ENGINEERING MECHANICS STATICS

Dams & Water Resources Department
First Stage – 2nd Semester
2017 - 2018

Lecture Notes Prepared by:

Assistant Professor
Dr. Ayad A. Sulaibi

Course Objectives

- **To understand and use the general ideas of force vectors and equilibrium of particle and rigid body.**
- **To understand and use the general ideas of structural analysis and internal force and friction.**
- **To understand and use the general ideas of center of gravity, centroids and moments of inertia.**

Subjects

1. General principles
2. Force vectors
3. Equilibrium of a particle
4. Force system resultants
5. Equilibrium of a Rigid Body
6. Structural Analysis
7. Internal Forces
8. Friction
9. Center of Gravity and Centroid of Areas
10. Moments of Inertia (Second Moment of Areas)

Textbook

- R. C. Hibbeler, "Engineering mechanics – Statics", 13th edition, 20013.

REFERENCES

1. Andrew Pytel and Jaan Kiusalaas, "Engineering Mechanics – Statics", Third Edition, 2010.
2. J. L. Meriam and L.G. Kraige, "Engineering Mechanics – Vol.1", Fifth Edition, 2002.

CHAPTER 1

CHAPTER OBJECTIVES

- ✓ *To provide an introduction to the basic quantities and idealizations of mechanics.*
- ✓ *To give a statement of Newton's Laws of Motion and Gravitation.*
- ✓ *To review the principles for applying the SI system of units.*
- ✓ *To examine the standard procedures for performing numerical calculations.*
- ✓ *To present a general guide for solving problems.*

1.1 Introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: rigid-body mechanics, deformable-body mechanics , and fluid mechanics.

The subject of statics developed very early in history because it's principles can be formulated simply from measurements of geometry and force. ***Statics deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity;*** whereas ***dynamics is concerned with the accelerated motion of bodies***. We can consider statics as a special case of dynamics, in which the acceleration is zero; however, statics deserves separate treatment in engineering education since many objects are designed with the intention that they remain in equilibrium..

1.2 Basic Concepts

Before we begin our study of engineering mechanics, it is important to understand the meaning of certain fundamental concepts and principles.

Length: Length is used to ***locate the position*** of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.

Time: Although the principles of statics are ***time independent***. This quantity plays an ***important*** role in the study of ***dynamics***.

Mass: Mass is a ***measure of a quantity of matter*** that is used to compare the action of one body with that of another.

Force: Force is considered as a "***push***" or "***pull***" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall. A force is completely characterized by its ***magnitude***, ***direction***, and ***point of application***.

Idealizations: Models or idealizations are used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations.

Three forces act on the ring. Since these forces all meet at point, then for any force analysis, we can assume the ring to be represented as a particle.

Particle: Particle has a ***mass***, but its ***size can be neglected***.

Rigid Body A rigid body can be considered as a ***combination*** of a ***large number*** of ***Particles.***

Concentrated Force: A *concentrated force* represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body. An example would be the contact force between a wheel and the ground.

Steel is a common engineering material that does not deform very much under load. Therefore, we can consider this railroad wheel to be a rigid body acted upon by the concentrated force of the rail.

Newton's Three Laws of Motion: Engineering mechanics is formulated on the basis of Newton's three laws of motion.

Newton's first law: A ***particle originally*** at ***rest*** or ***moving*** in a straight line with ***constant velocity***, ***tends*** to ***remain*** in this ***State*** provided the particle is not subjected to an unbalanced force (Fig.1-1).

$$\sum_{i=1}^{N} F_i = 0$$

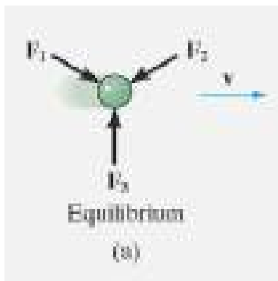


Fig 1-1

Newton's second law: A ***particle acted*** upon by an unbalanced ***force*** "***F***" ***experiences*** an ***acceleration*** "***a***" that has the same direction as the ***force*** and a ***magnitude*** that is directly ***proportional*** to the force (Fig.1-2).

If "F" is applied to a particle or mass "m", this law may be expressed mathematically as:

$$F = m.a \qquad ......(1.1)$$

Fig 1-2

Newton's third Law: The ***mutual forces*** of action between two particles are ***equal, opposite,*** and ***collinear*** (Fig. 1-3).

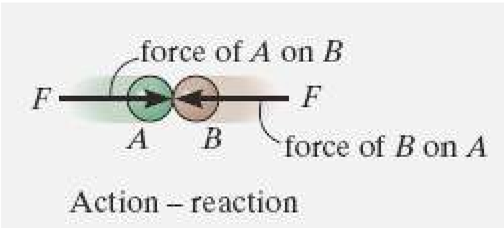


Fig 1-3

Newton's Law of Gravitational Attraction: Shortly after formulating his three laws of motion. Newton postulated a law governing the ***gravitational attraction between any two particles.*** Stated mathematically.

$$F = G\frac{m_1 m_2}{r^2} \qquad .... (1.2)$$

Where:

F: Force of ***gravitational*** between the two particles.

G: ***Universal constant of gravitation***, according to experimental evidence.

$$G = 66.73\ 10^{-12}\ \frac{m^3}{kg\ s^2}$$

m_1 , m_2 : mass of each of the two particles.
r **:** distance between the two particles.

Weight: Weight refers to the ***gravitational attraction*** of the ***earth*** on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

$$W = mg \quad \text{........ (1.3)}$$

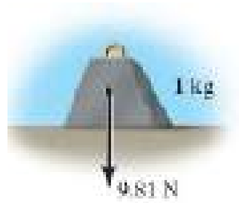


Fig 1-4

Measurements give: $g = 9.8066 \ m/s^2$
Therefore, a body of ***mass 1 kg*** has a ***weight of 9.81*** **N**, a 2 kg body weights 19.62 N, and so on (Fig. 1-4).

Units of Measurement:

- **SI units:** The ***international System of units***. Abbreviated **SI** is a ***modern version*** which has received worldwide recognition. As shown in Tab 1.1. The SI system defines ***length in meters (m)***, ***time in seconds (s)***, and ***mass in kilograms (kg).*** In the SI system the unit of force, the ***Newton*** is a ***derived unit***. Thus, 1 Newton (**N**) is equal to a force required to give 1 kilogram of mass and acceleration of $1 \ m/s^2$.
- **US customary:** In the ***U.S. Customary*** system of units (***FPS***) ***length*** is measured in ***feet (ft)***, ***time*** in ***seconds (s)***, and ***force*** in ***pounds (lb)***. The unit of ***mass***, called a ***slug***, ***1 slug*** is equal to the amount of ***matter*** accelerated at $1 \ ft/s^2$ when acted upon by a ***force of 1 lb*** (1 $slug$=1 $lb \ s^2/ft$). Therefore, if the measurements are made at the "standard location," where g = 32.2 ft/s^2 , then from Eq. 1.3 ,

$$m = W / g \qquad (g = 32.2 \text{ ft/s}^2) \quad \text{.......(1.4)}$$

And so a body weighing 32.2 lb has a mass of 1 slug, a 64.4-lb body has a mass of 2 slugs, and so on.

TABLE 1–1 Systems of Units				
Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	newton* N $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary FPS	foot ft	second s	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound lb

*Derived unit.

Conversion of Units:

Table 1.2 provides a set of direct conversion factors between FPS and SI units for the basic quantities. Also in the FPS system, recall that:

1 ft=12 in inches 1 mile=5280 ft 1 kp kilo pound =1000 lb 1 ton=2000 lb

TABLE 1–2 Conversion Factors			
Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

Prefixes: When a ***numerical quantity*** is either very ***Large*** or very ***small***, the units used to define its size may be modified by using a ***prefix***. Some of the prefixes used in the SI system are shown in Table 1.3. Each represents a ***multiple*** or ***submultiples*** of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place. For example, 4000000 N = 4000 kN (kilo-Newton) = 4MN (Mega-Newton), or 0.005 m = 5 mm (millimeter).

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

*The kilogram is the only base unit that is defined with a prefix.

Important Points

- Statics is the study of bodies that are at rest or move with constant velocity.
- A particle has a mass but a size that can be neglected, and a rigid body does not deform under load.
- Concentrated forces are assumed to act at a point on a body.
- Newton's three laws of motion should be memorized.
- Mass is measure of a quantity of matter that does not change from one location to another. Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located.
- In the SI system the unit of force, the newton, is a derived unit. The meter, second, and kilogram are base units.
- Prefixes G, M, k, m, μ, and n are used to represent large and small numerical quantities. Their exponential size should be known, along with the rules for using the SI units.
- Perform numerical calculations with several significant figures, and then report the final answer to three significant figures.
- Algebraic manipulations of an equation can be checked in part by verifying that the equation remains dimensionally homogeneous.
- Know the rules for rounding off numbers.

EXAMPLE 1.1

Convert 2 km/h to m/s How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\text{ km/h} = \frac{2\text{ km}}{\text{h}}\left(\frac{1000\text{ m}}{\text{km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$

$$= \frac{2000\text{ m}}{3600\text{ s}} = 0.556\text{ m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\text{ m/s} = \left(\frac{0.556\text{ m}}{\text{s}}\right)\left(\frac{1\text{ ft}}{0.3048\text{ m}}\right)$$

$$= 1.82\text{ ft/s} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

EXAMPLE 1.2

Convert the quantities 300 lb · s and 52 slug/ft³ to appropriate SI units.

SOLUTION

Using Table 1–2, 1 lb = 4.448 2 N.

$$300\text{ lb}\cdot\text{s} = 300\text{ lb}\cdot\text{s}\left(\frac{4.448\text{ N}}{1\text{ lb}}\right)$$

$$= 1334.5\text{ N}\cdot\text{s} = 1.33\text{ kN}\cdot\text{s} \qquad \textit{Ans.}$$

Since 1 slug = 14.593 8 kg and 1 ft = 0.304 8 m, then

$$52\text{ slug/ft}^3 = \frac{52\text{ slug}}{\text{ft}^3}\left(\frac{14.59\text{ kg}}{1\text{ slug}}\right)\left(\frac{1\text{ ft}}{0.304\,8\text{ m}}\right)^3$$

$$= 26.8(10^3)\text{ kg/m}^3$$

$$= 26.8\text{ Mg/m}^3 \qquad \textit{Ans.}$$

EXAMPLE 1.3

Evaluate each of the following and express with SI units having an appropriate prefix: (a) (50 mN)(6 GN), (b) (400 mm)$(0.6\text{ MN})^2$, (c) 45 MN^3/900 Gg.

SOLUTION

First convert each number to base units, perform the indicated operations, then choose an appropriate prefix.

Part (a)

$$
\begin{aligned}
(50\text{ mN})(6\text{ GN}) &= [50(10^{-3})\text{ N}][6(10^{9})\text{ N}] \\
&= 300(10^{6})\text{ N}^2 \\
&= 300(10^{6})\text{ N}^2\left(\frac{1\text{ kN}}{10^3\text{ N}}\right)\left(\frac{1\text{ kN}}{10^3\text{ N}}\right) \\
&= 300\text{ kN}^2 \qquad \textit{Ans.}
\end{aligned}
$$

NOTE: Keep in mind the convention $\text{kN}^2 = (\text{kN})^2 = 10^6\text{ N}^2$.

Part (b)

$$
\begin{aligned}
(400\text{ mm})(0.6\text{ MN})^2 &= [400(10^{-3})\text{ m}][0.6(10^{6})\text{ N}]^2 \\
&= [400(10^{-3})\text{ m}][0.36(10^{12})\text{ N}^2] \\
&= 144(10^{9})\text{ m}\cdot\text{N}^2 \\
&= 144\text{ Gm}\cdot\text{N}^2 \qquad \textit{Ans.}
\end{aligned}
$$

We can also write

$$
\begin{aligned}
144(10^{9})\text{ m}\cdot\text{N}^2 &= 144(10^{9})\text{ m}\cdot\text{N}^2\left(\frac{1\text{ MN}}{10^6\text{ N}}\right)\left(\frac{1\text{ MN}}{10^6\text{ N}}\right) \\
&= 0.144\text{ m}\cdot\text{MN}^2 \qquad \textit{Ans.}
\end{aligned}
$$

Part (c)

$$
\begin{aligned}
\frac{45\text{ MN}^3}{900\text{ Gg}} &= \frac{45(10^6\text{ N})^3}{900(10^6)\text{ kg}} \\
&= 50(10^{9})\text{ N}^3/\text{kg} \\
&= 50(10^{9})\text{ N}^3\left(\frac{1\text{ kN}}{10^3\text{ N}}\right)^3\frac{1}{\text{kg}} \\
&= 50\text{ kN}^3/\text{kg} \qquad \textit{Ans.}
\end{aligned}
$$

Exercises

Exercise 1.1:

Convert 2 $\frac{km}{h}$ to $\frac{m}{s}$. How many $\frac{ft}{s}$ is this?

Ans: $2\frac{km}{h} = 0.556\frac{m}{s} = 1.82\frac{ft}{s}$

Exercise 1.2:

Convert the quantities 300 lb. s and 52 $\frac{slug}{ft^3}$ to appropriate SI units.

Ans: $300\ lb.s = 1.33\ kN.s$ $\quad 52\frac{slug}{ft^3} = 26.8\frac{Mg}{m^3}$

Exercise 1.3:

Evaluate each of the following and express with SI units having an appropriate prefix:

(a) $(50\ mN)(6\ GN)$ (b) $(400\ mm)(0.6\ MN)^2$ (c) $\frac{45\ MN^3}{900\ Gg}$

Ans: $(50\ mN)(6\ GN) = 300\ kN^2$ $\quad (400\ mm)(0.6\ MN)^2 = 144\ Gm.N^2$ $\quad 45\frac{MN^3}{900Gg} = 50\frac{kN^3}{kg}$

Exercise 1.4:

Round off the following numbers to three significant figures:

(a) 4.65735 m (b) 55.578 s (c) 4555 N (d) 2768 kg

Ans: $(a)\ 4.66\ m$ $\quad (b) 55.6\ s$ $\quad (c) 4.56\ kN$ $\quad (d) = 2.77 Mg$

Exercise 1.5:

Represent each of the following combinations of units in the correct SI form using an appropriate prefix:

(a) μMN (b) N/μm (c) MN/ks² (d) kN/ms.

Ans: $(a)\ N$ $\quad (b)\frac{MN}{m}$ $\quad (c)\frac{N}{s^2}$ $\quad (d)\frac{MN}{s}$

Exercise 1.6:

Represent each of the following combinations of units in the correct SI form:

(a) Mg/ms (b) N/mm (c) mN/(kg. μs).

Ans: (a) $\frac{Mg}{ms} = \frac{Gg}{s}$ $\quad$ (b) $\frac{N}{mm} = \frac{kN}{m}$ $\quad$ (c) $\frac{mN}{kg\ \mu s} = \frac{kN}{kg.s}$

Exercise 1.7:

A rocket has a mass of 250 10^3 slugs on earth. Specify (a) its mass in SI units and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is g_m=5.30 ft/s², determine to 3 significant figures (c) its weight in units, and (d) its mass in SI units.

Ans: (a) 3.65 Gg $\quad$ (b) $W_e = 35.8$ MN $\quad$ (c) $W_m = 5.89$ MN $\quad$ $m_m = m_e = 3.65$ Gg

Exercise 1.8:

If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

Ans: $(a)\ 88.514\frac{km}{h}$ $\quad (b) 24.6\ \frac{m}{s}$

Exercise 1.9:

The Pascal (Pa) is actually a very small units of pressure. To show this, convert 1 Pa=1 N/m² to lb/ft². Atmospheric pressure at sea level is 14.7 lb/in². How many Pascals is this?

Ans: $(a)\ 1\ Pa = 20.9\ 10^{-3}\ \frac{lb}{ft^2}$ $\quad (b) 1\ ATM = 101.34\ kPa$

CHAPTER – 2

FORCE VECTORS

CHAPTER OBJECTIVES

- To show how to add forces and resolve them into components using the Parallelogram Law.
- To express force and position in Cartesian vector form and explain how to determine the vector's magnitude and direction.
- To introduce the dot product in order to determine the angle between two vectors or the projection of one vector onto another.

2.1 Scalars and Vectors

All physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar: **A *scalar*** is any positive or negative physical quantity that can be completely defined only by its *magnitude*. Examples of scalar quantities are: length, mass, and time.

Vector: **A *vector*** is any physical quantity that requires both a *magnitude* and a d*irection* for its complete description. Examples of vectors in statics are: force, position, and moment. A vector is shown graphically by an ***arrow***. The length of the arrow represents the *magnitude* of the vector, and the angle ***θ*** between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector, Fig. 2–1 .

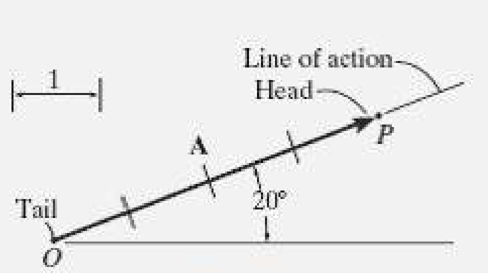
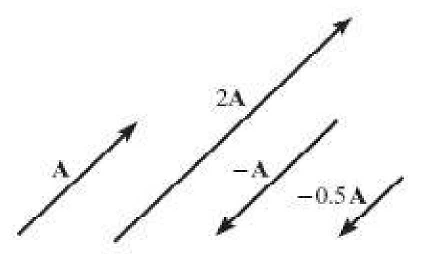


Fig. 2–1

- In print, vector quantities are represented by boldface letters such as **A** , and the magnitude of a vector is italicized, ***A*** . For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it, $\vec{A}$.

2.2 Vector Operations

Multiplication and Division of a Vector by a Scalar: If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. Multiplying by a negative scalar will also change the directional sense of the vector. Graphic examples of these operations are shown in Fig. 2–2 .



Scalar multiplication and division

Fig. 2–2

Vector Addition: All vector quantities obey the *parallelogram law of addition* . To illustrate, the two " *component* " *vectors* **A** and **B** in Fig. 2–3 *a* are added to form a " *resultant* " *vector* **R** = **A** + **B** using the following procedure:

• First join the tails of the components at a point to make them concurrent, Fig. 2–3 *b* .

• From the head of **B** , draw a line parallel to **A** . Draw another line from the head of **A** that is parallel to **B** . These two lines intersect at point *P* to form the adjacent sides of a parallelogram.

• The diagonal of this parallelogram that extends to *P* forms **R** , which then represents the resultant vector **R** = **A** + **B** , Fig. 2–3 *c* .

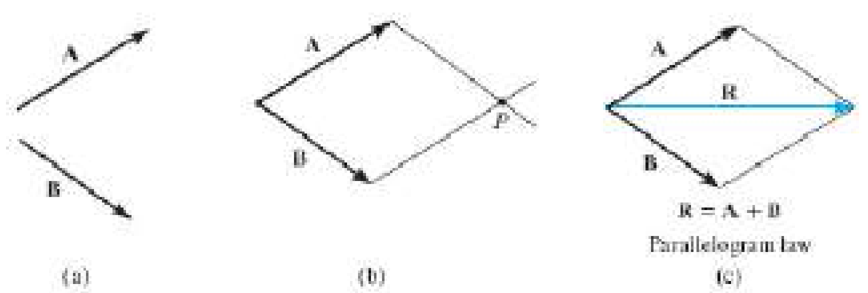
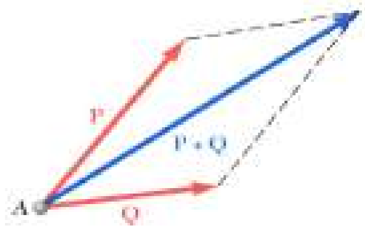
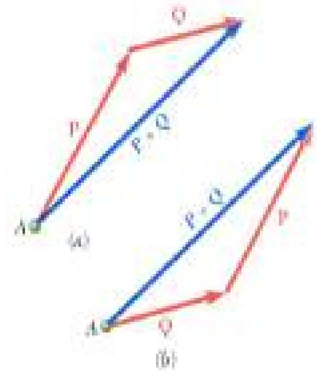


Fig. 2–3



- Trapezoid rule for vector addition.
- Triangle rule for vector addition.
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

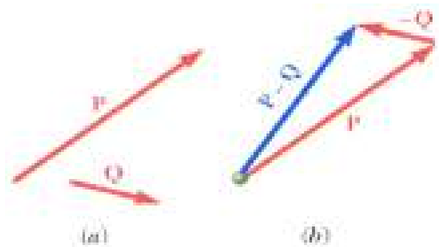


- Law of sines,

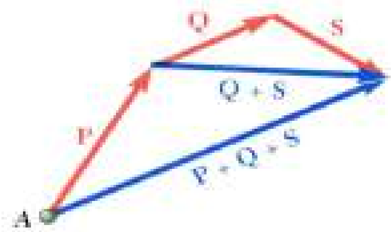
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

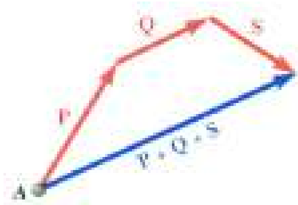
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$



- Vector subtraction.



- Addition of three or more vectors through repeated application of the triangle rule.

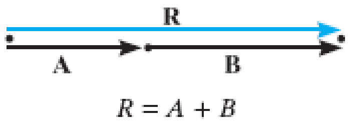


- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P} + \vec{Q} + \vec{S} = \left(\vec{P} + \vec{Q}\right) + \vec{S} = \vec{P} + \left(\vec{Q} + \vec{S}\right)$$

- Multiplication of a vector by a scalar

As a special case, if the two vectors A and B are collinear , i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B .



Addition of collinear vectors

2.3 Vector Addition of Forces

Force is the action of one body on another; characterized by its ***point of application, magnitude, line of action*, and *sense*.** Therefore it is a vector and it adds according to the parallelogram law. Two common problems in statics involve either finding the *resultant force*, knowing its *components,* or resolving a known force into *two components*. We will now describe how each of these problems is solved using the parallelogram law.

Finding a Resultant Force. The two component forces **F**1 and **F**2 acting on the pin in Fig. 2–7 *a* can be added together to form the resultant force **F***R* = **F**1 + **F**2 , as shown in Fig. 2–7 *b* . From this construction, or using the triangle rule, Fig. 2–7 *c* , we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.

The parallelogram law must be used to determine the resultant of the two forces acting on the hook.

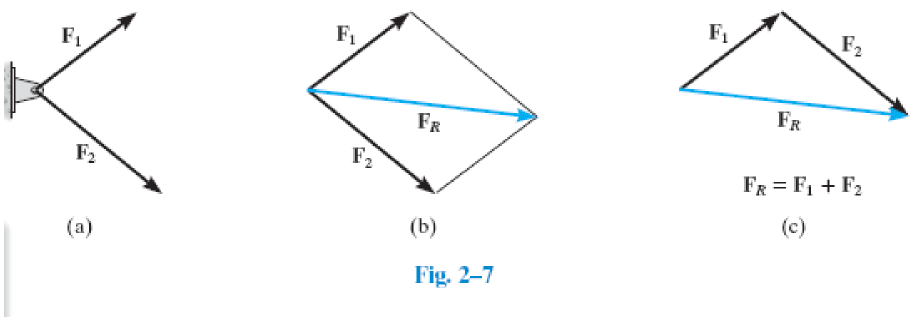
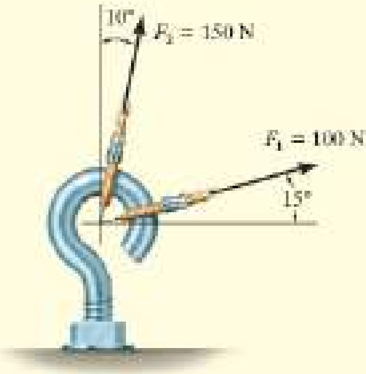


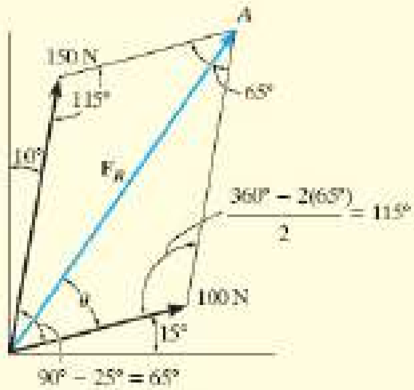
Fig. 2–7

EXAMPLE 2.1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)



(b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point *A*, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.

(c)

Fig. 2–11

Finding the Components of a Force. Sometimes it is necessary to resolve a force into two components in order to study its pulling or pushing effect in two specific directions. For example, in Fig. 2–8 a , F is to be resolved into two components along the two members, defined by the u and v axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of F , one line parallel to u , and the other line parallel to v . These lines then intersect with the v and u axes, forming a parallelogram. The force components Fu and Fv are then established by simply joining the tail of F to the intersection points on the u and v axes, Fig. 2–8 b . This parallelogram can then be reduced to a triangle, which represents the triangle rule, Fig. 2–8 c . From this, the law of sines can then be applied to determine the unknown magnitudes of the components.

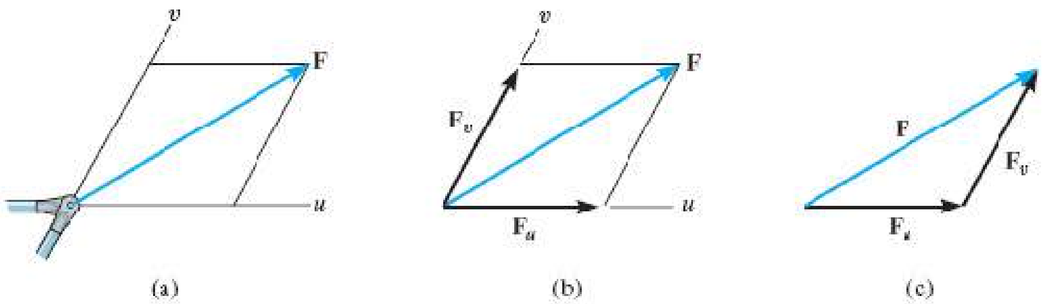


Fig. 2–8

Addition of Several Forces. If more than two forces are to be added, successive applications of the parallelogram law can be carried out in order to obtain the resultant force.

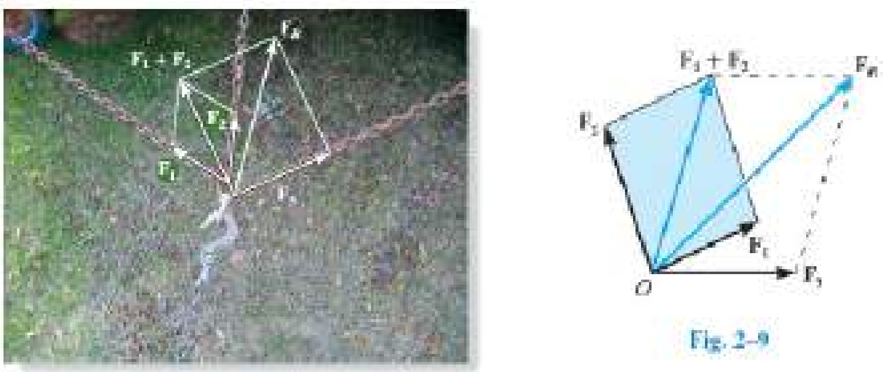
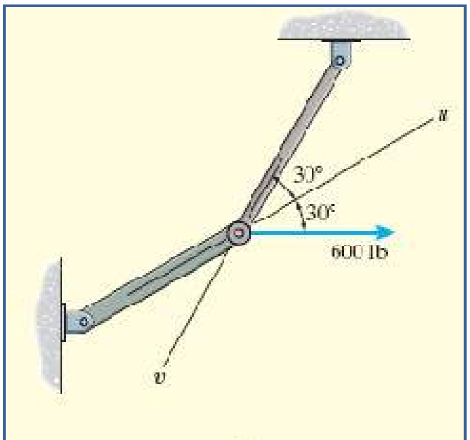


Fig. 2–9

Important Points

- A scalar is a positive or negative number.
- A vector is a quantity that has a magnitude, direction, and sense.
- Multiplication or division of a vector by a scalar will change the magnitude of the vector. The sense of the vector will change if the scalar is negative.
- As a special case, if the vectors are collinear, the resultant is formed by an algebraic or scalar addition.

EXAMPLE 2.2

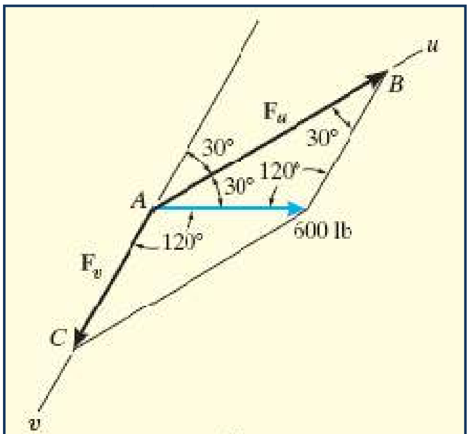


(a)

Resolve the horizontal 600-lb force in Fig. 2–12 *a* into components acting along the *u* and v axes and determine the magnitudes of these components.

Solution:

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the v axis until it intersects the *u* axis at point *B* , Fig. 2–12 *b* . The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the v axis at point *C,* which gives $\mathbf{F}_v$. The vector addition using the triangle rule is shown in Fig. 2–12 *c* . The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,



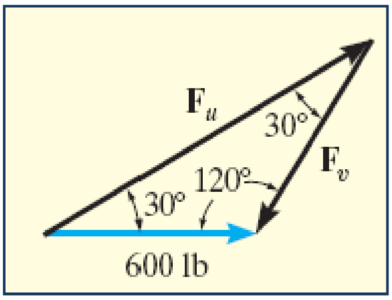
(b)

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$



(c)

<u>NOTE</u>: *The result for Fu shows that sometimes a component can have a greater magnitude than the resultant.*

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

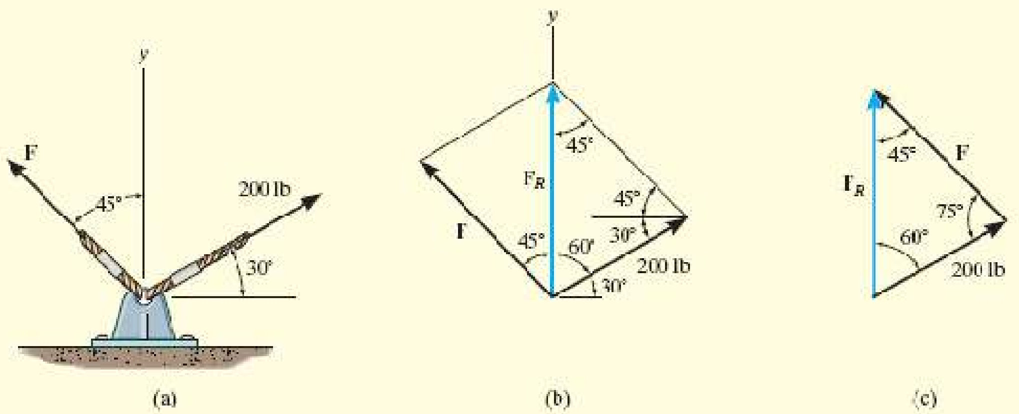


Fig. 2–13

<u>SOLUTION:</u> The parallelogram law of addition is shown in Fig. 2–13 b , and the triangle rule is shown in Fig. 2–13 c . The magnitudes of F_R and F are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb}$$

EXAMPLE 2.4

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive *x* axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

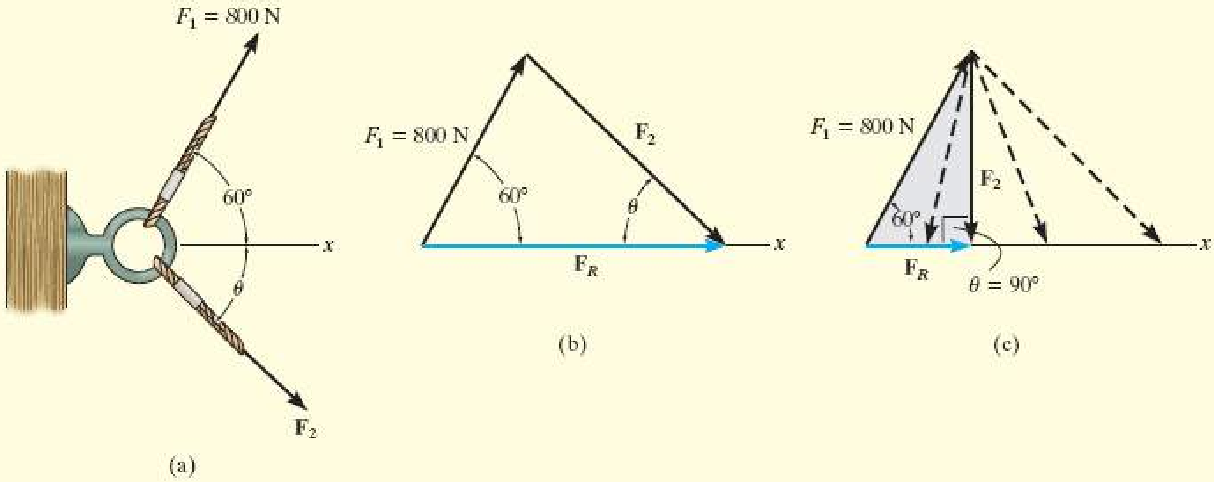
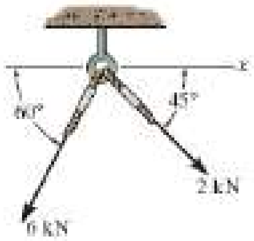


Fig. 2–14

Answers: $F_R = 400$ N , $F_2 = 693$ N

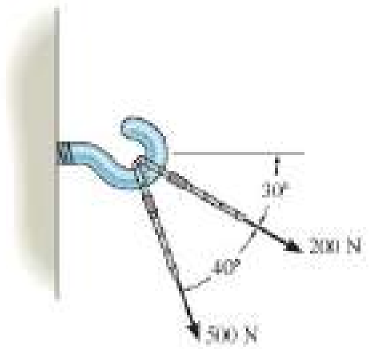
FUNDAMENTAL PROBLEMS*

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



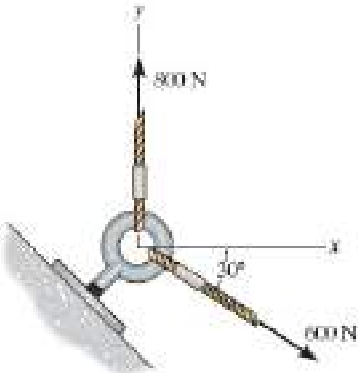
F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



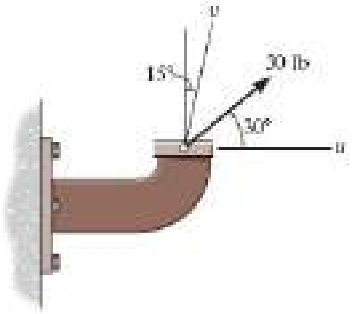
F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



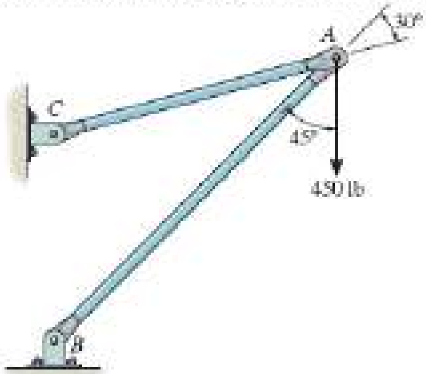
F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



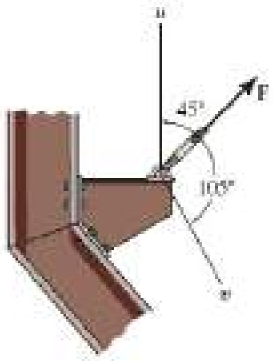
F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



F2–5

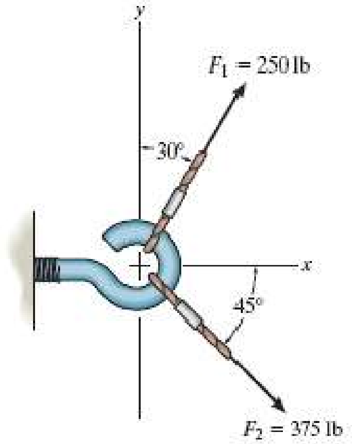
F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the v axis.



F2–6

PROBLEMS

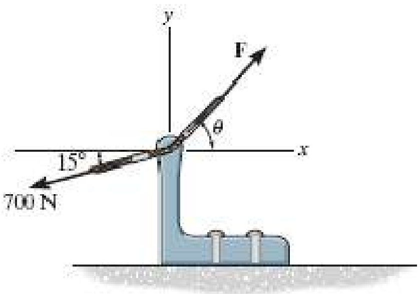
2–1. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



Prob. 2–1

2–2. If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

2–3. If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.

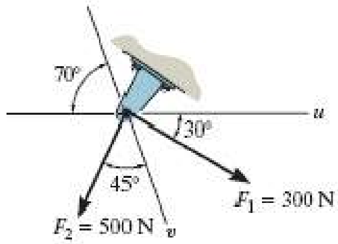


Probs. 2–2/3

*2–4. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

2–5. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

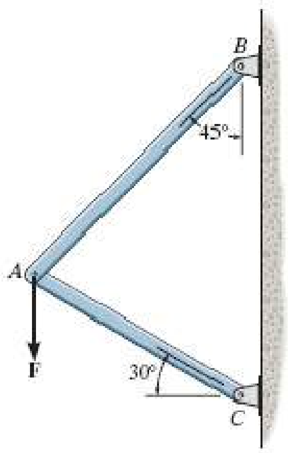
2–6. Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



Probs. 2–4/5/6

2–7. The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.

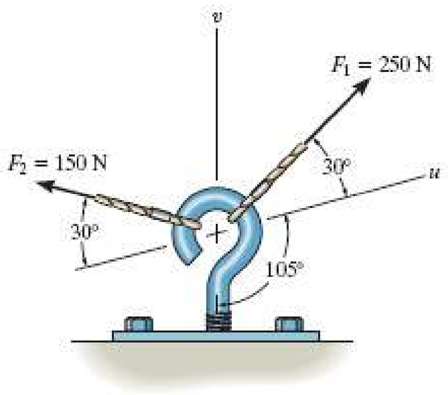
*2–8. Solve Prob. 2–7 with $F = 350$ lb.



Probs. 2–7/8

2–9. Resolve $\mathbf{F}_1$ into components along the u and v axes and determine the magnitudes of these components.

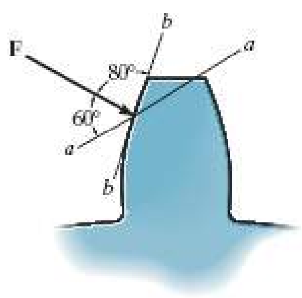
2–10. Resolve $\mathbf{F}_2$ into components along the u and v axes and determine the magnitudes of these components.



Probs. 2–9/10

2–11. The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines aa and bb.

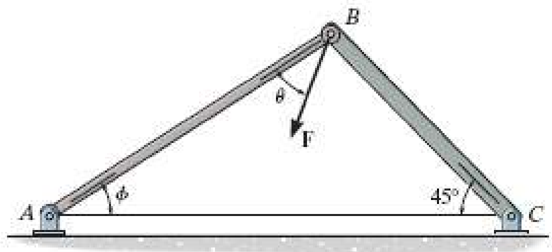
***2–12.** The component of force **F** acting along line aa is required to be 30 lb. Determine the magnitude of **F** and its component along line bb.



Probs. 2–11/12

2–13. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.

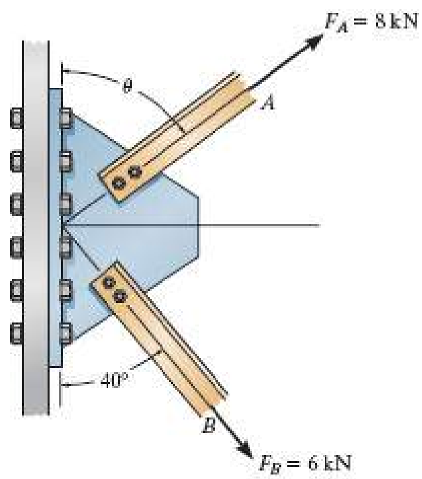
2–14. Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \le \phi \le 90°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



Probs. 2–13/14

2–15. The plate is subjected to the two forces at A and B as shown. If $\theta = 60°$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

***2–16.** Determine the angle θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



Probs. 2–15/16

2.4 Addition of a System of Concurrent Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called *rectangular components*. For analytical work we can represent these components in one of two ways, using either scalar notation or Cartesian vector notation.

Scalar Notation: The rectangular components of force **F** shown in Fig.2–15*a* are found using the parallelogram law, so that: $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, they can be determined from,

$$F_x = F \cos\theta \qquad \text{and} \qquad F_y = F \sin\theta$$

Instead of using the angle θ, however, the direction of **F** can also be defined using a small "*slope*" triangle, as in the example shown in Fig. 2–15 *b* . Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives:

$$\frac{F_x}{F} = \frac{a}{c} \quad \text{or} \quad F_x = F\left(\frac{a}{c}\right)$$

and

$$\frac{F_y}{F} = \frac{b}{c} \quad \text{or} \quad F_y = -F\left(\frac{b}{c}\right)$$

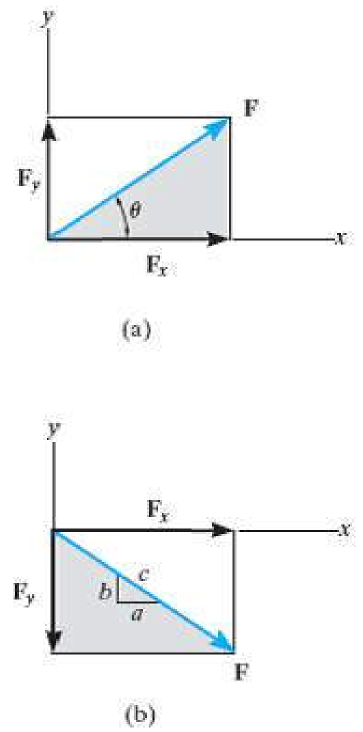


Fig. 2–15

Here the y component is a *negative scalar* since F_y is directed along the negative y axis.

Cartesian Vector Notation: It is also possible to represent the ***x*** and ***y*** components of a force in terms of Cartesian unit vectors **i** and **j** . They can be used to designate the *directions* of the ***x*** and ***y*** axes, respectively, Fig. 2–16.

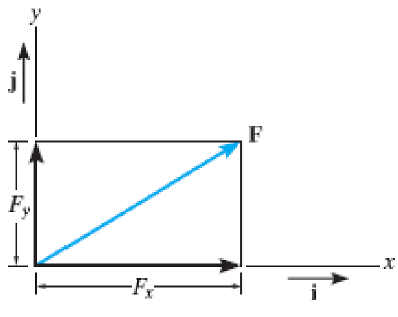


Fig. 2–16

Since the *magnitude* of each component of **F** is *always a positive quantity,* which is represented by the (positive) scalars F_x and F_y , then we can express **F** as a *Cartesian vector* ,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

Concurrent Coplanar Force Resultants: We can use the method just described to determine the resultant of several *Concurrent coplanar forces*. To do this, each force is first resolved into its *x* and *y* components, and then the respective components are added using *scalar algebra* since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law.

For example, consider the three concurrent forces in Fig. 2–17 *a* , which have *x* and *y* components shown in Fig. 2–17 *b* . Using *Cartesian vector notation* , each force is first represented as a Cartesian vector, i.e.,

$$F_1 = F_{1x}\,i + F_{1y}\,j$$
$$F_2 = -F_{2x}\,i + F_{2y}\,j$$
$$F_3 = F_{3x}\,i - F_{3y}\,j$$

The vector resultant is therefore,

$$\begin{aligned} F_R &= F_1 + F_2 + F_3 \\ &= F_{1x}\,i + F_{1y}\,j - F_{2x}\,i + F_{2y}\,j + F_{3x}\,i - F_{3y}\,j \\ &= (F_{1x} - F_{2x} + F_{3x})\,i + (F_{1y} + F_{2y} - F_{3y})\,j \\ &= (F_{Rx})i + (F_{Ry})j \end{aligned}$$

If *scalar notation* is used, then from Fig. 2–17 *b* , we have:

$$(\rightarrow+) \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(\uparrow+) \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

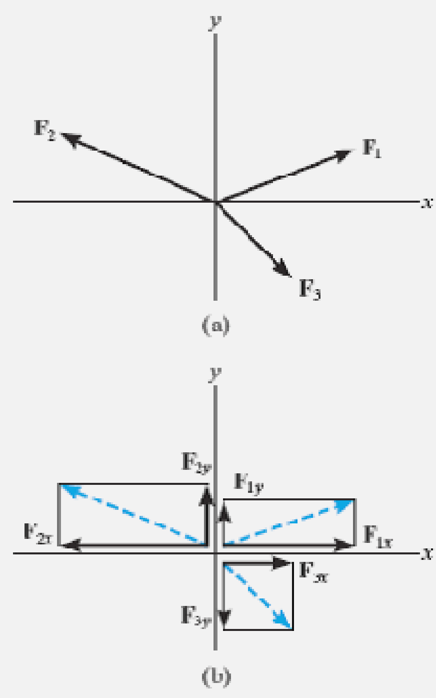


Fig. 2–17

These are the *same* results as the **i** and **j** components of $\mathbf{F}_R$ determined above.

We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e.,

$$(F_R)_x = \Sigma F_x \qquad (F_R)_y = \Sigma F_y \qquad (2\text{–}1)$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition, as shown in Fig. 2–17c .

From this sketch, the magnitude of $\mathbf{F}_R$ is then found from the Pythagorean Theorem; that is,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

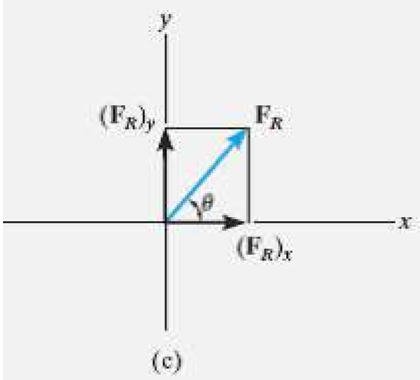


Fig. 2–17 (cont.)

Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

EXAMPLE 2.5

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–18*a*. Express each force as a Cartesian vector.

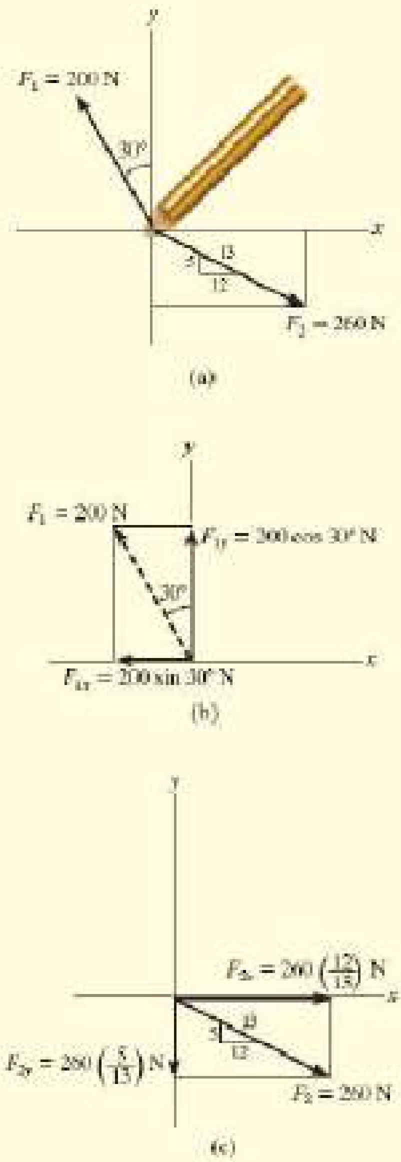


Fig. 2–18

SOLUTION

Scalar Notation. By the parallelogram law, $\mathbf{F}_1$ is resolved into x and y components, Fig. 2–18*b*. Since $\mathbf{F}_{1x}$ acts in the $-x$ direction, and $\mathbf{F}_{1y}$ acts in the $+y$ direction, we have

$$F_{1x} = -200 \sin 30° \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow \qquad \textit{Ans.}$$

$$F_{1y} = 200 \cos 30° \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow \qquad \textit{Ans.}$$

The force $\mathbf{F}_2$ is resolved into its x and y components, as shown in Fig. 2–18*c*. Here the *slope* of the line of action for the force is indicated. From this "slope triangle" we could obtain the angle θ, e.g., $\theta = \tan^{-1}(\frac{5}{12})$, and then proceed to determine the magnitudes of the components in the same manner as for $\mathbf{F}_1$. The easier method, however, consists of using proportional parts of similar triangles, i.e.,

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

Similarly,

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

Notice how the magnitude of the *horizontal component*, $\mathbf{F}_{2x}$, was obtained by multiplying the force magnitude by the ratio of the *horizontal leg* of the slope triangle divided by the hypotenuse; whereas the magnitude of the *vertical component*, F_{2y}, was obtained by multiplying the force magnitude by the ratio of the *vertical leg* divided by the hypotenuse. Hence, using scalar notation to represent these components, we have

$$F_{2x} = 240 \text{ N} = 240 \text{ N} \rightarrow \qquad \textit{Ans.}$$

$$F_{2y} = -100 \text{ N} = 100 \text{ N} \downarrow \qquad \textit{Ans.}$$

Cartesian Vector Notation. Having determined the magnitudes and directions of the components of each force, we can express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

EXAMPLE 2.6

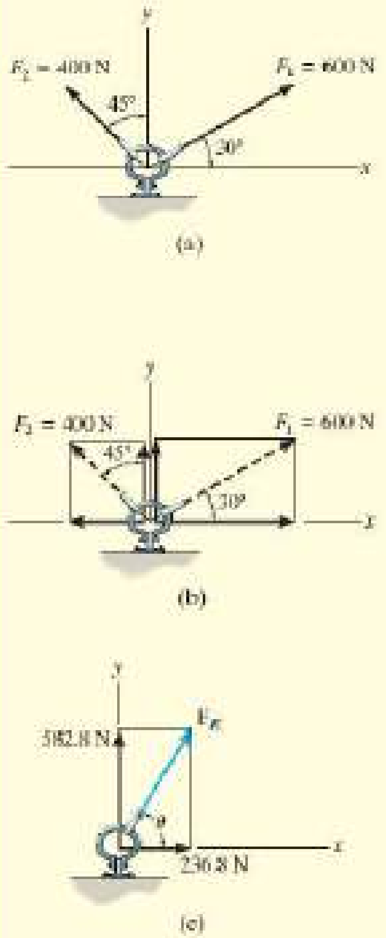


Fig. 2–19

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.

SOLUTION I

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19*b*, then we sum these components algebraically.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N}$$
$$= 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N}$$
$$= 582.8 \text{ N} \uparrow$$

The resultant force, shown in Fig. 2–18*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2}$$
$$= 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$

SOLUTION II

Cartesian Vector Notation. From Fig. 2–19*b*, each force is first expressed as a Cartesian vector.

$$\mathbf{F}_1 = \{600 \cos 30^\circ \mathbf{i} + 600 \sin 30^\circ \mathbf{j}\} \text{ N}$$
$$\mathbf{F}_2 = \{-400 \sin 45^\circ \mathbf{i} + 400 \cos 45^\circ \mathbf{j}\} \text{ N}$$

Then,

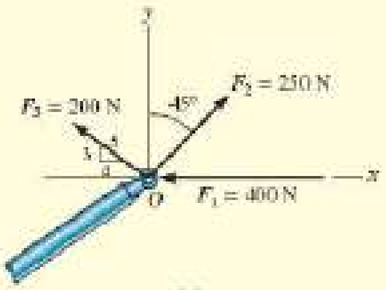
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N})\mathbf{i}$$
$$+ (600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N})\mathbf{j}$$
$$= \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N}$$

The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

NOTE: Comparing the two methods of solution, notice that the use of scalar notation is more efficient since the components can be found *directly*, without first having to express each force as a Cartesian vector before adding the components. Later, however, we will show that Cartesian vector analysis is very beneficial for solving three-dimensional problems.

EXAMPLE 2.7

The end of the boom O in Fig. 2–20a is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



(a)

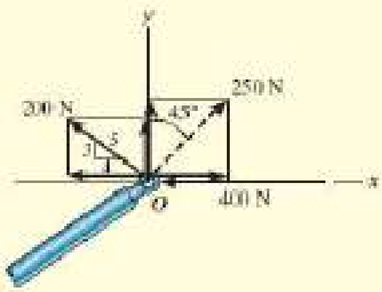
SOLUTION

Each force is resolved into its x and y components, Fig. 2–20b. Summing the x components, we have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400\text{ N} + 250\sin 45^\circ\text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$
$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$

The negative sign indicates that F_{Rx} acts to the left, i.e., in the negative x direction, as noted by the small arrow. Obviously, this occurs because F_1 and F_3 in Fig. 2–20b contribute a greater pull to the left than F_2 which pulls to the right. Summing the y components yields

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250\cos 45^\circ\text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$
$$= 296.8\text{ N}\uparrow$$



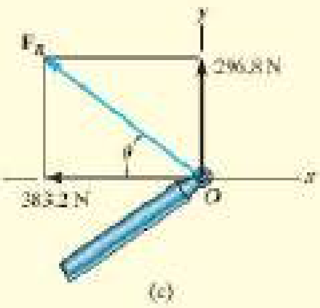
(b)

The resultant force, shown in Fig. 2–20c, has a *magnitude* of

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$
$$= 485\text{ N} \qquad \textit{Ans.}$$

From the vector addition in Fig. 2–20c, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ \qquad \textit{Ans.}$$



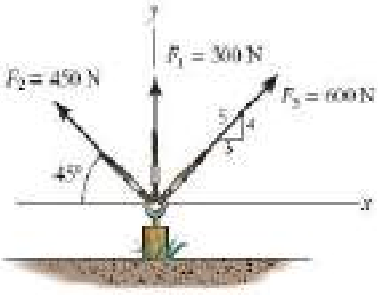
(c)

Fig. 2–20

NOTE: Application of this method is more convenient, compared to using two applications of the parallelogram law, first to add $\mathbf{F}_1$ and $\mathbf{F}_2$ then adding $\mathbf{F}_3$ to this resultant.

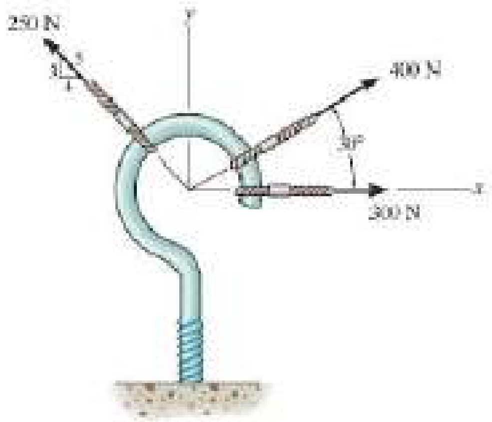
FUNDAMENTAL PROBLEMS

F2–7. Resolve each force acting on the post into its x and y components.



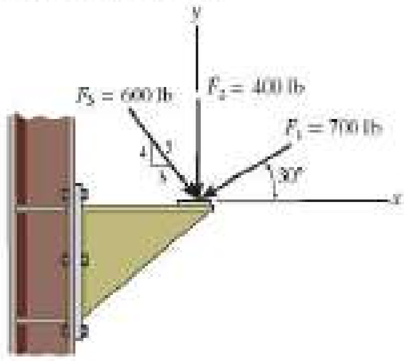
F2–7

F2–8. Determine the magnitude and direction of the resultant force.



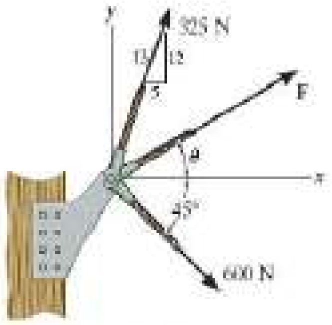
F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



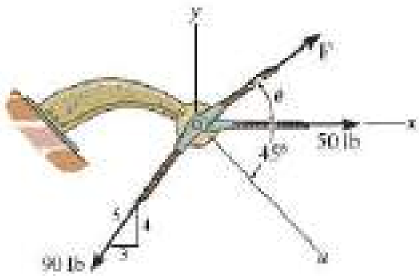
F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



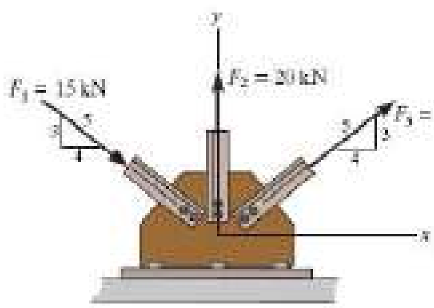
F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



F2–11

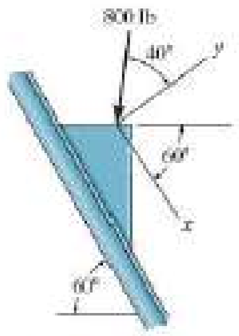
F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



F2–12

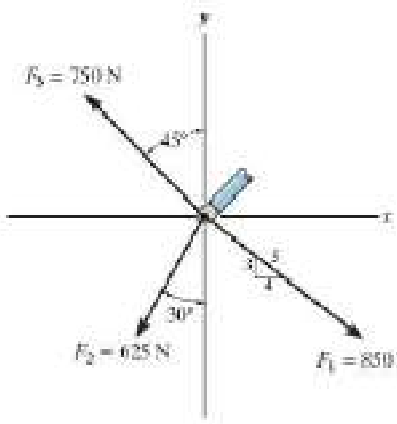
PROBLEMS

*2–32. Determine the x and y components of the 800-lb force.



Prob. 2–32

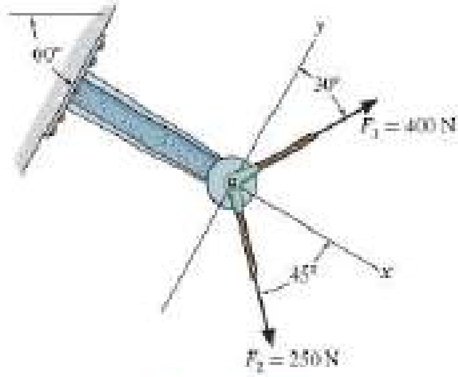
2–33. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–33

2–34. Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.

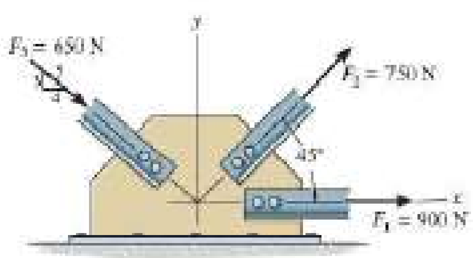
2–35. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Probs. 2–34/35

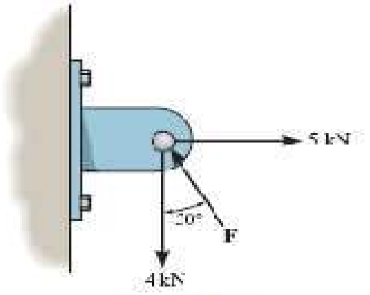
*2–36. Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

2–37. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



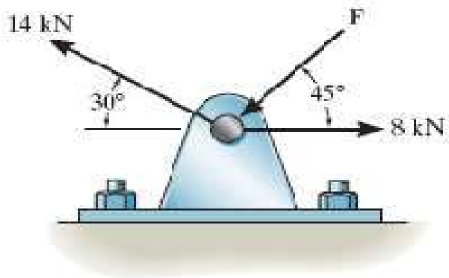
Probs. 2–36/37

*2–52. Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?



Prob. 2–52

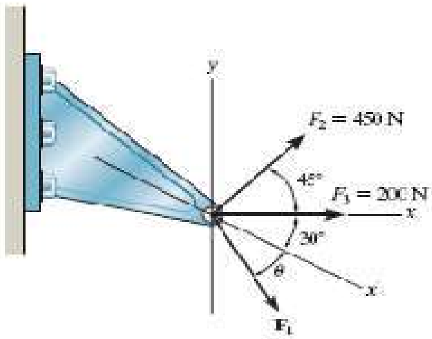
2–53. Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



Prob. 2–53

2–54. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.

2–55. If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.



Probs. 2–54/55

2.5 Cartesian Vectors (Vectors in 3-Dimensions)

- **Right-Handed Coordinate System:** We will use a right-handed coordinate system to develop the theory of vector algebra that follows. A rectangular coordinate system is said to be *right-handed* if the thumb of the right hand points in the direction of the positive *z* axis when the right-hand fingers are curled about this axis and directed from the positive *x* towards the positive *y* axis, Fig. 2–21.

Fig. 2–21

- **Rectangular Components of a Vector:** A vector **A** may have one, two, or three rectangular components along the *x, y, z* coordinate axes, depending on how the vector is oriented relative to the axes. In general, though, when **A** is directed within an octant of the *x, y, z* frame, Fig. 2–22 , then by two successive applications of the parallelogram law, we may resolve the vector into components as: $\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$ and then $\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$. Combining these equations, to eliminate **A'**, **A** is represented by the vector sum of its *three* rectangular components,

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z \qquad (2\text{–}2)$$

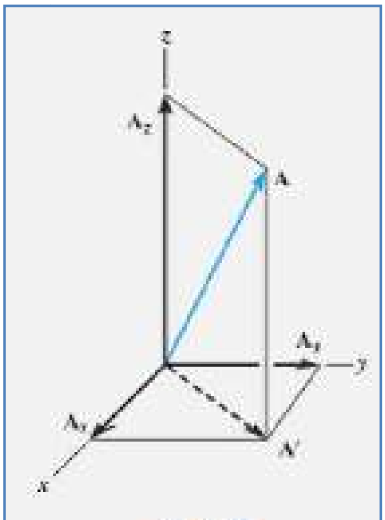


Fig. 2–22

- **Cartesian Unit Vectors:** In three dimensions, the set of Cartesian unit vectors, **i** , **j** , **k** , is used to designate the directions of the *x, y, z* axes, respectively.

The positive Cartesian unit vectors are shown in Fig. 2–23.

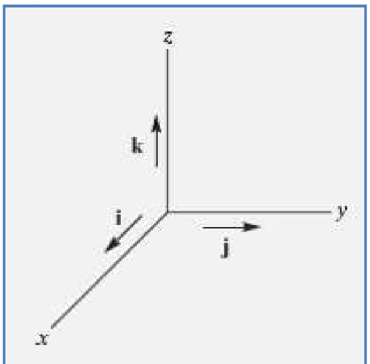


Fig. 2–23

- Since the three components of **A** in Eq. 2–2 act in the positive **i** , **j** , and **k** directions, Fig. 2–24 , we can write **A** in Cartesian vector form as:

$$\mathbf{A} = A_x\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k} \qquad (2\text{–}3)$$

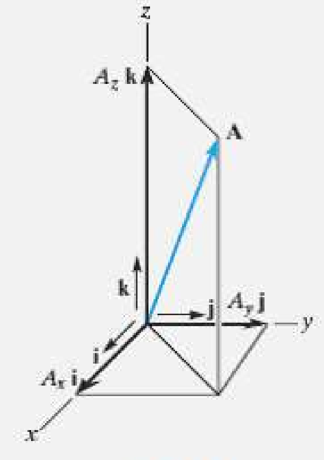


Fig. 2–24

- **Magnitude and direction of a Cartesian Vector:**

As shown in Fig. 2–25 , from the blue right triangle,

$A = \sqrt{A'^2 + A_z^2}$, , and from the gray right triangle,

$A' = \sqrt{A_x^2 + A_y^2}$.

Combining these equations to eliminate *A*' yields:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad (2\text{–}4)$$

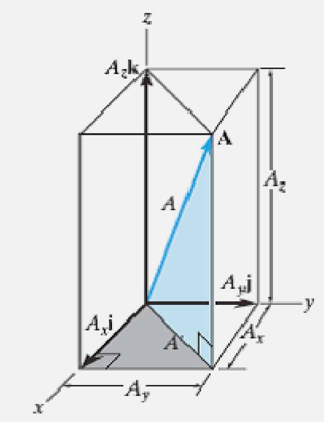


Fig. 2–25

- Now the *direction* of **A** will be defined by the *coordinate direction angles* *α* (alpha), *β* (beta), and *γ* (gamma), measured between the *tail* of **A** and the *positive x, y, z* axes provided they are located at the tail of **A** , Fig. 2-26 and Fig. 2-27 .

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A} \qquad (2\text{–}5)$$

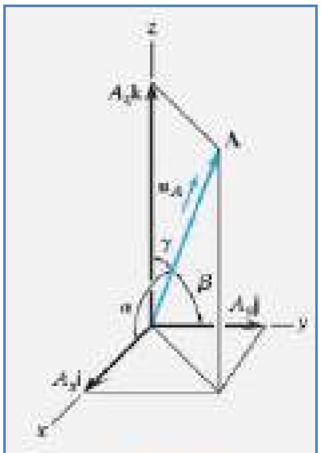


Fig. 2–26

These numbers are known as the ***direction cosines of A***. Once they have been obtained, the coordinate direction angles α, β, and γ can then be determined from the inverse cosines.

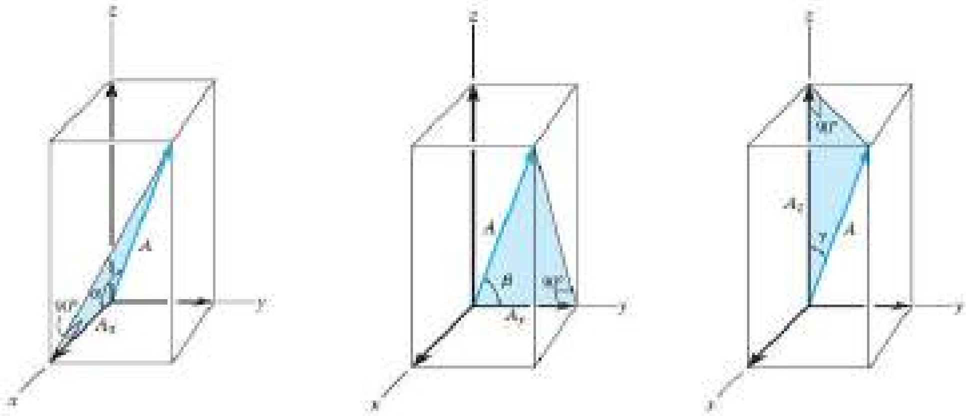


Fig. 2-27

- An easy way of obtaining these direction cosines is to form a unit vector u *A* in the direction of *A* , Fig. 2–26 . If A is expressed in Cartesian vector form, $\mathbf{A} = A_x\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k}$, then $\mathbf{u}_A$ will have a magnitude of one and be dimensionless provided **A** is divided by its magnitude, i.e. :

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k} \qquad (2\text{–}6)$$

Where: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

By comparison with Eqs. 2–5 , it is seen that *the* **i** , **j** , **k** *components* of $\mathbf{u}_A$ *represent the direction cosines of* **A** , i.e.,

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k} \qquad (2\text{–}7)$$

Since the magnitude of a vector is equal to the positive square root of the sum of the squares of the magnitudes of its components, and $\mathbf{u}_A$ has a magnitude of one, then from the above equation an important relation among the direction cosines can be formulated as:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad (2\text{–}8)$$

Finally, if the magnitude and coordinate direction angles of **A** are known, then **A** may be expressed in Cartesian vector form as:

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned} \qquad (2\text{–}9)$$

2.6 Addition of Cartesian Vectors

Let $\mathbf{A} = A_x\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k}$ and $\mathbf{B} = B_x\mathbf{i} + B_y\,\mathbf{j} + B_z\mathbf{k}$, Fig. 2–29 , then the resultant vector, **R** , has components which are the scalar sums of the **i** , **j** and **k** components of **A** and **B** , i.e.,

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (Ax + Bx)\mathbf{i} + (Ay + By)\mathbf{j} + (Az + Bz)\mathbf{k}$$

If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as:

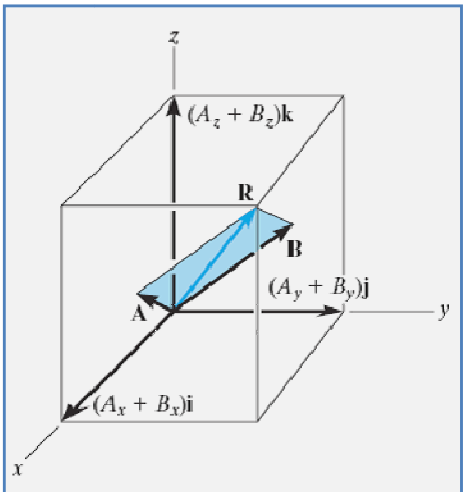


Fig. 2–29

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k} \qquad (2\text{–}10)$$

EXAMPLE 2.8

Express the force **F** shown in Fig. 2–30 as a Cartesian vector.

SOLUTION

Since only two coordinate direction angles are specified, the third angle α must be determined from Eq. 2–8; i.e.,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos\alpha = \sqrt{1 - (0.5)^2 - (0.707)^2} = \pm 0.5$$

Hence, two possibilities exist, namely,

$$\alpha = \cos^{-1}(0.5) = 60^\circ \quad \text{or} \quad \alpha = \cos^{-1}(-0.5) = 120^\circ$$

By inspection it is necessary that $\alpha = 60^\circ$, since $\mathbf{F}_x$ must be in the $+x$ direction.

Using Eq. 2–9, with $F = 200$ N, we have

$$\mathbf{F} = F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$$

$$= (200\cos 60^\circ\text{ N})\mathbf{i} + (200\cos 60^\circ\text{ N})\mathbf{j} + (200\cos 45^\circ\text{ N})\mathbf{k}$$

$$= \{100.0\mathbf{i} + 100.0\mathbf{j} + 141.4\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

Show that indeed the magnitude of $F = 200$ N.

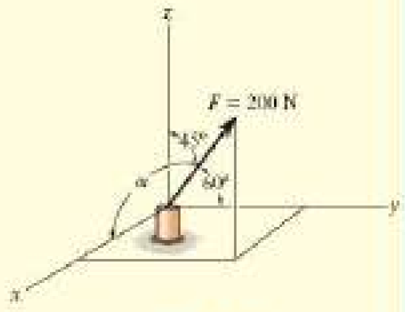


Fig. 2–30

EXAMPLE 2.9

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig. 2–31*a*.

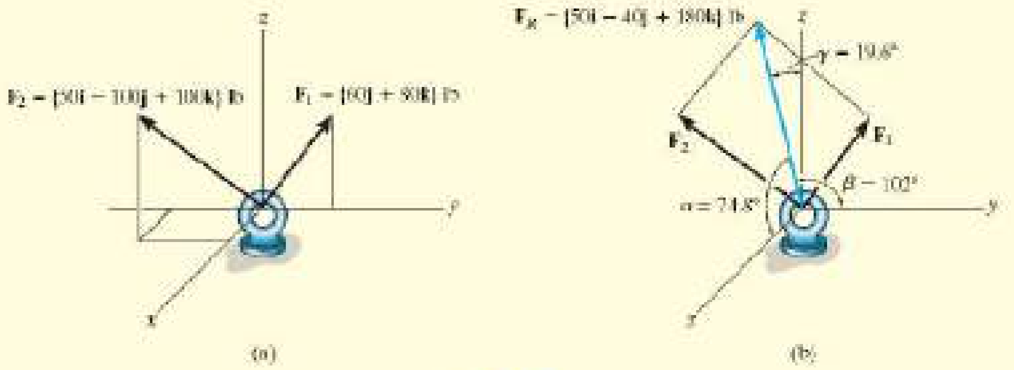


Fig. 2–31

SOLUTION

Since each force is represented in Cartesian vector form, the resultant force, shown in Fig. 2–31*b*, is

$$\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \{60\mathbf{j} + 80\mathbf{k}\}\text{ lb} + \{50\mathbf{i} - 100\mathbf{j} + 100\mathbf{k}\}\text{ lb}$$

$$= \{50\mathbf{i} - 40\mathbf{j} + 180\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(50\text{ lb})^2 + (-40\text{ lb})^2 + (180\text{ lb})^2} = 191.0\text{ lb}$$

$$= 191\text{ lb} \qquad \textit{Ans.}$$

The coordinate direction angles α, β, γ are determined from the components of the unit vector acting in the direction of $\mathbf{F}_R$.

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{50}{191.0}\mathbf{i} - \frac{40}{191.0}\mathbf{j} + \frac{180}{191.0}\mathbf{k}$$
$$= 0.2617\mathbf{i} - 0.2094\mathbf{j} + 0.9422\mathbf{k}$$

so that

$\cos\alpha = 0.2617 \qquad \alpha = 74.8^\circ$ *Ans.*

$\cos\beta = -0.2094 \qquad \beta = 102^\circ$ *Ans.*

$\cos\gamma = 0.9422 \qquad \gamma = 19.6^\circ$ *Ans.*

These angles are shown in Fig. 2–31*b*.

NOTE: In particular, notice that $\beta > 90^\circ$ since the $\mathbf{j}$ component of $\mathbf{u}_{F_R}$ is negative. This seems reasonable considering how $\mathbf{F}_1$ and $\mathbf{F}_2$ add according to the parallelogram law.

EXAMPLE 2.10

Express the force **F** shown in Fig. 2–32*a* as a Cartesian vector.

SOLUTION

The angles of 60° and 45° defining the direction of **F** are *not* coordinate direction angles. Two successive applications of the parallelogram law are needed to resolve **F** into its *x, y, z* components. First $\mathbf{F} = \mathbf{F}' + \mathbf{F}_z$, then $\mathbf{F}' = \mathbf{F}_x + \mathbf{F}_y$, Fig. 2–32*b*. By trigonometry, the magnitudes of the components are

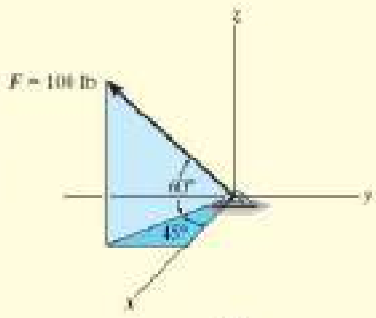
$$F_z = 100 \sin 60^\circ \text{ lb} = 86.6 \text{ lb}$$
$$F' = 100 \cos 60^\circ \text{ lb} = 50 \text{ lb}$$
$$F_x = F' \cos 45^\circ = 50 \cos 45^\circ \text{ lb} = 35.4 \text{ lb}$$
$$F_y = F' \sin 45^\circ = 50 \sin 45^\circ \text{ lb} = 35.4 \text{ lb}$$

Realizing that $\mathbf{F}_y$ has a direction defined by $-\mathbf{j}$, we have

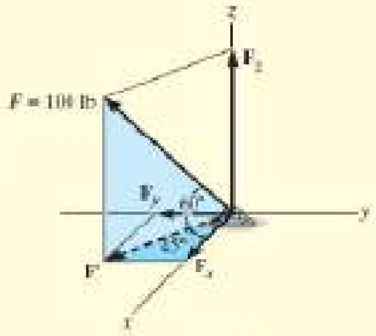
$$\mathbf{F} = \{35.4\mathbf{i} - 35.4\mathbf{j} + 86.6\mathbf{k}\} \text{ lb}$$ *Ans.*

To show that the magnitude of this vector is indeed 100 lb, apply Eq. 2–4,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb}$$

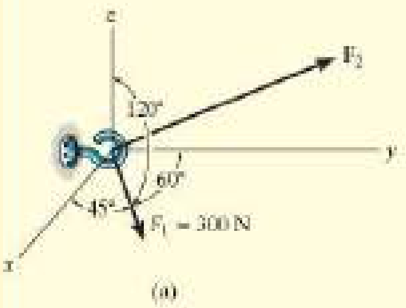


(a)



(b)

EXAMPLE 2.11



Two forces act on the hook shown in Fig. 2–33*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles so that the resultant force $\mathbf{F}_R$ acts along the positive *y* axis and has a magnitude of 800 N.

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–33*b*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

$$\mathbf{F}_1 = F_1 \cos\alpha_1 \mathbf{i} + F_1 \cos\beta_1 \mathbf{j} + F_1 \cos\gamma_1 \mathbf{k}$$
$$= 300 \cos 45^\circ \mathbf{i} + 300 \cos 60^\circ \mathbf{j} + 300 \cos 120^\circ \mathbf{k}$$
$$= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\text{ N}$$
$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800\text{ N})(+\mathbf{j}) = \{800\mathbf{j}\}\text{ N}$$

We require

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$800\mathbf{j} = 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$
$$800\mathbf{j} = (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}$$

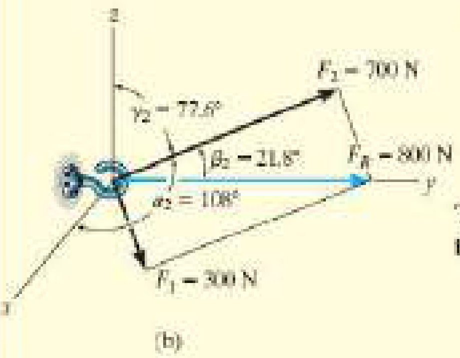


Fig. 2–33

To satisfy this equation the **i**, **j**, **k** components of $\mathbf{F}_R$ must be equal to the corresponding **i**, **j**, **k** components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1\text{ N}$$
$$800 = 150 + F_{2y} \qquad F_{2y} = 650\text{ N}$$
$$0 = -150 + F_{2z} \qquad F_{2z} = 150\text{ N}$$

The magnitude of $\mathbf{F}_2$ is thus

$$F_2 = \sqrt{(-212.1\text{ N})^2 + (650\text{ N})^2 + (150\text{ N})^2}$$
$$= 700\text{ N} \qquad \textit{Ans.}$$

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos\alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ \qquad \textit{Ans.}$$
$$\cos\beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ \qquad \textit{Ans.}$$
$$\cos\gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ \qquad \textit{Ans.}$$

These results are shown in Fig. 2–33*b*.

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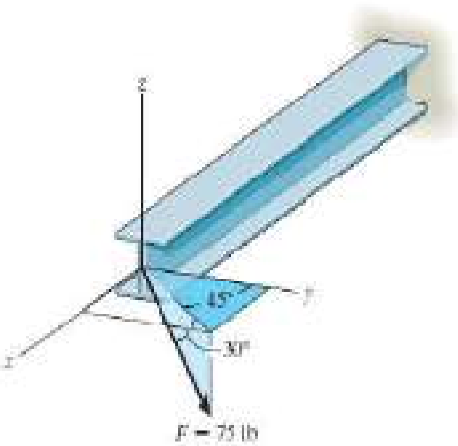
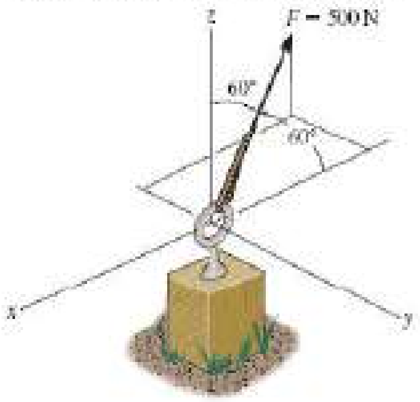
FUNDAMENTAL PROBLEMS

F2–13. Determine the coordinate direction angles of the force.



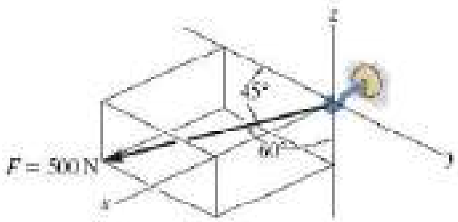
F2–13

F2–14. Express the force as a Cartesian vector.



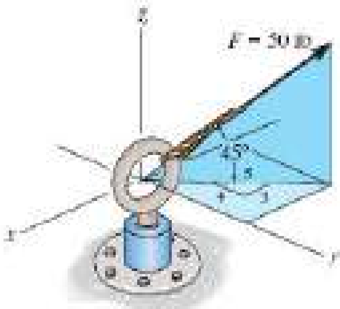
F2–14

F2–15. Express the force as a Cartesian vector.



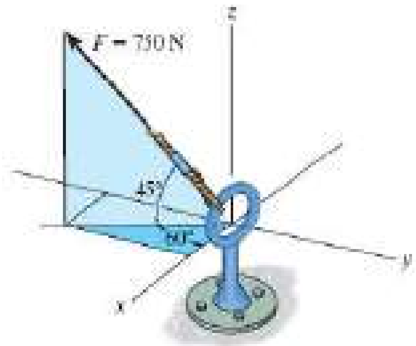
F2–15

F2–16. Express the force as a Cartesian vector.



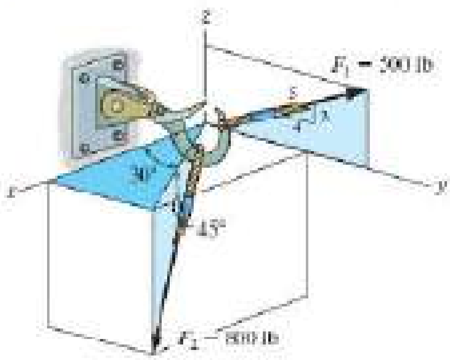
F2–16

F2–17. Express the force as a Cartesian vector.



F2–17

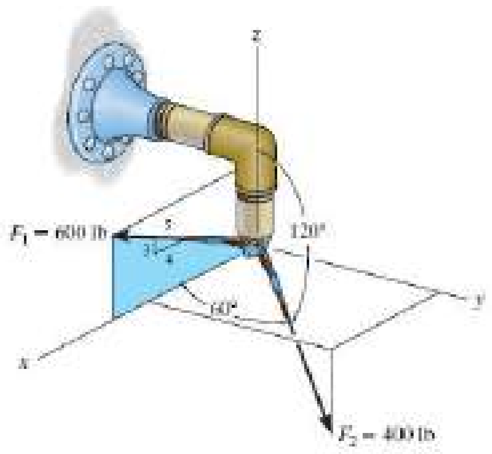
F2–18. Determine the resultant force acting on the hook.



F2–18

2–66. Express each force acting on the pipe assembly in Cartesian vector form.

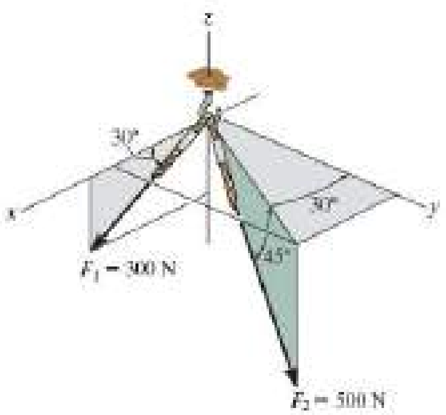
2–67. Determine the magnitude and the direction of the resultant force acting on the pipe assembly.



Probs. 2–66/67

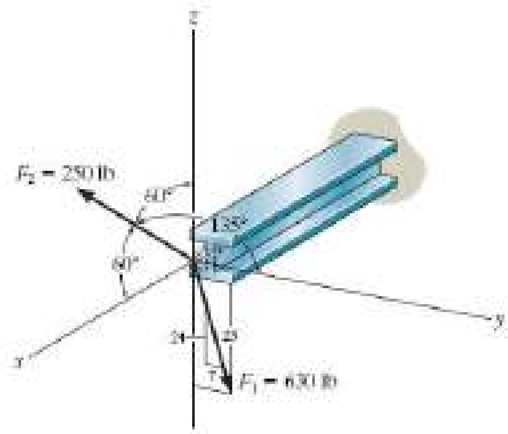
*2–68. Express each force as a Cartesian vector.

2–69. Determine the magnitude and coordinate direction angles of the resultant force acting on the hook.



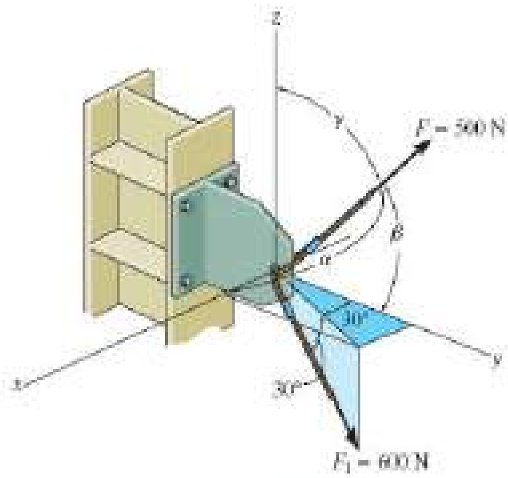
Probs. 2–68/69

2–70. The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



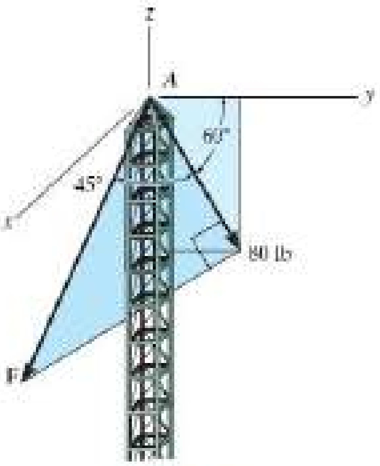
Prob. 2–70

2–71. If the resultant force acting on the bracket is directed along the positive y axis, determine the magnitude of the resultant force and the coordinate direction angles of F so that $\beta < 90°$.



Prob. 2–71

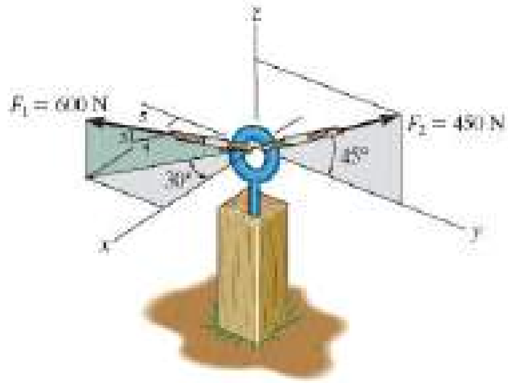
*2–72. A force **F** is applied at the top of the tower at *A*. If it acts in the direction shown such that one of its components lying in the shaded *y*-*z* plane has a magnitude of 80 lb, determine its magnitude *F* and coordinate direction angles α, β, γ.



Prob. 2–72

2–75. Determine the coordinate direction angles of force $\mathbf{F}_1$.

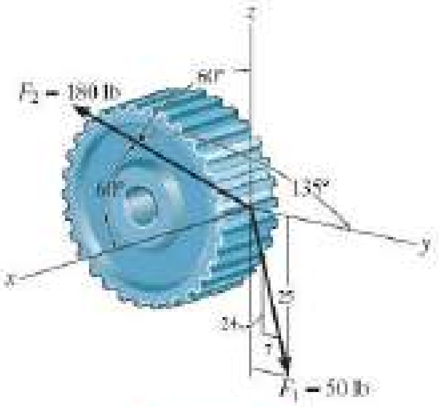
*2–76. Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



Probs. 2–75/76

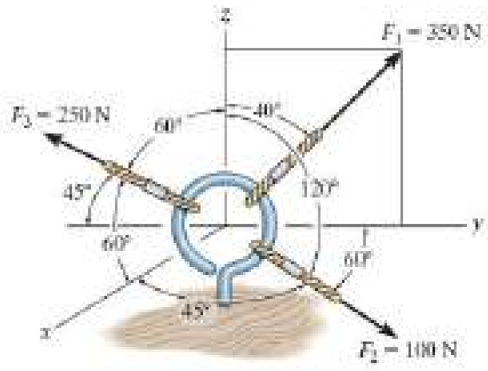
2–73. The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.

2–74. The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



Probs. 2–73/74

2–77. The cables attached to the screw eye are subjected to the three forces shown. Express each force in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



Prob. 2–77

2.7 Position Vectors:

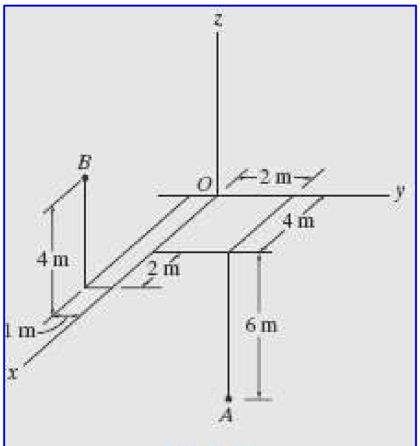


Fig. 2–34

In this section we will introduce the concept of a position vector. It will be shown that this vector is of importance in formulating a Cartesian force vector directed between two points in space.

***x*, *y*, *z* Coordinates,** we will use a *right-handed* coordinate system to reference the location of points in space, Fig. 2–34. Points in space are located relative to the origin of coordinates, *O* , by successive measurements along the *x, y, z* axes. For example, the coordinates of point *A* are obtained by starting at *O* and measuring x_A = +4 m along the *x* axis, then y_A = +2 m along the *y* axis, and finally z_A = - 6 m along the *z* axis. Thus, *A* (4 m, 2 m, - 6 m). In a similar manner, measurements along the *x, y, z* axes from *O* to *B* yield the coordinates of *B* , i.e., *B* (6 m, -1 m, 4 m).

Position Vector, A *position vector* **r** is defined as a fixed vector which locates a point in space relative to another point. For example, if **r** extends from the origin of coordinates, *O* , to point *P* (*x, y, z*), Fig. 2–35 *a* , then **r** can be expressed in Cartesian vector form as:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

Note how the head-to-tail vector addition of the three components yields vector **r** , Fig. 2–35 *b* . Starting at the origin *O* , one "travels" *x* in the +**i** direction, then *y* in the +**j** direction, and finally *z* in the +**k** direction to arrive at point ***P*** (*x, y, z*).

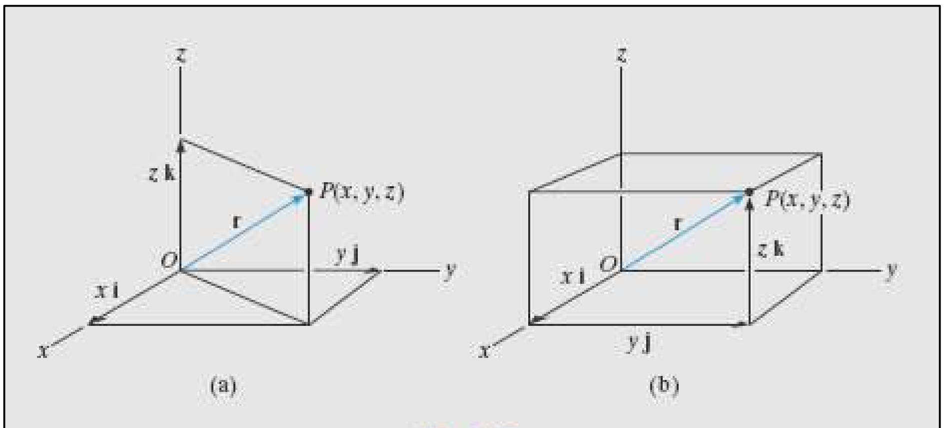
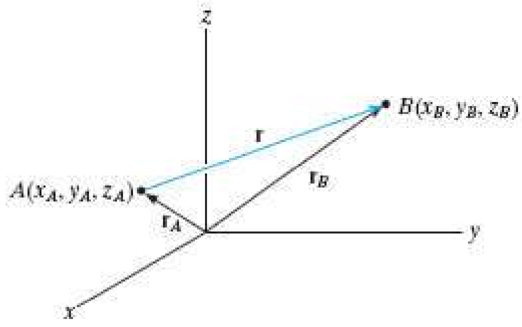


Fig. 2–35

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2–36 a . From Fig. 2–36 a , by the head-to-tail vector addition, using the triangle rule, we require:

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$



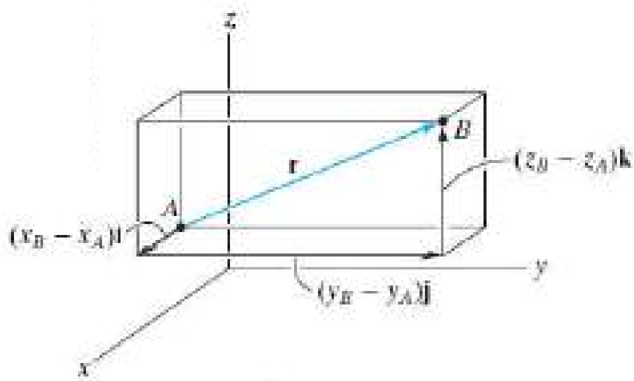
(a)

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

or

$$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k} \quad (2\text{–}11)$$

We can also form these components *directly* , Fig. 2–36 b , by starting at A and moving through a distance of (x_B - x_A) along the positive x axis (+**i**), then (y_B - y_A) along the positive y axis (+**j**), and finally (z_B - z_A) along the positive z axis (+**k**) to get to B .



(b)

Fig. 2–36

EXAMPLE 2.12

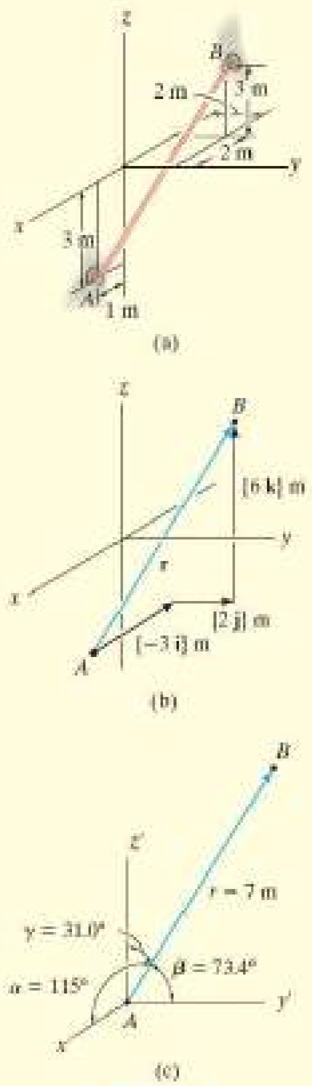


Fig. 2–37

An elastic rubber band is attached to points A and B as shown in Fig. 2–37a. Determine its length and its direction measured from A toward B.

SOLUTION

We first establish a position vector from A to B, Fig. 2–37b. In accordance with Eq. 2–11, the coordinates of the tail $A(1\text{ m}, 0, -3\text{ m})$ are subtracted from the coordinates of the head $B(-2\text{ m}, 2\text{ m}, 3\text{ m})$, which yields

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

These components of $\mathbf{r}$ can also be determined *directly* by realizing that they represent the direction and distance one must travel along each axis in order to move from A to B, i.e., along the x axis $\{-3\mathbf{i}\}$ m, along the y axis $\{2\mathbf{j}\}$ m, and finally along the z axis $\{6\mathbf{k}\}$ m.

The length of the rubber band is therefore

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m} \qquad \textit{Ans.}$$

Formulating a unit vector in the direction of $\mathbf{r}$, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

The components of this unit vector give the coordinate direction angles

$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115° \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4° \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0° \qquad \textit{Ans.}$$

NOTE: These angles are measured from the *positive axes* of a localized coordinate system placed at the tail of $\mathbf{r}$, as shown in Fig. 2–37c.

2.8 Force Vector Directed Along a Line

Quite often in three-dimensional statics problems, the direction of a force is specified by two points through which its line of action passes. Such a situation is shown in Fig. 2–38, where the force **F** is directed along the cord *AB*. We can formulate **F** as a Cartesian vector by realizing that it has the *same direction* and *sense* as the position vector **r** directed from point *A* to point *B* on the cord. This common direction is specified by the *unit vector* **u** = **r**>*r*. Hence,

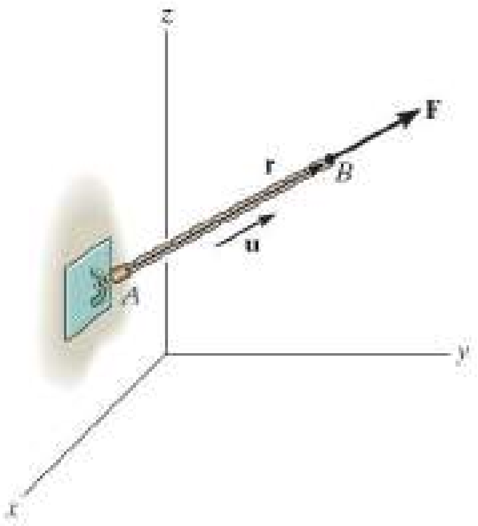


Fig. 2–38

$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

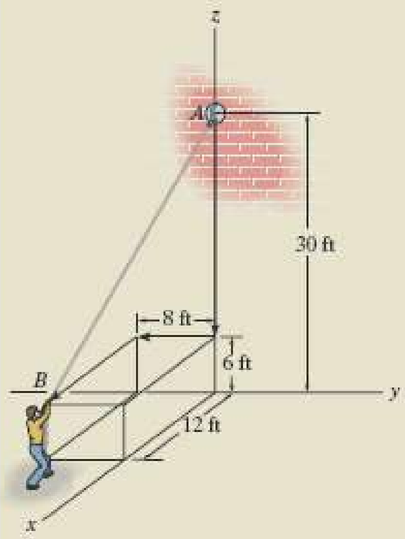
The force **F** acting along the rope can be represented as a Cartesian vector by establishing *x, y, z* axes and first forming a position vector **r** along the length of the rope. Then the corresponding unit vector **u** = **r**/*r* that defines the direction of both the rope and the force can be determined. Finally, the magnitude of the force is combined with its direction, **F** = *F***u**.

Important Points

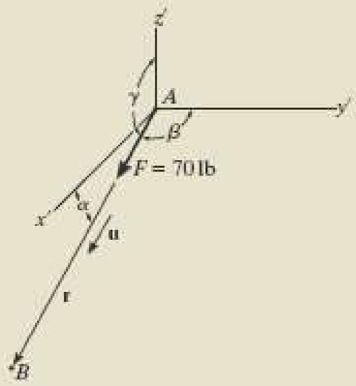
- A position vector locates one point in space relative to another point.
- The easiest way to formulate the components of a position vector is to determine the distance and direction that must be traveled along the *x, y, z* directions—going from the tail to the head of the vector.
- A force **F** acting in the direction of a position vector **r** can be represented in Cartesian form if the unit vector **u** of the position vector is determined and it is multiplied by the magnitude of the force, i.e., **F** = *F***u** = *F* (**r**>*r*).

EXAMPLE 2.13

The man shown in Fig. 2–39*a* pulls on the cord with a force of 70 lb. Represent this force acting on the support *A* as a Cartesian vector and determine its direction.



(a)



(b)

Fig. 2–39

SOLUTION

Force **F** is shown in Fig. 2–39*b*. The *direction* of this vector, **u**, is determined from the position vector **r**, which extends from *A* to *B*. Rather than using the coordinates of the end points of the cord, **r** can be determined *directly* by noting in Fig. 2–39*a* that one must travel from *A* {−24**k**} ft, then {−8**j**} ft, and finally {12**i**} ft to get to *B*. Thus,

$$\mathbf{r} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\}\text{ ft}$$

The magnitude of **r**, which represents the *length* of cord *AB*, is

$$r = \sqrt{(12\text{ ft})^2 + (-8\text{ ft})^2 + (-24\text{ ft})^2} = 28\text{ ft}$$

Forming the unit vector that defines the direction and sense of both **r** and **F**, we have

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$

Since **F** has a *magnitude* of 70 lb and a *direction* specified by **u**, then

$$\mathbf{F} = F\mathbf{u} = 70\text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$

$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\}\text{ lb} \qquad \textit{Ans.}$$

The coordinate direction angles are measured between **r** (or **F**) and the *positive axes* of a localized coordinate system with origin placed at *A*, Fig. 2–39*b*. From the components of the unit vector:

$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6^\circ \qquad \textit{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107^\circ \qquad \textit{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149^\circ \qquad \textit{Ans.}$$

NOTE: These results make sense when compared with the angles identified in Fig. 2–39*b*.

EXAMPLE 2.14

The force in Fig. 2–40*a* acts on the hook. Express it as a Cartesian vector.

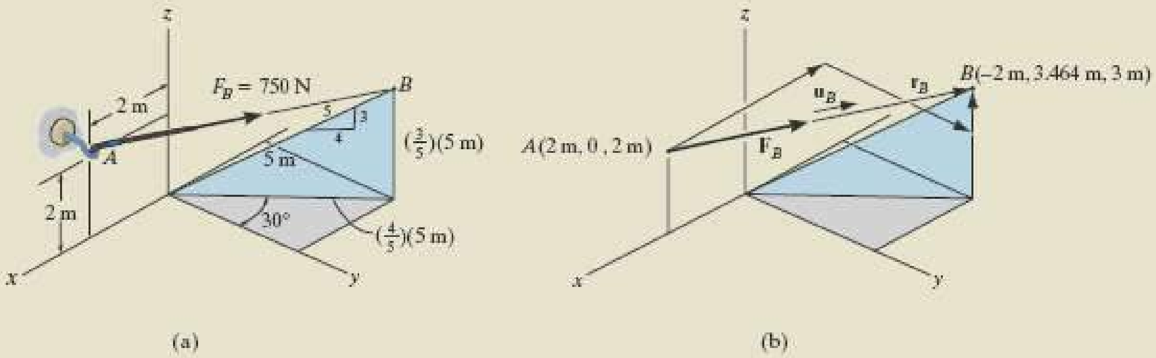


Fig. 2–40

SOLUTION

As shown in Fig. 2–40*b*, the coordinates for points *A* and *B* are

$$A(2\text{ m}, 0, 2\text{ m})$$

and

$$B\left[-\left(\frac{4}{5}\right)5\sin 30^\circ\text{ m}, \left(\frac{4}{5}\right)5\cos 30^\circ\text{ m}, \left(\frac{3}{5}\right)5\text{ m}\right]$$

or

$$B(-2\text{ m}, 3.464\text{ m}, 3\text{ m})$$

Therefore, to go from *A* to *B*, one must travel $\{-4\mathbf{i}\}$ m, then $\{3.464\mathbf{j}\}$ m, and finally $\{1\mathbf{k}\}$ m. Thus,

$$\mathbf{u}_B = \left(\frac{\mathbf{r}_B}{r_B}\right) = \frac{\{-4\mathbf{i} + 3.464\mathbf{j} + 1\mathbf{k}\}\text{ m}}{\sqrt{(-4\text{ m})^2 + (3.464\text{ m})^2 + (1\text{ m})^2}}$$

$$= -0.7428\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k}$$

Force $\mathbf{F}_B$ expressed as a Cartesian vector becomes

$$\mathbf{F}_B = F_B\mathbf{u}_B = (750\text{ N})(-0.74281\mathbf{i} + 0.6433\mathbf{j} + 0.1857\mathbf{k})$$

$$= \{-557\mathbf{i} + 482\mathbf{j} + 139\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 2.15

The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A as shown in Fig. 2–41*a*, determine the resultant force acting at A. Express the result as a Cartesian vector.

SOLUTION

The resultant force $\mathbf{F}_R$ is shown graphically in Fig. 2–41*b*. We can express this force as a Cartesian vector by first formulating $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ as Cartesian vectors and then adding their components. The directions of $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$ are specified by forming unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ along the cables. These unit vectors are obtained from the associated position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$. With reference to Fig. 2–41*a*, to go from A to B, we must travel $\{-4\mathbf{k}\}$ m, and then $\{4\mathbf{i}\}$ m. Thus,

$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AB} = \sqrt{(4\ \text{m})^2 + (-4\ \text{m})^2} = 5.66\ \text{m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100\ \text{N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N}$$

To go from A to C, we must travel $\{-4\mathbf{k}\}$ m, then $\{2\mathbf{j}\}$ m, and finally $\{4\mathbf{i}\}$. Thus,

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(4\ \text{m})^2 + (2\ \text{m})^2 + (-4\ \text{m})^2} = 6\ \text{m}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120\ \text{N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\}\ \text{N}$$ *Ans.*

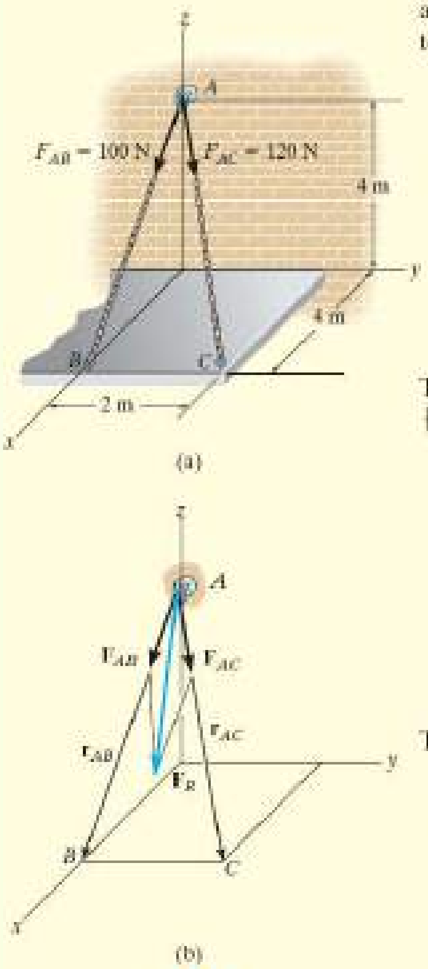
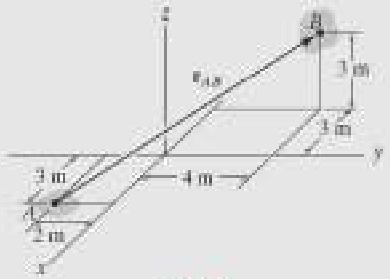


Fig. 2–41

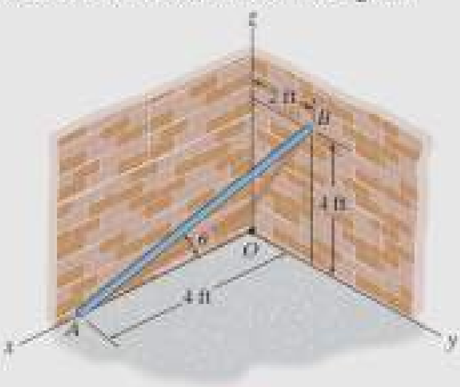
FUNDAMENTAL PROBLEMS

F2–19. Express the position vector $\mathbf{r}_{AB}$ in Cartesian vector form, then determine its magnitude and coordinate direction angles.



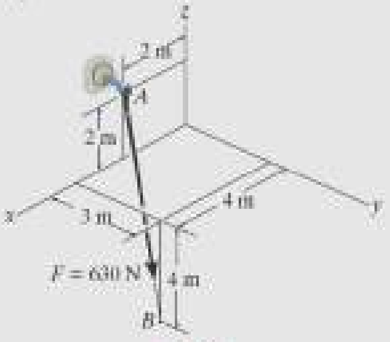
F2–19

F2–20. Determine the length of the rod and the position vector directed from A to B. What is the angle θ?



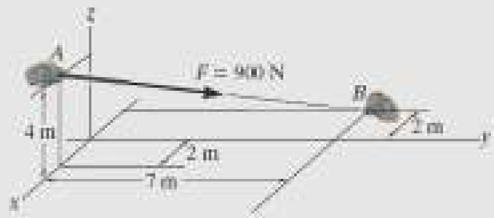
F2–20

F2–21. Express the force as a Cartesian vector.



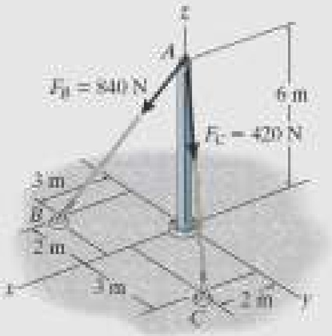
F2–21

F2–22. Express the force as a Cartesian vector.



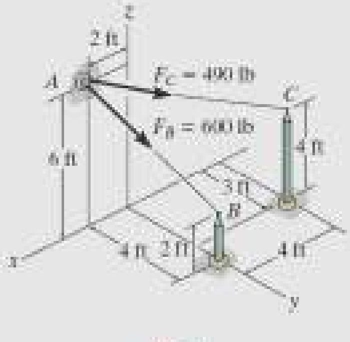
F2–22

F2–23. Determine the magnitude of the resultant force at A.



F2–23

F2–24. Determine the resultant force at A.



F2–24

CHAPTER-3
Equilibrium of a Particle

CHAPTER OBJECTIVES:

- ***To introduce the concept of the free-body diagram for a particle.***
- ***To show how to solve particle equilibrium problems using the equations of equilibrium.***

3.1 Condition for the equilibrium of a particle.

A particle is said to be in ***equilibrium*** if it ***remains at rest*** *if* ***originally at rest, or has a constant velocity*** *if* ***originally in motion***. To maintain equilibrium, it is necessary to satisfy Newton's first law of motion which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as:

$$\Sigma \mathbf{F} = 0 \qquad \ldots\ldots\ldots\ldots (3.1)$$

Where ΣF is the vector sum of all the forces acting on the particle.

3.2 The free body diagram

A **drawing** that **shows** the particle with **all the forces** that act on it is called a **free body diagram** (**FBD**).

We will consider ***a springs connections*** often encountered in particle equilibrium problems.

Springs: If a linearly elastic spring of undeformed length l0 is used to support a particle, **the length of the spring will change in direct proportion to the force F acting on it**, Fig 3.1. A **characteristic** that defines the **elasticity of a spring** is the **spring constant** or **stiffness** *k*. The magnitude of force exerted on a linearly elastic spring is stated as:

$$F = k\,s$$

Where:

$s = l - l_0$, measured from its *unloaded* position.

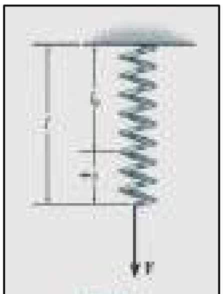
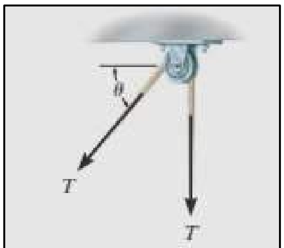


Fig. 3-1

Cables and Pulleys: All cables (or cords), unless otherwise mentioned, will be assumed to have negligible weight and they cannot stretch. Also, a cable can support ***only*** a tension or "pulling" force, and this force always acts in the direction of the cable. It will be shown that the tension force developed in a continuous cable which passes over a frictionless pulley must have ***a constant magnitude*** to keep the cable in equilibrium. Hence, for any angle u, shown in Fig. 3–2 , the cable is subjected to a constant tension ***T*** throughout its length.



Cable is in tension

Fig. 3–2

Procedure for Drawing a Free-Body Diagram

Since we must account for *all the forces acting on the particle* when applying the equations of equilibrium, the importance of first drawing a free-body diagram cannot be overemphasized. To construct a free-body diagram, the following three steps are necessary.

Draw Outlined Shape.

Imagine the particle to be *isolated* or cut "free" from its surroundings by drawing its outlined shape.

Show All Forces.

Indicate on this sketch *all* the forces that act *on the particle*. These forces can be *active forces*, which tend to set the particle in motion, or they can be *reactive forces* which are the result of the constraints or supports that tend to prevent motion. To account for all these forces, it may be helpful to trace around the particle's boundary, carefully noting each force acting on it.

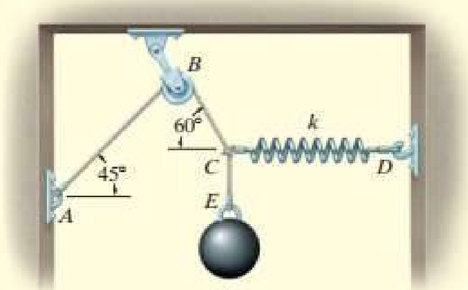
Identify Each Force.

The forces that are *known* should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.

The bucket is held in equilibrium by the cable, and instinctively we know that the force in the cable must equal the weight of the bucket. By drawing a free-body diagram of the bucket we can understand why this is so. This diagram shows that there are only two forces *acting on the bucket*, namely, its weight **W** and the force **T** of the cable. For equilibrium, the resultant of these forces must be equal to zero, and so $T = W$.

EXAMPLE 3.1

The sphere in Fig. 3–3*a* has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord *CE*, and the knot at *C*.



(a)

$\mathbf{F}_{CE}$ (Force of cord *CE* acting on sphere)

58.9 N (Weight or gravity acting on sphere)

(b)

$\mathbf{F}_{EC}$ (Force of knot acting on cord *CE*)

$\mathbf{F}_{CE}$ (Force of sphere acting on cord *CE*)

(c)

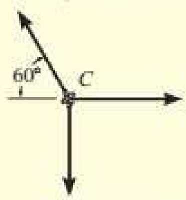
SOLUTION

Sphere. By inspection, there are only two forces acting on the sphere, namely, its weight, 6 kg $(9.81\ \text{m/s}^2) = 58.9$ N, and the force of cord *CE*. The free-body diagram is shown in Fig. 3–3*b*.

Cord *CE*. When the cord *CE* is isolated from its surroundings, its free-body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig. 3–3*c*. Notice that $\mathbf{F}_{CE}$ shown here is equal but opposite to that shown in Fig. 3–3*b*, a consequence of Newton's third law of action–reaction. Also, $\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull on the cord and keep it in tension so that it doesn't collapse. For equilibrium, $F_{CE} = F_{EC}$.

Knot. The knot at *C* is subjected to three forces, Fig. 3–3*d*. They are caused by the cords *CBA* and *CE* and the spring *CD*. As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord *CE* subjects the knot to this force.

$\mathbf{F}_{CBA}$ (Force of cord *CBA* acting on knot)



$\mathbf{F}_{CD}$ (Force of spring acting on knot)

$\mathbf{F}_{CE}$ (Force of cord *CE* acting on knot)

(d)

Fig. 3–3

3.3 Coplanar Force Systems

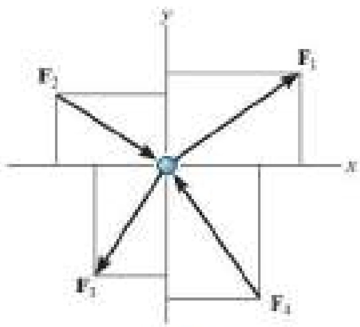


Fig. 3–4

If a particle is subjected to a system of coplanar forces that lie in the *x–y* plane, as in Fig. 3–4 , then each force can be resolved into its **i** and **j** components. For equilibrium, these forces must sum to produce a zero force resultant, i.e.,

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma F_x \, \mathbf{i} + \Sigma F_y \, \mathbf{j} = \mathbf{0}$$

For this vector equation to be satisfied, the resultant force's *x* and *y* components must both be equal to zero. Hence,

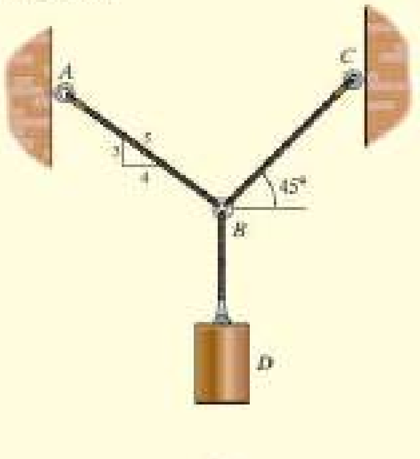
$$\Sigma Fx = 0 \quad \text{and} \quad \Sigma Fy = 0 \qquad \text{.......(3–3)}$$

These two equations can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

Note: When applying each of the two equations of equilibrium, we must account for the sense of direction of any component by using an ***algebraic sign*** which corresponds to the arrowhead direction of the component along the *x* or *y* axis. It is important to note that if a force has an ***unknown magnitude***, then the arrowhead sense of the force on the free-body diagram can be ***assumed.*** Then if the ***solution*** yields **a *negative scalar***, this indicates that the sense of the force is opposite to that which was assumed.

EXAMPLE 3.2

Determine the tension in cables *BA* and *BC* necessary to support the 60-kg cylinder in Fig. 3–6*a*.



(a)

(b)

SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable *BD* to be $T_{BD} = 60(9.81)$ N, Fig. 3–6*b*. The forces in cables *BA* and *BC* can be determined by investigating the equilibrium of ring *B*. Its free-body diagram is shown in Fig. 3–6c. The magnitudes of $\mathbf{T}_A$ and $\mathbf{T}_C$ are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the *x* and *y* axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

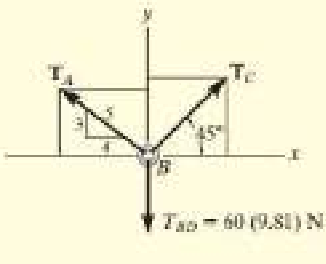
so that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.

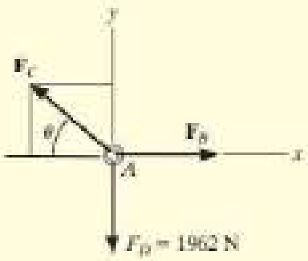


(c)

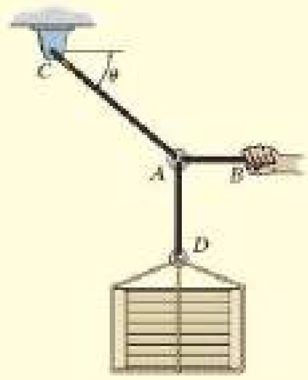
Fig. 3–6

EXAMPLE 3.3

The 200-kg crate in Fig. 3–7*a* is suspended using the ropes *AB* and *AC*. Each rope can withstand a maximum force of 10 kN before it breaks. If *AB* always remains horizontal, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.



(b)



(a)

Fig. 3–7

SOLUTION

Free-Body Diagram. We will study the equilibrium of ring *A*. There are three forces acting on it, Fig. 3–7*b*. The magnitude of $\mathbf{F}_D$ is equal to the weight of the crate, i.e., $F_D = 200\,(9.81)\text{ N} = 1962\text{ N} < 10\text{ kN}$.

Equations of Equilibrium. Applying the equations of equilibrium along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_C\cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta} \qquad (1)$$

$$+\uparrow\Sigma F_y = 0; \qquad F_C\sin\theta - 1962\text{ N} = 0 \qquad (2)$$

From Eq. (1), F_C is always greater than F_B since $\cos\theta \leq 1$. Therefore, rope *AC* will reach the maximum tensile force of 10 kN *before* rope *AB*. Substituting $F_C = 10$ kN into Eq. (2), we get

$$[10(10^3)\text{ N}]\sin\theta - 1962\text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ \qquad \textit{Ans.}$$

The force developed in rope *AB* can be obtained by substituting the values for θ and F_C into Eq. (1).

$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

$$F_B = 9.81\text{ kN}$$

EXAMPLE 3.4

Determine the required length of cord AC in Fig. 3–8a so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.

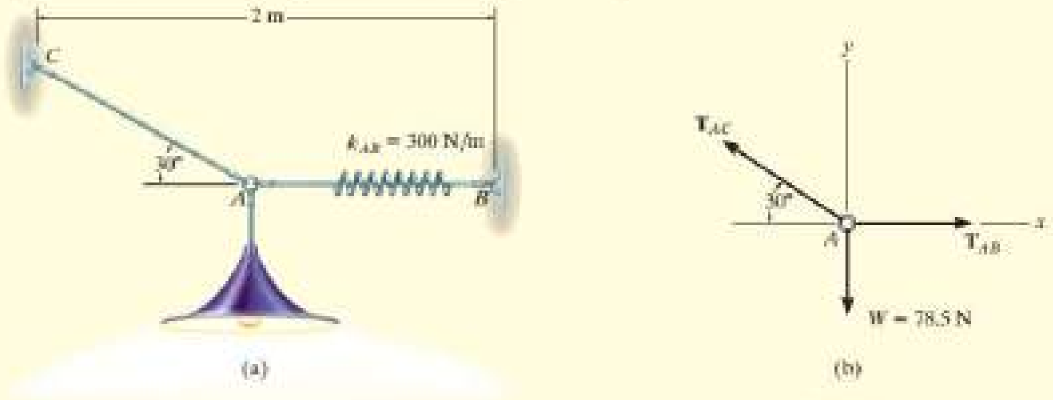


Fig. 3–8

SOLUTION

If the force in spring AB is known, the stretch of the spring can be found using $F = ks$. From the problem geometry, it is then possible to calculate the required length of AC.

Free-Body Diagram. The lamp has a weight $W = 8(9.81) = 78.5$ N and so the free-body diagram of the ring at A is shown in Fig. 3–8b.

Equations of Equilibrium. Using the x, y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{AB} - T_{AC}\cos 30^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{AC}\sin 30^\circ - 78.5\text{ N} = 0$$

Solving, we obtain

$$T_{AC} = 157.0\text{ N}$$
$$T_{AB} = 135.9\text{ N}$$

The stretch of spring AB is therefore

$$T_{AB} = k_{AB}s_{AB}; \qquad 135.9\text{ N} = 300\text{ N/m}(s_{AB})$$
$$s_{AB} = 0.453\text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$
$$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$$

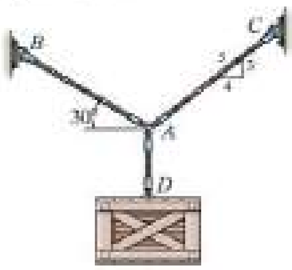
The horizontal distance from C to B, Fig. 3–8a, requires

$$2\text{ m} = l_{AC}\cos 30^\circ + 0.853\text{ m}$$
$$l_{AC} = 1.32\text{ m} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

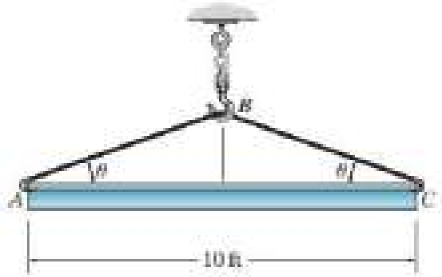
All problem solutions must include an FBD.

F3–1. The crate has a weight of 550 lb. Determine the force in each supporting cable.



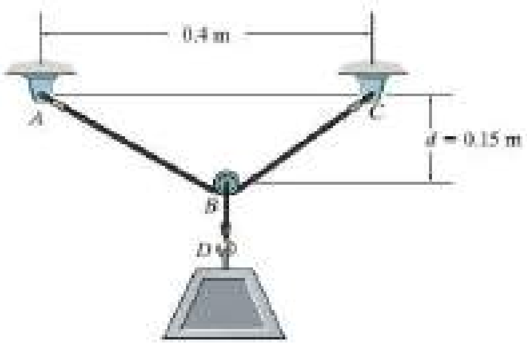
F3-1

F3–2. The beam has a weight of 700 lb. Determine the shortest cable *ABC* that can be used to lift it if the maximum force the cable can sustain is 1500 lb.



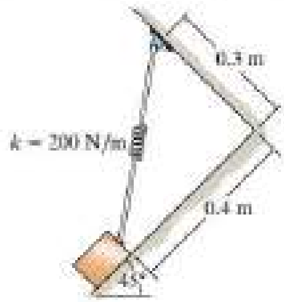
F3–2

F3–3. If the 5-kg block is suspended from the pulley *B* and the sag of the cord is $d = 0.15$ m, determine the force in cord *ABC*. Neglect the size of the pulley.



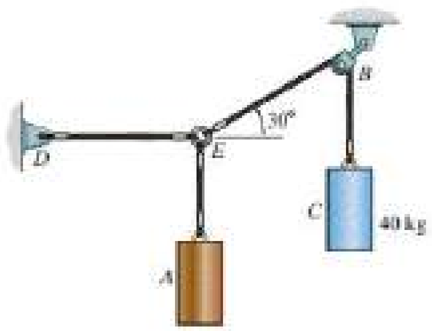
F3–3

F3–4. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.



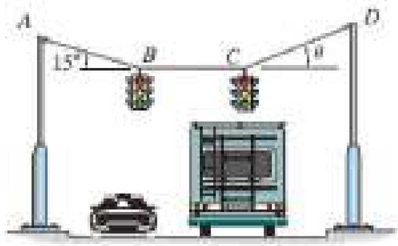
F3–4

F3–5. If the mass of cylinder *C* is 40 kg, determine the mass of cylinder *A* in order to hold the assembly in the position shown.



F3–5

F3–6. Determine the tension in cables *AB*, *BC*, and *CD*, necessary to support the 10-kg and 15-kg traffic lights at *B* and *C*, respectively. Also, find the angle *θ*.



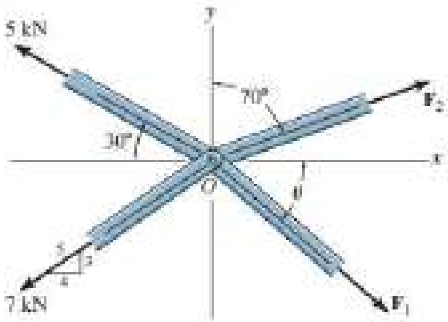
F3–6

PROBLEMS

All problem solutions must include an FBD.

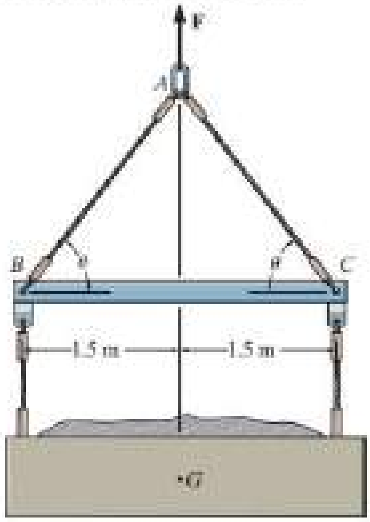
3–1. The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$.

3–2. The members of a truss are pin connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.



Probs. 3–1/2

3–3. The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ. If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.



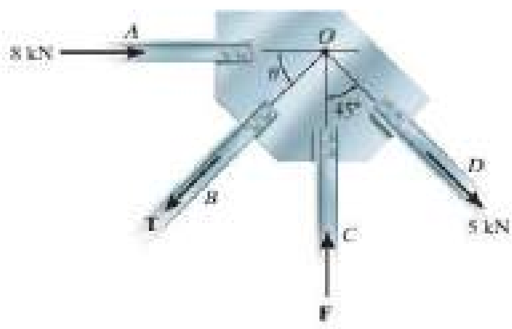
Prob. 3–3

***3–4.** Cords AB and AC can each sustain a maximum tension of 800 lb. If the drum has a weight of 900 lb, determine the smallest angle θ at which they can be attached to the drum.

Prob. 3–4

3–5. The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of $\mathbf{F}$ and $\mathbf{T}$ for equilibrium. Take $\theta = 30°$.

3–6. The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O. Take $F = 12$ kN.

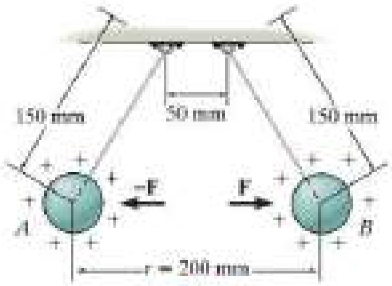


Probs. 3–5/6

3–7. The device shown is used to straighten the frames of wrecked autos. Determine the tension of each segment of the chain, i.e., *AB* and *BC*, if the force which the hydraulic cylinder *DB* exerts on point *B* is 3.50 kN, as shown.

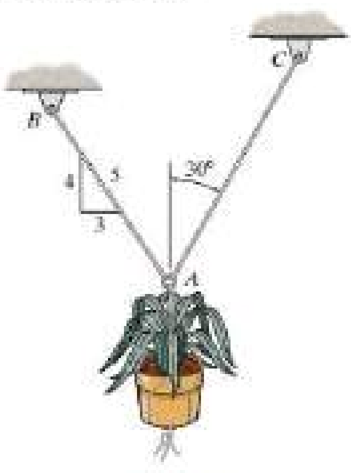
Prob. 3–7

***3–8.** Two electrically charged pith balls, each having a mass of 0.2 g, are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, *F*, acting on each ball if the measured distance between them is $r = 200$ mm.



Prob. 3–8

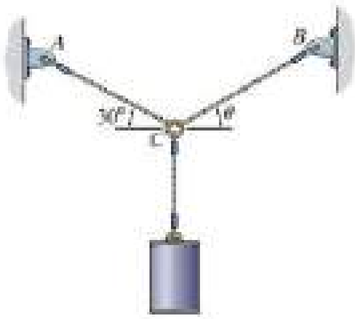
3–9. Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of 50 lb in either cable *AB* or *AC*.



Prob. 3–9

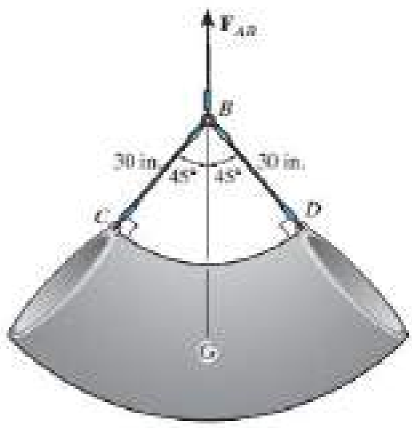
3–10. Determine the tension developed in wires *CA* and *CB* required for equilibrium of the 10-kg cylinder. Take $\theta = 40°$.

3–11. If cable *CB* is subjected to a tension that is twice that of cable *CA*, determine the angle θ for equilibrium of the 10-kg cylinder. Also, what are the tensions in wires *CA* and *CB*?



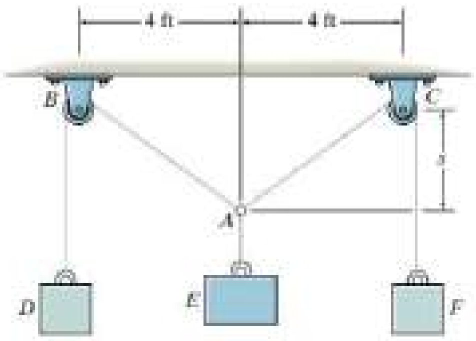
Probs. 3–10/11

***3–12.** The concrete pipe elbow has a weight of 400 lb and the center of gravity is located at point *G*. Determine the force $\mathbf{F}_{AB}$ and the tension in cables *BC* and *BD* needed to support it.



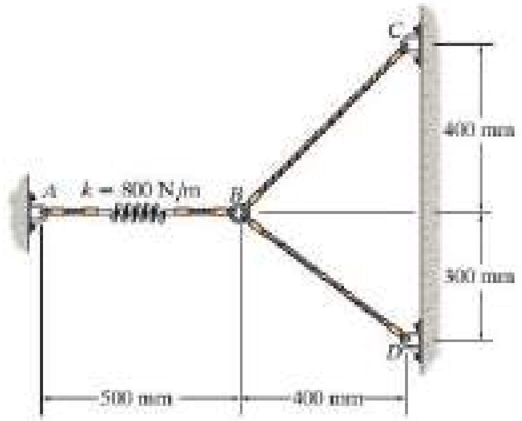
Prob. 3–12

3–14. If blocks D and F weigh 5 lb each, determine the weight of block E if the sag $s = 3$ ft. Neglect the size of the pulleys.



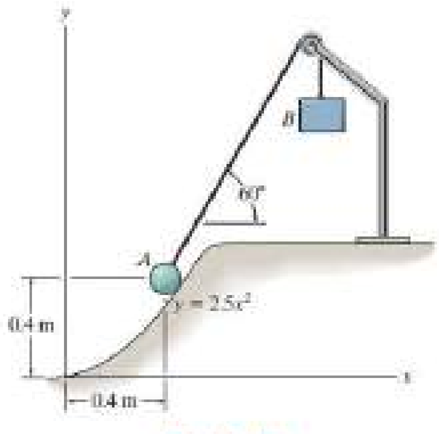
Probs. 3–13/14

3–15. The spring has a stiffness of $k = 800$ N/m and an unstretched length of 200 mm. Determine the force in cables BC and BD when the spring is held in the position shown.



Prob. 3–15

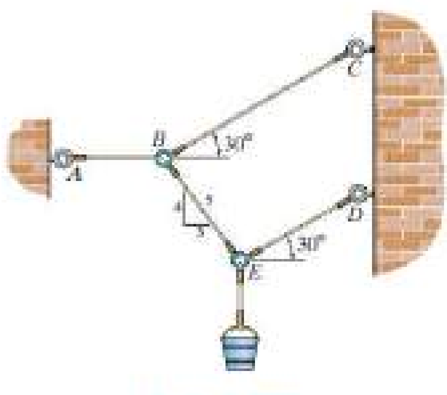
3–30. A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.



Prob. 3–30

3–31. If the bucket weighs 50 lb, determine the tension developed in each of the wires.

***3–32.** Determine the maximum weight of the bucket that the wire system can support so that no single wire develops a tension exceeding 100 lb.



Probs. 3–31/32

3.4 Three-Dimensional Force Systems

In Section 3.1 we stated that the necessary and sufficient condition for particle equilibrium is:

$$\Sigma \mathbf{F} = \mathbf{0} \qquad \text{............ (3–4)}$$

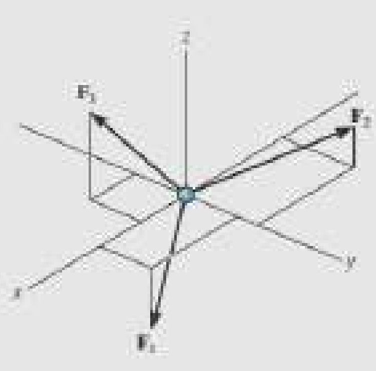


Fig. 3-9

In the case of a three-dimensional force system, as in Fig. 3–9 , we can resolve the forces into their respective **i**, **j**, **k** components, so that: $\Sigma F_x\mathbf{i} + \Sigma F_y\,\mathbf{j} + \Sigma F_z\mathbf{k} = \mathbf{0}$. To satisfy this equation we require:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0 \qquad \text{........ (3–5)}$$
$$\Sigma F_z = 0$$

These three equations state that the *algebraic sum* of the components of all the forces acting on the particle along each of the coordinate axes must be zero. Using them we can solve for at most three unknowns, generally represented as coordinate direction angles or magnitudes of forces shown on the particle's free-body diagram.

Procedure for Analysis

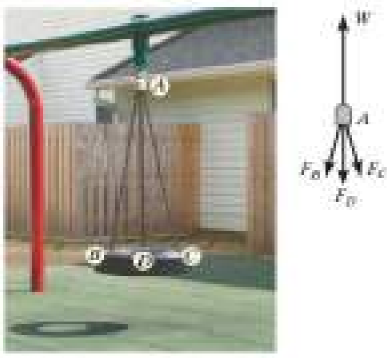
Three-dimensional force equilibrium problems for a particle can be solved using the following procedure.

Free-Body Diagram.

- Establish the x, y, z axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium.

- Use the scalar equations of equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, in cases where it is easy to resolve each force into its x, y, z components.
- If the three-dimensional geometry appears difficult, then first express each force on the free-body diagram as a Cartesian vector, substitute these vectors into $\Sigma \mathbf{F} = \mathbf{0}$, and then set the **i**, **j**, **k** components equal to zero.
- If the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.



The joint at A is subjected to the force from the support as well as forces from each of the three chains. If the tire and any load on it have a weight W, then the force at the support will be **W**, and the three scalar equations of equilibrium can be applied to the free-body diagram of the joint in order to determine the chain forces, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$.

EXAMPLE 3.6

The 10-kg lamp in Fig. 3–11*a* is suspended from the three equal-length cords. Determine its smallest vertical distance *s* from the ceiling if the force developed in any cord is not allowed to exceed 50 N.

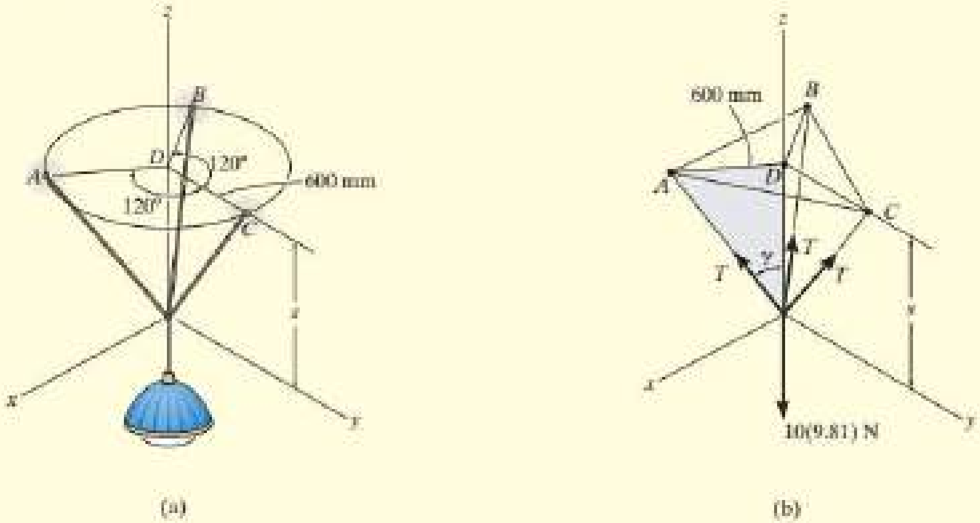


Fig. 3-11

SOLUTION

Free-Body Diagram. Due to symmetry, Fig. 3–11*b*, the distance $DA = DB = DC = 600$ mm. It follows that from $\sum F_x = 0$ and $\sum F_y = 0$, the tension T in each cord will be the same. Also, the angle between each cord and the z axis is γ.

Equation of Equilibrium. Applying the equilibrium equation along the z axis, with $T = 50$ N, we have

$$\sum F_z = 0; \qquad 3[(50\text{ N})\cos\gamma] - 10(9.81)\text{ N} = 0$$

$$\gamma = \cos^{-1}\frac{98.1}{150} = 49.16^\circ$$

From the shaded triangle shown in Fig. 3–11*b*,

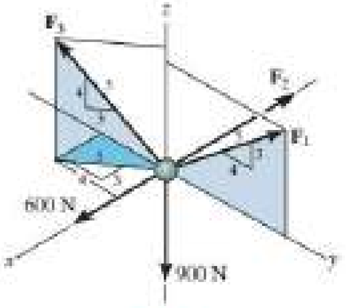
$$\tan 49.16^\circ = \frac{600\text{ mm}}{s}$$

$$s = 519\text{ mm} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

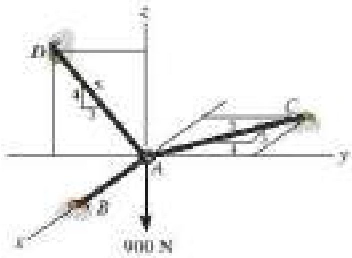
All problem solutions must include an FBD.

F3–7. Determine the magnitude of forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, so that the particle is held in equilibrium.



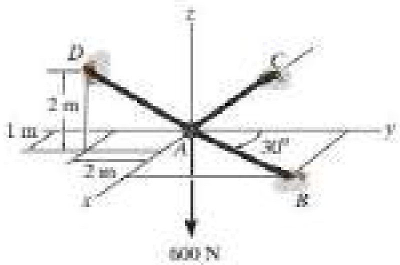
F3–7

F3–8. Determine the tension developed in cables *AB*, *AC*, and *AD*.



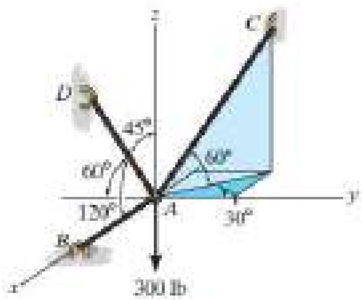
F3–8

F3–9. Determine the tension developed in cables *AB*, *AC*, and *AD*.



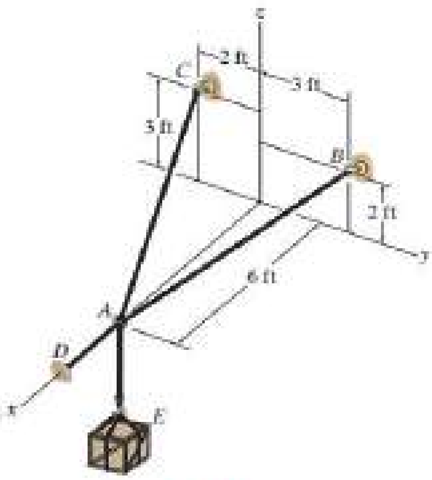
F3–9

F3–10. Determine the tension developed in cables *AB*, *AC*, and *AD*.



F3–10

F3–11. The 150-lb crate is supported by cables *AB*, *AC*, and *AD*. Determine the tension in these wires.



F3–11

CHAPTER - 4

Force System Resultants

CHAPTER OBJECTIVES:

- ***To discuss the concept of the moment of a force and show how to calculate it in two and three dimensions.***
- ***To provide a method for finding the moment of a force about a specified axis.***
- ***To define the moment of a couple.***
- ***To present methods for determining the resultants of non-concurrent force systems.***
- ***To indicate how to reduce a simple distributed loading to a resultant force having a specified location.***

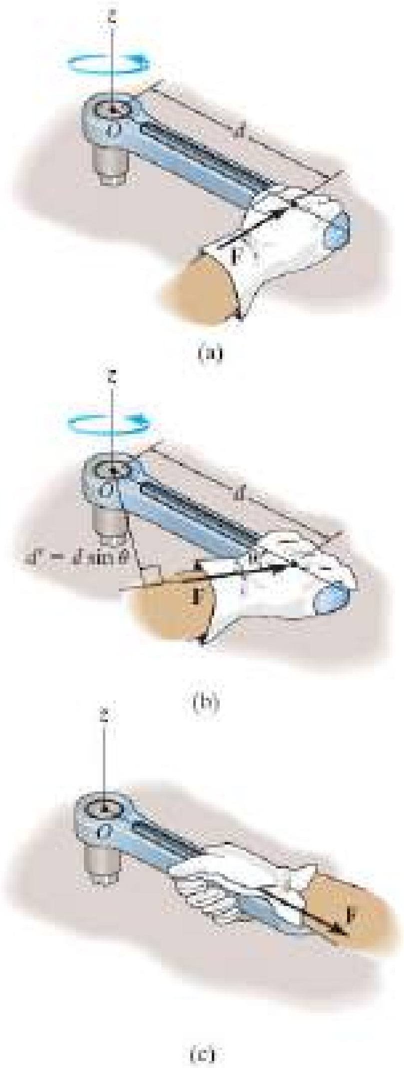


Fig. 4-1

4.1 Moment of a Force - Scalar Formulation:

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.

Consider the force **F** and point *O* which lie in the shaded plane as shown in Fig. 4–2 *a*. The moment $\mathbf{M}_O$ about point *O*, or about an axis passing through *O* and perpendicular to the plane, is a *vector quantity* since it has a specified magnitude and direction.

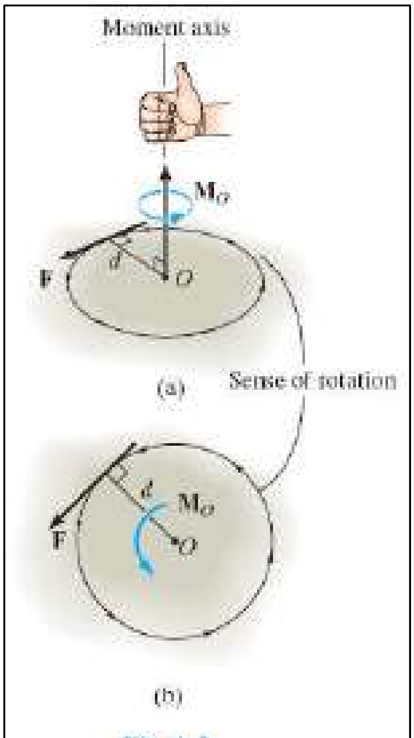


Fig. 4–2

Magnitude: The magnitude of $\mathbf{M}_O$ is:

$$M_O = Fd \tag{4–1}$$

Where ***d is the moment arm*** or ***perpendicular distance from the axis at point*** *O* to the line of action of the force. **Units** of moment are **N.m** or **lb.ft**.

Direction: The direction of $\mathbf{M}_O$ is defined by its *moment axis,* which is perpendicular to the plane that contains the force **F** and its moment arm ***d*** . The right-hand rule is used to establish the sense of direction of $\mathbf{M}_O$.

Resultant Moment: For two-dimensional problems, where all the forces lie within the *x–y* plane, Fig. 4–3 , the resultant moment $(\mathbf{M}_R)_O$ about point *O* (the *z* axis) can be determined by *finding the algebraic sum* of the moments caused by all the forces in the system. As a convention, we will generally consider *positive moments* as *counterclockwise* since they are directed along the positive *z* axis (out of the page). *Clockwise moments* will be *negative*. Therefore:

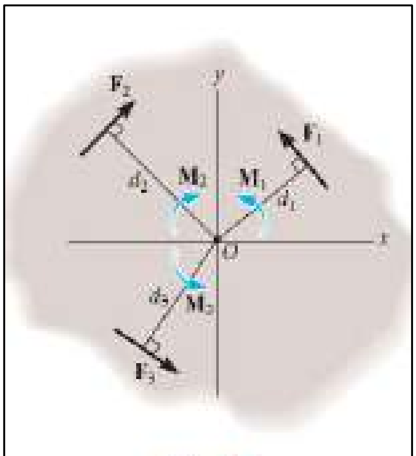


Fig. 4–3

$$\circlearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1d_1 - F_2d_2 + F_3d_3$$

If the numerical result of this sum is a positive scalar, $(\mathbf{M}_R)_O$ will be a Counterclockwise moment (out of the page); and if the result is negative, $(\mathbf{M}_R)_O$ will be a clockwise moment (into the page).

EXAMPLE 4.1

For each case illustrated in Fig. 4–4, determine the moment of the force about point O.

SOLUTION (SCALAR ANALYSIS)

The line of action of each force is extended as a dashed line in order to establish the moment arm d. Also illustrated is the tendency of rotation of the member as caused by the force. Furthermore, the orbit of the force about O is shown as a colored curl. Thus,

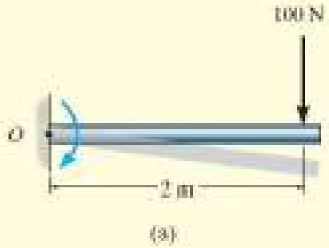
Fig. 4–4*a* $\quad M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$ *Ans.*

Fig. 4–4*b* $\quad M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m} \circlearrowright$ *Ans.*

Fig. 4–4*c* $\quad M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft} \circlearrowright$ *Ans.*

Fig. 4–4*d* $\quad M_O = (60\text{ lb})(1\sin 45^\circ\text{ ft}) = 42.4\text{ lb}\cdot\text{ft} \circlearrowleft$ *Ans.*

Fig. 4–4*e* $\quad M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m} \circlearrowleft$ *Ans.*



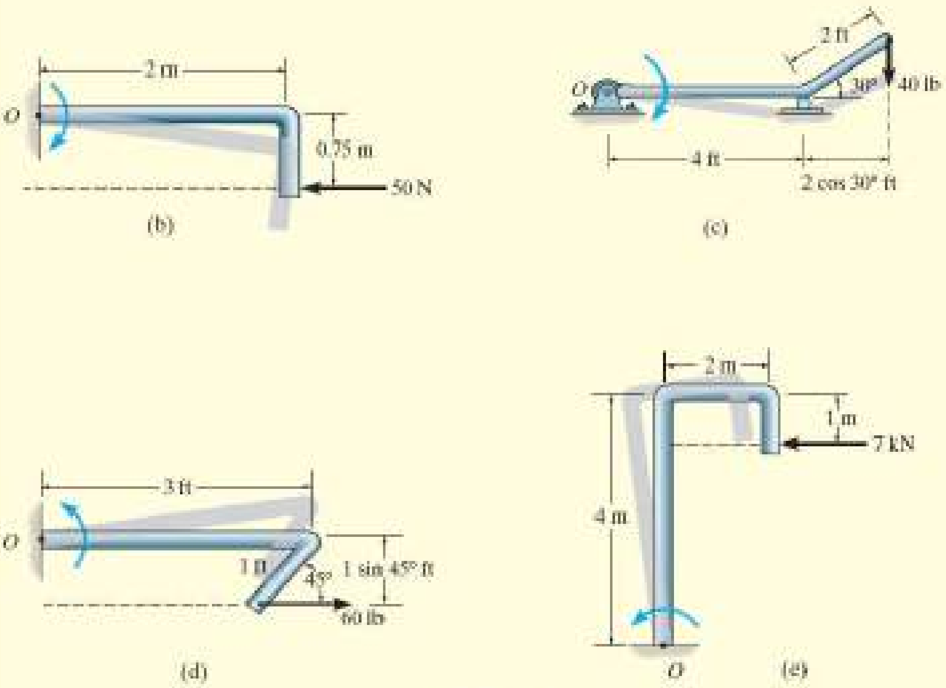


Fig. 4–4

EXAMPLE 4.2

Determine the resultant moment of the four forces acting on the rod shown in Fig. 4–5 about point O.

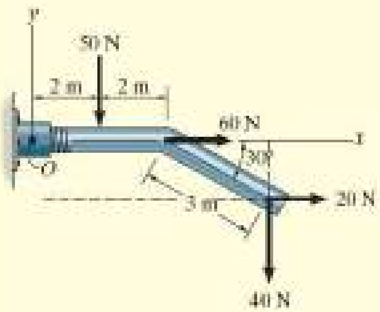


Fig. 4–5

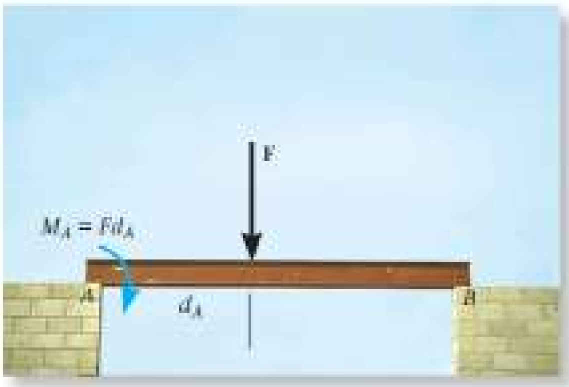
SOLUTION

Assuming that positive moments act in the $+\mathbf{k}$ direction, i.e., counterclockwise, we have

$$\circlearrowleft + \quad (M_R)_o = \Sigma Fd;$$

$$(M_R)_o = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$
$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$
$$(M_R)_o = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright \qquad \textit{Ans.}$$

For this calculation, note how the moment-arm distances for the 20-N and 40-N forces are established from the extended (dashed) lines of action of each of these forces.



As illustrated by the example problems, the moment of a force does not always cause a rotation. For example, the force **F** tends to rotate the beam clockwise about its support at A with a moment $M_A = Fd_A$. The actual rotation would occur if the support at B were removed.

The ability to remove the nail will require the moment of $\mathbf{F}_H$ about point O to be larger than the moment of the force $\mathbf{F}_N$ about O that is needed to pull the nail out.

4.2 Cross Product:

The moment of a force will be formulated using Cartesian vectors in the next section. Before doing this, however, it is first necessary to expand our knowledge of vector algebra and introduce the cross-product method of vector multiplication.

The *cross product* of two vectors **A** and **B** yields the vector **C** , which is written as:

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} \qquad (4\text{–}2)$$

and is read " **C** equals **A** cross **B** ."

Magnitude: The *magnitude* of **C** is defined as the product of the magnitudes of **A** and **B** and the sine of the angle u between their tails ($0° \leq \theta \leq 180°$). Thus, **C**=$A\,B\,\sin\theta$.

Direction: Vector **C** has a ***direction*** that is perpendicular to the plane containing **A** and **B** such that **C** is specified by the right-hand rule; i.e., curling the fingers of the right hand from vector **A** (cross) to vector **B**, the thumb points in the direction of **C** , as shown in Fig. 4–6 .

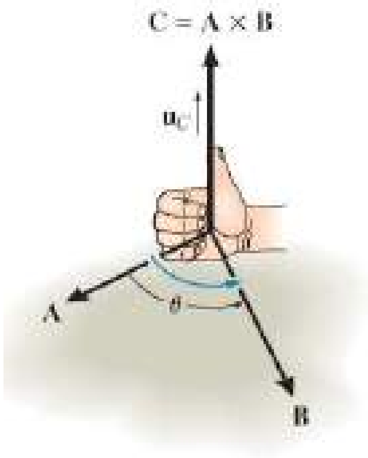


Fig. 4–6

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB\sin\theta)\mathbf{u}_C \qquad (4\text{–}3)$$

Laws of Operation:

• The commutative law is *not* valid; i.e. , **A** x **B** ≠ **B** x **A** Rather, **A** x **B** = -**B** x **A.** This is shown in Fig. 4–7 by using the right-hand rule. The cross product **B** x **A** yields a vector that has the same magnitude but acts in the opposite direction to **C** ; i.e., **B** x **A** = - **C**.

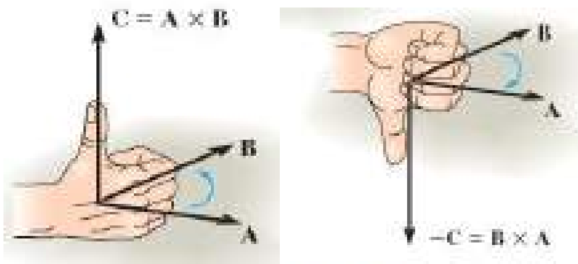


Fig. 4–7

• If the cross product is multiplied by a scalar a, it obeys the associative law;

$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

• The distributive law of addition,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

Cartesian Vector Formulation: Equation 4–3 may be used to find the cross product of any pair of Cartesian unit vectors. For example, to find **i** x **j**, the magnitude of the resultant vector is $(i)(j)(\sin 90^0) = (1)(1)(1) = 1$, and its direction is determined using the right-hand rule. As shown in Fig. 4–8 , the resultant vector points in the +**k** direction. Thus, **i** x **j** = (1)**k**. In a similar manner,

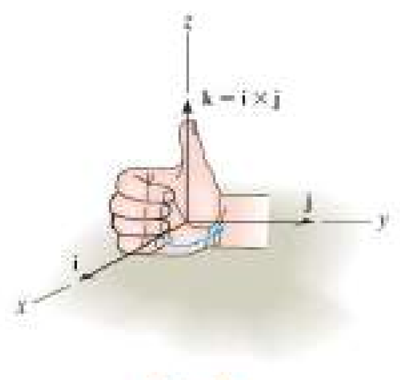


Fig. 4–8

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

These results should *not* be memorized; rather, it should be clearly understood how each is obtained by using the right-hand rule and the definition of the cross product. A simple scheme shown in Fig. 4–9 is helpful for obtaining the same results when the need arises. If the circle is constructed as shown, then "crossing" two unit vectors in a *counterclockwise* fashion around the circle yields the *positive* third unit vector; e.g., **k** x **i** = **j**. "Crossing" *clockwise* , a *negative* unit vector is obtained; e.g., **i** x **k** = **-j**.

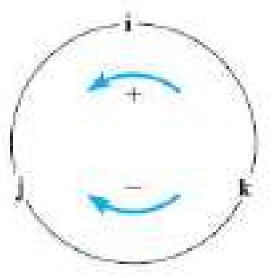


Fig. 4–9

Let us now consider the cross product of two general vectors **A** and **B**,

$$\mathbf{A}\times\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

$$\mathbf{A}\times\mathbf{B} = A_xB_x\,(\mathbf{i}\times\mathbf{i}) + A_xB_y(\mathbf{i}\times\mathbf{j}) + A_xB_z(\mathbf{i}\times\mathbf{k}) + A_yB_x(\mathbf{j}\times\mathbf{i}) + A_yB_y\,(\mathbf{j}\times\mathbf{j}) + A_yB_z(\mathbf{j}\times\mathbf{k}) + A_zB_x(\mathbf{k}\times\mathbf{i}) + A_zB_y(\mathbf{k}\times\mathbf{j}) + A_zB_z(\mathbf{k}\times\mathbf{k})$$

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

This equation may also be written in a more compact determinant form as:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad (4\text{–}5)$$

4.3 Moment of a Force - Vector Formulation

The moment of a force **F** about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and **F**, Fig. 4–10 *a*, can be expressed using the vector cross product, namely,

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \qquad (4\text{–}6)$$

Here **r** represents a position vector directed *from O* to *any point* on the line of action of **F** .

The magnitude of the cross product is defined from Eq.4–3 as $M_O = rF\sin\theta$, where the angle θ is measured between the *tails* of **r** and **F**. From Fig. 4–10 *b,* since the moment arm $d = r\sin\theta$, then:

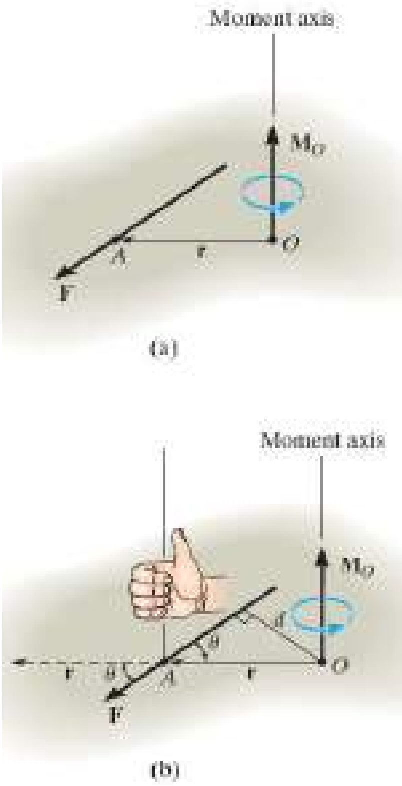


Fig. 4–10

$$M_O = rF\sin\theta = F(r\sin\theta) = Fd$$

The direction and ***sense*** of M_O in Eq. 4–6 are determined by the right-hand rule as it applies to the cross product.

Principle of Transmissibility: The cross product operation is often used in three dimensions since the perpendicular distance or moment arm from point *O* to the line of action of the force is not needed. In other words, we can use any position vector **r** measured from point *O* to any point on the line of action of the force **F**, Fig. 4–11 . Thus,

$$\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$$

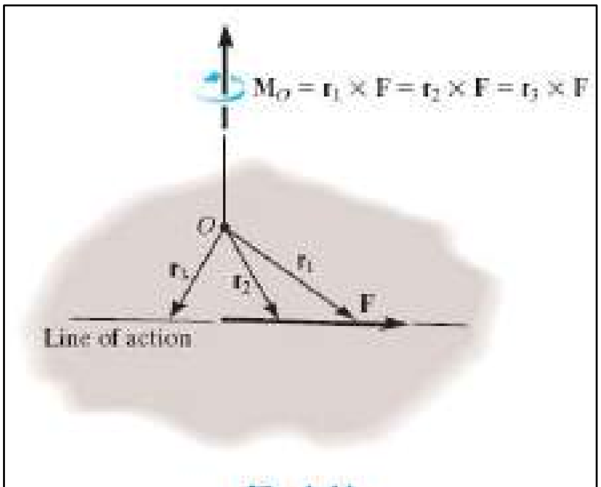
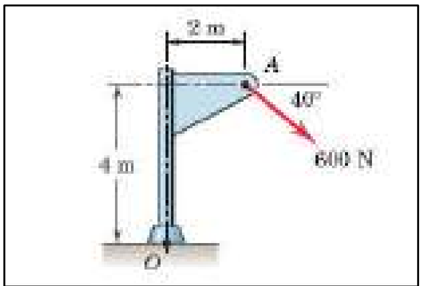


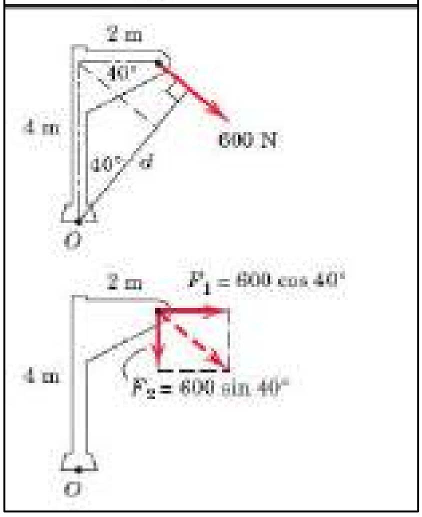
Fig. 4–11

Since **F** can be applied at any point along its line of action and still create this ***same moment*** about point *O* , then **F** can be considered a ***sliding vector*** . This property is called the ***principle of transmissibility*** of a force.

Example -1

Calculate the magnitude of the moment about the base point *O* of the 600-N force in five different ways.





Solution:

(I) The moment arm to the 600-N force is $d = 4\cos 40^o + 2\sin 40^o = 4.35$ m

By $M = Fd$ the moment is clockwise and has the magnitude:

$M_O = 600(4.35) = 2610$ *N.m* *Ans.*

(II) Replace the force by its rectangular components at *A*,

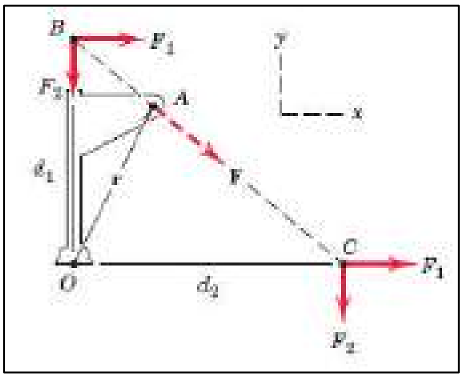
$F_1 = 600\cos 40^o = 460$ N, $F_2 = 600\sin 40^o = 386$ N

The moment becomes:

$M_O = 460(4) + 386(2) = 2610$ *N.m* *Ans.*

(III) By the principle of transmissibility, move the 600-N force along its line of action to point *B*, which eliminates the moment of the component F_2. The moment arm of F_1 becomes: $d_1 = 4 + 2\tan 40^o = 5.68$ m and the moment is:

$M_O = 460(5.68) = 2610$ *N.m* *Ans.*



(IV) Moving the force to point *C* eliminates the moment of the component F_1. The moment arm of F_2 becomes: $d_2 = 2 + 4 \cot 40^o = 6.77$ m

and the moment is:

$$M_O = 386(6.77) = 2610 \text{ N.m} \qquad \textit{Ans.}$$

(IV) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have:

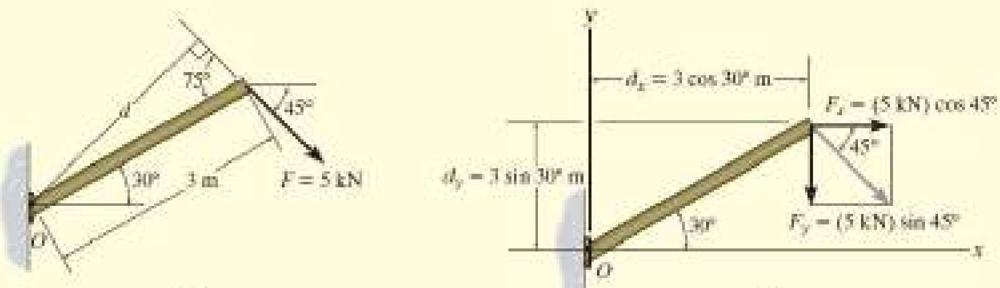
$$\mathbf{M_O} = \mathbf{r} \times \mathbf{F} = (2i + 4j) \times 600(i \cos 40^o - j \sin 40^o)$$
$$= -2610\mathbf{k} \text{ N.m}$$

The minus sign indicates that the vector is in the negative *z*-direction. The magnitude of the vector expression is:

$$M_O = 2610 \text{ N.m} \qquad \textit{Ans.}$$

EXAMPLE 4.5

Determine the moment of the force in Fig. 4–18*a* about point *O*.



(a) (b)

SOLUTION I

The moment arm *d* in Fig. 4–18*a* can be found from trigonometry.

$$d = (3 \text{ m}) \sin 75^\circ = 2.898 \text{ m}$$

Thus,

$$M_O = Fd = (5 \text{ kN})(2.898 \text{ m}) = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point *O*, the moment is directed into the page.

continue Example (4.5)

SOLUTION II

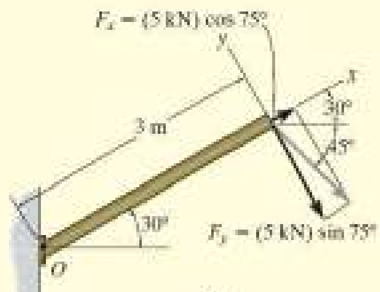
The x and y components of the force are indicated in Fig. 4–18*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\zeta + M_O = -F_x d_y - F_y d_x$$
$$= -(5 \cos 45^\circ \text{ kN})(3 \sin 30^\circ \text{ m}) - (5 \sin 45^\circ \text{ kN})(3 \cos 30^\circ \text{ m})$$
$$= -14.5 \text{ kN}\cdot\text{m} = 14.5 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4–18*c*. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\zeta + M_O = -F_y d_x$$
$$= -(5 \sin 75^\circ \text{ kN})(3 \text{ m})$$
$$= -14.5 \text{ kN}\cdot\text{m} = 14.5 \text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$



(c)

Fig. 4–18

Cartesian vector formulation:

If we establish *x, y, z* coordinate axes, then the position vector **r** and force **F** can be expressed as Cartesian vectors (Fig 4-12-a) then we can write:

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \qquad (4\text{–}7)$$

Where r_x, r_y,r_z represent the *x, y, z* components of the position vector drawn from point O to any point on the line of action of the force. F_x, F_y, F_z represent the *x, y, z* ***components*** of the force vector. If the determinant is expanded, then like Eq. 4–4 we have:

$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k} \qquad (4\text{–}8)$$

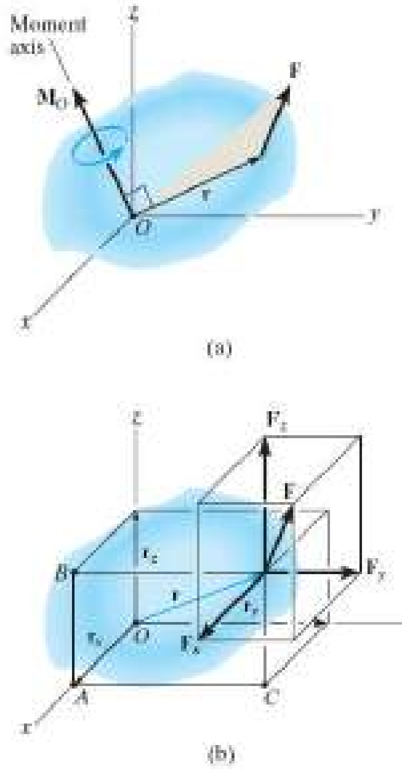


Fig. 4–12

Resultant Moment of a system of forces:

If a body is acted upon by a system of forces (Fig 4-13), the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as:

$$(\mathbf{M_R})_o = \Sigma\,(\mathbf{r} \times \mathbf{F})$$

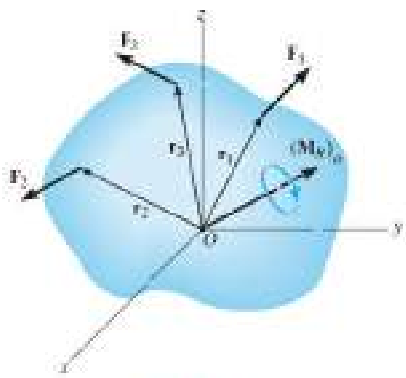


Fig. 4–13