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Electric physics II Assist. Lac. Yasameen Kamil 2020-2021

## Electric physics II

 Electric chargeBy
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2020-2021

## 1.The electric charge

- Empirically it was known since ancient times that if amber is rubbed on fur, it acquires the property of attracting light objects such as feathers.
- This phenomenon was attributed to a new property of matter called "electric charge".
- electron is the Greek name for amber because of its electrostatic properties and whilst analyzing elementary charge for the first time.
- More experiments show that they are two distinct type of electric charge: positive (color code: red), and negative (color code: black). The names "positive" and "negative" were given by Benjamin Franklin.


## The electric charge on

(1) a glass rod rubbed with silk is positive.
(2) an amber (plastic) rod rubbed with fur is negative.
((Note))
Rubber rubbed with cat fur: rubber becomes negative, while the fur becomes positive.

Amber rod (-)
Plastic rod (-)
Rubber (-)
Glass rod (+)


(a) Uncharged amber rod exerts no force on papers
(b) Amber rod is rubbed against a dry cloth (a fur)
(c) Amber rod becomes charged and attracts the papers.

Further experiments on charged objects showed that as :

1. Charges of the same type (either both positive or both negative) repel each other as in fig a.
2. Charges of opposite type on the other hand attract each other as in fig b.
3. The force direction allows us to determine the sign of an unknown electric charge


## 2.Charge is quantized

- The experiments strongly suggested that the electric charge, $q$, is said to be quantized. $q$ is the standard symbol used for charge as a variable. Electric charge exists as discrete packets The SI Unit of charge is the coulomb (c).
- The charge of the electron:
- $q=n e$
- where $n$ is an integer(no. of electron or proton), and $e$ is the fundamental unit of charge.
- e $=1.602176487 \times 10^{-19} \mathrm{C}$
- For electron $q=-e$
- For proton $\mathrm{q}=+\mathrm{e}$
- For neutron $\mathrm{q}=0$
- How many electrons are there to form 1 C ?

$$
\mathrm{n}=\frac{q}{e}=\frac{1 c}{e}=\frac{1}{1.602 * 10^{-19}}=6.24 * 10^{18}
$$

- $1 \mu \mathrm{C}=10^{-6} \mathrm{C}$ ( $\mu$ : micro)
- $1 \mathrm{nC}=10^{-9} \mathrm{C}$ ( n : nano)
- $1 \mathrm{pC}=10^{-12} \mathrm{C}$ (p: pico)
- $1 \mathrm{fC}=10^{-15} \mathrm{C}$ (f: femto)
- $1 \mathrm{aC}=10^{-18} \mathrm{C}$ (a: atto)
- ((Note)) Relation between 1 C (SI units) and 1 esu (cgs gaussian unit of charge, electrostatic unit)

We consider a force between two charges with $q=$ 1 c , The separation between two charges is $r=1 \mathrm{~m}$.

$$
F_{s t}=\frac{q^{2}}{4 \pi \varepsilon_{0} r^{2}}=\frac{(1 C)^{2}}{4 \pi \delta_{0}(1 m)^{2}} \quad[\mathrm{~N}]
$$

In egs units, the corresponding force between $A$ (esu) $[=1 C]$ is

$$
F_{\mathrm{se}}=\frac{q^{2}}{r^{2}}=\frac{(A \mathrm{esu})^{2}}{(100 \mathrm{~cm})^{2}} \quad[\text { dyne }]
$$

Note that $F_{s y}=F_{c o v}$, and $1 \mathrm{~N}=10^{5}$ dyne. Then we have

$$
\frac{1}{4 \pi s_{0}} \times 10^{5}=\frac{A^{2}}{10^{4}}, \quad \text { or } \quad A=\sqrt{\frac{1}{4 \pi \delta_{0}} \times 10^{9}}=2.99792 \times 10^{9}
$$

So we have

$$
1 \mathrm{C}=2.99792 \times 10^{9} \mathrm{esu}
$$

The charge of electron is


## 3. Charge is conserved

- Consider a glass rod and a piece of silk cloth (both uncharged) shown in the upper figure.
- If we rub the glass rod with the silk cloth we know that positive charge appears on the rod (see the figure).
- At the same time an equal amount of negative charge appears on the silk cloth

- so that the net rod-cloth charge is actually zero. This suggests that rubbing does not create charge but only transfers it from one body to the other.
- Charge conservation can be summarized as follows: In any process the charge at the beginning equals the charge at the end of the process.

- The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never change.

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## Some concepts

Due to the movement of electrons, charge is transferred from one object to another.
Positive ion: the atom that loses an electron is said to be a positive ion;
Negative ion: the atom that receives an extra electron is said to be a negative ion.


Na


Cl


War-

ehiorine ion
er ${ }^{+}$

| H | (1s) |
| :---: | :---: |
| He | (1s) ${ }^{2}$ |
| Li | (1s) ${ }^{2}(2 s)^{1}$ |
| Ba | (1s) ${ }^{2}(2 s)^{2}$ |
| B | ( 1 s$)^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{1}$ |
| C | (1s) ${ }^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{2}$ |
| N | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{3}$ |
| 0 | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{4}$ |
| F | $(1 \mathrm{~s})^{2} \cdot(2 \mathrm{~s})^{2}(2 \mathrm{p})^{5}$ |
| Ne | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}$ |
| Na | (1s) ${ }^{1}\left(2 s s^{2}(2 \mathrm{p})^{(1)}(3 s)\right.$ |
| Mg | (1s $)^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}$ |
| AI | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{1}$ |
| Si | (1s) ${ }^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{2}$ |
| P | (1ss) $)^{1}(2 s)^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{3}$ |
| S | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{4}$ |
| CI | (1ss) ${ }^{(2 s s)^{3}(2 p)^{6}(3 s)^{2}(3 p)^{3}}$ |
| Ar | $(1 \mathrm{~s})^{1}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}$ |
| K | (1s) ${ }^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}(3 \mathrm{~d})^{1}$ |
| Ca | $(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}(3 \mathrm{~d})^{2}$ |

$\mathrm{Na}^{+}$(sodium ion)
$\left.\left.\mathrm{Na} \quad(1 \mathrm{~s})^{2}\right)(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right)(3 \mathrm{~s})$
$\mathrm{Na}^{+} \quad(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6} \mid$
(11 electrons)
( 10 electrons)
$\mathrm{Cl}^{-}$(chloride ion)

| Cl | $\left.(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right)(3 \mathrm{~s})^{2}(3 \mathrm{p})^{5}$ | $(17$ electrons $)$ |
| :--- | :--- | :--- |
| Cl | $\left.(1 \mathrm{~s})^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right)(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}$ | $(18$ electrons $)$ |

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## 4.Coulomb's law

- Charles-Augustin de Coulomb was a French physicist. He is best known for developing Coulomb's law

- Coulomb's law which is the definition of the electrostatic force of attraction and repulsion.
- coulomb's law state that :"Two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them".

$$
f_{1,2}=\frac{k_{e} q_{1} q_{2}}{r^{2}} e_{1,2}
$$

Here q 1 and q 2 are numbers (scalars) giving the magnitude and sign of the respective charges, e is the unit vector in the direction from
 charge 1 to charge 2 , and F12 is the force acting on charge 2.

Note that

```
F21= - F12
```

The constant of proportionality (ke) is written as
$k_{e}=\frac{1}{2 \pi \varepsilon_{0}}=c^{2} \times 10^{-7}=8.98755^{*} 10^{9} \mathrm{~N} \mathrm{~m}^{2} / c^{2}($ or $\mathrm{Vm} / \mathrm{c})$

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where $c$ is the speed of light,

$$
c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}
$$

Note that $\varepsilon_{0}$ is the permittivity of free space and $\mu_{0}$ is the permeability of free space,

$$
\begin{aligned}
& \varepsilon_{0}=8.8541878176 \times 10^{-12} \frac{\mathrm{C}^{2}}{\mathrm{Nm}^{2}} \\
& \mu_{0}=4 \pi \times 10^{-7}\left(\mathrm{~N} / \mathrm{A}^{2}\right)
\end{aligned}
$$

The coulomb is an extremely large unit. The force between two charges of 1 C each a distance of 1 m apart is

$$
F=\frac{1}{4 \pi \varepsilon_{0}} \frac{1 C \times 1 C}{1 m^{2}}=8.98755 \times 10^{9} \mathrm{~N}
$$

((Note)) It is easy for you to memorize the value of $k_{\mathrm{c}}$.

$$
k_{e}=9 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}(\text { or } \mathrm{V} \mathrm{~m} / \mathrm{C})
$$

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((Note))

$$
\begin{aligned}
& \frac{N m^{2}}{C^{2}}=\frac{N m}{C} \frac{m}{C}=\frac{J}{C} \frac{m}{C}=\frac{V A s}{A s} \frac{m}{C}=\frac{V m}{C} \\
& \begin{array}{l}
N m=\mathrm{J}, \\
W=V A
\end{array} \quad C=A s \\
& J=W s=V A s
\end{aligned}
$$

where
J (Joule), A (Ampere), V (Volt), C (Coulomb),
s (second), N (Neuton), and W (Watt).
((Note))
The SI unit of charge is coulomb. The coulomb unit is derived from the SI unit A (Ampere) for the electric current $i$. The current $i$ is the rate $\mathrm{d} q / \mathrm{d} r$ at which the amount of charge $(\mathrm{d} q)$ moves past a point or through a region in time $\mathrm{d} t$ (second).

$$
i=\frac{d q}{d t}
$$

This relation implies that.

$$
1 \mathrm{C}=(1 \mathrm{~A})(1 \mathrm{~s})
$$

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## 5.Bohr model

- consider the Bohr model shown in this figure.
- The system consists of a proton and an electron.
- These two particles are coupled with an attractive Coulomb interaction.

- The electrical force between the electron (charge q1 $=-e$ ) and proton (charge $q 2=+e$ ) is found from Coulomb's law,
- $f_{e}=\frac{k_{e} q_{1} q_{2}}{r_{B}^{2}}=8.19 \times 10^{-8} \mathrm{~N}$
where $e=1.602176487 \times 10^{-19} \mathrm{C}$ and $r_{\mathrm{B}}$ is the Bohr radius given by

$$
r_{\mathrm{B}}=5.2917720859 \times 10^{-11}(\mathrm{~m})=0.52917720859 \AA .
$$

This can be compared with the gravitational force between the electron and proton

$$
F_{x}=\frac{G m_{e} m_{p}}{r_{B}^{2}}=3.63153 \times 10^{-47} \mathrm{~N}
$$

What is the angular frequency $\omega$ for electrons rotating the circular orbit?

$$
\begin{aligned}
& F_{e}=\frac{1}{4 \pi s_{0}} \frac{e^{2}}{r_{B}^{2}}=m m \frac{v^{2}}{r_{B s}}=m r_{B B^{2}} \\
& \omega=\sqrt{\frac{1}{4 \pi s_{0}} \frac{e^{2}}{m r_{B}^{3}}}=4.13414 \times 10^{16} \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

where $m$ is the mass of electron, $m=9.1093821545 \times 10^{-31} \mathrm{~kg}$.
The period is

$$
T=\frac{2 \pi}{\omega}=1.51983 \times 10^{-16} s
$$

((Note))
An important difference between the electric force and the gravitational force is that the gravitational force is always attractive, while the electric force can be repulsive, or attractive, depending on the charges of the particles

## 6.Conductors and insulators

## (a) Conductors

A conductor is a material that permits the motion of electric charge through its volume. Examples of conductors are copper, aluminum and iron. An electric charge placed on the end of a conductor will spread out over the entire conductor until an equilibrium distribution is established.

## (b) Insulators

Electric charge placed on an insulator stays in place: an insulator (like glass, rubber and mylar) does not permit the motion of electric charge.
(c) Superconductors

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance

## 7. Principle superposition

- When there are more than two charges present we must supplement the Coulomb's law with one other fact of nature. This fact is called "the principle of superposition."
- principle of superposition state that The force on any charge is the vector sum of the Coulomb forces(electrostatic force) from each of the other charges. This fact is called "the principle of superposition."
- If we combine the Coulomb's law and the principle of superposition, That is all there is to electrostatics.

Suppose we have some arrangement of charges q1, q2, q3, ... qN, fixed in space. From the principle of superposition, the resultant force on the charge $q 0$ is expressed by

$$
\boldsymbol{F}_{0}=\sum_{j=1}^{N} \boldsymbol{F}_{j 0}=\sum_{j=1}^{N} \frac{1}{4 \pi \varepsilon_{0}} \frac{q_{0} q_{j}}{r_{j 0}{ }^{2}} \boldsymbol{e}_{j 0}
$$

The resultant force Fo on the charge qo is given by

$$
F 0=F 10+F 20+F_{30}+F_{40}
$$



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## Example :1

Four point charge at the corners of a square of side (a) shown in fig . Determine the magnitude and direction of the resultant electric force on $q$ in symbolic form q, ke, a ?



Free body diagram (F.B.D) for force on charge A

## Solution



Fig 1.1

As shown in fig 1.1.
1- The force FBA is the force acting from charge $B$ to charge $A$ and the direction of this force upward because the charge ( +A ) move away from charge ( +B ) because of the repel.

2- The force FDA is the force acting from charge D to charge A and the direction of this force to the right because the charge ( $+A$ ) move away from charge (+D) because of the repel.

3- The force FcA is the force acting from charge $c$ to charge $A$ and the direction of this force diagonal component in the north east because the charge $(+$ A) move away from charge $(+c)$ because of the repel. And this force make an angle 45 deg. Because of the symmetry as shown below


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- By applying Coulombe's law to find the net electrostatic force

$$
f_{1,2}=\frac{k_{e} q_{1} q_{2}}{r^{2}}
$$

| Forces | $x$-direction | Y-direction |
| :---: | :---: | :---: |
| FBA | 0 | $\frac{k(2 q)(q)}{a^{2}}$ |
| FDA | $\frac{k(2 q)(q)}{a^{2}}$ | 0 |
| FCA | $\frac{k(3 q)(q)}{(a \sqrt{2})^{2}} \cos 45$ | $\frac{k(3 q)(q)}{(a \sqrt{2})^{2}} \cos 45$ |

The resultant electrostatic force in x -direction (FNX) $=\frac{k(2 q)(q)}{a^{2}}+\frac{k(3 q)(q)}{(a \sqrt{2})^{2}} \cos 45$
The resultant electrostatic force in Y -direction (FNY) $=\frac{k(2 q)(q)}{a^{2}}+\frac{k(3 q)(q)}{(a \sqrt{ })^{2}} \cos 45$
Then find the resultant electrostatic force in q is $\mathrm{FN}=\sqrt{F_{N X}^{2}+F_{N Y}^{2}}$

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## Electric physics II

The Electric Field
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## The Electric Field

- An electric field is said to exist in the region of space around a charged objectthe source charge. When another charged object-the test charge-enters this electric field, an electric force acts on it. As an example, consider Figure 23.11, which shows a small positive test charge 90 placed near a second object carrying a much greater positive charge $Q$. We define the electric field due to the source charge at the location of the test charge to be the electric force $Q$
on the test charge per unit charge, or to be more specific the electric field vector E at a point in space is defined as the electric force $\mathrm{F}_{e}$ acting on a positive test charge $\boldsymbol{q}_{0}$ placed at that point divided by the test charge:

$$
\begin{align*}
\mathbf{E} \equiv \frac{\mathbf{F}_{e}}{q_{0}}  \tag{23.7}\\
\boldsymbol{F}_{e}=\mathrm{q} \mathbf{E}
\end{align*}
$$

Figure 23.11 A small positive test
charge $q_{0}$ placed near an object carrying a much larger positive charge Qexperiences an electric field $\mathbf{E}$ directed as shown.

- Notice the similarity between Equation 23.8 and the corresponding equation for a particle with mass placed in a gravitational field, $\mathrm{F}_{g}=m \mathrm{~g}$
- The vector $E$ has the SI units of newtons per coulomb (N/C).
- The direction of E , as shown in Figure 23.11
. For a positive point charge the lines of electric field are directed outward
. For a negative charge the lines of electric field are directed inward


Figure (23.11)

- According to Coulomb's law, the force exerted by $q$ on the test charge is

$$
\mathbf{F}_{e}=k_{e} \frac{q q_{0}}{r^{2}} \hat{\mathbf{r}}
$$

- where $r^{\wedge}$ is a unit vector directed from $q$ toward $q 0$. This force in Figure 23.13a is directed away from the source charge $q$. Because the electric field at $P$, the position of the test charge, is defined by $\mathrm{E}=\mathrm{F}_{e} / q_{0}$, we find that at $P$, the electric field created by $q$ is

$$
\begin{equation*}
\mathbf{E}=k_{e} \frac{q}{r^{2}} \hat{\mathbf{r}} \tag{23.9}
\end{equation*}
$$



To calculate the electric field at a point $P$ due to a group of point charges, we first calculate the electric field vectors at $P$ individually using Equation 23.9 and then add them vectorially. In other words, at any point $P$, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$
\mathbf{E}=k_{e} \sum_{i} \frac{q_{i}}{r_{i}{ }^{2}} \hat{\mathbf{r}}_{i}
$$

where $r_{i}$ is the distance from the $i$ th source charge $q_{i}$ to the point $P$ and $r^{\wedge}$ is a unit vector directed from $q_{i}$ toward $P$.

## Example 23.5 Electric Field Due to Two Charges

A charge $q_{1}=7.0 \mu \mathrm{C}$ is located at the origin, and a second charge $q_{2}=-5.0 \mu \mathrm{C}$ is located on the $x$ axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point $P$, which has coordinates $(0,0.40) \mathrm{m}$.


Figure 23.14 (Example 23.5) The total electric field E at $P$ equals the vector $\operatorname{sum} \mathbf{E}_{1}+\mathbf{E}_{2}$, where $\mathbf{E}_{1}$ is the field due to the positive charge $q_{1}$ and $\mathbf{E}_{2}$ is the field due to the negative charge $q_{2}$.

Solution First, let us find the magnitude of the electric field at $P$ due to each charge. The fields $\mathbf{E}_{1}$ due to the $7.0-\mu \mathrm{C}$ charge and $\mathbf{E}_{2}$ due to the $-5.0-\mu \mathrm{C}$ charge are shown in Figure 23.14. Their magnitudes are

$$
\begin{aligned}
E_{1} & =k_{e} \frac{\left|q_{1}\right|}{r_{1}^{2}}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(7.0 \times 10^{-6} \mathrm{C}\right)}{(0.40 \mathrm{~m})^{2}} \\
& =3.9 \times 10^{5} \mathrm{~N} / \mathrm{C} \\
E_{2} & =k_{e} \frac{\left|q_{2}\right|}{r_{2}{ }^{2}}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right) \frac{\left(5.0 \times 10^{-6} \mathrm{C}\right)}{(0.50 \mathrm{~m})^{2}} \\
& =1.8 \times 10^{5} \mathrm{~N} / \mathrm{C}
\end{aligned}
$$

The vector $\mathbf{E}_{1}$ has only a $y$ component. The vector $\mathbf{E}_{2}$ has an $x$ component given by $E_{2} \cos \theta=\frac{3}{5} E_{2}$ and a negative $y$ component given by $-E_{2} \sin \theta=-\frac{4}{5} E_{2}$. Hence, we can express the vectors as

$$
\begin{aligned}
& \mathbf{E}_{1}=3.9 \times 10^{5} \hat{\mathbf{j}} \mathrm{~N} / \mathrm{C} \\
& \mathbf{E}_{2}=\left(1.1 \times 10^{5} \hat{\mathbf{i}}-1.4 \times 10^{5} \hat{\mathbf{j}}\right) \mathrm{N} / \mathrm{C}
\end{aligned}
$$

The resultant field $\mathbf{E}$ at $P$ is the superposition of $\mathbf{E}_{1}$ and $\mathbf{E}_{2}$ :

$$
\mathbf{E}=\mathbf{E}_{1}+\mathbf{E}_{2}=\left(1.1 \times 10^{5} \hat{\mathbf{i}}+2.5 \times 10^{5} \hat{\mathbf{j}}\right) \mathrm{N} / \mathbf{C}
$$

From this result, we find that $\mathbf{E}$ makes an angle $\phi$ of $66^{\circ}$ with the positive $x$ axis and has a magnitude of $2.7 \times 10^{5} \mathrm{~N} / \mathrm{C}$.

An electric dipole is defined as a positive charge $q$ and a negative charge - $q$ separated by a distance $2 a$. For the dipole shown in Figure 23.15, find the electric field E at $P$ due to the dipole, where $P$ is a distance $y \gg a$ from the origin.

Solution At $P$, the fields $\mathbf{E}_{1}$ and $\mathbf{E}_{2}$ due to the two charges are equal in magnitude because $P$ is equidistant from the charges. The total field is $\mathbf{E}=\mathbf{E}_{1}+\mathbf{E}_{2}$, where

$$
E_{1}=E_{2}=k_{e} \frac{q}{r^{2}}=k_{e} \frac{q}{y^{2}+a^{2}}
$$



Figure 23.15 (Example 23.6) The total electric field E at Pdue to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $\mathbf{E}_{1}+\mathbf{E}_{2}$. The field $\mathbf{E}_{1}$ is due to the positive charge q. and $\mathbf{E}_{2}$ in the field due to the negative charge $-q$

The $y$ components of $\mathbf{E}_{1}$ and $\mathbf{E}_{2}$ cancel each other, and the $x$ components are both in the positive $x$ direction and have the same magnitude. Therefore, $\mathbf{E}$ is parallel to the $x$ axis and has a magnitude equal to $2 E_{1} \cos \theta$. From Figure 29.15 we see that $\cos \theta=a / r=a /\left(y^{2}+a^{2}\right)^{1 / 2}$. Therefore,

$$
\begin{aligned}
E & =2 E_{1} \cos \theta=2 k_{e} \frac{q}{\left(y^{2}+a^{2}\right)} \frac{a}{\left(y^{2}+a^{2}\right)^{1 / 2}} \\
& =k_{e} \frac{2 q a}{\left(y^{2}+a^{2}\right)^{3 / 2}}
\end{aligned}
$$

Because $y \gg A$, we can neglect $a^{2}$ compared to $y^{2}$ and write

$$
E=k_{n} \frac{2 q a}{y^{3}}
$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the fine joining the two charges, the magnituade of the electric field created by the dipole varies as $1 / r^{3}$, whereas the more slowly varying field of at point charge varies as $1 / r^{2}$ (see Eq. 23.9). This is because at distant points, the fields of the two charges of equal magnitude and opposite sign almost cancel each other. The $1 / r^{3}$ variation in $E$ for the dipole also is obtained for a distant point along the $x$ axis (see Problem 22) and for any general distant point.

The electric dipole is a good model of many molecules, such as bydrochloric acid (HC1). Neutral atoms and molecules behave as dipoles when placed in an external clectric field. Furthermore, many molecules, such as HCl, are permanent dipoles. The effect of stach dipoles on the behavior of materials subjected to eleceric fields is discussed in Chapter 26.

## Electric Field of a Continuous Charge Distribution

The electric field at $P$ due to one charge element carrying charge $\Delta q$ is

$$
\Delta \mathbf{E}=k_{e} \frac{\Delta q}{r^{2}} \hat{\mathbf{r}}
$$

where $r$ is the distance from the charge element to point $P$ and $r^{\wedge}$ is a unit vector directed from the element toward $P$. The total electric field at $P$ due to all elements in the charge distribution is approximately

$$
\mathbf{E} \approx k_{e} \sum_{i} \frac{\Delta q_{i}}{r_{i}^{2}} \hat{\mathbf{r}}_{i}
$$

where the index $i$ refers to the $i$ th element in the distribution. Because the charge distribution is modeled as continuous, the total field at $P$ in the limit $\Delta q_{i} \rightarrow 0$ is

$$
\begin{equation*}
\mathbf{E}=k_{e} \lim _{\Delta q_{i} \rightarrow 0} \sum_{i} \frac{\Delta q_{i}}{r_{i}^{2}} \hat{\mathbf{r}}_{i}=k_{e} \int \frac{d q}{r^{2}} \hat{\mathbf{r}} \tag{23.11}
\end{equation*}
$$

where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:
If a charge $Q$ is uniformly distributed throughout a volume $V$, the volume charge density $\rho$ is defined by
where $\rho$ has units of coulombs per cubic meter ( $\mathrm{C} / \mathrm{m}^{3}$ ).

$$
\rho \equiv \frac{Q}{V}
$$

Figure 23.16 The electric field at $P$ due to a continuous charge distribution is the vector sum of the fields $\Delta \mathbf{E}$ due to all the elements $\Delta q$ of the charge distribution.

- If a charge $Q$ is uniformly distributed on a surface of area $A$, the surface charge density $\sigma$ (lowercase Greek sigma) is defined by

$$
\sigma \equiv \frac{Q}{A}
$$

- where $\sigma$ has units of coulombs per square meter ( $\mathrm{C} / \mathrm{m} 2$ ).
- If a charge $Q$ is uniformly distributed along a line of length $\ell$, the linear charge density $\lambda$ is defined by

$$
\lambda \equiv \frac{Q}{\ell}
$$

- where 3 has units of coulombs per meter (C/m).
- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge $d q$ in a small volume, surface, or length element are

$$
d q=\rho d V \quad d q=\sigma d A \quad d q=\lambda d \ell
$$

## Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge $Q$. Calculate the electric field due to the ring at a point $P$ lying a distance $x$ from its center along the central axis perpendicular to the plane of the ring (Fig. 23.18a).

Solution The magnitude of the electric field at $P$ due to the segment of charge $d q$ is

(a)

$$
d E=k_{e} \frac{d q}{r^{2}}
$$

This field has an $x$ component $d E_{x}=d E \cos \theta$ along the $x$ axis and a component $d E_{\perp}$ perpendicular to the $x$ axis. As we see in Figure 23.18b, however, the resultant field at $P$ must lie along the $x$ axis because the perpendicular com-


Figure 23.18 (Example 23.8) A uniformly charged ring of radius $a$. (a) The field at $P$ on the $x$ axis due to an element of charge $d q$. (b) The total electric field at $P$ is along the $x$ axis. The perpendicular component of the field at $P$ due to segment 1 is canceled by the perpendicular component due to segment 2 .

- components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r=\left(x^{2}+a^{2}\right)^{1 / 2}$ and $\cos \theta=x / r_{1}$, we find that:

$$
d E_{x}=d E \cos \theta=\left(k_{e} \frac{d q}{r^{2}}\right) \frac{x}{r}=\frac{k_{e} x}{\left(x^{2}+a^{2}\right)^{3 / 2}} d q
$$

- we can integrate to obtain the total field at $P$ :

$$
\begin{aligned}
E_{x} & =\int \frac{k_{e} x}{\left(x^{2}+a^{2}\right)^{3 / 2}} d q=\frac{k_{e} x}{\left(x^{2}+a^{2}\right)^{3 / 2}} \int d q \\
& =\frac{k_{e} x}{\left(x^{2}+a^{2}\right)^{3 / 2}} Q
\end{aligned}
$$

- This result shows that the field is zero at $x=0$. Does this finding surprise you?
- What If? Suppose a negative charge is placed at the center of the ring in Figure 23.18 and displaced slightly by a distance $x$ !! a along the $x$ axis. When released, what type of motion does it exhibit? (it will be a harmonic motion due to the different type of charges )


## Example 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius $R$ has a uniform surface charge density $\sigma$. Calculate the electric field at a point $P$ that lies along the central perpendicular axis of the disk and a distance $x$ from the center of the disk (Fig. 23.19).

Solution If we consider the disk as a set of concentric rings, we can use our result from Example 23.8-which gives the field created by a ring of radius $a$-and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

The ring of radius $r$ and width $d r$ shown in Figure 23.19 has a surface area equal to $2 \pi r d r$. The charge $d q$ on this ring is equal to the area of the ring multiplied by the surface charge density: $d q=2 \pi \sigma r d r$. Using this result in the equation given for $E_{x}$ in Example 23.8 (with $a$ replaced by $r$ ), we have for the field due to the ring

$$
d E_{x}=\frac{k_{e} x}{\left(x^{2}+r^{2}\right)^{3 / 2}}(2 \pi \sigma r d r)
$$



Figure 23.19 (Example 23.9) A uniformly charged disk of radius $R$ The electric field at an axial point $P$ is directed along the central axis, perpendicular to the plane of the disk.

To obtain the total field at $P$, we integrate this expression over the limits $r=0$ to $r=R$, noting that $x$ is a constant.

This gives

$$
\begin{aligned}
E_{x} & =k_{e} x \pi \sigma \int_{0}^{R} \frac{2 r d r}{\left(x^{2}+r^{2}\right)^{3 / 2}} \\
& =k_{e} x \pi \sigma \int_{0}^{R}\left(x^{2}+r^{2}\right)^{-3 / 2} d\left(r^{2}\right) \\
& =k_{e} x \pi \sigma\left[\frac{\left(x^{2}+r^{2}\right)^{-1 / 2}}{-1 / 2}\right]_{0}^{R} \\
& =2 \pi k_{e} \sigma\left(1-\frac{x}{\left(x^{2}+R^{2}\right)^{1 / 2}}\right)
\end{aligned}
$$

A convenient way of visualizing electric field patterns is to draw curved lines that are
parallel to the electric field vector at any point in space. These lines, called electric field
lines and first introduced by Faraday, are related to the electric field in a region of space
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parallel to the electric field vector at any point in space. These lines, called electric field
lines and first introduced by Faraday, are related to the electric field in a region of space in the following manner:

- The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

This result is valid for all values of $x>0$. We can calculate the field close to the disk along the axis by assuming that $R \gg x$; thus, the expression in parentheses reduces to unity to give us the near-field approximation:

$$
E_{x}=2 \pi k_{c} \sigma=\frac{\sigma}{2 \epsilon_{0}}
$$

where $\epsilon_{0}$ is the permittivity of free space. In the next chapter we shall obtain the same result for the field created by a uniformly charged infinite sheet.


Figure 23.20 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

These properties are illustrated in Figure 23.20. The density of lines through surface $A$ is greater than the density of lines through surface $B$. Therefore, the magnitude of the electric field is larger on surface A than on surface B. Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.

Is this relationship between strength of the electric field and the density of field lines consistent with Equation 23.9, the expression we obtained for $E$ using Coulomb's law? To answer this question, consider an imaginary spherical surface of radius $r$ concentric with a point charge. From symmetry, we see that the magnitude of the electric field is the same everywhere on the surface of the sphere. The number of lines $N$ that emerge from the charge is equal to the number that penetrate the spherical surface. Hence, the number of lines per unit area on the sphere is $N / 4 \pi r^{2}$ (where the surface area of the sphere is $4 \pi r^{2}$ ). Because $E$ is proportional to the number of lines per unit area, we see that $E$ varies as $1 / r^{2}$; this finding is consistent with Equation 23.9.


Figure 23.21 The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane of the page. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.

- The rules for drawing electric field lines are as follows:
- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.
- We choose the number of field lines starting from any


Positively charged object to be $C q$ and the number of lines ending on any negatively charged object to be $C / q /$ where $C$ is an arbitrary proportionality constant. Once $C$ is chosen, the number of lines is fixed. For example, if object 1 has charge $Q 1$ and object 2 has charge $Q 2$, then the ratio of number of lines is $N 2 / N 1=Q 2 / Q 1$. The electric field lines for two point charges of equal magnitude but opposite signs (an electric dipole)

## Motion of Charged Particles in a Uniform Electric Field

- When a particle of charge $q$ and mass $m$ is placed in an electric field $E$, the electric force exerted on the charge is $q \mathrm{E}$ according to Equation 23.8. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$
\mathbf{F}_{e}=q \mathbf{E}=m \mathbf{a}
$$

- The acceleration of the particle is therefore

$$
\mathbf{a}=\frac{q \mathbf{E}}{m}
$$

- If $E$ is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.


## Example 23.10 An Accelerating Positive Charge

A positive point charge $q$ of mass $m$ is released from rest in a uniform electric field $\mathbf{E}$ directed along the $x$ axis, as shown in Figure 23.25. Describe its motion.

Solution The acceleration is constant and is given by $q \mathbf{E} / m$. The motion is simple linear motion along the $x$ axis. Therefore, we can apply the equations of kinematics in one dimension (see Chapter 2):

$$
\begin{aligned}
x_{f} & =x_{i}+v_{i} t+\frac{1}{2} a t^{2} \\
v_{f} & =v_{i}+a t \\
v_{f}{ }^{2} & =v_{i}^{2}+2 a\left(x_{f}-x_{i}\right)
\end{aligned}
$$

Choosing the initial position of the charge as $x_{i}=0$ and assigning $v_{j}=0$ because the particle starts from rest, the position of the particle as a function of time is

$$
x_{f}=\frac{1}{2} a t^{2}=\frac{q E}{2 m} t^{2}
$$

The speed of the particle is given by

$$
v_{f}=a t=\frac{q E}{m} t
$$

The third kinematic equation gives us

$$
v_{f}^{2}=2 a x_{f}=\left(\frac{2 q E}{m}\right) x_{f}
$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x=x_{j}-x_{i}$ :

$$
K=\frac{1}{2} m y_{f}^{2}=\frac{1}{2} m\left(\frac{2 q E}{m}\right) \Delta x=q E \Delta x
$$

We can also obtain this result from the work-kinetic energy theorem because the work done by the electric force is $F_{e} \Delta x=q E \Delta x$ and $W=\Delta K$.


Figure 23.25 (Example 23.10) A positive point charge $q$ in a uniform electric field $\mathbf{E}$ undergoes constant acceleration in the direction of the field.

- The electric field in the region between two oppositely charged flat metallic plates is approximately uniform (Fig. 23.26). Suppose an electron of charge -e is projected horizontally into this field from the origin with an initial velocity ' $v_{i} \hat{\mathbf{i}}$ at time $t=0$. Because the electric field E in Figure 23.26 is in the positive $y$ direction, the acceleration of the electron is in the negative $y$ direction. That is,

$$
\begin{equation*}
\mathbf{a}=-\frac{e E}{m_{e}} \hat{\mathbf{j}} \tag{23.13}
\end{equation*}
$$

Because the acceleration is constant, we can apply the equations of kinematics in two dimensions with $v_{x i}=v_{i}$ and $v_{v i}=0$. After the electron has been in the


Active Figure 23.26 An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite $\mathbf{E}$ ), and its motion is parabolic while it is between the plates.

- electric field for a time interval, the components of its velocity at time $t$ are

$$
\begin{aligned}
& v_{x}=v_{i}=\text { constant } \\
& v_{y}=a_{y} t=-\frac{e E}{m_{e}} t \\
& x_{f}=v_{i} t \\
& y_{f}=\frac{1}{2} a_{y} t^{2}=-\frac{1}{2} \frac{e E}{m_{e}} t^{2}
\end{aligned}
$$

- Substituting the value $t=x_{f} / v_{i}$ from Equation 23.16 into Equation 23.17, we see that $y_{f}$ is proportional to $x_{f}{ }^{2}$. Hence, the trajectory is a parabola. This should not be a surpriseconsider the analogous situation of throwing a ball horizontally in a uniform gravitational field. After the electron leaves the field, the electric force vanishes and the electron continues to move in a straight line in the direction of $v$ in Figure 23.26 with a speed $v>v_{i}$.
- Note that we have neglected the gravitational force acting on the electron. This is a good approximation when we are dealing with atomic particles. For an electric field of $10^{4} \mathrm{~N} / \mathrm{C}$, the ratio of the magnitude of the electric force $e E$ to the magnitude of the gravitational force $m g$ is on the order of $10^{14}$ for an electron and on the order of $10^{11}$ for a proton.

An electron enters the region of a uniform electric field as shown in Figure 23.26, with $v_{i}=3.00 \times 10^{6} \mathrm{~m} / \mathrm{s}$ and $E=200 \mathrm{~N} / \mathrm{C}$. The horizontal length of the plates is $\ell=0.100 \mathrm{~m}$.
(A) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of $1.60 \times 10^{-19} \mathrm{C}$, and $m_{e}=9.11 \times 10^{-31} \mathrm{~kg}$. Therefore, Equation 23.13 gives

$$
\begin{aligned}
\mathbf{a} & =-\frac{e E}{m_{e}} \hat{\mathbf{j}}=-\frac{\left(1.60 \times 10^{-19} \mathrm{C}\right)(200 \mathrm{~N} / \mathrm{C})}{9.11 \times 10^{-31} \mathrm{~kg}} \hat{\mathbf{j}} \\
& =-3.51 \times 10^{13} \hat{\mathbf{j}} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

(B) If the electron enters the field at time $t=0$, find the time at which it leaves the field.

Solution The horizontal distance across the field is $\ell=$ 0.100 m . Using Equation 23.16 with $x_{f}=\ell$, we find that the time at which the electron exits the electric field is

$$
t=\frac{\ell}{v_{i}}=\frac{0.100 \mathrm{~m}}{3.00 \times 10^{6} \mathrm{~m} / \mathrm{s}}=3.33 \times 10^{-8} \mathrm{~s}
$$

(C) If the vertical position of the electron as it enters the field is $y_{i}=0$, what is its vertical position when it leaves the field?

Solution Using Equation 23.17 and the results from parts (A) and (B), we find that

$$
\begin{aligned}
y_{f} & =\frac{1}{2} a_{y} t^{2}=-\frac{1}{2}\left(3.51 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}\right)\left(3.33 \times 10^{-8} \mathrm{~s}\right)^{2} \\
& =-0.0195 \mathrm{~m}=-1.95 \mathrm{~cm}
\end{aligned}
$$

If the electron enters just below the negative plate in Figure 23.26 and the separation between the plates is less than the value we have just calculated, the electron will strike the positive plate.

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Electric physics II
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Electric physics II<br>Electric dipole moment , torque, potential energy<br>By<br>Assist. Lac. Yasameen Kamil<br>2020-2021

## 1.Electric dipole moment

- An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge $q$ and a negative charge $-q$ ) separated by a distance ( $\mathrm{d}=2 \mathrm{a}$ ) which is the dipole axis as shown in figure (3-1) .

- Definition of electric moment:In physics, the electric dipole moment (or electric dipole for short) is a measure of the polarity of a system of electric charges.
In figure(3-2) $r=2 a=d$, In the simple case of two point one with charge $+q$ and one with charge $-q$

Electric dipole moment

- the electric dipole moment is:
- $p=r \times q \quad$.............. (3-1) (the magnitude of dipole moment)
- The unite of dipole moment is Coulomb. meter (C. m)
- where $r$ is the displacement vector pointing from the negative charge to the positive charge ( $r=2 a=d$ ). This implies that the electric dipole moment vector points from the negative charge to the positive charge.
- What happen if we put a dipole in electric field?
- If the dipole is parallel to the electric field as shown in figure (3-3 (a),(b)
- The + ve charge produces a force accelerated in the direction of E, while the -ve charge produces a force accelerated opposite to the direction of E .
- In this case there are two forces having the same magnitude but opposite in their direction, then the resultant force acting on dipole=0



## figure 3-3(a)

the electric field parallel to the dipole axis, the forces having the same magnitude but opposite direction , The resultant force acting on dipole $=0$

Figure 3-3(b)
the electric field parallel to the dipole axis, the forces having the same magnitude but opposite direction, The resultant force acting on dipole =0


Figure 3-3 (c)
the electric field perpendicular to the dipole axis, the forces having the same magnitude but opposite direction, The resultant force acting on dipole generate a torque

- Figure 3-3c represent the dipole axis vertical to the electric field (E). In this case there is a torque will be produces
- $\tau=f \times 1 \quad$......... (3-2)
- Where $\tau$ represent the torque and $\ell$ the arm of the acting force
- $\tau_{\text {net }}=\tau_{1}+\tau_{2}$ becuse the two force in clock wise (C.W) direction
- $\quad=f \times \ell+f \times \ell$
- $=f \times \frac{d}{2}+f \times \frac{d}{2}$
- $\tau_{n e t}=f \times \mathrm{d}$
- $F=q \times E$
- $\tau_{n e t}=q \times E \times \mathrm{d}$


## 2. Torque

- We consider the behavior of the electric dipole moment in the presence of an electric field
- An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge $q$ and a negative charge $-q$ ) separated by a distance $d(=2 a)$ as in figure (3-4).
- The electric dipole moment is defined by
- $P=2 a q$ $\qquad$
- The unit of $p$ is Cm . The magnitude of the torque $\tau$ exerted by the field E is according to equation (3-3):
- $\tau=(q E)(2 a) \sin \varnothing=p E \sin \varnothing$ $\qquad$ .(3-5)
where $\varnothing$ is the angle between $E$ and the dipole axis
The torque directed to clock -wise
Figure (3-5) represent the vector form of the torque ( $\tau$ )


Figure (3-5)

## 3-Potential energy of electric dipole

- The work done on the electric dipole moment (p) by the electric field (E) is given by $\omega=\tau \varnothing$ (3-6)
-Where $\tau$ is the torque, $\varnothing$ is the angular displacement
- $\mathrm{d} \omega=\tau d \emptyset, \quad \tau=-p E \sin \emptyset$
- Not that This torque $(-\tau)$ is the counterclockwise direction and in the direction of increasing $\emptyset$ as shown in
figure (3-6)(a),(b).


Figure (3-6)

- When $\emptyset$ is increases from $0^{\circ}$ to $90^{\circ}$ then the torque will be negative sign $(-\tau)$
- $\int d w=-\int p E \sin \varnothing=-p E \int \sin \emptyset \mathrm{~d} \emptyset$
$\mathrm{W}=p E \cos \emptyset$
$\mathrm{W}=-\mathrm{U}=p E \cos \varnothing$
- Since $\Delta U=-\Delta \mathrm{W}$, we have the expression for the potential energy ( U ) (in units of J)
- $U=-p E$
- Note-1 Work-energy theorem; $\Delta \mathrm{K}=\Delta \mathrm{W}=-\Delta \mathrm{U}$, Where $\Delta \mathrm{K}$ is the kinetic energy
- note-2) The kinetic energy work theorem; $\mathrm{W}=+\Delta \mathrm{K}$
- Note-3)) The potential energy has its minimum value where $p$ and $E$ are parallel $(\varnothing=0)$. The potential energy has its maximum value where $p$ and $E$ are antiparallel $(\varnothing=\pi)$


## Example: -

An electric dipole has a charge of $+/-3.2 \times 10^{-19}$ and separation distance of 0.25 mm , the electric field is ( $4 \times 10^{6} \mathrm{~N} / \mathrm{C}$ ).
A) What is the magnitude and the direction of the electric dipole moment? b) what is the force on each charge and the net force on the entire dipole? C) calculate the potential energy at an angle of $90^{\circ}$ and $30^{\circ}$ ? d) calculate the work required to move the dipole from $90^{\circ}$ to $30^{\circ}$ ? e) what is the magnitude of the net torque at $90^{\circ}$ ?
Solution: -
a) What is the magnitude and the direction of the electric dipole moment?

$$
p=a \times d
$$

$$
\mathrm{P}=3.2 \times 10^{-19} \times 0.25 \times 10^{-9}=8 \times 10^{-20} \mathrm{C} . \mathrm{m}
$$ the direction of $p$ in $y$ _ direction + according to the figure

b) what is the force on each charge and the net force on the entire dipole?


$$
f=q \times E
$$

$\mathrm{F}=3.2 \times 10^{-10} \times 4 \times 10^{6}=1.28 \times 10^{-12} \mathrm{~N}$ the force on each charge
The two force have the same magnitude but in the opposite direction, then the net
force $=0$

$$
f_{\text {nee }}=0
$$


c) calculate the potential energy at an angle of $90^{\circ}$ and $30^{\circ}$ ?

```
U}=-P\timesE\operatorname{cos}
    V _ { 1 } = - ( 8 \times 1 0 ^ { - 2 9 } ) \times ( 4 \times 1 0 ^ { 6 } ) \operatorname { c o s 9 0 }
    U2}=-(8\times1\mp@subsup{0}{}{-29})\times(4\times1\mp@subsup{0}{}{6})\operatorname{cos}30=-2.771\times1\mp@subsup{0}{}{-22J
```

(d) calculate the work required to move the dipole from $90^{\circ}$ to $30^{\circ}$ ?

$$
\begin{aligned}
& W=P \times E\left(\cos \theta_{2}-\cos \theta_{1}\right) \\
& W=\left(8 \times 10^{-29}\right) \times\left(4 \times 10^{6}\right)(\cos 30-\cos 90) \\
& W=2.771 \times 10^{-22}
\end{aligned}
$$

This mean $W=-\Delta U$

e) what is the magnitude of the net torque at $90^{\circ}$ ?

$$
\begin{aligned}
& \tau=P \times E \sin \theta \\
& \tau=\left(8 \times 10^{-29}\right) \times\left(4 \times 10^{6}\right) \sin 90 \\
& \tau=3.2 \times 10^{-22} N \mathrm{~N}
\end{aligned}
$$

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Electric physics II
Electric flux\& Gauss's law
By
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2020-2021

## Summary

- Electric Flux
- Gauss's Law
- Examples of using Gauss's Law
- Properties of Conductors


## Flux of an electric field

- Is the measure of electric field line passing through the surface area " S "

$$
\phi=E \times A \quad \text { Electric flux } \quad \text { unite }: \mathrm{N} \cdot \mathrm{~m}^{2} / C
$$

Where:
A: is a vector perpendicular to the surface area,
$E$ : is the electric field

(A) parallel to the (E)

(A) perpendicular to the( E )

- Electric flux depend on the strength of the E on the surface area, and depend on the relative orientation of the field and the surface ( $\varnothing$
$\phi=E \times A \cos \emptyset$


Where $\emptyset$ is the angle between $E$ and $A$

Calculate the flux of the electric field $E$, through the surface $A$, in each of the three cases shown:
a) $\Phi=$
b) $\mathbb{D}=$
c) $\mathbb{D}=$

## Gauss's Law

- Gauss' law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface. The closed surface is often called a Gaussian surface.

Gaussian surface : The closed surface is often called a Gaussian surface. If the Gaussian surface has a net electric charge $q_{i n}$ within it, then the electric flux through the surface is $q_{\text {in }} / \varepsilon_{0}$, that is

$$
\Phi=\oint E . d A=\frac{Q_{i n}}{\varepsilon_{0}}
$$



Flux through a sphere from a point charge
The electric field around a point charge

$$
|\mathbf{E}|=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left|\mathbf{r}_{1}\right|^{2}}
$$

Thus the flux on a sphere is $\mathrm{E} \times$ Area

$$
\Phi=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left|\mathbf{r}_{1}\right|^{2}} \times 4 \pi\left|\mathbf{r}_{1}\right|^{2}
$$

Cancelling we get

$$
\Phi=\frac{Q}{\varepsilon_{0}}
$$



Now we change the radius of sphere

$$
|\mathbf{E}|=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left|\mathbf{r}_{2}\right|^{2}}
$$

$$
\Phi_{2}=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{\left|\mathbf{r}_{2}\right|^{2}} \times 4 \pi\left|\mathbf{r}_{2}\right|^{2}
$$

$$
\Phi_{2}=\frac{Q}{\varepsilon_{0}}
$$

The flux is the same as before

$$
\Phi_{2}=\Phi_{1}=\frac{Q}{\varepsilon_{0}}
$$



Since the flux is related to the number of field lines passing through a surface the total flux is the total from each charge


$$
\Phi_{S}=\frac{Q_{1}}{\varepsilon_{0}}+\frac{Q_{2}}{\varepsilon_{0}}
$$

$$
\Phi_{S}=\sum \frac{Q_{i}}{\varepsilon_{0}}
$$

For any
surface

Gauss's Law

## Quiz:-



Quiz)
Calculate the electric flux in each closed surface

## $\phi_{s 1}=$

$\phi_{s 2}=$
$\phi_{s 3}=$
$\phi_{s 4}=$


Find the flux in each surface if the electric field is pass through the closed surface and its direction as shown in figure?

## 1- electric flux on the upper surface is

```
Dout=+E A
```

Because E Il the tine perpendecoler to $A$ and its sign is positive ( $E$ is pointing outward the surface)
2- electric flux on the lower surface is
$\square$
Because $E$ II the lineperpendecoler to $A$ and its sign is negative ( $E$ is pointing inward the surface)

3- the electric flux on the other 4-surface of the cubic is O Because E $\perp$ the line perpendecoler to $A$
$\Phi=E A \cos 90=0$
4 -The net $\Phi$ is

$$
\begin{aligned}
\Phi \text { net } & =\Phi_{i n}+\Phi_{o u t} \\
\Phi_{\text {net }} & =-E A+E A=0
\end{aligned}
$$

$\Phi_{i n}=\Phi_{\text {out }}$ (but in the opposite direction

## Conductors in Electric Fields

- $\mathrm{E}=0$ everywhere inside the conductor.
- 2. There is no net charge inside the conductor.
- 3. E is everywhere perpendicular to the bounding surface of the conductor.
- 4. The electric potential V is constant insider the conductor.
- 5. Any net charge must reside on the surface of conductor.
-6. The tangential component of the electric field E is zero on the surface of conductor.


## 1-E is zero within conductor

If there is a field in the conductor, then the free electrons would feel a force and be accelerated. They would then move and since there are charges moving the conductor would not be in electrostatic equilibrium
Thus E=0
2. There is no net charge inside the conductor.

Because of the repulsive force inside the conductor the charge would reside on the surface of conductor
3-E is everywhere perpendicular to the bounding surface of the conductor.
If the tangential component of the $\mathrm{E}_{| |}>0$, it would cause surface charge $q$ to move thus it would not be in electrostatic equilibrium, thus $E_{| |}=0$, for this reason only the vertical component of $E$ bounded the surface of conductor

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Dept. of Electrical Engineering

Electric physics II
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# Electric physics II <br> Application of the Gauss' law 

By
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2020-2021

## Application of the Gauss' law:

1.Electric field due to infinite point of charge $\Phi=\oint E . d A=\frac{Q_{i n}}{\varepsilon_{0}}$
$E \oint d A \cos 0=\frac{Q_{\text {in }}}{\varepsilon_{0}}$
$E \times 4 \pi R^{2}=\frac{Q_{\text {in }}}{\varepsilon_{0}}$
$E=\frac{Q_{\text {in }}}{4 \pi R^{2} \varepsilon_{0}}=\frac{K Q_{\text {in }}}{R^{2}}$


## Application of the Gauss' law:

2.Electric field due to infinite line of charge

$$
\begin{aligned}
& \oint E . d A=\frac{Q_{i n}}{\varepsilon_{0}} \\
& =\oint_{1} \quad E \cdot d A+\oint_{2} \quad E . d A+\oint_{3} \quad E . d A \quad=\frac{Q_{i n}}{\varepsilon_{0}}
\end{aligned}
$$

$$
E \oint d A \cos 0=\frac{\lambda l}{\varepsilon_{0}}
$$

$$
E .2 \pi r l=\frac{\lambda l}{\varepsilon_{0}} \quad \text { where the area of cylindrical surface is } 2 \pi r l
$$



$$
\lambda=\frac{Q}{l}
$$

Linear charge density

## Application of the Gauss' law:

3.Nonconducting sheet ( surface of charge)
$\oint E . d A=\frac{Q_{\text {in }}}{\varepsilon_{0}}$

$$
\begin{aligned}
& =\oint_{1} E . d A+\oint_{2} E . d A+\oint_{3} E . d A \quad=\frac{Q_{i n}}{\varepsilon_{0}} \\
& \text { E.A }+ \text { E.A }=\frac{\sigma A}{\varepsilon_{0}}
\end{aligned}
$$



2A.E $=\frac{\sigma A}{\varepsilon_{0}}$
$\mathrm{E}=\frac{\sigma}{2 \varepsilon_{0}}$

$$
\sigma=\frac{Q}{A}\left(\mathrm{C} / m^{2}\right) \text { the surface }
$$

charge density)


## Application of the Gauss' law:

4.Solid nonconducting sphere
a)The electric field outside the sphere
$\oint E . d A=\frac{Q_{\text {in }}}{\varepsilon_{0}}$
$E \times 4 \pi r^{2}=\frac{Q_{i n}}{\varepsilon_{0}}$
$E=\frac{Q_{\text {in }}}{4 \pi r^{2} \varepsilon_{0}}$

$\rho=\frac{Q}{V}\left(\mathrm{c} / m^{3}\right)($ the volume charge density)

## Application of the Gauss' law:

Solid nonconducting sphere
b)The electric field inside the sphere( $r<R$ )
$\oint E . d A=\frac{Q_{\text {in }}}{\varepsilon_{0}}$
$E \times 4 \pi r^{2}=\frac{Q_{i n}}{\varepsilon_{0}}$
$E=\frac{Q^{\prime}}{4 \pi r^{2} \varepsilon_{0}}$
$E=\frac{Q^{\prime}}{4 \pi r^{2} \varepsilon_{0}}$
$\rho=\frac{Q}{V}=\frac{Q^{\prime}}{V}$
$E=\frac{K Q r}{R^{3}}$
$\frac{Q}{4 / 3 \pi R^{3}}=\frac{Q^{\prime}}{4 / 3 \pi r^{3}}$
$Q^{\prime}=\frac{Q r^{3}}{R^{3}}$


Gaussian surface

## Application of the Gauss' law:

c) The electric field on the surface of the sphere( the radius is R )

$$
E=\frac{Q R}{4 \pi R^{3} \varepsilon_{0}}=\frac{Q}{4 \pi R^{2} \varepsilon_{0}}
$$



## Application of the Gauss' law:

-5) conducting sphere and thin shell

- A) $r>R$
$\Phi=\oint E \cdot d A=\frac{Q}{\varepsilon_{0}}$
$E \oint d A \cos 0=\frac{Q}{\varepsilon_{0}}$
$E \times 4 \pi R^{2}=\frac{Q}{\varepsilon_{0}}$
$E=\frac{Q_{\text {in }}}{4 \pi R^{2} \varepsilon_{0}}=\frac{K Q}{R^{2}}$

b) $r<R$
$E \times 4 \pi R^{2}=\frac{Q}{\varepsilon_{n}}=0 \quad$ (there is no charge in side the conductor)


## Application of the Gauss' law:

6)Thick conducting shell

- E outside
$\oint E . d A=\frac{Q}{\varepsilon_{0}}$
$E \times 4 \pi R^{2}=\frac{Q}{\varepsilon_{0}}$
$E \times 4 \pi R^{2}=\frac{(+10-5) \times 10^{-6}}{\varepsilon_{0}}$
$\mathrm{E}=\frac{5 \times 10^{-6}}{4 \pi R^{2} \varepsilon_{0}}$
.E inside

$E \times 4 \pi R^{2}=\frac{-5 \times 10^{-6}}{\varepsilon_{0}}$


## Application of the Gauss' law:

- Find E when $\mathrm{r}>5$ ?
$E \times 4 \pi R^{2}=\frac{-5 \times 10^{-6}}{\varepsilon_{0}}$
- Find E when $5<\mathrm{r}<3$ ?
$\mathrm{E}=0$
- Find E when $\mathrm{r}<3$ ?
$\mathrm{E}=0$ (there is no charge inside the shell)


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Electric physics II
Potential difference and electric potential
And capacitors
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## Potential difference and electric potential

When a test charge $q_{0}$ is placed in an electric field $\mathbf{E}$ created by some source charge distribution, the electric force acting on the test charge is $q_{0} \mathbf{E}$. The force $q_{0} \mathbf{E}$ is conservative because the force between charges described by Coulomb's law is conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. This is analogous to the situation of lifting an object with mass in a gravitational field-the work done by the external agent is $m g h$ and the work done by the gravitational force is $-m g h$.

When analyzing electric and magnetic fields, it is common practice to use the notation $d l$ to represent an infinitesimal displacement vector that is oriented tangent to a path through space. This path may be straight or curved, and an integral performed along this path is called either a path integral or a line integral (the two terms are synonymous).

For a given position of the test charge in the field, the charge-field system has a potential energy $U$ relative to the configuration of the system that is defined as $U=0$. Dividing the potential energy by the test charge gives a physical quantity that depends only on the source charge distribution. The potential energy per unit charge $U / q_{0}$ is independent of the value of $q_{0}$ and has a value at every point in an electric field. This quantity $U / q_{0}$ is called the electric potential (or simply the potential) $V$. Thus, the electric potential at any point in an electric field is

$$
\begin{equation*}
V=\frac{U}{q_{0}} \tag{25.2}
\end{equation*}
$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

## Work and Potential (V)

The work done by the electric force in moving a test charge from point $a$ to point $b$ is given by

$$
W_{a \rightarrow b}=\int_{a}^{b} \vec{F} \cdot d \vec{l}=\int_{a}^{b} q_{0} \vec{E} \cdot d \vec{l}
$$

Dividing through by the test charge $\mathrm{q}_{0}$ we have

$$
V_{a}-V_{b}=\int_{a}^{b} \vec{E} \cdot d \vec{l}
$$

Rearranging so the order of the subscripts is the same on both sides

$$
V_{b}-V_{a}=-\int_{a}^{b} \vec{E} \cdot d \vec{l}
$$

## Electric Potential

From this last result $\quad V_{b}-V_{a}=-\int_{a}^{b} \vec{E} \cdot d \vec{l}$
We get $\quad d V=-\vec{E} \cdot d \vec{l}$ or $\frac{d V}{d x}=-E$
We see that the electric field points in the direction of decreasing potential Work (W) = Ua - Ub = q (Va - Vb)
We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

## Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of $e\left(1.6 \times 10^{-19} \mathrm{C}\right)$ that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$
W=q \Delta V=1.6 \times 10^{-19} \text { joules }=1 \mathrm{eV}
$$

## Electric Potential

General Points for either positive or negative charges
The Potential increases if you move in the direction
opposite to the electric field
and


The Potential decreases if you move in the same direction as the electric field

$$
\begin{aligned}
\Delta V & =-E d \cos 0 \\
\Delta V & =\Theta E d
\end{aligned}
$$

## Example 1

Points $\mathrm{A}, \mathrm{B}$, and C lie in a uniform electric field.


What is the potential difference between points A and B ? $\Delta V_{\mathrm{AB}}=V_{\mathrm{B}}-V_{\mathrm{A}}$
a) $\Delta V_{\mathrm{AB}}>0$
b) $\Delta V_{A B}=0$
c) $\Delta V_{\mathrm{AB}}<0$

The electric field, $E$, points in the direction of decreasing potential

Since points $A$ and $B$ are in the same relative horizontal location in the electric field there is no potential difference between them

## Example 2



Point C is at a higher potential than point A .
True
False

As stated previously the electric field points in the direction of decreasing potential

Since point C is further to the right in the electric field and the electric field is pointing to the right, point C is at a lower potential

The statement is therefore FALSE

## Example 3

Points $A, B$, and $C$ lie in a uniform electric field.


If a negative charge is moved from point $A$ to point $B$, its electric potential energy
a) Increases.
b) decreases.
c) doesn't change.

The potential energy of a charge at a location in an electric field is given by the product of the charge and the potential at the location ( $\mathrm{pE}=\mathrm{q} \Delta v$ )

As shown in Example 1, the potential at points $A$ and $B$ are the same

Therefore the electric potential energy also doesn't change

## Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of $e\left(1.6 \times 10^{-19} \mathrm{C}\right)$ that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$
W=q \Delta V=1.6 \times 10^{-19} \text { joules }=1 \mathrm{eV}
$$

## Electric Potential (V) due to point of charge

We define the term to the right of the summation as the electric potential at point $a$

$$
\begin{aligned}
\text { Electric_Potential }{ }_{a} & =\sum_{i} \frac{1}{4 \pi \varepsilon_{0}} \frac{q_{i}}{r_{i}} \\
\mathrm{~V} & =\sum \frac{k q_{i}}{r_{i}}
\end{aligned}
$$

Like energy, potential is a SCALAR
We define the potential of a given point charge as being

$$
\text { Potential }=V=\frac{1}{4 \pi \varepsilon_{0}} \underline{q}
$$

This equation has the convention that the potential is zero at infinite distance

## Example

Question: A particle of charge $q_{1}=+6.0 \mu \mathrm{C}$ is located on the $x$-axis at the point $x_{1}=5.1 \mathrm{~cm}$. A second particle of charge $q_{2}=-5.0 \mu \mathrm{C}$ is placed on the $x$-axis at $x_{2}=-3.4 \mathrm{~cm}$. What is the absolute electric potential at the origin $(x=0)$ ? How much work must we perform in order to slowly move a charge of $q_{3}=-7.0 \mu \mathrm{C}$ from infinity to the origin, whilst keeping the other two charges fixed?

$$
\begin{aligned}
& V_{1}=k_{\mathrm{e}} \frac{q_{1}}{x_{1}}=\left(8.988 \times 10^{9}\right) \frac{\left(6 \times 10^{-6}\right)}{\left(5.1 \times 10^{-2}\right)}=1.06 \times 10^{6} \mathrm{~V} \\
& V_{2}=k_{e} \frac{q_{2}}{\left|x_{2}\right|}=\left(8.988 \times 10^{9}\right) \frac{\left(-5 \times 10^{-6}\right)}{\left(3.4 \times 10^{-2}\right)}=-1.32 \times 10^{6} \mathrm{~V}
\end{aligned}
$$

The net potential V at the origin is simply the algebraic sum of the potentials due to each charge taken in isolation. Thus,

$$
V=V_{1}+V_{2}=-2.64 \times 10^{5} \mathrm{~V}
$$

The work W which we must perform in order to slowly moving a charge q 3 from infinity to the origin is simply the product of the charge and the potential difference. Thus,

$$
W=q_{3} V=\left(-7 \times 10^{-6}\right)\left(-2.64 \times 10^{5}\right)=1.85 \mathrm{~J}
$$

## Electric Potential(v) due to dipole

Line $B A$ is on the $z$ axis. The positive charge is at ( $0,0, a$ ) and the negative charge is at ( $0,0,-a$ )
We consider an electrical potential at the point $P$, due to the electric dipole moment

$$
\begin{aligned}
& v_{A P}=\frac{k q}{r_{1}} \\
& v_{B P}=\frac{k q}{r_{2}} \\
& v_{\text {total }}=k q\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)
\end{aligned}
$$



## Equipotential Surfaces

- It is possible to move a test charge from one point to another without having any net work done on the charge.
- This occurs when the beginning and end points have the same potential
- It is possible to map out such points and a given set of points at the same potential form an equipotential surface


## Equipotential Surfaces

- The electric field does no work as a charge is moved along an equipotential surface
- Since no work is done, there is no force, qE, along the direction of motion
- The electric field is perpendicular to the equipotential surface

- Capacitor


## Capacitance and Dielectrics

Consider two conductors carrying charges of equal magnitude and opposite sign, as shown in Figure 26.1. Such a combination of two conductors is called a capacitor. The conductors are called plates. A potential difference $\Delta V$ exists between the conductors due to the presence of the charges.

What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge $Q$ on a capacitor ${ }^{1}$ is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors. ${ }^{2}$ We can write this relationship as $Q=C \Delta V$ if we define capacitance as follows:

The capacitance $C$ of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$
\begin{equation*}
C \equiv \frac{Q}{\Delta V} \tag{26.1}
\end{equation*}
$$

Note that by definition capacitance is always a positive quantity. Furthermore, the charge $Q$ and the potential difference $\Delta V$ are always expressed in Equation 26.1 as positive quantities. Because the potential difference increases linearly with the stored charge, the ratio $Q / \Delta V$ is constant for a given capacitor. Therefore, capacitance is a measure of a capacitor's ability to store charge. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

From Equation 26.1, we see that capacitance has SI units of coulombs per volt. The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday:

$$
1 \mathrm{~F}=1 \mathrm{C} / \mathrm{V}
$$

Suppose that we have a capacitor rated at 4 pF . This rating means that the capacitor can store 4 pC of charge for each volt of potential difference between the two conductors. If a $9-\mathrm{V}$ battery is connected across this capacitor, one of the conductors wimber ends up with a net charge of -36 pC and the other ends up with a net charge of +36 pC .

Figure 26.2 A parallel-plate capacitor consists of two parallel conducting plates, each of area $A$, separated by a distance $d$. When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.


## Types of capacitors

- 1- parallel plate capacitor $c=\frac{Q}{v}=\frac{\varepsilon_{0} A}{d} \quad \ldots . . .(26-3) \quad$ due to the geometry

parallel plate capacitor

2- cylindrical capacitor
$c=\frac{Q}{v}=\frac{2 \pi \varepsilon_{0} l}{\ln \frac{a}{b}} \quad$ due to the geometry
Where $\mathrm{a}=$ the small radios, $\mathrm{b}=$ the bigger radios and $l$ : the length of the cylinder

## Types of capacitors

## 3- Spherical capacitor

$c=\frac{Q}{v}=\frac{4 \pi \varepsilon^{\circ} a b}{b-a}$
due to the geometry

- Where $a=$ the small radios, $b=$ the bigger radios



## Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area $A=2.00 \times 10^{-4} \mathrm{~m}^{2}$ and a plate separation $d=1.00 \mathrm{~mm}$. Find its capacitance.

Solution From Equation 26.3, we find that

$$
\begin{aligned}
C & =\frac{\epsilon_{0} A}{d}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(2.00 \times 10^{-4} \mathrm{~m}^{2}\right)}{1.00 \times 10^{-3} \mathrm{~m}} \\
& =1.77 \times 10^{-12} \mathrm{~F}=1.77 \mathrm{pF}
\end{aligned}
$$

## Connection of Capacitors

## 1- parallel combination

The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

$\mathrm{V}=v_{1}=v_{2}=v_{3}$
. The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitor

$$
\mathrm{Q}=Q_{1}+Q_{2}+Q_{3}
$$

The equivalent capacitance of parallel connected is the algebraic sum of individual capacitance
$C_{e q}=c_{1}+c_{2}+c_{3} \quad$ parallel connection

$$
\begin{aligned}
& \mathrm{Q} 1=c_{1} v \\
& \mathrm{Q} 2=c_{2} v \\
& \mathrm{Q} 3=c_{3} v \\
& C_{e q} v=c_{1} v+c_{2} v+c_{3} v \\
& \mathrm{Q}=Q_{1}+Q_{2}+Q_{3} \\
& C_{e q}=c_{1}+c_{2}+c_{3}
\end{aligned}
$$

## Series combination

- The charges on capacitors connected in series are the same
- $Q=Q_{1}=Q_{2}=Q_{3}$
- The total potential differences across any number of capacitors connected in series is the sum of the potential difference across the individual capacitors

- $\mathrm{v}=v_{1}+v_{2}+v_{3}$

$$
\begin{aligned}
& Q=c_{1} v_{1}=c_{2} v_{2}=c_{3} v_{3} \\
& \mathrm{v}=v_{1}+v_{2}+v_{3}
\end{aligned}
$$

- The inverse of the equivalent capacitance of series connection is the algebraic sum of inverse of the individual capacitances
- $\frac{1}{c_{e q}}=\frac{1}{c_{1}}+\frac{1}{c_{2}}+\frac{1}{c_{3}}$

$$
\begin{aligned}
& \frac{Q}{c_{e q}}=\frac{Q}{c_{1}}+\frac{Q}{c_{2}}+\frac{Q}{c_{3}} \\
& \frac{1}{c_{e q}}=\frac{1}{c_{1}}+\frac{1}{c_{2}}+\frac{1}{c_{3}}
\end{aligned}
$$

## Example

Find the equivalent capacitance between $a$ and $b$ for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.

Solution Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The $1.0-\mu \mathrm{F}$ and $3.0-\mu \mathrm{F}$ capacitors are in parallel and combine according to the expression $C_{\mathrm{eq}}=C_{1}+C_{2}=4.0 \mu \mathrm{~F}$. The $2.0-\mu \mathrm{F}$ and $6.0-\mu \mathrm{F}$ capacitors also are in parallel and have an equivalent capacitance of $8.0 \mu \mathrm{~F}$. Thus, the upper branch in Figure 26.11 b consists of two $4.0-\mu \mathrm{F}$ capacitors in series, which combine as follows:

$$
\begin{aligned}
\frac{1}{C_{\mathrm{eq}}} & =\frac{1}{C_{1}}+\frac{1}{C_{2}}=\frac{1}{4.0 \mu F}+\frac{1}{4.0 \mu F}=\frac{1}{2.0 \mu F} \\
C_{\mathrm{eq}} & =2.0 \mu \mathrm{~F}
\end{aligned}
$$

The lower branch in Figure 26.11b consists of two $8.0-\mu \mathrm{F}$ capacitors in series, which combine to yield an equivalent capacitance of $4.0 \mu \mathrm{~F}$. Finally, the $2.0-\mu \mathrm{F}$ and $4.0-\mu \mathrm{F}$ capacitors in Figure 26.11c are in parallel and thus have an
equivalent capacitance of $6.0 \mu \mathrm{~F}$.


Figure 26.11 (Example 26.4) To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b). (c), and (d), using the series and parallel rules described in the text.

## Energy Stored in a Charged Capacitor

- The work done in charging the capacitor appears as electric potential energy $U$ stored in the capacitor as in the following forms
- $U=\frac{1}{2} c v^{2} \ldots . . . . . .1$
- $U=Q V$... .... 2
- $U=\frac{1}{2} \frac{Q^{2}}{C}$ .3
- The unite of energy is joule (J)


## Energy density

- The energy per unite volume known as the energy density
- $u_{E}=\frac{U}{v}$
- $u_{E}=\frac{\frac{1}{2} c v^{2}}{A d}=\frac{\frac{1}{2} \varepsilon_{0} A v^{2}}{d A d}$
- $u_{E}=\frac{1}{2} \varepsilon_{\circ} E^{2} \quad$ the unite is $\left(J / m^{3}\right)$

The energy density in any electric field is proportional to the square of magnitude of the electric field at a given point.

## Dielectric

- The capacitance of a set of charged parallel plates is increased by the insertion of a dielectric material.
- $C_{\circ}=\frac{\varepsilon_{\circ} A}{d}$
- Also $C_{\circ}=\frac{Q_{\circ}}{v_{\circ}}$

The dielectric is the free space (air)

- if we put the dielectric between two plates of the capacitors, then the capacitance is increased as in the
 following form
- $\mathrm{C}=\frac{k \varepsilon_{0} A}{d}$ where $\mathrm{k}=$ the dielectric constant
- $\mathrm{K}=\frac{c}{c_{0}}$


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Electric physics II
Electric circuits
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## Basic concepts

Electricity: Physical phenomenon arising from the existence and interactions of electric charge * Charge

* Current
* Voltage
* Power and Energy



## Electric current



An ampere (A) is the number of electrons having a total charge of 1 C moving through a given cross section in 1 sec .

As defined, current flows in direction of positive charge flow

## Current density

- It is the amount of current flowing in a unit area and its symbol (J).
- $\mathrm{J}=\frac{I}{A}$
- The unite of current density is $\left(\frac{A}{m^{2}}\right)$


## Electric circuit

An electric circuit is an interconnection of electrical elements linked together in a closed path so that electric current may flow continuously

Circuit diagrams are the standard for electrical engineers


## Voltage

The voltage across an element is the work (energy) required to move a unit of positive charge from the " - " terminal to the " + " terminal


A volt is the potential difference (voltage) between two points when $\mathbf{1}$ joule of energy is used to move 1 coulomb of charge from one point to the other

## Power

The rate at which energy is converted or work is performed


A watt results when $\mathbf{1}$ joule of energy is converted or used in $\mathbf{1}$ second

Power Dissipated in Resistor

$$
\mathrm{V}\}^{\mathrm{I}} \mathrm{P}=\mathrm{VI}=\frac{\mathrm{V}^{2}}{\mathrm{R}}=\mathrm{I}^{2} \mathrm{R}
$$

## Resistors



Resistance ( R ) is the physical
property of an element that impedes the flow of current. The units of resistance are $\mathbf{O h m s}(\Omega)$

Resistivity ( $\rho$ ) is the ability of a material to resist current flow. The units of resistivity are Ohm-meters ( $\Omega$-m)

Example:

| Resistivity of copper | $1.68 \times 10^{-8} \Omega \cdot \mathrm{~m}$ |
| :--- | :--- |
| Resistivity of glass | $10^{10}$ to $10^{14} \Omega \cdot \mathrm{~m}$ |

## Resistors



## Resistors

Standard ELA Color Code Table 4 Band $-2^{2}, 45^{2} /$ and $\pm 10^{2}$


| Coter | 10 Eand <br>  | Znol Band Fnd hignem | 3rd Band (mathiplar | sth Bind Pobonarical |
| :---: | :---: | :---: | :---: | :---: |
| Elich | 4 | 0 | 14- |  |
| Erawn | 1 | 1 | 101 |  |
| Rod | 2 | 2 | 141 | Yx |
| Dnint | 3 | 3 | 19 |  |
| Yollow | 4 | 4 | 14 |  |
| Stopy | 5 | 5 | 10 |  |
| Bive | 6 | $\theta$ | 14 |  |
| Hatrt | 7 | 7 | 40 |  |
| Crmy | 9 | F | $10^{2}$ |  |
| White | 9 | 9 | $10^{19}$ |  |
| Gold |  |  | 101 | 159 |
| Elum |  |  | $10^{4}$ | $\pm 10 \%$ |

## Ohm's Law



$$
R=\frac{\rho L}{A}
$$

(remember, R is in $\Omega$ and $\rho$ is in $\Omega$.m)

## Ohm's Law

$$
V=R I
$$

- The resistor consume energy this energy is consumed as a heat

If the temperature increase the resistivity ( $\rho$ ) also increase du to the following formula

$$
\rho=\rho_{\circ}\left(1+\alpha\left(T-T_{\circ}\right)\right.
$$

Where $\alpha$ is the temperature coefficient of resistivity and its unite $\left(\frac{1}{c^{c}}\right)$

And ( $T$ ) measured in kelvin or centigrade
Can find the resistance from the above formula above

$$
R=R_{\circ}\left(1+\alpha\left(T-T_{\circ}\right)\right.
$$

## Electrical sources



An electrical source is a voltage or current generator capable of supplying energy to a circuit

Examples:
-AA batteries
-12-Volt car battery
-Wall plug

## Ideal voltage source

An ideal voltage source is a circuit element where the voltage across the source is independent of the current through it.


Recall Ohm's Law: V=IR

The internal resistance of an ideal voltage source is zero.


If the current through an ideal voltage source is completely determined by the external circuit, it is considered an independent voltage source

Figure 1: An ideal voltage source, V,
driving a resistor, $R$, and creating a current $I$

## Ideal current source

An ideal current source is a circuit element where the current through the source is independent of the voltage across it.


## Recall Ohm's Law: I = V/R

The internal resistance of an ideal current source is infinite.

If the voltage across an ideal current source is completely determined by the external circuit, it is considered an independent current source

## Dependent Sources

A dependent or controlled source depends upon a different voltage or current in the circuit


## Electric Circuit Design Principles

## Resistors in series

The resistors in a series circuit are $680 \Omega, 1.5 \mathrm{k} \Omega$, and $2.2 \mathrm{k} \Omega$. What is the total resistance?


## Series circuits



A series circuit with a voltage source (such as a battery) and 3 resistors

A series circuit has only one current path

Current through each component is the same

In a series circuit, all elements must function for the circuit to be complete


## Multiple elements in a series circuit



$$
\stackrel{+\|_{1}}{\circ}
$$

$$
\begin{aligned}
& R_{t o t a l}=R_{1}+R_{2}+\ldots+R_{n} \\
& L_{\text {total }}=L_{1}+L_{2}+\ldots+L_{n}
\end{aligned}
$$

$$
\frac{1}{C_{t o t a l}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\ldots+\frac{1}{C_{n}}
$$

$$
V_{\text {total }}=V_{1}+V_{2}+\ldots+V_{n}
$$

## Example: Resistors in series

The resistors in a series circuit are $680 \Omega, 1.5 \mathrm{k} \Omega$, and $2.2 \mathrm{k} \Omega$. What is the total resistance?


$$
\begin{aligned}
R_{\text {total }} & =R_{1}+R_{2}+R_{3} \\
& =680 \Omega+1500 \Omega+2200 \Omega \\
& =4380 \Omega \\
& =4.38 k \Omega
\end{aligned}
$$

The current through each resistor?

$$
I=\frac{V}{R_{\text {total }}}=\frac{12 \mathrm{~V}}{4380 \Omega}=2.74 \mathrm{~mA}
$$

## Example: Voltage sources in series

Find the total voltage of the sources shown

$$
V_{\text {total }}=V_{1}+V_{2}+V_{3}=27 \mathrm{~V}
$$



## Example: Resistors in parallel

The resistors in a parallel circuit are $680 \Omega, 1.5 \mathrm{k} \Omega$, and $2.2 \mathrm{k} \Omega$.
What is the total resistance?


## Parallel circuits



Voltage across each pathway is the same


In a parallel circuit, separate current paths function independently of one another

## Multiple elements in a parallel circuit



For parallel voltage sources, the voltage is the same across all batteries, but the current supplied by each element is a fraction of the total current

## Example: Resistors in parallel

The resistors in a parallel circuit are $680 \Omega, 1.5 \mathrm{k} \Omega$, and $2.2 \mathrm{k} \Omega$. What is the total resistance?


$$
\begin{aligned}
R_{\text {total }} & =\frac{1}{\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}} \\
& =386 \Omega
\end{aligned}
$$

Voltage across each resistor? Dissipated power?
Current through each resistor?

## Circuit Definitions

- Node - any point where 2 or more circuit elements are connected together
- Wires usually have negligible resistance
- Each node has one voltage (w.r.t. ground)
- Branch - a circuit element between two nodes
- Loop - a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice


## Example

- How many nodes, branches \& loops?



## Example

- Three nodes


Example

- 5 Branches



## Example

- Three Loops, if starting at node A



## Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
- Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a - sign first)
- $\Sigma$ voltage drops $-\Sigma$ voltage rises $=0$
- Or $\Sigma$ voltage drops $=\Sigma$ voltage rises


## Example

- Kirchoff's Voltage Law around $1^{\text {st }}$ Loop


Assign current variables and directions
Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

## Example

- Kirchoff's Voltage Law around $1^{\text {st }}$ Loop



## Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchhoff's two laws and Ohm's law to determine all branch voltages and currents



## Series Resistors

- KVL: $+\mathrm{I} \cdot 10 \Omega-12 \mathrm{v}=0$, $\quad$ So $\mathrm{I}=1.2 \mathrm{~A}$
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



## Circuit Analysis

- By Ohm's law: $\mathrm{V}_{\mathrm{AB}}=\mathrm{I} \cdot 7 \Omega$ and $\mathrm{V}_{\mathrm{BC}}=\mathrm{I} \cdot 3 \Omega$
- By KVL: $\mathrm{V}_{\mathrm{AB}}+\mathrm{V}_{\mathrm{BC}}-12 \mathrm{v}=0$
- Substituting: $\mathrm{I} \cdot 7 \Omega+\mathrm{I} \cdot 3 \Omega-12 \mathrm{v}=\mathrm{Q}$
- Solving: I = 1.2 A

Since $V_{A B}=I \cdot 7 \Omega$ and $V_{B C}=I \cdot 3 \Omega$


And $\mathrm{I}=1.2 \mathrm{~A}$
So $\mathrm{V}_{\mathrm{AB}}=8.4 \mathrm{v}$ and $\mathrm{V}_{\mathrm{BC}}=3.6 \mathrm{v}$

## Kirchoff's Current Law (KCL)

- The algebraic sum of currents entering a node is zero
- Add each branch current entering the node and subtract each branch current leaving the node
- $\Sigma$ currents in $-\Sigma$ currents out $=0$
- Or $\Sigma$ currents in $=\Sigma$ currents out


## Example

- Kirchoff's Current Law at B


Assign current variables and directions
Add currents in, subtract currents out: $I_{1}-I_{2}-I_{3}+I s=0$

## Example: Find VAB for the Figure below

By KVL:

$$
-I_{1} \cdot 8 \Omega+I_{2} \cdot 4 \Omega=0 \longmapsto I_{2}=2 \cdot I_{1}
$$

By KCL:

$$
10 A=I_{1}+I_{2}
$$

Substituting:

$$
10 A=I_{1}+2 \cdot I_{1}=3 \cdot I_{1}
$$

So

$$
I_{1}=3.33 \mathrm{~A} \quad I_{2}=6.67 \mathrm{~A}
$$

And $V_{A B}=I_{2} \cdot 4=26.33$ volts

## Another Way



## By Ohm's Law: $\mathrm{V}_{\mathrm{AB}}=10 \mathrm{~A} \cdot 2.667 \Omega$

So $V_{A B}=26.67$ volts
Replacing two parallel resistors (8 and $4 \Omega$ ) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

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Electric physics II
Alternating current(A.C) circuits

## By

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2020-2021

## Alternating current circuits (A.C) circuits

- Electrical appliances in the house use alternating current (AC) circuit
- If an AC sources applies an alternating voltage to a series circuit containing resistor, inductor, and capacitor, what are the amplitude and time characteristics of the alternating current.
- An AC circuit consist of combination of circuit element and power source
- The power source provides an alternating voltage, $\Delta v$


## Alternating voltage (A.C voltage)

- The output from an AC power source is sinusoidal and varies with time according to the equation
- $\Delta v=\Delta V_{\max } \sin \omega t$
- Where:
- $\Delta v$ : instantaneous voltage
- $\Delta V_{\text {max }}$ : maximum output voltage of source also called the voltage amplitude
- $\omega$ : the angular frequence of the $A C$ voltage

The angular frequency is
$\omega=2 \pi f=\frac{2 \pi}{T}$
F is the frequency of the source
T is the period of source
The voltage is positive during one half of the cycle and negative during the other half


- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time.
- Commercial electric power plants in the us use a frequency of 50 Hz and 60 at the USA and Saudi Arabia .


## Resisters in an AC circuit

Consider a circuit consisting of an AC source and a resistor
The AC source is symbolized by


$$
\begin{gathered}
\text { Apply Kirchhoff's loop rule Therefore, } \\
\Delta v+\Delta v_{R}=0 \\
\Delta v-i_{R} R=0 \\
\Delta V_{\max } \sin (\omega t)-i_{R} R=0 \\
i_{R}=\frac{\Delta V_{\max } \sin (\omega t)}{R}=I_{\max } \sin (\omega t)
\end{gathered}
$$

Where: $I_{\max }=\frac{\Delta V_{\max }}{R}$

$$
\Delta v_{R}=I_{\max } R \sin \omega t
$$

## Resisters in an AC circuit

- The graph shows the current through and the voltage across the resistor
- The current and the voltage reach their maximum value at the same time
- The current and the voltage are said to be in phase
- For a sinusoidal applied voltage , the current in a resistor is always in phase with the voltage
- The direction of the current has no effect on the behavior of the resistor


## Phaser diagram

-To simplify the analysis of AC circuits, a graphical constructor called a phasor diagram can be used.
-A phasor is a vector whose length is proportional to the maximum value of the variable it represents.
-The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.
-The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.


## RMS current and voltage

- The average current in one cycle is zero.
- Resistors experience a temperature increase which depends on the magnitude of the current, but not the direction of the current.
- The power is related to the square of the current.
- The rms current is the average of importance in an ac circuit.
- RMS STANDS FOR ROOT MEAN SQUARE

$$
I_{\operatorname{ma}}=\frac{I_{\max }}{\sqrt{2}}=0.707 I_{\max }
$$

- Alternating voltages can also be discussed in terms of rms values.

$$
\Delta V_{m=}=\frac{\Delta V_{\max }}{\sqrt{2}}=0.707 \Delta V_{\max }
$$

## Note about RMS value

- RMS values are used when discussing alternating current and voltage because
- AC ammeter and voltmeter are designed to read RMS value


## Example

The voltage output of an AC source is given by the expression $\Delta v=$ $200 \sin (\omega t)$, where $\Delta v$ is in volts. Find the rms current in the circuit when this source is connected to a $100-\Omega$ resistor.

$$
\begin{aligned}
& \Delta v=200 \sin \omega t \\
& \quad v_{r m s}=0.707 * 200=141.4 \mathrm{v} \\
& I_{r m s}=\frac{v_{r m s}}{R}=\frac{141.4}{100}=1.414 \mathrm{~A}
\end{aligned}
$$

## Inductors in an AC circuit

-Kirchhoff's loop rule can be applied and gives:

$$
\begin{aligned}
& \Delta v+\Delta V_{L}=0, \text { or } \\
& \Delta v-L \frac{d i}{d t}=0 \\
& \Delta v=L \frac{d i}{d t}=\Delta V_{m a} \sin \omega t
\end{aligned}
$$

$i_{L}=\frac{\Delta V_{\text {max }}}{L} \int \sin \omega t d t=-\frac{\Delta V_{\text {max }}}{\omega L} \cos \omega t$
$i_{L}=\frac{\Delta V_{\max }}{\omega L} \sin \left(\omega t-\frac{\pi}{2}\right) \quad I_{\max }=\frac{\Delta V_{\max }}{\omega L}$


This shows that the instantaneous current $i_{l}$ in the inductor and the instantaneous voltage $\Delta v_{l}$ across the inductor are out of phase by $(\pi / 2) \mathrm{rad}=90^{\circ}$

## phase relationship of inductors in an AC circuit

- For a sinusoidal applied voltage, the current in an

The current lags the voltage by one-fourth of a cycle. inductor always lags behind the voltage across the inductor by $90^{\circ}(\pi / 2)$.

- The current is a maximum when the voltage across the inductor is zero.


## Phasor diagram for an inductor

- The phasors are at $90^{\circ}$ with respect to each other.
- This represents the phase difference between the current and voltage.
- Specifically, the current lags behind the voltage by $90^{\circ}$.

The current and voltage phasors are at $90^{\circ}$ to each other.


## Inductive reactance

- The factor $\omega L$ has the same units as resistance and is related to current and voltage in the same way as resistance.
- Because $\omega L$ depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.
- The factor is the inductive reactance and is given by:

$$
x_{L}=\omega l
$$

## Inductive reactance

- Current can be expressed in terms of the inductive reactance:

$$
I_{\text {max }}=\frac{\Delta V_{\text {max }}}{X_{L}} \text { or } I_{\text {max }}=\frac{\Delta V_{\text {ma }}}{X_{L}}
$$

- As the frequency increases, the inductive reactance increases

The voltage across inductive
-The instantaneous voltage across the inductor is

$$
\begin{aligned}
\Delta v_{L} & =-L \frac{d i}{d t} \\
& =-\Delta V_{\max } \sin \omega t \\
& =-I_{\max } X_{L} \sin \omega t
\end{aligned}
$$

## example

Ex. 31.2 In a purely inductive AC circuit, $L=25.0 \mathrm{mH}$ and the rms voltage is 150 V . Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz .

$$
\begin{aligned}
& X_{l}=\omega L=2 \pi f l=2 * 60 * \pi * 25 * 10^{-3}=3^{*} 10^{3} \pi \text { ohm } \\
& I_{r m s}=\frac{\Delta v_{r m s}}{x_{L}}=\frac{150}{3 * 10^{3} \pi}=15.92 \mathrm{~mA}
\end{aligned}
$$

## Capacitors in an AC circuit

- The circuit contains a capacitor and an AC source.
- Kirchhoff's loop rule gives:

$$
\begin{gathered}
\Delta V+\Delta V_{\mathrm{C}}=0 \text { And so } \\
i_{C}=\frac{d q}{d t}=\omega C \Delta V_{\max } \cos \omega t \\
\text { or } i_{c}=\omega C \Delta V_{\max } \sin \left(\omega t+\frac{\pi}{2}\right) \\
i_{c}=\omega c \Delta V_{\max } \sin \left(\omega t+\frac{\pi}{2}\right)
\end{gathered}
$$



The current is $\pi / 2 \mathrm{rad}=90^{\circ}$ out of phase with the voltage

## Capacitors in an AC circuit

- The current reach its maximum value before the voltage reach's its maximum value
- The current lead the voltage by $90^{\circ}$


## Phaser diagram for capacitor

The current leads the voltage by one-fourth of a cycle.

The current and voltage phasors are at $90^{\circ}$ to each other.


## Capacitive reactance

- The maximum current in the circuit occurs at $\cos \omega t=1 \quad$ which given by

$$
\begin{gathered}
I_{\text {max }}=\omega C \Delta V_{\max }=\frac{\Delta V_{\max }}{(1 / \omega C)} \\
X_{C} \equiv \frac{1}{\omega C} \text { which gives } I_{\operatorname{mxx}} \frac{\Delta V_{\max }}{X_{C}}
\end{gathered}
$$

- The impeding effect of capacitor on the current in an AC circuit is called the capacitive reactance and given by

$$
x_{c} \equiv \frac{1}{\omega C}
$$

## Voltage across capacitor

-The instantaneous voltage across the capacitor can be written as

$$
\Delta v_{c}=\Delta V_{\max } \sin (\omega t)=I_{\max } X_{C} \sin (\omega t)
$$

-As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.
-As the frequency approaches zero, $\mathrm{x}_{\mathrm{c}}$ approaches infinity and the current approaches zero.

- This would act like a dc voltage and the capacitor would act as an open circuit.


## example

## An $8.00 \mu \mathrm{~F}$ capacitor is connected to the terminals of a 60.0 Hz AC source whose rms voltage is 150 V . Find the capacitive reactance and the rms current in the circuit.

1- To find the capacitive reactance

$$
\begin{aligned}
& x_{c}=\frac{1}{2 \pi f c} \\
& x_{c}=\frac{1}{2 \pi * 60 * 8 * 10^{-6}}=\frac{1}{\pi} * 10^{3} \mathrm{ohm}
\end{aligned}
$$

2-To find the rms current

$$
I_{r m s}=\frac{v_{r m s}}{x_{c}}=\frac{150}{\frac{1}{\pi} * 10^{3}}=0.47 \mathrm{~A}
$$

