Course Title: **Theory of machines I**

Course Code: **ME 3303**

Year: **Third Year**

Tutor: **Dr. Ahmed N. Uwayed**

Syllabus:

1. Velocity and acceleration diagrams

2. Hooke’s joint

3.Steering gear mechanism

4. Gyroscopic couple

5.Turning moment diagrams and flywheel

6. Governors

Text books:

1. ***Mechanics of Machines: Elementary theory and examples. By: J. Hannah and R.C. Stephens.***
2. ***Mechanics of Machines: Advanced theory and examples. By: J. Hannah and R.C. Stephens.***

CHAPTER (1)

1- Velocity in Mechanisms.

1.1. Relative velocity of two bodies moving in straight lines

* Consider two bodies A and B moving along parallel lines in the same direction with abslout velocities v_A and v_B such that $v_A > v_B$ as shown in Fig.(1.1a).

The relative velocity of A with respect to B,

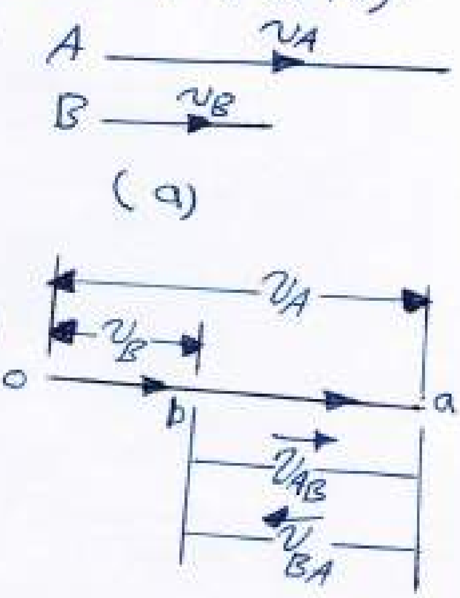
$$v_{AB} = \overline{v_A} - \overline{v_B}$$

From Fig.(1.1b) the relative velocity of A with respect to B (v_{AB}):

$$\overline{ab} = \overline{oa} - \overline{ob}$$

also $v_{BA} = \overline{v_B} - \overline{v_A}$

$$\overline{ab} = \overline{ob} - \overline{oa}$$



(b)

Fig.(1.1)

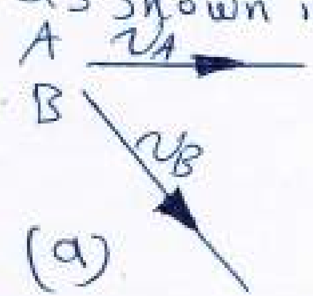
** If body B moving in an inclined direction as shown in Fig.(1-2a). The relative velocity A with respect to B is

$$v_{AB} = \overline{v_A} - \overline{v_B}$$

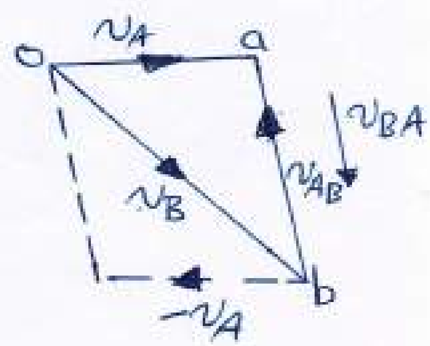
$$\overline{ba} = \overline{oa} - \overline{ob}$$

Similarly: $v_{BA} = \overline{v_B} - \overline{v_A}$

$$\overline{ab} = \overline{ob} - \overline{oa}.$$



(a)



1.2. Motion of a link:

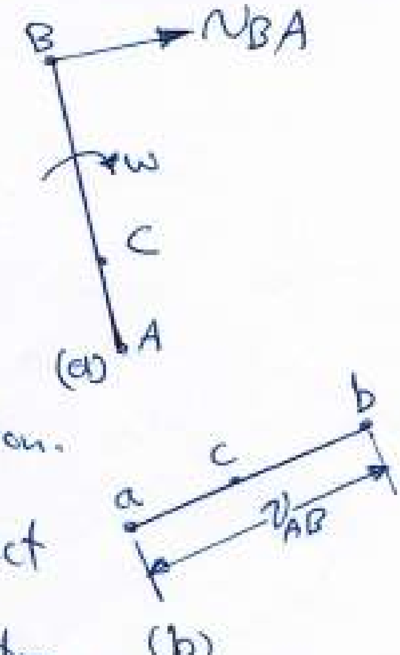


Fig.(1-3)

Let A and B two points on a rigid link AB as shown in fig (1.3 a). Let one of the extremities (B) of the link moves relative to (A), in a clockwise direction. The relative velocity of A with respect to B (v_{AB}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig.(1.3b).

if: w = Angular velocity of the link AB about A.

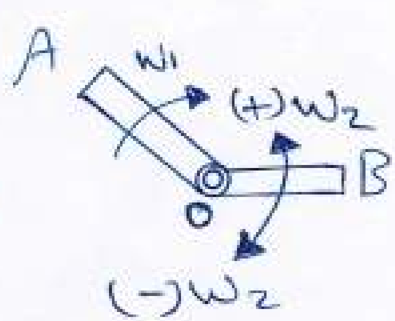
$$v_{BA} = \overline{ab} = w \cdot AB.$$

Similarly, the velocity of any point (C) on link AB with respect to A

$$v_{CA} = \overline{ac} = w AC$$

From above
$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{w \cdot AC}{w \cdot AB} = \frac{AC}{AB}.$$

1.3. Rubbing velocity at a pin Joint:



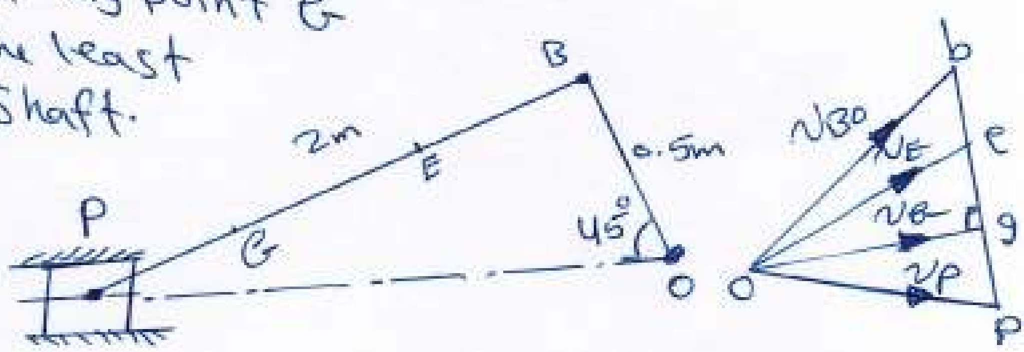
The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links wich are connected by bin points, mutiplied by the radius of the pin.

$$R.V. = (w_1 \mp w_2) \cdot r.$$

Ex.1: The crank and connecting rod of a steam engine are 0.5m and 2m long respectively. The crank makes 180 r.p.m. in the clockwise direction as shown in Fig. determine (a) velocity of piston, (b) angular velocity of connecting rod, (c) velocity of point E on the connecting rod 1.5m from the gudgeon pin, (d) velocity of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 5cm, 6cm and 3cm respectively, (e) position and linear velocity of any point G on C.r. which has the least velocity to crank shaft.

Sol.

$$\omega_{BO} = \frac{2\pi \times 180}{60} = 18.852 \text{ rad/s.}$$



(a) Space diagram

$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5$
$= 9.426$ m/s.

(a) $ob = v_{BO} = 9.426$ m/s.

$v_P = op = 8.15$ m/s.

(b) $v_{PB} = bp = 6.8$ m/s.

Length of C.r. $BP = 2m$.

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s.}$$

(c) $\frac{BE}{BP} = \frac{be}{bp} \Rightarrow be = \frac{BE * bp}{BP} = \frac{0.5 \times 6.8}{2}$
$= 1.7$ m/s

$\therefore$ Velocity of point $E = oe = 8.5$ m/s.

(d) $d_O = 5cm$, $d_B = 6cm$, $d_C = 3cm$

Velocity of rubbing of crank shaft
$= \frac{d_O}{2} \omega_{BO} = \frac{5}{2} \times 18.85 = 47.125$ cm/s.

V. of rubbing of crank
$= \frac{d_B}{2} \times \omega_{PB} = \frac{6}{2} \times 3.4$
$= 10.2$ cm/sec.

V. of rubbing of cross head
$= \frac{d_C}{2}(\omega_{BO} + \omega_{PB})$
$= \frac{3}{2}(18.85 + 3.4)$
$= 33.75$ cm/s.

(d) $\frac{bg}{bp} = \frac{BG}{BP}$

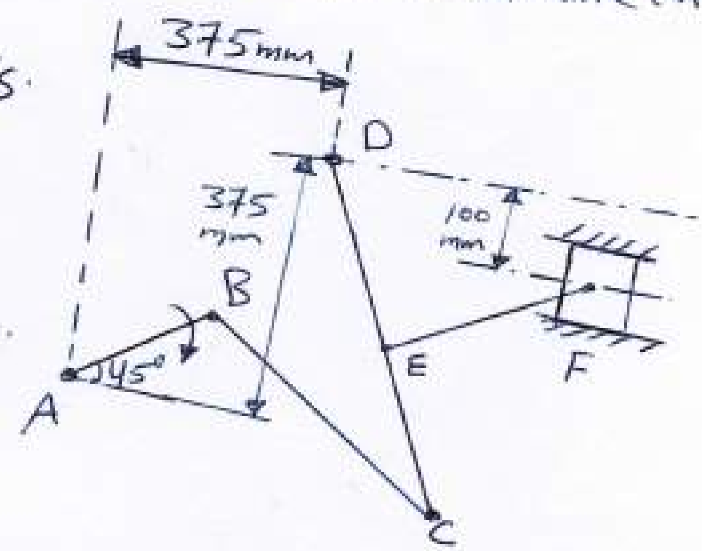
$\Rightarrow BG = \frac{5}{6.8} \times 2 = 1.47$ m

Ans.

EX. 2: The mechanism shown in Fig. has the dimensions of various links as follows: AB = DE = 15cm ; BC = CD = 45cm ; EF = 375mm

The crank AB rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. Determine (a) velocity of block F, (b) angular velocity of DC and (c) rubbing speed at the pin (C) which is 50mm in diameter.

SoL: $\omega_{BA} = \frac{2\pi \times 120}{60} = 4\pi$ rad/s.



$\therefore v_{BA} = v_B = \omega_{BA} * AB$

$= 4\pi * 15 = 188$ cm/s.

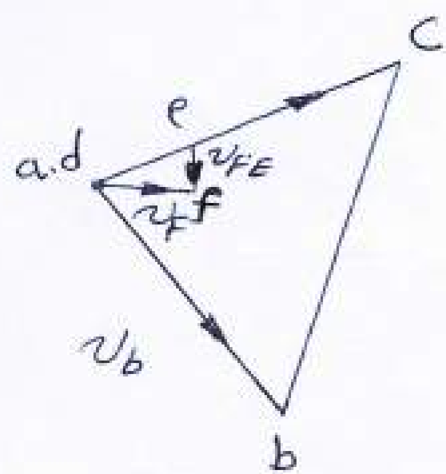
(a) $\frac{ce}{cd} = \frac{CE}{CD}$

$v_F = fd = 69$ cm/s.

(b) $\omega_{CD} = \frac{v_{CD}}{DC}$

$v_{CD} = dc = 225$ cm/s.

$\Rightarrow \omega_{CD} = \frac{225}{45} = 5$ rad/s.



(c) dc = 5cm.

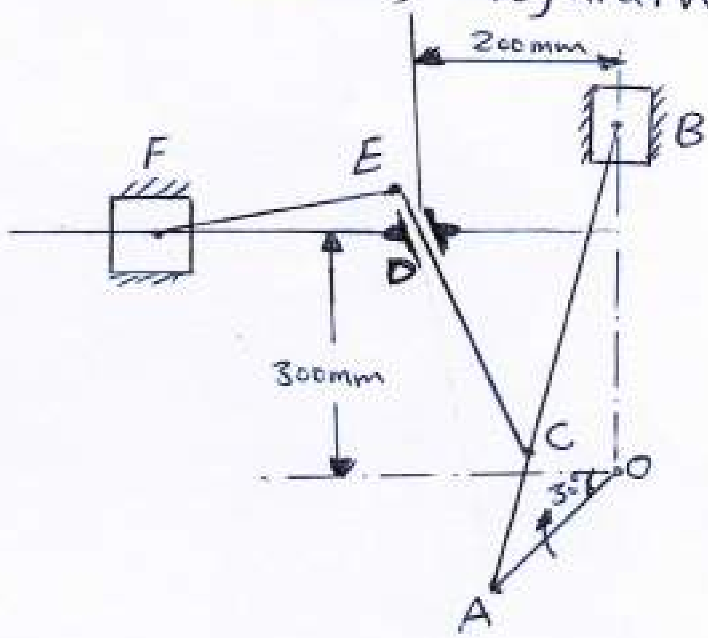
$v_{CB} = bc = 225$ cm/s.

$\omega_{CB} = \frac{v_{CB}}{CB} = \frac{225}{45} = 5$ rad/s.

rubbing velocity $= (\omega_{CB} - \omega_{CD}) r_c = (5-5)\, 2.5 = 0$. Ans.

<u>Ex.3</u> In a mechanism shown in Fig. the crank OA is 100 mm long and rotates clockwise about O at 120 r.p.m. The Connecting rod AB is 400 mm Long, AC = 150 mm, EF = 300 mm. CE = 350 mm. The rod CE slides a slot in a trunnion at D. For this position find (a) velocity of F, (b) velocity of sliding of CE in the trunnion and (C) angular velocity of CE.

<u>SoL.</u>



a)

$$W_{AO} = \frac{2\pi \times 120}{60} = 4\pi \ rad/s.$$

$$v_{AO} = v_A = W_{AO} \times AO$$

$$= 4\pi \times 0.1 = 1.26 \ m/s.$$

$ab \perp AB$

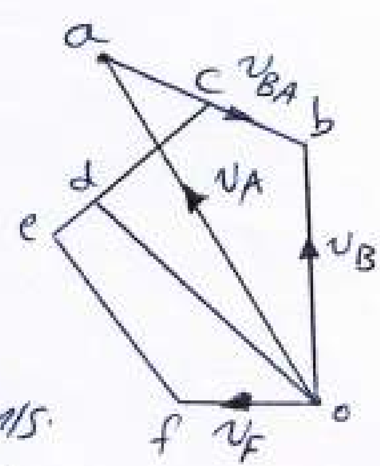
$$\frac{ac}{ab} = \frac{AC}{AB}.$$

$cd \perp CD$

$$\frac{cd}{ce} = \frac{CD}{CE}$$

$ef \perp EF$

$v_F = of = 0.53 \ m/s$. <u>Ans.</u>



(b) velocity of Sliding of CE = od = 1.08 m/s. <u>Ans.</u>

(C) V_{CE} = ec = 0.44 m/s.

$$W_{CE} = \frac{V_{CE}}{CE} = \frac{0.44}{0.35} = 1.26 \ rad/s. \ \underline{Ans.}$$

Ex. 4: A quick return mechanisum of the crank and slotted lever type shown in Fig.

$O_1O_2 = 350mm$; $O_1B = 100mm$; $O_2D = 550mm$, $DR = 125mm$.

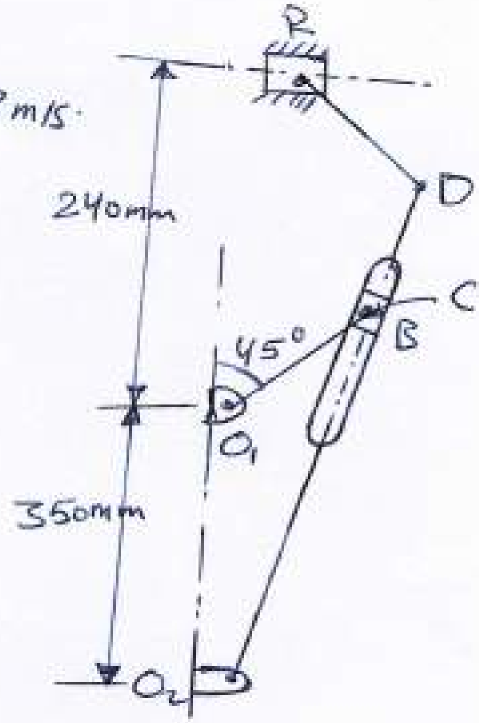
The crank O_1B makes an angle of 45° with the vertical and rotates at 60 r.p.m. in clockwise direction.

Find ① velocity of the ram R, ② angular velocity of link O_2D.

SoL.

$$\omega_{BO_1} = \frac{60 * 2\pi}{60} = 6.28 \text{ rad/s}$$

$$v_B = \omega_{BO_1} * O_1B = 6.28 * 0.1 = 0.628 \text{ m/s}$$



$o_1b \perp O_1B$ (B on crank)

$o_1b = 0.628$ m/s.

$o_2c \perp O_2C$

$$\frac{cd}{o_2d} = \frac{CD}{O_2D}$$

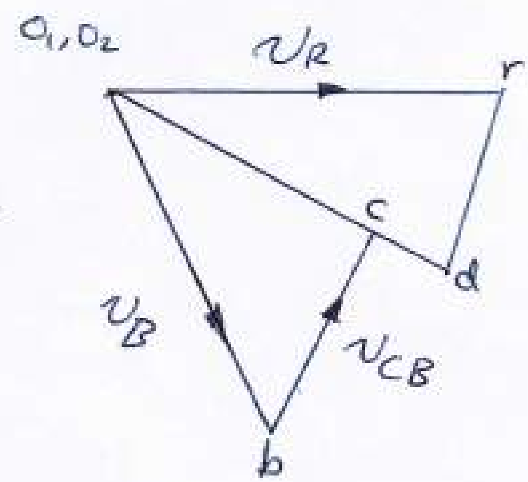
$dr \perp DR$.

$\Rightarrow v_R = \text{vector } o_1r = 0.7$ m/s.

② $\omega_{DO_2} = \dfrac{v_{DO_2}}{O_2D}$

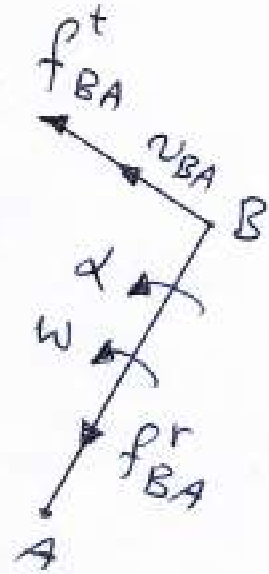
$v_{DO_2} = v_D = o_2d = \cancel{\ldots}$ 0.665 m/s.

$$\omega_{DO_2} = \frac{0.665}{0.55} = 1.2 \text{ rad/s. } \underline{\text{Ans.}}$$



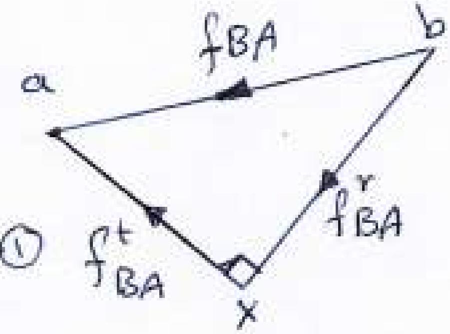
2. Acceleration in Mechanism

2.1 Acceleration Diagram for a Link:



a- Radial Component of B with respect to A

$f^r_{BA} = w^2 \cdot AB = \dfrac{v^2_{BA}}{\overline{BA}}$ —①



b- Tangential Component B with respect to A

$$f^t_{BA} = \alpha \cdot \overline{BA} \quad —②$$

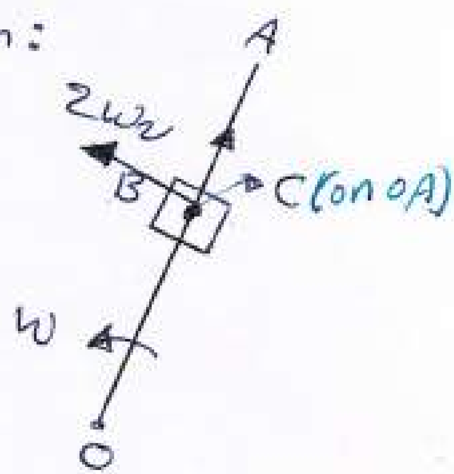
Total acceleration of B with respect to A:

$$f_{BA} = f^r_{BA} + f^t_{BA}$$

$$= w^2 \, \overline{BA} + \alpha \, \overline{BA}$$

$$= \frac{v^2_{BA}}{\overline{BA}} + \alpha \overline{BA}$$

2.2. Coriolis Component of Acceleration:



When a point on one link is sliding along another rotating link, then the Coriolis component of the acceleration must be calculated

$$f^c_{BC} = f^t_{BC} = 2wv$$

w: Angular velocity of the link

v = velocity of slider B with the respect to Point C.

Exa. 1: The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 15 cm and the connecting rod is 60 cm. Determine (a) linear velocity and acceleration of the midpoint of the connecting rod and (b) angular velocity and angular acceleration of the connecting rod.

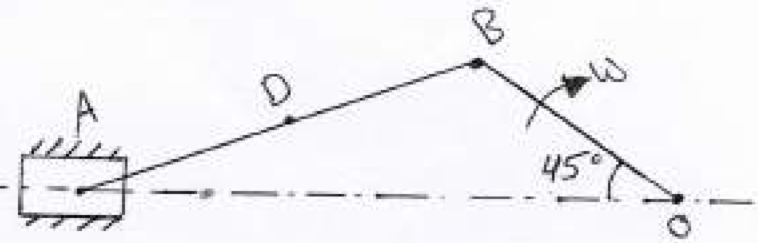
Sol.

from o $ob \perp oB = v_B$

from b $ba \perp AB = v_{AB}$

d point on connecting rod

$$\frac{BD}{AB} = \frac{bd}{ab}.$$



(a) Space diagram

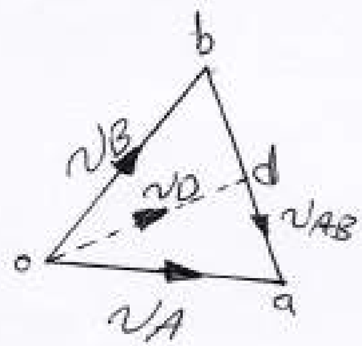
acceleration diagram.

$$f^r_{Bo} = f_B = \frac{v^2_{Bo}}{oB}$$

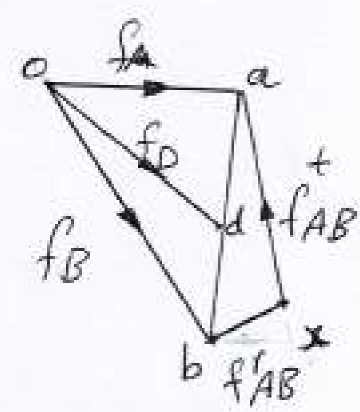
$$= 14808 \text{ cm/s}^2$$

$$f^r_{AB} = \frac{v^2_{AB}}{BA}$$

$$= 1926.7 \text{ cm/s}^2$$



(b) Velocity diagram.

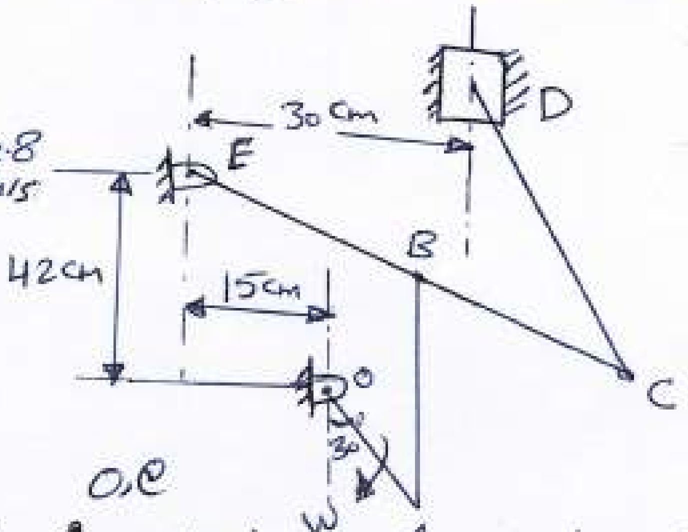


(C) accn. diagram

by measurment $\Rightarrow f^t_{AB} = 10300 \text{ cm/s}^2$

$$\alpha_{AB} = \frac{f^t_{AB}}{AB} = \frac{10300}{60} = 171.6 \text{ rad/s}^2.$$

Exa-2: Find out the acceleration of the slider D and the angular acceleration of Link CD for the engine mechanism shown in Fig. The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various Length are: OA = 15 cm ; AB = 45 cm , PB = 24 cm ; BC = 21 cm CD = 66 cm.



Sol:

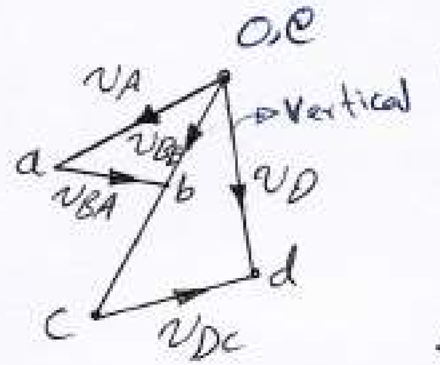
$V_{AO} = W_{AO} * AO = 6\pi * 15 = 282.8$ cm/s

$ab \perp BA$

$be \perp EC$

$\frac{eb}{ec} = \frac{EB}{EC}$

$cd \perp DC$



velocity diagram

Acceleration

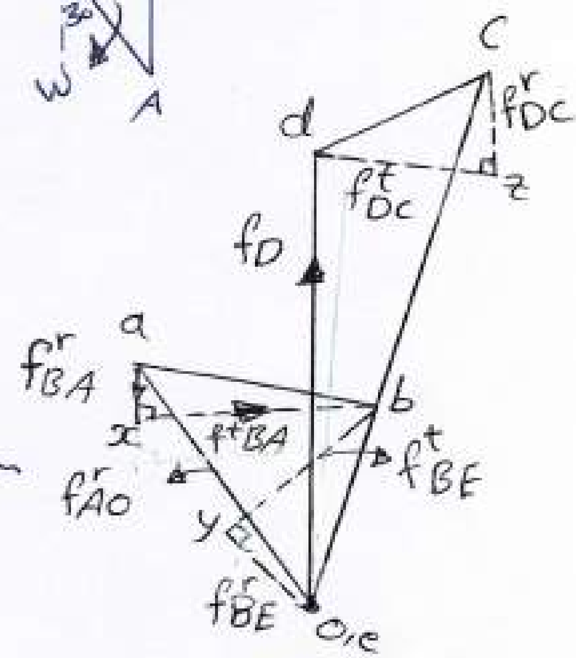
$f^r_{AO} = w^2_{AO} * AO$

$= (6\pi)^2 * 15 = 5330$ cm/s²

$f^r_{BA} = \frac{v^2_{BA}}{BA}$

$f^r_{BE} = \frac{v^2_{BE}}{BE}$

$f^r_{DC} = \frac{v^2_{DC}}{DC}$



acc^n diagram

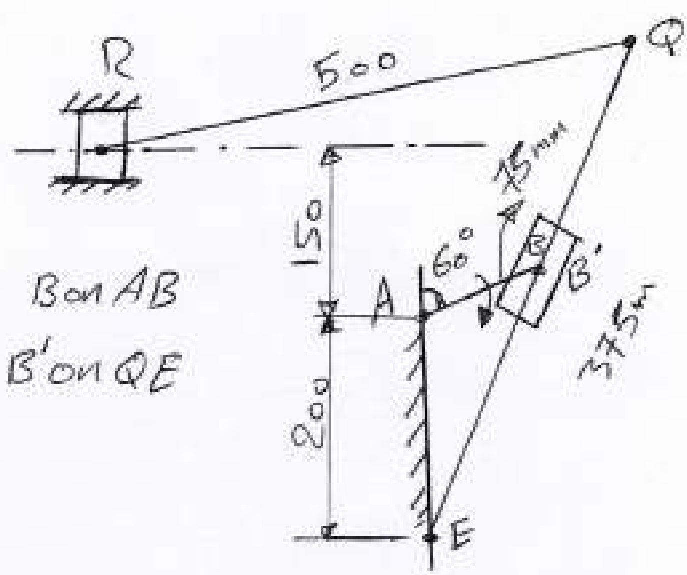
from diagram

$\Rightarrow \alpha_{CD} = \frac{f^t_{DC}}{DC} = \frac{1740}{66} = 26.3$ rad/s² Ans.

Exa.3 The driving crank AB of the quik-return mechanism. revolves at a uniform speed of 200 r.p.m. Find the velocity and acceleration of the tool-box R in the position shown. what is the acceleration of sliding of the block at B along the slotted lever EQ?

Sol.

$$W_{BA} = \frac{2\pi \times 200}{60} = 20.95 \text{ rad/s}$$



$v_{AB} = w_{AB} \times AB$

$= 20.95 \times 0.075$

$= 1.57 \text{ m/s}$

B on AB

B' on QE

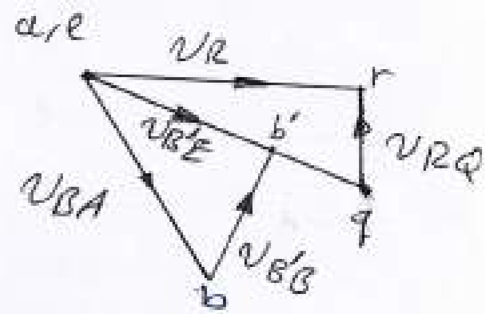
$ab \perp AB$

$b'e \perp QE$

$b'b \parallel QE$

$$\frac{eb'}{eq} = \frac{EB'}{EQ}$$

$qr \perp QR$



Velocity diagram.

$f^r_{BA} = W^2_{BA} \times AB = 20.95^2 \times 0.075 = 32.9 \text{ m/s}^2$

$f^c_{BB'} = 2wv = 2 W_{QE} \times v_{BB'}$

$$f^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{0.4^2}{0.5} = 0.32 \text{ m/s}^2$$

$$f^r_{B'E} = \frac{v^2_{B'E}}{EB'} = \frac{1.13^2}{0.248} = 5.15 \text{ m/s}^2$$

$ab \parallel AB$

$ey \parallel B'E$

bx Coriolis $\perp B'E$

$xb' \perp bx$

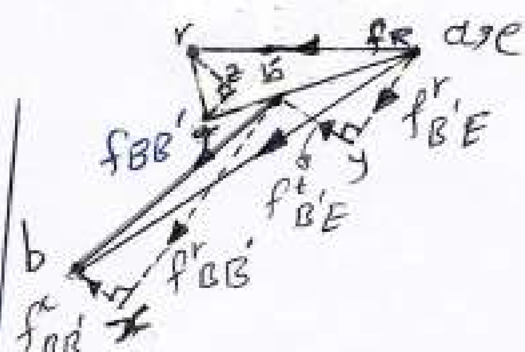
$yb' \perp ey$

Acceleration of sliding of the Block B along the slotted lever EQ

$= f^r_{BB'} =$ vector $b'x$

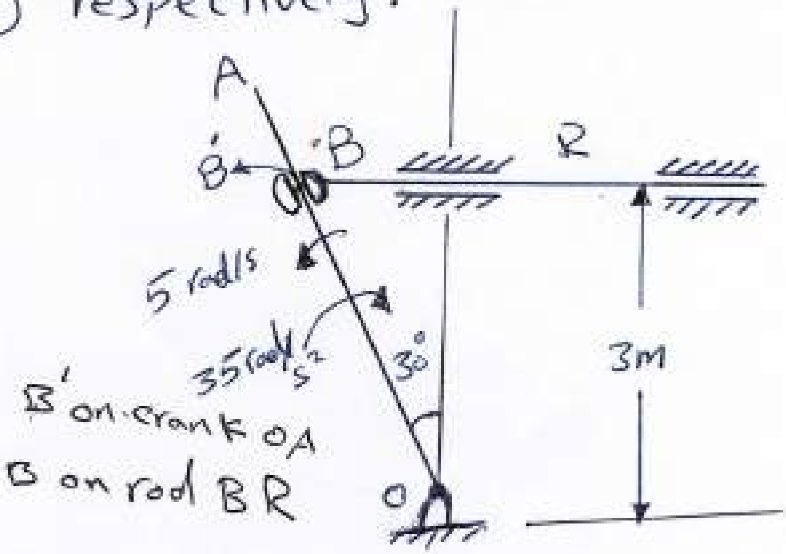
* Acc. of tool box R

$= f^r =$ vector ar.



acceleration.

Ex. 4 A rod BR is constrained by guides to move horizontally and driven by a crank OA and sliding block at B. For the given configration, determine the acceleration of BR when OA has angul velocity and acceleration of (5 rad/s) and (−35 rad/s²) respectively.



Sol.

$V_{B'O} = 5 \times \frac{3}{\cos 30} = 17.3\ m/s.$

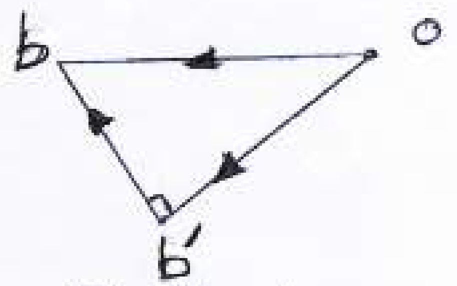
$ob' \perp AO$

ob horizontal.

$bb' \parallel OA.$

$f^{t}_{B'O} = \alpha_{OB'} * \overline{OB} = 35 * 3.46 = 121.2\ m/s^2 = b_1' b'$

$f^{r}_{B'O} = \frac{v^2_{B'O}}{\overline{OB}} = \frac{(17.3)^2}{3.46} = 86.6\ m/s^2 = ob_1'$



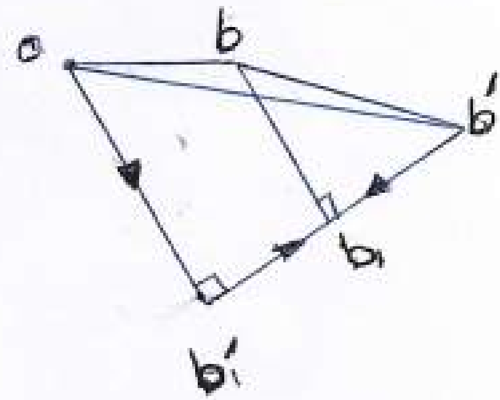
Velocity diagram

$f^{c}_{BB'} = 2 W_{OB'} * V_{BB'}$

$= 2 * 10 * 5 = 100\ m/s^2 = b' b_1$

from acceleration dia.

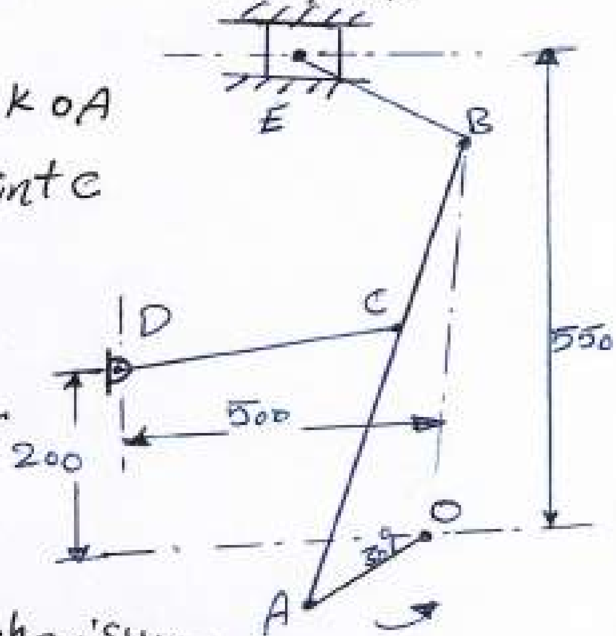
$a_{BR} = 25\ m/s^2.$



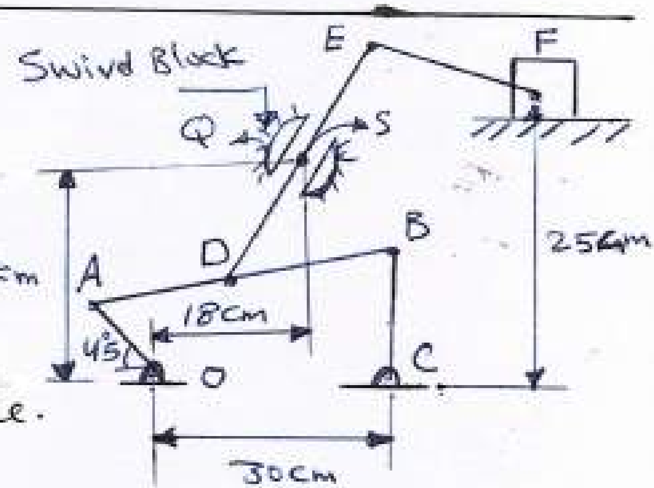
acceleration diagram.

Theory of machines – Sheet No.(1) / Third Year.

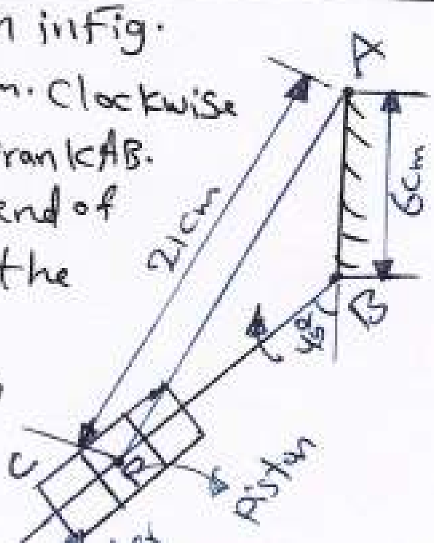
1- For the mechanism shown in Fig. Crank OA turns uniformly at 150 r.p.m. The point C in this rod is guided in the circular path with D. The dimensions of various links are: OA = 150 mm; AB = 550 mm; AC = 450 mm; DC = 500 mm, BE = 350 mm. Determine velocity and acceleration of ram E for given position of the mechanisum.



2- In swivelling joint mechanisum as shown in Fig. the driving crank OA is rotating clockwise at 100 r.p.m. The lengths of the various links are OA = 5 cm, AB = 35 cm; AD = BD, DE = EF = 25 cm and BC = 12.5 cm. For the given configration, determine.
① Velocity of the slider block F
② Angular velocity of the link DE ③ Velocity of sliding of the link DE in swivel block ④ Acceleration of sliding the link DE in the trunnion.



③ The cylinder of rotary engine as shown in fig. rotate at a uniform speed of 900 r.p.m. clockwise about the lower end B of a fixed vertical crank AB. The connecting rod rotate about the upper end of the crank and reciprocate the pistons in the cylinder.
Determine the acceleration of the piston relative to the cylinder and angular acceleration of the connecting rod for cylinder



2- Hooke's Joint (Universal joint)

A Hooke's joint is used to connect two shafts, which are intersection at a small angle, as shown Fig.(1).

The motion is transmitted from the driving shaft to driven shaft through across. The inclination of the two shafts may be constant, but in actual practice it varies.

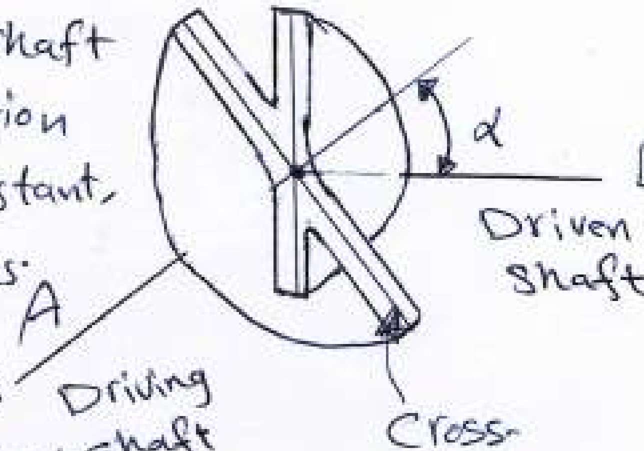


Fig.(1) Hooke's joint

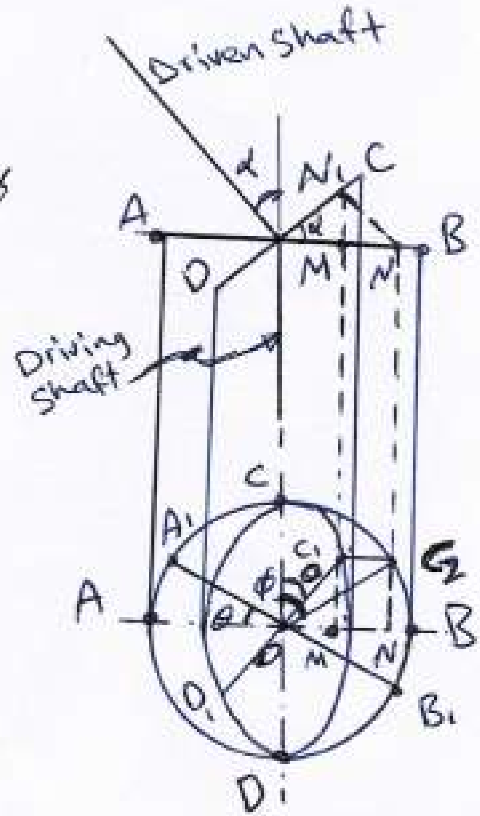
Let the driving shaft rotates through an angle (θ) and the angle (ϕ) turned through by shaft (B) then:

In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \tan\theta = \frac{OM}{MC_1} \quad \text{——} (1)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1} \quad \text{——} (2)$$



Dividing eq. (1) by (2)

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $OM = ON_1 \cos\alpha = ON \cos\alpha$

$$\therefore \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\phi\cos\alpha \quad \text{——} (3)$$

Let ω = Angular velocity of the driving shaft $= \frac{d\theta}{dt}$

ω_1 = Angular velocity of the driven shaft $= \frac{d\phi}{dt}$

Differentiating both sides of eq. (3)

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \cos\alpha \cdot \sec^2\phi \, \frac{d\phi}{dt}$$

$$\sec^2\theta \times w = \cos\alpha \cdot \sec^2\phi \times w_1$$

$$\frac{w}{w_1} = \frac{\cos\alpha \cdot \sec^2\phi}{\sec^2\theta}$$

$$= \cos\alpha \cdot \sec^2\phi \cdot \cos^2\theta \quad \text{——— (4)}$$

$$\Rightarrow \frac{w}{w_1} = \frac{1 - \cos^2\theta \sin^2\alpha}{\cos\alpha} \quad \text{——— (5)} \quad \text{prove that}^{*}$$

** Maximum and Minimum Speeds of Driven shaft:

$$\frac{w}{w_1} = \frac{1 - \cos^2\theta \sin^2\alpha}{\cos\alpha}$$

$$w_1 = \frac{w\cos\alpha}{1 - \cos^2\theta \sin^2\alpha}$$ *[when $\cos^2\theta = 1$ i.e. when $\theta = 0, \pi, 2\pi$ etc.

$$w_{1(max)} = \frac{w\cos\alpha}{1 - \sin^2\alpha} = \frac{w\cos\alpha}{\cos^2\alpha} = \frac{w}{\cos\alpha} \quad *$$

$w_{2(min)} = w\cos\alpha$ [when $\cos^2\theta = 0$ i.e. when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

*** Angular Acceleration of the Driven Shaft:

$$\frac{w}{w_1} = \frac{1 - \cos^2\theta \sin^2\alpha}{\cos\alpha}$$

$$w_1 = \frac{w\cos\alpha}{1 - \cos^2\theta \cdot \sin^2\alpha} = w\cos\alpha \left(1 - \cos^2\theta \cdot \sin^2\alpha\right)^{-1}$$

$$\boxed{\frac{dw_1}{dt} = \frac{-w^2\cos\alpha \times \sin2\theta \cdot \sin^2\alpha}{\left(1 - \cos^2\theta \sin^2\alpha\right)^2}}$$

Maximum angular acceleration:

when $\cos 2\theta = \dfrac{\sin^2\alpha\,(2-\cos^2 2\theta)}{2-\sin^2\alpha}$

If the value of α is less than 30°

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$

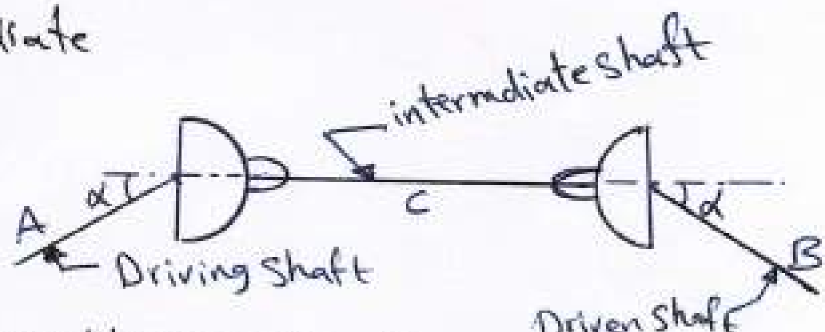
**** Maximum Fluctuation of Speed:

$$q = \omega_{1max} - \omega_{1min} = \frac{\omega}{\cos\alpha} - \omega\cos\alpha = \omega\left(\frac{1}{\cos\alpha} - \cos\alpha\right)$$

$$\Rightarrow q = \omega \tan\alpha \cdot \sin\alpha$$

** Double Hooke's Joint:

If the driving, intermediate and driven shafts, in the same time rotate through angles θ and ϕ, from the position then:



$$\tan\theta = \tan\phi \cos\alpha$$

$\therefore$ For shaft A and C

$$\tan\theta_A = \tan\phi_C \cdot \cos\alpha$$

and for shaft B and C,

$$\tan\theta_B = \tan\phi_C \cdot \cos\alpha$$

$$\Rightarrow \tan\theta_A = \tan\theta_B$$

$$\Rightarrow \theta_A = \theta_B \text{ or } \omega_A = \omega_B$$

Ex.(1) Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of 12 kg and 10 cm radius of gyration. Find the maximum acceleration of the driven shaft and the maximum torque required.

Sol.

$$w = \frac{1500 * 2\pi}{60} = 157 \text{ rad/sec.}$$

$$I = mK^2 = 12 * (0.1)^2 = 0.12 \text{ kg.m}^2$$

$\frac{dw_1}{dt}$ = maximum acceleration of driven shaft

$$\text{when } \cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha} = \frac{2\sin^2 20}{2-\sin^2 20} = 0.124$$

$$\Rightarrow 2\theta = 82.9° \text{ or } \theta = 41.45°$$

$$\frac{dw_1}{dt} = \frac{-w^2\cos\alpha \cdot \sin 2\theta \cdot \sin^2\alpha}{(1-\cos^2\theta \cdot \sin^2\alpha)^2} \quad (\alpha = 20°, \theta = 41.45°)$$

$$\Rightarrow \frac{dw_1}{dt} = -3040 \text{ rad/sec}^2.$$

∴ Maximum torque required

$$T_{max} = I \cdot \alpha$$

$$= 0.12 * -3040 = -364.8 \text{ N.m.}$$

or $2\theta = 277.1°$

$$\Rightarrow \frac{dw_1}{dt} = 3040 \text{ rad/sec}^2, \quad T_{max} = 364.8 \text{ N.m} \quad \text{Ans.}$$

Exa:(2) Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 r.p.m. Determine the greatest permissible angle between the shaft axes so that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft.

Sol^n

$$q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$100 = 1200\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\frac{1-\cos^2\alpha}{\cos\alpha} = \frac{100}{1200} = 0.083$$

$$\Rightarrow \cos^2\alpha + 0.083\cos\alpha - 1 = 0$$

$$\therefore \cos\alpha = \frac{-0.083 \mp \sqrt{(0.083)^2+4}}{2}$$ (Taking +ve sign)

$$\Rightarrow \alpha = 16.4^\circ$$

$$N'_{max} = \frac{N}{\cos\alpha} = \frac{1200}{\cos 16.4} = 1251 \text{ r.p.m.}$$

$$N'_{min.} = N\cos\alpha = 1200\cos 16.4 = 1151 \text{ r.p.m.}$$

Exa(3): The driving Shaft of a Hooke's joint runs at a uniform speed of 240 r.p.m. and the angle α between the Shafts is 20° The driven shaft attached masses has a weight of 55 kg at a radius of gyration of 15 cm.

1- If a steady torque of (20 kg.m) resists rotation of the driven Shaft, find the torque required at the driving Shaft, when $\theta = 45^\circ$.

2- At what value of "α" will be total fluctuation of speed of driven Shaft be limited to 24 r.p.m.

Sol.: ① Torque required at driving shaft $= T'$

$$\frac{d\omega_1}{dt} = -\frac{\omega^2 \cos\alpha \sin 2\theta \sin^2\alpha}{(1-\cos^2\theta \sin^2\alpha)^2}$$

$$= -78.4 \text{ rad/s}^2$$

Torque required to accelerate the driven Shaft $= T_2$

$$T_2 = I * \frac{d\omega_1}{dt} = m k^2 * \frac{d\omega_1}{dt} = 55*(0.15)^2 * -78.4$$

$$= -97.02 \text{ N.m}$$

Total torque required on driven shaft

$$T = T_1 + T_2 = (20*9.81) - 97.02 = 99.18 \text{ N.m}$$

$$\therefore T' \cdot \omega = T \cdot \omega_1 \Rightarrow T' = \frac{T \cdot \omega_1}{\omega} = \frac{T\cos\alpha}{(1-\cos^2\theta \sin\alpha^2)}$$

$$= 99.081 \text{ N.m.}$$

② $$q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$24 = 240\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$$

$$\cos^2\alpha + 0.1\cos\alpha - 1 = 0$$

$$\Rightarrow \alpha = 18.2^\circ$$

Sheet No.(2)

Q.1/ Three shafts A, B and C are supported in bearings and are connected end to end, by Two Hooke's Joints. The axes of A and C are parallel but not in the same line, and axis of B makes angles of 20° with the axes of A and C. The moments of inertia of three shafts, with the rotating parts mounted upon them, are: for A, 0.9 gm^2, B, 0.15 gm^2 and C 1.2 gm^2. If the shafts are set in motion and allowed to rotate freely with out friction, what is the percentage fluctuion of speed of the shafts A and C? Ans.(0.885)

Q2/ Two shafts, the axes of which intersect but are inclined at 20° to each other, are connected by a Hooke's joint. If the driving shaft has a uniform speed of 1000 r.p.m. The driven shaft carries a rotating mass 15 kg, and radius of gyration 250 mm. Find the variation in speed of the driven shaft and the accelerating torque on the driven shaft for the position when the driven shaft has turned 45° from the position in which its fork end is in the plane containing the two shafts. Ans.(126.5 r.p.m.; 12.7 N.m).

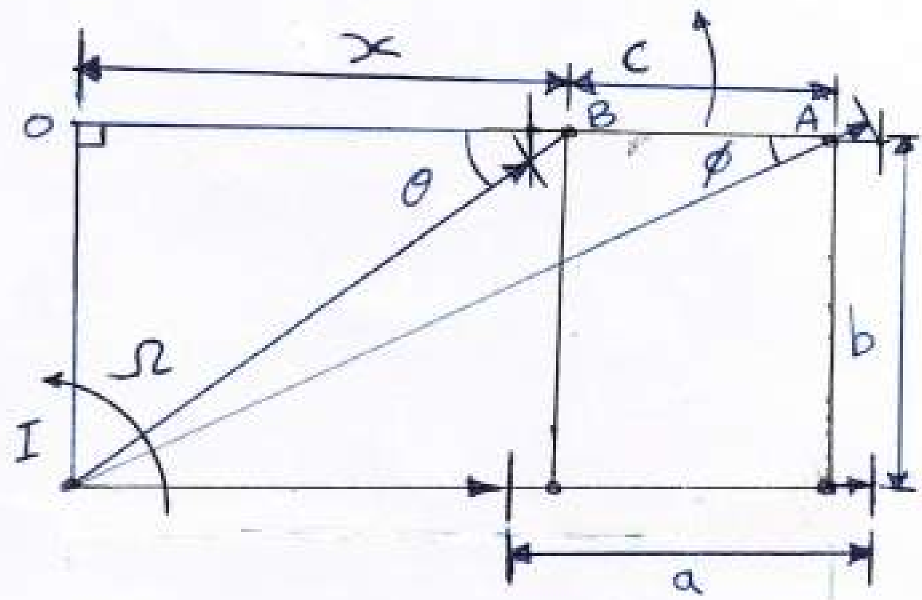
Q3/ Two shafts are connected by a universal joint. The driving shaft revolves uniformly at 500 r.p.m. If the total Permissible variation in speed of the driven shaft is not to exceed ±6% of the mean speed, find the greatest permissible angle between the shafts. Also, determine the max. and min. speeds of the driven shaft.

(Ans. 19.6°; 530, 470 rpm)

3- Steering gear mechanism:

It used for changing the direction of two or more of the wheel axles with interference to chassis, So as to move the automobile in any desired path.

The condition for correct steering is that all the four wheels must turn about the same instantaneous center, otherwise the wheels slip sideways.



If

a = wheel track

b = wheel base

Correct steering.

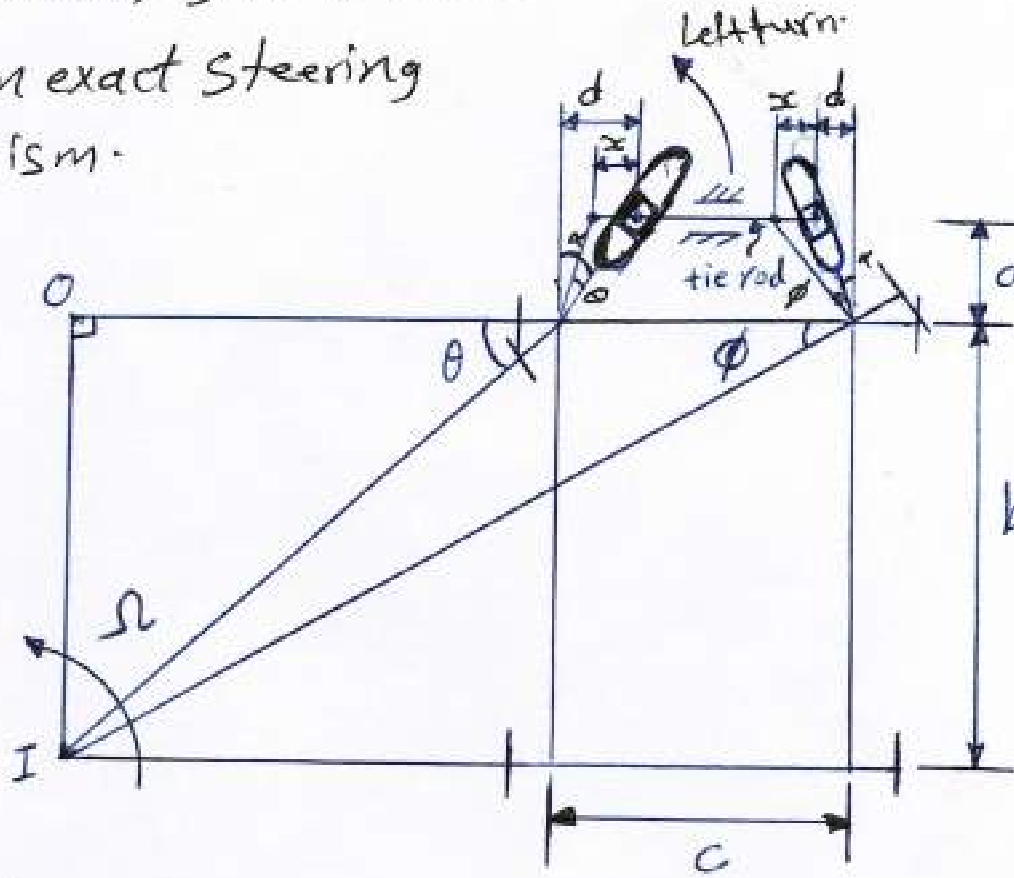
c = Distance between pivots on front axle, or distance between king pins.

$$\cot\theta = \frac{x}{b} \; ; \; \cot\phi = \frac{x+c}{b}$$

$$\therefore \cot\phi - \cot\theta = \frac{c}{b}$$

Davis Steering gear:

The davis steering gear is shown in Fig. It is an exact steering gear mechanism.



If $\tan\alpha = \dfrac{d}{a}$ —— ①

$\tan(\alpha+\phi) = \dfrac{d+x}{a}$ —— ②

$\tan(\alpha-\theta) = \dfrac{d-x}{a}$ —— ③

from above eqs. $\Rightarrow \tan\theta = \dfrac{ax}{a^2+d^2-dx}$ —— ④

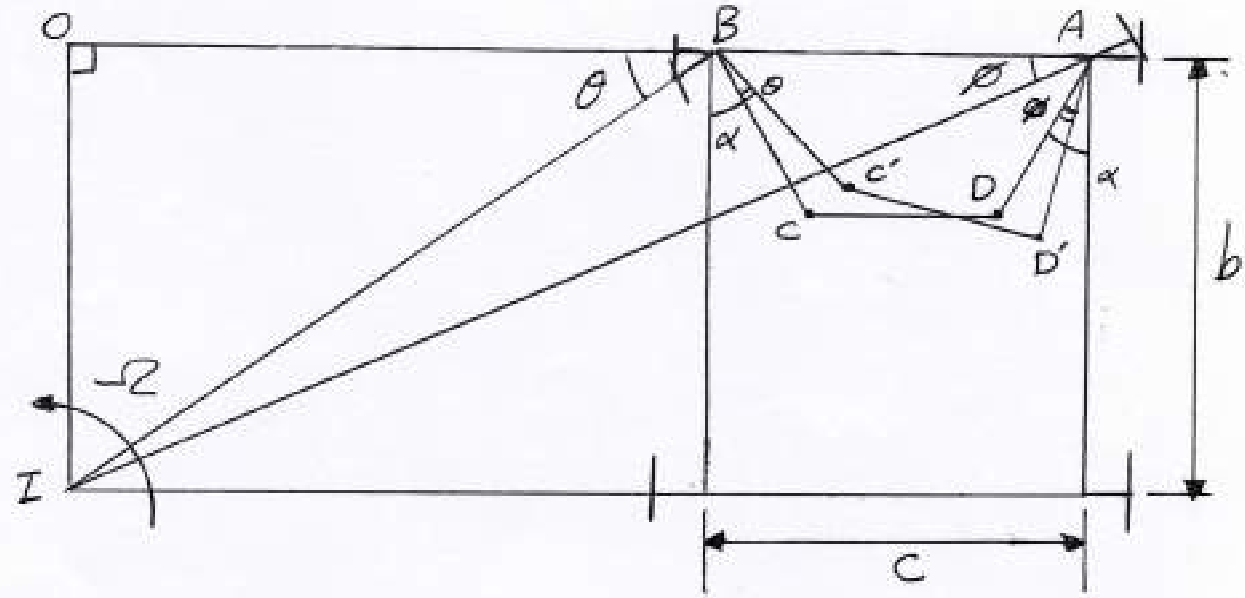
$\tan\phi = \dfrac{ax}{a^2+d^2+dx}$ —— ⑤

from correct steering $\cot\phi - \cot\theta = \dfrac{c}{b}$ —— ⑥

$\Rightarrow \boxed{\tan\alpha = \dfrac{c}{2b}}$ → prove that.

Ackermann Steering gear :-

It is a four bar mechanism in which $\overline{BC} = \overline{AD}$. This gear gives correct steering for three positions only, one when moving straight and two when moving at one correct angle to the right and to the left.

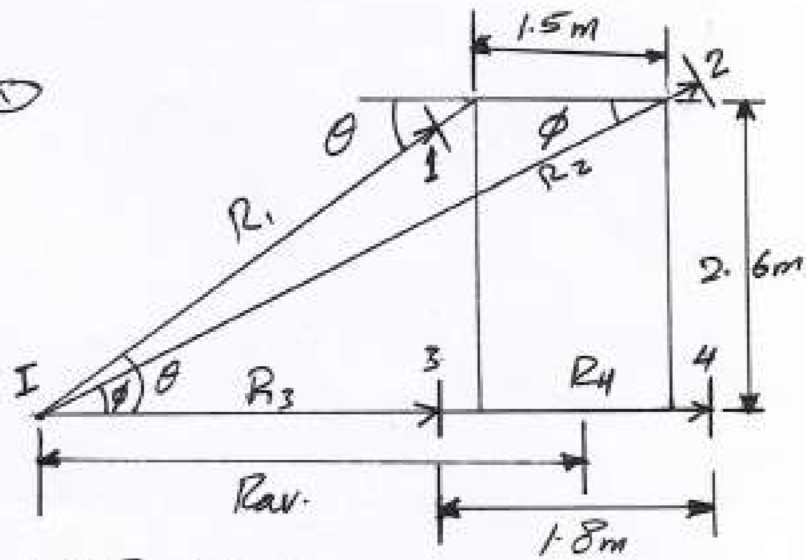


In order to staisfy the fundamental equation for correct steering, the links AD and DC are suitably proportioned. The value of θ and φ may be obtained graphically.

Ex.1 :- An automobile has a correct steering mechanism with king pins (1.5m) apart and wheel base of (2.6m). The auto turns leftwards about an average radius of rotation (10m).

If the wheel track is (1.8 m), determine the radii of rotation of each wheel.

Sol.



$$R_{av.} = \frac{R_3 + R_4}{2} = 10 \quad \text{—— (1)}$$

$$R_4 - R_3 = 1.8\,m \quad \text{—— (2)}$$

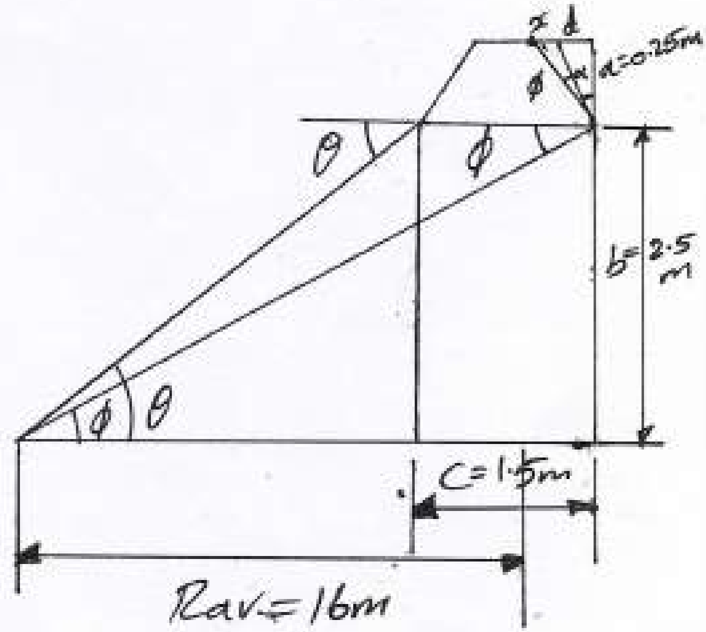
$$\Rightarrow R_3 = 9.1\,m \; ; \; R_4 = 10.9\,m$$

$$\left(R_1 + \frac{1.8-1.5}{2}\right)^2 = 2.6^2 + \left(R_3 + \frac{1.8-1.5}{2}\right)^2 \Rightarrow R_1 = 9.46\,m.$$

$$\left(R_2 - \frac{1.8-1.5}{2}\right)^2 = 2.6^2 + \left(R_4 - \frac{1.8-1.5}{2}\right)^2 \Rightarrow R_2 = 11.21\,m.$$

EX.2:- In a Davis steering mechanism, the distance between the tie rod and the line joining the king pins is (0.25m), the wheel base is (2.5m) and the distance between king pins is (1.5m).

If it is desired to turn leftwards about a radius of rotation of (16m) without slipping of any wheel. What is the horizontal displacement required for the tie rod.



Sol:-

$$\tan\alpha = \frac{c}{2b} = \frac{1.5}{2*2.5} = 0.3$$

$$\alpha = 16.7^\circ$$

$$\tan\alpha = \frac{d}{a}$$

$$d = 0.25 \times 0.3 = 0.075m$$

$$\tan\phi = \frac{b}{R_{av.} + \frac{c}{2}} = \frac{2.5}{16 + \frac{1.5}{2}}$$

$$\Rightarrow \phi = 8.5^\circ$$

$$\tan(\alpha+\phi) = \frac{d+x}{a}$$

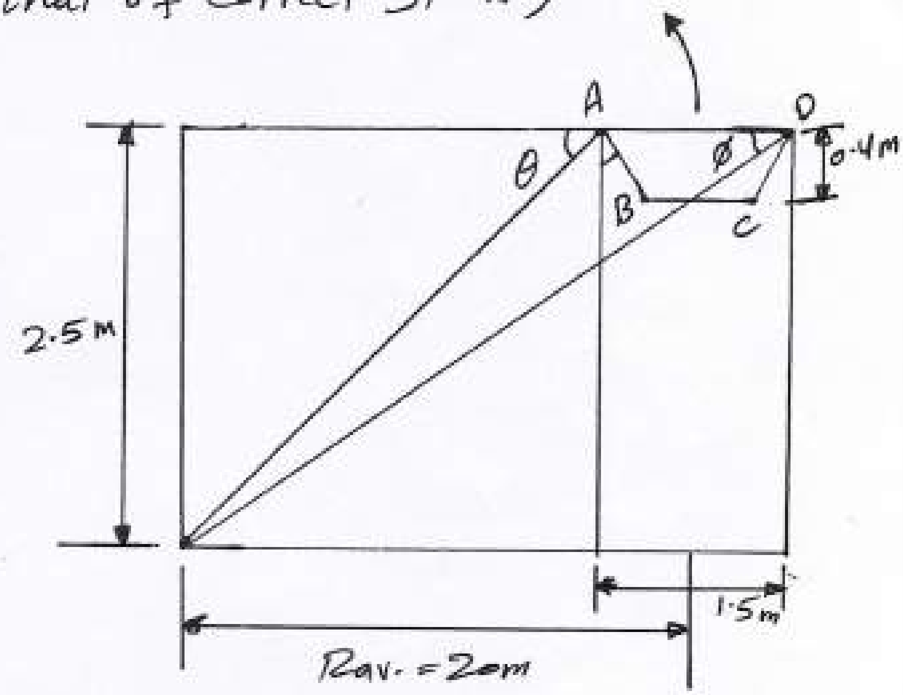
$$x = a\tan(\alpha+\phi) - d = 0.25\tan(16.7+8.5) - 0.075$$

$$= 0.0426m.$$

Ex. 3:- An automobile employs the akermann Steering gear inwhich the distance between the tie rod of (1.2m) length and line joining the King pins is (0.4m). The wheel base is (2.5m) and the distance between King pins is (1.5m).

If the auto is turning leftwards about an average radius of rotation of (20m), what will be the percent deviation of this steering from that of correct steering?

Sol$^{(n)}$:-



$$\theta = \tan^{-1} \frac{2.5}{20 - \frac{1.5}{2}}$$

$$= 7.4^\circ$$

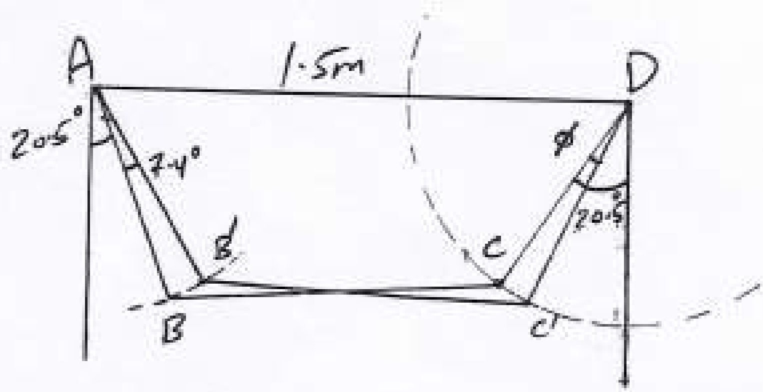
$$\phi = \tan^{-1} \frac{2.5}{20 + \frac{1.5}{2}}$$

$$= 6.9^\circ = \phi_{correct}$$

$$\alpha = \tan^{-1} \frac{1.5 - 1.2}{2 \times 0.4}$$

$$= 20.5^\circ$$

$$\overline{AB} = \overline{CD} = \frac{0.4}{\cos\alpha} = 0.427m$$



From the graph:

$\phi = ----$

deviation of Steering

$\phi_{correct} - \phi$ ... ?

Sheet No.(3)

1- An automobile, of wheel has of (2.8m) and wheel track (1.9m), has a correct steering mechanism with king pins (1.6m) apart. It turns rightwards about an average radius of rotation of (10m) with a speed of (36 km/hr). If the mean diameter of each wheel is (0.6m), determine the angular speed of each wheel.

2. The tie rod in Davis steering mechanism is (1.3m) long, the wheel base is (2.55m) and the distance between king pins is (1.4m)

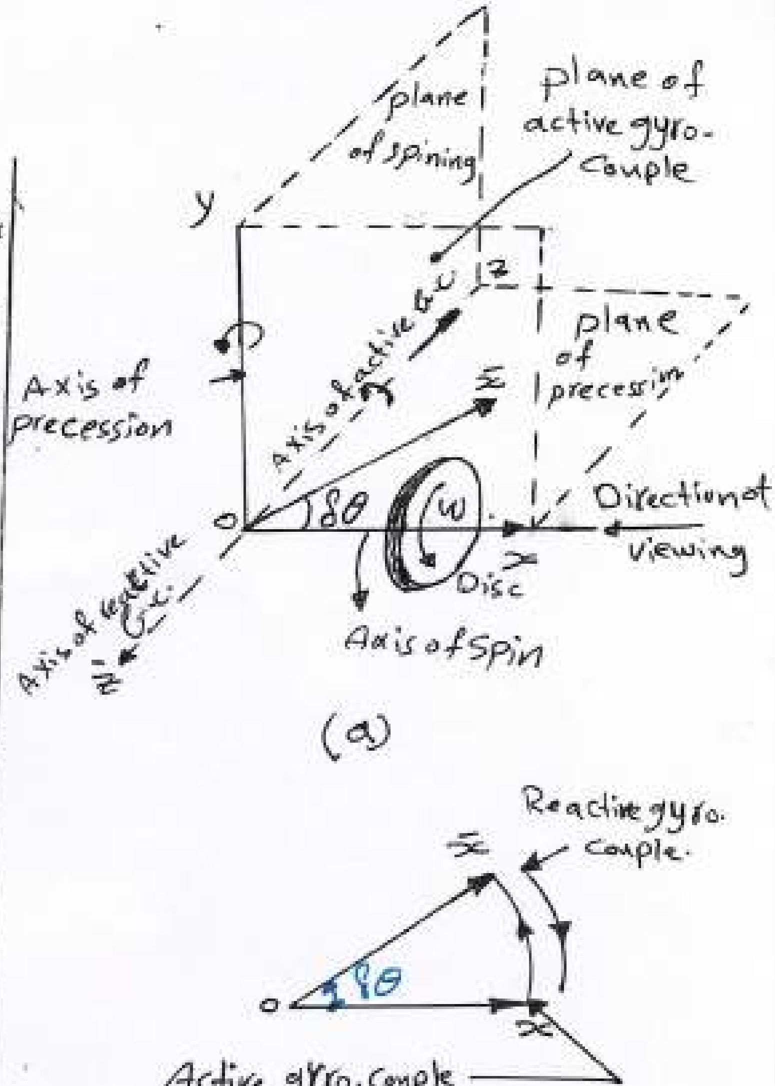
Determine the distance between the tie rod and the line joining the king pins to achieve correct steering. what is the minimum radius of rotation the car can turn if the maximum horizontal displacement of the tie rod is limited to (90 mm).

3. In an auto employing the Ackermann steering gear, the side links make (20°) with the vertical. The wheel base is (2.6m) and the distance between king pins is (1.6m).

If the auto is desired to turn about an average radius of rotation of (25m) without slipping of any wheel, what is the required length of the tie rod.

Gyroscope:-

Let a disc spining with an angular velocity W(rad/s) about the axis of spin OX, Fig.(1.a). Since the plane in which the disc is rotating is parallel to the plane (YOZ), therfore it is called plane of spining. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be processing about axis OY at an angular velocity W_p(rad/s). This plane XOZ is called plane of precession and OY is the axis of precession.



(a)

(b)

Fig(1)

I = Mass moment of Inertia of disc

W = Angular Velocity of the disc.

$\therefore$ Angular momentum of the disc $= I w$.

Change in angular momentum $= \overline{ox'} - \overline{ox} = xx'$

$= \overline{ox} \cdot \delta\theta$

$= I w \cdot \delta\theta$

Rate of change of angular momentum $= I \cdot w \cdot \dfrac{\delta\theta}{dt}$ = Couple applied

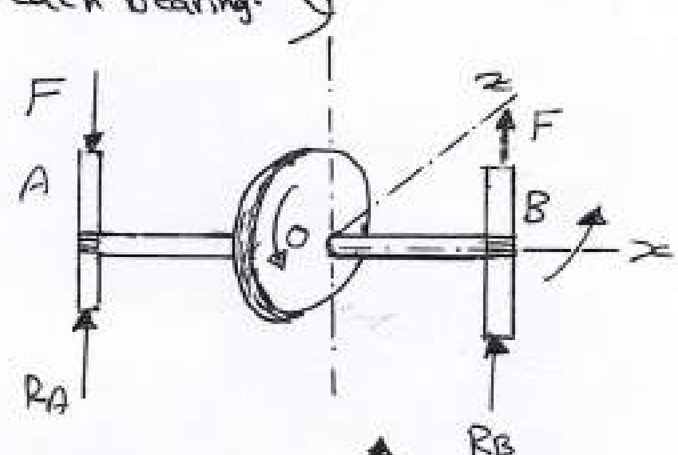
$\therefore C = \lim_{\delta t \to 0} I \cdot w \cdot \dfrac{\delta\theta}{\delta t} = I \cdot w \cdot \dfrac{d\theta}{dt} \quad \left(\because \lim_{\delta t \to 0} \dfrac{\delta\theta}{\delta t} = \dfrac{d\theta}{dt}\right)$

$C = I$

Ex. 1:— A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about its axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotation are as shown in Fig. If the distance between bearings 0.1 m find the resultant reaction at each bearing.

Sol. ①

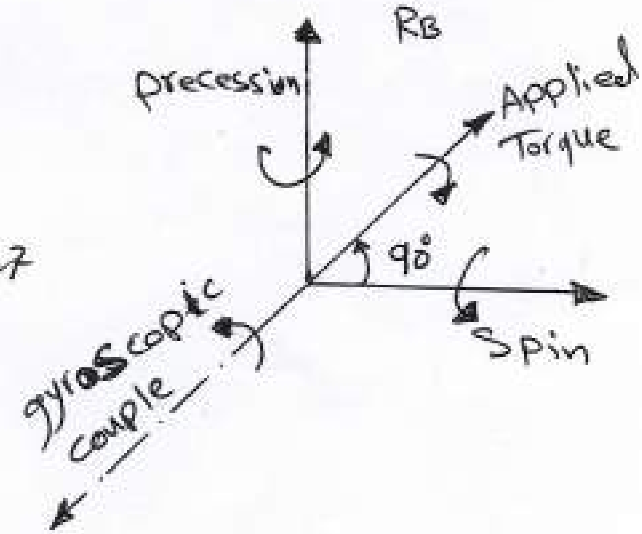
$N = 1000$ r.p.m.



$$w = \frac{2\pi * 1000}{60} = 104.7 \text{ rad/s.}$$

$Np = 60$ r.p.m. $\Rightarrow w_p = \dfrac{2\pi * 60}{60}$

$= 6.28$ rad/s.



$C = I \cdot w \cdot w_p$

$I = mk^2 = 5 * 0.075 = 0.375$ kg.m²

$C = 246.6$ Nm.

$$F = \frac{C}{x}$$

x: distance between bearings.

$$F = \frac{246.6}{0.1} = 2466\ N$$

$\because R_A = R_B = \frac{5}{2} = 2.5 \text{ kg} \times 9.81 = 24.5 N$

$\therefore$ Resultant reaction at each bearing

$R_{A1} = F - R_A = 2466 - 24.5 = 2441.5\ N.$ (downwards)

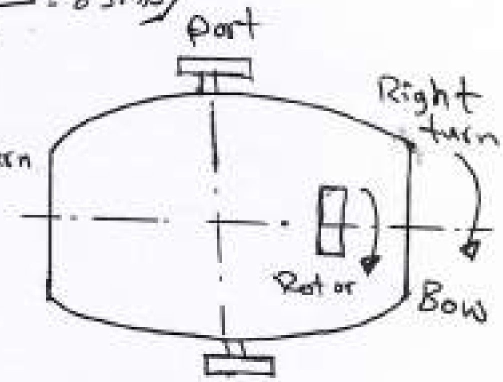
EX.2:— The turbine rotor of a ship has a mass of 2000 kg and rotates at a speed of 3000 r.p.m. clockwise when looking from astern. The radius of gyration of the rotor is 0.5 m. Determine g.c. and its effects upon the ship when the ship is steering to the right in a curve of 100 m radius at a speed of 16.1 knots (1 knot = 1855 m/hour).

Calculate also the torque and its effects when the ship is pitching in simple harmonic motion (S.H.M.). The period of pitching is 50 sec. and the total angular displacement between the two extreme positions of pitching is 12°. Find the maximum acceleration during pitching motion.

Sol:—

$$W = 3000 * \frac{2\pi}{60} = 314.2 \text{ rad/s.}$$

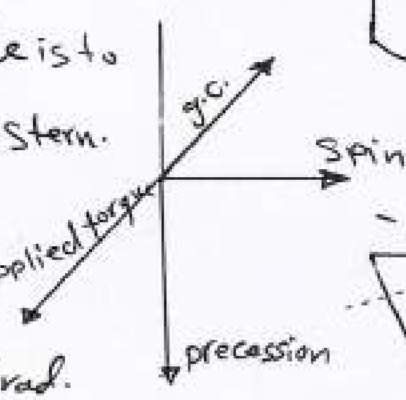
$$W_p = \frac{v}{R} = \frac{8.3}{100} = 0.083 \text{ rad/s.} \quad \left[\frac{16.1 * 1855}{60} = 83 \text{ m/s}\right]$$

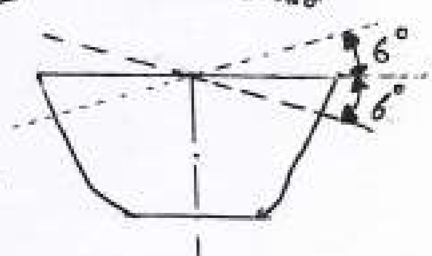


* Gyroscopic Couple:—

$$C = I W W_p = 500 * 314.2 * 0.083 = 13040 \text{ N.m}$$

The effect of gyroscopic couple is to lower the Bow and to raise the Stern.





** Torque during pitching

$$2\phi = 12° \text{ or } \phi = 6° = 6 * \frac{\pi}{180} = \frac{\pi}{30} \text{ rad.}$$

$$C_{max} = I W * W_{p_{max}}$$

$$W_p = \phi\, w_1 \cos w_1 t$$

w_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{t_p}$$

$$W_{p_{max}} = \phi w_1 \quad [\cos w_1 t = 1]$$

$$C_{max} = 500 * 314.2 * \frac{2\pi}{50} * \frac{\pi}{30} = 2042 \text{ N.m.}$$

Since the pitching is downwards, therefore the ship has a tendency to

EX.3 A rear engine automobile is traveling along a track of 100m. Each of the four wheels has a moment of inertia of 2 kg m² and an effective diameter of 0.6m. The rotating parts of the engine have a moment of inertia of 1 kg·m². The engine axis is parallel to the rear axle and the crank shaft rotates in the same sense as the road wheels. The gear ratio of engine to the back wheel is 3 to 1. The vehicle has a mass of 2000 kg and its center of gravity is 0.5m above road level. The width of the track of the vehicle is 1.5m. Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface if it is not combined.

Solⁿ:– Road reaction over each wheel

$$= \frac{W}{4} = \frac{mg}{4} = \frac{2000 \times 9.81}{4} = 4905\,N.$$

Let v = limiting speed of vehicle in m/s.

$\omega_W = \frac{v}{r_W} = \frac{v}{0.3}$ rad/s.

$\omega_P = \frac{v}{R} = \frac{v}{100}$ rad/s.

G.C. due to 4 wheels

$C_W = 4 I_W \omega_W \cdot \omega_P$

$= 4 \times 2 \times \frac{v}{0.3} \times \frac{v}{100} = 0.27 v^2$ N·m

$C_E = I_E \omega_E \cdot \omega_P = 1 \times 3 \times \frac{v}{0.3} \times \frac{v}{100} = 0.1 v^2$ N·m

$\left[G = \frac{\omega_E}{\omega_W}\right]$

Total G.C. $= C_W + C_E = (0.27 + 0.1) v^2 = 0.37 v^2$ N·m

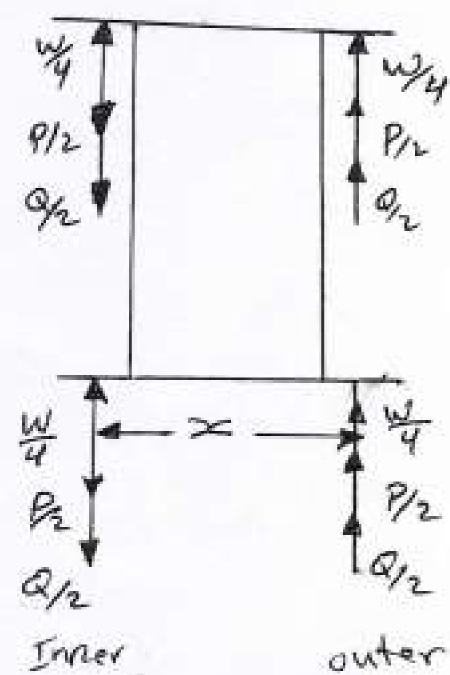
$P \times x = C \Rightarrow P = \frac{C}{x} \Rightarrow \frac{P}{2} = \frac{C}{2x}$

Centrifugal force $F_c = m\frac{v^2}{R} = 2000 \times \frac{v^2}{100} = 20 v^2$ N.

Overturning couple $= F_c \times h = 20 v^2 \times 0.5 = 10 v^2$ N·m

$Q \times x =$ O.T. $\Rightarrow \frac{Q}{2} = \frac{O.T}{2x} = \frac{10 v^2}{2 \times 1.5} = 3.3 v^2$ N.

$P_o = \frac{W}{4} + \frac{P}{2} - \frac{Q}{2}$



$\Rightarrow \frac{P}{2} + \frac{Q}{2} \leq \frac{W}{4}$

$0.123 v^2 + 3.3 v^2 \leq 4905$

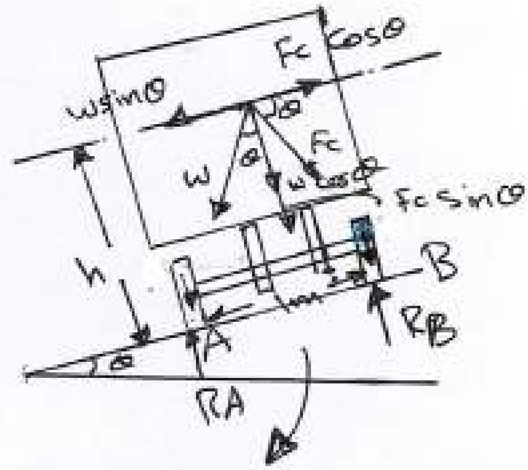
$\Rightarrow v \leq 37.8$ m/s

$v = \frac{37.8 \times 3600}{1000}$

$= 136$ km/hr

EX.4 :- A four-wheeled trolley car of total mass 2000 kg runing on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 Km/hr. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The hieght of center of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

Sol :- $\sum F_y = 0$



$\Rightarrow R_A + R_B = W\cos\theta + F_c \sin\theta$ —①

$= 2000 \times 9.81 \cos 8° + \frac{2000 \times 15^2}{30} \times \sin 8°$

$= 21518 N.$

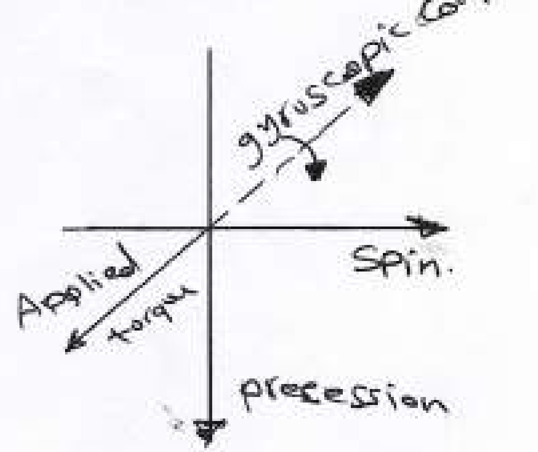
taking moments about B,

$R_A \times x = (W\cos\theta + F_c \sin\theta)\frac{x}{2} + W\sin\theta h - F_c \cos\theta \times h.$ —②

$\Rightarrow R_A = 3182 N.$

sub in eq. ① $\Rightarrow R_B = 18336 N.$

$\omega_w = \frac{v}{r_w} = \frac{15}{0.35} = 42.8 rad/s.$



$\omega_p = \frac{v}{R} = \frac{15}{30} = 0.5 rad/s.$

G.C. $= I \omega_w \cos\theta \omega_p.$

$= 200 \times 0.3^2 \times 42.8 \times \cos 8° \times 0.5$

$= 381.5 N.m$

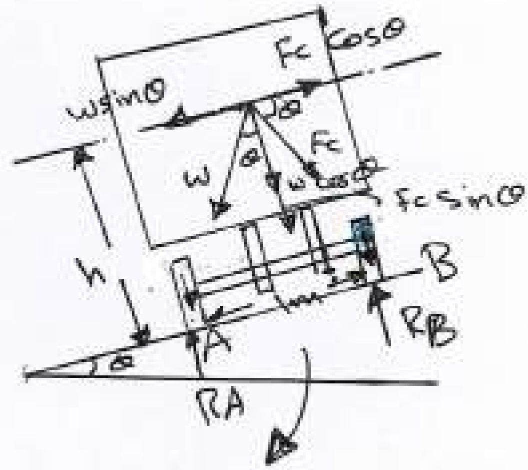
$P \times x = 381.5 \Rightarrow P = \frac{381.5}{1.6} = 238.4 N$

The total pressure on the inner rail

$P_I = R_A - P = 3182 - 238.4 = 2943.6 N$

EX.4:— A four-wheeled trolley car of total mass 2000 kg runing on rails of 1.6m gauge, rounds a curve of 30m radius at 54 Km/hr. The track is banked at 8°. The wheels have an external diameter of 0.7m and each pair with axle has a mass of 200kg. The radius of gyration for each pair is 0.3m. The hieght of center of gravity of the car above the wheel base is 1m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

SoL:— $\sum F_y = 0$



$\Rightarrow R_A + R_B = W\cos\theta + F_c\sin\theta$ — ①

$= 2000 * 9.81 \cos 8° + \frac{2000 * 15^2}{30} * \sin 8°$

$= 21518 N.$

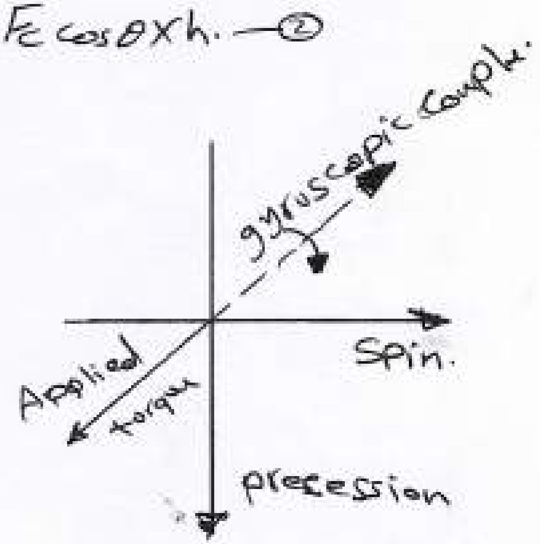
taking moments about B.

$R_A * x = (W\cos\theta + F_c\sin\theta)\frac{x}{2} + W\sin\theta h - F_c\cos\theta \times h.$ — ②

$\Rightarrow R_A = 3182 N.$

sub in eq. ① $\Rightarrow R_B = 18336 N.$

$\omega_w = \frac{v}{r_w} = \frac{15}{0.35} = 42.8 rad/s.$



$\omega_p = \frac{v}{R} = \frac{15}{30} = 0.5 rad/s.$

$G.C. = I\omega_w \cos\theta \omega_p.$

$= 200 * 0.3^2 * 42.8 * \cos 8° * 0.5$

$= 381.5 N.m$

$P * x = 381.5 \Rightarrow P = \frac{381.5}{1.6} = 238.4 N$

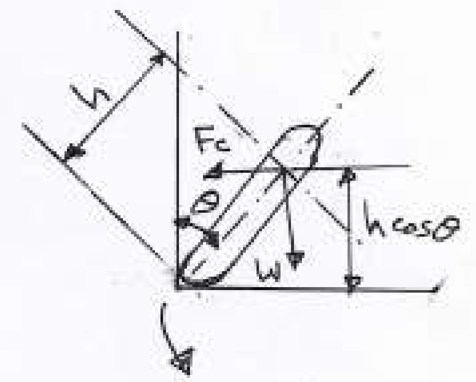
The total pressure on the inner rail

$P_I = R_A - P = 3182 - 238.4 = 2493.6 N$

EX.5:— A motor cycle and its rider together weigh 200 kg and their combined center of gravity is 60 cm above the ground level when the motor cycle is upright. Each road wheel is of 60 cm diameter and has a moment of inertia of 10000 kg cm². The rotating parts of the engine have a moment of inertia of 1700 kg cm². The engine rotates at 5.5 times the speed of the road wheels and in the same sense.

Determine the angle of heel necessary when the motorcycle is rounding a curve of 30 m radius at a speed of 55 km/hr.

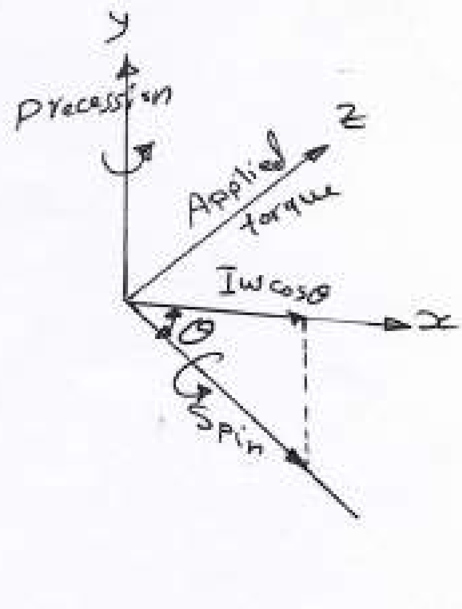
Sol.: Let θ Angle of heel



$$G.C. = \frac{v^2}{R r_W}(2 I_W + G I_E)\cos\theta$$

$$C_1 = \frac{15.3^2}{30*0.3}(2*0.102 + 5.5*0.017)\cos\theta$$

$$= 7.74 \cos\theta \ N.m.$$



Centrifugal Couple, $C_2 = \frac{mv^2}{R} * h \cos\theta$

$C_2 = 936.36 \cos\theta$ N.m.

Total overturning couple $= C_1 + C_2$

$= 936.36\cos\theta + 7.74\cos\theta$ — ①

$= 944.1 \cos\theta$

Balancing couple $= Wh \sin\theta$ — ②

$= 200*9.81*0.6 \sin\theta$

$$\therefore \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{944.1}{1177.2} \Rightarrow \theta = 38.73^\circ. \ \underline{Ans}$$

Sheet No.(4)

Q1/ A horizontal axle AB, 1m long, is pivoted at the mid point C. It carries a weight of 2kg at A and a wheel weighing 5kg at B. The wheel is made to spin at a speed of 600 r.p.m. in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 60cm. Calculate the angular velocity of precession of system around the vertical axis through C.

Q2/ The rotor of a turbine installed in boat with its axis along the longitudinal axis of the boat makes 1500 r.p.m. clockwise when viewed from the stern. The rotor weighs 730 kg and has a radius of gyration 30 Cm. If at an istant the boat pitches in the longitudinal vertical plane so that the bow rises from the horizontal plane with angular velocity of 1 rad/s, determine the torque acting on the boat and the direction in which it tends to turn the boat at the instant.

Q3/ A pair of flanged wheels 1.2 meters in diameter mounted on an axle, rolles along the rails spaced 1.5 meters apart and at the same level. The axle negotiates a curve of 150 m. mean radius at a speed 64 km/hr. Determine the reactions of the rail on the wheel and the horizontal force on the outer rail. Assume that the mass of wheel is 270 kg.

Q4/ The rotor of a traction motor of an electric train has a mass of 450 kg and is supported on two bearings 750 mm apart. The radius of gyration of the rotor is 180 mm and its axis is parallel to the axle of the track wheels. The motor rotates at 1500 r.p.m. When the train negotiates a curve of 200 m radius at 100 km/hr. Determine the magnitude and direction of change in bearing reactions while negotiating the curve.

Q5/ The wheel of motor cycle and the engine parts have a moment of inertia 2.5 $kg.m^2$ and 0.15 $kg.m^2$ respectively. The axis of rotation of the engine crank shaft is parallel to that of the road wheels. If the gear ratio is 5:1 and the diameter of the road wheels is 0.65 m, find the magnitude and direction of the gyroscopic couple when the motor cycle rounds a curve of 30 m radius at 16 m/s.

5. Turning Moment Diagrams and Flywheel :-

The turning moment diagram (crank-effort diagram) is the graphical representation of the turning moment or crank effort for various positions of the crank.

1. Turning Moment Diagram for Single Cylinder Double Acting Steam Engine:-

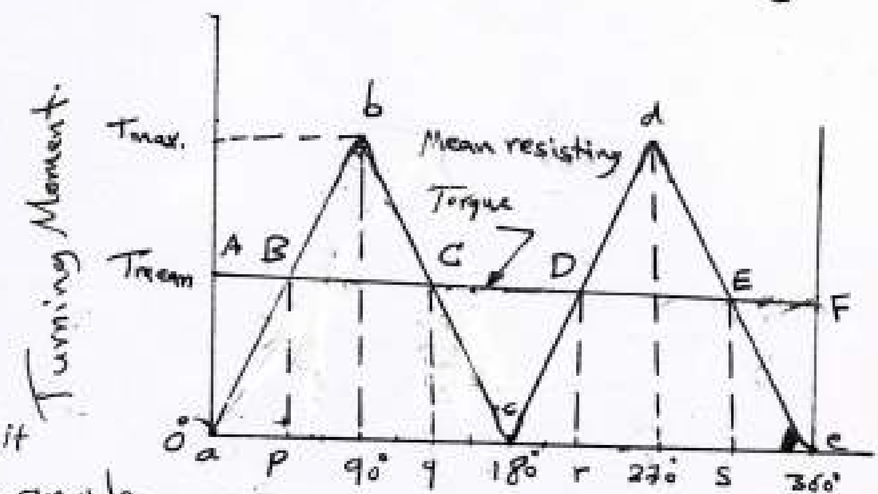


Fig. (1)

The value of turning moment shown in Fig. (1) is zero when the crank angle is zero. It rises to a maximum valu when crank angle reaches (90°) and it is again zero when crank angle is (180°). This is shown in curve abc and it represents the turning moment diagram for outstroke. The curve (cde) is the turning moment diagram for instroke.

The workdone is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. The height of the ordinate aA represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle aAFe is proportional to the work done against the mean resisting torque.

5.2. Turning Moment Diagram for a Four Stroke Cycle:-

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig.(2). In a four stroke cycle there is one working stroke after the crank has turned through 720°. The positive and negative loop is obtained as shown in Fig.(2)

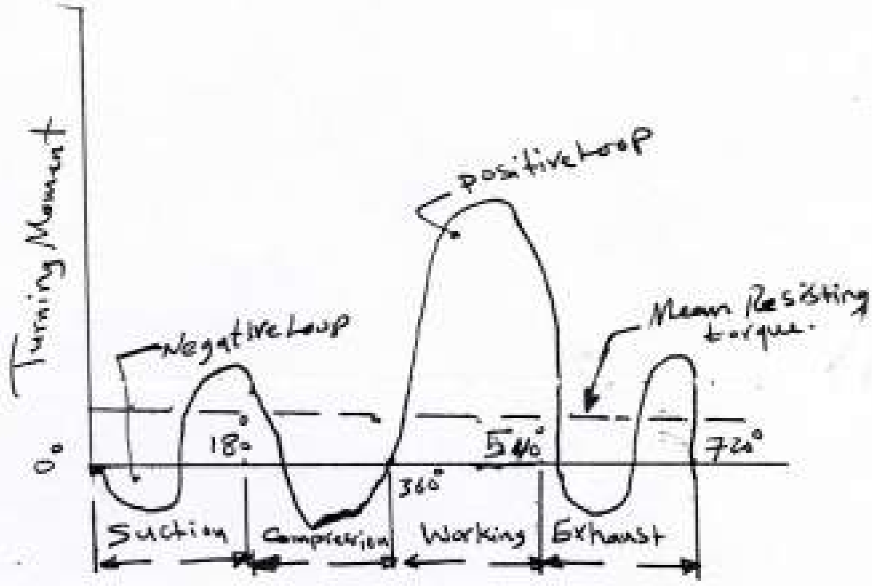


Fig.(2)

5.3. Maximum Fluctuation of Energy:-

A turning moment diagram for a multi-cylinder engine shown in Fig.(3). The horizontal line AG represents the mean torque line. Let $a_1, a_2, \ldots, a_6$ be the areas above and below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

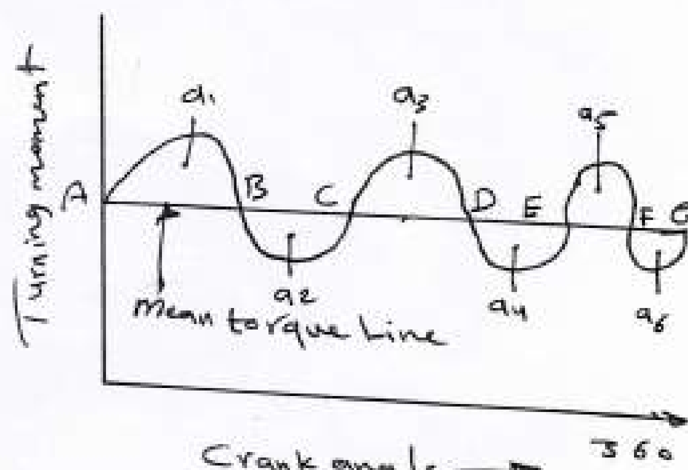


Fig.(3).

Let E the energy in the fly wheel at A.

Energy at B $= E + a_1$

Energy at C $= E + a_1 - a_2$

Energy at D $= E + a_1 - a_2 + a_3$

Energy at E $= E + a_1 - a_2 + a_3 - a_4$

Energy at F $= E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at G $= E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6 =$ Energy at A.

Let us now suppose that the greatest of these energies at B and least at E. Therefore.

Maximum energy in the flywheel $= E + a_1$

Minimum energy in the flywheel $= E + a_1 - a_2 + a_3 - a_4$

Maximum fluctuation of energy = Max. energy − Min. energy

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4)$$

$$= (a_2 - a_3 + a_4)$$

* Cofficient of fluctuation of Energy: (C_E)

$$C_E = \frac{\text{Maximum fluctation of energy}}{\text{Workdone per cycle.}}$$

Workdone per cycle $= T_{mean} \times \theta$

θ: Angle turned (in radians), in one revolution

5.4. Flywheel :-

A Flywheel is a device to control the variations in speed during each cycle of engine. It serves as a reservoir, which stores energy during the period when the supply of energy is more the requirement, and releases it during the period when the requirement of energy is more than the supply. The speed of flywheel increases when it absorbs energy and decreases when it releases energy. In this way, the flywheel keeps the speed of the engine whitin prescribed limits during each cycle.

* Cofficient of Fluctuation of Speed :- (C_s)

Let N_1 and N_2 = Max. and Min. Speeds in r.p.m. during the cycle

$$N_{mean} = \frac{N_1 + N_2}{2}$$

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

** Energy Stored in a Flywheel :-

Let: m = mass of flywheel.

k = Radius of gyration.

I = Mass moment of inertia.

Kinetic energy of flywheel, $E = \frac{1}{2} I \omega^2$

Max. fluctuation of energy e = Max. K.E − Min. K.E.

$$e = \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$$

$$= I\omega(\omega_1 - \omega_2) = I\omega^2 C_s = 2EC_s$$

5.5 Dimentions of the flywheel:—

Consider a rim of the flywheel as shown.

D = Mean diameter of rim.

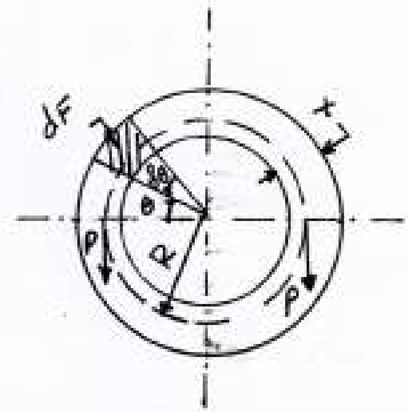
A = Cross-sectional area of rim.

ρ = Density of rim material.

w = Angular velocity of the flywheel.

v = Linear velocity at the mean radius.

f = Centrifugal or hoop stress.



Volume of the small element $= A \times R\delta\theta$

$$dW = \rho A R \delta\theta \quad \text{(weight of element)}$$

Centrifugal force $$dF = \frac{dW}{g} w^2 R = \frac{\rho A R \delta\theta . w^2 R}{g}$$

$$= \frac{\rho A R^2 w^2}{g} \delta\theta$$

Vertical component of $dF = dF \sin\theta = \frac{\rho A R^2 w^2}{g} \delta\theta \times \sin\theta$

$\therefore$ Total vertical force $$= \frac{\rho A R^2 w^2}{g} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{2\rho A R^2 w^2}{g}$$

$\Sigma F_y = 0 \Rightarrow 2P = 2fA$

$$\frac{2\rho A R^2 w^2}{g} = 2fA \Rightarrow f = \frac{\rho}{g} w^2 R^2 = \frac{\rho}{g} v^2$$

$$v = \sqrt{\frac{f \times g}{\rho}} = \frac{\pi D N}{60}$$

Ex 1:- A horizontal cross compound steam engine develops 400 H.p. at 900 r.p.m. The coefficient of fluctuation of energy as found from the turning moment diagram is (0.1) and speed is to be kept within 0.5% of the mean speed. Find the weight of the flywheel required, if the radius of gyration is 2m.

Sol: w = mean speed $= \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 3\pi$ rad/s

fluctuation of speed $= w_1 - w_2 = 0.01\,w = C_s$

Work done per cycle, $= \frac{P \times 4500}{N} = \frac{400 \times 4500}{90}$

$= 20 \times 10^3$ kg.m

Max. fluctuation of energy, e = work done $\times C_E$

$= 20 \times 10^3 \times 0.1$

$= 2 \times 10^3$ kg.m

$$e = \frac{W}{g} k^2 w^2 C_s$$

$$W = \frac{g \cdot e}{k^2 w^2 C_s} = \frac{2 \times 10^3 \times 9.81}{2^2 \times (3\pi)^2 \times 0.01}$$

$= 5500$ kg

EX.2: A machine has to carry out punching at the rate of 10 holes per minute. It does 52 Kgm of work per sq.cm of sheared area in cutting 2.5cm holes in 2 cm thick plates. A flywheel fitted to the machine shaft which is driven by a constant torque, fluctuates in speed between 180 and 200 r.p.m. The actual punching takes 1.5 seconds. The friction losses are equivalunt to $\frac{1}{6}$th of the work done during punching. Find (1) Horsepower required to drive the punching machine (2) weight of the flywheel (raduis of gyration is 50cm).

Sol. Sheared area per hole $= \pi d t = \pi \times 2.5 \times 2 = 15.71\ cm^2$

Total energy required per hole $= 52 \times 15.71 + \frac{1}{6} \times 52 \times 15.71$

$= 953$ Kg.m

Energy required for punching $= 953 \times 10 = 9530$ Kg.m/min

Horse power required to drive the punching machine

$$= \frac{9530}{4500} = 2.11\ h.p.$$

Since the time required to punch a hole is 1.5 second therefore the energy supplied by the motor in 1.5 second

$= 2.11 * 75 \times 1.5 = 237.4$ Kg.m

Energy supplied by the flywheel during punching of one hole

$= 953 - 237.4 = 715.6$ Kg.m = max. fluctuation of energy

$$\therefore e = \frac{\pi^2}{900} \frac{W}{g} k^2 N (N_1 - N_2)$$

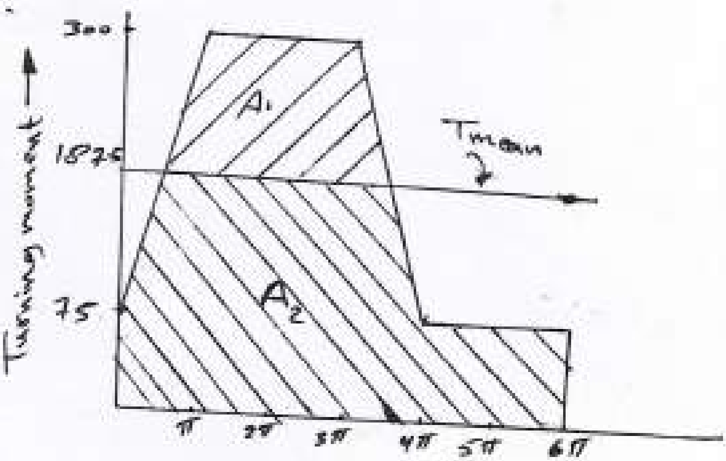
$$\Rightarrow W = 674\ Kg.$$

EX-3: A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 75 kg.m to 300 kg.m uniformly during $\frac{1}{2}$ revolution and remains constant for the following revolution. It then falls uniformly to 75 kg.m during the next $\frac{1}{2}$ revolution and remains constant for one revolution, the cycle being repeated thereafter.

Determine (1) the horsepower required to drive the machine

(2) the percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the flywheel weighs 500 kg with radius of gyration of 600 mm.

Sol:



Work done per one cycle = Area under the curve

$= A_1 + A_2$

$= 6\pi \times 75 + \frac{1}{2} \times \pi \times 225 + 2\pi \times 225 + \frac{1}{2} \times \pi \times 225 = 1125\pi$ kg.m

Work done = $T_{mean} \times \theta$

$\Rightarrow T_{mean} = \frac{1125\pi}{6\pi} = 187.5$ kg.m

$$H.p. = \frac{2\pi N\, T_{mean}}{4500} = 65.46\ h.p$$

Since $A_1 = e = 2(\frac{1}{2} \times 0.5\pi) + 2\pi \times 112.5$ = fluctuation of energy.

$= 708.4$ kg.m

$$e = \frac{W}{g} k^2 w^2 C_s \Rightarrow C_s = 0.056$$

$$\frac{w_1 - w_2}{w} = 5.6\%$$

and percentage fluctuation of speed $= \pm 2.81$ of

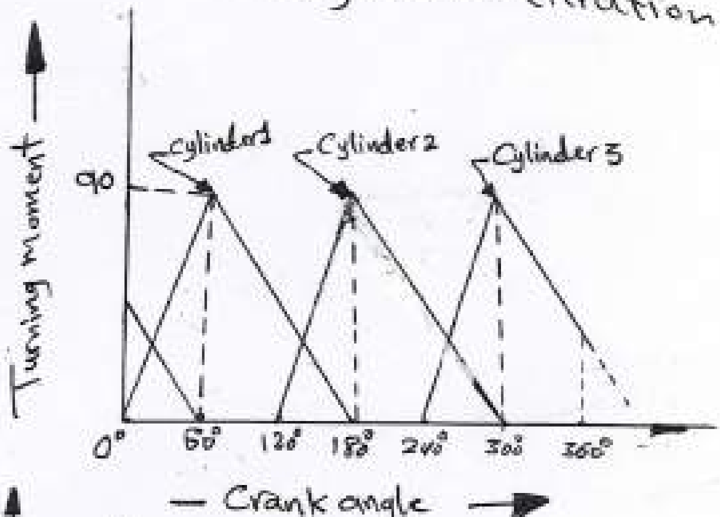
Ex. 4:- A three cylinder single action engine has its cranks set equally at $120°$ and it runs at 600 r.p.m. The torque crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque of 90 Nm at $60°$ from the dead centere of corresponding crank. The torque on the return stroke is senibly zero. Determine ① power developed ② coefficient of fluctuation of speed, if the mass of flywheel is 8 kg and radius of gyration of 60 mm, ③ coefficient of fluctuation of energy and ④ maximum angular acceleration of the flywheel.

Sol:-

1. Power developed :-

$$\text{Work done} = 3 \times \frac{\pi}{2} \times 90 = 424 \text{ N.m}$$

$$\text{power} = 424 \times \frac{600}{60} = 4240 \text{ W}$$



2. Coefficient of fluctuation of speed.

$$T_{mean} = \frac{\text{Work done}}{\text{Crank angle}} = \frac{424}{2\pi} = 67.5 \text{ N.m}$$

Let energy at $A = E$

Energy at $B = E - \frac{22.5}{2} \times \frac{\pi}{6} = E - 1.875\pi$

" $C = E - 1.875\pi + \frac{22.5}{2} \times \frac{\pi}{3} = E + 1.875\pi$

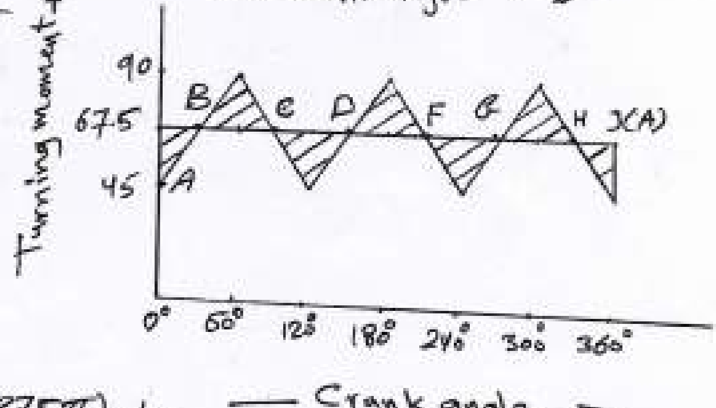
E at $D = E - 1.875\pi$

" $F = E + 1.875\pi$

" $G = E - 1.875\pi$

" $H = E + 1.875\pi$

" $J = E$



Maximum fluctuation of energy

$$e = (E + 1.875\pi) - (E - 1.875\pi) \text{ N.m}$$
$$= 3.75\pi = 11.78 \text{ N.m}$$

$$e = I\omega(\omega_1 - \omega_2) \Rightarrow \omega_1 - \omega_2 = 6.6 \text{ rad/s}$$

$$C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.105$$

3. Coefficient of fluctuation of energy

$$C_E = \frac{\text{Max. fluctuation of energy}}{\text{Work done}} = \frac{11.78}{424} = 0.0278$$

4. Maxim...

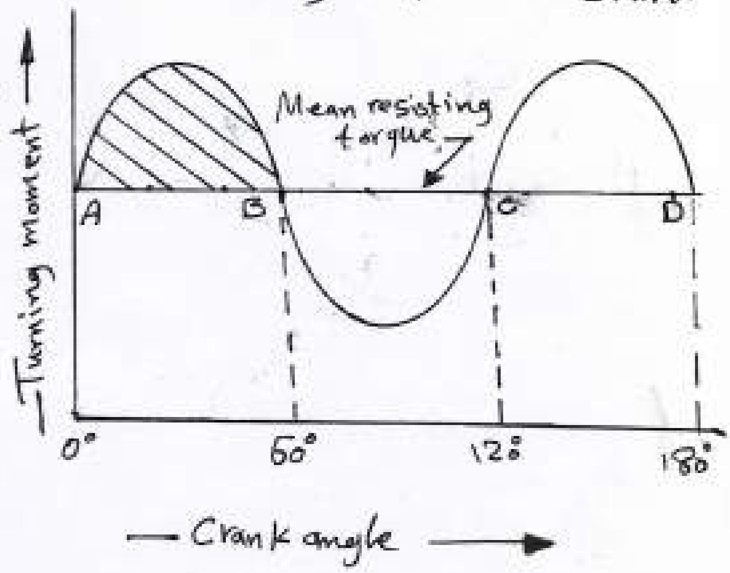
Ex. 5:– The equation of the turning moment curve of a three crank engine is $500 + 150\sin 3\theta$ kgm, where θ radians is the crank angle. The moment of inertia of the flywheel is $1000\ \text{kgm}^2$ and the mean engine speed is 300 r.p.m. Calculate:
(a) The h.p. of the engine and (b) The total fluctuation of speed of the flywheel in percentage if the resisting torque is constant.

Sol: Work done per revolution

$$= \int_0^{2\pi} (500 + 150\sin 3\theta)\, d\theta$$

$$= \left[500\theta - \frac{150}{3}\cos 3\theta\right]_0^{2\pi}$$

$$= 1000\pi \text{ kg.m}$$



$$H.P = \frac{1000\pi \times 300}{4500} = 209.46 \text{ h.p.}$$

$$T_{mean} = \frac{\text{Workdone}}{2\pi} = \frac{1000\pi}{2\pi}$$

$$= 500 \text{ kg.m}$$

$\because$ Resisting torque is constant $\Rightarrow T = T_{mean}$

$$500 + 150\sin 3\theta = 500$$

$$\Rightarrow \sin 3\theta = 0 \Rightarrow 3\theta = 0^\circ \text{ or } 180^\circ$$

$$\theta = 0^\circ \text{ or } 60^\circ$$

Maximum fluctuation of energy

$$e = \int_0^{60^\circ} (T - T_{mean})\, d\theta$$

$$= \int_0^{60^\circ} (500 + 150\sin 3\theta - 500)\, d\theta$$

$$= \left[-\frac{150\cos 3\theta}{3}\right]_0^{60}$$

$$= 100 \text{ kg.m}$$

$$e = I.\omega^2 C_s \Rightarrow C_s = \frac{e}{I.\omega^2} = \frac{100}{(1000/9.81) \times 31.42^2}$$

$$C_s = 0.001 \text{ or } 0.1\%$$

Ex. 6:- A single cylinder double acting steam engine develops 150 KW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean speed. If the mean diameter of the flywheel rim is 2 m and the hub and spokes provide 5% of the rotational inertia of the flywheel, find the mass and cross sectional area of the flywheel rim, if the density of the flywheel is 7200 kg/m³.

SoL:- $C_S = \frac{w_1 - w_2}{w} \Rightarrow w_1 - w_2 = 0.04\, W$

work done per cycle $= \frac{P \times 60}{N} = \frac{150 \times 10^3 \times 60}{80}$

$= 112.5 \times 10^3\ Nm$

Max. fluctuation of energy (e)

$e = C_E \times \text{work done}$

$= 0.1 \times 112.5 \times 10^3 = 11.25 \times 10^3\ Nm$

Mass of the flywheel rim: $e = E\, 2C_s \Rightarrow E = \frac{11.25 \times 10^3}{2 \times 0.04} = 140\,625\ N.m$

Since 5% of the rotational inertia is to be provided by the hub and spokes $\Rightarrow E_{rim} = 0.95E = 0.95 \times 140625 = 133594\ Nm$

$E_{rim} = \frac{1}{2} I w^2 = \frac{1}{2} m k^2 w^2$

$\therefore m = \frac{2 \times E_{rim}}{k^2 w^2} = \frac{2 \times 133594}{1^2 \times 8.4^2} = 3790\ kg$

weight of the flywheel $W = mg = A \times 2\pi \times R \times \rho \times g$

$\Rightarrow A = \frac{m}{2\pi R \rho} = \frac{3790}{2\pi \times 1 \times 7200} = 0.083\ m^2$

Sheet no.( )/ Flywheel - Theory of machines

1- A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10000 J of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find (a) the number of rivets that can be closed per hour, and (b) the reduction in speed after the riveting operation is over.

2- Two stroke engine, in forward stroke, the turning moment has the maximum value of 200 N.m when the crank makes an angle of 80° and during the backward stroke, the maximum value is 150 N.m when the crank makes 80° with the outer dead center. The turning moment diagram for the engine may be assumed for simplicity to be represented by two triangles.

If the crank makes 100 r.p.m. and the radius of gyration of flywheel is 1.75 m, find the coefficient of fluctuation of energy and the weight of the flywheel to keep the speed within ± 0.75% of the mean speed. Also determine the crank angles at which the speed has its minimum and maximum values.

3. The turning moment diagram for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows:

Expansion stroke = 35.5 cm^2; exhaust stroke = 5 cm^2; Suction = 3.5 cm^2 and Compression = 14 cm^2 - Each cm^2 represents 30 kg m.

Assuming the resisting moment to be uniform, find the weight of a flywheel required to keep the mean speed 200 r.p.m within ± 2%. The mean radius of gyration of flywheel 0.75 m. Also find the positions of crank at which the value of speed maximum and minimum.