Chapter one

Physics and measurments

Classical Physics: Includes the theories, concept laws and experiments in classical mechanics, thermodynamics, optics and electromagnatism.

Modern physics: The two most important developments in this medern era were the theories of relativity and quantum mechanics (Einsteins theory of relativity).

Standard of length, mass and time: The three basic quantities are length, mass and time. An international committee established a set of standards for the fundamental quantities of science. It is called the (SI) system international and it is units of length, mass and time are the (meter, kilogram and second) respectively. Other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere) luminous intensity (the candela), and the amount of the substance (the mole).

Length: The meter was defined as (the distance between two lines on a specific platinum-iridium bar stored under controlled conditions in france). In 1960, the meter was defined as (1,650,763.73 wave length of orange-red light emitted from a krypton 86 Kr^{86} lamp). In 1983, the meter (m) was defined as (the distance traveled by light in vacuum during a time of 1/299,792,458 second). This last definition established that the speed of light in vacuum is precisely (299792458 meters per second).

Examples:

- Mean redius of the earth = $6.73*10^6$m
- Mean distance from the earth to the moon = $3.84*10^8$ m
- One light year = $9.46*10^{15}$ m
- Size of smallest dust particles ≈ 10^{-4} m
- Diameter of a hydrogen atom ≈ 10^{-10} m

Mass: The SI unit of mass, (kg) is defined as (the mass of a specific platinum-iridum alloy cylinder kept at the international bureau of weights and measures at sevres, france).

Examples: Sun mass = $1.99*10^{30}$ kg, Earth mass= $5.98*10^{24}$ kg.

Time:

Before 1960, the standard of time was defined in terms of (the mean **solar day**: defined as the time interval between successive appearances of the sun at the highest point it reaches in the sky each day).

* **The second** was defined as (1/60) (1/60)(1/24) of a mean solar day, or, defined as (9,192,631,1770) times the period of variation from the cesium atom.

* **Period**: the time interval needed for one complete vibration.

Atomic clock, uses the characteristic frequency of the cesium 133 atom as the "reference clock"

Customary system: Another system of units that is still used in the USA. In this system the units of length, mass and time are foot (ft), slug and second respectively.

Density and atomic mass:

Density (ρ): is the mass per unit volume ($\rho = m/v$)

Examples: $\rho_{aluminum} = 2.7\ g/cm^3$, $\rho_{lead} = 11.39\ g/cm^3$

Atomic mass: is the mass of a single atom of the element measured in atomic mass units (u) where: $1u = 1.6605387*10^{-27}$ kg.

Examples: the atomic mass of lead= 207u, for aluminum=27u.

The ratio of atomic masses is (207u/27u=7.67), does not correspond to the ratio of densities ($11.3*10^3/2.7*10^3=4.19$), this discrepancy is due to the difference in atomic spacing and atomic arrangements in the crystal structure of the two elements.

Example: A solid cube of aluminum ($\rho = 2.7\ g/cm^3$) has a volume of ($0.200\ cm^3$). It is known that (27.0 g) of aluminum contains ($6.02*10^{23}$ atoms). How many aluminum atoms are contained in the cube?

Solution: the mass of cube (m) = ρv = ($2.7\ g/cm^3$)($0.200\ cm^3$) = 0.540g.

$m_{sample} / m_{27g} = N_{sample} / N_{27g}$ ⟶ $(0.540g/27g) = (N_{sample}/6.02*10^{23} atoms)$

$N_{sample} = 1.20*10^{22}$ atoms

power	perfix	abbreviation	power	perfix	abbreviation
10^{-24}	yocto	y	10^{-1}	deci	d
10^{-21}	zepto	z	10^{3}	kilo	k
10^{-18}	atto	a	10^{6}	mega	M
10^{-15}	femto	f	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{15}	peta	P
10^{-6}	micro	μ	10^{18}	exa	E
10^{-3}	milli	m	10^{21}	zetta	Z
10^{-2}	centi	c	10^{24}	yotta	Y

Dimensional analysis:

* Used to check the final expression

* Dimentions treated as algebraic quantities. Quantities can be added or subtracted only if they have the same dimentions. The terms on both sides of an equation must have the same dimentions. The relationship can be correct only if the dimentions on both sides of the equation are the same.

Example: check the validity of $\mathbf{x = (1/2)at^2}$

Solution: The right side = $(L/T^2)\, T^2 = L$ = left side

Example: $\mathbf{x = a^n . t^m}$ where $[\mathbf{a^n.t^m}] = L = L^1\ T^0$

$(L / T^2)^n\, (T)^m = L^1\ T^0 \longrightarrow (L^n.T^{m-2n}) = L^1\ T^0$

$n = 1$ and $m-2n = 0 \longrightarrow m = 2$, but $(\mathbf{x = a^n . t^m})$, so that $(\mathbf{x = a.t^2})$

This result differs by a factor of (1/2) from the correct equation $\mathbf{x = (1/2)\ a.t^2}$

system	Area(L^2)	Volume(L^3)	speed (L/T)	acceleration(L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S customary	ft^2	ft^3	ft/s	ft/s^2

Conversion:

* **Length**

1 in = 2.54 cm, 1 m = 39.37 in = 3.281 ft, 1 ft = 0.3048 m, 1 yd = 3 ft, 1 ft = 12 in, 1 mi = 1.609 km, 1 km = 0.621m, 1 light year = $9.461*10^{15}$ m.

* **Mass**

1 ton=1000kg, 1 kg= $6.852*10^{-2}$slug, 1 slug=14.59 kg

* **Time**

1 year = 365 days=$3.16*10^{7}$ s,, 1 day= 24 hr =$1.44*10^{3}$min=$8.64*10^{4}$s

Convertion of units:

1 mile = 1609 m = 1.609 km

1 ft = 0.3048 m = 30.48 cm

1 m = 39.37 in = 3.281 ft

1 in = 0.0254 m = 2.54 cm

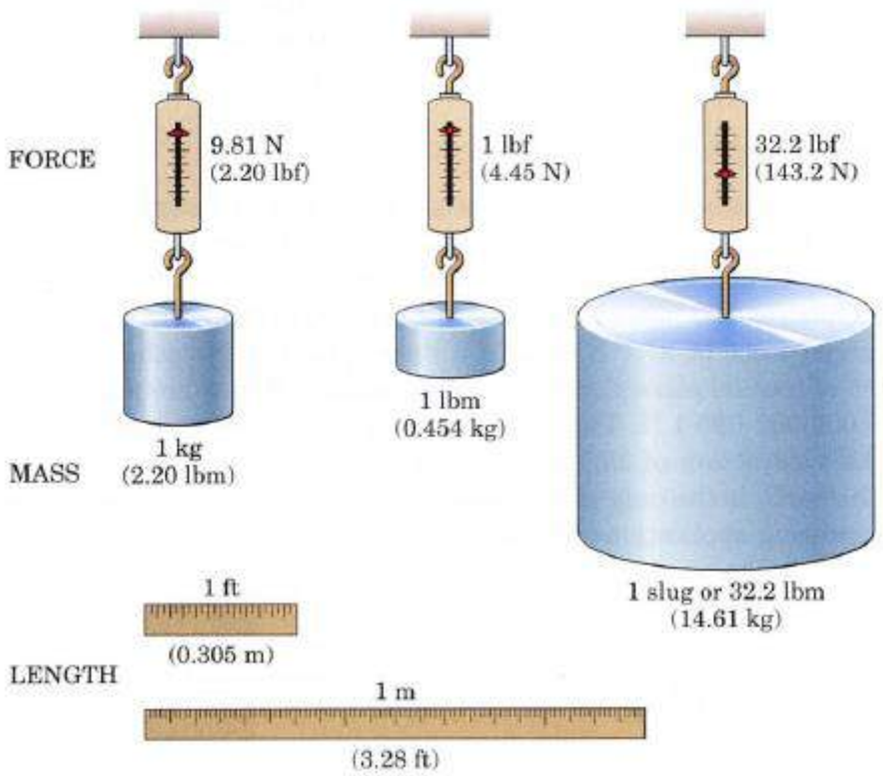
Example: convert 15 in to centimeter

Sloution: 15 in (2.54 cm / 1 in) = 38.1 cm

Example: A car is traveling at a speed of 38.0 m/s. Is this car exceeding the speed limit of 75.0 mil/h ?

Solution: 38.0 m/s (1 mil / 1609 m) = $2.36*10^{-2}$ mil/s

($2.36*10^{-2}$ mil / s) (60 s / 1 min) (60 min / 1 hr) = 85.0 mi/hr



Estimates and order of magnitude calculations:

It is often useful to compute an approximate answer to a given physical problem. Such approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will some times refer to an (order of magnitude) of a certain quantity as the power of ten of the number that describs that quantity. We use the symbol,~, for‶is on the order of″.

Thus: $0.0087 \sim 10^{-2}$ & $0.0021 \sim 10^{-3}$ & $740 \sim 10^{3}$. The results are reliable to within about a factor of (10).

Example: Estimate the number of steps a person would take walking from new york to los angeles?

Solution:

1- The distance between these two cities is about 3000 mil.

2- Each steps covers about 2 ft

3- 1 mil = 5280 ft or 1 mil ≈ 5000 ft

(5000 ft/mil) / (2 ft/steps) = 2500 steps/mil

Steps = ($3*10^3$ mi) ($2.5*10^3$ steps/mi) = $7.5*10^6$ steps ≈ 10^7 steps.

Example: Estimate the number of gallone used each year by all the cars in the united state?

Solution:

No. of people in the united state ≈ 280 million

No. of cars = 100 million

Quessing that there are between 2-3 people/car.

Average distance each car travels per year is 10000 mi

Gazoline consumption of 0.05 gal/mi

Each car uses about 500 gal/year

Total consumption = 500 ga/yr * $100*10^6$ car = $10^{10}*5$ gal ≈ 10^{11}gal/yr

Significant Figures:

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainity. The value of this uncertainity can depend on various factors such as

1- Quality of apparatus

2- The skill of the experimenter

3- The number of the measurements performed

An example of signifigant figures: suppose that we are asked in a laboratory experiment to measure the area of a computer disk label using a meter stick as a measuring instrument. If the length measured to be 5.5 cm and the accuracy to which we can measure the length of the label is ± 0.1cm. the length lies between 5.4 cm and 5.6 cm. If the label width is 6.4 cm, the actual value lies between 6.3 cm and 6.5 cm. Thus we could write the measured values as 5.5 ± 0.1 cm and 6.4 ± 0.1 cm. The area of the label is 5.5*6.4 = 35.2 cm^2 , it is taken as 35 cm^2. This value can range between (5.4*6.3 = 34 cm^2) and (5.6*6.5 = 36 cm^2).

Example: The radius of a circle is measured to be (12 ± 0.2) m, Calculate:

a) The area of circle

b) The circumference of the circle

c) Give the uncertainity in each value?

Answer:

a) Large radius is (12 + 0.2) ⟶ $\mathbf{r_1}$ = 12.2m & smaller radius is (11.8) m.

$A_1 = \pi (r_1)^2 = \pi *(12.2)^2 = 467.59\ m^2$

$A_2 = \pi(r_2)^2 = \pi *(11.8)^2 = 437.43\ m^2$

Average area (A) = $(A_1 + A_2) / 2 = 452.51\ m^2$

Uncertainity of area is $(452.51 \pm 15.08)\ m^2$

b) Large circumference $(S_1) = 2 \pi r_1 = (2* \pi *12.2) = 76.65$ m

Smaller circumference $(S_2) = 2 \pi r_2 = (2* \pi *11.8) = 74.14$ m

Average circumference = $(2* \pi *12) = 75.39$ m

Uncertainity of circumference = 1.26 m

Chap .2

Motion in One Direction

particle model:

A particle is "a point like object that is an object with mass but having infinitesimal size".

For example : to describe the motion of the Earth around the sun , we can treat the Earth as a particle and obtain reasonably accurate data about its orbit .

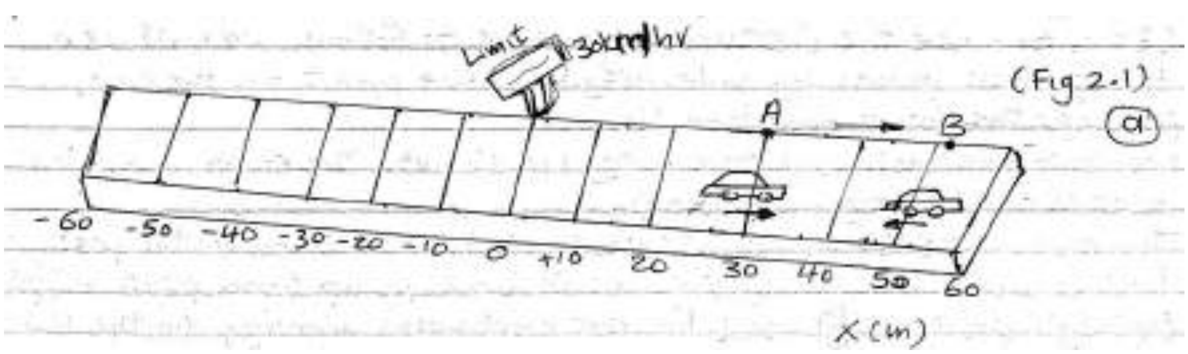
This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the sun.

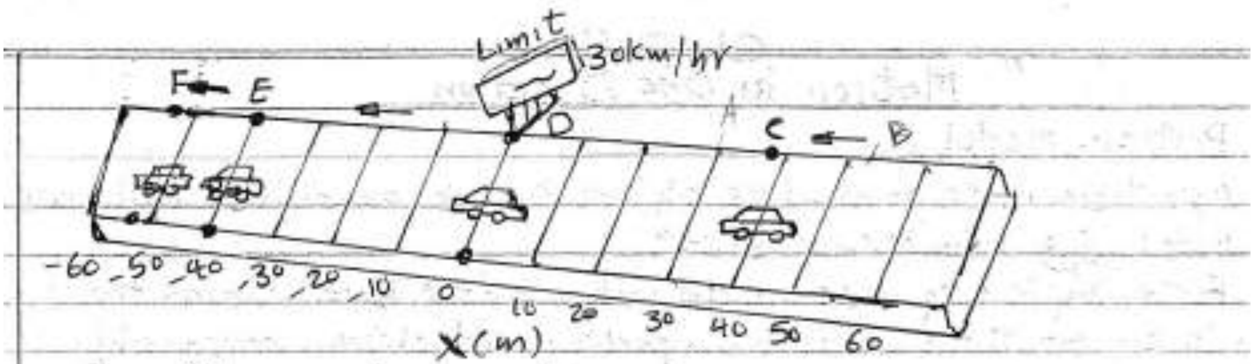
As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules .

To describe the moving object as a particle we can use what is called the (particle model) .

Position, Velocity and Speed:

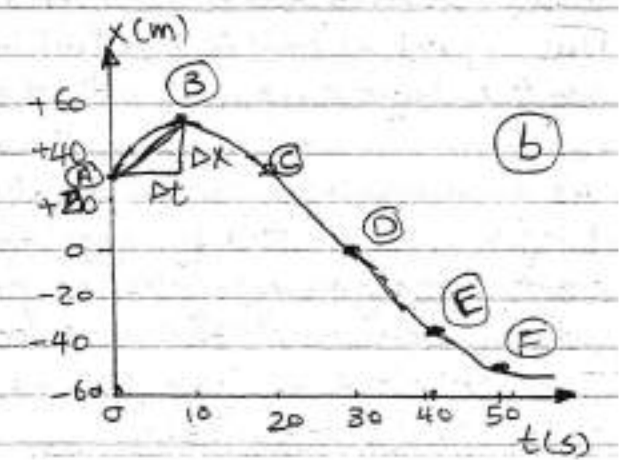
The motion of a particle is completely known if the particles position in space is known at all times. A *particle's position* is "the location of the particle with respect to chosen reference point that we can consider to be the origin of a coordinate system.





position of the car at various time

position	t(s)	X(m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	-37
F	50	-53



Consider car moving back and forth along the x- axis as in figure(2.1/a). The car is at (30m) to the right of a road sign which we well use to identify the reference position (x=0). (Assume all data are known to two significant figures). The initial position is (3.0*10m) it is written in the simpler form (30m) to make the discussion easier to follow. We will use the particle the model by identifying some point on the car, perhaps the front door handle.

We start our clock and once every (10s) note the car's position relative to the sign at (x=0).

The car moves to the right (which we have defined as the positive direction) (from A to B), the car is backing up from position (B) through position (F) and the car continues moving to the left and is more than (50m) to the left of the sign when we stop recording information is as shown in figure. It is called (a position-time graph).

The displacement of a particle is defined as " its change in position in some time interval".

$\Delta x = x_f - x_i$

Δx=displacement or change in position of the particle .

x_i =initial position, x_f =final position

If $x_f > x_i$ $\longrightarrow$ Δx is positive if $x_f < x_i$ $\longrightarrow$ Δx is negative

** Displacement differ from distance that (distance is length of a path followed by a particle. Displacement is an example of a vector quantity.*

Position, velocity and acceleration also are vectors. In general a vector quantity requires the specification of both direction and magnitude. By contrast, a scalar quantity has a numerical value and no direction.

Distance is a scalar quantity and it is always represented as a positive number while displacement can be either positive or negative.

The average velocity (V_x) of a particle as (the particle's displacement Δx divided by the time interval Δt during which that displacement occur).

$$\tilde{V}x = \frac{\Delta x}{\Delta t}$$

The unite of ($\tilde{V}_x$) is (m/s) in SI units.

The average of a particle moving in one direction can be positive or negative depending on the sing of the displacement.

The slope of the line between the points (A) and (B) on the position-time graph in (fig 2.1/b), represents the ratio ($\Delta x/\Delta t$) which is defined as (average velocity).

Ex: the average velocity of the car between points (A) and (B)

$$\tilde{V}x = \frac{(52-30)m}{(10-0)s} = 2.2 \text{ m/s}$$

The terms (speed) and (velocity) are interchangeable. In physics however, there is a clear distinction between these two quantities. The (average speed) of a particle , a scalar quantity, is defined as (the total distance traveled divided by the total time interval required to travel that distance).

Average speed = total distance / total time

unlike average velocity, average speed has no and hence carrying no algebraic sign, they have the same units and the magnitude of the average velocity is not the average speed.

Example

Find the displacement, average velocity and average speed of the car in (fig 2.1/a) between points (A) and (F)

Solution:

from the position –time graph given in (fig 2.1/b) note that (x_A =30m) at (t_A=o) , and that (x_f = - 53m) at (t_f=50s).

$\Delta x = -53m - 30m = -83m$ displacement.

$$\tilde{V}x = \frac{\Delta x}{\Delta t} = \frac{xf - xi}{tf - ti} = \frac{xf - xA}{tf - tA} = \frac{(-53-30)m}{(50-0)s} = \frac{-83m}{50s} = -1.7m/s$$

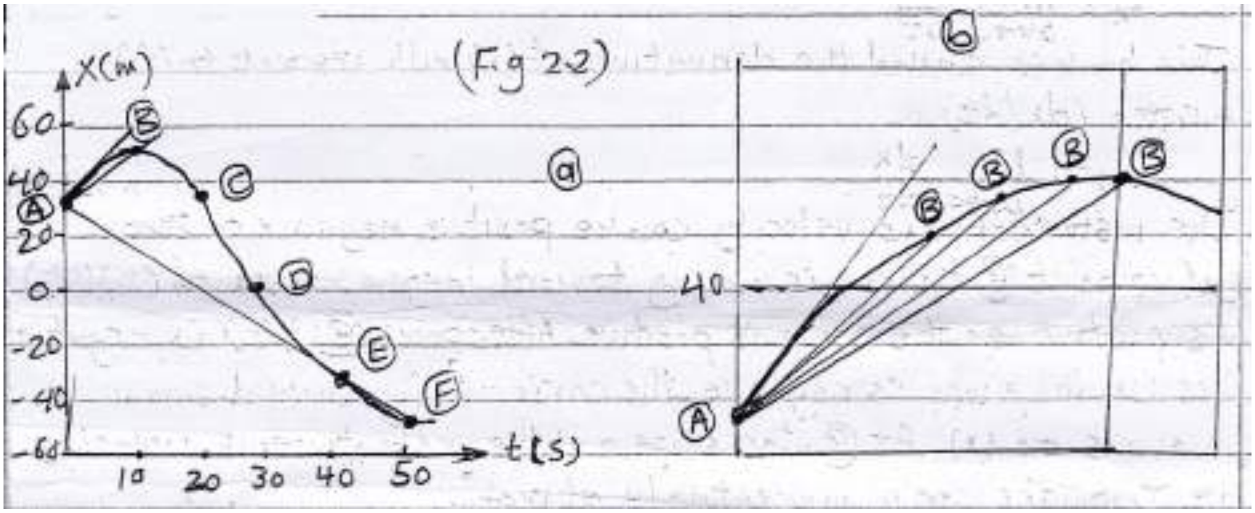
To calculate the average speed we need the complete details about the distance between the points (from (A) to (B) is (22m) plus from (B) to (F) for a total of 127 m)

Average speed = 127 m /50 s =2.54 m/s

Instantaneous velocity and speed

It is the velocity of a particle at a particular instant in time. Consider (fig 2.2/a) which is a reproduction of the graph in (fig 2.1/b) The slope of the line (AB) represent the average velocity.

For the interval during which the car moved from position (A) position (F)



The slope of the line (AF) represents the average velocity of the interval during which it moved from (A) to (F). which of these two lines do you think is a closer approximation of the initial velocity of the car ?

The car starts out by moving to the right (positive direction), the value of the average velocity during the (A) to (B) interval is more representative of the initial value than is the value that is the value of the average velocity during the (A) to (F) interval, which is negative (Ex-1).

Now let us focus on the line (AB) and side point (B) to the left along the curve toward point (A) as in (Fig 2.2/b). The line between points becomes sleeper and sleeper as the two points become extremely close to gather, the line becomes a tangent line to the curve. The slope of this tangent line represents the velocity of the car at the moment we started taking data at point (A)

The instantaneous velocity (V_x) equals: "the limiting value of the ratio ($\Delta x/\Delta t$) as (Δt) approaches zero".

[Note: the displacement (Δx) also approaches zero as (Δt) approaches zero, so that the ratio look like 0/0. As (Δx) and (Δt) become smaller and smaller, the ratio ($\Delta x/\Delta t$) approaches a value equal to the slope of the line tangent to the (x) versus(t) curve.

$V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

This limit is called the derivative of (x) with respect to (t), written (dx/dt)

$Vx \equiv \lim_{\Delta t \to 0} \frac{dx}{dt}$

The instantaneous velocity can be positive, negative or zero.

Before point (B) the car is moving toward larger values of (X), (V_x) is positive so the slope is positive. After point (B), (V_x) is negative because the slope is negative, the car is moving toward smaller values of (X). At (B) ,the slope and the instantaneous velocity are zero, the car is momentarily at rest.

The instantaneous speed of a particle is defined as the magnitude of its instantaneous velocity. It has no direction and hence carries no algebraic sign.

For example if one particle has an instantaneous velocity of (+35m/s) along a given line and another particle has an instantaneous velocity of (-35m/s) along the same line, both have a speed of (35m/s).

Ure car use the word velocity to designate instantaneous velocity, Usr car use the word speed to designate instantaneous speed .

Ex-2

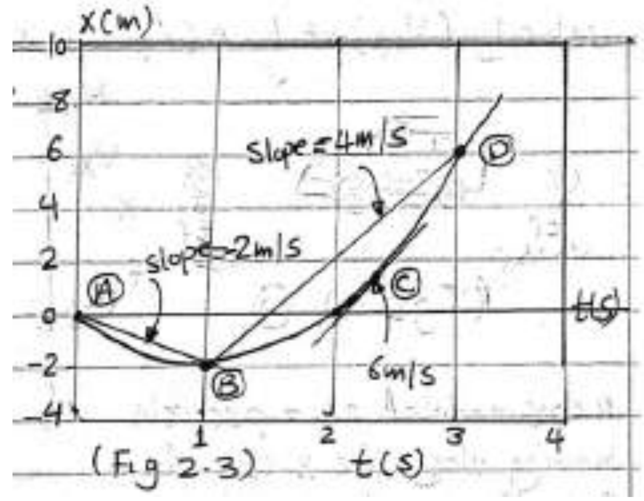
A particle moves along the x-axis. It position vanes with time according to the expression ($x=-4t+2t^2$) where (x) is in meters and (t) is in seconds. The position-time graph is shown in (fig 2.3). Note that the particle moves in the negative (x) direction for the first second of motion, is momentarily at rest at the moment (t=1s) and moves in the positive direction at times (t >1s).

a) Determine the displacement of the particle in the time intervals (t=0), to (t=1s) and (t =1s to t =3s).

Solu:

During the first time interval, the slope is negative and hence the average velocity is negative .

Thus, we known that the displacement between A and B must be negative. The displacement between B and D is positive . $t_i=t_A=0$, $t_f=t_B=1s$ and $x=-4t+2t^2$



$\Delta x_{A\rightarrow B} = x_f - x_i = x_B - x_A = [-4(1) + 2(1)^2]-[-4(0)+2(0)^2] = -2m$ between t=0 and t=1s.

Between t =1s and t =3s , $t_i=t_B=1s$ and $t_f=t_D=3s$.

$\Delta x_{B\rightarrow D} = x_f - x_i = x_D - x_B = [-4(3) + 2(3)^2]-[-4(1)+2(1)^2] = +8m$

b) Calculate the average velocity during these two time intervals.

In the first time interval $\tilde{V}x_{A\rightarrow B} = \frac{\Delta x_{A\rightarrow B}}{\Delta t} = \frac{x_B - x_A}{t_B - t_A} = \frac{-2M}{1s} = -2m/s$

In the second time interval $\Delta t = t_D - t_B = 3-1 = 2s$

$$\tilde{V}x_{B\rightarrow D} = \frac{\Delta x_{B\rightarrow D}}{\Delta t} = \frac{8m}{2s} = 4m/s$$

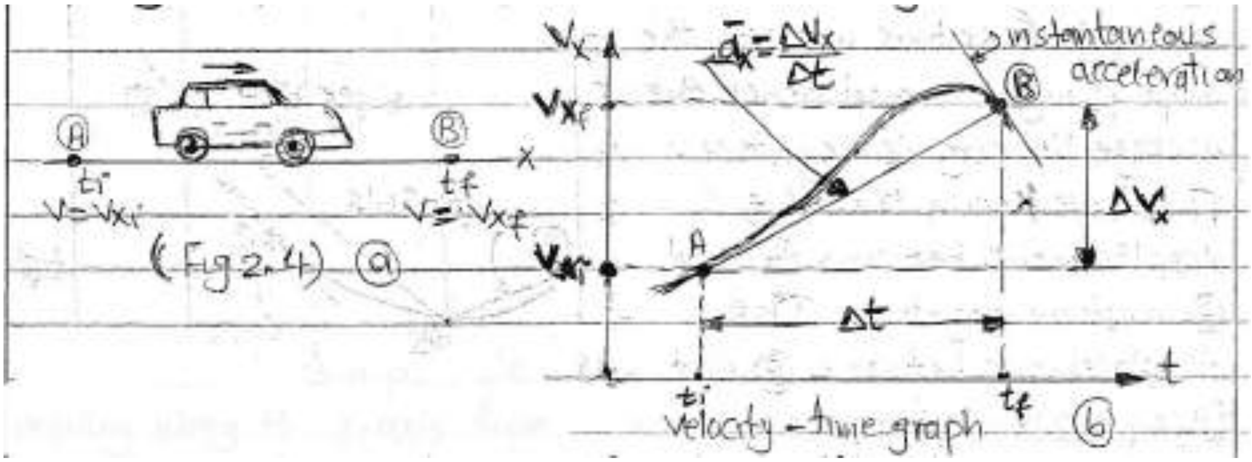
c) Find the instantaneous velocity of the particle at (t = 2.5s).

Solu: By measuring the slope of the line at (t=2.5s) we find that $v_x = +6m/s$

Acceleration

When the velocity of a particle changes with time the particle is said to be accelerating. How to quantify acceleratain?

Suppose an object that can be modeled as a particle moving along the x-axis has initial velocity (V_{xi}) at time (t_i) and a final velocity (V_{xf}) at time (t_f) as shown in (fg2.4/a).



A car modeled as a particle Moving along the x-axis from A to B. $\Delta t = t_f - t_i$, the slope of the line in (fig 2.4/b) is the average acceleration ($\Delta V_x = V_{xf} - V_{xi}$).

The average acceleration ($\tilde{a}_x$) of the particle is defined as (the change in velocity ΔV_x divided by the time interval Δt during which that change occurs).

$$\tilde{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{V_{xf} - V_{xi}}{t_f - t_i}$$

The SI unit of acceleration is (m/s^2).

The instantaneous acceleration is defined as "the limit of the average acceleration as (Δt) approach (zero) It is equal to :-

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

That is, the instantaneous acceleration equals'' the derivative of the velocity with respect to time. It is the slope of the velocity-time graph (the slope of the line at point B in fig 2.4/b). If (**a_x**) is positive, the acceleration is in the positive (x) direction, If (**a_x**) is negative, the acceleration is in the negative (x) direction. Negative acceleration does not necessarily mean that an object is slowing down. If the acceleration is negative and the velocity is negative, the object is speeding up.

when the objects velocity and acceleration are in the same direction , the object is speeding up. On the other hand , when the objects veloaty and acceleration are in opposite directions , the object is slowing down.

The acceleration is caused by force exerted on the object. F ∝ a

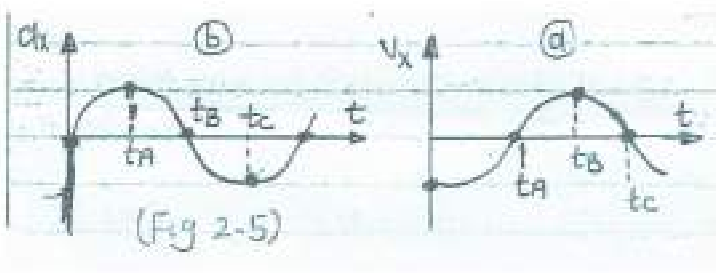
Force and acceleration are both vectors and the vectors act in the same direction .It is very useful to equate the direction of the acceleration to the direction of a force. We use the term(acceleration to mean(instantaneous acceleration), and when use (average) we mean(average acceleration).

The acceleration can also be written.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

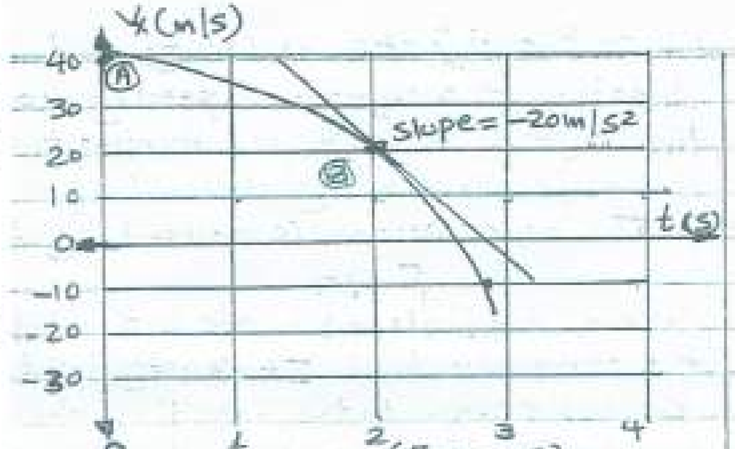
That is in one-dimensional motion , the acceleration equals the second derivative of (x) with respect to time.

The acceleration at any time is the slope of the velocity-time graph at that time .positive values of acceleration corresponds to those points in(fig 2.5/a)where the velocity is increasing in the positive (x) direction. The acceleration reaches a maximum at time (ta), when the slope of the (velocity-time graph)is a maximum. The acceleration then goes to zero at time (t_B), when the velocity is maxmum (slope of V_x-t graph) is zero. The acceleration is negative when the velocity is decreasing in the positive (x) direction.



The instantaneous acceleration can be obtained from the velocity-time graph (a) e.g

The velocity of a partied moving along the axis varies in time according to the expression(v_x=40-5t^2)m/s, where(t) is in seconds.



(Fig 2-6)

(a) Find the average acceleration in the time interval:-

T=0 to t=2.0s .

Solu

$t_i = t_A = 0$ and $t_f = t_B = 2.0\,s$

$V_{xA} = (40 - 5t_A^2) = [40 - 5(0)^2]\,m/s = +40\,m/s$

$V_{xB} = (40 - 5t_B^2) = [40 - 5(2.0)^2] = +20\,m/s$

The average acceleration in the specified time interval

$\Delta t = t_B - t_A = 2.0 - 0.0 = 2.0\,s$ is:-

$$\bar{a}_x = \frac{V_{xf} - V_{xi}}{t_f - t_i} = \frac{V_{xB} - V_{xA}}{t_B - t_A} = \frac{(20-40)}{(2.0-0.0)} = -10\,m/s^2$$

(B) Determine the acceleration at (t=2.0s).

The velocity at any time (t) is ($V_{xi} = 40 - 5t^2$) m/s and the velocity at any later time ($t+\Delta t$) is

$V_{xf} = 40 - 5(t+\Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5\,\Delta t^2$

The change in velocity over the time interval (Δt) is :-

$\Delta V_x = V_{xf} - V_{xi} = (-10t\,\Delta t - 5\Delta t^2)$ m/s

$a_x = \lim_{\Delta t \to 0} \frac{\Delta V_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t$ m/s^2

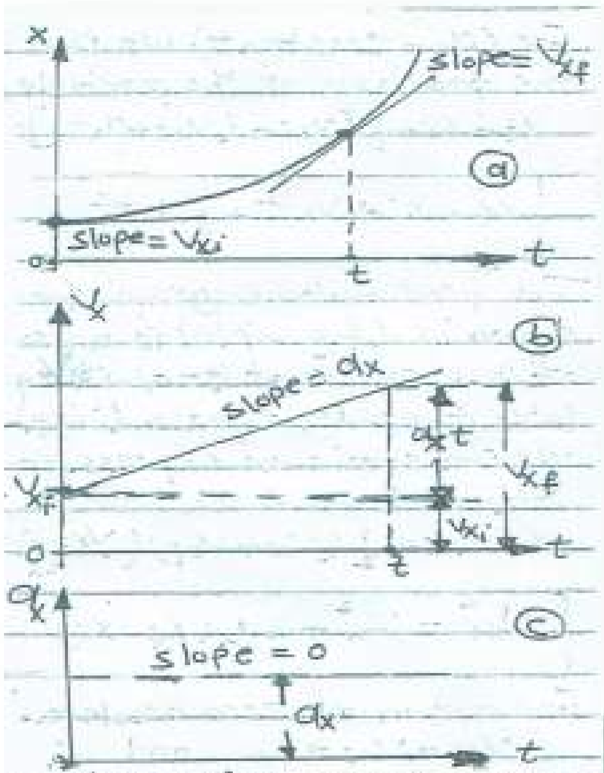
Therefore at $t = 2.0$ s, $a_x = -10(2.0) = -20$ m/s^2, because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

(Note): the average acceleration in (A) is the slope of the line

Connecting points (A) and (B) . The instantaneous acceleration in (B) is the slope of the line tangent to the carve at point (B) the acceleration in this example is not constant.

One-Dimensional Motion With constant Acceleration

In this case , the average acceleration ($\bar{a}_x$) over any time intrvel is numerically equal to the instantaneous acceleration (a_x) at any instant within the interval and the velocity changes at the same rate throughout the motion.



(Fig 2-7) A particle moving along x axis with constant acceleration a_x

velocity-time graph

If we replace ($\bar{a}_x$) by (a_x) and take (t_i=0) and (t_f=t) by any later time (t) , the equ.

$\bar{a}_x = \frac{V_{xf} - V_{xi}}{t_f - t_i}$ becomes:- $a_x = \frac{v_{xf} - v_{xi}}{t - 0}$ OR

$V_{xf} = V_{xi} + a_x t$ powerful expression

The powerful expression enables us to determine an object's velocity at any time (t) if we know the object's initial velocity (V_x) and its (constant) acceleration(ax).

In(fig 2.7/b) (velocity-time graph) , the slope of the straight line is the constant slope ($a_x = dv_x/dt$). When the acceleration is constant , the graph of acceleration – time is a straight line having slope of Zero.

The average velocity in any time interval at constant acceleration

(a_x) is: $\bar{V}_x = \frac{v_{xi}+v_{xf}}{2}$ it is applied only for (a_x=constant).The position of an object as function of time when ($\Delta_x = x_f - x_i$) and ($\Delta t = x_f - x_i = t - 0 = t$)is:-

$x_f - x_i = \bar{V}_x t = \frac{1}{2}(\bar{V}_{xi} + \bar{V}_{xf})t$ ➡ $x_f = x_i + \frac{1}{2}(\bar{V}_{xi} + \bar{V}_{xf})t$

For (a_x= constant) we can obtain another useful expression for the position of the partical , moving with constant acceleration .

$x_f = x_i + \frac{1}{2}\{V_{xi} + (V_{xi} + a_x t)\}t$

$x_f = x_i + V_{xi} + \frac{1}{2}a_x t^2$ for (a_x=constant)

The postion-time graph for motion at constant (positive) acceleration shown in (fig 2.7/a) is a parabola , the slope of the tangent line of this cuwe at (t=0) is aqual to (V_{xf}) .

We can obtain an expression for the final velocity that dose not contain time:-

$$x_f = x_i + \frac{1}{2}(V_{xi} + V_{xf})\left(\frac{V_{xf} - V_{xi}}{a_x}\right) = \frac{V_{xf}^2 - V_{xi}^2}{2a_x} + x_i$$

$$\boxed{V_{xf}^2 = V_{xi}^2 + 2a_x(x_f - x_i)} \quad \text{for } (a_x = \text{constant})$$

For motion at zero acceleration we see :-

$V_{xf} = V_{xi} = V_x$ and $x_f = x_i + V_x t$ when a_x=0

That is when (a_x=0) , the velocity of the partical is conetant and its position changes linearly with time.

[these relationships are (kinematic equations) that may ve use to solve any problem involving one-dimensional motion at constant acceleration].

Note: for these equations the motion is a long x-axis.

e.g: A car traveling at a constant speed of (45m/s) passes a trooper.

some time it's more convenient to represent a point in a Plane by it's Plane coordinates (r, θ) as shown in Fig (1)

r:- is the distance from origin to the point having cartisian coordinate (x, y)

θ:- is the angle between a line drawn from the origin to the Point and a Fixed axis.

$x = r\cos\theta$ $\quad y = r\sin\theta$ and $r = \sqrt{x^2 + y^2}$

Ex:- The cartesian coordinates of a Point in the xy Plane are $x = -3.5\,m$ find the Polar coordinates of this Point. Take $y = -2.5\,m$

Solution

$r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (2.5)^2} = 4.30\,m$

$\tan\theta = y/x = -2.5\,m / -3.5\,m = 0.714$

$\Rightarrow \theta = \tan^{-1} 0.714 \longrightarrow \theta = 216^\circ$

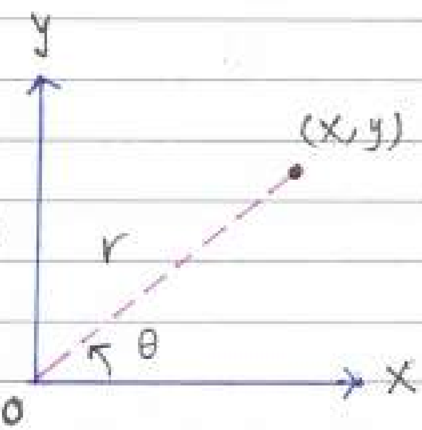
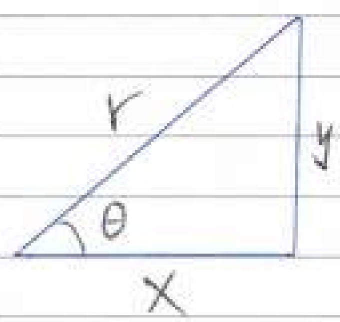


Fig (1)



$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$\tan\theta = \frac{y}{x}$

vector and scalar quantities.

Scalar quantity :- is completely specified by a single value with appropriate unit and has one direction.

Ex (volume , speed , time , mass
temperature , [illegible])

vector quantity :- is completely specified by a - number and appropriate unit plus a direction
Ex (wind , velocity , displacement , speed)

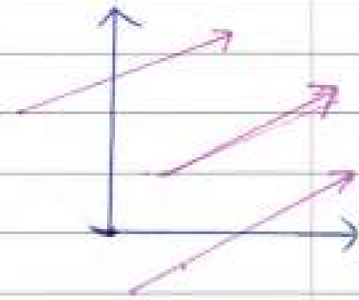
For the particle moves (Fig-2) the displacement represnted by drawing an arrow from (A) to (B) the direction of the arrow head represents the direction of the displacement and the length of arrow represents the magnitude of the displacement.

Note :- if the particle travels along some other path from (A) to (B) such as (broken line), it's displacement is still arrow dram from (A) to (B)

* **Displacement :-** depends only on the initial and Final positions, its independent on the path taken between these two point.

Properties of vectors.

1)- $\vec{A} = \vec{B}$ only if $A = B$ and have same direction along particel lines.



2) Adding vector:
«graphical method»

The result vector $\vec{R} = \vec{A} + \vec{B}$ is the vector drawn from the tail of $\vec{A}$ to the tip of $\vec{B}$

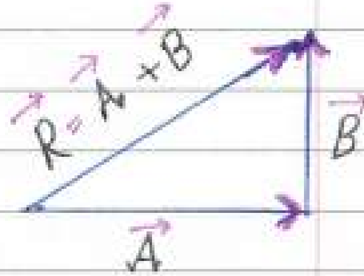
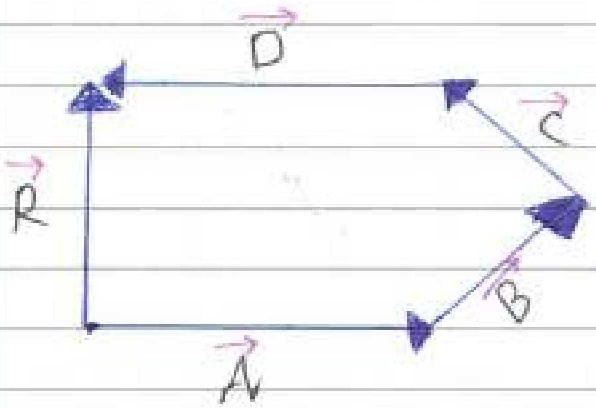
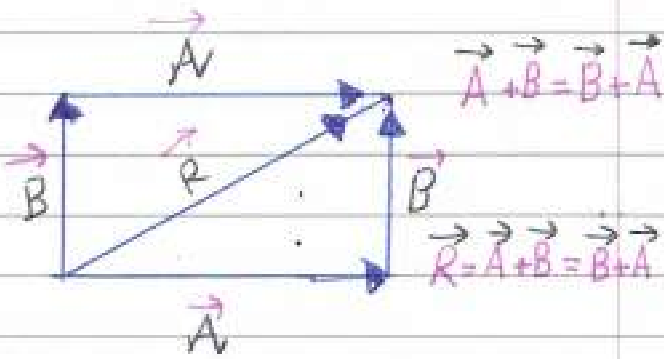
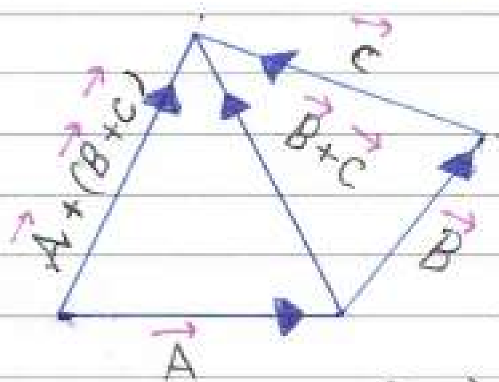


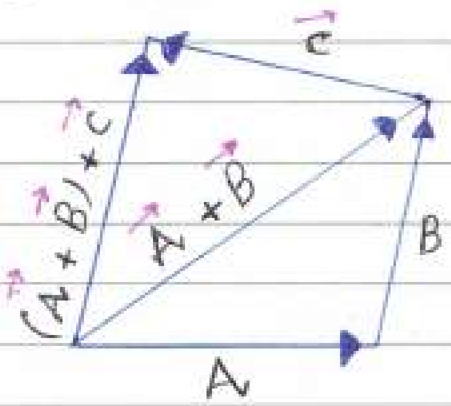
Figure-3 represent add more than two vectors





R:- is the vector drown from tail of first vector to the tip of the last vector





$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

Negetive of vector.

the negative of vector $\vec{A}$

is the vector that when added to $\vec{A}$ gives zero for the vector sum.

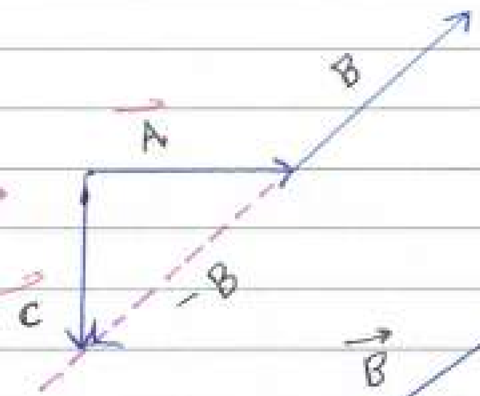
$\vec{A} + (-\vec{A}) = 0$

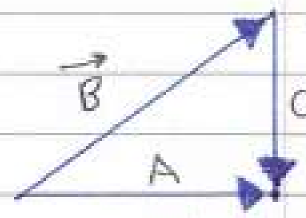
the vectors $\vec{A}$ and $-\vec{A}$ have the same magnitude but Point in opposite direction.

subtracting vectors

① $A - B = A + (-B)$

② from tiP of $\vec{A}$ to tail $\vec{B}$





c = A - B

Multiplying vector by a scalar

if $\vec{A}$ multiplying by Positive scalar quantity (m) them $(m\vec{A})$ is the vector has the same direction of $\vec{A}$ and has a magnitude = mA

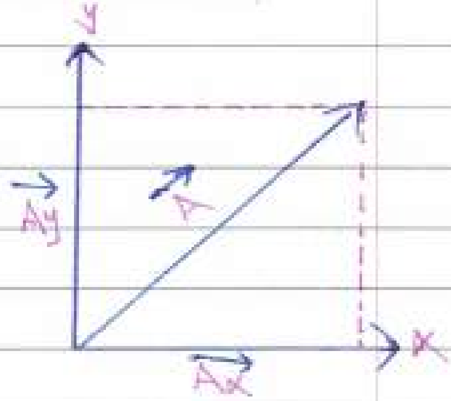
If $\vec{A} \times (-m)$ then $(-mA)$ is the vector has opposite direction and $(-mA)$ magnitude.

components of a vector and unit vectors

$\vec{A} = \vec{A_x} + \vec{A_y}$

where

$A_x = A\cos\theta$, $A_y = A\sin\theta$



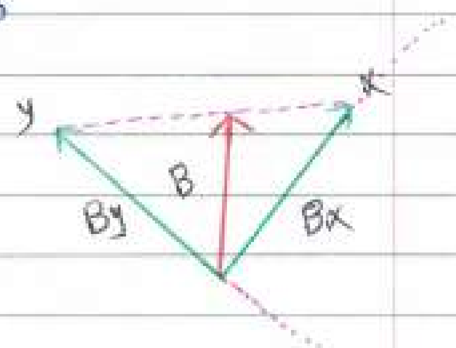
the sign of the components A_x and A_y depend on the angle θ.

Ex:- if $\theta = 120^\circ$ → A_x is negtive, A_y is Positive

if $\theta = 225^\circ$ → A_x is negtive, A_y is negtive

$A = \sqrt{A_x^2 + A_y^2}$, $\theta = \tan^{-1}\frac{A_y}{A_x}$

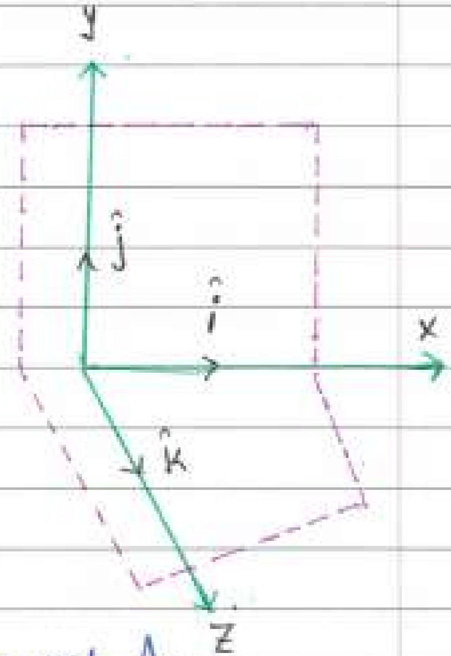
Note:- when we are working a Physics problem that required resolving a vector into it's component. it's convenient to the component in accordinate system having axes perpendicular to each other.



$B_x = B\cos\theta$, $B_y = B\sin\theta$

$B = \sqrt{B_x^2 + B_y^2}$, $\theta = \tan^{-1}\frac{B_y}{B_x}$

* vectors unit

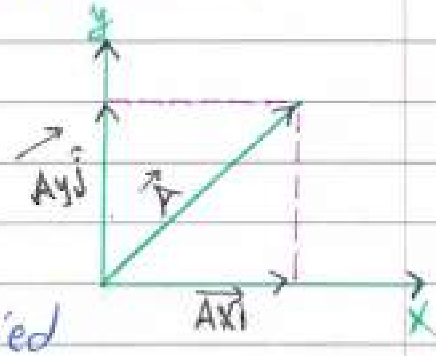


It's a dimensionless vector having a magnitude of exactly (1)

* used to specify a given direction.
* The symbols $\hat{i}$, $\hat{j}$ and $\hat{k}$ are represent unit vectors pointing in the Positive x, y and z direction respectivily.
* $|\hat{i}| = |\hat{j}| = |\hat{k}|$

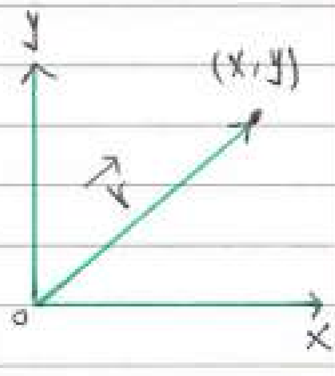
* The prodution of vector the component A_x and the unit vector $(\hat{i})$ is the vector $\overrightarrow{A_x \hat{i}}$
* the vector $\overrightarrow{A_x \hat{i}}$ is the alternative representation of vector $\overrightarrow{A_x}$.
* the unit vector for the [redacted] vector $\vec{A}$ is.

$\vec{A} = \overrightarrow{A_x \hat{i}} + \overrightarrow{A_y \hat{j}}$



* The point in figure can be specified by the position vector which is given by

$\vec{r} = x\hat{i} + y\hat{j}$, $\vec{r}$ = Position vector



If we want to add $\vec{A}$ to $\vec{B}$ where ...

$\vec{B} = \vec{B_x} + \vec{B_y}$ and $\vec{A} = \vec{A_x} + \vec{A_y}$.

The result is $\vec{R} = \vec{A} + \vec{B}$

$\vec{R} = (\overrightarrow{A_x \hat{i}} + \vec{A_y \hat{j}}) + (B_x \hat{i} + B_y \hat{j})$

or $R = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$

$\vec{R} = \vec{R_x i} + \vec{R_y j}$

$R_x = A_x + B_x \quad , \quad R_y = A_y + B_y$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$

$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$

Considering three component

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

The sum of $\vec{A}$ and $\vec{B}$ is

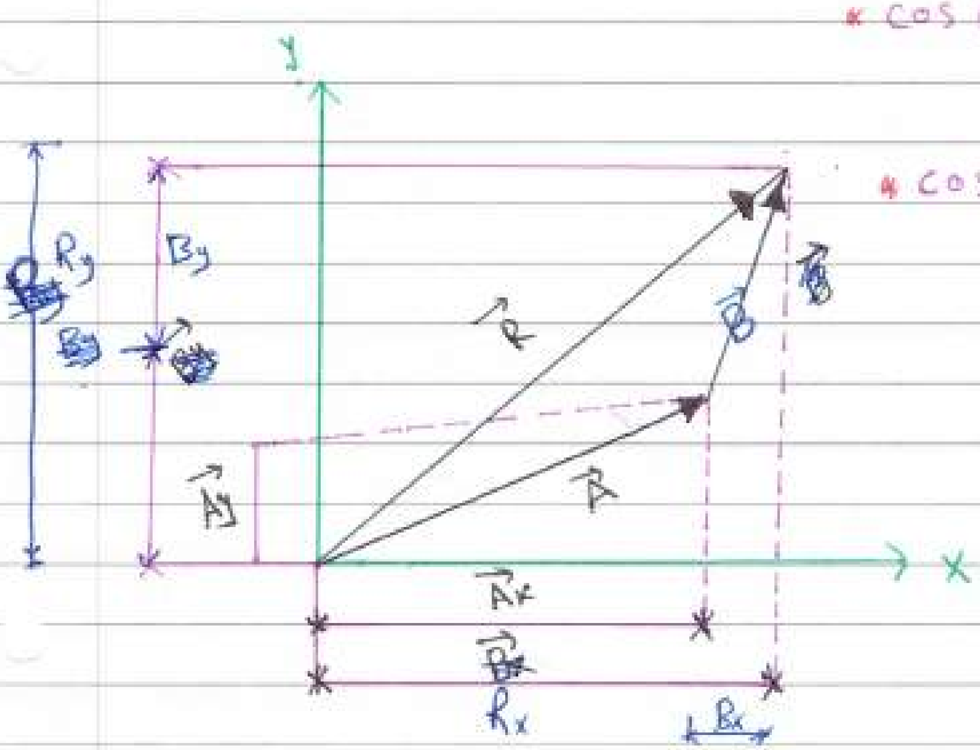
$R = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad , \quad R_z = A_z + B_z$

θ_x :- angle that R makes with x-axis :- * $\cos\theta_x = \frac{R_x}{R}$

* $\cos\theta_y = \frac{R_y}{R}$

* $\cos\theta_z = \frac{R_z}{R}$



Ex :- Find the sum of two vector $\vec{A}$ and $\vec{B}$ lying in the xy plane and given by $A = (2.0\hat{i} + 2.0\hat{j})$ m and $\vec{B} = (2.0\hat{i} - 4.0\hat{j})$ m

Sol.

($A_x = 2.0$ m , $A_y = 2.0$ m , $B_x = 2.0$ m , $B_y = -4.0$ m)

$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i}\,m + (A_y + B_y)\hat{j}\,m$

$R = (4.0\hat{i} - 2.0\hat{j})$ m or $R_x = 4.0$ m and $R_y = -2.0$ m

The magnitude of $\vec{R}$ is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4)^2 + (-2)^2} = 4.5$ m

The direction of $\vec{R}$ is :-

$\tan\theta = \frac{R_y}{R_x} = \frac{-2\,m}{4\,m} = -0.5$

$\theta = \tan^{-1}(-0.5) \Longrightarrow \theta = -27^\circ$

$\theta = -27^\circ$:- It means clockwise from the x-axis thus the angle for this vector $= (-27 + 360)$

$\theta = 333^\circ$ counter clockwise from the x-axis

Example :- A hiker begins a trip by first walking (25 km) Southeast from her car. She stops and set up her tent for the night. one the second day she walks (40 km) in direction $\theta = (60.0^\circ)$ north of east, at which point she discovers a forest ranger's tower.

a) Determine the components of the hiker's displacement for each day ?

b) Determine the components of the hiker's resultant displacement $\vec{R}$ for the trip. find the eapression $\vec{R}$ in terms of unit vectors.

Solution :- * the displacement in first day = $\vec{A}$
* the displacement is second day = $\vec{B}$
* use the Car the origin coordinates

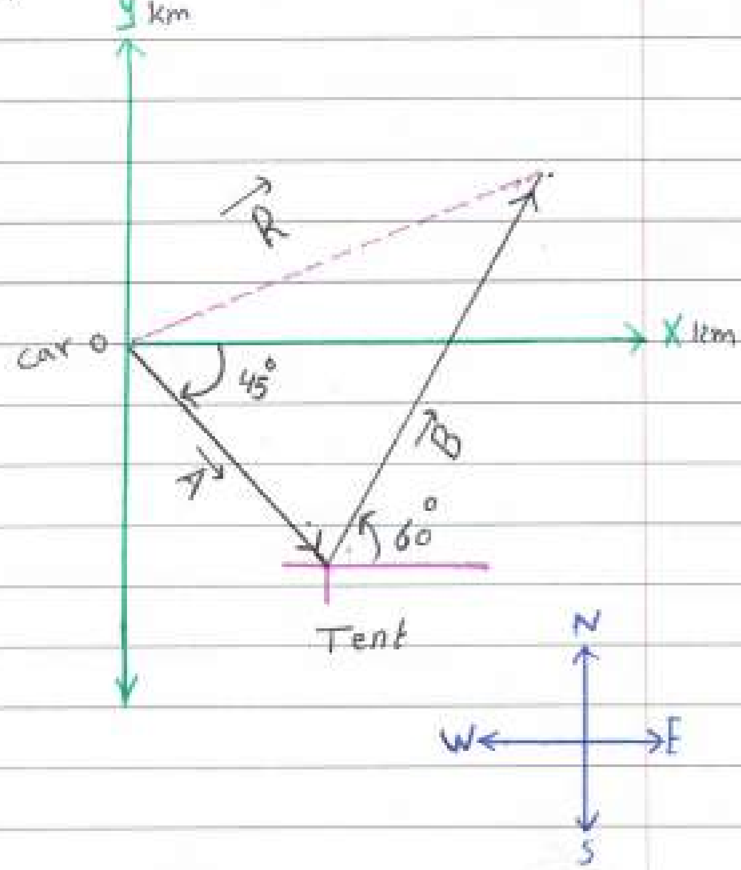
a)

$A_x = A\cos(-45^\circ) = 25 \times 0.707 = 17.7$ km

$A_y = A\sin(-45^\circ) = -17.7$ km

$B_x = B\cos(60^\circ) = 20$ km

$B\sin(60^\circ) = 34.6$ km



b)

$\vec{R} = \vec{A} + \vec{B}$

$R_x = A_x + B_x = 17.7\text{ km} + 20\text{ km} = 37.7\text{ km}$

$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \sout{52.9\text{ km}}\ 16.9\text{ km}$

In unit vector from

$\vec{R} = (37.7\hat{i} + 16.9\hat{j})\text{ km}$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(37.7)^2 + (16.9)^2} \Rightarrow R = 41.3$

$\tan\theta = \frac{R_y}{R_x} = \frac{16.9\text{ km}}{37.7\text{ km}}$

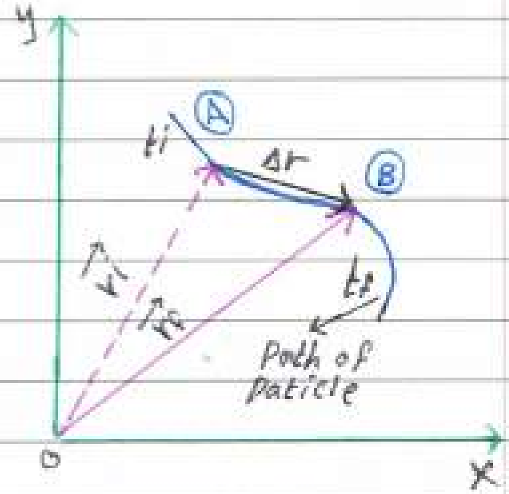
$\theta = 24.1^\circ$ north of east

Chapter Four - Motion in two Dimension

The Position, velocity and acceleration vectors

The displacement of the Particle is
((displacement vector; Δr))

$\Delta r = r_f - r_i$



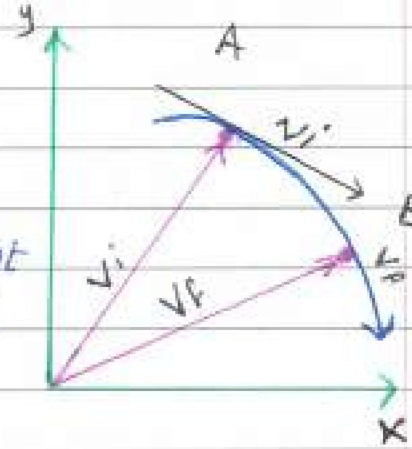
* The magnitude of $\vec{\Delta r}$ < the distance traveled along the curve path

* average velocity, $\vec{\bar{V}} = \frac{\vec{\Delta r}}{\vec{\Delta t}}$
where $\vec{\bar{V}}$:- vector quantity

* The average velocity :- is independent of the path taken because it's proportional to displacement

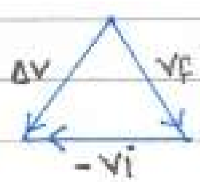
* The instantaneous velocity ($\vec{V}$) :- is defined as ((The limit of the average velocity $\vec{\Delta r}/\Delta t$ as Δt approach Zero))

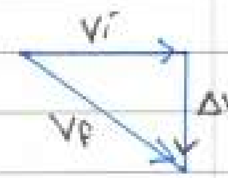
$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$ or $V = \frac{dr}{dt}$



* The direction of $\vec{V}$ is along a line tangent to the path in the direction of motion.

* The magnitude of $V = |\vec{V}|$ is called speed (scalar quantity)





The average acceleration ($\vec{a}$): is the change in stantanous velocity $\vec{\Delta v}$ derided by. The time interval during which the change occurs.

$$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

* The average acceleration is a vector quantity directed along $\vec{\Delta v}$

* The istantanous acceleration is (The limiting value of the ratio ($\vec{\Delta v} / \Delta t$) at Δt approach zero

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\vec{\Delta v}}{\Delta t} \quad \text{or} \quad \vec{a} = \frac{dv}{dt}$$

* Various changes can occur when particle accelerates.

1- The magnitude of velocity vector
(speed → change → stright) line
(one direction motion).

2- the direction of speed and it's magnitude remain constant
(curved Path) → (two dimentional motion)

3- Both direction and magnitude an change

* Two dimensional motion with constant acceleration.

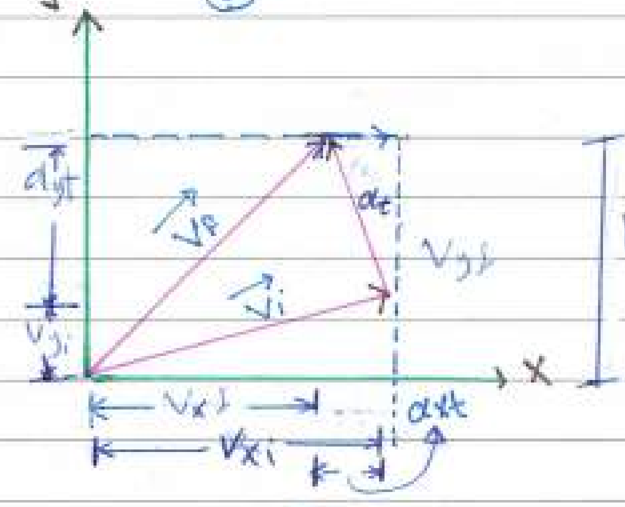
The Position vector → $\vec{r} = x\hat{i} + y\hat{j}$

The Velocity vector → $\vec{v} = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$
$= v_x\hat{i} + v_y\hat{j}$

* $\vec{v}_f = \vec{v}_i + \vec{a}t$ ⇒ quation of velocity vector as a function of time

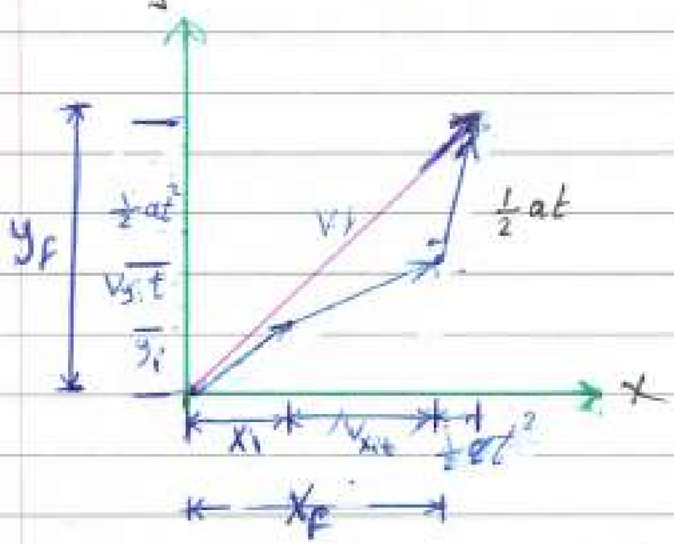
$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}at^2$ ⇒ quation of ~~velocity~~ position vector as a function of time

ⓐ



note that $\vec{v}_f$ not along the direction of $\vec{v}_i$ or $\vec{a}$

ⓑ



EX:- A Particle starts from origin at ($t=0$) with initial velocity having an (x) component of ($20\ m/s$) and a y component of ($-15\ m/s$). The Particle moves in the xy = Plane with an x component of acceleration only by « $a_x = 4.0\ m/s^2$ »

a:- Determine the component of the Velocity at any time and total velocity Vector at any time.

Sol:

$V_{xi} = 20\ m/s$, $V_{yi} = -15\ m/s$ $\quad a_x = 4\ m/s^2$, $a_y = 0$

$\therefore V_{xf} = V_{xi} + a_x . t \Longrightarrow V_{xf} = 20 + 4t$

$V_{yf} = V_{yf} + a_y . t \Longrightarrow V_{yf} = -15 + 0 = -15\ m/s$

$\therefore V_f = V_{xf}\hat{i} + V_{yf}\hat{j} \Longrightarrow V_f = [(20 + 4t)\hat{i} - 15\hat{j}]\ m/s$

b:- Calculate the velocity and speed of the Particle at ($t = 5.0\ s$)

Sol:

$V_f = [20 + 4t\ \hat{i} - 15\hat{j}]\ m/s = [40\hat{i} - 15\hat{j}]\ m/s$

or] at $t = 5s \Longrightarrow V_{xf} = 40\ m/s$ and $V_{yf} = -15\ m/s$

To determine in the angle (θ) that $\vec{V}$ makes with x-axis at ($t = 5\ s$) $\Rightarrow \tan\theta = \frac{V_{yf}}{V_{xf}} \Longrightarrow \theta = -21^\circ$

$\tan\theta = \frac{V_{yf}}{V_{xf}}$

$\tan\theta = \frac{-15}{40} = -0.375$

The speed is the magnitude of $\vec{V_f}$

$\therefore V_f = |\vec{V_f}| = \sqrt{V_{xf}^2 + V_{yf}^2}$

$= \sqrt{(40)^2 + (-15)^2} = 43\ m/s$

C :- Determine the x and y coordinates of the particle at any time (t) and the position at this time.

Sol:- $x_i = y_i = 0$ at $t = 0$

$$x_f = V_{xi}\, t + \frac{1}{2} a_x t^2 = [20t + 2t^2]\ m$$

$$y_f = V_{yi}\, t = [-15\ t]\ m$$

$$r_f = x_f \hat{i} + y_f \hat{j} = [(20t + 2t^2)(\hat{i}) - 15 t \hat{j}]\ m$$

At $t = 5.0\ s \Longrightarrow x = 150\ m$, $y = -75\ m$

$\therefore r_f = (150\hat{i} - 75\hat{j})\ m$

The magnitude of the displacement of a particle from the origin at ($t = 5.0\ s$) is the magnitude of ($\vec{r_f}$) at this time

$$r_f = |\vec{r_f}| = \sqrt{(150)^2 + (-75)^2} = 170\ m$$

Projectile motion

* The two assumptions are

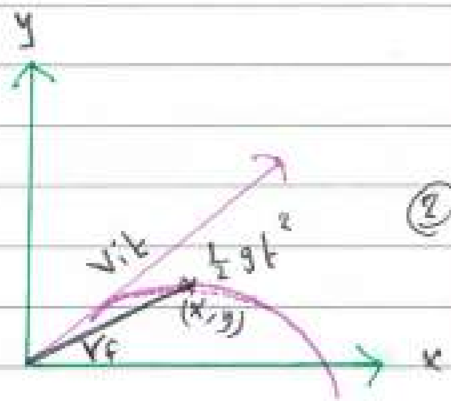
1- The free fall acceleration is constant $\langle g \rangle$ and direct down ward

2- The effect of air resistance is neglible

$x_f = (v_i \cos\theta_i)\, t$

$y_f = v_{yi} t + \frac{1}{2} a_y t^2$

$\quad = (v_i \sin\theta_i)\, t - \frac{1}{2} g t^2$

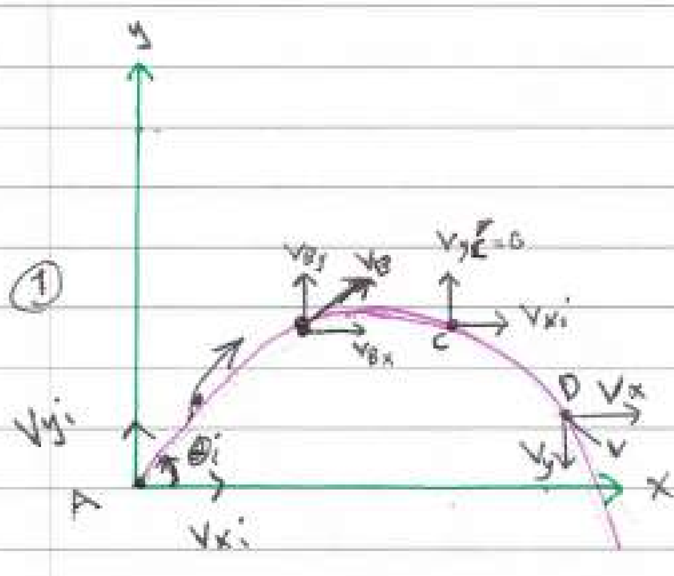


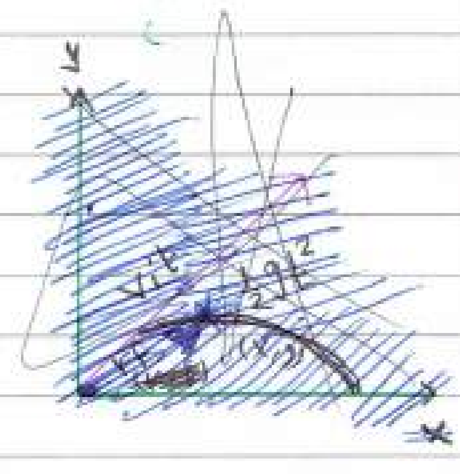
$$y = (\tan\theta_i)\, x - \left(\frac{g}{2 v_i^2 \cos^2\theta_i}\right) x^2$$

$\left[\text{for } 0 < \theta < \frac{\pi}{2}\right]$

$$\vec{r_f} = \vec{r_i} + \vec{v_i t} + \frac{1}{2} g t^2$$

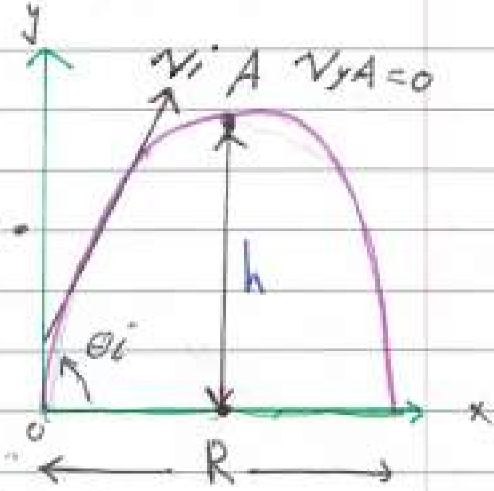
↳ Position vector of the Projectile





Projectile motion is a special case of two dimensional motion with constant acceleration ($a_y = 0$) in the (y) direction and with zero acceleration in the (x)- direction

Horizontal ~~rage~~ range and maximum high of a Projectile



Vi: initial Velocity

h: maximum height of the Projectile.
R: horizontal range.

At Point (A), the Particle has coordinate $(\frac{R}{2}, h)$ → (Peak Point)

$$t_A = \frac{v_i \sin\theta_i}{g}$$: The time with the Projectile reaches the Peak

$$h = \frac{V_i^2 \sin^2\theta_i}{2g}$$

$$R = \frac{V_i^2 \sin 2\theta_i}{g}$$

* (The maximum value of (R) is) $R_{max} = \frac{V_i^2}{2g}$

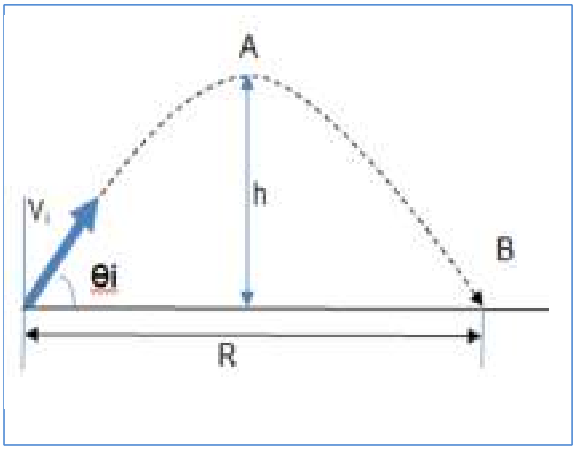
at $\sin 2\theta = 1$
or] $2\theta_i = 90° = \theta_i = 45°$

Example: A long-jumper leaves the ground at an angel of (20^0) above the horizontal and at speed of 11.0 m/s.

a) How far does the jump in the horizontal direction?

Solution: $X_f = X_B = (V_i \cos \Theta_i)\, t_B = (11\ m/s)\,(\cos 20^0)\, t_B$

$V_{yA} = 0$ at the top of the jump



$V_{yf} = V_{yA} = (V_i \sin \Theta_i) - g * t_A$

($\mathbf{a}_y = -g$)

$0 = (11\ m/s) \sin 20^0 - 9.8\ m/s^2 * t_A$

$t_A = 0.384$ S

$t_B = 2*t_A = 2*0.384 = 0.768$ S.

$X_F = X_B = (11\ m/s)(\cos 20^0)(0.768\ S) = 7.94$ m

b) What is the maximum height reached

Solution: $\mathbf{y}_{max} = \mathbf{y}_A = (V_i \sin \Theta_i)\, t_A - (1/2)\, g * t^2_A$

$= (11\ m/s)(\sin 20^0)(0.384\ s) - (1/2)(9.8\ m/s^2)(0.384)^2$

$= 0.722$ m

❖ Uniform circular motion

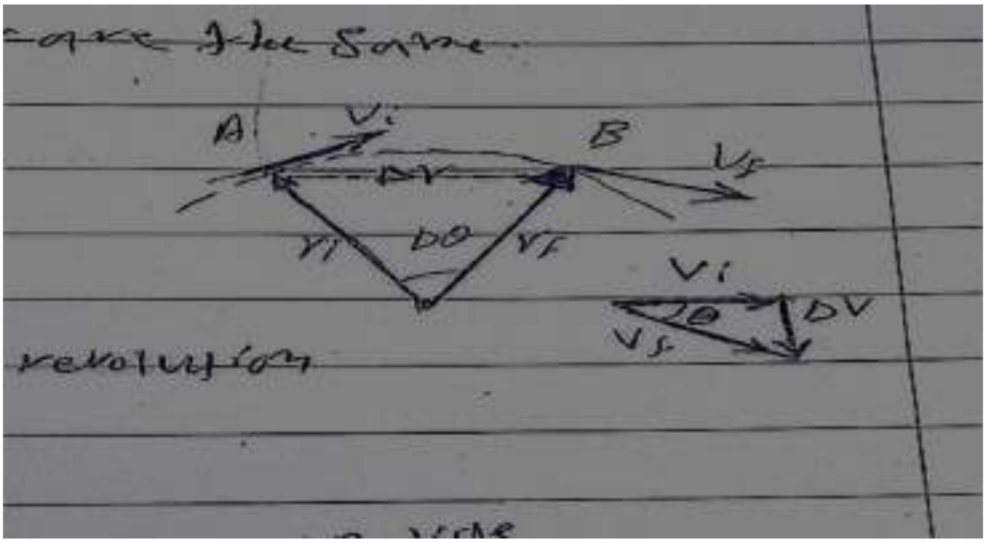
It is a motion of an object in a circular path with constant speed (It still has an acceleration).

❖ The acceleration is called (centripetal acceleration)

$a_c = \frac{v^2}{r}$ (perpendicular to the path)

$r = circle\ radious$

$v = v_i = v_f$ (the magnitude of v is the same)



$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

$$r = r_i = r_f$$

The time required for one complete revolution

$$T = \frac{2\,\pi\,r}{v}$$

❖ <u>**Tangent and radial acceleration**</u>

- The velocity change in direction and magnitude.

a_t causes the change in the speed

a_t tangential component perpendicular to $\overrightarrow{a_r}$

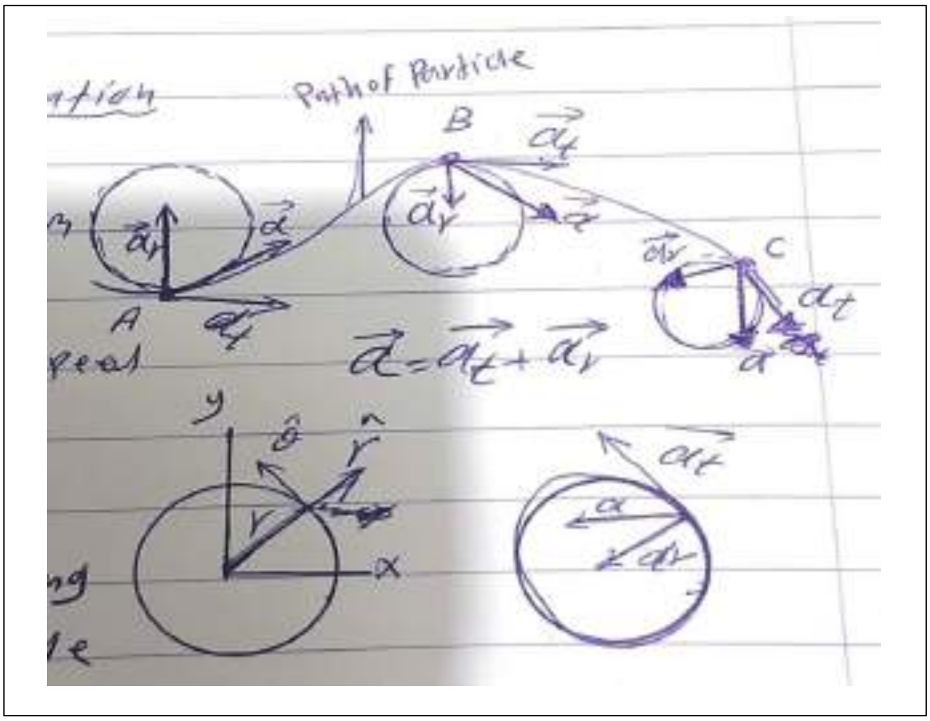
$\overrightarrow{a_r}$ = radial component along the radius of model circle.

$$\overrightarrow{a_t} = \frac{d_{/v/}}{d_t}$$

$\overrightarrow{a_r} = -a_c = \frac{-v^2}{r}$ (the change in direction of velocity).

r = radius of curvature

The magnitude of (a) is $a = \sqrt{a_r^2 + a_t^2}$



The direction of ($\mathbf{a}_t$) either the same direction of $\vec{v}$ (if v is increasing or opposite (if v is decreasing).

- ❖ Uniform circular motion where v is constant

$$a_t = 0 \; and \; a = a_r$$

- ❖ If the direction of v is not changing ⇒

$$a_r = 0 \;\; \& \;\; a_t \neq 0$$

- ❖ $\vec{a} = \overrightarrow{a_t} + \overrightarrow{a_r} = \frac{d/v/}{d_t}\widehat{\Theta} - \frac{v^2}{r}\hat{r}$ (In term of unit vector).

  $\hat{v} =$ unit vector lying around of the radius vector and directed radially outward from the center.

  $\widehat{\Theta} =$ unit rector tangent to the circle in the direction of increasing Θ.

❖ **Relative velocity and relative acceleration**

The example of this concept is the motion of a package dropped from an airplane flying with a constant velocity.

The two vectors are related to each other through the expression

$\vec{r} = \vec{\bar{r}} + \overrightarrow{v_o}t \quad or \quad \bar{r} = \vec{r} - \overrightarrow{v_o}t$ if v_o is constant.

$$\frac{d_{\bar{r}}}{d_t} = \frac{d_r}{d_t} - v_o \Rightarrow \vec{\bar{v}} = \vec{v} - \overrightarrow{v_o}$$

$\vec{\bar{v}} =$ Velocity of practical observed in the $\bar{s}$ frame,

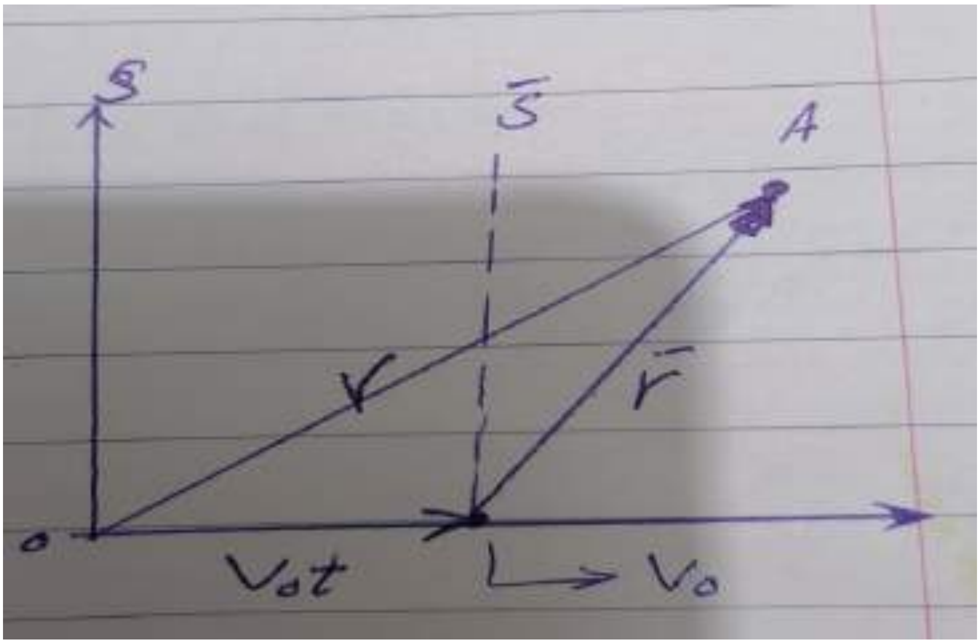
$\vec{v} =$ Velocity of practical observed in the s frame

These two equations are called (Galilean transformation equations).

$\frac{d_{\bar{v}}}{d_t} = \frac{d_v}{d_t} - \frac{dv_o}{d_t} \Rightarrow \frac{dv_o}{d_t} = 0$ (because v_o is constant)

v_o is constant and $\bar{a} = \frac{d_{\bar{v}}}{d_t}$, $a = \frac{d_v}{d_t}$

We conclude: $\vec{\bar{a}} = \vec{a}$



Example: A stone is throw from the top of a building upward at an angle of (30.0°) to the horizontal with an initial speed of (20.0 m/s) and the height of the building is (45m).

a) How long does the stone take to reach the ground?

Solution:

$v_{xi} = v_i c_o s\Theta i = (20.0m/s)(cos 30.0^{\circ}) = 17.3m/s$

$v_{yi} = v_i s_{in}\Theta i = (20.0m/s)(s_{in}30.0^{\circ}) = 10.0m/s$

To find (t) we use $y_f = y_i + v_{yi} + \frac{1}{2}a_y t^2$

$$-45.0m = (10.0m/s)t - \frac{1}{2}(9.80m/s^2)t^2 \Rightarrow t = 4.22s$$

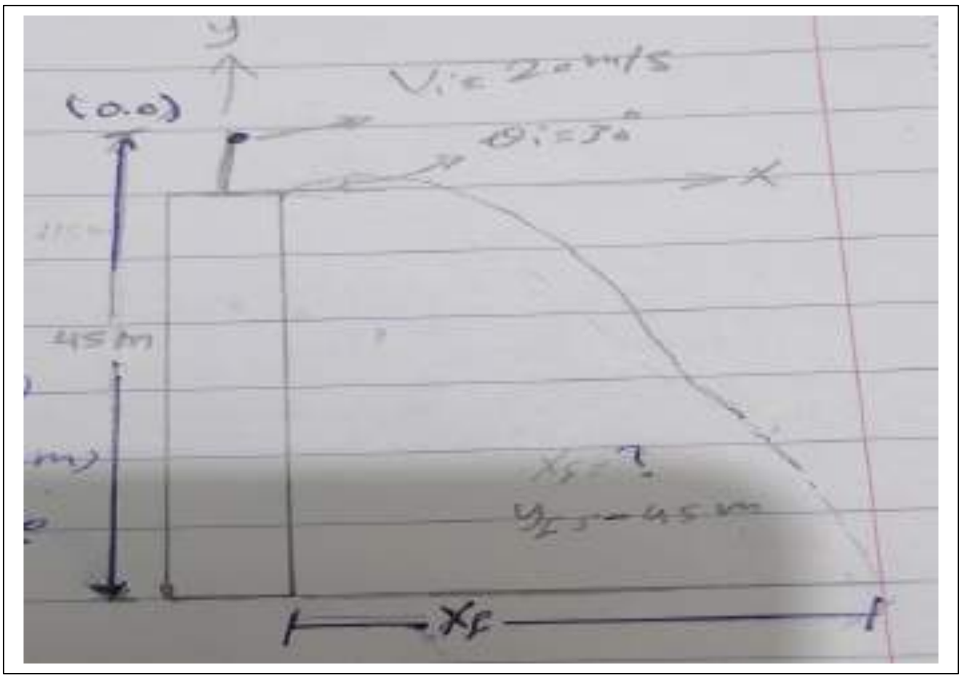
b) What is the speed of the stone just before it strikes the ground?

Solu: $v_{yf} = 10\, m/s - (9.80m/s^2)(4.22s) = -31.4m/s$

Because $v_{xf} = v_{xi} = 17.3m/s$,

The required speed is

$$v_f = \sqrt{v_{xf^2} + v_{yf^2}} = \sqrt{(17.3)^2 + (31.4)^2} = 35.9m/s$$



Examples: a Sky-jumper leaves the sky track moving in the horizontal direction with speed of (25.0m/s). The landing incline below him falls off with a slope of (35^0). where does he land on the incline?

Solution:

d = distance travelled along the incline.

$v_{xi} = 25.0$ m/s & $v_{yi} = 0$

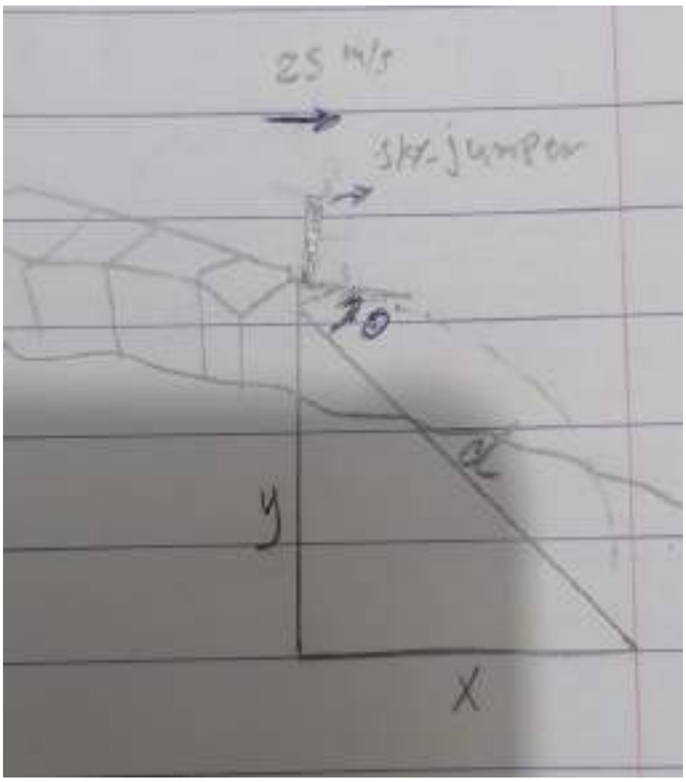
$x_f = v_{xi}t = (25.0m/s)t$

$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = -\frac{1}{2}(9.8\,m/s^2)t^2$

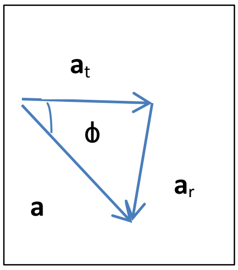
$x_f = dc_os35^0$ & $y_f = \text{d} \sin 35^\circ$

$d * \cos 35 = \left(25\frac{m}{s}\right) t \;\; and \;\; -d \sin 35 = -\frac{1}{2}(9.80 m/s^2)\, t^2$

$\Rightarrow d = 109m, \quad x_f = 89.3m, \quad y_f = -62.5m$



Example: A car exhibits a constant acceleration of (0.3 m/s^2) parallel to the road way. The car passes over a rise in the road such that the top of the rise is shaped like a circle of radius (500m). at the moment the car is at the top of the rise, it's velocity vector is horizontal and has a magnitude of (6.0 m/s). what is the direction of the total acceleration vector for the car at this instant?



Solution: $\mathbf{a_r} = \mathbf{-v^2/r}$ = -(0.6 m/s) / 500m = -0.072 m/s^2

$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$

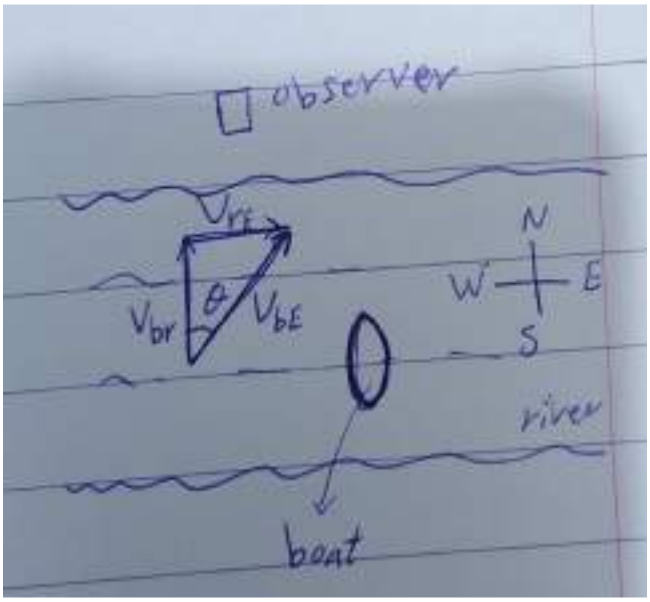
The magnitude of a is :

$\vec{\mathbf{a}} = (\vec{\mathbf{a}}_r^2 + \vec{\mathbf{a}}_t^2)^{1/2} = [(-0.072)^2 + (0.3)^2]^{1/2} = 0.309\ m/s^2$

ϕ = the angle between (**a**) and horizontal

$\phi = \tan^{-1}(\mathbf{a_r/a_t}) = \tan^{-1}(-0.072/0.3) = -13.5^{\mathbf{0}}$

Example: A boat heading due to the north crosses a wide river with a speed of (10.0 km/hr.) relative to the water. The water in the river has a uniform speed of (5.00 km/hr.) due east relative to the earth. Determine the velocity of the boat relative to an observer standing on other bank?



Solution:

$\mathbf{V}_{br}$ = the velocity of the boat relative to the river

$\mathbf{V}_{rE}$ = the velocity of the river relative to earth

$\mathbf{V}_{bE}$ = the velocity of the boat relative to earth

$$\mathbf{V}_{bE} = \mathbf{V}_{br} + \mathbf{V}_{rE}$$

$$\mathbf{V}_{bE} = [(\mathbf{V}_{br})^2 + (\mathbf{V}_{rE})^2]^{1/2} = (10^2 + 5^2)^{1/2} = 11.2 \text{ km/hr}$$

The direction of $\mathbf{V}_{bE} = \Theta = \tan^{-1}(\mathbf{V}_{rE} / \mathbf{V}_{br})$

$\Theta = 26.6^0$ (the boat is moving at a speed of 11.2 km/hr

in the direction of 26.60 east of north relative to earth)

Chapter Five

The Laws of Motions

When several forces act simultaneously on an object, it's accelerates if the net force acting on it is not equal to zero.

If the net force exerted on an object is zero, the acceleration of an object is zero and it’s velocity remains constant.

When the velocity is constant (or at rest), the object said to be in equilibrium.

The force may be: 1- Contact forces   2- Field forces

Because force is a vector we use the symbol $\vec{F}$ and the rules of vector addition to obtain the net force on an object.

Newton's First Law and Inertial Frames (Law of Inertia)

Newton's first law sometimes called the (law of inertia). This law can be stated as (if an object does not interact with other objects, it's possible to identify a reference frame in which the object has zero acceleration).

Another statement of newton's first law is that an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is with a constant speed on a straight line).

Inertia, define as the tendency of an object to resist any attempt to change it's velocity.

Mass

Mass defined as: the resistance an object exhibits to change in it's velocity.
Weight: the magnitude of the gravitational force exerted on the object, various with location.
For example: a person who weight 180 Ib on the Earth weight only 30 Ib on the Moon, on the other hand the mass of an object is the same everywhere.

- When a force acting on an object of mass ($\mathbf{m_1}$) produces an acceleration ($\mathbf{a_1}$) and the same force acting on another object of mass ($\mathbf{m_2}$) produces an acceleration ($\mathbf{a_2}$). The ratio of the two masses is defined as the inverse ratio of the magnitudes of the accelerations produced by the force:

$$\mathbf{m_1/m_2 = a_2/a_1}$$

Newton's Second Law

The magnitude of the acceleration of an object is inversely proportional to it's mass.

$$\mathbf{\Sigma \vec{F} = m \cdot \vec{a}}$$

Compound form

$$\mathbf{\Sigma \vec{F}_x = m \cdot \vec{a}_x \quad \& \quad \Sigma \vec{F}_y = m \cdot \vec{a}_y \quad \& \quad \Sigma \vec{F}_z = m \cdot \vec{a}_z}$$

The unit of force in SI system is (**newton**)
Newton defined as, the force that when acting on an object of mass (1kg) produced an acceleration of ($1 m/s^2$). (***1N = 1kg.m/s²***).

In the US customary system, the unit of force is (pound)
Pound is defined as: the force that, when acting on an object of mass (1slug) produces an acceleration of ($1 ft/s^2$). (***1Ib = 1slug.ft/s²***) and (***1N = 0.25 Ib***).

The Gravitational Force and Weight:

Gravitational force ($\mathbf{F_g}$): is the attractive force exerted by the Earth on an object and it's directed toward the center of the Earth. It's magnitude called (the weight) of the object.

Applying newton's second law $\mathbf{\Sigma \vec{F} = m \vec{a}}$ to a freely falling object of mass (**m**) with $\mathbf{\vec{a} = g}$ and $\mathbf{\Sigma \vec{F} = m\, g}$ we obtain

$$\mathbf{\vec{F}_g = m\, \vec{g}}$$

Where (m) called gravitational mass

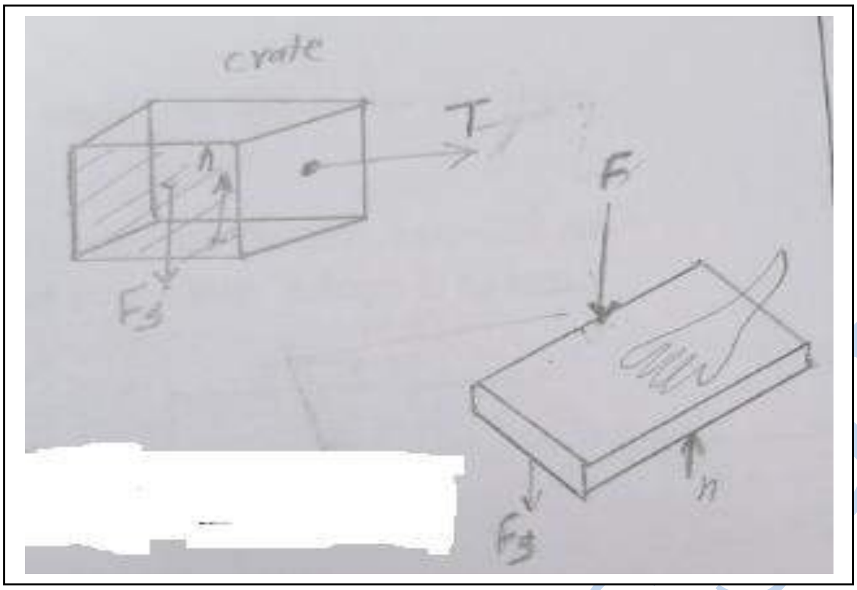
Note: the kilogram is the unit of mass not unit of weight.

Newton's third law:

If two objects interact, the force $\mathbf{F_{12}}$ exerted by object (1) on object (2), is equal in magnitude and opposite in direction to the force $\mathbf{F_{21}}$ exerted by object (2) on object (1) ➡

$\mathbf{F_{12}}$ (action force) **= -** $\mathbf{F_{21}}$ (reaction force)

Objects Experiencing A Net Force:



$\Sigma Fx = m\ a_x = T$ or $a_x = T/m$

The acceleration occurs in the **y** direction

$\Sigma f_y = m\ a_y$ with $a_y = 0$ $\Longrightarrow \Sigma f_y = 0$

$n + (-F_g) = 0 \Longrightarrow n = F_g$

For constant acceleration

$V_{xf} = V_{xi} + a_x\ t$

$V_{xf} = V_{xi} + (T/m)\ t$

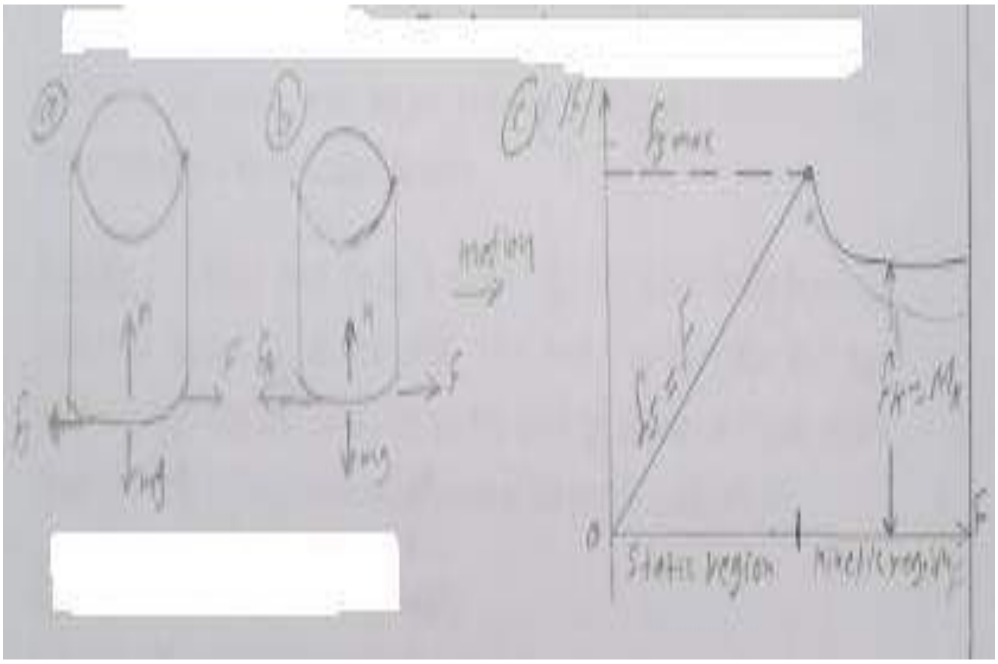
And $X_f = X_i + V_{xi}\ t + (1/2).(T/m).t^2$

Note: **n** = the normal force is not always equal to the magnitude of (F_g)

For example suppose a book is lying on a table and you push down on the book with a force **F**, the book is at rest and therefore not accelerating

$n - F_g - F = 0$ or $n = F_g + F$ Note : ($n > F_g$)

Force of Friction:



When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. This resistance called (force of fraction).

Consider you try to drag a trash can filled with yard clippings across the surface of your concrete patio **figure (a)** below it's a real surface not an idealized (friction less surface), If we apply an external horizontal force (**F**) to the trash can acting to the right:

1- the trash can be remain stationary if (**F**) is small. The force that counteracts force (**F**) and keep the trash can from moving acts to the left and is called the force of friction ($\mathbf{f_s}$), as long as the trash can is not moving ($\mathbf{f_s = F}$).

Note: If **(F)** increased, $\mathbf{f_s}$ also increases.

2- If we increase the magnitude of (**F**) as in **figure (b)**, the trash can eventually slips when ($\mathbf{f_s}$) reaches its max value ($\mathbf{f_{s\ max}}$), as in **figure c**.

3- The trash can moves if (**F** > $\mathbf{f_{s\ max}}$), when the trash is in motion, the friction force is called (*force of kinetic friction* $\mathbf{f_k}$). The net force ($\mathbf{F\text{-}f_k}$) is in the x-direction and produces acceleration to the right.

4- If **F=** $\mathbf{f_s}$, the acceleration is zero and the trash can moves to the right with constant speed.

5- If the force is removed, the friction force acting to the left, provides an acceleration of the trash in the x-direction and eventually brings it to rest.

Experimental Observations:

1- The magnitude of the force of static friction between any two surfaces in contact can have the values: ($\mathbf{f_s < \mu_s.n}$) where
μ_s= coefficient of friction (dimensionless constant)
n= the normal force exerted by one surface on the other
When the surfaces are on the average of slipping, $\mathbf{f_s = f_{max} = \mu_s.n}$, (impending motion).

2- The magnitude of the kinetic friction force acting between two surfaces is: $\mathbf{f_k = \mu_k.n}$
μ_k = coefficient of kinetic friction, it is vary with speed.

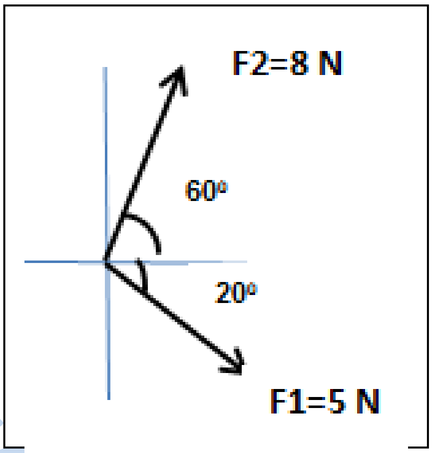
3- The values of μ_s and $\mathbf{m_k}$ are depend on the nature of the surfaces, but generally: ($\mu_k < \mu_s$).

4- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction), or the impending motion (static friction) of the object relative to the surface.

5- The coefficient of friction is independent of the area of contact between the surfaces.

6- The equations ($\mathbf{f_s \le \mu_s.n}$) and ($\mathbf{f_k \le \mu_k.n}$) are not vector equations, they are relationships between the magnitude of the friction and normal forces. Because the friction and normal forces are perpendicular to each other, the vectors cannot be related by a multiplicative constant.

Example1: A hockey puck having a mass of (0.3 kg) slides on the horizontal frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck as shown in Figure below. Determine both the magnitude and direction of the puck's acceleration?



$\mathbf{\Sigma F_x = f_{x1} + f_{x2} = f_1 \cos(-20°) + f_2 \cos(60)}$

$(5.0\ N)(0.940) + (8.0\ N)(0.5) = 8.7\ N$

$\mathbf{\Sigma F_y = f_{y1} + f_{y2} = f_1 \sin(-20°) + f_2 \sin(60)}$

$(5.0\ N)(-\ 0.342) + (8.0\ N)(0.866) = 5.2\ N$

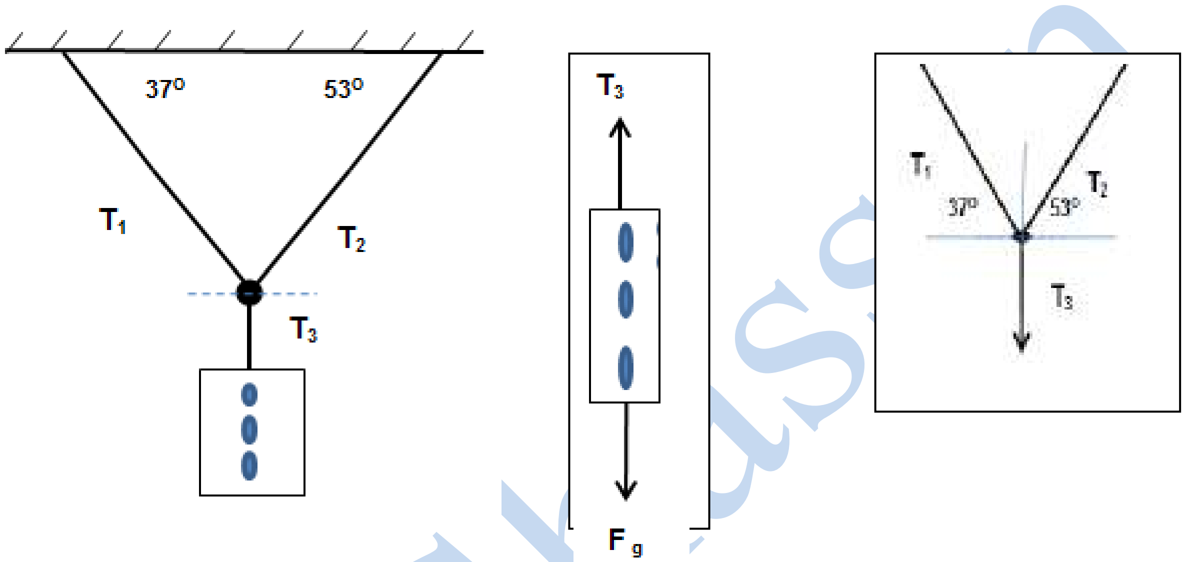
$\mathbf{a_x = (\Sigma f_x) / (m)} = 8.7\ N / 0.3\ kg = 29\ m/s^2$

$\mathbf{a_y = (\Sigma fy)/(m)} = 5.2\ N / 0.3\ kg = 17\ m/s^2$

$\mathbf{a = \sqrt{a^2{}_X + a^2{}_y}} = \sqrt{(29^2) + (17^2)} = 34\ m/s^2$

$\mathbf{\theta = \tan^{-1}(a_y/a_x)} = \tan^{-1}(17/29) = 30°$

Example 2: A traffic light weight (122 N) hangs from a cable tied to two other cables fastened to a support as in Figure-6. The upper cables are not as strong as the vertical cable and will break if the tension in them exceeds (100 N). Will the traffic light remains hanging in this situation, or will one of the cables break?



$\Sigma f_y = 0$

$T_3 - F_g = 0$

$T_3 = F_g = 122$ **N**

From the free body diagram for the knot

$\Sigma F_x = T_{2X} - T_{1X}$

$= T_2 \cos(53) - T_1 \cos(37) = 0$ - - - - - (1)

$\Sigma F_y = T_{2y} + T_{1y} + (-122) = 0$

$= T_2 \sin(53) + T_1 \sin(37) - 122 = 0$ - - - - - (2)

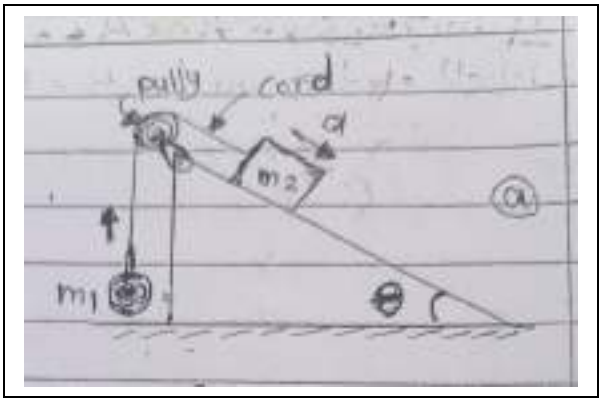
From equation (1)

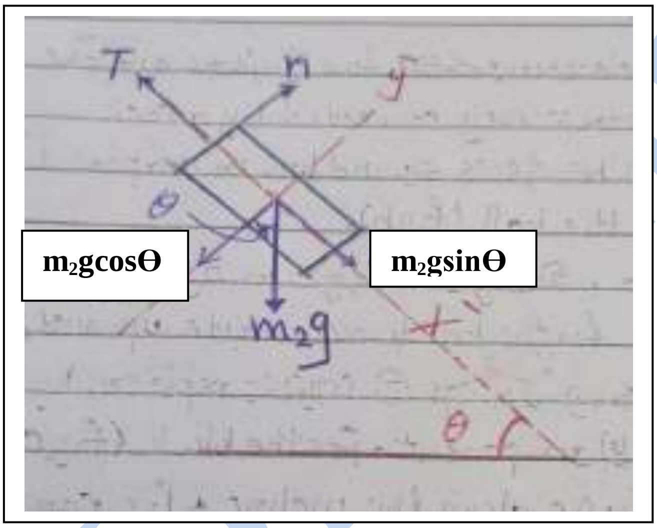
$T_2 = 1.33\, T_1$ substitute in equation (2) to obtain

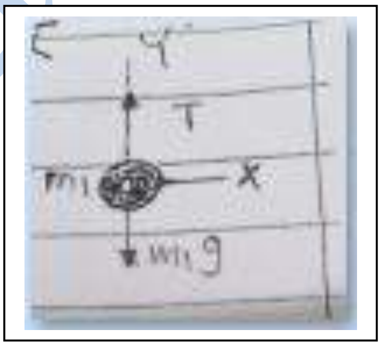
$T_1 = 73.4$ N & $T_2 = 97.4$ N

Both of these values are less than (100 N), so the cables will not break

Example 3: A ball of mass (m_1) and a block of mass (m_2) are attached by a light weight cord that passes over a frictionless pulley negligible mass (Figure-7). Find the magnitude of the acceleration of the two objects and the tension in the cord?







SOL:

Applying newton's second law in component form to the ball

$$\Sigma F_x = 0 \text{ - - - - - } (1)$$

$$\Sigma F_y = T - m_1.g = m_1.\, a_y = m_1.a$$

$$T - m_1.g = m_1.a \text{ - - - - - } (2)$$

For the block

$\Sigma F_x = m_2.g.\sin\Theta - T$

But

$\Sigma F_x = m_2\, a_x = m_2.a$

Then

$[m_2.g.\sin\Theta - T = m_2.a]$ **- - - - - (3)**

$\Sigma F_y = n - m_2.g.\cos\Theta = 0$

From equation (2)

$T = m_1.g + m_1.a$ sub in eq. (3)

$a = (m_2.g.\sin\Theta - m_1.g) / (m_1 + m_2)$ **- - - - - (5)**

substitute in eq. (2)

$T = (m_1.m_2.g)(\sin\Theta+1) / (m_1 + m_2)$ **- - - - - (6)**

IF $m_2.\sin\Theta > m_1$, the block accelerates down the incline

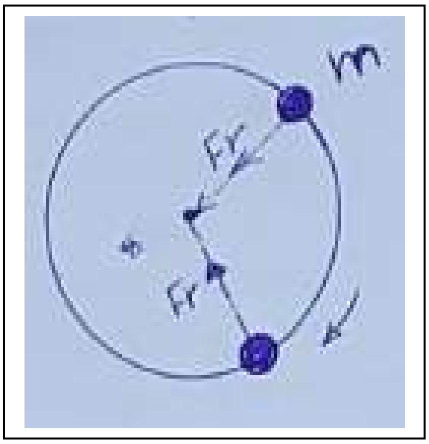
IF $m_2.\sin\Theta < m_1$, the acceleration is up the incline for the block and down for the ball.

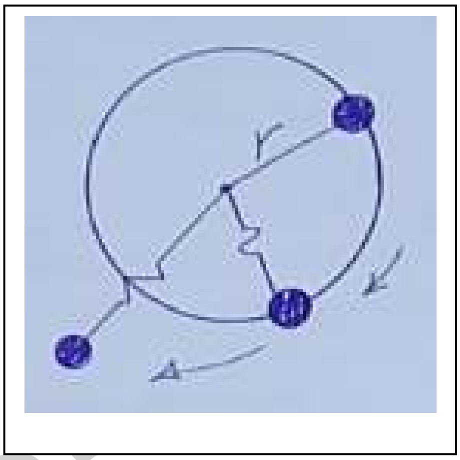
Chapter 6

Circular Motion and Other Applications of Newton's laws

Newton's Second Law Applied to Uniform Circular Motion

$$\mathbf{a_c = v^2/r}$$



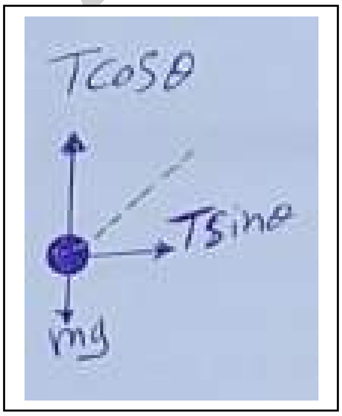


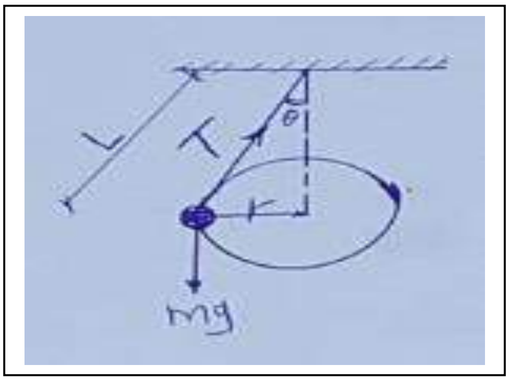
Where ($\mathbf{a_c}$): the centripetal acceleration of a particle moving with uniform speed
Consider figure (1), when we apply the newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

$$\mathbf{\Sigma F = m.a_c = m(v^2/r)}$$

* This force acts toward the center of the circular path and causes a change in the direction of a velocity vector.
* The force causing centripetal acceleration is called a (centripetal force).

Example 1: A small object of mass (m) is suspended from a string of length (L). The object revolves with constant speed (v) in a horizontal circle of radius (r). The system is known as a (conical pendulum). Find the expression of (v)?





Solution:

Because the object does not accelerate in the vertical direction, $\Sigma F_y = 0$

$T\cos\theta - m.g = 0 \Longrightarrow T\cos\theta = m.g$ .... (1)

$\Sigma F_x = ma_c \Longrightarrow T\sin\theta = m(v^2/r)$ .... (2)

Solve eq (1) and (2) $\Longrightarrow \tan\theta = v^2/g.r$ **or** $v = \sqrt{r\, g \tan\theta}$

But $r = L \sin\theta \Longrightarrow v = \sqrt{Lg \sin\theta \tan\theta}$

So the speed is independent of the mass of the object.

<u>Example 2</u>: A ball of mass (0.5 kg) is attached to the end of the cord (L = 1.5 m) long. The ball is whirled in a horizontal circle as shown in fig-(3). If the cord can withstand a maximum tension of (50 N). **(a)** What is the max speed at which the ball can be whirled before the cord breaks? Assume that the string remains horizontal during motion. **(b)** Suppose that the ball is whirled in a circle of larger radius at the same speed (v), is the cord more likely to break or less likely?

Solution:

(a)

$T = m(v^2/r)$ .... (1) and $v = \sqrt{T.r/m}$

This equation shows that (v) increases with (T) and decreases with larger (m).

$v_{max} = \sqrt{T_{max}(r)/m} = \sqrt{(50\ N)(1.5\ m)/(0.5\ kg)} = 12.2$ m/s

(b) The larger radius means that the change in direction of the velocity vector will be smaller for a given time interval. Thus the acceleration is smaller and the required force from the string is small. As a result, the string is less likely to break when the ball travels in the in a circle of larger radius.

$T_1 = mv^2 / r_1$ **or** $T_2 = mv^2 / r_2$

$T_2 / T_1 = (mv^2 / r_2) / (mv^2 / r_1) \Longrightarrow T_2 / T_1 = r_1 / r_2$

If we choose $r_2 > r_1$ we see that $T_2 < T_1$

* Less tension required to whirl the ball in the larger circle and the string is less likely to break.

Example 3: A civil engineer wish to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction around the curve without skidding. In other words a car moving at the designated speed can negotiate the curve even when the road is covered with ice. The roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be (13.4 m/s) and radius of the curve is (50 m). At what angle should the curve be banked?

Solution: Because the ramp is to be designed so that the force of static friction is zero. Only the component (**$n_x = n \sin\theta$**) causes the centripetal acceleration.

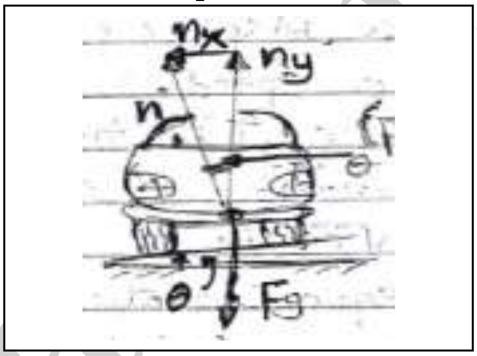
$\Sigma F_r = n \sin\theta = m (v^2/r)$ .... (1)

$\Sigma F_y = 0 \Longrightarrow n \cos\theta = m g$ .... (2)

From equations (1) and (2)

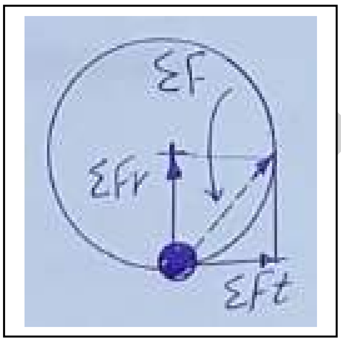
$\tan\theta = v^2 / r.g$ .... (3)

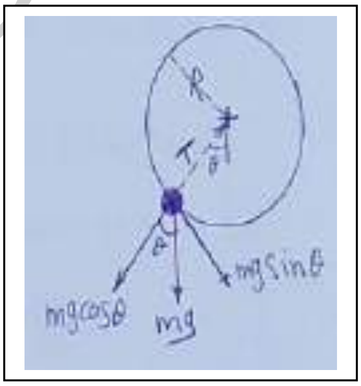
$\theta = \tan^{-1}[(13.4 / (50*9.8)] = 20.1°$

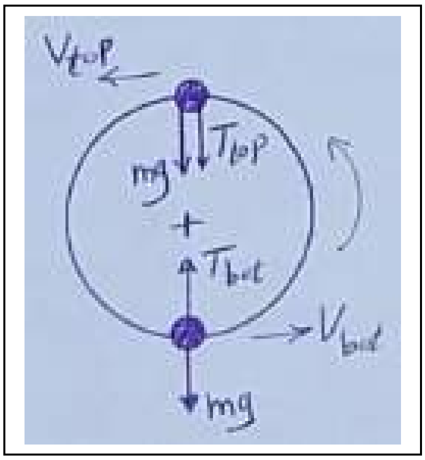


* **Note**: If a car round the curve at a speed less than (13.4 m/s), friction is needed to keep it from sliding down the bank (to the left). At speed greater than (13.4 m/s) friction required to keep it from sliding up the bank (to the right).

Non Uniform Circular Motion:







* In addition to radial component of acceleration (**a_r**), there is a tangential component (**a_t**) having a magnitude of (**dv/dt**).

* The total force exerted on the particle is:

$\Sigma F = \Sigma F_r + \Sigma F_t$

* **a_t:** represent the change in the speed of particle with time.

<u>Example 4</u>: A small sphere of mass (m) is attached to the end of a cord of length (R) and set into the motion in a vertical circle about a fixed point (o). (a): Determine the tension in the cord at any instant when the speed of the sphere is (v) and the cord makes an angle (ɵ) with the vertical? (b): If we set the ball in motion of a slower speed, what would the ball have as it passes over the top of the circle if the tension in the goes to zero instantaneously at this point?

Solution:

(a): $\Sigma F_t = m\, g \sin\theta = m\, a_t \Longrightarrow a_t = g \sin\theta$

$\Sigma F_r = T - m\, g \cos\theta = m(v^2/R)$ or $T = m\,(v^2/R + g\cos\theta)$

(b): At the top of the path where (ɵ = 180°), we have (cos 180°) = -1, so:

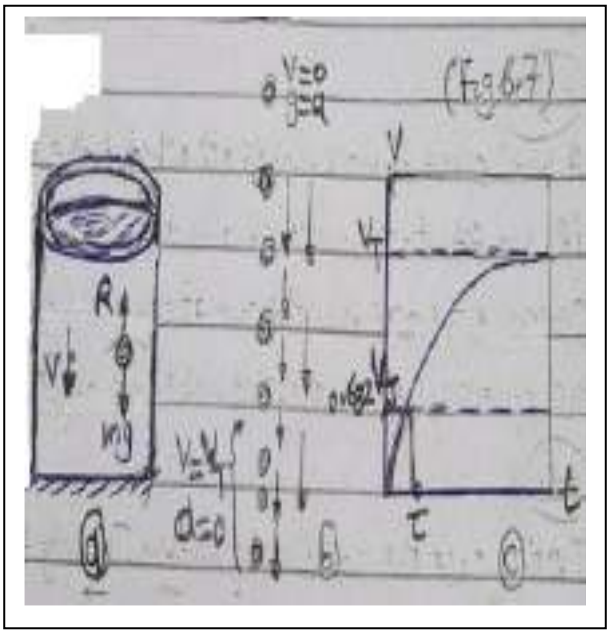
$T_{top} = m\,[(v^2_{top} / R) - g]$

Let us set $T_{top} = 0 \Longrightarrow 0 = m\,[(v^2_{top} / R) - g]$

$V_{top} = \sqrt{g\,R}$

<u>Motion in the Presence of Resistive Forces:</u>

Any medium exerts a resistive force (R) on the object moving through it. The magnitude of (R) depends on factors such as the speed of the object. The direction of (R) is always opposite the direction of motion of the object relative to the medium.



<u>Resistive force proportion to object speed:</u>

$\vec{R} = -b\,\vec{v}$ where

v = velocity of object

b = constant

(depend on properties of medium and the shape and dimensions of the object).

For the Fig,Small sphere released from rest.

$\Sigma f_y = m.g - b.v \Longrightarrow m.g - b.v = m.a$

$m.g - b.v = m(dv/dt)$

$dv/dt = g - (bv/m)$

where dv/dt = acceleration

* When initially v = 0, the magnitude of resistive is also zero $\Longrightarrow dv/dt = g$ or (a = g).

* When (**R**) approaches the sphere's weight, the acceleration approaches zero. In this situation, the speed of the sphere approaches it's (terminal speed,$\mathbf{v}_T$).

We can obtain the terminal speed by setting $a = dv/dt = 0$ ⟹ $m.g - b.V_T = 0$

$$V_T = m.g/b$$

* The expression of of (v)with (v=0) at (t=0) is:

$V = m.g/b\ (1-e^{-bt/m})$ ⟹ $= V_T\ (1- e^{-t/\tau})$.

- e = 2.71828

The time constant (**$\tau = m/b$**), is the time at which the sphere released from rest reaches (63.2%) of its terminal speed.

When (**$t=\tau$**), then **$V = 0.632V_T$**

Note: there is (a buoyant force) acting on the submerged object, this force is constant, and it's magnitude equal to weight of the displaced liquid. This force change the apparent weight of the sphere by a constant factor.

<u>Example 5</u>: A small sphere of mass (2.0 g) is released from rest in a large vessel filled with oil. The sphere reaches a terminal speed of (5.00 cm/s). Determine the time constant (**τ**) and the time at which the sphere reaches (90.0%) of it's terminal speed?

<u>Solution</u>:

$V_T = m.g / b$ ⟹ $b = mg / V_T = [(2.0\ g * 9.8\ cm/s^2)] / [5\ cm/s] = 392\ g/s$

$\tau = m / b = (2\ gr) / (392\ gm/s) = 5.1*10^{-3}\ s$

To find the time at which the sphere reaches a speed of (**$0.9V_T$**), we set ($v= 0.9v_T$)

$0.9\ v_T = v_T\ (1-e^{-t/\tau})$ ⟹ $e^{-t/\tau} = 0.1$ ⟹ $-t/\tau = \ln(0.1) = -0.2.3$

$t = -2.3* \tau = 2.3\ (5.1*10^{-3}\ s)$ ⟹ $= 11.7*10^{-3}\ s = 11.7\ s$

<u>Air Drag at High Speed</u>

The resistive force is

$R = 0.5\ D.\rho.A.v^2$ where

ρ = air density

A = cross sectional area of of the moving particle measured in plane perpendicular to it's velocity.

D = drag coefficient (**0.5** for spherical object and **2** for irregularly shaped).

* To analyze the motion of an object in free-fall

$\Sigma F = m.g$ (F_g, downward gravitational force) $= 0.5\ D.\rho.A.v^2$ (upward air resistive force)

But $\Sigma F = m.a$ ⟹ the object has a downward acceleration of magnitude

$a = g - (D.\rho.A/2m)\, v^2$

we can calculate the terminal speed by using the fact that, when the gravitational force is balanced by resistive force, the net force on the object is **zero** and therefore it's acceleration is zero. (**a=0**)

$g - (D.\ \rho.A/2m)\, v^2 = 0 \implies$ $\mathbf{V_T = \sqrt{(2m.g) / (D.\rho.A)}}$

Object	Mass (kg)	Cross Sectional Area (m^2)	V_T (m/s)
Sky Driver	75	0.7	60
Bass Ball (r =3.7 cm)	0.145	$4.2*10^{-3}$	43
Golf Ball (r =2.1 cm)	0.046	$1.4*10^{-3}$	44
Rain Drop (r =0.2 cm)	$3.4*10^{-5}$	$1.3*10^{-5}$	9

<u>Example 6</u>: A pitcher hurls a (0.145 kg) bassball past a batter at (4.02 m/s). Find the resistive force acting on the ball at this speed?

<u>Solution:</u>

$D = (2m.g) / (V^2_T.\rho.A) = (2*0.145kg*9.8m/s^2) / (43m/s^2*1.2kg/m^3*4.2*10^{-3}m^2)$

$D = 0.305$

$\mathbf{R = 0.5\ D.\rho.A.v^2} = 0.5*0.305*1.2kg/m^3*4.2*10^{-3}m^2*40.2m/s^2$

R= 1.2 N