Reinforced Concrete Structures

INTRODUCTION

Concrete Materials:

Concrete is a mixture of sand, gravel, crushed rock, or other aggregates held together in a rocklike mass with a paste of cement and water. Sometimes one or more admixtures are added to change certain characteristics of the concrete such as its workability, durability, and time of hardening.

- Cement: (Ordinary Portland Cement, Sulphate Resistance Cement, Low Heat Cement ... etc.)
- Water.
- Coarse and Fine Aggregate: (Gravels + Sand), 75 % of concrete mix.
- Admixtures: (Water-reducing Admixtures, Accelerating Admixtures, Coloring Admixtures ... etc.)

Types of Concrete:

Plain Concrete, Reinforced Concrete, Lightweight Concrete, High-Density Concrete , Precast Concrete, Pre-stressed Concrete, Glass Concrete, Rapid Hardening, Roller Compacted, Vacuum, Self-Compact Concrete.


Plain Concrete


Reinforced Concrete


Precast Concrete


Pre-stress Concrete


Foam Concrete

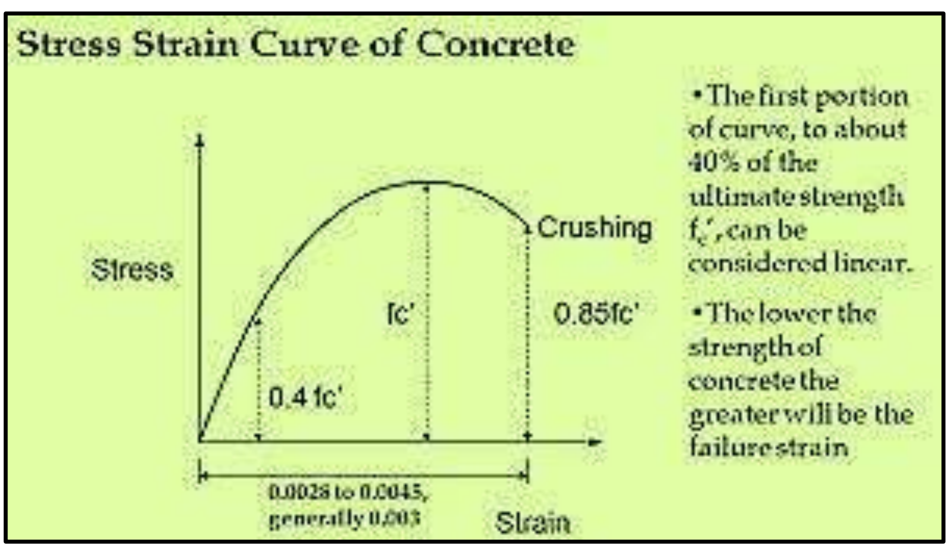

Rolled Compact Concrete

Mechanical Properties of Concrete:

1- **Compressive Strength:** compressive strength is one of the most important engineering properties of concrete. It is a standard industrial practice that the concrete is classified based on grades. This grade is nothing but the Compressive Strength of the concrete cube or cylinder. Cube or Cylinder samples are usually tested under a compression testing machine (7 and 28 days curing) to obtain the compressive strength of concrete.

2- **Tensile Strength:** Also important because it effect on cracks that occurred in structures. It's very low in concrete about (10 – 15 %) compared with compressive strength. There are two ways to test the tensile strength in concrete:
- Splitting Cylinder Test.
- Modulus of Rupture.


Stress - Strain Curve for Concrete :



3- **Modulus of Elasticity for Concrete:** Modulus of Elasticity of Concrete can be defined as the slope of the line drawn from a stress of zero to a compressive stress of 0.45f'c. As concrete is a heterogeneous material. The strength of concrete is dependent on the relative proportion and modulus of elasticity of the aggregate.

For normal-weight concrete (2300 Kg/m3),

$$E_c = 4700\sqrt{f'_c}\,\text{MPa}$$

Reinforced concrete (RC): is a composite material in which concrete's relatively low tensile strength and ductility are counteracted by the inclusion of reinforcement having higher tensile strength or ductility.

★ The overall goal is to be able to design reinforced concrete structures that are:

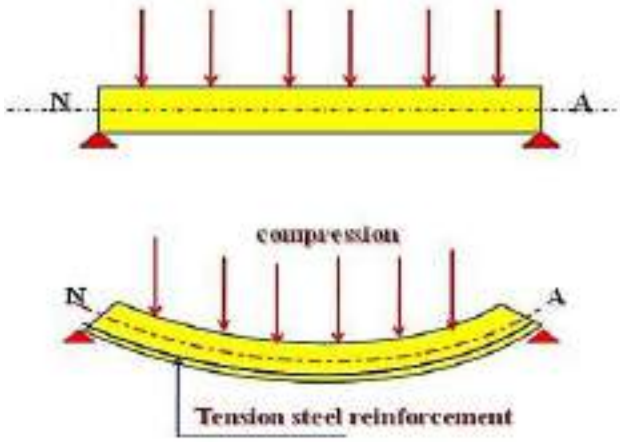
- Safe.
- Economical.
- Efficient.

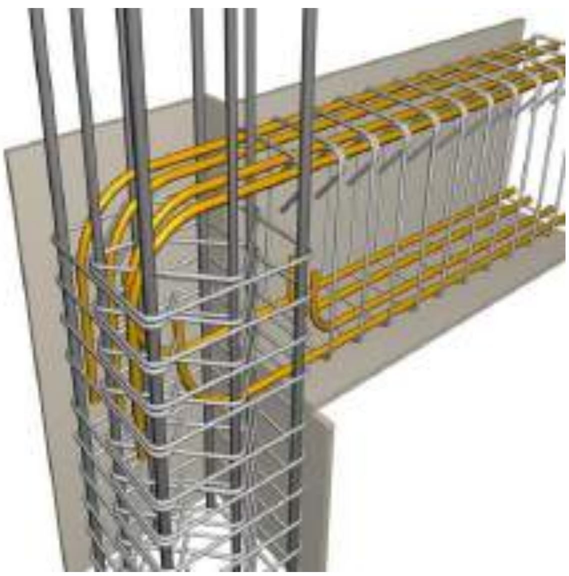
★ Reinforced concrete is one of the principal building materials used in engineered structures because:

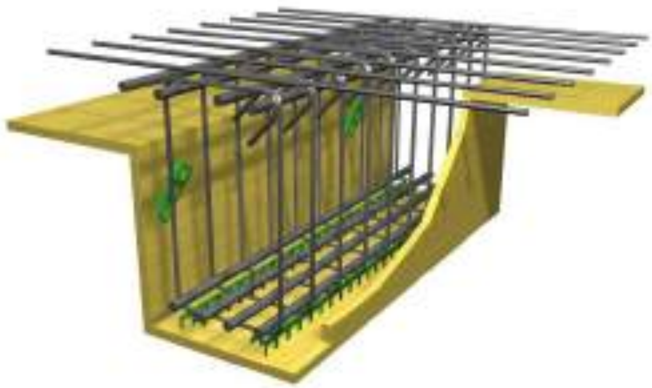
- Low cost.
- Weathering and fire resistance.
- Good compressive strength.
- Formability.

Reinforcing schemes are generally designed to resist tensile stresses in particular regions of the concrete that might cause unacceptable cracking and/or structural failure. Modern reinforced concrete can contain varied reinforcing materials made of steel, polymers or alternate composite material in conjunction with bars or not.

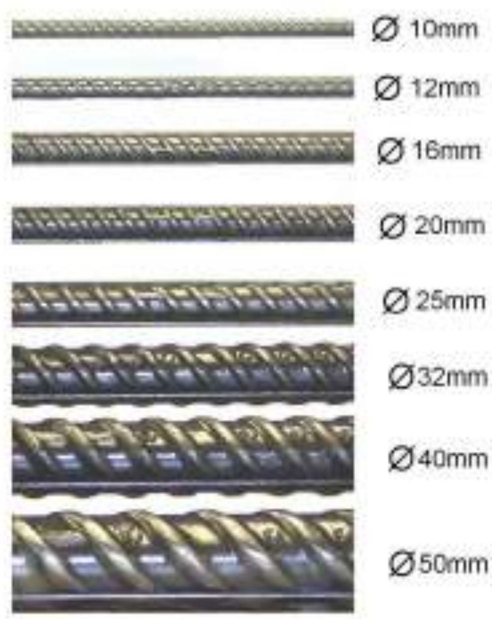
★Steel Location: "Place reinforcing steel where the concrete is in tension"







★Steel Bars Sizes:



★**Grades: (fy / fu in N/mm²):**

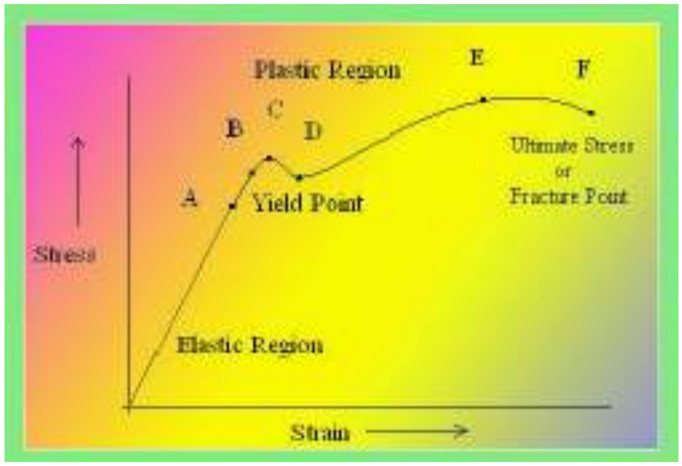
240/360, 280/420, 350/520 and 400/600

fy : Steel Yield Strength

fu: Steel Ultimate Strength

Bar Diameter (mm)	Unit Weight (kg/m)	Sectional Area (mm2)
6	0.222	28.3
8	0.395	50.3
10	0.617	78.5
12	0.888	113.1
16	1.578	201.1
20	2.466	314.2
25	3.853	490.9
32	6.313	804.2
40	9.864	1256.6

Stress – Strain Diagram for Steel:



Typical Structural Elements of a Skeletal R.C. Building:

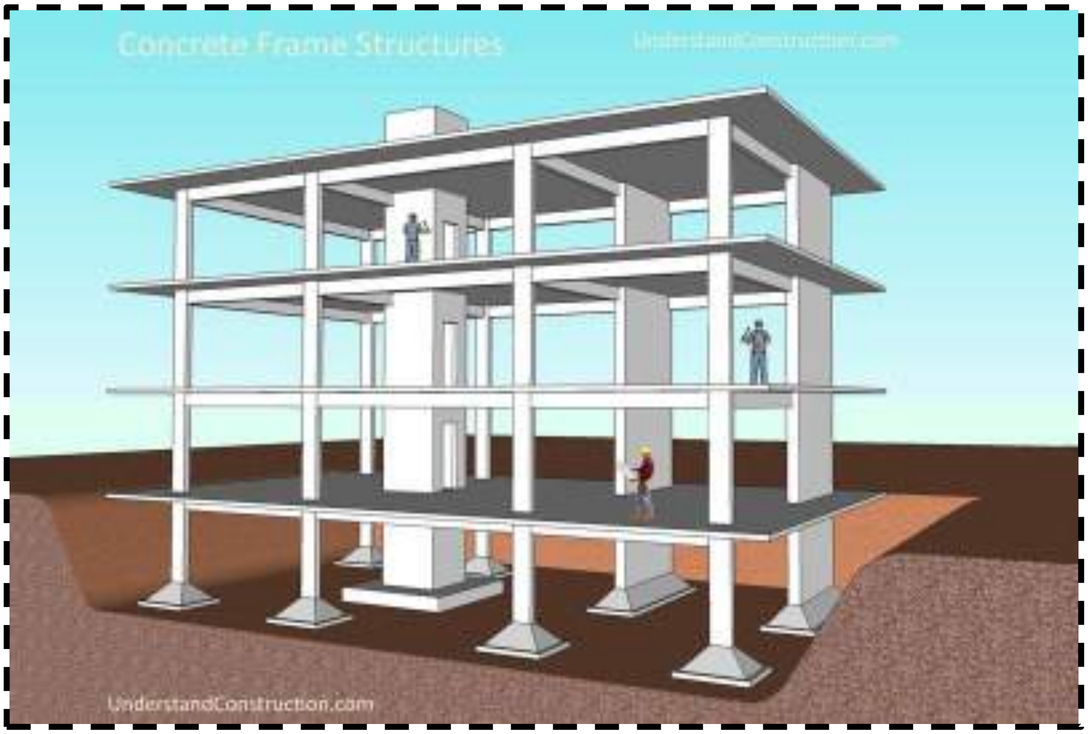
Concrete frame structures are the most common type of modern building. It usually consists of a frame or a skeleton of concrete. Horizontal members are beams and vertical ones are the columns. Concrete Buildings structures also contain slabs which are used as base, as well as roof / ceiling. Among these, the column is the most important as it carries the primary load of the building.

1- Slabs: These are the plate element and carry the loads primarily by flexure. They usually carry the vertical loads. Under the action of horizontal loads, due to a large moment of inertia, they can carry quite large wind and earthquake forces, and then transfer them to the beam.

2- Beams: These carry the loads from slabs and also the direct loads as masonry walls and their Self-Weights. The beams may be supported on the other beams or may be supported by columns forming an integral part of the frame. These are primarily the flexural members.

3- Columns: These are the vertical members carrying loads from the beams and from upper columns. The loads carried may be axial or eccentric. Columns are the most important when compared with beams and slabs. This is because, if one beam fails, it'll be a local failure of one floor but if one column fails, it can lead to the collapse of the whole structure.

4- Foundation: These are the load transmitting members. The loads from the columns and walls are transmitted to the solid ground through the foundations.


Concrete Frame Structures
UnderstandConstruction.com

Structural Loads:

1- **Dead load:** The dead load includes loads that are relatively constant over time, including the weight of the structure itself, in addition to walls, plasterboard or carpet. The roof is also a dead load. Dead loads are also known (static loads).

Density of Some Materials that using in Construction:

Material	Density (kg/m³)
concrete	2300
Asphalt conc.	2400
Bricks	1900
Cement	1400
Clay (wet)	2080
Cement mortar	1440
Concrete (reinforced)	2400
Gypsum	1200
Sand	1650
Concrete Blocks	1400
Gravel	1800
Steel	7850
Wood(average)	400-700
Water	1000

2- **Live load:** Live loads are temporary of short duration, or a moving load. These dynamic loads may involve considerations such as impact, momentum, and vibration.

Live load for Deferent Types of Structures:

Occupancy or Use	Live Load lb/ft^2 (kN/m^2)	Occupancy or Use	Live Load lb/ft^2 (kN/m^2)
Air-conditioning	200[1] (9.58)	Kitchens, other than domestic	150[2] (7.18)
(machine space)		Laboratories, scientific	100[1] (4.79)
Amusement park	100[1] (4.79)	Laundries	150[1] (7.18)
structure		Libraries, corridors	80[1] (3.83)
Attic, nonresidental		Manufacturing, ice	300 (14.36)
Nonstorage	25 (1.20)	Morgue	125 (6.00)
Storage	80[1] (3.83)	Office Buildings	
Bakery	150 (7.18)	Business machine equipment	100[1] (4.79)
Exterior	100 (4.79)	Files (see file room)	
Interior (fixed seats)	60 (2.87)	Printing Plants	
Interior (movable seats)	100 (4.79)	Composing rooms	100 (4.79)
Boathouse, floors	100[1] (4.79)	Linotype rooms	100 (4.79)
Boiler room, framed	300[1] (14.36)	Paper storage	—[4]
Broadcasting studio	100 (4.79)	Press rooms	150[1] (7.18)
Catwalks	25 (1.20)	Public rooms	100 (4.79)
Ceiling, accessible furred	10[6] (0.48)	Railroad tracks	—[5]
Cold storage		Ramps	
No overhead system	250[2] (11.97)	Driveway (see garages)	
Overhead system		Pedestrian (see sidewalks and	
Floor	150 (7.18)	corridors in Table 4–1)	
Roof	250 (11.97)	Seaplane (see hangars)	
Computer equipment	150[1] (7.18)	Rest rooms	60 (2.87)
Courtrooms	50–100 (2.40–4.79)	Rinks	
Dormitories		Ice skating	250 (11.97)
Nonpartitioned	80 (3.83)	Roller skating	100 (4.79)
Partitioned	40 (1.92)	Storage, hay or grain	300[1] (14.36)
Elevator machine room	150[1] (7.18)	Telephone exchange	150[1] (7.18)
Fan room	150[1] (7.18)	Theaters:	
File room		Dressing rooms	40 (1.92)
Duplicating equipment	150[1] (7.18)	Grid-iron floor or fly gallery:	
Card	125[1] (6.00)	Grating	60 (2.87)
Letter	80[1] (3.83)	Well beams, 250 lb/ft per pair	
Foundries	600[1] (28.73)	Header beams, 1,000 lb/ft	
Fuel rooms, framed	400 (19.15)	Pin rail, 250 lb/ft	
Garages—trucks	—[3]	Projection room	100 (4.79)
Greenhouses	150 (7.18)	Toilet rooms	60 (2.87)
Hangars	150[3] (7.18)	Transformer rooms	200[1] (9.58)
Incinerator charging floor	100 (4.79)	Vaults, in offices	250[1] (11.97)

[1] Use weight of actual equipment or stored material when greater.
[2] Plus 150 lb/ft^2 (7.18 kN/m^2) for trucks.
[3] Use American Association of State Highway and Transportation Officials lane loads. Also subject to not less than 100% maximum axle load.
[4] Paper storage 50 lb/ft (2.40 kN/m^2) of clear story height.
[5] As required by railroad company.
[6] Accessible ceilings normally are not designed to support persons. The value in this table is intended to account for occasional light storage or suspension of items. If it may be necessary to support the weight of maintenance personnel, this shall be provided for.

3- **Environmental loads**: Environmental Loads are structural loads caused by natural forces such as:

- Wind loads.
- Snow, rain and ice loads.
- Temperature changes leading to thermal expansion because thermal loads.
- Lateral pressure of soil, groundwater or bulk materials.
- Loads from fluids or floods.
- Dust loads.

4- **Other loads:** Engineers must also be aware of other actions that may affect a structure, such as:

- Foundation settlement or displacement.
- Fire.
- Corrosion.
- Explosion.
- Creep or shrinkage.
- Impact from vehicles or machinery vibration.

How Loads Flow Through a Building?

Elements of building are used to transmit and resist external loads within a building. These elements define the mechanism of load transfer in a building known as the load path. The load path extends from the roof through each structural element to the foundation. An understanding of the critical importance of a complete load path is essential for everyone involved in building design and construction.

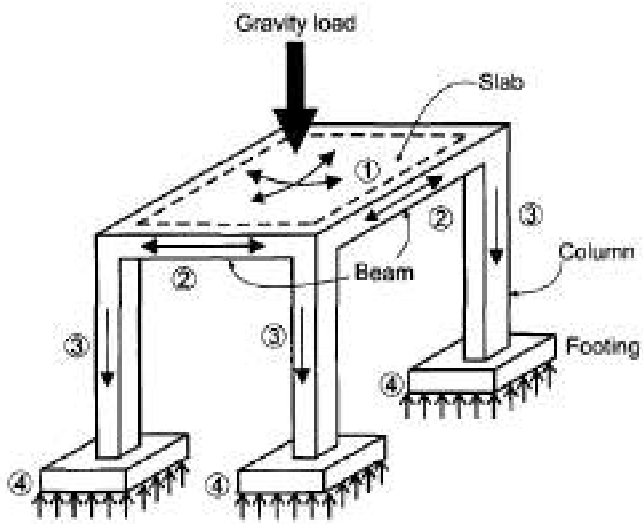
The load path can be identified by considering the elements in the building that contribute to resisting the load and by observing how they transmit the load to the next clement. Depending on the type of load to be transferred, there are two basic load paths:

- **Gravity load path**
- **Lateral load path**

Both the gravity and lateral load paths utilize a combination of horizontal and vertical structural components, as explained below

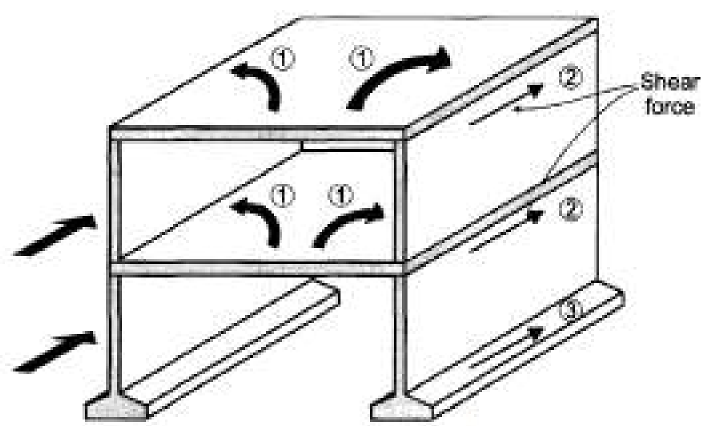
1. Gravity Load Path: Gravity load is the vertical load acting on a building structure, including

dead load and live load due to occupancy or snow. Gravity load on the floor and roof slabs is transferred to the columns or walls, down to the foundations, and then to the supporting soil beneath. Figure shows an isometric view of a concrete structure and a gravity load path.
The gravity load path depends on the type of floor slab, that is, whether a slab is a one way or a two-way system.



2. Lateral Load Path: The lateral load path is the way lateral loads (mainly due to wind and earthquakes) are transferred through a building. The primary elements of a lateral load path are as follows,

- **Vertical components: shear walls and frames;**
- **Horizontal components: roof, floors, and foundations.**



Text Books:

1	تصميم المنشات الخرسانية المسلحة / د.جمال عبدالواحد فرحان
2	Reinforced Concrete Structures/Dr. I.C. Syal.
3	Reinforced Concrete Structures/N. Krishna Rajo
4	Reinforced Concrete Structures/ Supramanian
5	Others

First Semester Subjects

No.	Subject
1	WORKING STRESS DESIGN METHOD (WSDM)
2	ULTIMATE STRESS DESIGN METHOD (USDM)
2-1	Design and Analysis of Singly Reinforced Beam
2-2	Design and Analysis of Doubly Reinforced Beam
2-3	Design and Analysis of (T-Beam)
2-4	Design for Shear Requirements

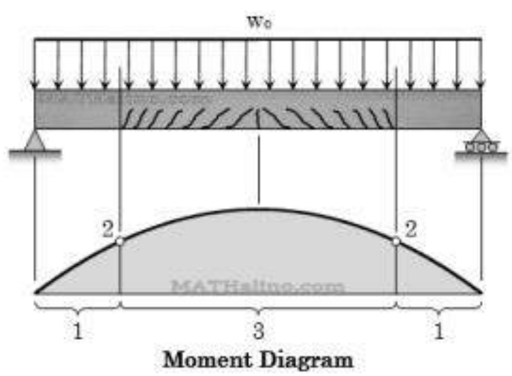
Flexural Analysis of Beam by Working Stress Method

Behaviour of Reinforced Concrete Beam under Loading:

Working Stress Analysis for Concrete Beams Consider a relatively long simply supported beam shown below. Assume the load (Wo) to be increasing progressively until the beam fails.

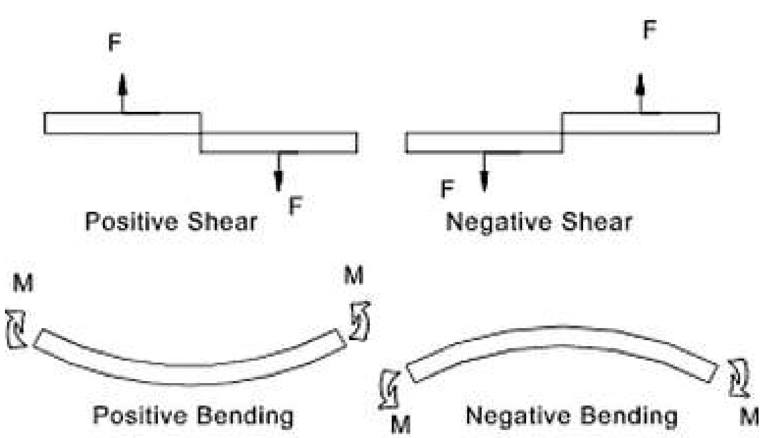
The beam will go into the following three stages:

1. **Uncrack Concrete Stage.**
2. **Crack Concrete Stage (Elastic).**
3. **Ultimate Stress Stage – Beam Failure.**



At section 1: Uncrack stage:

1. Actual moment, (M) < Cracking moment (Mcr).
2. No cracking occur.
3. The gross section resists bending.
4. The tensile stress of concrete is below rupture.



fc < o.5 fc' .... Concrete is Elastic

fs < fy .... Steel is Elastic

fct < fr .... Un-cracked

$$n = \frac{Es}{Ec} = \frac{200000}{4700\sqrt{fc'}}$$

<u>Where:</u>

fc: Actual compressive Strength for Concrete.

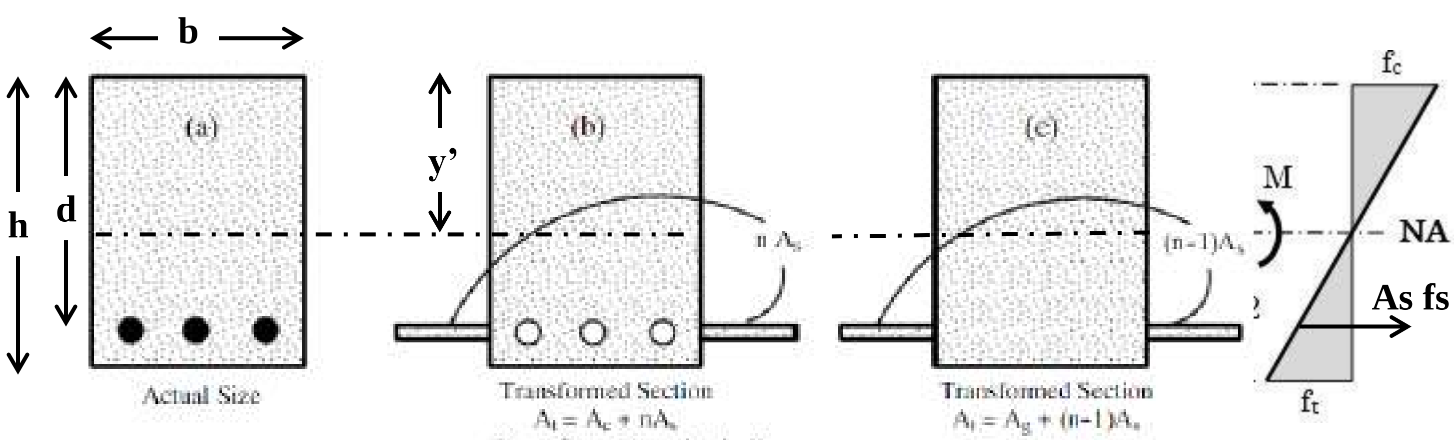
fc': Maximum compressive Strength for Concrete.

fs: Actual tensile strength for steel.

fy: Yield strength for steel.

fr: Modulus of rupture.

n: Modulus ratio.



Transformed section in Beams

$$\bar{y} = \frac{\sum A y}{\sum A} = \frac{(bh)(h/2) + (n-1)A_s * (d)}{bh + (n-1)A_s}$$

$$I_{N.A.} = \frac{b\bar{y}^3}{3} + \frac{b(h-\bar{y})^3}{3} + (n-1)A_s(d-\bar{y})^2$$

$$f_{ct} = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(h-\bar{y})}{I_{N.A.}} \quad \text{check} < f_r = 0.7\sqrt{f'_c}$$

$\therefore$ Uncracked ok

$$f_c = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(\bar{y})}{I_{N.A.}} \quad \text{check} < f_c \text{ allowable} = 0.5 f'_c$$

$\therefore$ ok Elastic

$$f_s = \frac{n.M(d-\bar{y})}{I_{N.A.}} \quad \text{check} < f_y$$

$\therefore$ ok Elastic

<u>At Section 2 : Crack concrete stage</u>:

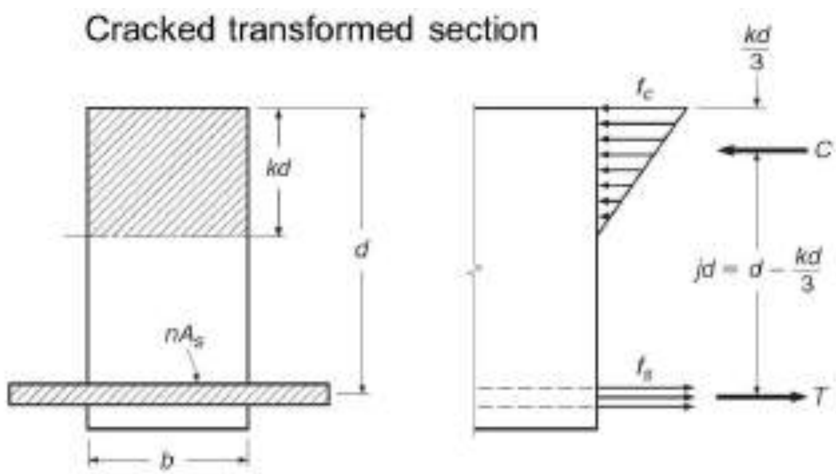
1- Actual moment, (M) > Cracking moment (Mcr).
2- Elastic stress stage.
3- Cracks developed at the tension fiber of the beam and spreads quickly to the neutral axis.
4- The tensile stress of concrete is higher than the rupture strength.
5- Ultimate stress stage can occur at failure.

$f_c < 0.5 f_c'$ .... Concrete is Elastic

$f_s < f_y$ .... Steel is Elastic

$f_{ct} > f_r$ .... Cracked

$$n = \frac{Es}{Ec} = \frac{200000}{4700\sqrt{fc'}}$$



Find N.A Position

Area of Compression about N.A = Area of Tension about N.A

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n A_s (d - kd) \quad \text{—— (2)}$$

$$\text{Steel Ratio} = \frac{A_s}{bd} \Longrightarrow A_s = \rho b d$$

Sub. in eq.(2)

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n \rho b d (d - kd)$$

$$\frac{k^2}{2} = n\rho(1-k)$$

$$k^2 = 2n\rho - 2k\rho n$$

$$k^2 + 2\rho n k - 2\rho n = 0 \quad \text{—— (3)}$$ تحل بالدستور

$$k = \sqrt{(\rho n)^2 + 2(\rho n)} - \rho n \quad \text{—— (4)}$$

$$I_{N.A} = \frac{b(kd)^3}{3} + n A_s (d - kd)^2$$

$$f_c = \frac{MC}{I_{N.A}} = \frac{M_{max}\, kd}{I_{N.A}} < 0.5 f'_c \quad \therefore \text{(Elastic)}$$

$$f_s = n\frac{MC}{I_{N.A}} = \frac{n M_{max}(d - kd)}{I_{N.A}} < f_y \quad \therefore \text{(Elastic)}$$

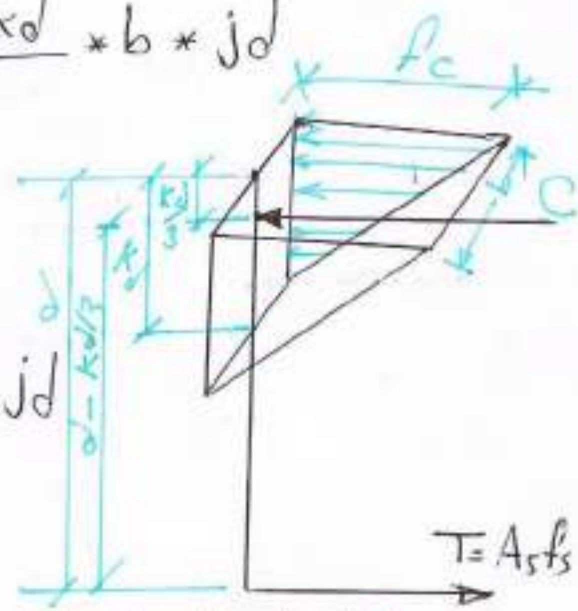
Allowable Stresses of Materials according to ACI-Code

- Concrete $f_c = 0.45 f'_c$
- Steel Reinforcement $f_y = 300\,MPa \Rightarrow f_s = 140\,MPa$

  $f_y = 400\,MPa \Rightarrow f_s = 170\,MPa$

- Method of Internal Moment أسلوب العزم الداخلي

$$M = C \cdot jd = \frac{f_c \cdot kd}{2} * b * jd$$



$$f_c = \frac{2M}{kjbd^2}$$

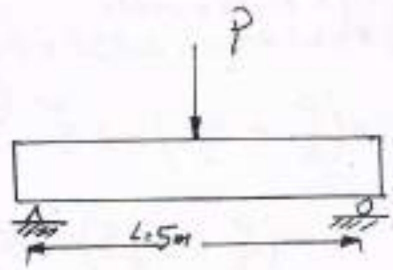
$$M = T \cdot jd = A_s f_s \cdot jd$$

$$\therefore f_s = \frac{M}{A_s j d}$$

$$d = \frac{kd}{3} + jd \implies$$ (المقطع المستطيل فقط)

$$j = 1 - \frac{k}{3}$$

Ex:- Find Maximum Load (P) Can be applied at the Center of the beam Shown below for these information:-

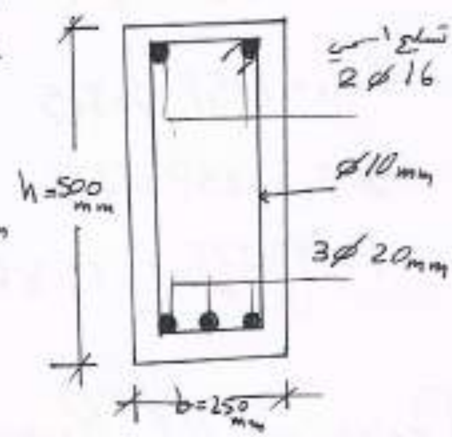


$b = 250\ mm$, $h = 500\ mm$

$A_s = 3\phi\ 20mm$, $E_s = 200\,000\ N/mm^2$

$E_c = 22\,000\ N/mm^2$, $\gamma_c = 24\ kN/m^3$

$f_y = 300\ MPa$, $f'_c = 20\ MPa$



Solution:- $d = 500 - (40 + 10 + \frac{20}{2}) = 440\ mm$

$$A_s = 3 * \frac{\pi}{4}(20)^2 = 942\ mm^2$$

$$\rho = \frac{A_s}{bd} = \frac{942}{250 * 440} = 0.0088$$

$$n = \frac{E_s}{E_c} = \frac{200\,000}{22\,000} \simeq 9.09$$

$$\rho n = 0.0088 * 9.09 = 0.08$$

$$K = \sqrt{(0.08)^2 + 2(0.08)} - (0.08) = 0.328$$

$$j = 1 - \frac{K}{3} = 0.891$$

$$jd = d - \frac{kd}{3} = 440 - \frac{0.328 * 440}{3} = 391.9\ mm$$

$$f_c = \frac{2M}{Kjbd^2} \quad , \quad f_{c_{all}} = 0.45 * 20 = 9.0\ MPa$$

$$M = 0.5 * f_{c_{all}} Kjbd^2 = 0.5 * 9 * 0.328 * 0.891 * 250 * (440)^2$$
$$= 63.652 * 10^6\ N.mm$$

$$f_s = \frac{M}{A_s jd} \quad , (f_y = 300\ MPa) \quad f_{s_{all}} = 140\ MPa$$

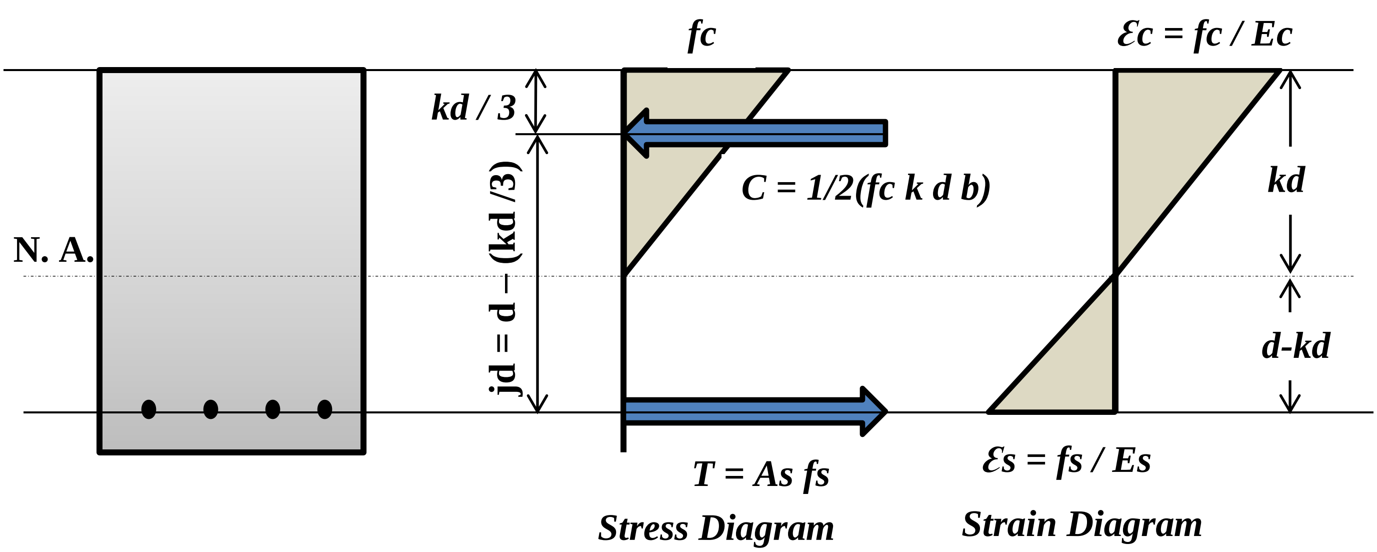
$$M = f_{s_{all}} A_s jd = 140 * 942 * 0.891 * 440 = 51.7 * 10^6\ N.mm$$
$$= 51.7\ kN.m$$

M allowable is the smaller of the moments that make $f_c = f_{c\ all}$

Design of R.C. Rectangular Beam by W.D. Method:

Notes:

1- **Analysis:** Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.
2- **Design**: Given a factored design moment, normally designated as select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.
3- **Balance Section**: is economical section because it is used both of steel and concrete properties in high level.



From Strain Diagram:

$$\frac{\frac{f_c}{E_c}}{kd} = \frac{\frac{f_s}{E_s}}{d - kd} \Longrightarrow \frac{\frac{E_s}{E_c}}{k} = \frac{\frac{f_s}{f_c}}{1-k}$$

Let $r = \frac{f_{s\,all}}{f_{c\,all}}$, $\frac{n}{k} = \frac{\frac{f_s}{f_c}}{1-k}$

$\frac{f_s}{f_c} = \frac{n(1-k)}{k}$, in balance conditions $r = \frac{n(1-k_b)}{k_b}$

$$r k_b = n - n k_b \Longrightarrow r k_b + n k_b = n$$

$$k_b (r+n) = n \Longrightarrow \boxed{k_b = \frac{n}{n+r}}$$

From Stress Diagram

$$T = C \Longrightarrow A_s f_s = \frac{1}{2} f_c k d b$$

$$\frac{A_s}{bd} * \frac{f_s}{f_c} = \frac{1}{2} * k \Longrightarrow \rho \frac{f_s}{f_c} = \frac{k}{2}$$

in balance conditions $\rho_b \frac{f_{s\,all}}{f_{c\,all}} = \frac{k_b}{2} \Rightarrow \boxed{\rho_b = \frac{k_b}{2r}}$

$$\rho_{min} = \frac{1.4}{f_y}$$ according to ACI-code

$$\rho_b > \rho > \rho_{min}$$

EX.

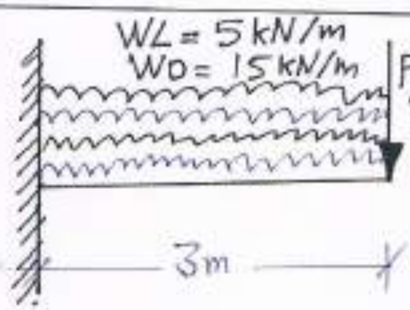


Fig.

Design the cantilever shown in the Fig. below using the following data. $f'_c = 20\,N/mm^2$, $f_y = 275\,N/mm^2$; $E_s = 200\,000\,N/mm^2$, $\gamma_c = 24\,kN/m^3$

Solution:-

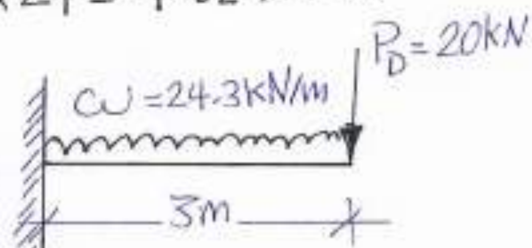
- Assume depth of cantilever $= \frac{L}{5} = h$
- Assume width of cantilever $(b) = \frac{h}{2} = \frac{L}{5}/2 = \frac{L}{10}$

$$\omega_{self} = b \times h \times 1 \times \gamma = \frac{h}{2} \times h \times 24$$

$$= \frac{L}{5} \times \frac{L}{10} \times 24 = 4.32\ kN/m$$

$$\omega_{total} = \omega_L + \omega_D + \omega_{self}$$

$$= 5 + 15 + 4.3 = 24.3\,kN/m$$



$$M_{max} = P_D \cdot L + \frac{\omega L^2}{2} = 169.44\,kN.m$$

$$\rho_b = \frac{K_b}{2r}, \quad k_b = \frac{n}{n+r}, \quad r = \frac{f_{s\,all}}{f_{c\,all}} = \frac{140}{0.45 \times 20} = 15.55$$

$$n = \frac{200\,000}{4700\sqrt{20}} = 9.52$$

$$k_b = \frac{9.52}{9.52 + 15.55} \simeq 0.38, \quad j = 1 - \frac{k}{3} = 1 - \frac{0.38}{3} = 0.8733$$

$\rho_b = \frac{0.38}{2 * 15.55} = 0.0122$, $\rho_{min} = \frac{1.4}{275} = 0.005$

Use $\rho = 0.01$

$M = M_s = \rho f_s j b d^2 \Rightarrow 169.44 * 10^6 = 0.01 * 140 * 0.8733 * b * (2b)^2$

$4b^3 = 138.587 * 10^6$

$b = \sqrt[3]{34.64 \times 10^6} = 326\ mm$ USE $b = 330 mm$

$d = 2b = 2 * 330 = 660 mm$

$A_s = \rho b d = 0.01 * 330 * 660 = 2178 mm^2$

Use ϕ 22 mm , $A_b = \frac{\pi}{4} * 22^2 = 380\ mm^2$

No of bars $= \frac{2178}{380} = 5.73$

Use $6\phi22$

- The distanse between each bar must not be less than 25mm
- The concrete cover from each sides must not be less than 100mm (i.e. 50mm for each side)
- If we put all bars in the same layer ∴ the width of the beam will be equal to

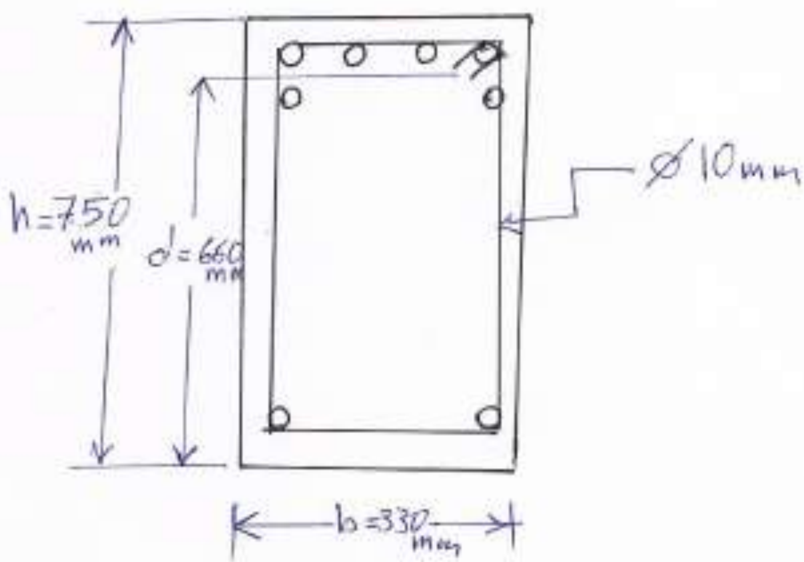
  $6 \times 22 + 5 \times 25 + 100 = 357 mm > b = 330 mm$

- ∴ We distribute the bars into (2) layer one of them contains $4\phi22$ & the other contains $2\phi22$
- $4 \times 22 + 3 \times 25 + 100 = 263 mm < b = 330 mm$ ∴ ok

$$h = d + \frac{\text{the distance between 2 bars}}{2} + d_{bar} + d_{stirup} + \text{Cover}$$

$$h = 660 + \frac{25}{2} + 22 + 10 + 40$$

$$= 744.5_{mm} \Longrightarrow \text{USE } h = 750\,mm$$



ULTIMATE STRENGTH DESIGN METHOD (S.D.M)

The assumptions which are used in this method:

1- **Stress in reinforcement** varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
2- **Concrete sections** are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
3- **Strains in reinforcement** and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
4- **Tensile strength of concrete is neglected.**
5- **Compressive stress** distribution of concrete can be represented by the corresponding stress-strain relationship of concrete.

Safety Factors: S.F.

a- S.F. = Max. Stress / Allowable Stress (W.S.D.M)
b- S.F. = Max Load / Service Load (S.D.M)

Load Factors:

U= 1.2 D + 1.6 L

U= 1.2 D + 1.6 L +0.5 (Lr or S or R)

U= 1.2 D + 1.6 (Lr or S or R) + (1.0 L or 0.5 W)

U= 1.2 D + 1.0 W +1.0 L + 0.5 (Lr or S or R)

D: Dead Load, L: Live Load, W: Wind Load, S: Snow Load, Lr: Roof Load, R: Rain Loaf.

Strength Reduction Factors:

⋆ Tension .... Ǿ = 0.9 ⟹ Mu = Ǿ Mn

Mu: Ultimate moment capacity.

Mn: Nominal (Actual) moment capacity.

⋆ Shear, Torsion .... Ǿ = 0.75 ⟹ Vu = Ǿ Vn

Vu: Ultimate shear capacity.

Vn: Nominal shear capacity.

⋆ Compression:

a- Ǿ = 0.70 for spiral reinforced member like column.
b- Ǿ = 0.65 for other reinforced member like column.

Stress and Strain Distribution:

Resultant of concrete compressive force :

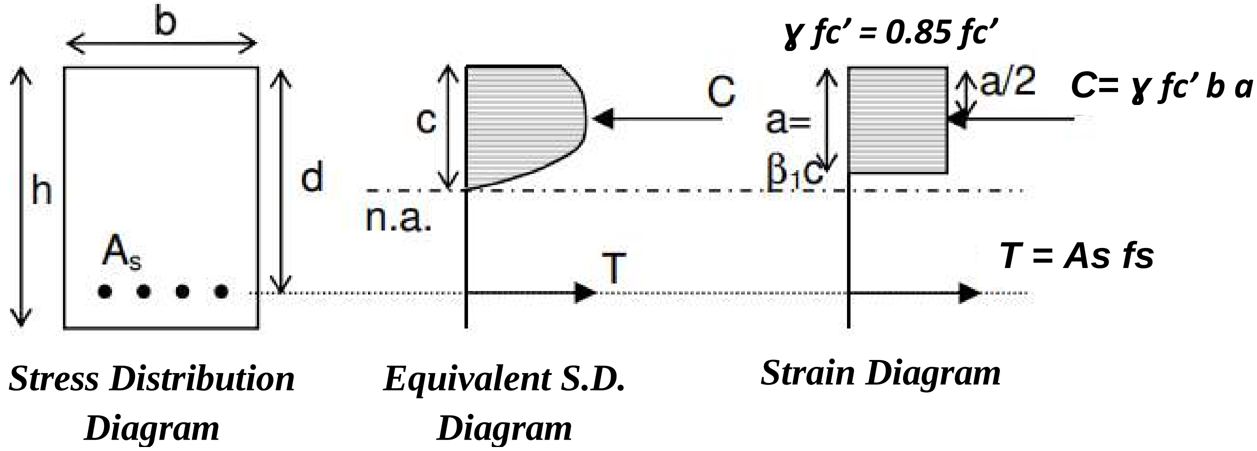
C = fav . b . c

Where:

fav: average compressive stress.

b: the width of section.

c: the depth of Neutral Axis.



$$C = \alpha\ fc'\ b\ c$$

Where:

$$\alpha = \frac{avaerage\ concrtet\ stress}{concrete\ compressive\ stress}$$

The location of the resultant is usually represented by βc.

Where:

$$\beta = \frac{compressive\ resultant\ depth}{N.A.depth}$$

α: 0.72 for fc' ≤ 30 MPa

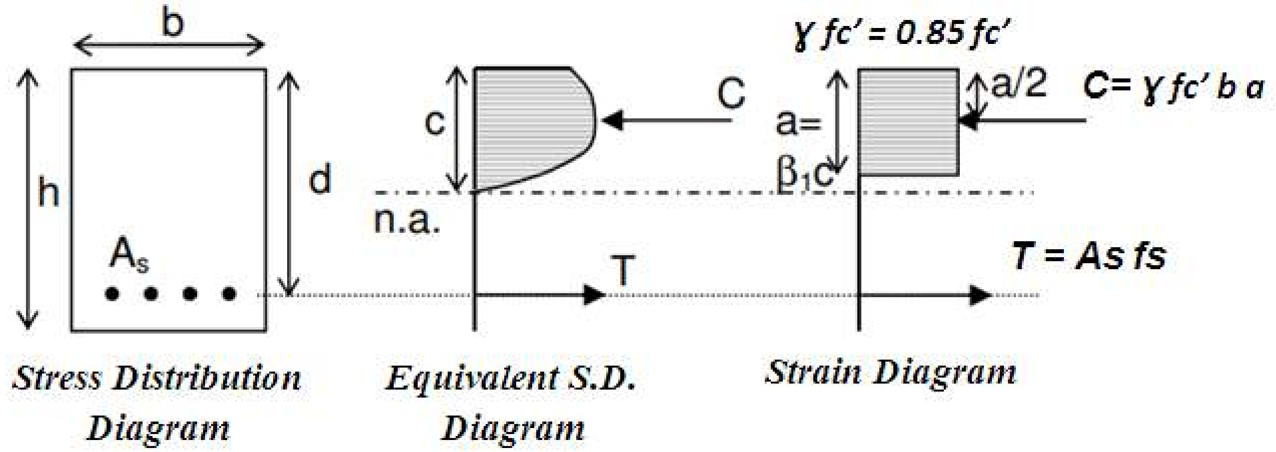
α: decreased by (0.04) for every (7 MPa) increasing in compressive strength of concrete.

α: Value must not be less than (0.56).

β: 0.425 for fc'≤ 30MPa

β: decreased by (0.025) for every (7 MPa) increasing in compressive strength of concrete.

β: value must be less than (0.325).



Equivalent rectangular stress block is used for analysis of reinforced concrete sections:

$$C = \alpha\, fc'\, b\, c = \gamma\, fc'\, a\, b \quad ....(1)$$

Let $a = \beta 1 \cdot c$ .... (2)

We can find γ , $\beta 1$, α , β

$a/2 = \beta \cdot c$ .... $a = 2(\beta \cdot c)$

from eq. (2) .... $\beta 1 \cdot c = 2(\beta \cdot c)$ .... $\beta 1 = 2\beta$ ...... (3)

sub. in eq. (1) :

$$\alpha\, fc'\, b\, c = \gamma\, fc'\, a\, b \Longrightarrow \gamma = \alpha\, c/a \Longrightarrow \gamma = \alpha\, c/2(\beta\, c) \Longrightarrow$$

$\gamma = \alpha / 2(\beta_1/2) \Longrightarrow \gamma = \alpha / \beta_1$ ..... (4)

From the above equation and from the value of (β, γ) we can find the value of (β_1, γ):

$\beta_1 = 2\,\beta$ .... $\beta_1 = 2 * 0.425 = 0.85$

$\gamma = (\alpha / \beta_1) = (0.72/0.85) = 0.85$ .... (5)

$\beta_1 = 0.85$ *for* $f_c' \leq 30$ MPa,

β_1: decreased by (0.05) for every (7 MPa) increasing in compressive strength of concrete.

β_1: value must not be less than (0.65).

$\beta_1 = 0.85 - [0.05(f_c' - 30)/7]$

Analysis and Design of Singly Reinforced Rectangular Beam:

a- **Balance or Under Reinforced.**

$\rho \leq \rho_b \Rightarrow f_s = f_y$

from the equilebrium conditions

$C = T$

$0.85 f_c' * b * a = A_s f_y$

$a = \dfrac{A_s f_y}{0.85 f_c' b}$ —(a), $A_s = \rho b d$ —(b)

$a = \dfrac{\rho b d f_y}{0.85 f_c' b} \Rightarrow a = \dfrac{\rho f_y d}{0.85 f_c'}$ —(1)

$M_n = A_s f_y * (d - \frac{a}{2})$ —(2)

$M_n = 0.85 f_c' \cdot a \cdot b (d - \frac{a}{2})$ —(3)

sub a & b in equ (2)

$M_n = \rho b d f_y \left[d - \dfrac{\rho f_y d}{2\ 0.85 f_c'}\right]$

$M_n = \rho b d^2 f_y \left[1 - \dfrac{0.59 \rho f_y}{f_c'}\right]$

$$M_u = \phi M_n$$

$$M_u = \phi \rho b d^2 f_y \left[1 - \frac{0.59 \rho f_y}{f'_c}\right] \quad \text{--- (4)}$$

b- **Over Reinforced Beam:**

b- Over Reinforced Beams $\therefore \rho > \rho_b$

f_s = unknown $> f_y$

$$A_s f_s = 0.85 f'_c \cdot a \cdot b$$

$$A_s f_s = 0.85 f'_c \cdot (\beta_1 c) \cdot b$$

There are (2) unknowns f_s and c

After many steps:-

$$m = \frac{600}{0.85 \beta_1 f'_c}, \quad k_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$$

Then we can find the nominal strength by the following procedure:-

1- find ρ, m were $\rho = \frac{A_s}{bd}$, $m = \frac{600}{0.85 \beta_1 f'_c}$

2- submit in $K_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$

3- calculate c value were $c = k_u \cdot d$

4- calculate a value were $a = \beta_1 c$

5- find f_s were $f_s = 600 - \frac{(d-c)}{c}$

6- find the nominal bending moment M_n by using on of the three equations (2), (3) and (4)

متطلبات الكود الامريكي للعتبات ناقصة التسليح:

Maximum Steel Ratio

- Tension failure occurs when $f_s = f_y$ before concrete strain reachs Max. Strain = 0.003, and this failure occurs gradually.
- Compression failure occurs when the strain of concrete reach max. strain = 0.003 before steel stress reach the yeild strength = f_y
- Tension failure hapend when $\rho < \rho_b$
- Tension failure is better than compression failure.

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

($\varepsilon_s = 0.004$ according to ACI code 2002)

Determination of Reduction Factors (ϕ)

a- For members with tension controlled

$$\varepsilon_t \geq 0.005 \Rightarrow \phi = 0.9$$

$$\rho_t = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

b- For compression controlled members

$$\varepsilon_t \leq 0.002 \Rightarrow \phi = 0.7 \text{ (spiral Reinforcement)}$$

$$\phi = 0.65 \text{ (other kind of Reinforcement)}$$

C- Transition between tension and compression

- Spiral reinforcement

$$\phi = 0.7 + \frac{0.2}{0.003}(\varepsilon_t - 0.002) = 0.567 + 66.7\,\varepsilon_t$$

- Other reinforcement

$$\phi = 0.65 + \frac{0.25}{0.003}(\varepsilon_t - 0.002) = 0.483 + 83.3\,\varepsilon_t$$

ACI-Code encourages the designers to reduce the (ρ) value to increase the magnitude of (ϕ)

Minimum Steel Ratio

$$\rho_{min} = \frac{A_{s\,min}}{b_w d} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y}$$

<u>Design by Ultimate Design Method:</u>

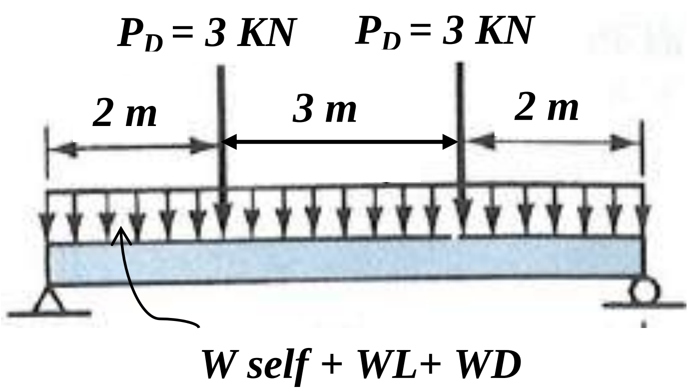
1- The design of R.C. members means finding the adequate dimensions for these members and the reinforcement magnitude to enable the member to withstand maximum loads applied on it safety.
2- Sometime, all dimensions or some of them are determined by architectures.
3- Complete design for the beam requires determine the shear reinforcement, torsion reinforcement and check deflections; check development lengths and points of cuts or bend of steel reinforcement. All these details must be put on the beam sketch or diagram.

Ex.: Design the beam shown for the following data:

fc' = 20 N/mm2 and fy= 300 N/mm2

WL= 6 KN/m and WD= 12 KN/m

γc = 24 KN/m3



Solution:

1- Find Moment (Mu):

* for cantilever, assume $h = \frac{L}{5}$

for simply supported beam & continuous beam, assume $h = \frac{L}{10}$

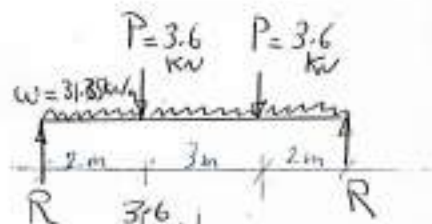
* $b = \frac{h}{2}$, $\therefore b = \frac{L}{10}/2 = \frac{L}{20}$

$$\omega_{self} = b*h*1*\gamma_c = \frac{L}{20}*\frac{L}{10}*1*24 = \frac{L^2}{8.33} \approx \frac{L^2}{8} \; (kN/m)$$

$$\omega_{total} = \omega_{self} + \omega_D + \omega_L$$

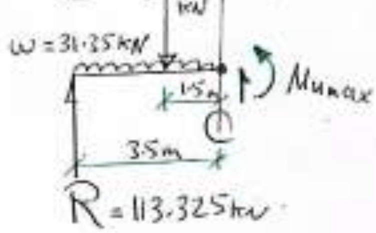
$$= 1.2\left(\frac{(7)^2}{8} + 12\right) + 1.6(6) = 31.35 \; kN/m$$

$$P = 1.2 P_D = 1.2*3 = 3.6 \; kN$$



$\Sigma F_y = 0$

$$R = \frac{3.6*2 + 31.35*7}{2} = 113.325 \; kN$$



$\Sigma M_{(c)} = 0$

$$Mu_{max} = R*3.5 - P*1.5 - \frac{\omega(3.5)^2}{2}$$

$$= 113.325*3.5 - 3.6*1.5 - \frac{31.35(3.5)^2}{2}$$

$$Mu_{max} = 199.219 \; kN \cdot m$$

2- ρ_{min}, ρ_{max} & ρ_t

* from Table(3), $\rho_{min} = 0.0047$

or $\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{300} = 0.00467$ ✓

$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{20}}{4*300} = 0.00373$

use the bigst value

* ρ_{max} :- from Table(3) :- $\rho_{max} = 0.0206$

or by the eq. :- $\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t}$ ($\varepsilon_t = 0.004$)

($\beta_1 = 0.85$ for $f'_c \leq 30MPa$) $\rho_{max} = 0.85*(0.85)*\frac{20}{300} * \frac{0.003}{0.003+0.004}$

$\rho_{max} = 0.02064$

• We must use $\rho \Longrightarrow \rho_{min} \leq \rho \leq \rho_{max}$

3- • for use $\phi = 0.9$ ρ must be $\leq \rho_t$

$\rho_t = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t}$ ($\varepsilon_t = 0.005$)

$\rho_t = 0.85(0.85)*\frac{20}{300} * \frac{0.003}{0.003+0.005} = 0.01806$

or from Table(3) $\rho_t = 0.0180$

∴ USE $\rho = 0.0170$

4- $M_u = \phi \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right)$

$199.22 \times 10^6 = 0.9 \times 0.0170 \times bd^2 \times 300 \left(1 - 0.59 \times 0.017 \times \frac{300}{20}\right)$

$199.22 \times 10^6 = 4.59\, bd^2 - 0.6906\, bd^2$

$bd^2 = 51089459.3$

assume $b = \frac{d}{2}$

$\frac{d}{2} \times d^2 = 51089459.3$

$d^3 = 102.179 \times 10^6$

$d = \sqrt[3]{102.179 \times 10^6} = 467.5\ mm \Rightarrow$ USE $d = 470\ mm$

5- $A_s = \rho b d = 0.0170 \times 240 \times 470 = 1917.6 \simeq 1918\ mm^2$

Use $\phi_{bar} = 22\ mm$ $\quad A_{s\,bar} = \frac{\pi}{4} \times (22)^2 = 380.13\ mm^2$

(or from Table (1) :-

No of bars $= \frac{A_s}{A_{s\,bar}} = \frac{1918}{380} = 5.047 \simeq 5$

6- • $S \geq \begin{cases} 25\ mm \\ \phi_{bar} \\ \frac{4}{3} \times \text{max. Size of aggregate} \end{cases}$ distance between bars

- Thickness of the covers:-

a- cover $\geq$ 75mm (on the ground)

b- cover $\geq$ 25mm (concrete in contact with soil or with invironment conditions)

(for slabs & عقدات & wall slabs)

* for other concrete members = 40 mm

c- Cover $\geq$ 20mm (concret is not in contact with soil or other conditions)

(slabs, عقدات & walls)

* for beams & columns = 40mm & for secondary = 25mm

$$S = \left(b - 2\left(\overset{40}{cover} + \phi_{s_{bar}}\right) - n\phi_{bar}\right) / (n-1)$$

n = № of bars

b = width of the section

ϕ_{bar} = bar diameter

ϕ_s or $\phi_{s_{bar}}$ = diameter of shear reinf.

assume the steel bars are distributed in one layer.

$S_{actuale} = [240 - 2*(40+10) - 5*22] / (5-1) = 7.5 mm$

$S = \begin{cases} 25 mm ✓ \\ \phi_b = 22 mm \\ \frac{4}{3} * \text{max. of agg.} \end{cases}$ $\therefore S = 25 mm$

$\because S_{act} < S_{min} = 25 mm$

$\therefore$ let us use the reinf. in two layers

5 bars < 3 bars, 2 bars

$S_{act} = [240 - 2(40+10) - 3*22] / (3-1) = 37 mm > 25 mm$

$\therefore$ O.K.

$h = d + 70 mm$ (one layer)

$h = d + 100 mm$ (two layers)

$h = d + 130 mm$ (3 layers)

or actual $h = d + \frac{S}{2} + d_{bar} + d_s + Cover$

$h = 470 + 25/2 + 22 + 10 + 40$

$= 554.5 mm$

Use $h = 560 mm$

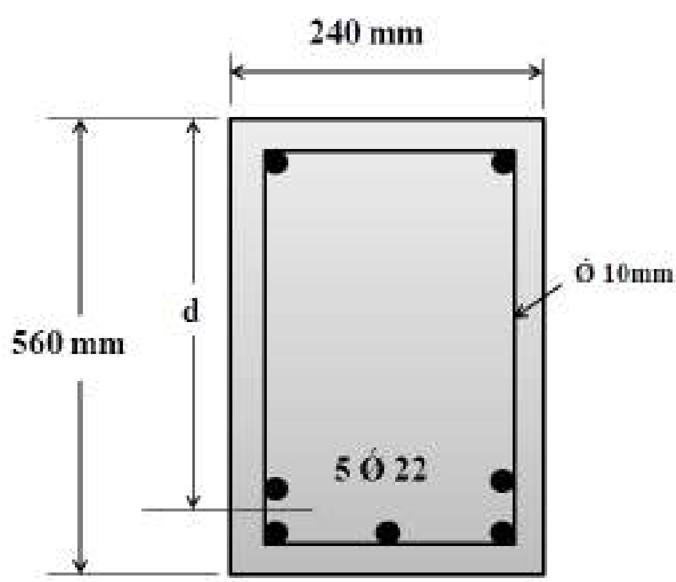
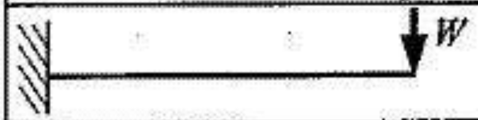
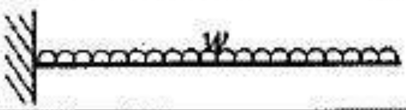
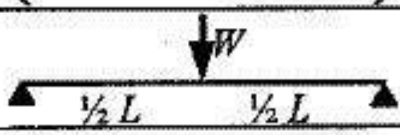
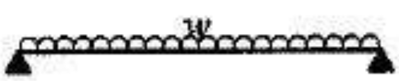
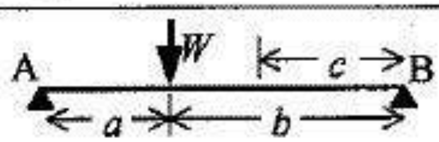


Table (3) : ρ_{min} and ρ_{max} values

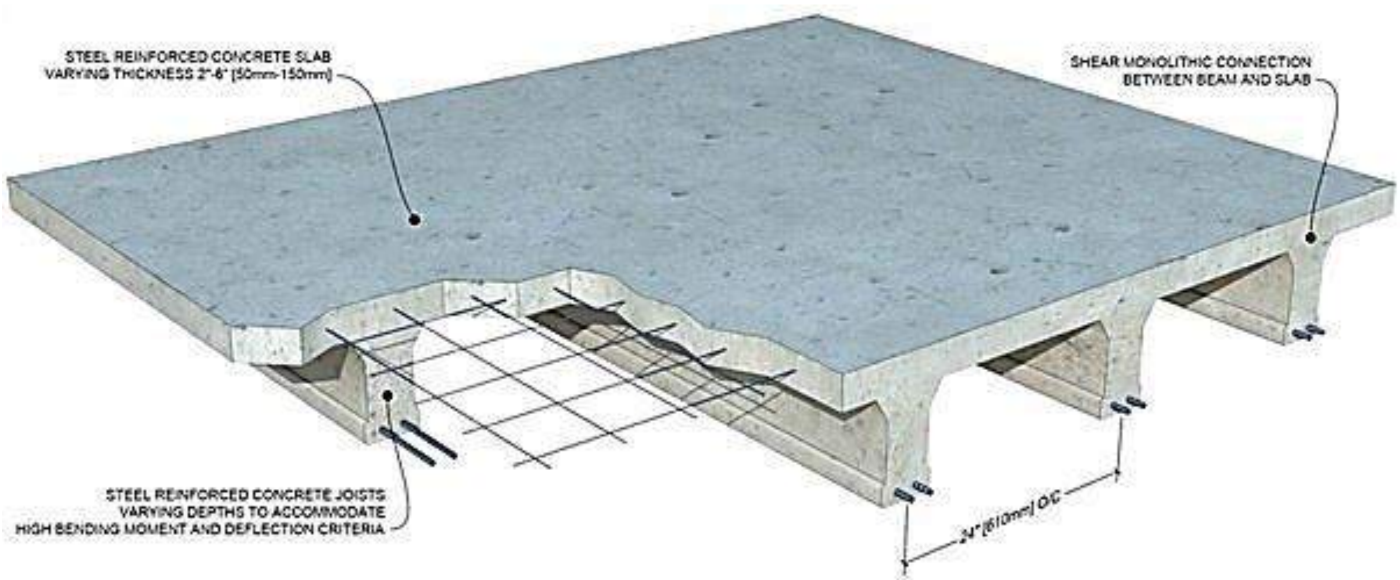
f_y (Mpa)	f'_c (Mpa)	β_1	ρ_b	ρ_{max}	ρ_t	$\rho_{min} = \frac{1.4}{f_y}$	$\rho_{min} = \frac{\sqrt{f'_c}}{4f_y}$
300	20	0.85	0.0321	0.0206	0.018	0.0047	0.0037
	25	0.85	0.0401	0.0258	.0226	0.0047	0.0042
	30	0.85	0.0482	0.031	0.0271	0.0047	0.0046
	35	0.814	0.0538	0.0346	.0303	0.0047	0.0049
	40	0.779	0.588	0.0378	.0331	0.0047	0.0053
350	20	0.85	0.0261	0.0177	.0155	0.004	0.0032
	25	0.85	0.0326	0.0221	0.0193	0.004	0.0036
	30	0.85	0.0391	0.0265	0.0232	0.004	0.0039
	35	0.814	0.0437	0.0296	0.0259	0.004	0.0042
	40	0.779	0.0478	0.0324	0.0284	0.004	0.0045
400	20	0.85	0.0217	0.0155	0.0136	0.0035	0.0028
	25	0.85	0.0271	0.0194	0.017	0.0035	0.0031
	30	0.85	0.0325	0.0232	0.0203	0.0035	0.0034
	35	0.814	0.0363	0.026	0.0228	0.0035	0.0036
	40	0.779	0.0397	0.0284	0.0249	0.0035	0.0039

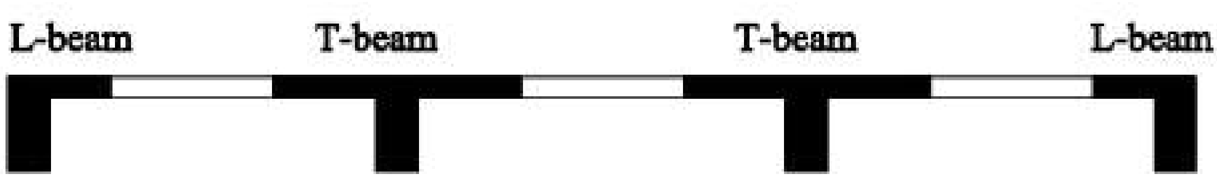
BEAM BENDING

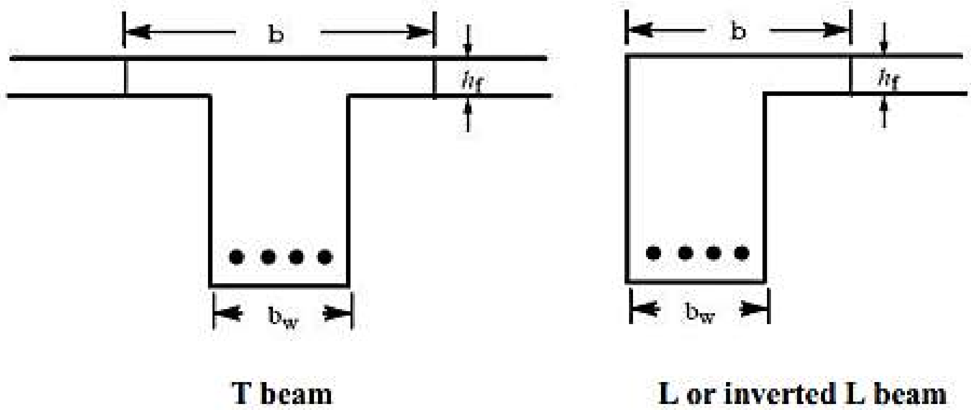
L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \leq b, \quad c = \sqrt{\frac{1}{3}b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

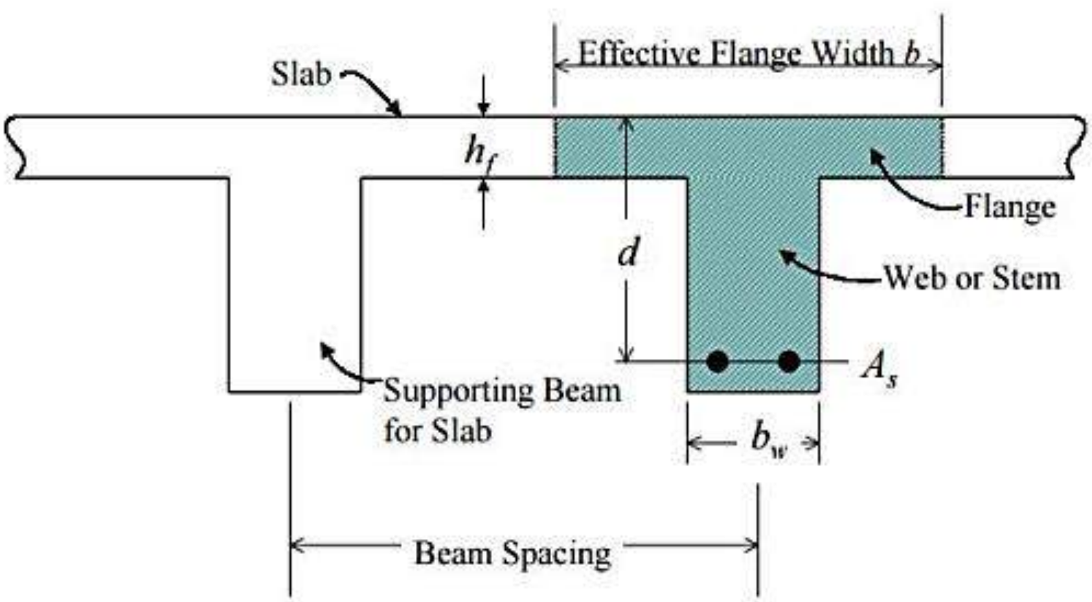
ANALYSIS AND DESIGN OF T- BEAM

When floor slabs and their supporting beams are cast monolithically, they deflect along with the beams under the action of external loads. Therefore, slabs in the vicinity of the beams act as flanges for the beam. Interior beams have a flange on both sides, which are called T-beams. Edge beams have a flange on one side only, and referred to as L-beams as shown in Figure 8. Isolated T-beams, which are produced as precast concrete elements, are used in concrete construction.






b
hf
bw
T beam
b
hf
bw
L or inverted L beam


Slab
Effective Flange Width b
hf
d
Flange
Web or Stem
As
Supporting Beam for Slab
bw
Beam Spacing

Effective width of the flange can be calculated as per ACI 318 section 8.10.2 which is given in the following table:

T-Beam	L-Beam
1. $b \le \frac{Span}{4}$ 2. $b \le b_w + 16h_f$ 3. $b \le$ average clear distance to adjacent webs + b_w **The smallest of three values control**	1. $b \le b_w + \frac{Span}{12}$ 2. $b \le b_w + 6h_f$ 3. $b \le b_w + \frac{C/C\ beam\ distance}{2}$ **The smallest of three values control**

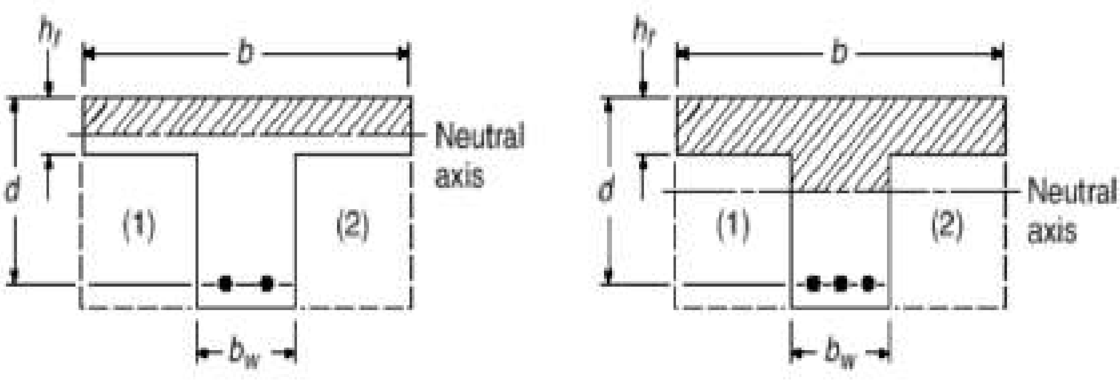
Isolated non pre-stressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to 0.5bw and an effective flange width less than or equal to 4bw.

$$h_f > \frac{1}{2} * bw \quad and\ bf < 4 * bw$$

<u>**Analysis of T or L Beams**</u>

The calculation of the design strengths of T beams depend on the neutral axis position,

a- If it falls in the flange then is considered as rectangular sections,
b- While it is T section if the neutral axis is at the web.



(a) (b)

Analysis of T-beam

1- Find the depth of compressive area

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

2- If $a \leq h_f$ $\xrightarrow{then}$ the analysis will be as rectangular beam with (width = b) and (depth = d).

3- If $a > h_f$ $\xrightarrow{then}$ $A_{sf} = \dfrac{0.85 f'_c (b - b_w) h_f}{f_y}$

A_{sf}: Area of steel required to equilified the compressive stress of flange

Find ρ_w , $\rho_w = \dfrac{A_s}{b_w d}$

Find ρ_{wb} , $\rho_{wb} = 0.85 \beta_1 \dfrac{f'_c}{f_y} \dfrac{600}{600 + f_y} + \rho_f$

$$\rho_{wb} = \rho_b + \rho_f$$

4- If $\rho_w \leq \rho_{wb} \Longrightarrow a = \dfrac{A_{sw} f_y}{0.85 f'_c b_w}$

and find Mn from one of the two eqs

$$M_n = M_{n1} + M_{n2} = A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right)$$

or

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c \left[(b - b_w) h_f * \left(d - \frac{h_f}{2}\right) + a \cdot b_w \cdot \left(d - \frac{a}{2}\right)\right]$$

5- If $\rho_w > \rho_{wb}$ then we must calculate (c) by this equation

$$A_s \cdot 600 \cdot \left(\frac{d-c}{c}\right) = 0.85 f'_c (b-b_w) h_f + 0.85 \beta_1 f'_c\, c\, b_w$$

$$a = \beta_1 / c$$

then find M_n :-

$$M_n = 0.85 f'_c (b-b_w) h_f \cdot \left(d - \frac{h_f}{2}\right) + 0.85 f'_c\, a\, b_w \left(d - \frac{a}{2}\right)$$

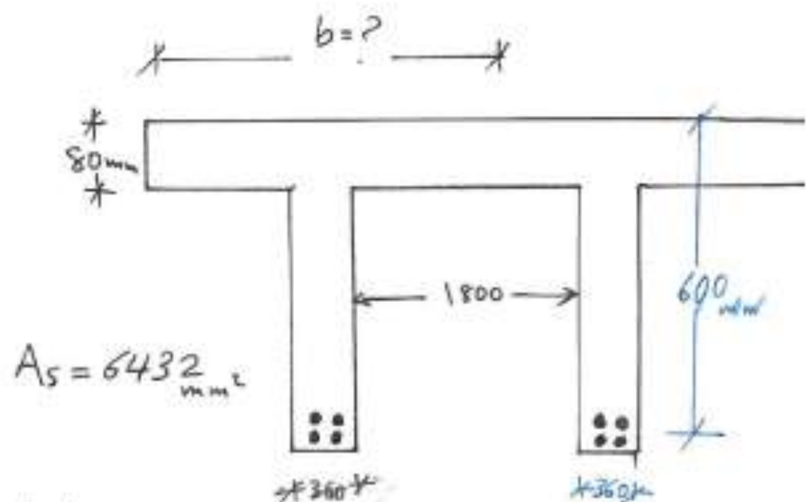
6- If $\varepsilon_t = 0.005 \xrightarrow{\text{then}} \phi = 0.9$

$$\rho_{wt} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} + \rho_f = \rho_t + \rho_f$$

ρ_{wt} : Reinforcement ratio caused strain = (0.005) for T-beam

ρ_t : Reinforcement ratio cause strain = (0.005) for rectangular portion

Ex. :- An 80 mm thick continous slab is supported by rectangular beams as shown in the Fig. The span of the beam is 5m, $f'_c = 20.7$ MPa, $f_y = 345$ MPa. Find the design strength of the T-beam



Solution :-

- $b = \frac{L}{4} = \frac{5000}{4} = 1250$ mm
- $b = 16 h_f + b_w$

$b = 80 \times 16 + 360 = 1640$ mm

- $b = 360 + 1800$

$= 2160$ mm

Use $b = 1250$ mm (The smallest value)

- $a = \frac{A_s f_y}{0.85 f'_c b}$

- $a = \frac{A_s f_y}{0.85 * 20.7 * 1250} = \frac{6432 * 345}{0.85 \times 20.7 \times 1250} \Longrightarrow a = 100.9\text{mm} > 80$ mm

∴ The beam is behaive as T-beam.

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 * 20.7 (1250 - 360) \times 80}{345} = 3631 \text{ mm}^2$$

$$\rho_f = \frac{A_{sf}}{b_w d} = \frac{3631}{360 \times 600} = 0.0168$$

$$\rho_b = (0.85)^2 \frac{f'_c}{f_y} \frac{600}{600+f_y} = (0.85)^2 * \frac{20.7}{345} * \frac{600}{600+345} = 0.0289$$

$$\rho_{wb} = \rho_b + \rho_f = 0.0289 + 0.0168 = 0.0457$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{6432}{360 * 600} = 0.0298 < \rho_{wb} = 0.0457$$

∴ The beam is under reinforced

$$A_{sw} = A_s - A_{sf} = 6432 - 3631 = 2801 \text{ mm}^2$$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 \times f'_c \times b_w} = \frac{2801 * 345}{0.85 \times 20.7 \times 360} = 152.56 \text{ mm}$$

$$M_u = \phi M_n$$

$$\rho_{wt} = \rho_t + \rho_f = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.005} + \rho_f = (0.85)^2 * \frac{20.7}{345} * \frac{0.003}{0.003+0.005} + 0.0168$$

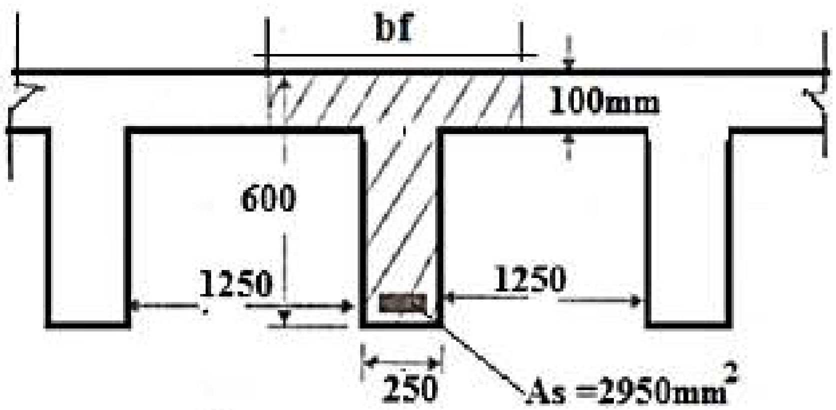
$\because \rho_{wt} > \rho \quad \therefore \phi = 0.9$

$$M_u = \phi \left[A_{sf} f_y \left(d - \frac{h_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) \right]$$

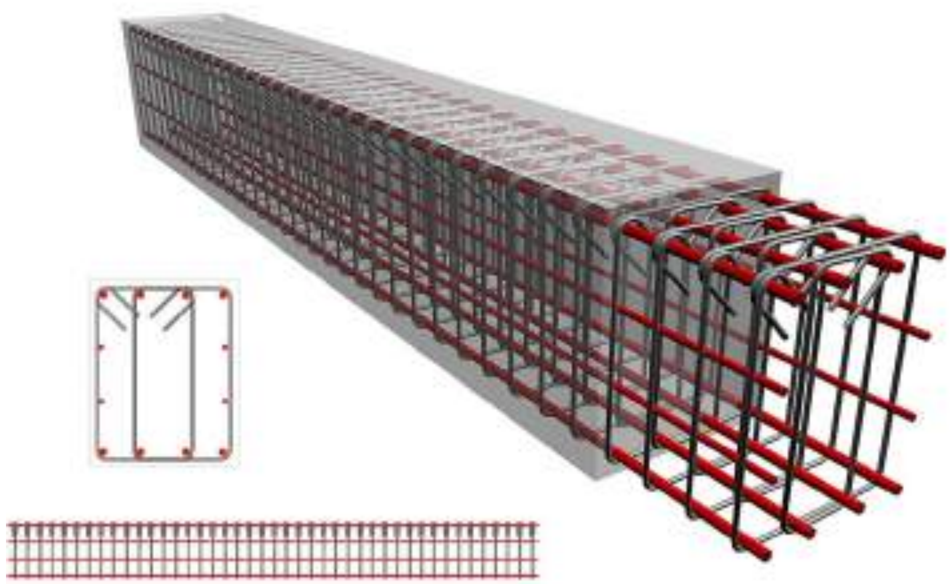
$$M_u = 0.9 \left[3631 \times 345 \left(600 - \frac{80}{2}\right) + 2801 \times 345 \left(600 - \frac{152.56}{2}\right) \right]$$

$$M_u = 1086.8 * 10^6 \text{ N.mm} = 1086.8 \text{ kN.m}$$

H.W: Determine the design strength of the (T beam) shown in Figure below, with f'c'= 25 MPa and fy = 420 MPa. The beam has a (10 m) span and is cast integrally with a floor slab that is (100 mm) thick. The clear distance between webs is (1250 mm).



DESIGN OF DOUBLY REINFORCED RECTANGULAR BEAM



In design of singly reinforced beams (ρ) is be taken equal to (ρ_{max}) to insure tension failure. When the cross-section of beam is limited because of Architectur reasons or service reasons and its resistance strength is not enough to withstand Applied Moment. In this case, the solution is by adding compression steel instead of an equivalent tensile steel to keep the Neutral Axis (N.A.) in the same position in the case of ($\rho = \rho_{max}$) to ensure tensile failure.

To calculate the steel reinforcement of both, tension and compression the next procedure must be do.

1- Calculate the design moment from structural analysis.
2- Find (ρ_{max}) from equation or table(P3).
3- Find (ρ) value from equation or table (P4), and if $\rho \leq \rho_{max}$ then the section is singly beam designed as singly reinforced beam, or the s is doubly and it will be designed as the next steps.
4- Find maximum design moment (Mu_{max}) which will be genrate by Maximum allowd Steel reinforcement area (As_{max}) and here will be call it (As_1), and we will call Mu_{max} (M_{u1}). for this case $\phi = 0.483 + 83.3\,\varepsilon_t = 0.816$

$$A_{s1} = \rho_{max} \cdot b \cdot d$$

We can use $\rho = \rho_t \Longrightarrow$ to ensure $\phi = 0.9$

$$a = \frac{A_{s1}\, f_y}{0.85 f_c' b}$$

$$M_{u1} = \phi M_n = \phi A_{s1} f_y \left(d - \frac{a}{2}\right)$$

5- Calculate design moment which withstand compression Steel (A'_s) and the equivelant tensile Steel reinforcement and the design moment must equal to:-

$$M_u = M_{u1} + M_{u2}$$

$$M_{u2} = M_u - M_{u1}$$

where:- M_u = design moment results from Str. analysis
M_{u1} = design moment results from tension reinforcement steel and concrete compress.
M_{u2} = design moment results from compression steel reinforcement and the equivelant tensile steel reinforcement.

6- Calculate compression steel stress.

$$C = a/\beta_1$$

$$\varepsilon'_s = \frac{c-d'}{c}\,\varepsilon_u$$

$$f'_s = E_s \varepsilon'_s = 600\,\frac{c-d'}{c} \leq f_y$$

7- Calculate compression steel area from equilebrium eq.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d-d')}$$

8- Calculate equivelant tensile steel area (balanced by comp. Steel)

$$A_{s2} f_y = A'_s f'_s \quad \therefore \quad A_{s2} = \frac{A'_s f'_s}{f_y}$$

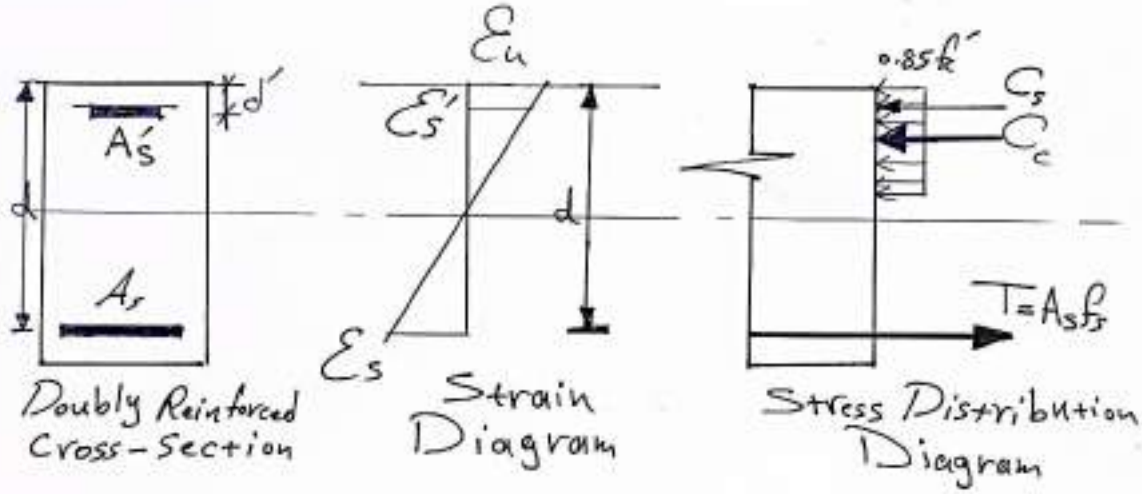
9- Find total area of tensile steel

$$A_s = A_{s1} + A_{s2}$$

10- Chose the diameter of steel reinforcement bar and find the number of these bars, then check the distances among bars according to ACI-Code requirements.

Analysis and Design of Doubly Reinforced Rectangular Beams

If a beam cross section is limited because of architectural or other considerations, it may happen that the concrete can not develop the compression force required to resist the given bending moment. In this case, reinforcement is added in the compression zone, resulting in a so-called doubly reinforced beam, i.e., one with compression as well as tension reinforcement.



A_s' : area of steel reinforcement for compression

$$\rho' = \frac{A_s'}{bd}$$

ρ'_{cy} : minimum tensile reinforcement ratio that will ensure yielding of the compression steel at failure.

$$\rho'_{cy} = 0.85\beta_1 \frac{600}{600-f_y} \cdot \frac{f_c'}{f_y} \cdot \frac{d'}{d} + \rho'$$

if $\rho \geq \rho'_{cy}$ $\Longrightarrow$ Compression Steel yield at failure.

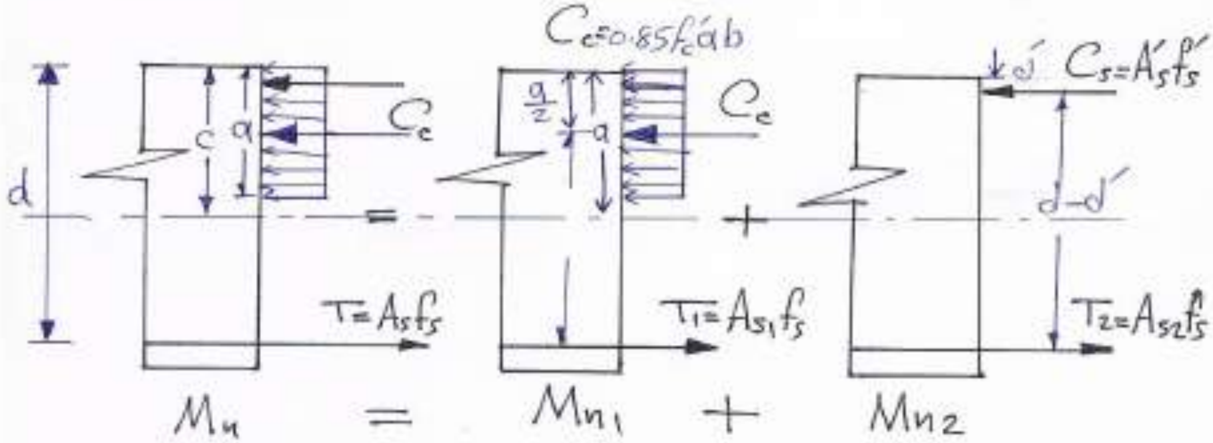
if $\rho < \rho'_{cy}$ $\Longrightarrow$ Compression Steel Stress will not reach (f_y) at failure.

Compression reinforcement may be added according to another consideration like:-

- Decreasing the deflection resulting from creep.
- Fixing the shear reinforcement.
- Resistance of tensile force resulting from changing of moment

Analysis of Doubly Reinforced Rectangular Beams:-

a- Tension and Compression Steel Both at Yield Stress:-



$$A'_s f_y = A_{s2} f_y \qquad \therefore A'_s = A_{s2}$$

$$A_{s1} = A_s - A_{s2} \qquad \therefore A_{s1} = A_s - A'_s$$

from force equilibrium:

$$A_{s1} f_y = 0.85 f'_c ab \Rightarrow a = \frac{A_{s1} f_y}{0.85 f'_c b}$$

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b}$$

$$M_n = M_{n1} + M_{n2} = A_{s1} f_y \left(d - \frac{a}{2}\right) + A_{s2} f_y (d - d')$$

$$M_n = M_{n1} + M_{n2} = 0.85 f'_c ab \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

ρ_b' : balanced reinforcement ratio for doubly reinforced beam

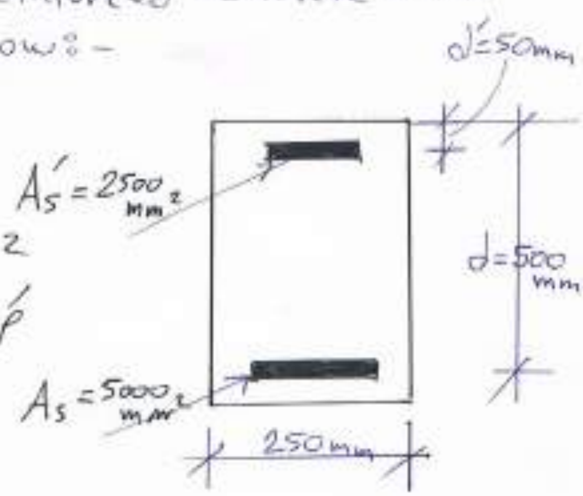
$\rho_b' = \rho_b + \rho'$

ρ_b: balanced reinforcement ratio for corresponding singly reinforced beam

$\rho_{max}' = \rho_{max} + \rho'$

Ex.:- Find the nominal moment for the cross section of doubly rectangular reinforced concrete beam shown in the figure below:-

$f_y = 350\,MPa$, $f_c' = 30\,MPa$



Solution

$\rho = \frac{5000}{250 \times 500} = 0.04$, $\rho' = \frac{2500}{500 \times 250} = 0.02$

$\rho_{cy}' = 0.85 \beta_1 \frac{f_c'}{f_y} \cdot \frac{d'}{d} \cdot \frac{600}{600 - f_y} + \rho'$

$\rho_{cy}' = 0.0349$

$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \cdot \frac{600}{600 + f_y}$

$\rho_b = 0.039$ $\quad \rho = 0.04 > \rho_{cy}' = 0.0349$ ∴ both tension & compression steel at yield stress

$\rho_b' = \rho_b + \rho' = 0.059$

$\rho = 0.04 < \rho_b'$, $a = \frac{(5000 - 2500) * 350}{0.85 * 30 * 250} = 137.254\ mm$

$\therefore M_n = 2500 \times 350 \left(500 - \frac{137.254}{2}\right) + 2500 \times 350 (500 - 50)$

$$M_n = 771.2 \times 10^6 \ N.mm$$
$$= 771.2 \ kN.m$$

b- Compression Steel below Yield Stress

$\rho < \rho'_{cy} \longrightarrow$ Compression steel will not reach f_y

$$\rho'_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{600}{600+f_y} + \rho' \frac{f'_s}{f_y}$$

$$\rho'_b = \rho_b + \rho' \frac{f'_s}{f_y}$$

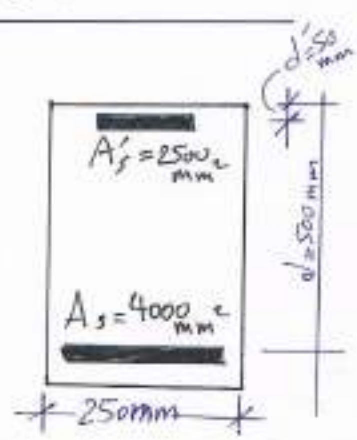
$\rho \leq \rho'_b \Longrightarrow$ Tension steel will yield

- Find (C) $\quad k_1 = \frac{A_s f_y - 600 A'_s}{0.85\beta_1 f'_c b}$, $k_2 = \frac{600 A'_s d'}{0.85\beta_1 f'_c b}$

$$C = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2} \ , \ f'_s = \frac{c-d}{c}$$

- Find $M_n = 0.85 f'_c a b (d - \frac{a}{2}) + A'_s f'_s (d - d')$

E.X. :- Find the nominal moment for cross-section of doubly rect. reinforced concrete beam shown in the figure, if, $f'_c = 30 MPa$, $f_y = 350 MPa$



Solution:-

$$\rho = \frac{4000}{250\times500} = 0.032 \quad , \quad \rho' = \frac{A'_s}{250\times500} = 0.02$$

$$\rho_b = 0.85\beta \frac{f'_c}{f_y} \frac{600}{600+f_y} = 0.85^2 \cdot \frac{30}{350} \cdot \frac{600}{600+350}$$

$$\rho_b = 0.039$$

$$\rho'_{cy} = 0.85\beta_1 \frac{600}{600-f_y} \frac{f'_c}{f_y} \frac{d'}{d} + \rho' = 0.0349$$

$$\rho = 0.032 < \rho'_{cy} = 0.0349$$

∴ Compression steel will not reach f_y

Check Tension Steel :-

$(\rho'_b) = ?$

find $f'_s \Rightarrow f'_s = 600 - (600+f_y)\frac{d'}{d}$

$$= 600 - (600+350) * \frac{50}{500} = 505\,MPa > f_y = 350\,MPa$$

$$\therefore f'_s = 350\,MPa$$

$$\rho'_b = \rho_b + \rho' \cdot \frac{f'_s}{f_y} = 0.039 \times \frac{1}{1} + 0.02 = 0.059$$

∵ $\rho < \rho'_b$ ∴ Tension Steel reach f_y (yield)

Find (c) value

$$K_1 = \frac{A_s f_y - 600 A'_s}{0.85\beta_1 f'_c b} = \frac{4000(350) - 600*(2500)}{(0.85)^2 \cdot 30 \cdot 250} = -18.45$$

$$K_2 = \frac{600 A'_s d'}{0.85\beta_1 f'_c b} = \frac{600*2500*50}{(0.85)^2 \cdot 30 \cdot 250} = 13840.83$$

$$C = \frac{k_1 + \sqrt{k_1^2 + 4k_2}}{2} = \frac{-18.45 + \sqrt{(-18.45)^2 + 4 \times 13840.83}}{2} = 108.8 \text{ mm}$$

$$a = \beta_1 c = 0.85 * 108.8 = 92.5 \text{ mm}$$

$$f_s' = \frac{c - d'}{c} 600 = \frac{(108.8 - 50)}{108.8} * 600 = 324.3 \text{ MPa}$$

$$M_n = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

$$= 0.85 \times 30 \times 92.5 \times 250 \left(500 - \frac{92.5}{2}\right) + 2500 \times 324.3 (500 - 50)$$

$$M_n = 632.408 * 10^6 \text{ N·mm}$$

$$= 632.4 \text{ kN·m}$$

C- Tensile steel below the yield stress

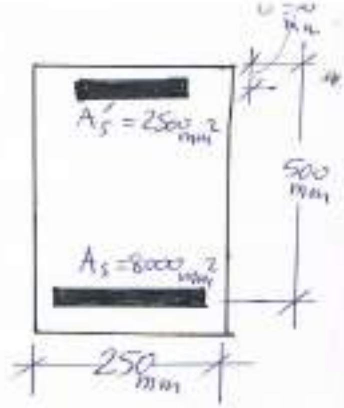
In this case $\rho > \rho_b'$ then we must find (C) by the following equation:-

$$A_s * \frac{(d-c)}{c} 600 = 0.85 \beta_1 f_c' C b + A_s' * 600 * \frac{(c-d')}{c}$$

Then find f_s', f_s

$$M_n = 0.85 f_c' a b \left(d - \frac{a}{2}\right) + A_s' f_s' (d - d')$$

EX.:- Find the nominal moment for cross-section of doubly rectangular reinforced beam shown below, for the following data: $f'_c = 30$ MPa, $f_y = 350$ MPa



Solution:-

$$\rho = \frac{8000}{250 \times 500} = 0.064 \quad , \quad \rho' = \frac{A'_s}{bd} = 0.02$$

$$\rho_b = 0.039 \quad , \quad \rho'_{cy} = 0.0349$$

$$\rho'_b = \rho_b + \rho' = 0.059$$

$\rho = 0.064 > \rho'_b = 0.059$ ∴ The failure will be compression failure

$\rho = 0.064 > \rho'_{cy}$ ∴ Compression steel will reach f_y i.e. $f'_s = f_y$

Find (C):-

$$A_s \frac{(d-C)}{C}(600) = 0.85 \beta_1 f'_c c\, b + A'_s f_y$$

$$8000 \cdot \frac{(500-C)}{C} \cdot (600) = (0.85)^2 * 30 * C * 250 + 2500 * 350$$

$C = 323$ mm (الحل بواسطة طريقة الدستور)

$$f_s = 600\left(\frac{500-C}{C}\right) = 600\left(\frac{500-323}{328}\right) = 328.8 \text{ MPa}$$

$$a = \beta_1 C = 0.85 * 323 = 274.6 \text{ mm}$$

$$M_n = 0.85 f'_c a\, b\left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

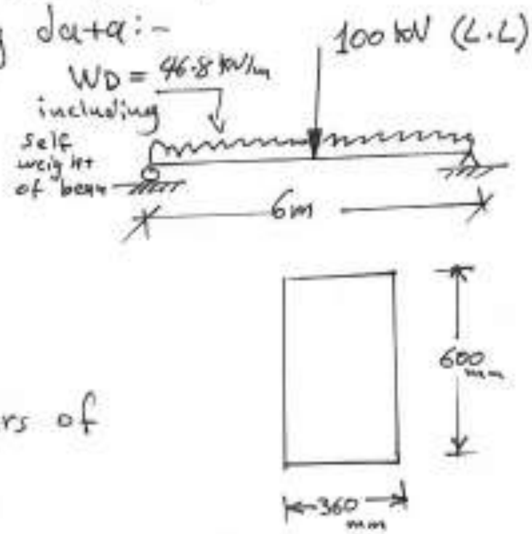
$$M_n = 1028.7 \text{ kN.m}$$

Design:-

Ex. :- For a simply supported beam shown in the Fig. shown below, Find the area of Steel & its details for the following data:-

$f_y = 400$ MPa, $f'_c = 20$ MPa

Note:- If there is need for compression Steel use $d' = 65$ mm.



Solution:- Assume 2 layers of steel Reinf.

* $d = h - 100$
  $= 500$ mm

* $P_u = 100 * 1.6 = 160$ kN

* $W_u = 46.8 * 1.2 = 56.16$ kN/m

• $M_u = \frac{P_u * L}{4} + \frac{W_u * L^2}{8}$

$= 160 * \frac{6}{4} + 56.16 * \frac{(6)^2}{8} = 492.72$ kN.m

• from Table (r3) $\rho_{max} = 0.0155$

• $R = \frac{M_u}{\phi b d^2}$, $m = \frac{f_y}{0.85 f'_c}$

$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right)$

$$m = \frac{400}{0.85 * 20} = 23.5 \quad , \quad R = \frac{492.72 * 10^6}{0.9 * 360 * (500)^2} = 6.08$$

$$\rho = \frac{1}{23.5}\left(1 - \sqrt{1 - \frac{2 * 23.5 * 6.08}{400}}\right) = 0.0198$$

$\because \rho > \rho_{max} \Longrightarrow$ Design the beam as a Doubly Reinforced Beam.

- Find A_{s1}

$$A_{s1} = \rho_{max}\, b d = 0.155 * 360 * 500 = 2790\ mm^2$$

$$a = \frac{A_{s1} f_y}{0.85 f'_c b} = \frac{2790 * 400}{0.85 * 20 * 360} = 182\ mm$$

$$M_{u1} = \phi M_{n1} = \phi A_{s1} f_y \left(d - \frac{a}{2}\right)$$

$\because \rho > \rho_{max} \longrightarrow \therefore \rho > \rho_t$

$\therefore \phi < 0.9$

$$\phi = 0.483 + 83.3\,\varepsilon_t = 0.483 + 83.3 * 0.004 = 0.816$$

$$\therefore M_{u1} = 0.816 * 2790 * 400\left[500 - \frac{182}{2}\right] * 10^{-6} = 372.5\ kN.m$$

- Find M_{u2}

$$M_{u2} = M_u - M_{u1} = 492.72 - 372.5 = 120\ kN.m$$

- Calculate the compressive Steel Reinf. Stress.

$$C = a/\beta_1 = \frac{182}{0.85} \approx 214\ mm$$

$$f'_s = \frac{c - d'}{c} 600 = \left(\frac{214 - 65}{65}\right) * 600 = 418 \text{ MPa} > f_y = 400 \text{ MPa}$$

- Finding area of compressive steel reinf.

$$A'_s = \frac{M_{u2}}{\phi f'_s (d - d')} = \frac{120.22 * 10^6}{0.816 * 400 (500 - 65)} = 847 \text{ mm}^2$$

- Area of tension steel insted of comp. steel
(مساحة حديد الشد الذي يقابل حديد الانضغاط)

$$A_{s2} = A'_s = 847 \text{ mm}^2$$

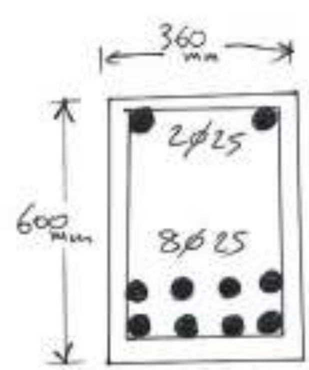
- Total Tension Steel Reinforcement

$$A_s = A_{s1} + A_{s2} = 2790 + 847 = 3637 \text{ mm}^2$$

- Use $\phi 25$ → No of bars $= \dfrac{3637}{\frac{\pi}{4} * (25)^2} = 7.4$

∴ Use $8 \phi 25$

$$S = \frac{360 - 100 - 4*25}{3} = 53 \text{ mm} > 25 \text{ mm} \quad \therefore \text{o.k.}$$



for Steel Reinf. in Comp Zone

$A'_s = 847$ No of bars $= \dfrac{847}{491} = 1.725$

∴ Use $2 \phi 25$

Ex.: A rectangular beam has width 250mm & Effective depth 460mm. $f_y = 300$ MPa, $f'_c = 20$ MPa. What is the maximum moment that can be utilized in design, according to the ACI Code,

when: – a- $A_s = 2000\ mm^2$ b- $A_s = 5160\ mm^2$

Solution:–

a-

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{600}{600+f_y} = (0.85)^2 \times \frac{20}{300} \times \frac{600}{600+300}$$

$\rho_b = 0.032$ or from Table (A3) Page 350

$$\rho = \frac{A_s}{bd} = \frac{2000}{250 * 460} = 0.0174 < \rho_b = 0.032$$

∴ The section is underreinforced

To calculate ϕ value we must find ρ_t

$$\rho_t = 0.85\beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003 + \varepsilon_t} = (0.85)^2 * \frac{20}{300} * \frac{0.003}{0.003 + 0.005}$$

$\rho_t = 0.018$ or from Table (A3), Page 350

$\rho = 0.0174 < 0.0180$ ∴ $\phi = 0.9$

$M_u = \phi M_n$

$$M_u = \phi \rho b d^2 f_y \left[1 - 0.59 \frac{\rho f_y}{f'_c}\right]$$

$$M_u = 0.9 * 0.0174 * 250 * 460^2 * 300 \left[1 - \frac{0.59 * 0.0174 * 300}{20}\right]$$

$= 210,253,958$ N.mm

or

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2000 * 300}{0.85 * 20 * 250} = 141.176 \text{ mm}$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 * 2000 * 300 \left(460 - \frac{141.176}{2}\right)$$

$$= 210,282,480 \text{ N.mm}$$

$$\simeq 210.282 \text{ kN.m}$$

or $M_u = 0.85 * \phi f'_c a b \left(d - \frac{a}{2}\right)$

$$= 0.85 * 0.9 * 20 * 141.176 * 250 \left(460 - \frac{141.176}{2}\right)$$

$$= 210,281,779 \text{ N.mm} \simeq 210.281 \text{ kN.m}$$

b- $A_s = 5160 \text{ mm}^2$ $\quad \rho = \frac{A_s}{bd} = \frac{5160}{250 * 460} = 0.04487 \simeq 0.045$

$\rho_b = 0.032$ $\quad \rho = 0.045 > \rho_b = 0.032$

∴ The beam is over reinforced

1- find $m = \frac{600}{0.85 \beta_1 f'_c} = \frac{600}{0.85 * 0.85 * 20} = 41.522$

$$\rho * m = 0.045 * 41.522 = 1.869$$

$$k_u = \sqrt{\left(\frac{\rho m}{2}\right)^2 + \rho m} - \frac{\rho m}{2}$$

$$= \sqrt{\left(\frac{1.869}{2}\right)^2 + 1.869} - \frac{1.869}{2} = 0.721$$

3- Find c

$$c = k_u d = 0.721 * 460 = 331.66\ mm$$
$$\simeq 331.7\ mm$$

4- Find a

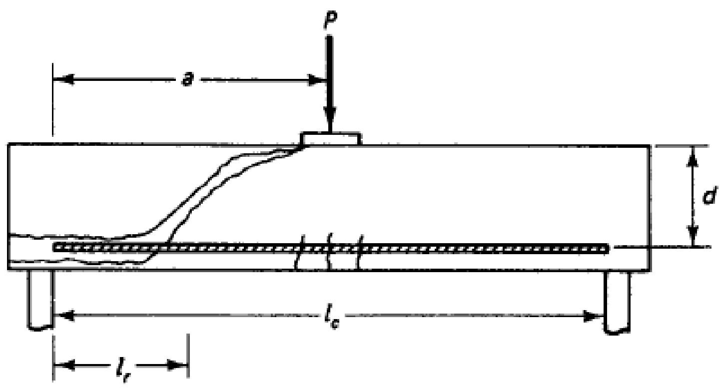
$$a = 0.85c = 0.85 * 331.7 = 281.945\ mm$$
($0.85 \rightarrow \beta_1$)
$$a \simeq 282\ mm$$

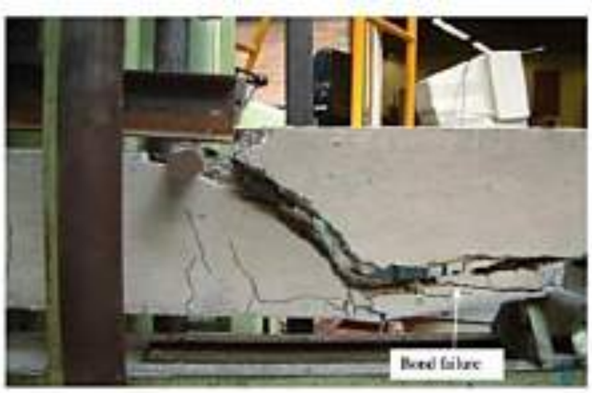
5- Find M_n

$$M_n = 0.85 f'_c * a * b\left(d - \frac{a}{2}\right)$$
$$= 0.85 * 20 * 282 * 250 * \left(460 - \frac{282}{2}\right)$$
$$= 382,321,500\ N.mm$$
$$= 382.321 \times 10^6\ N.mm = 382.321\ kN.m$$

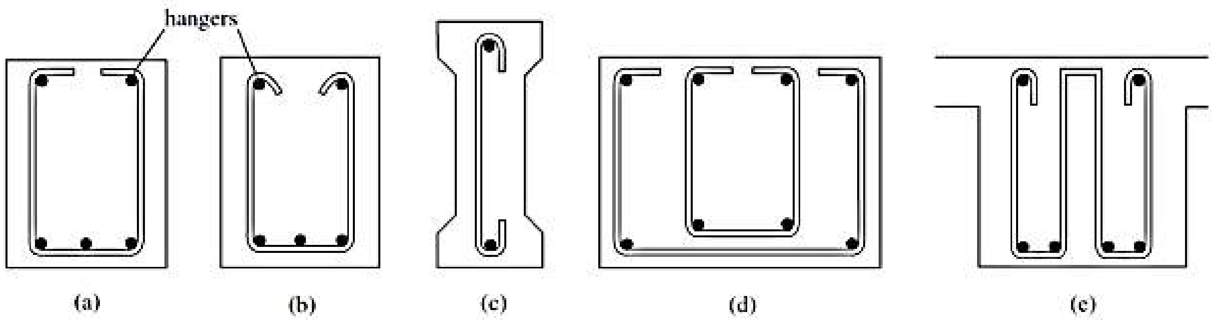
SHEAR AND DIAGONAL TENSION

When a simple beam is loaded, as shown in Fig. bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters. The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.

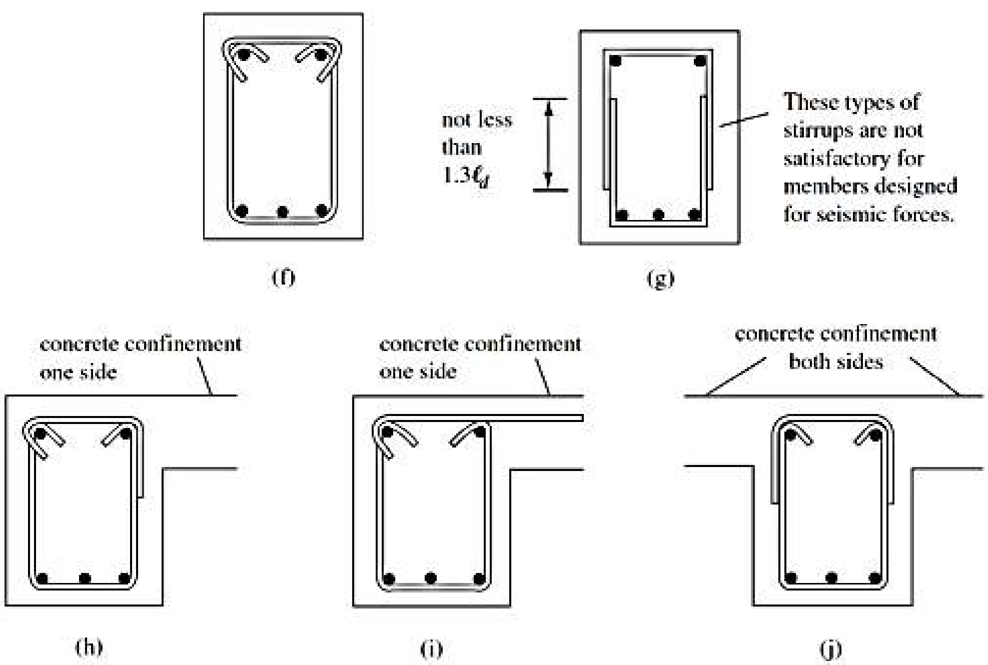




Web Reinforcement

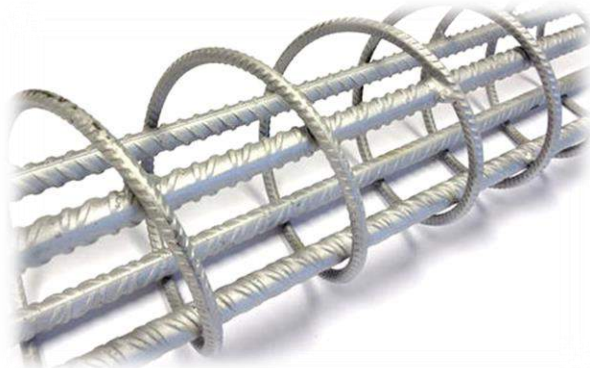


Closed stirrups for beams with significant torsion (see ACI 11.5.2.1)

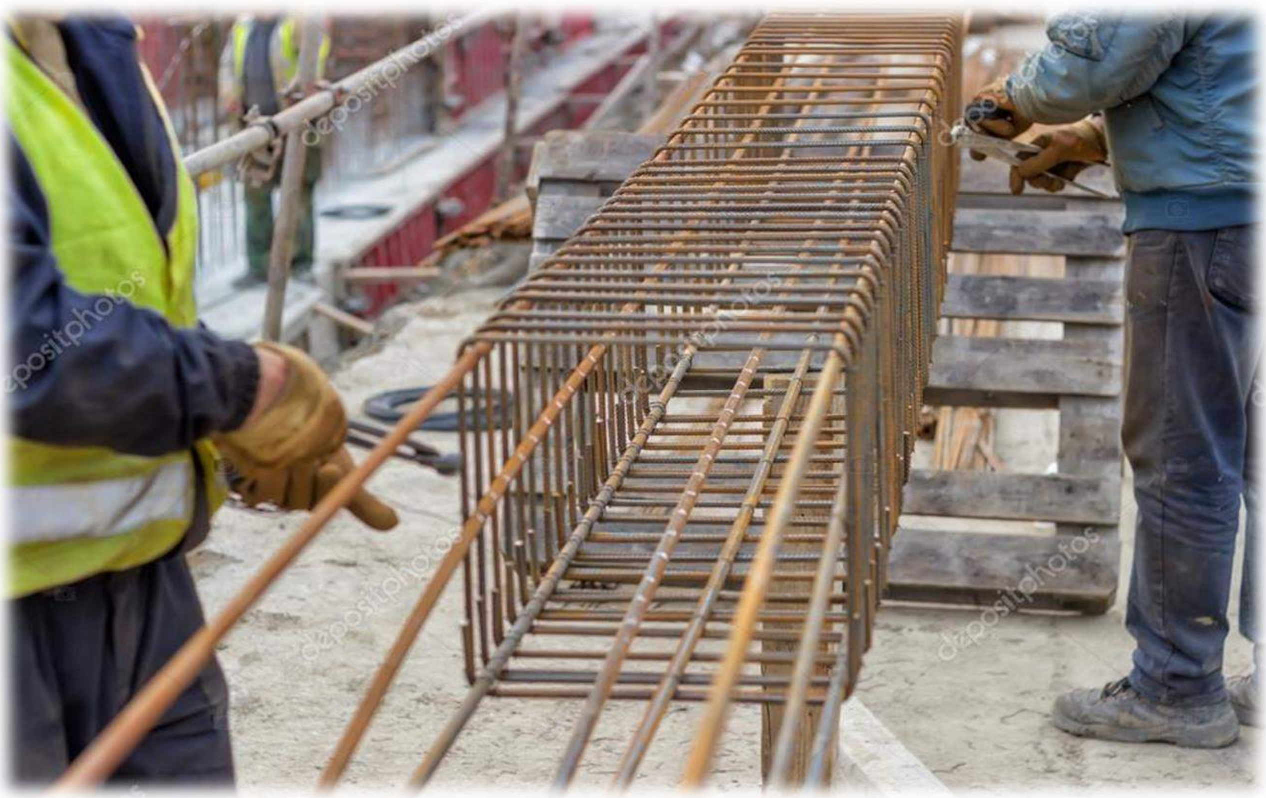


Types of stirrups.

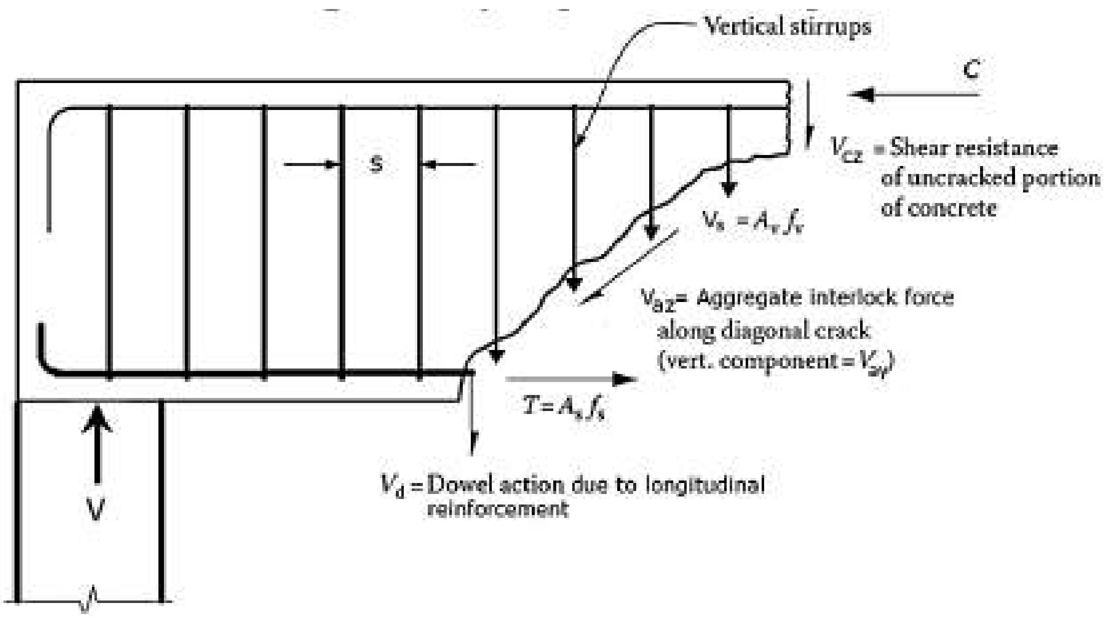


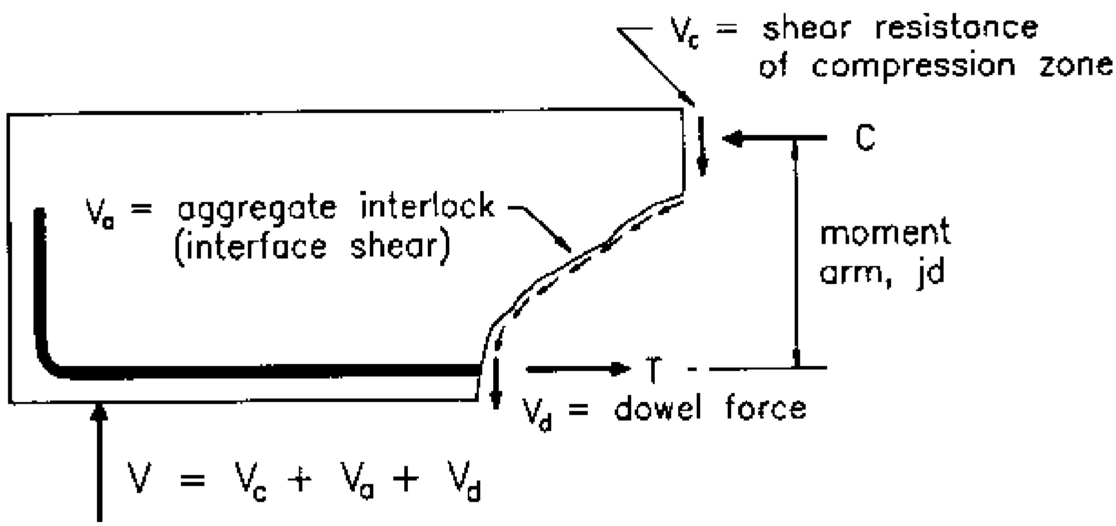






Shear Strength of Beam





C- Design of Web Reinforcement :-

* $S = (A_v f_y d)/V_s$
* $V_s = V_n - V_c = \frac{V_u}{\phi} - V_c$

$$V_s = \frac{V_u}{\phi} - V_c$$

* $S = \frac{A_v f_y (\sin\alpha + \cos\alpha)}{V_s}$

* When $V_s \leq \frac{1}{3}\sqrt{f'_c}\, b_w d \; (=2V_c)$

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \frac{3A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* when $V_s > \frac{1}{3}\sqrt{f'_c}\, b_w d \; (=2V_c)$

$$S_{max} \leq \begin{cases} d/4 \\ 300\,mm \\ \frac{3A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases}$$

* If $V_s > \frac{2}{3}\sqrt{f'_c}\, b_w d \; (=4V_c)$ The Section must be changed.

Design Procedure for Web Reinforcement

1- Analayzing the beam and draw S.F. Diagram

2- Find shear force design (V_{ud}) and find (ϕV_c) from the equations below according to the kind of loadings:-

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w d$$

$$V_c = \left(1+\frac{N_u}{14 A_g}\right)\left[\frac{\sqrt{f_c'}}{6} b_w d\right]$$

$$V_c = \left(1+\frac{0.3 N_u}{A_g}\right)\left[\frac{\sqrt{f_c'}}{6} b_w d\right]$$

3- If • $V_{ud} \leq \phi V_c/2 \Rightarrow$ No need for shear reinforcement

• $\phi V_c/2 \leq V_{ud} \leq \phi V_c \Rightarrow$ Minimum shear reinf.

The maximum distance between stirrups is calculated by

$$S_{max} \leq \begin{cases} d/2 \\ 600\text{ mm} \\ \dfrac{3 A_v f_y}{b_w} \\ \dfrac{16 A_v f_y}{\sqrt{f_c'}\, b_w} \end{cases} \quad [\text{the min. value}]$$

The stirrups will be continued to $V_u = \dfrac{\phi V_c}{2}$ and after that there is no need for shear reinf.

4- If $(V_{ud} > \phi V_c)$, then find shear force design for steel (ϕV_s). If this force is greater than $(4\phi V_c)$ then the beam section must be changed, if not the distance between stirrups (Max distance) will be find from the equation, below according to (V_s) value

$$S_{max} \leq \begin{cases} d/2 \\ 600\,mm \\ \frac{3A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases} \quad \text{the min Value} \quad \text{if } V_s \leq \frac{1}{3}\sqrt{f'_c}\, b_w d \; (=2V_c)$$

$$S_{max} \leq \begin{cases} d/4 \\ 300\,mm \\ \frac{3A_v f_y}{b_w} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \end{cases} \quad \text{the min Value} \quad \text{if } V_s > \frac{1}{3}\sqrt{f'_c}\, b_w d \; (4V_c)$$

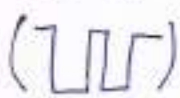
5- Calculating the distance between the stirrups at critical section (S_o), if thes distance greater or equal to (S_{max}), then the distance from support face to the point, which at this point $(V_u = \phi V_c/2)$ will found and using $(S = S_{max})$. If

$(S_o < S_{max})$, then we find the distance which, after this distance we will reinforce by minmum reinforcement, and after that we find the distance which there is no need to shear reinforcement.

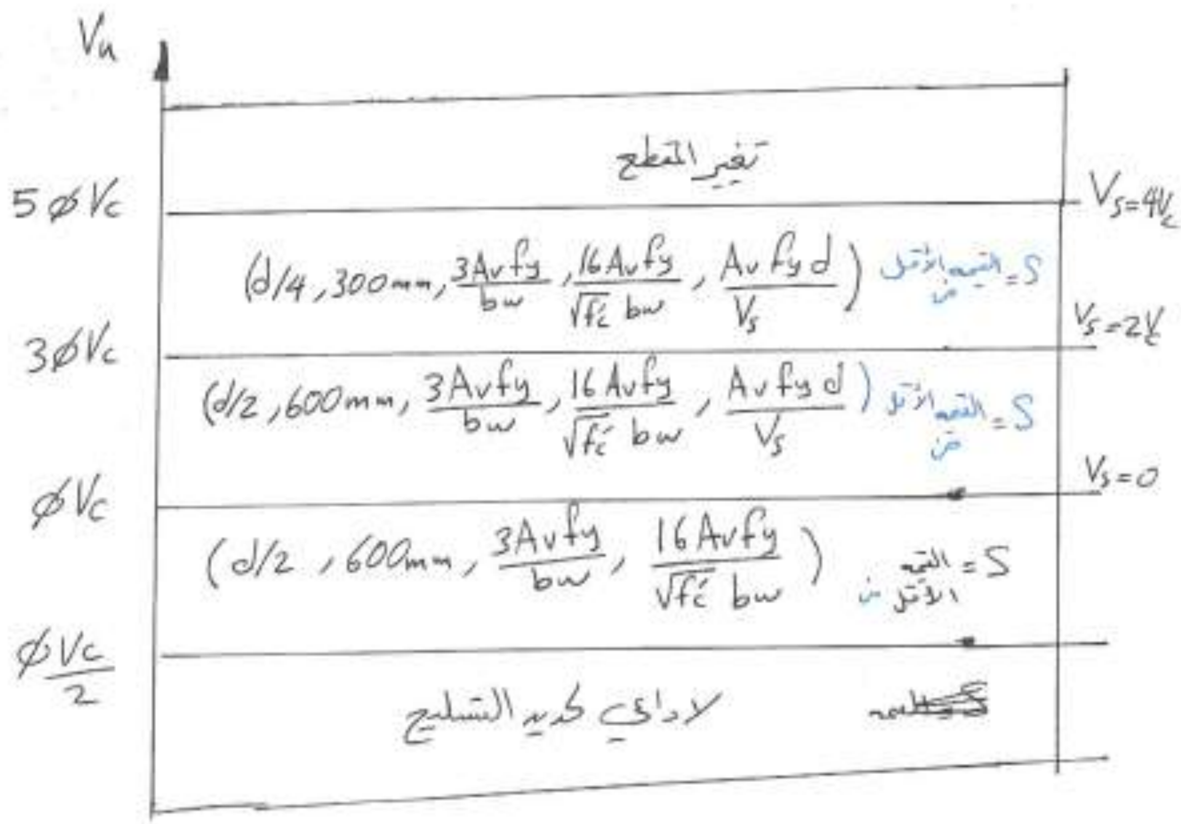
6- Find the distance between stirrups for the region between critical section and the point of min. reinf. by using the following eq.

$$S = \frac{A_v f_y d}{V_s}$$

and the distance between stirrups will be changed according to the methods used before.

- If the distance between stirrups is small, then we use bigger (ϕ bar) or use stirrups with () shape.

7. Clarify the position, kind & radius of stirrups (ϕ bar of stirrups) on the beam diagram



مخطط تسليح القص حسب المقاومة التصميمية للقص (Vu)

Shear Strength of Concrete :-

$$v_{cr} = \frac{V_{cr}}{b_w d} = 0.3\sqrt{f_c'}$$

because of reduction in area which was caused by flexural cracking, the shear strength of beam is less than that found in the equation above, & it is find by the following equation.

$$v_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{6}\sqrt{f_c'}$$

That means bending moment may caused a decreasing in shear strength to about half its magnitude.

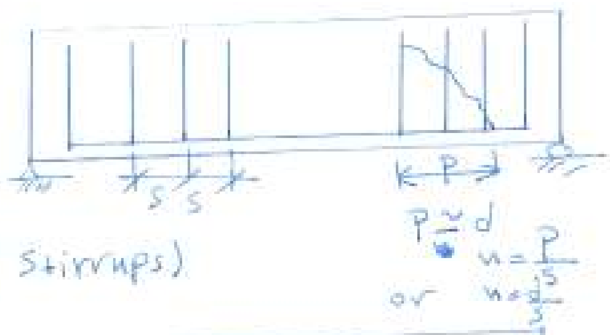
The stresses of shear in the case of cracking depend on the ratio between bending moment to shear & it is also depend on the longitudenal steel reinforcement ratio, because this steel reinforcement lead to decrease the cracks caused by bending & then increase of concrete resistance to radial cracks, i.e. increasing shear strength of concrete.

$$v_{cr} = \frac{V_{cr}}{b_w d} = \frac{1}{7}\left(\sqrt{f_c'} + 120\rho\frac{Vd}{M}\right) \leq 0.3\sqrt{f_c'}$$

Shear Strength of Web Reinforcement:-

$V_s = n A_v f_y$

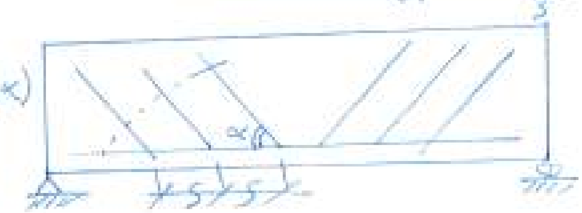
(n = No. of stirrups; A_v = area of shear reinforcement)



$V_s = \frac{A_v f_y d}{S}$ (vertical stirrups)

$V_s = \frac{A_v f_y d}{S} (\sin\alpha + \cos\alpha)$

(Inclined stirrups)



$V_n = V_c + V_s = V_c + \frac{A_v f_y d}{S}$

$V_u \leqslant \phi V_n$

In the case of there is no concentrated force between the face of the support & in the distance equal to (d), so the critical section for maximum shear force is taken in distance about (d) from the face of the support. The distance (S) from the face of the support to (d) is equal to the space calculated at the distance (d) from the face of the support.

If the conditions above are not occure then the critical section is taken at the face of the support.

According to ACI-code

$$V_c = \left(\sqrt{f'_c} + 120\rho_w \frac{V_u d}{M_u}\right)\frac{b_w d}{7} \leq 0.3\sqrt{f'_c}\, b_w d$$

& the $\frac{V_u d}{M_u}$ must be ≤ 1.0

The equation above is used for researchs & Programming but for design the code give this eq.:-

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d$$

If there is an axial compressive force, the shear resistance will increase and can be found by this eq.

$$V_c = \left(1 + \frac{N_u}{14 A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right) b_w d$$

where:- N_u is a compressive force (N)
A_g is a total section area.

If there is an axial tension force, then, the shear resistance will decrease & can be found by the following eq.:-

$$V_c = \left(1 + \frac{0.3 N_u}{A_g}\right)\left(\frac{\sqrt{f'_c}}{6}\right) b_w d$$

where:- N_u is tension force in (N) with negative sign (-).

Shear Design of Beams:-

a- Minimum Shear Reinforcement:-

Theoretically there is no need to shear reinforcement when the shear force is less than concrete strength design:- $V_u \leq \phi V_c$

& the following equation is used to find shear strength of concrete

$$V_c = \frac{1}{6}\sqrt{f'_c}\, b_w d$$

But the ACI code requires provision of at least a minimum area of web reinforcement equal to:-

$$A_v = \frac{1}{16}\sqrt{f'_c}\,\frac{b_w S}{f_y} \geq \frac{b_w S}{3 f_y}$$

when $V_u > \frac{\phi V_c}{2}$

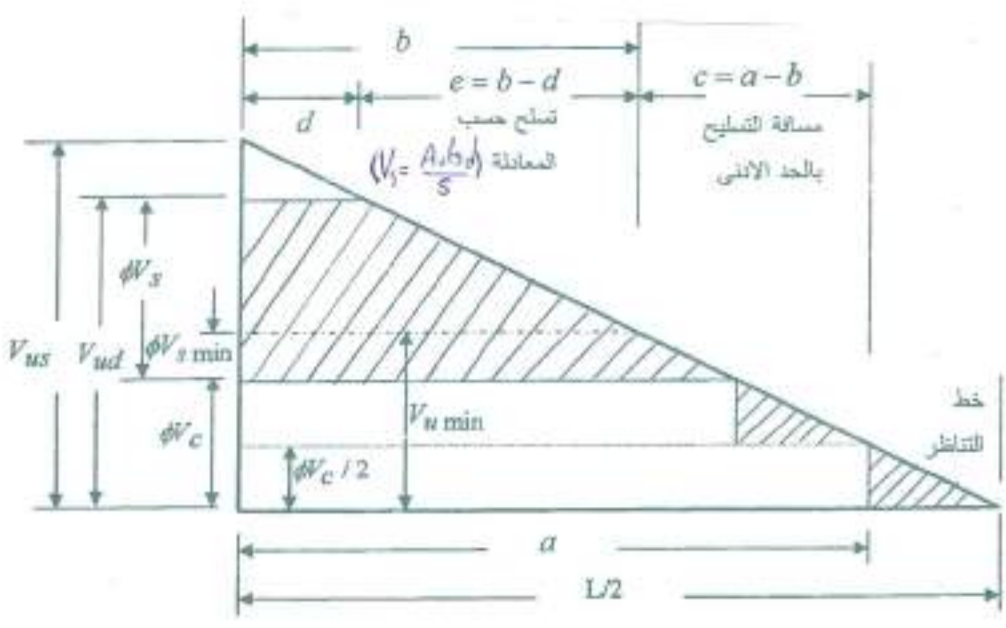
$$S_{max} \leq \begin{cases} \dfrac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \\ \dfrac{3 A_v f_y}{b_w} \end{cases}$$

There is no need for shear reinforcement when

$$V_u \leq \phi \frac{V_c}{2}$$

B- Region of Web Reinforcement

- When (V_{ud}) Shear force design at critical section less than ($\phi V_c/2$), there is no need to stirrups
- When (V_{ud}) larger than ($\phi V_c/2$) & less or equal (ϕV_c) then the beam must reinforced by minmum shear reinforcement for the distance varied from the face of the support to the point, that at this point the shear force equal to ($\phi V_c/2$).
- If the (V_{ud}) (shear force design) at critical section is greater than (ϕV_c), then there will be categories according to the Fig. below.



This Fig. represents the shear force diagram for a half uniformly distributed load simply supported beam. These catogaries are:-

1- The distance between critical section and face of the support. The shear reinforcement at this distance equal to the same amount of reinforcement for critical section. That means the distance between the stirrups at critical section (S_o). The first stirrup will be putted at distance equal to $(S_o/2)$ from the support face.

2- The distance from the point reinforced with minimum shear reinforcement (b) to the critical section which is called (e) and it reinforced according to equation $\left(V_s = \frac{A_v f_y d}{s}\right)$. The minmum shear reinforcement means that, the distance between stirrups, is the maximum distance (S_{max}). The distance (b) is determined by calculating the minimum shear strength of reinforcement (i.e. $S = S_{max}$)

$$V_{s\,min} = \frac{A_v f_y d}{S_{max}}$$

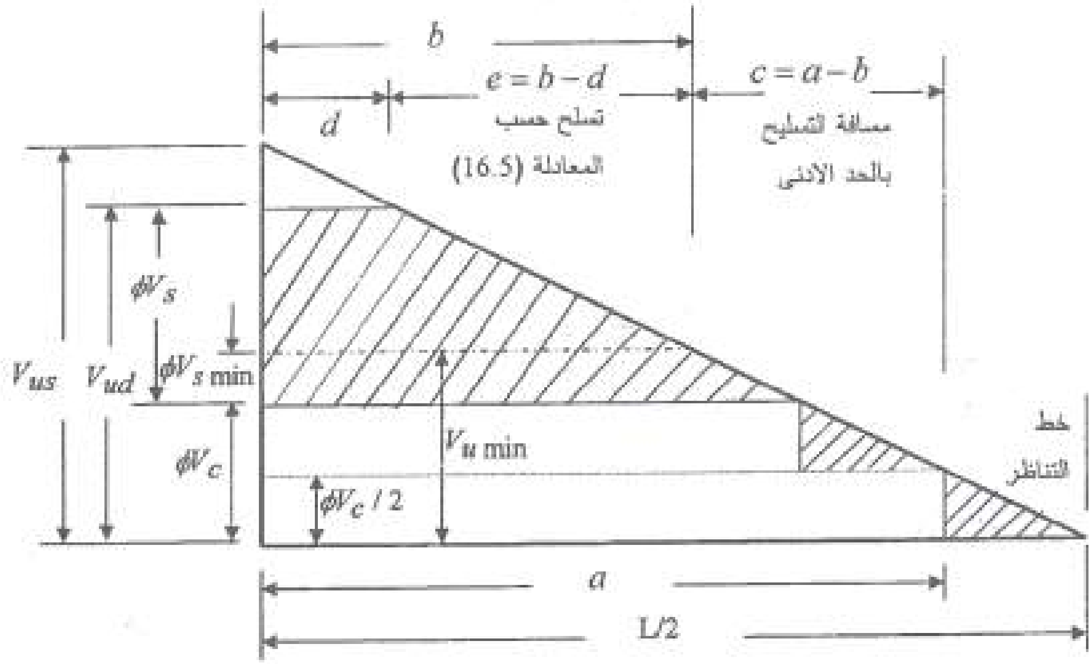
After that, minmum shear strength design (V_{umin}) will calculated.

$$V_{u\,min} = \phi V_{s\,min} + \phi V_c$$

From equilibrium or the triangnls theory (b) can be found. At this point shear strength design equal to ($V_{u\,min}$).

3- The distance from the point which there is no need to shear reinf. (at distance (a) from support face) to the point, which the shear reinforcement at this point is equal to minimum reinforcement (Point (C)). This distance will reinforce by minimum reinforcement ($S = S_{max}$). (a) can be found by force equilibrium or the trinanguls theory. Shear Force design at distance (a) equal to ($\phi V_c/2$)

- There is no need for shear reinforcement between point (a) and the point which, at this point the shear force equal to zero.


b
e = b − d
c = a − b
d
تسلح حسب
المعادلة (16.5)
مسافة التسليح
بالحد الأدنى
ϕVs
ϕVs min
Vus
Vud
ϕVc
Vu min
ϕVc / 2
خط
التناظر
a
L/2

E.x.:- Design shear reinforcement for the beam shown below for the following data:-

$b = 300\,mm$, $d = 500\,mm$, $LL = 40\,kN/m$, DL including self weight $= 34\,kN/m$, $f_y = 300\,MPa$, $f'_c = 30\,MPa$.

Solution:-

* $W_u = 1.6 \times 40 + 1.2 \times 34 = 104.8\,kN/m$

* finding shear force at the face of the support.

$V_{us} = 104.8 * \frac{5.5}{2} = 288.2\,kN$

* Finding shear force at critical section.

$V_{ud} = V_{us} - W_u d = 288.2 - 0.5 \times 104.8 = 235.8\,kN$

* $\phi V_c = 0.75 \left(\frac{1}{6}\sqrt{f'_c} \times b \times d\right)$

$= 0.75\left(\frac{1}{6}\sqrt{30} \times 300 \times 500\right) * 10^{-3} = 102.698\,kN$

Check if there is need for shear reinforcement

$V_{ud} = 235.8 > \phi V_c = 102.698$ ∴ there is a need for shear reinforcement

* $\phi V_s = V_{ud} - \phi V_c = 235.8 - 102.698 = 133.1\,kN$

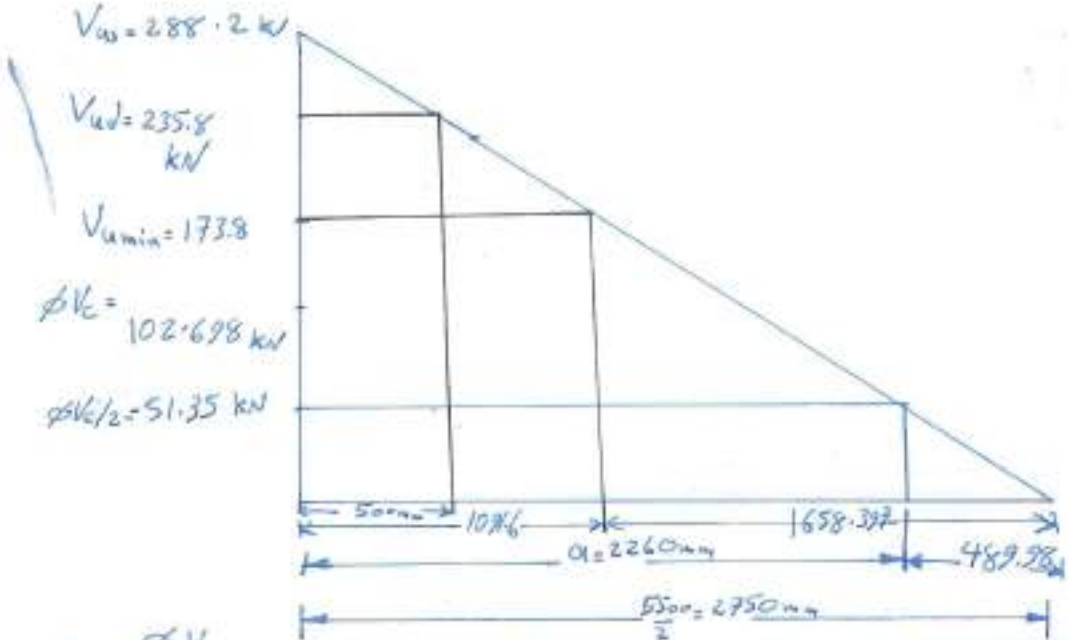
$V_s = \frac{133.1}{\phi} = \frac{133.1}{0.75} = 177.47\,kN$

* $4 * \phi V_c = 4 * 102.698 = 410.792\,kN$

∵ $\phi V_s < 4\phi V_c$

$133.1\,kN < 410.792\,kN$

∴ The section is adiquate for shear reinforcement.



* $\frac{\phi V_c}{2} =$

$\frac{102.698}{2} = 51.35$ kN

$\because \phi V_s < 4\phi V_c$

$* \therefore S_{max} \leq \begin{cases} d/2 = 250 \text{ mm} \checkmark \\ 600 \text{ mm} \\ \frac{3 A_v f_y}{b_w} = \frac{3 \times 2 \times 79 \times 300}{300} = 474 \text{ mm} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} = \frac{16 \times 2 \times 79 \times 300}{\sqrt{30} \times 300} = 461 \text{ mm} \end{cases}$

$\therefore$ Use $S_{max} = 250$ mm

* Find spacing of reinforcement at critical section.

$S_o = \frac{A_v f_y d}{V_s} \longrightarrow$ use ϕ 10mm for stirrups.

$\therefore A_v = 2 \times \frac{\pi}{4} \times 10^2 = 157.0 \text{ mm}^2$

$S_o = \frac{157 \times 300 \times 500}{177.47 \times 10^{-3}} = 132.7 \text{ mm} < S_{max} = 250 \text{ mm}$

$\therefore$ use $S_o = 130$ mm c/c

* Finding the distance in which there is no need for reinf.

$\frac{x}{51.35} = \frac{2750}{288.2} \Rightarrow x = 489.98$ mm

$a = 2750 - 489.98 = 2260$ mm

- We can find the distance (a) by other method.

$$V_{us} - W_u * a = \phi V_c / 2$$

$$288.2 - 104.8 * a = 51.35 \Longrightarrow a = \frac{51.35 - 288.2}{-104.8}$$

$\therefore a = 2.260$ m

* Determine the distance, which it is after reinforced by minimum reinforcement (shear reinforcement).

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 * 157 * 300 * 500}{250} * 10^{-3} = 70.650 \text{ kN}$$

$$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8 \text{ kN}$$

$$V_{us} - W_u b = 173.8$$

$$288.2 - 104.8 * b = 173.8 \Longrightarrow b = \frac{173.8 - 288.2}{-104.8}$$

$b = 1.0916$ m

or $\frac{x}{173.8} = \frac{2750}{288.2} \Longrightarrow x = 1658.397 \Longrightarrow x_1 = 2750 - 1658.397$

$\Longrightarrow x_1 = 1091.6$ mm

* Distribution of shear reinforcement along the beam

a- Put the first stirrups at a distance equal to $\frac{S_o}{2} = \frac{130}{2} = 65$ mm

$\therefore$ Put the first stirrups at a distance equal to 60 mm from the face of the support.

b- Number of the stirrups (130 mm)

$$n = \frac{1092 - 60}{130} = 7.938$$ Use 8 ϕ10 stirrup @ 130 mm

c- So, the distance from the face of the support which reinforced for shear until now equal to:-

$60 + 8 * 130 = 1100$ mm

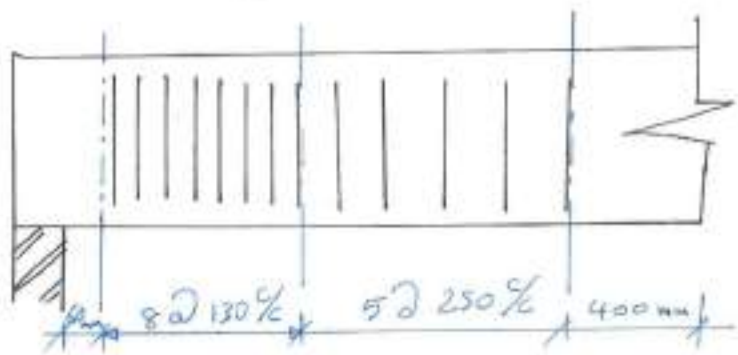
$\therefore$ № of stirrups of 250mm c/c $= \frac{2260 - 1100}{250}$

$= 4.64$

$\therefore$ use $5 \phi 10$ mm stirrups @ 250 c/c

So, the space which reinforced to shear equal to $1100 + 5(250) = 2350$ mm

So, the region which not reinforced for shear is equal to $2750 - 2350 = 400$ mm



or from equilibrium

$V_{us} - w_u a = \phi V_c/2$

$288.2 - 104.8a = 51.35\ kN \Rightarrow a = 2260\ mm$

finding the distance which after this distance the shear reinforcement in minmum magnitude

$$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 500 \times 10^{-3}}{250}$$
$$= 71.1\ kN$$

$V_{u\,min} = \phi V_{s\,min} + \phi V_c = 71.1 + 102.7 = 173.8\ kN$

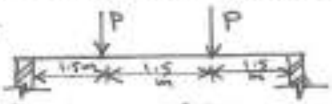
$V_{us} - w_u b = 288.2 - 104.8b = 173.8 \Rightarrow b = 1092\ mm$

the distance between critical section & the point of minmum shear reinf. is such

$e = b - 500 = 1092 - 500 = 592\ mm$

So use the same shear reinf. for critical section

Ex.:- A reinforced concrete girder with a rectangular section, loaded by two concentrated loads, each of them consist of 80 kN service live load & 60 kN service dead load. The width of this girder equal to 300 mm & its effective depth equal to 550 mm. Design this girder for shear.



Solution:- 1- $h = 550 + 100 = 650$ mm (if we assume the reinforcement in 2 layrs)

$\therefore W_g = 0.65 \times 0.3 \times 24 = 4.68$ kN/m

$W_u = 4.68 \times 1.2 = 5.62$ kN/m

$P_u = 1.2 * 60 + 1.6 * 80 = 200$ kN

$V_{us} = 200 + 4.5 \times \frac{5.62}{2} = 212.65$ kN

2- Calculate V_{ud}

$V_{ud} = 212.65 - 5.62 \times 0.55 = 209.6$ kN

$\phi V_c = 0.75 \left(\frac{1}{6}\right)\sqrt{30} \times 300 \times 550 * 10^{-3} = 112.96$ kN

$\frac{\phi V_c}{2} = 56.48$ kN

3- $V_{ud} = 209.6$ kN $> \phi V_c = 112.96$ kN $\therefore$ There is need for shear reinf.

Calculate $\phi V_s = 209.6 - 112.96 = 96.6$ kN

$\therefore V_s = 128.8$ kN

$\because \phi V_s < 4\phi V_c \; \therefore$ the section is adiqute for shear

$\because \phi V_s < 2\phi V_c$

Use ϕ bar 10 mm for strirups

$$\therefore S_{max} \begin{cases} 600 \text{ mm} \\ d/2 = 275 \text{ mm} \\ \frac{3 A_v f_y}{b_w} = 3 \times 2 \times \frac{\pi}{4} \times 10^2 \times 300 = 474 \text{ mm} \\ \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} = \frac{16 \times 2 \times 79 \times 300}{\sqrt{30} \times 300} = 462 \text{ mm} \end{cases}$$

Distance from (0-1.5) m

∴ from shear force diagram

$V_u = 204.22\ kN > \phi V_c = 112.96\ kN$

∴ all the distance will reinforced for shear

$\phi V_{s\,min} = \frac{\phi A_v f_y d}{S_{max}} = \frac{0.75 \times 2 \times 79 \times 300 \times 550}{275} \times 10^{-3} = 71.1\ kN$

$V_{u\,min} = 71.1 + 112.96 = 184.06\ kN$

$V_{u\,min} < 204.22\ kN$

So, we don't use $S = S_{max}$

Distance (3-15) m

$V_u = 4.22 < \phi V_c/2 = 56.48\ kN$

∴ There is no need for shear Reinf.

Note :- (Because the variation in shear in the region (0-1.5) is small, the distance between the stirrups is still with the same reinforcement for shear for S_o)

Put the firs stirrup in the distance equal to $S_o/2 = \frac{200}{2} = 100\ mm$ from the face of the support

So, the No other stirrups

$n = \frac{1500 - 100}{200} = 7$

i.e. (7 @ 200 mm c/c)

Use $S_{max} = 275\,mm$

5- Calculate (S_o) $\quad S_o = \frac{A_v f_{yd}}{V_s} = \frac{2 * \frac{\pi}{4} * 300 * 550}{128.8 * 1000} = 202\,mm$

USE ⟹ $S_o = 200\,mm < S_{max}$