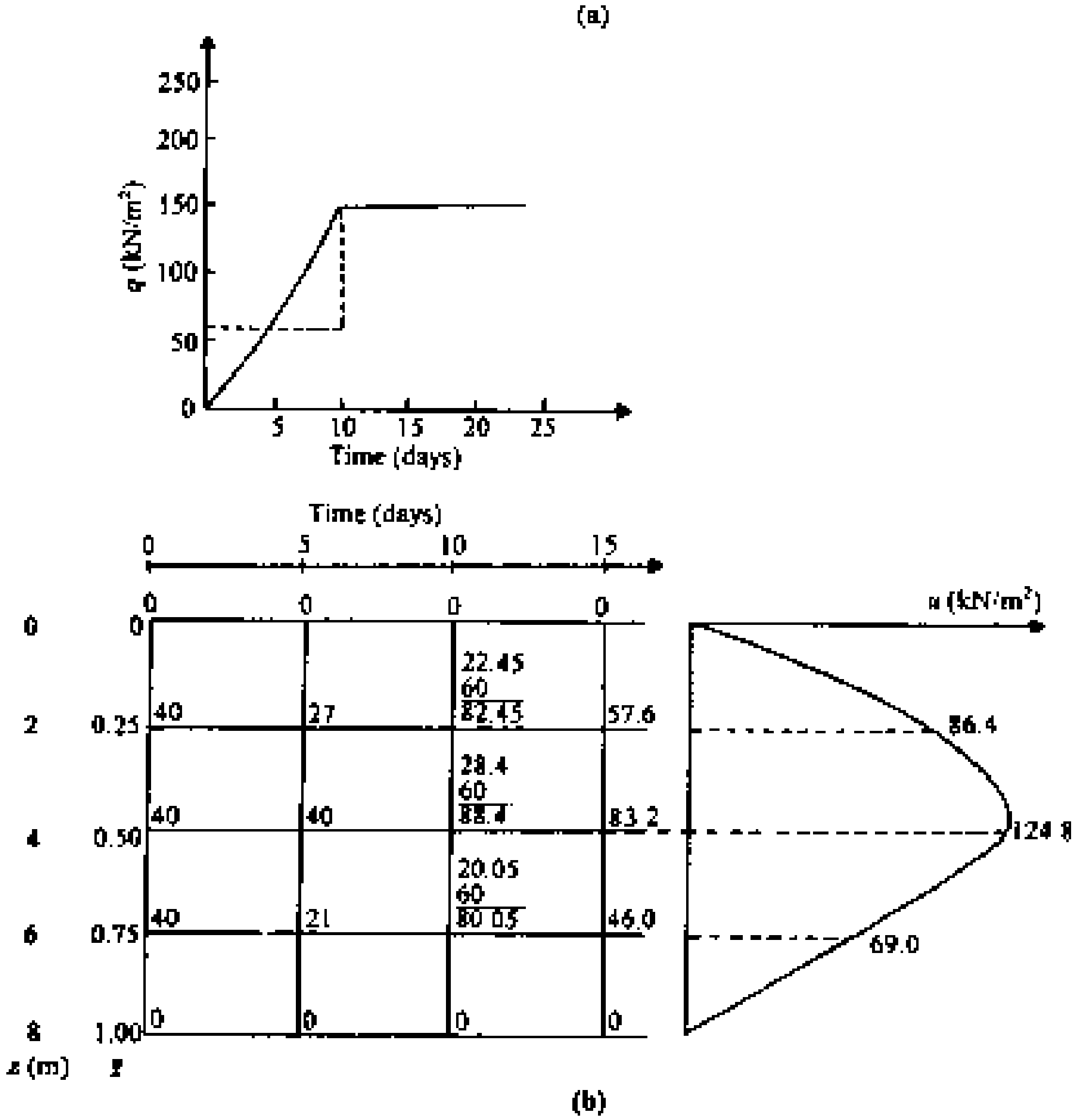


Fig. 6.11 Numerical solution for ramp loading.

The continuous loading can be divided into step loads such as 60 kN/m^2 from 0 to 10 days and an added 90 kN/m^2 from the tenth day on. This is shown by dashed lines in Fig. 6.11*a*.

At $t = 0$ days,

$$\bar z = 0 \qquad \bar u = 0$$

$$\bar z = 0.25 \qquad \bar u = 60/1.5 = 40$$

$$\bar z = 0.5 \qquad \bar u = 40$$

$$\bar z = 0.75 \qquad \bar u = 40$$

$$\bar z = 1 \qquad \bar u = 0$$

At $t = 5$ days,

At $\bar{z} = 0$,

$$\bar{u} = 0$$

At $\bar{z} = 0.25$, from Eq. (6.61),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.325[0 + 40 - 2(40)] + 40 = 27$$

At $\bar{z} = 0.5$, from Eq. (6.66),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = (1.532)(0.325)\left[\frac{2 \times 2.8}{2 + 2.8}(40) + \frac{2 \times 2}{2 + 2.8}(40) - 2(40)\right] + 40 = 40$$

At $\bar{z} = 0.75$, from Eq. (6.61),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.475[40 + 0 - 2(40)] + 40 = 21$$

At $\bar{z} = 1$,

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0$$

At $t = 10$ days,
At $\bar{z} = 0$,

$$\bar{u} = 0$$

At $\bar{z} = 0.25$, from Eq. (6.61),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.325[0 + 40 - 2(27)] + 27 = 22.45$$

At this point, a new load of 90 kN/m^2 is added, so $\bar{u}$ will increase by an amount 90/1.5 = 60. The new $\bar{u}_{0,\bar{t}+\Delta\bar{t}}$ is 60 + 22.45 = 82.45. At $\bar{z} = 0.5$, from Eq. (6.66),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = (1.152)(0.325)\left[\frac{2 \times 2.8}{2 + 2.8}(27) + \frac{2 \times 2}{2 + 2.8}(21) - 2(40)\right] + 40 = 28.4$$

$$\text{New } \bar{u}_{0,\bar{t}+\Delta\bar{t}} = 28.4 + 60 = 88.4$$

At $\bar{z} = 0.75$, from Eq. (6.61),

$$\bar{u}_{0,\bar{t}+\Delta\bar{t}} = 0.475[40 + 0 - 2(21)] + 21 = 20.05$$

$$\text{New } \bar{u}_{0,\bar{t}+\Delta\bar{t}} = 60 + 20.05 = 80.05$$

At $\bar{z} = 1$,

$$\bar{u} = 0$$

At $t = 15$ days,
At $\bar{z} = 0$,

$$\bar{u} = 0$$

At $\bar{z} = 0.25$,

$$\bar{u}_{0,t+\Delta t} = 0.325[0 + 88.4 - 2(82.45)] + 82.45 = 57.6$$

At $\bar{z} = 0.5$,

$$\bar{u}_{0,t+\Delta t} = (1.152)(0.325)$$

$$\times\left[\frac{2 \times 2.8}{2 + 2.8}(82.45) + \frac{2 \times 2}{2 + 2.8}(80.05) - 2(88.4)\right] + 88.4 = 83.2$$

At $\bar{z} = 0.75$,

$$\bar{u}_{0,t+\Delta t} = 0.475[88.4 + 0 - 2(80.05)] + 80.05 = 46.0$$

At $\bar{z} = 1$,

$$\bar{u} = 0$$

The distribution of excess pore water pressure is shown in Fig. 6.11*b*.

6.5 STANDARD ONE-DIMENSIONAL CONSOLIDATION TEST AND INTERPRETATION

The standard one-dimensional consolidation test is usually carried out on saturated specimens about 1 in (25.4 mm) thick and 2.5 in (63.5 mm) in diameter (Fig. 6.12). The soil specimen is kept inside a metal ring, with a porous stone at the top and another at the bottom. The load P on the specimen is applied through a lever arm, and the compression of the specimen is measured by a micrometer dial gauge. The load is usually doubled every 24 hours. The specimen is kept under water throughout the test.

For each load increment, the specimen deformation and the corresponding time t

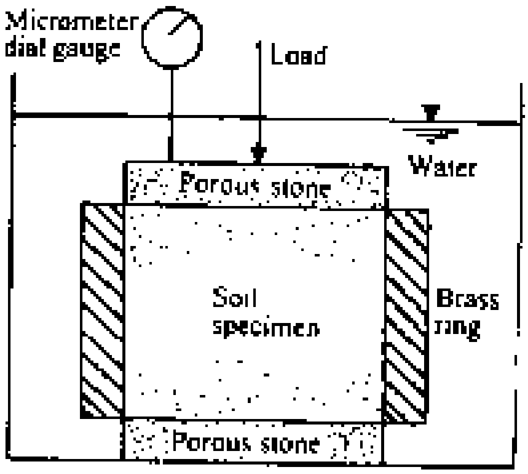


Fig. 6.12 Consolidometer

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Total Stresses and Effective stresses

Total stress:

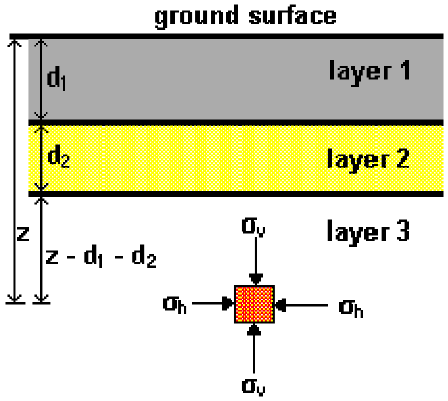
The total stress at depth z is the sum of the weights of soil in each layer thickness above.

Vertical total stress at depth z,

$$\sigma_v = \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3(z - d_1 - d_2)$$

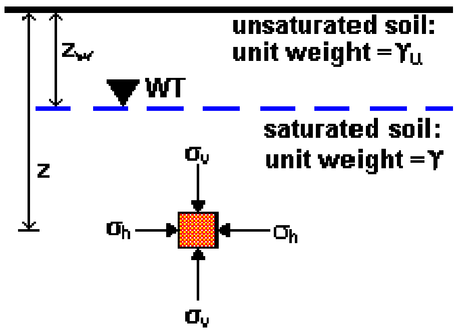
where

$\gamma_1, \gamma_2, \gamma_3$, etc. = unit weights of soil layers 1, 2 , 3, etc. respectively



Just above the water table the soil will remain saturated due to capillarity, but at some distance above the water table the soil will become unsaturated, with a consequent reduction in unit weight (unsaturated unit weight = γ_u)

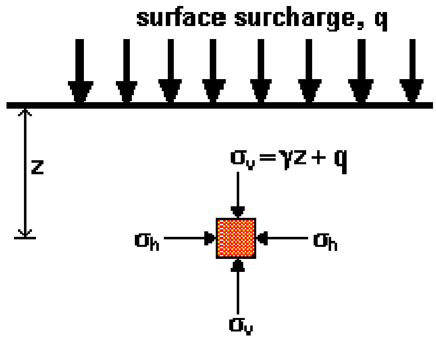
$$\sigma_v = \gamma_u \cdot z_w + \gamma_s (z - z_w)$$



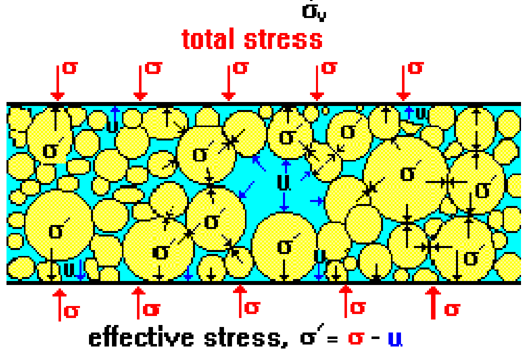
The addition of a surface surcharge load will increase the total stresses below it. If the surcharge loading is extensively wide, the increase in vertical total stress below it may be considered constant with depth and equal to the magnitude of the surcharge.

Vertical total stress at depth z,

$$\sigma_v = \gamma \cdot z + q$$



Pore Pressure:



- For saturated soil: *pore pressure = pore water pressure*
- For partially saturated soil: *pore pressure = pore water pressure + pore air pressure*
- For dry soil :*pore pressure = pore air pressure*
- In the case of partially saturated soil : *pore pressure depends on degree of saturation (S).*
- At level x-x *: pore water pressure (u) =h_2 γw (saturated soil)*

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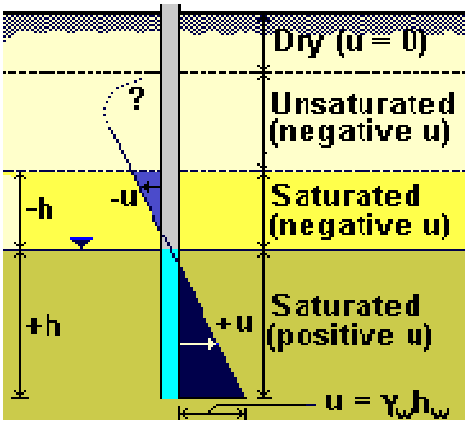

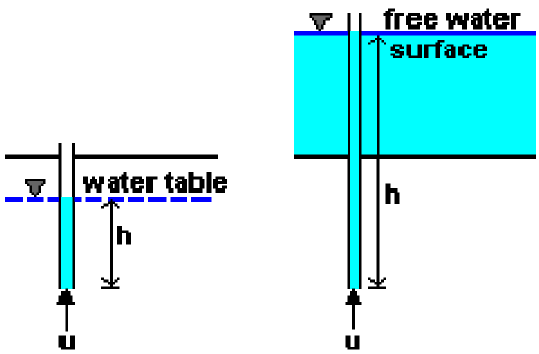

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Under hydrostatic conditions (no water flow) the pore pressure at a given point is given by the **hydrostatic pressure:**

$$\mathbf{u} = \gamma_w \, . \, \mathbf{h}_w$$

-The natural static level of water in the ground is called the **water table** or the **phreatic surface** (or sometimes the **groundwater level**). Under conditions of no seepage flow, the water table will be horizontal, as in the surface of a lake. The magnitude of the pore pressure at the water table is zero. Below the water table, pore pressures are positive.

$$\mathbf{u} = \gamma_w \, . \, \mathbf{h}_w$$

-Below the water table, pore pressures are **positive**. In dry soil, the pore pressure is **zero**. Above the water table, when the soil is saturated, pore pressure will be **negative**.

$$\mathbf{u} = - \gamma_w \, . \, \mathbf{h}_w$$

-The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and type (and thus the size of pores):

· in coarse soils capillary rise is very small
· in silts it may be up to 2m
· in clays it can be over 20m

- In conditions of seepage in the ground there is a change in pore pressure. Consider seepage occurring between two points P and Q.

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The hydraulic gradient, **i**, between two points is the head drop per unit length between these points. It can be though of as the "potential" driving the water flow.

Hydralic gradient P-Q $i = -\dfrac{\delta h}{\delta s} = \dfrac{\delta u}{\delta s} \cdot \dfrac{1}{\gamma_w}$

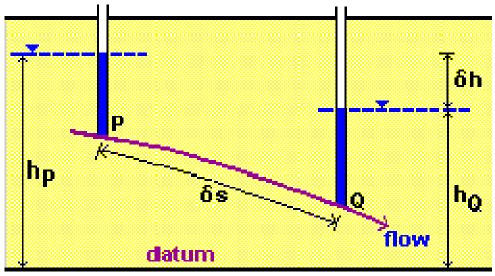
Thus $\delta u = i . \gamma_w . \delta s$

But in **steady-state** seepage, i = constant

Therefore the change in pore pressure due to seepage alone, $\delta u_s = i . \gamma_w . s$

For seepage flow vertically downward, i is negative

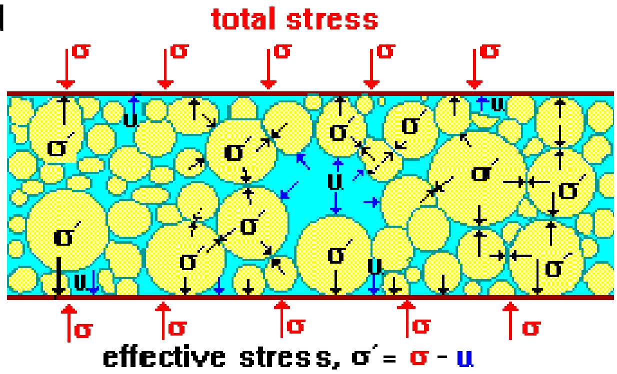
For seepage flow vertically upward, i is positive.



Effective Stress :

Ground movements and instabilities can be caused by changes in total stress (such as loading due to foundations or unloading due to excavations), but they can also be caused by changes in pore pressures (slopes can fail after rainfall increases the pore pressures).

In fact, it is the combined effect of total stress and pore pressure that controls soil behaviour such as shear strength, compression and distortion. The difference between the total stress and the pore pressure is called the effective stress:



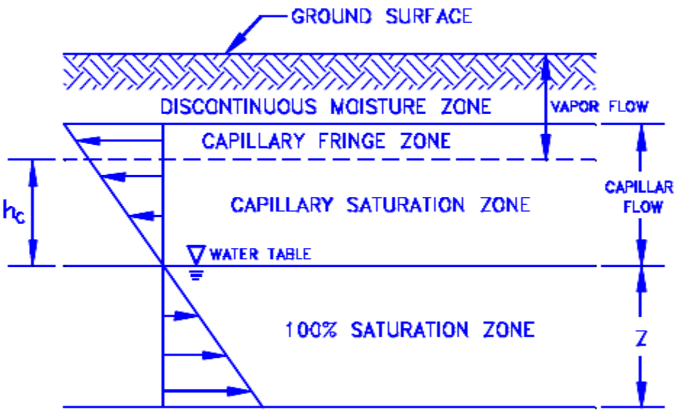
effective stress = total stress - pore pressure

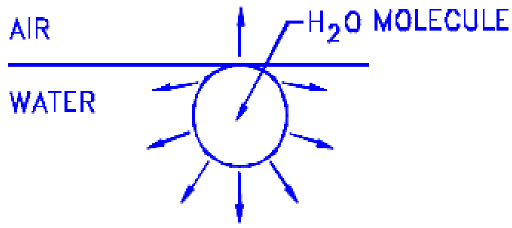
or $\sigma' = \sigma - u$

Note that the prime (dash mark ´) indicates effective stress.

Capillarity in Soils

Water can raise and remain above the line of atmospheric pressure (pheriatic line) in a very fine pores due to attraction (surface tension)between adjacent molecules in the surface.





Capillary Head or Capillary Rise

$W = F_T COS\alpha$

$\gamma_w \pi r^2 h = T_s . 2\pi r . COS\alpha$

Where Ts= Surface tension

$\therefore h = \frac{2Ts}{\gamma_w r} \cos\alpha$ = Capillary head

$$\Rightarrow \text{therefore } h_C = \frac{-4T_o}{\gamma_w d}$$

= Capillary rise $\quad h \propto \frac{1}{r}$

For example, how much does the water rise above the water table in a very fine sand (d = 0.1 mm) ?

$$h_C = \frac{-4T_o}{\gamma_w d} = \frac{-4(0.073\,{}^{N}\!/_{m})}{9.81\,{}^{kN}\!/_{m^3}\,(0.1mm)} = \underline{0.30\text{ m}}$$

Using typical values of T = 0.073 N/m, $\alpha = 0^{\circ}$ and γ_w = 9810 N/m^3 in Eq. 6.6, it can be shown that:

$$h_c(m) \approx \frac{0.03}{d(mm)}$$

What do these have to do with soils? The interconnected voids within the soil can act like capillary tubes (not straight though) and allow the water to rise well above the water table. The “capillary tube” diamater of a soil is approximately 1/5 of D_{10}. Therefore, the capillary rise within a soil can be written as:

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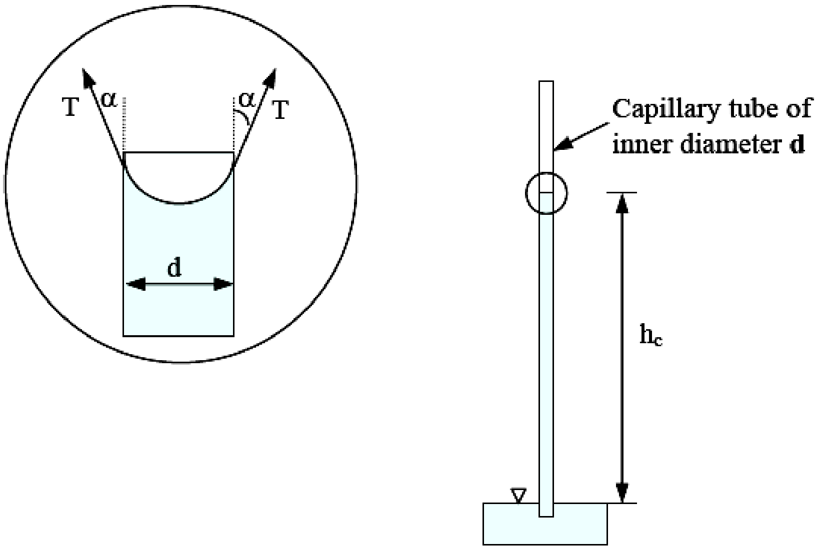
$$h_c(m) \approx \frac{0.15}{D_{10}(mm)}$$



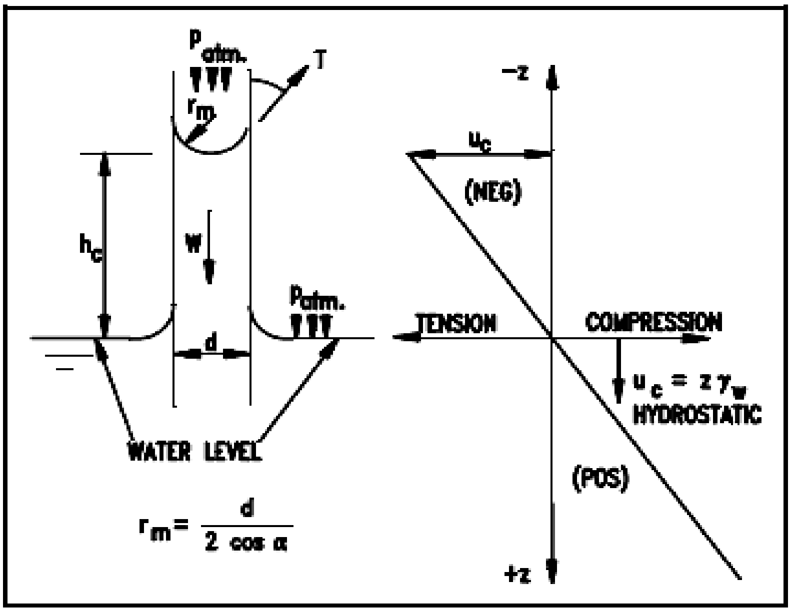


Figure 1. Capillarity water rise and pressure in capillary tube.

Effective Stress in the zone of Capillary Rise

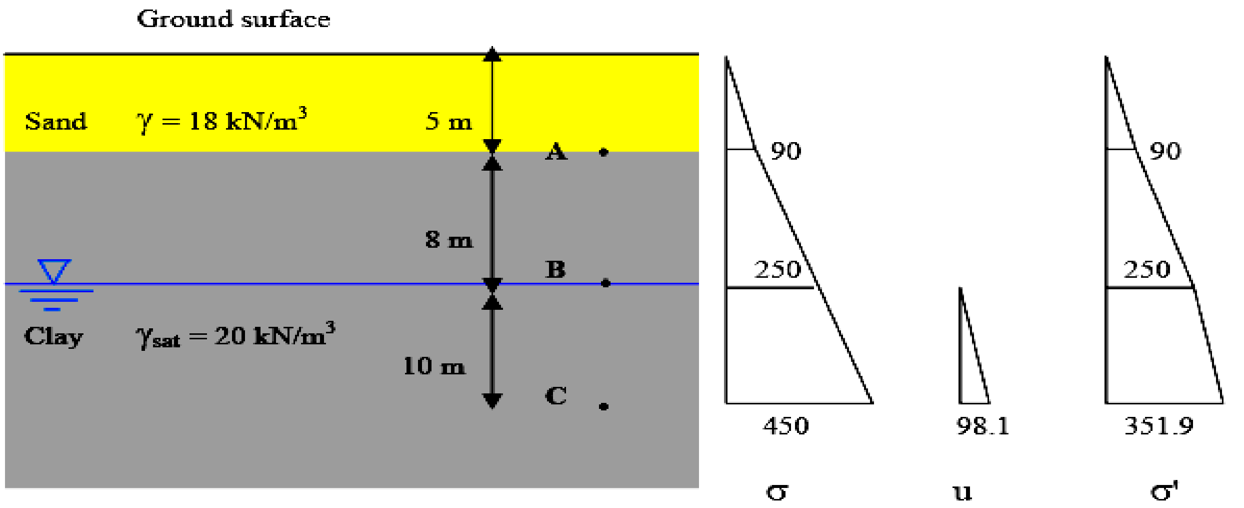
The pore water pressure at a point above W.T depends on degree of saturation due to capillary rise.

-For fully saturated zone by capillary rise $\mathbf{u = -h\, \gamma_w}$

-For partially saturated zone by capillary rise $\mathbf{u = -s\, h\, \gamma_w/100}$

-his measured from and above W.T.

1) For the cross section given in the figure, calculate the total vertical stresses, pore pressures, and effective vertical stresses at points A, B, C and sketch these stress distributions. Assume that the whole clay layer is fully saturated.



At point A:
$\sigma = 18x5 = 90$ kN/m^2 $= 90$ kPa $\quad u = 0 \quad \sigma' = 90-0 = 90$ kPa

At point B:
$\sigma = 18x5+20x8 = 250$ kPa $\quad u = 0 \quad \sigma' = 250-0 = 250$ kPa

At point C:
$\sigma = 18x5+20x(10+8) = 450$ kPa $\quad u = 9.81x10=98.1$ kPa
$\sigma' = 450-98.1 = 351.9$ kPa

2) The basin of a lake consists of uniform clay with saturated unit weight 19 kN/m^3. Calculate the effective stress at a depth of 20 m below ground surface under the lake when the water depth is 5 m. The elevation of water in the lake changes throughout the year, it rises to 10 m in the rain season. How does this affect the effective stress you have previously calculated?

When water depth = 5 m
$\sigma = 9.81x5+19x20= 429$ kPa
$u = 9.81x(5+20)= 245$ kPa
$\sigma' = 429-245 = 184$ kPa

When water depth = 10 m
$\sigma = 9.81x10+19x20= 478$ kPa
$u = 9.81x(10+20)= 294$ kPa
$\sigma' = 478-294 = 184$ kPa

Note that effective stress does not change with increasing water depth in the lake while total stress and pore water pressure increase. Effective stress on the ground surface is zero.

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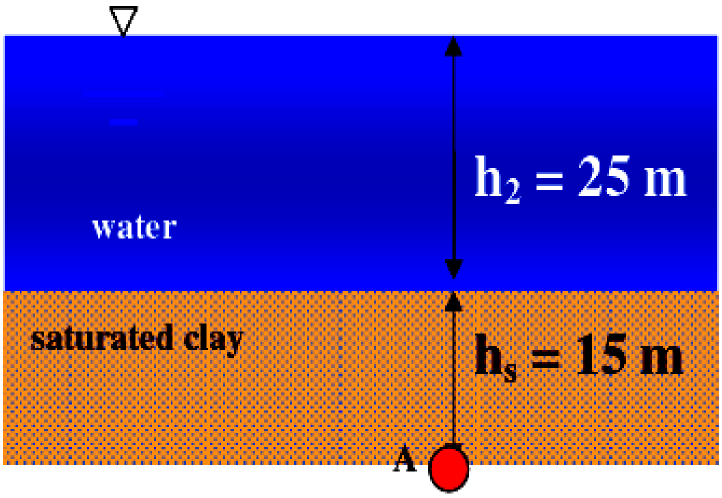
A sample was obtained from point A in the submerged clay layer shown below, and it was determined that it had a *w* = 54%, and a *Gs* = 2.78. What is the effective vertical stress at A?



Solution:

The effective stress σ' at the point A consists solely of the depth of the soil (not the water) multiplied by the soil buoyant unit weight.

$$\sigma' = \gamma' h_{soil} \quad where \quad \gamma' = \gamma_{SAT} - \gamma_W$$

In order to find γ' there are a number of derivation, such as this one,

$$\gamma' = \frac{(G_S + e)\gamma_W}{1+e} - \gamma_W$$ where the voids ratio e can be replace with $Se = wG_S$

and noticing that $S = 1$ because the soil is 100% saturated, $e = wG_S$

$$\sigma' = \left[\frac{(G_S + wG_S)}{1+wG_S}\gamma_W - \gamma_W\right]h_{soil} = \left[\frac{2.78+(0.54)(2.78)}{1+(0.54)(2.78)}(9.81) - 9.81\right](15\,m)$$

$$\sigma' = 105\,kPa$$

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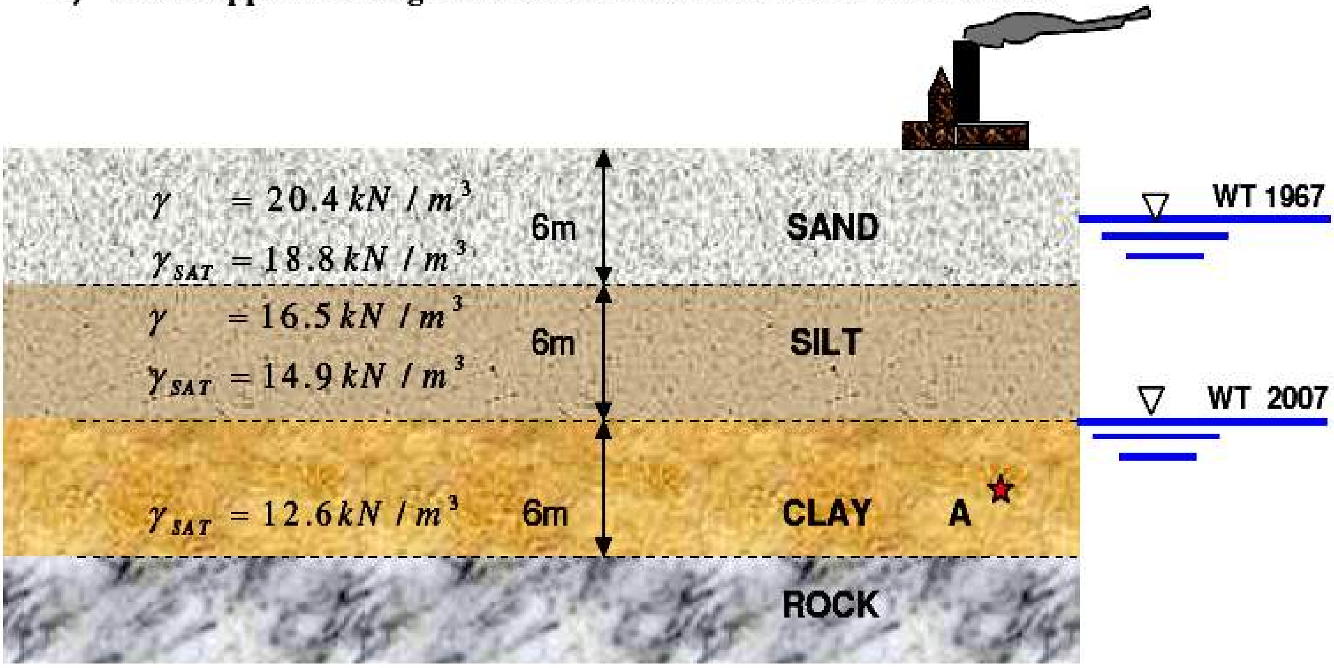
The city of Houston, Texas has been experiencing a rapid lowering of its phreatic surface during the past 40 years, due to large volumes of water pumped out of the ground by industrial users.

a) **What was the effective vertical stress at a depth of 15 m in 1967?**
b) **What is the effective stress at the same depth in 2007?**
c) **What happens to the ground surface as a result of the draw downs?**



Solution:

a)

$$\sigma_V' = [\gamma h + \gamma' h']_{SAND} + [\gamma' h']_{SILT} + [\gamma' h']_{CLAY}$$

$$\sigma_V' = \left[(20.4)(3) + (18.8 - 9.81)(3)\right] + [(14.9 - 9.81)(6)] + [(12.6 - 9.81)(3)]$$

$$\sigma_V' = 128\ kPa$$

b) $\sigma_V' = \left[(20.4)(6) + (16.5)(6)\right] + [(12.6 - 9.81)(3)] = 230\ kPa$

This is an 80% increase in stress due solely to a dropping water table.

c) **The ground surface has also been lowered due to the decreasing thickness of the clay and the silt strata due to their loss volume previously occupied by the water.**

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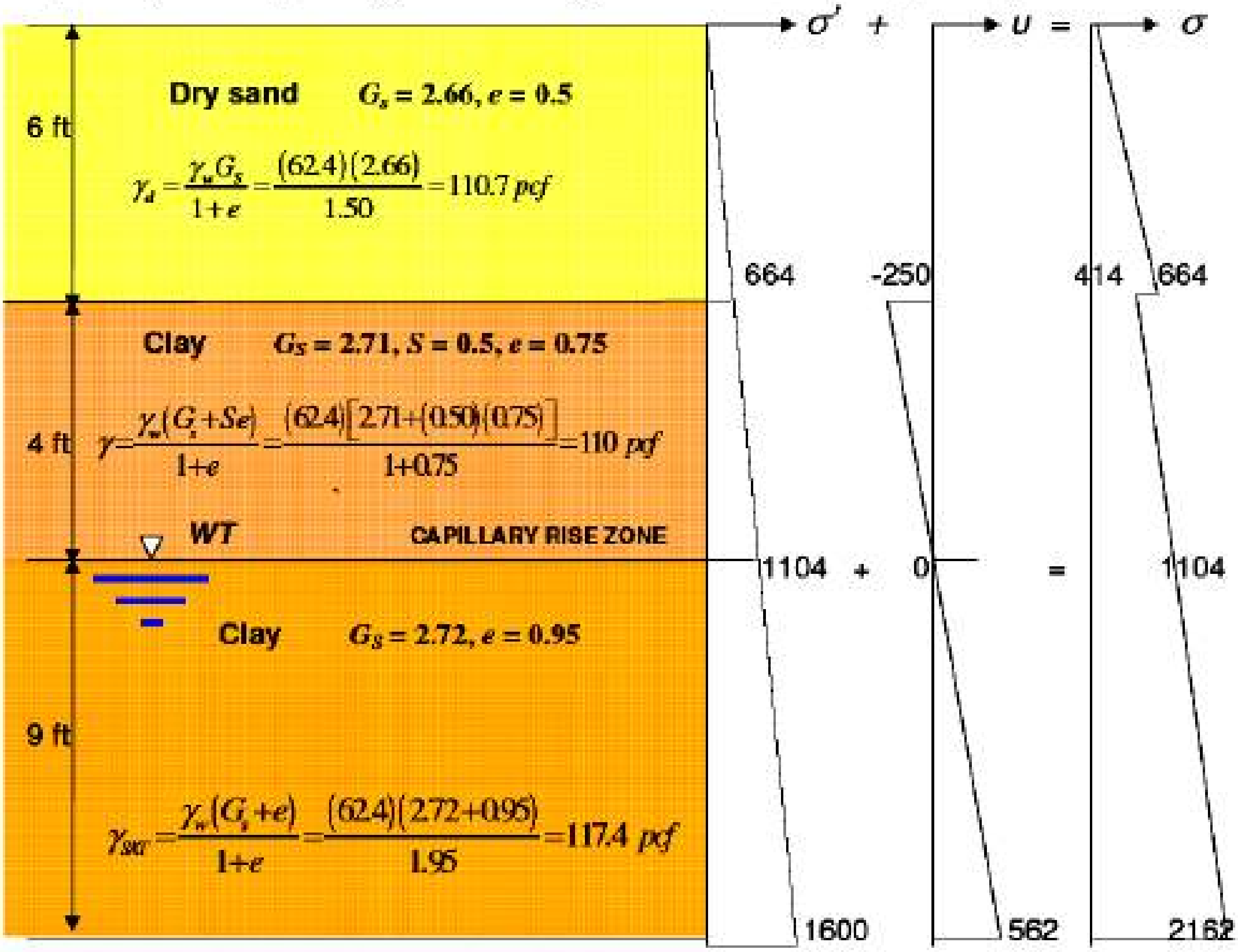
In the soil profile shown below, show a plot of the *pore water pressure* and the *effective stress* along the right margin of the figure with numerical values at each interface. Pay heed to the capillarity in the upper clay (d = 0.001 mm), where S = 50% in the upper clay stratum.



Depth, ft	σ ' +	u =	σ
0'	0	0	0
6'	(110.7)(6) = 664	0	664
		(62.4)(-4) = -250	414
10'	664+ (110)(4)=1104	0	1104
19'	1104+(117.4-62.4)(9)=1600	9(62.4) = 562	2162

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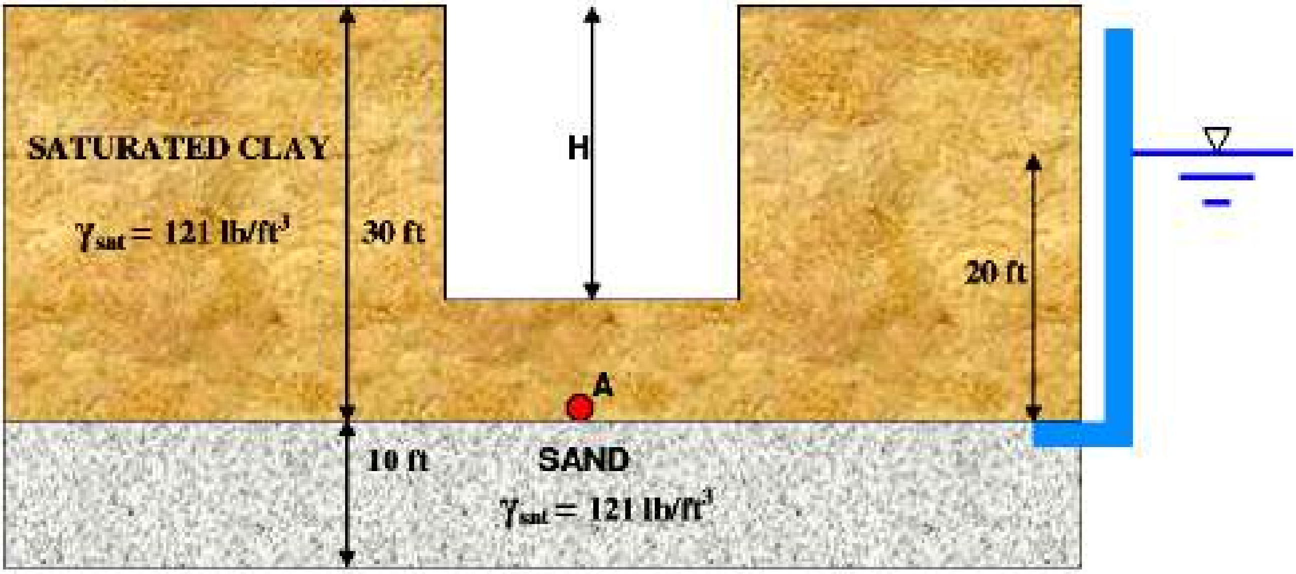
Calculate the maximum theoretical depth of excavation *H* below, before the remaining clay layer is uplifted by the vertical seepage pressure.

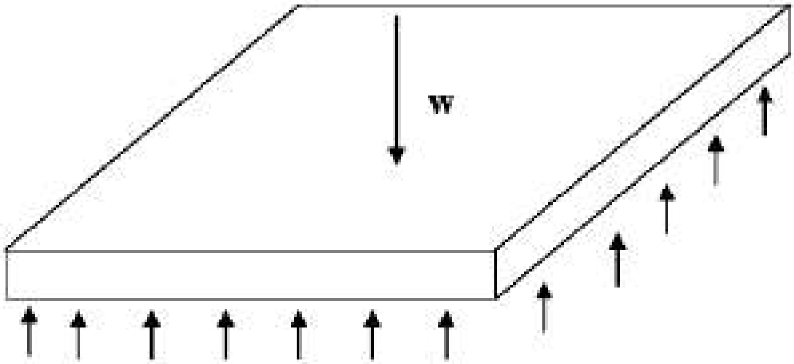


Solution:

The maximum depth of excavation *H* is reached when the effective stress σ' = 0 (that is, the upward seepage force is equal to the downward weight of the soil. Mathematically,

$$\sigma_A' = \sigma_A - u_A = \left(121\frac{lb}{ft^3}\right)(30' - H) - \left(62.4\frac{lb}{ft^3}\right)(20') = 0$$

$$30' - H = \frac{(62.4)(20')}{(121)} = 10.3 \ feet \quad \therefore \quad H = 19.7 \ feet$$



Seepage hydrostatic uplift

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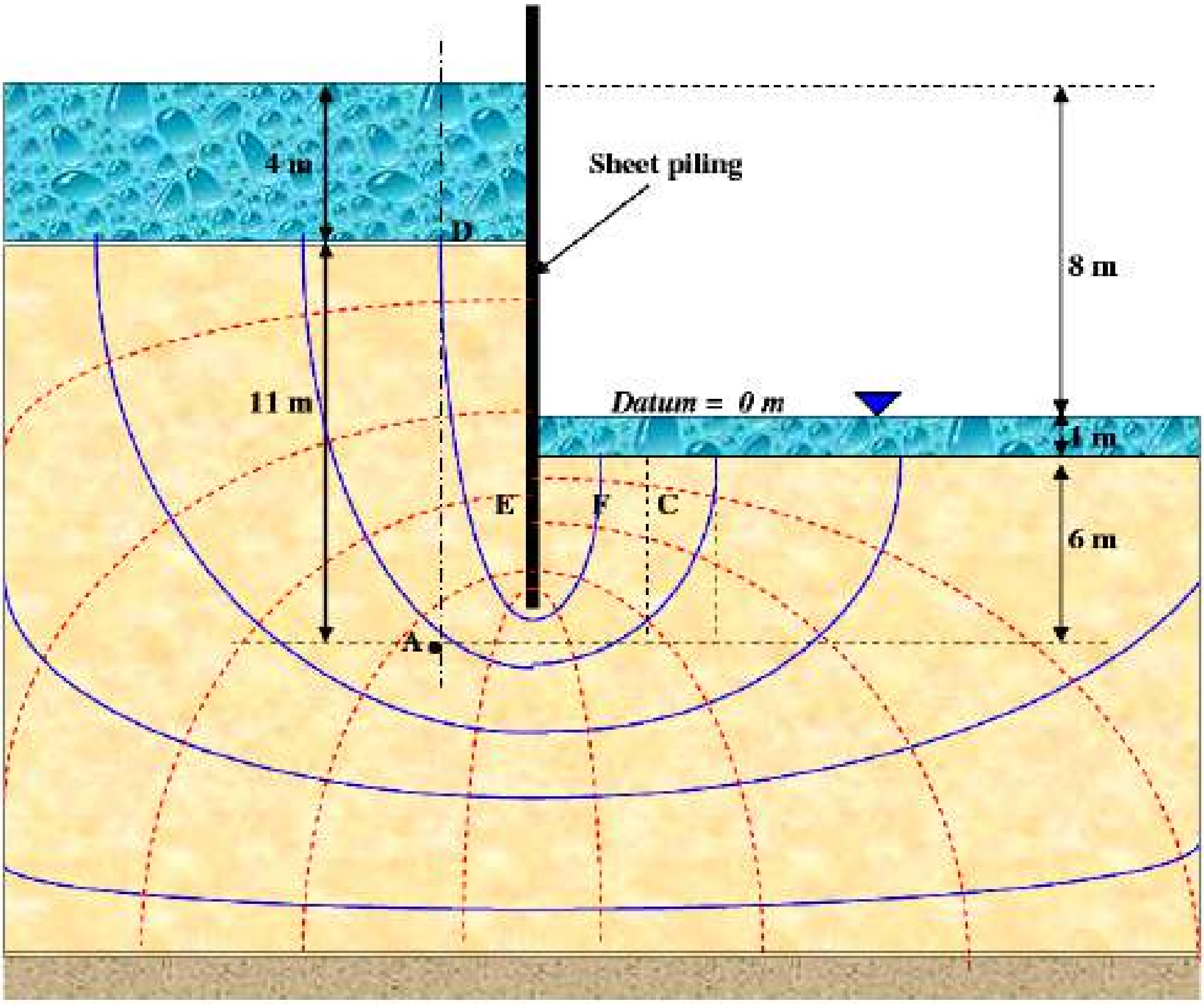
If the saturated unit weight of the soil in this shallow bay bottom is 20 kN/m^3, what is the effective vertical stress σ' at point A?



Solution:

$$\text{The total stress } \sigma_A = \gamma_w h_w + \gamma_{SAT} h_{SOIL} = \left(9.81\frac{kN}{m^3}\right)(4\,m) + \left(20\frac{kN}{m^3}\right)(11\,m) = 259\frac{kN}{m^2}$$

$$\text{The pore water pressure } u_A = \gamma_w (h_A - z_A) = \left(9.81\frac{kN}{m^3}\right)\left[\left(\frac{5.7}{10}\right)(8\,m) - (-7\,m)\right] = 122\frac{kN}{m^2}$$

$$\text{Therefore, the effective stress at } A:\ \sigma_A' = \sigma_A - u_A = 259 - 122 = 137\frac{kN}{m^2} = 137\,kPa$$

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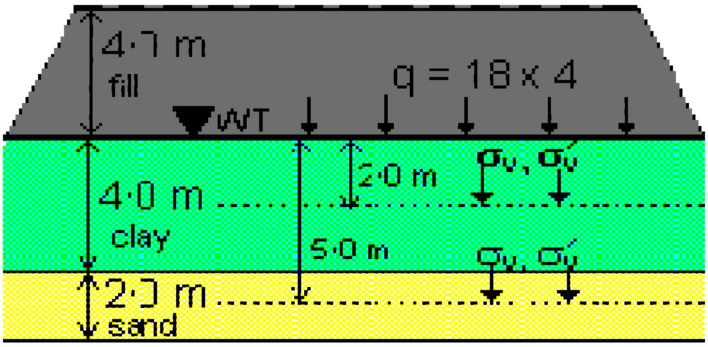
Example: The figure shows how an extensive layer of fill will be placed on a certain site. The unit weights are: clay and sand = 20kN/m³, rolled fills 18kN/m³, assume water = 10 kN/m³.

Calculations are made for the total and effective stress at the mid-depth of the sand and the mid-depth of the clay for the following conditions: initially, before construction; immediately after construction; many years after construction



Initially, before construction

Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

σ_v = 20.0 x 2.0 = **40.0kPa**

Pore pressure

u = 10 x 2.0 = 20.0kPa

Vertical effective stress

$\sigma'_v = \sigma_v$ - u = **20.0kPa**

Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress

σ_v = 20.0 x 5.0 = **100.0 kPa**

Pore pressure

u = 10 x 5.0 = 50.0 kPa

Vertical effective stress

$\sigma'_v = \sigma_v$ - u = **50.0 kPa**

immediately after construction

The construction of the embankment applies a surface surcharge:

q = 18 x 4 = 72.0 kPa.

The sand is drained (either horizontally or into the rock below) and so there is no increase in pore pressure. The clay is undrained and the pore pressure increases by 72.0 kPa.

Initial stresses at mid-depth of clay (z = 2.0m)
Vertical total stress
σ_v = 20.0 x 2.0 + 72.0 = **112.0kPa**
Pore pressure
u = 10 x 2.0 + 72.0 = 92.0 kPa
Vertical effective stress
$\sigma'_v = \sigma_v$ - u = **20.0kPa**
(i.e. no change immediately)

Initial stresses at mid-depth of sand (z = 5.0m)
Vertical total stress
σ_v = 20.0 x 5.0 + 72.0 = **172.0kPa**
Pore pressure
u = 10 x 5.0 = 50.0 kPa
Vertical effective stress
$\sigma'_v = \sigma_v$ - u = **122.0kPa**
(i.e. an immediate increase)

Many years after construction

After many years, the excess pore pressures in the clay will have dissipated. The pore pressures will now be the same as they were initially.

Initial stresses at mid-depth of clay (z = 2.0 m)
Vertical total stress
σ_v = 20.0 x 2.0 + 72.0 = **112.0 kPa**
Pore pressure
u = 10 x 2.0 = 20.0 kPa
Vertical effective stress
$\sigma'_v = \sigma_v$ - u = **92.0 kPa**
(i.e. a long-term increase)

Initial stresses at mid-depth of sand (z = 5.0 m)
Vertical total stress
σ_v = 20.0 x 5.0 + 72.0 = **172.0 kPa**
Pore pressure
u = 10 x 5.0 = 50.0 kPa

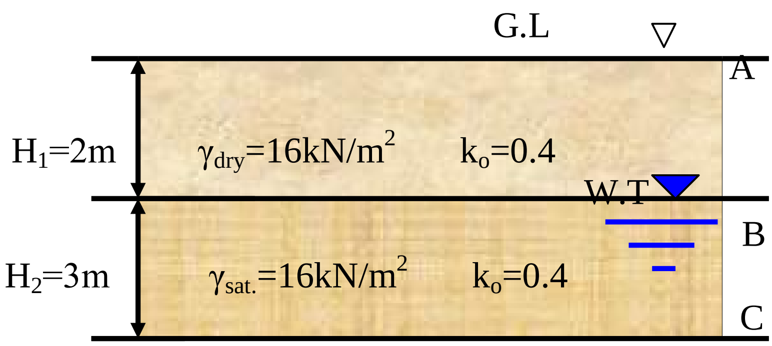
Vertical effective stress

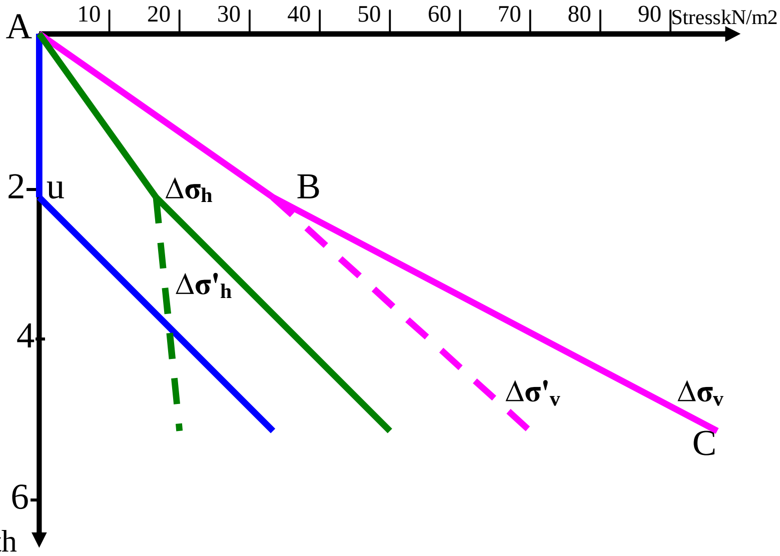
$\sigma'_v = \sigma_v$ - u = **122.0 kPa**

(i.e. no further change)

Example1. For the soil profile shown below find $\Delta\sigma_v$; $\Delta\sigma_h$; **u** ;$\Delta\sigma'_v$; $\Delta\sigma'_h$; at points A,B and C.

Solution:





1. at point A

$\Delta\sigma_v = 0$

$\Delta\sigma_h = 0$; $u = 0$;$\Delta\sigma'_v = 0$

$\Delta\sigma'_h = 0$

2. At point B

$\Delta\sigma_v = \gamma_{dry}$ x H_1=16 x 2= 32 kN/m^2

$\Delta\sigma_h = \Delta\sigma_v$ x k_o= 32 x 0.4 = 12.8 kN/m^2

$u = \gamma_w$x H_1=0 ;

$\Delta\sigma'_v = \Delta\sigma_v$ = 32 kN/m^2

$\Delta\sigma'_h = \Delta\sigma_h$= 12.8 kN/m^2

3. At point C

$\Delta\sigma_v = \gamma_{dry}$ x H_1 + $\gamma_{sat.}$ x H_2 =16 x 2 + 3 x 20 = 92 kN/m^2

$\Delta\sigma_h = \Delta\sigma_{v1}$ x k_o + $\Delta\sigma_{v2}$ x k_o = 32 x 0.4 + 60 x 0.5 = 42.8 kN/m^2

$u = \gamma_w$x H_2= 9.81 x 3 = 29.43 kN/m^2

$\Delta\sigma'_v = \Delta\sigma_v$ - u = 92 – 29.43 = 62.57 kN/m^2

$\Delta\sigma'_h = \Delta\sigma_h$ – u = 42.8 – 29.43=13.37 kN/m^2

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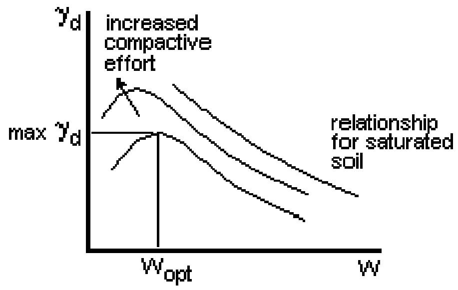
Compaction :

is the process of increasing soil dry unit weight by forcing soil solids into a tighter state and reducing the air voids .Compaction is measured in terms of dry unit weight.

Phase Diagram



The objective of compaction :
Content exhibits different engineering properties (strength ,compressibility and permeability) depending on their dry density .
-Water added to permit the soil particles to slip relative to one another (water acts as a lubricant)
-Water added to soil + compaction (energy)→rearrangement of the solid particles in to a denser state .

Compaction can be applied to improve the properties of an existing soil or in the process of placing fill. The main objectives are to:

- increase shear strength and therefore bearing capacity
- increase stiffness and therefore reduce future settlement
- decrease voids ratio and so permeability, thus reducing potential frost heave

Factors affecting compaction

A number of factors will affect the degree of compaction that can be achieved:

- Nature and type of soil, i.e. sand or clay, grading, plasticity
- Water content at the time of compaction
- Site conditions, e.g. weather, type of site, layer thickness
- Compactive effort: type of plant (weight, vibration, number of passes)

Types of compaction plant

Smooth-wheeled roller

- Self-propelled or towed steel rollers ranging from 2 - 20 tonnes
- Suitable for: well-graded sands and gravels
  silts and clays of low plasticity
- Unsuitable for: uniform sands; silty sands; soft clays

Grid roller

- Towed units with rolls of 30-50 mm bars, with spaces between of 90-100 mm
- Masses range from 5-12 tonnes
- Suitable for: well-graded sands; soft rocks; stony soils with fine fractions
- Unsuitable for: uniform sands; silty sands; very soft clays

Sheepsfoot roller

- Also known as a 'tamping roller'
- Self propelled or towed units, with hollow drum fitted with projecting club-shaped 'feet'
- Mass range from 5-8 tonnes
- Suitable for: fine grained soils; sands and gravels, with >20% fines
- Unsuitable for: very coarse soils; uniform gravels

Pneumatic-tyred roller

- Usually a container on two axles, with rubber-tyred wheels.
- Wheels aligned to give a full-width rolled track.
- Dead loads are added to give masses of 12-40 tonnes.
- Suitable for: most coarse and fine soils.

Unsuitable for: very soft clay; highly variable soils

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Vibrating plate

- Range from hand-guided machines to larger roller combinations
- Suitable for: most soils with low to moderate fines content
- Unsuitable for: large volume work; wet clayey soils

Power rammer

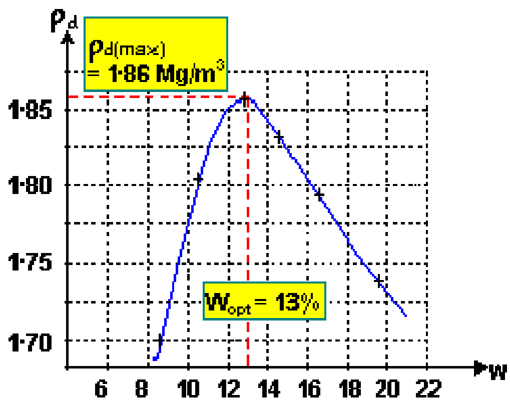
- Also called a 'trench tamper'
- Hand-guided pneumatic tamper
- Suitable for: trench back-fill; work in confined areas
- Unsuitable for: large volume work

Standard laboratory tests:

- Standard protect Test (ASTMD - 698)
- Modified Protect Test (ASTMD-1557)

Carried out on soil passing the No.40 sieve (opening size=0.425)

		Standard	**Modified**
Hammer wt.(Ib)		5.5	10
	(Kg)	2.49	4.54
Drop	(in)	12	18
	(mm)	305	457
Vol.	(ft^3)	1/30	1/30
	(m^3)	0.944×10^{-3}	0.944×10^{-3}
No. of layers		3	5
No. of blows		25	25
Comp.energy			
	(Kilo Joules/m^3)	593	2693
	(Ft-Ib/ft^3)	12375	56250



$$E = \frac{\text{Number of blows per layer} \times \text{Number of layers} \times \text{Weight of hammer} \times \text{Height of drop of hammer}}{\text{volume of mold}}$$

Higher comp. energy $\rightarrow \begin{cases} \text{higher } \gamma_d \\ \text{Smaller opt. moisture content} \end{cases}$

Since :

$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

$$S.e = G_s.w_c$$

$$\therefore \gamma_d = \frac{G_s}{1+\frac{G_s.w_c}{S}}\gamma_w$$

Total or wet density ρ :

$$\rho = \frac{M_t}{V_t} = \frac{M_s + M_w}{V_t}$$

Solid density ρ_s

$$\rho_s = \frac{M_s}{V_s}$$

dry density ρ_d

$$\rho_d = \frac{M_s}{V_t}, \quad \because V_t > V_s \quad \therefore \rho_d < \rho_s$$

Also we have

$$\rho_d = \frac{M_s}{V_t} = \frac{M_t - M_w}{V_t} = \frac{M_t}{V_t} - \frac{M_w}{V_t} = \rho - \frac{M_w}{M_s}\frac{M_s}{V_t} = \rho - w\rho_d$$

$$so \quad that \quad \rho_d + w\rho_d = \rho \quad and \quad \rho_d = \frac{\rho}{1+w}$$

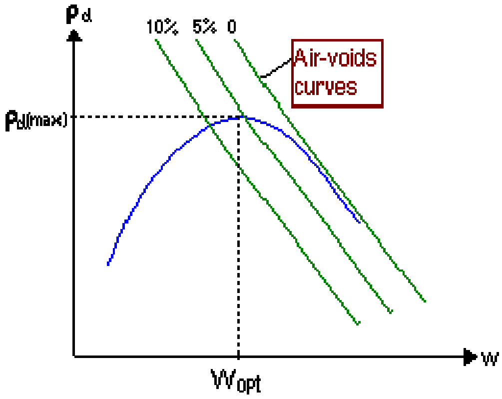
Hence for a given(ω_c) ,layer values of γ_d can be obtained by the use of higher comp. energy.

Example: A compacted soil sample has been weighed with the following results:
Mass = 1821 g Volume = 950 ml Water content = 9.2%
Determine the bulk and dry densities.

Bulk density ρ = 1821 / 950 = 1.917 g/ml or Mg/m³

Dry density ρ_d = 1.917 / (1+0.092) = 1.754 Mg/m³

Dry density and air-voids content



A fully saturated soil has zero air content. In practice, even quite wet soil will have a small air content

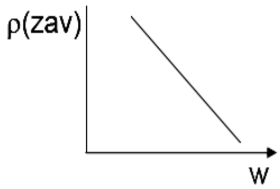
$$\text{Air - voids content, } A_v = \frac{\text{Volume of air}}{\text{Total volume}}$$

The maximum dry density is controlled by both the water content and the air-voids content. Curves for different air-voids contents can be added to the ρ_d / w plot using this expression:

$$\rho_d = \frac{G_s \rho_w}{1 + wG_s}(1 - A_v)$$

The air-voids content corresponding to the maximum dry density and optimum water content can be read off the ρ_d/w plot or calculated from the expression.

$$\rho(z.a.v) = \frac{G_s \rho_w}{1 + wG_s}$$



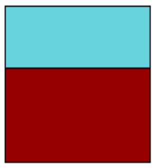
$$V_s = 1$$

$$\rho_s = M_s / V_s, then \quad M_s = V_s \rho_s = V_s G_s \rho_w$$

$$w = M_w / M_s, then \quad M_w = w M_s = wG_s \rho_w, by \quad V_s = 1$$

$$M_w = V_w \rho_w, or \quad V_w = M_w / \rho_w = wG_s \rho_w / \rho_w = wG_s$$

$$\rho_{dry} = \frac{\rho}{1+w} = \frac{M_t}{V_t(1+w)} = \frac{(1+w)\rho_w G_s}{(1+w)(1+wG_s)} = \frac{\rho_w G_s}{1+wG_s}$$



Example: Determine the dry densities of a compacted soil sample at a water content of 12%, with air-voids contents of zero, 5% and 10%. (G_s = 2.68).

$$\text{For } A_v = 0: \quad \rho_d = \frac{2.68 \times 1.0}{1 + 2.68 \times 0.12} = 2.03 \text{ Mg/m}^3$$

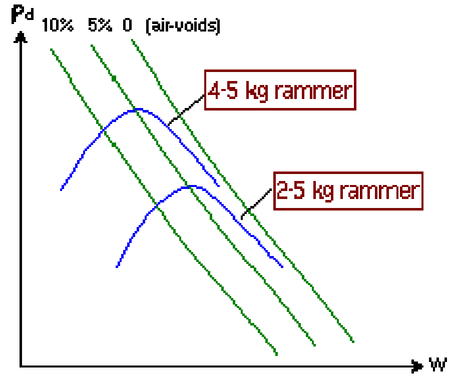
$$\text{For } A_v = 5\%: \quad \rho_d = \frac{2.68 \times 1.0}{1 + 2.68 \times 0.12}\left(1 - \frac{5}{100}\right) = 1.93 \text{ Mg/m}^3$$

$$\text{For } A_v = 10\%: \quad \rho_d = \frac{2.68 \times 1.0}{1 + 2.68 \times 0.12}\left(1 - \frac{10}{100}\right) = 1.83 \text{ Mg/m}^3$$

Effect of increased compactive effort

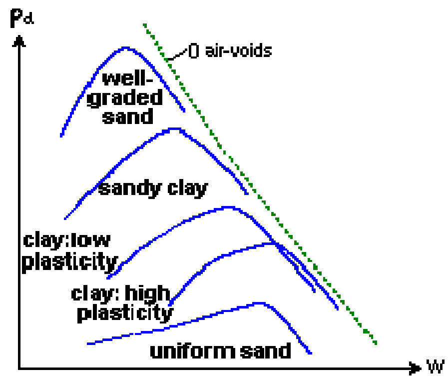
The compactive effort will be greater when using a heavier roller on site or a heavier rammer in the laboratory. With greater compactive effort:

- maximum dry density increases
- optimum water content decreases
- air-voids content remains almost the same.



Effect of soil type

- Well-graded granular soils can be compacted to higher densities than uniform or silty soils.
- Clays of high plasticity may have water contents over 30% and achieve similar densities (and therefore strengths) to those of lower plasticity with water contents below 20%.
- As the % of fines and the plasticity of a soil increses, the compaction curve becomes flatter and therefore less sensitive to moisture content. Equally, the maximum dry density will be relatively low



Interpretation of laboratory data

Example data collected during test

In a typical compaction test the following data might have been collected:
Mass of mould, M_o = 1082 g
Volume of mould, V = 950 ml
Specific gravity of soil grains, G_s = 2.70

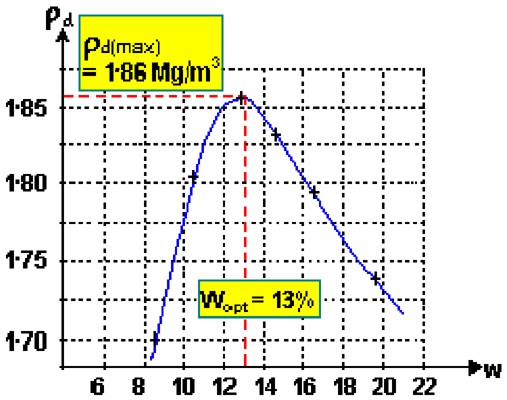
Mass of mould + soil (g)	2833	2979	3080	3092	3064	3027
Water content (%)	8.41	10.62	12.88	14.41	16.59	18.62

Calculated densities and density curve

The expressions used are:

$$\rho = \frac{M - M_o}{V} \quad \text{and} \quad \rho_d = \frac{\rho}{1+w}$$

Bulk density, ρ (Mg/m³)	1.84	2.00	2.10	2.12	2.09	2.05
Water content, w	0.084	0.106	0.129	0.144	0.166	0.186
Dry density, ρ_d (Mg/m³)	1.70	1.81	1.86	1.851	1.79	1.73

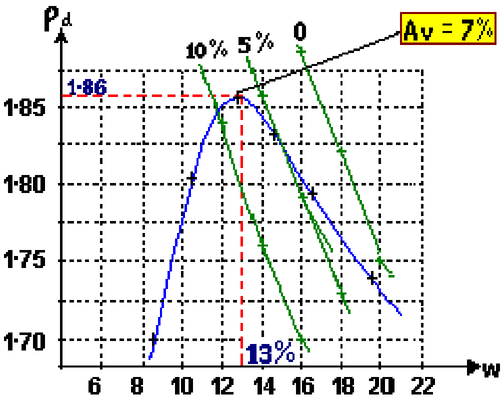


Air-voids curves

The expression used is:

$$\rho_d = \frac{G_s \rho_w}{1 + wG_s}(1 - A_v)$$

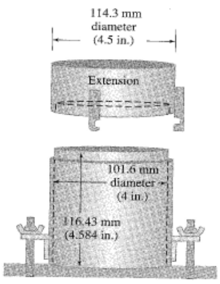
Water content (%)	10	12	14	16	18	20
ρ_d when A_v = 0%	2.13	2.04	1.96	1.89	1.82	1.75
ρ_d when A_v = 5%	2.02	1.94	1.86	1.79	1.73	1.67
ρ_d when A_v = 10%	1.91	1.84	1.76	1.70	1.64	1.58



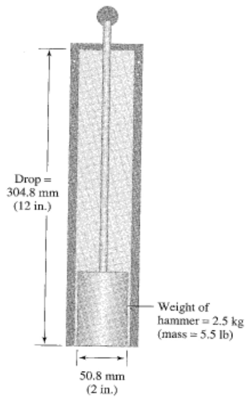
The **optimum air-voids content** is the value corresponding to the maximum dry density (1.86 Mg/m³) and optimum water content (12.9%).

$$A_{v(opt)} = 1 - \frac{1.86}{2.70 \times 1.0}(1 + 0.129 \times 2.70) = 0.071 \quad (7.1\%)$$

Procedure



(a)



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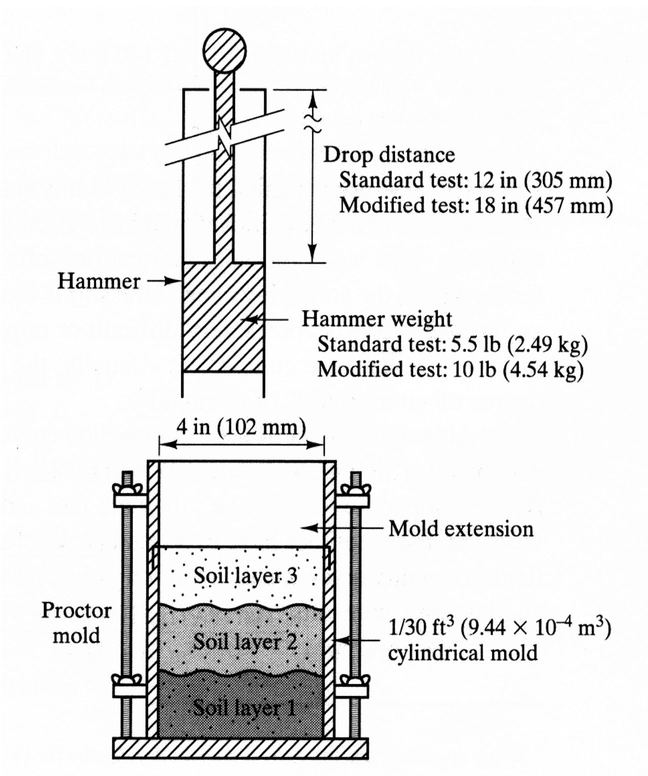

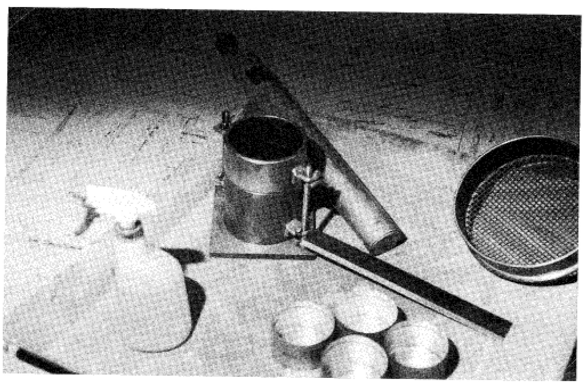
(c)

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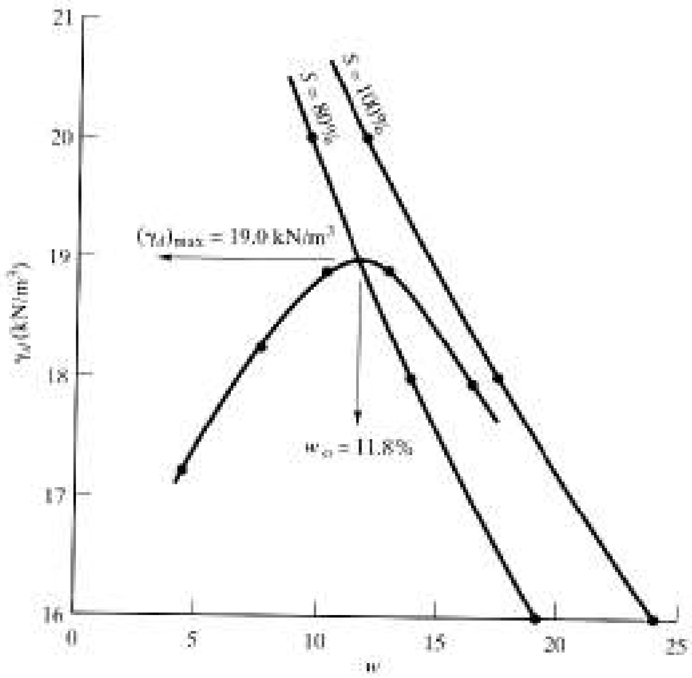
1. Obtain 10 lbs of soil passing No. 4 sieve
2. Record the weight of the Proctor mold without the base and the (collar) extension, the volume of which is 1/30 ft3.
3. Assemble the compaction apparatus.
4. Place the soil in the mold in 3 layers and compact using 25 well distributed blows of the Proctor hammer.
5. Detach the collar without disturbing the soil inside the mold
6. Remove the base and determine the weight of the mold and compacted soil.
7. Remove the compacted soil from the mold and take a sample (20-30 grams) of soil and find the moisture content
8. Place the remainder of the molded soil into the pan, break it down, and thoroughly remix it with the other soil, plus 100 additional grams of water.

Results

- Plot of dry unit weight vs moisture content
- Find gd (max) and *w*opt
- Plot Zero-Air-Void unit weight
  (only S=100%)



Specification for Field Compaction

- Specifications will refer to % Relative Compaction
- Relative to what?
  - Proctor Test – standard or modified

- % Relative Compaction

- If R > 100 % use Modified Proctor Test
- Soil will be compacted to 98% relative compaction as compared to a standard proctor test, ASTM D-698
- The soil moisture content will be ± 2% of optimum.
- 98% means the soil in the field should be 98% of the lab result
- For example, if the peak of the curve is at 100 pcf and 22% moisture
- The field compaction must be at least 98 pcf and within the stated moisture range (20 ~24%)

<u>Measurement of Field Compaction</u>

- Most common methods are
  - Nuclear Method
  - Sand Cone method
  - Rubber Balloon method

$$R(\%) = \text{Relative Compaction}$$

$$R(\%) = \frac{\gamma_d \text{ in the field}}{\text{maximum } \gamma_d \text{ from the Proctor test}}$$

Value of RC is specified according to importance and type of the project (about 90-95%)

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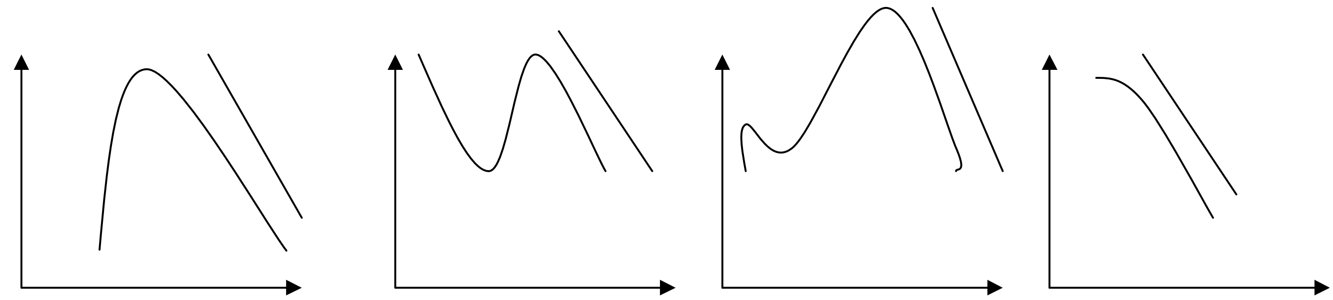
Types of Compaction Curves



Single peak	irregular Shape	double peak	oddly Shape
30 < L.L < 70	L.L < 30	L.L < 30 & > 70	L.L > 70

Properties of Compacted Soils :

Effect of molding moisture cont on **soil structure** :

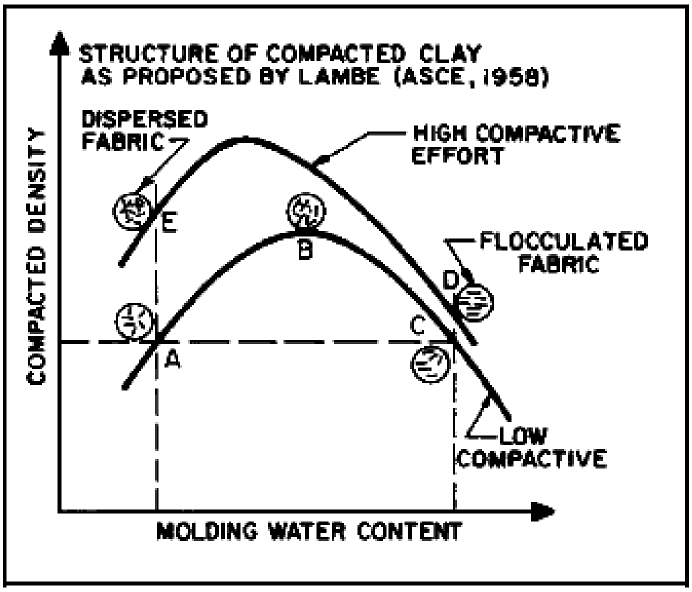
increasingWc for a given compaction effort tends to increase the repulsions and permitting a more orderly arrangement of the soil particles

Effect of molding moisture content on **permeability** :

increasing Wc results in a decrease in permeability on the dry of optimum and a slight increase in permeability on the wet side of optimum .(K1 > K2 since for dry side of opt.(flocculated st),the doainage paths are smaller then of the wet side of opt .(dispersed st .in which the drainage paths are longer)

Effect of molding moisture content on **stress-strain relationship** :

samples compacted dry of optimum tend to be more rigid and stronger than samples compacted wet of optimum



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Note :
In designing the **earth dam** , the engineer must consider not only the strength and compressibility of the soil element as compacted ,but also its properties after it has been subjected to increased total stresses and saturated by permeating water .

effect of moisture content on **compression characteristics** :
1-at **low. Stress consolidation** : the sample compacted on the wet side is more compressible than the one compactedon the dry side
2-at high-stress consolidation : the sample compacted on the dry side is more compressible than the compacted on the wet side .

Moisture condition value

This is a procedure developed by the Road Research Laboratory using only one sample, thus making laboratory compaction testing quicker and simpler. The minimum compactive effort to produce near-full compaction is determined. Soil placed in a mould is compacted by blows from a rammer dropping 250 mm; the penetration after each blow is measured.

Apparatus and sizes

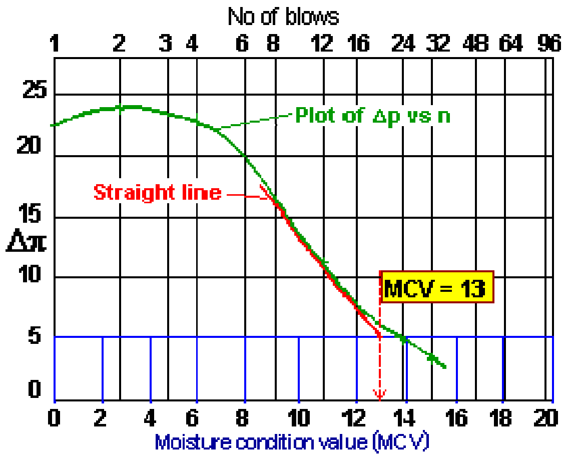
Cylindrical mould, with permeable base plate:
internal diameter = 100 mm, internal height at least 200 mm
Rammer, with a flat face:
face diam = 97 mm, mass = 7.5 kg, free-fall height = 250 mm
Soil:
1.5 kg passing a 20 mm mesh sieve

Test procedure and plot

- Firstly, the rammer is lowered on to the soil surface and allowed to penetrate under its own weight
- The rammer is then set to a height of 250 mm and dropped on to the soil
- The penetration is measured to 0.1 mm
- The rammer height is reset to 250 mm and the drop repeated until no further penetration occurs, or until 256 drops have occurred
- The change in penetration ($\Delta\pi$) is recorded between that for a given number of blows (**n**) and that for **4n** blows

- A graph is plotted of $\Delta\pi$ / **n** and a line drawn through the steepest part
- The **moisture condition value (MCV)** is give by the intercept of this line and a special scale

Example plot and determination of MCV



After plotting $\Delta\pi$ against the number of blows n, a line is drawn through the steepest part.

The intercept of this line and the 5 mm penetration line give the MCV

The defining equation is:**MCV = 10 log B**
(where B = number of blows corresponding to 5 mm penetration)

On the example plot here an MCV of 13 is indicated.

Significance of MCV in earthworks

The MCV test is rapid and gives reproducible results which correlate well with engineering properties. The relationship between MCV and water content for a soil is near to a straight line, except for heavily overconsolidated clays.A desired value of undrained strength or compressibility can be related to limiting water content, and so the MCV can be used as a control value after calibrating MCV vs w for the soil. An approximate correlation between MCV and undrained shear strength has been suggested by Parsons (1981).

$\text{Log } s_u = 0.75 + 0.11(\text{MCV})$

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Examples

1. The natural water content of a borrow material is known to be 10%. Assuming 6000 g of wet soil is used for each laboratory compaction test point, compute how much water is to be added to each of the other 6000 g samples to bring their water contents up to 13, 17, 20, 24, and 28%

Answer

GIVEN 6000 g samples at natural m/c = 10%

$$w = \frac{M_w}{M_s} = 0.1$$

$$M_t = M_s + M_w = 6000\ g$$

$$M_s = \frac{6000}{1.1} = 5455\ g$$

$$But\ M_w = w \times M_s$$

$$If\ w = 0.10\ then\ M_w = 545\ g$$

$$If\ w = 0.13\ then\ M_w = 709\ g$$

$$If\ w = 0.17\ then\ M_w = 927\ g$$

$$If\ w = 0.20\ then\ M_w = 1091\ g$$

$$If\ w = 0.24\ then\ M_w = 1309\ g$$

$$If\ w = 0.28\ then\ M_w = 1527\ g$$

water added given by

$$(M_w)_{moisture} - (M_w)_{0.10}$$

$$If\ w = 0.13\ water\ added = 164\ g$$

$$If\ w = 0.17\ water\ added = 382\ g$$

$$If\ w = 0.20\ water\ added = 546\ g$$

$$If\ w = 0.24\ water\ added = 764\ g$$

$$If\ w = 0.28\ water\ added = 982\ g$$

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2. For the data given below ($\rho S = 2.64\ Mg/m^3$):

C (Low Compaction)		B (Standard Proctor)		A (Modified Proctor)	
w (%)	ρd (Mg/m3)	w (%)	ρd (Mg/m3)	w (%)	ρd (Mg/m3)
10.9	1.627	9.3	1.691	9.3	1.873
12.3	1.639	11.8	1.715	12.8	1.910
16.3	1.740	14.3	1.755	15.5	1.803
20.1	1.707	17.6	1.747	18.7	1.699
22.4	1.647	20.8	1.685	21.1	1.641
		23.0	1.619		

(a) Plot the compaction curves.
(b) Establish the maximum dry density and optimum water content for each test.
(c) Compute the degree of saturation at the optimum point for the Modified Proctor test data.
(d) Plot the 100% saturation (zero air voids) curve. Also plot the 70, 80, and 90% saturation curves. Plot the line of optimums.

Answer:

(A) Plot Dry Density v Moisture Content

(B)From graph plot of DRY DENSITY V MOISTURE CONTENT

Data "A" $\rho_d = 1.91\ t/m^3$ $w_{opt} = 12.5\ \%$

Data "B" $\rho_d = 1.76\ t/m^3$ $w_{opt} = 15.5\ \%$

Data "C" $\rho_d = 1.75\ t/m^3$ $w_{opt} = 17.3\ \%$

(C)

$$\rho=\rho_d(1+w);$$

$$S=\frac{V_w}{V_v}=\frac{w\,\rho_d}{1-\frac{\rho_d}{\rho_s}}$$

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Data "A"

$$S = \frac{0.125 \times 1.91}{1 - \frac{1.91}{2.64}} = 0.863 \text{ or } 86.3\%$$

Data "B" gives S = 81.8%

Data "C" gives S = 89.8%

(D) For selected values of ρ_d and for s = 100% calculate w.

Then plot on graph. ρ_d (t/m^3)

ρ_d (t/m^3)	w (%)
2.0	12.1
1.9	14.8
1.8	17.7
1.7	10.9
1.6	24.6

3. A soil proposed for a compacted fill contains 40% fines and 60% coarse material by dry weight. When the coarse fraction has w = 1.5%, its affinity for water is completely satisfied (that is, it is saturated but surface dry). The Atterberg limits of the fines are LL = 27 and PL = 12. The soil is compacted by rolling to a ρd = 2.0 Mg/m^3 at w = 13%. Note: This is the water content of the entire soil mixture.

(a) What is the water content of the fines in the compacted mass?

(b) What is the likely USCS classification of the soil?

(c) What is the liquidity index of the fines?

(d) What can you say about the susceptibility of the fill to

(i) shrinkage-swelling potential?

(ii) potential for frost action?

(e) Is there a certain type of compaction equipment you would especially recommend for this job? Why?

Answer

Given: $\rho_d = 2.0\ t/m^3$ *and* $w = 13\%$

$[\rho]_{coarse} = 0.60 \times 2.0 = 1.2\ t/m^3$

$[\rho]_{fines} = 0.40 \times 2.0 = 0.8\ t/m^3$

Mass of water = 13% = .13 x 2.0 = 0.26 *t*

Mass of water in coarse = 1.5% = 0.15 x 1.2 = 0.018 *t*

Mass of water in fines = 0.26 − 0.018 = 0.242 *t*

(a)

$$[w]_{fines} = \frac{M_w}{[M_s]_{fines}} 100 = \frac{0.242}{0.80} 100 = 30.3\%$$

(b)

Unified Classification = GC or SC

(c)

$$\text{Liquidity Index} = \frac{w_L - w_p}{w_L - w_p} = \frac{30.25 - 12}{27 - 12} = 1.22$$

(d)

Shrinkage - Swell
Shrinkagefrom Plastic Limit = 12 *LOW*
Swell from page 55 *of notes Table* 1 *PI*<18 *LOW*
Frost from Figure 6-11 *of Holtz p* 183 *LOW - MODERATE*

(e)

Fines are CL. From page 159 *of Holtz*
Sheepsfoot or rubber tired roller
or page 37 *of notes (d) use:*
Sheepsfoot

4. As an earthwork construction control inspector you are checking the field compaction of a layer of soil. The laboratory compaction curve for the soil is identical to the test for the Standard Test of Question 4-2. Specifications call for the compacted density to be at least 95% of the maximum laboratory value and within ± 2% of the optimum water content. When you did the sand cone test, the volume of soil excavated was 1153 cm^3. It weighted 2209 g wet and 1879 g dry.

(a) What is the compacted dry density?
(b) What is the field water content?
(c) What is the relative compaction?
(d) Does the test meet specifications?
(e) What is the degree of saturation of the field sample?
(f) If the sample were saturated at constant density, what would be the water content?

Answer

(a)

$$[\rho_d]_{field} = \frac{1879}{1153} = 1.63\ g/cm^3$$

(b)

$$[w]_{field} = 100\ \frac{2209 - 1879}{1879} = 17.6\%$$

(c)

$$[Rel\ Comp] = \frac{[\rho_d]_{field}}{[\rho_d]_{max}}\ 100 = \frac{1.63}{1.73}\ 100 = 94.2\%$$

(d)

Spec.: Rel Comp > 95 % and m/c = $w_{opt} \pm 2$ %

Rel Comp at 94.2% FAILS

m/c at 17.6% PASSES

(e) $\rho_w S e = w \rho_s$ and $\rho_d = \dfrac{\rho_s}{1 + e}$

Assuming $\rho_s = 2.65$ $[e = 0.63]$ $S = 74.4\%$

Assuming $\rho_s = 2.70$ $[e = 0.66]$ $S = 72.2\%$

(f)

Assuming $\rho_s = 2.65$ $[e = 0.63]$ $w = 23.6\%$

Assuming $\rho_s = 2.70$ $[e = 0.66]$ $w = 24.4\%$

5. The following results were obtained from a standard Proctor Test in performed in Lab.

Weight of wet soil (lb)	Moisture content (%)
3.65	12.2
3.95	13.4
4.25	15.3
4.15	19.1

(a) Calculate dry unit weight for each set of the data

(b) Calculate dry unit weight for Zero-Air-Void (ZAV) at each moisture content assuming G_s to be 2.70.

(c) Plot moisture-unit weight relationship along with ZAV line. Also calculate and plot (on the same diagram) dry unit weight versus moisture content for S = 70 and 80 percent (assume G_s = 2.70)

(d) Determine maximum unit weight, $\gamma_{d(max)}$ and OMC from the diagram

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(e) If the specifications call for field compaction to be minimum of 95 percent of maximum dry unit weight of the soil, recommend the range of moisture content to be used in the field.

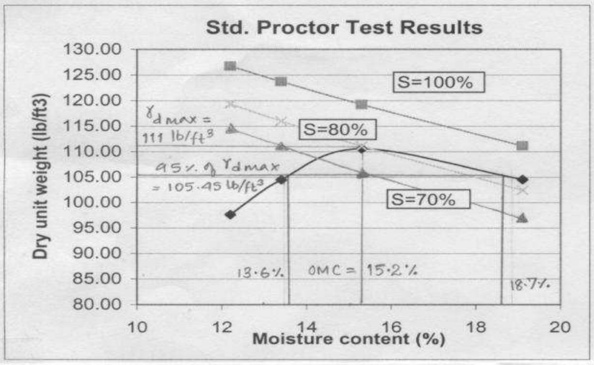
Draw the diagram to scale

(Note: If you select to use spreadsheet, please show at least one set of hand calculations for each problem)

PROBLEM #1

Wet wt.(lb)	Volume (ft3)	Wt unit wt.(lb/ft3)	Moisture (%)	Dry unit wt.(lb/ft3)	Zero void unit wt. (lb/ft3)
3.65	1/30	109.5	12.2	97.59	126.7338649
3.95	1/30	118.5	13.4	104.50	123.7186077
4.25	1/30	127.5	15.3	110.58	119.2272309
4.15	1/30	124.5	19.1	104.53	111.1565613

Dry unit wt (S=70%)	Dry unit wt (S=80%)
114.567709	119.341243
111.071765	116.013083
105.952745	111.107081
97.0107757	102.442806



Equations used

Wet unit wt, $\gamma_t = \dfrac{\text{wet wt}}{\text{Volume}} \Rightarrow$ eg. $\gamma_t = \dfrac{3.65\ lb}{1/30\ ft^3} = 109.5\ lb/ft^3$

(a) Dry unit wt. $\gamma_d = \dfrac{\gamma_t}{1+w} \Rightarrow$ eg. $\gamma_d = \dfrac{109.5}{1+0.122} = 97.59\ lb/ft^3$

(b) $\gamma_{zero\ air\ void} = \dfrac{\gamma_w}{w + 1/G_s} \Rightarrow$ eg. $\gamma_{zav} = \dfrac{62.4}{0.122 + 1/2.7} = 126.73\ lb/ft^3$

(c) $\gamma_{zav\,(s=70\%)} = \dfrac{G_s \gamma_w}{1 + \dfrac{G_s w}{s}} \Rightarrow$ eg. $\gamma_{zav\,(s=70\%)} = \dfrac{2.7 \times 62.4}{1 + \dfrac{2.7 \times 0.122}{0.70}} = 114.57\ lb/ft^3$

(d) From the figure, $\gamma_{d(max)} = 111\ lb/ft^3$

OMC = 15.2%

(e) For 95% of $\gamma_{d(max)}$, $\gamma_d = 0.95 \times 111 = 105.45\ lb/ft^3$

Equivalent MC range = 13.6% to 18.7%.