University of Anbar
College of Engineering
Department of Electrical Engineering

Lecture Notes in:

Fundamentals of Analog Communications and Noise

for the classes EE3208

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الثقافة هي ما يبقى بعد أن ننسى كل ما تعلمناه في المدرسة
"Education is what remains after one has forgotten what one has learned in school." Albert Einstein

2021-2022

References

- *Introduction to Communication Systems*, Ferrel G. Stremler.
- Instructor's Lectures.

Bibliography

[1] *Modern Digital and Analog Communication Systems*, B. P. Lathi.

[2] Communication Systems, *an Introduction to Signals and Noise in Electrical Communications*, Bruce Carlson.

[3] Communication Systems Engineering, John G. Proakis and Masoud Salehi.

[4] *Digital and Analog Communication Systems*, Leon W. Couch.

Course Outline

Week	First Semester: Class EE3?28
1	Introduction to Communication System, Channel, Signal Classification and Characteristics
2	Introduction to Waves Propagation, Multipath
3	Introduction to Fourier Transform, Filters & Bandwidth
4	Analog Communications, Amplitude Modulation Systems, AM-DSB-SC (Mod./Demod.)
5	AM-DSB-LC (Modulation-Demodulation)
6	AM-SSB & AM-VSB (Modulation-Demodulation)
7	FDM, Frequency Conversion, Super-Heterodyne Receiver
8	FM: Introduction, NBFM, WBFM
9	Spectrum Plotting Using Bessel Function, Power in FM
10	FM Generation: Direct (VCO) and Indirect Method (Armstrong)
11	FM Detection: Discriminator, Zero Crossing Detector, PLL
12	Introduction, Noise Sources, Mathematical Representation of Noise, Noise Figure
13	Thermal Noise, White & Filtered Noise, Equivalent Temperature, Noise in Multistage System
14	Noise & SNR in AM (DSB / SSB / normal AM), FM, Noise Reduction in FM Using Emphasis

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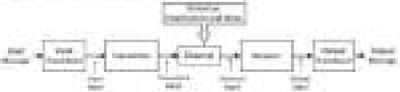
Part 1 Introduction to Communication Systems

A typical Communication system can be modeled as:



The source originates the message. If the message is non-electrical (such as human voice or a television picture), it must be converted by an input transducer into electrical waveform referred as the baseband signal.

For efficient transmission, the transmitter processes and amplifies the input signal to produce a transmitted signal suited to the characteristics of the transmission channel. Signal processing involves Modulation and Coding.

The channel is the medium that bridges the distance from source to destination, such as wire, coaxial cable, optical fiber, radio link, ...etc.

Generally, the signal passes a channel suffers from the following main problems:

(1) Attenuation: the signal power gradually decreases along the distance.
(2) Distortion: the channel changes the shape of the signal.
(3) Noise: the transmitted signal is corrupted by unwanted, random, and unpredictable electrical signals.
(4) Interference: is the contamination by extraneous signals, such as, other transmitters, power lines and machinery, switching circuits, ...etc.

The receiver compensates the received signal for the channel effects and reprocesses the received signal by undoing the signal modification made at the transmitter through the demodulation and the decoding. The receiver output is fed to the output transducer to convert the electrical signal to its original form as the message.

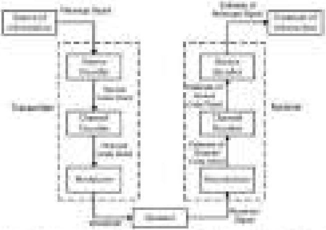
The basic objective of a communication engineer is to design and implement a maximum efficiency system which can send and receiving data with minimum required resources (power, frequency band, time, and cost) beside many other issues that need to be considered (such as data security, reliability, immunity to noise, lower cost of weight, size, etc.).

1.1 Basic Definitions

An Analogue signal is defined as a physical time varying quantity and is usually smooth and continuous e.g. acoustic pressure variation when speaking. The performance of an analogue communication system is often specified in terms of its fidelity or quality.

A digital signal on the other hand is made up of discrete symbols selected from a finite set, e.g. letters from the alphabet or binary data. The performance of a digital system is specified in terms of accuracy of information e.g. Bit Error Rate (BER) and Symbol Error Rate (SER).

The main elements of a digital system are shown in below diagram:



The source encoder removes redundant information from the message signal to facilitate the transmission. The opposite is true for the channel encoder. It adds redundant data to the transmission to provide the capability of the error correction and control at the receiver. Finally, the modulation represents each symbol of the channel code word by an analog symbol

The sequence of analog symbols is called a waveform, which is suitable for transmission over the physical channel.

At the receiver, the channel output (the received signal) is processed in the order reversed to that of the transmitter, thereby reconstructing a recognizable version of the original message signal to be finally delivered to the user.

From this description, it is apparent that the design of a digital communication system is rather complex, but nowadays electronics are inexpensive due to the ever increasing availability of VLSI circuits in the form of silicon chips. Besides being easy to build, digital communications offer greater tolerance of physical effects (e.g., temperature variations, aging, mechanical vibrations) than its analog counterparts.

Advantages of digital communication systems:

(1) Relatively inexpensive digital circuits may be used.
(2) Privacy is preserved by using data encryption.
(3) Voice, video, and data sources may be merged and transmitted over a common digital system.
(4) Errors can often be corrected using coding.
(5) It is easy to regenerate the transmitted signal to be able to extend the receiver distance. For example, in regenerative repeaters.

Digital communications also has disadvantages:

- Generally more bandwidth is required than that for analog systems.
- Synchronization is required.

1.2 Synchronous/Asynchronous Communications

Synchronous (clocked) transmission: A synchronous system is one in which the transmitter and receiver are operating simultaneously at the same number of symbols per second in the desired phase relationship.

Asynchronous (non-clocked) transmission: In an asynchronous system, no rigid timing constraint is applied between the transmitter and the receiver.

Advantages of Synchronous data communications:

- Superior noise immunity due to matched filtering; that is, the system as bit is averaged over the entire duration giving good noise immunity and interference rejection and maintaining signal power.
- Can accommodate higher data rates than asynchronous systems.

Disadvantages of synchronous data communications:

- Requires extra time for synchronization to occur.
- Is more expensive and complex than asynchronous systems.

1.3 Power Levels of Signals and Decibels

The decibel is a measure of signal level and its change. We use this logarithmic measure or its variants in the telecommunications network for many purposes, for example, to express the mean level of the transmission and reception power of radio systems such as mobile telephones or an optical line system.

1.3.1 Decibel, Gain, and Loss

Along the long-distance communication connection or channel, the power of the signal is reduced and amplified over and over again. The signal power needs to be rigidly controlled to keep it high enough in relation to background noise and low enough to avoid system overload and resulting distortion.

The reduction of signal strength, loss or attenuation, is expressed in terms of power loss. When the signal is regained, this is expressed in terms of power gain. Thus the absolute gain of loss corresponds to the loss of 1/10.

Alexander Graham Bell was the first to use logarithmic power measures. This was found to be handy and the unit for power gain was named to Bell's honor as decibel (dB). The gain in decibels is defined as follows:

$$\text{Gain or Loss} = P = \frac{\text{Measured Power}}{\text{Reference Power}} = \frac{P_{out}}{P_{in}} = \frac{P(2)}{P(1)}$$

$$G_{dB} = 10\log_{10}(P)$$

- If $P > 1$, it is gain, we label it as G.
- If $P < 1$, it is loss; we label it as L.
- If $P = 1$, i.e., the output = input power, then G_{dB} and L_{dB} is [illegible] 0 dB.

So, if an amplifier makes an output signal 100 times stronger than its input, then it has gain of:

$$G_{dB} = 10\log_{10}\left(\frac{100 P_1}{P_1}\right) = 20\,dB$$

Also, if a signal reduced to 1/100 of its original strength when passing through a cable. Then the cable has dB loss (negative gain) of:

$$G_{dB} = 10\log_{10}\left(\frac{1}{100}\right) = -20\,dB$$

The figure below presents an element in a telecommunications network with a certain input power and an output power. The formulas of loss (attenuation) and gain are given in the figure as well.

Gain | Gain in dB

$g = P_{out}/P_{in}$ | $G_{dB} = 10\log_{10} P_{out}/P_{in}$

Loss | Loss in dB

$L = P_{in}/P_{out}$ | $L_{dB} = 10\log_{10} P_{in}/P_{out}$

In telecommunications systems we usually have many elements in cascade:

Resulting gain: $g = g_1 \cdot g_2$

Resulting gain in dB: $G_{dB} = G_1 + G_2$

Note that the decibel is the measure of power gain/ratio. If we are interested in how voltage level changes, the impedances must be considered. The voltage and power gains are the same only if the impedances at the points where the power and voltage are measured are the same. The following formula gives the power gain in terms of input and output voltages and impedances as follows:

$$G_{dB} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log_{10}\left(\frac{V_{out}^2/Z_{out}}{V_{in}^2/Z_{in}}\right) = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) + 10\log_{10}\left(\frac{Z_{in}}{Z_{out}}\right)$$

The impedances in the preceding equation are assumed to be real numbers.

(3) Calculate the value of the input power that must be applied to the following system to get an output power of 0.002W if the system was: (a) An amplifier having a gain of 12dB (b) A transmission channel having an attenuation of 13dB.

(4) Calculate the value of the output power of the following system if the input power was 1.5W and the sub-systems were: (a) S_1 is an amplifier having the gain of 9dB and S_2 is also an amplifier with 25dB gain. (b) S_1 is an amplifier having the gain of 7 and S_2 is a transmission channel with 13dB losses. (c) S_1 and S_2 are two successive different channels with the attenuation 15dB and 0.2 respectively.

(5) The input power of an amplifier is 2mW and output power is 1W. What are the power levels (dBm) at the input and output and what is the gain of the amplifier in decibels?

(6) A copper channel has an attenuation of 0.8dB per 100 meters. Calculate the total losses occurred when the cable was 1.5km long.

1.8 Communication Modes

In any communication link connecting two devices, data can be sent in one of three communication modes. These are:

A *simplex* line communication flow (usually occur in one direction) e.g. broadcast radio.

A *half-duplex* communication in both directions, but not at the same time, e.g. walkie-talkies.

A *full-duplex* system can support simultaneous two-way communication e.g. telephone.

1.5 Transmission Characteristics

1.5.1 Signalling Rate

The signalling rate of a communication link is a measure of how many times the physical signal changes per second and it is represented by the baud rate. An oscilloscope trace of the data transfer would show pulses at the baud rate.

1.5.2 Data Rate

The data rate is measured in bits per second (bps). This represents the actual number of data bits transferred per second.

1.5.3 Signal to Noise Ratio

The signal to noise (S/N) ratio of a communication link is an important limiting factor of the quality and rate of the communication. Noise sources may be internal or external, as discussed in Part 1 of this document.

1.6 Wave Propagation

Generally, there are three types of traveling waves in wireless channels: the ground waves, the sky waves, and the Line-of-Sight (LoS).

The ground waves communication is efficient using LF band. While the sky waves communication is working only in the HF band. Higher frequency (VHF or higher) is not reflected by ionosphere and they pass directly to space, so they are used in the satellite communications.

The ionosphere is a region of ionized gas that extends from 100km → 700km above the earth surface. The ionization is caused by the action of the sun's radiation on the upper atmosphere of the earth. The free electrons act as reflectors for HF radio waves, and their density varies with height, period of the day and year. During day time the reflection capability is strong, but it decreases during night. Most AM signals are transmitted over the HF band [illegible]. The transmitted HF signal reaches the receiver via the sky waves that have been reflected by the ionosphere.

While, the FM signals are transmitted over the VHF band which pass through the ionosphere layers (not reflected), hence the FM broadcasting coverage is limited. High frequencies, therefore, are transmitted through the LoS communications.

(a) Ground-Wave Propagation (below 2 MHz)

(b) Sky-Wave Propagation (2 to 30 MHz)

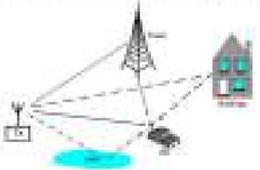
(c) Line-of-Sight (LOS) Propagation (above 30 MHz)

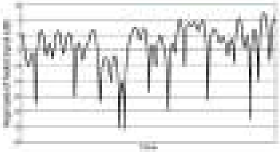
Propagation Types of Radio Frequencies

1.7 Multipath Effect

When two or more radio waves arrive at the receiver point along different paths, their phases (and sometimes frequencies) are not the same; results in cancellation of one signal by the other, which results in a loss of the signal strength. This phenomenon is called *fading*.

The fading effect is usually divided into two types, namely large-scale fading, mainly due to path loss as a function of distance and shadowing by large objects such as mountains and tall buildings, and small-scale fading due to the constructive and destructive combination of randomly scattered, reflected, diffracted, and delayed multipath signals.

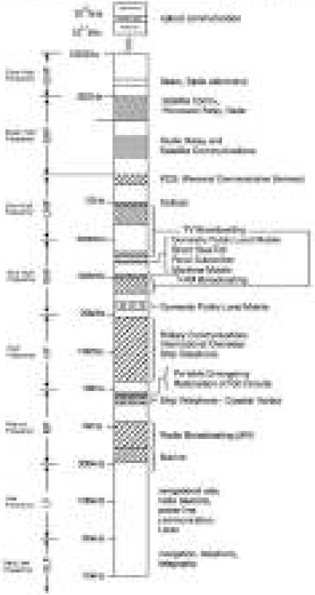




1.8 Standards and Government Regulations

In telecommunications, perhaps more than any other field, the need for standardization to ensure correct interoperation of equipment is paramount. Communication systems are always interworking with other devices, possibly located on the other side of the world.

The drawing up of standards falls to a small number of national and international bodies, with, for example, ITU (International Telecommunications Union) being responsible for the drafting standards of most of the new wireless and mobile communications.



Part 2 Signals & Spectra

2.1 Basic Definitions

2.1.1 Classification of Signals

- Deterministic Signal vs. Random Signal.
- Periodic Signal vs. Nonperiodic Signal.
- Analog Signal vs. Discrete Signal.
- Energy Signal vs. Power Signal: where the average power and the total energy are:

$$P = x_{rms}^2 = \frac{1}{T}\int x^2(t)\,dt \qquad E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

- Power signal if $x(t)$ is periodic, it has finite mean power but infinite energy.
- Energy signal if $x(t)$ is non-periodic, it has finite energy and zero mean power.

2.1.2 Correlation

The correlation function between $x_1(t)$ and $x_2(t)$ is:

For finite energy signals:

$$R(\tau) = \int_{-\infty}^{\infty} x_1(t)\,x_2(t+\tau)\,dt$$

For finite power signals:

$$R(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} x_1(t)\,x_2(t+\tau)\,dt$$

2.1.3 Convolution

The convolution between $x_1(t)$ and $x_2(t)$ is:

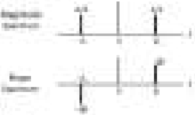
$$x_3(t) = x_1 * x_2 = \int_{-\infty}^{\infty} x_1(\tau)\,x_2(t-\tau)\,d\tau$$

2.2 Fourier Transform

Although an electrical signal physically exists in the time domain, we can also represent it in the frequency domain. We view the signal as a mixture of sinusoidal components at various frequencies. This frequency domain description is called spectrum.

To understand the frequency spectrum, let's consider the signal $x(t) = A\cos(\omega_0 t + \varphi)$ where A is the peak value or the amplitude, ω_0 the radian frequency and φ the phase angle.

The spectrum of $x(t)$ will be look like:



Next, consider the periodic function:

$$y(t) = 7 + 10\sin(40\pi t + 60^\circ) + 4\sin(120\pi t)$$

which is plotted in the time domain as:

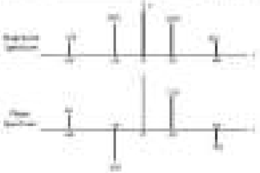
To get the frequency spectrum of $y(t)$, first, we convert $y(t)$ to cosines using:

$\sin\omega t = \cos(\omega t - 90^\circ)$ and $-\cos\omega t = \cos(\omega t \pm 180^\circ)$.

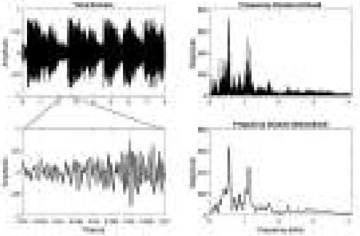
This yields to:

$$y(t) = 7\cos(0\pi t) + 10\cos(40\pi t - 30^\circ) + 4\cos(120\pi t - 90^\circ)$$

Whose spectrum will be:



Practical signals, such as speech, consist of large number of frequency components, and they [illegible]



The basic equations relating the time-domain version $x(t)$ and the frequency-domain version $X(f)$ are known as the Fourier Transforms, which are:

$$X(f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = F^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

These general equations are valid for non-periodic signals whose frequency domain are continuous. The spectrum exponential functions is discrete, and the Fourier Transformation takes

$$X_k = F(x(t)) = \frac{1}{T_0}\int_{T_0} x(t)e^{-j2\pi k f_0 t}dt$$

$$x(t) = F^{-1}(X_k) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k f_0 t}$$

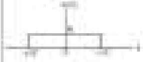
Examples:

(1) Find the Fourier transformation of a non-periodic single square pulse.

Solution:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi f t}dt = A\frac{e^{-j2\pi f t}}{-j2\pi f}\Big|_{-\tau/2}^{\tau/2} = A\frac{e^{j\pi f\tau}-e^{-j\pi f\tau}}{j2\pi f}$$

$$= A\tau\frac{\sin(\pi f\tau)}{\pi f\tau} = A\tau\,\mathrm{sinc}(f\tau)$$

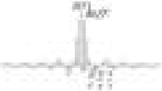


(2) Find the Fourier transformation of periodic square pulses.

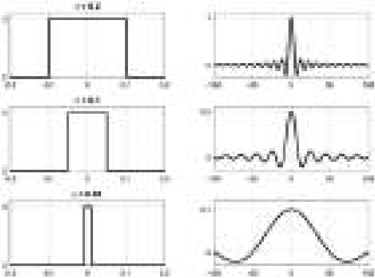
Solution:

$$X_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi k f_0 t}dt = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A e^{-j2\pi k f_0 t}dt = \frac{A}{T_0}\frac{e^{-j2\pi k f_0 t}}{-j2\pi k f_0}\Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{A\tau}{T_0}\frac{\sin(\pi k f_0\tau)}{\pi k f_0\tau} = \frac{A\tau}{T_0}\mathrm{sinc}(k f_0\tau)$$



For example, when $A = 1/\tau$ and $\tau = [0.2, 0.1, 0.02]$ seconds, we get the first nulls respectively located at (5, 10, 50) Hz.



Main properties of the Fourier Transform (See Fig. 3.3, Tables 3.1 and 3.2)

Signal (in time)	Spectrum (in Freq)
$x(t)$	$X(f)$
$a \cdot x(t)$	$a \cdot X(f)$
$x_1(t) + x_2(t)$	$X_1(f) + X_2(f)$
$x_1(t) * x_2(t)$	$X_1(f) \cdot X_2(f)$
$x_1(t) \cdot x_2(t)$	$X_1(f) * X_2(f)$

2.3 Bandwidth of a System

The set of all frequencies that are present in a signal is the frequency content, and if the frequency content consists of all frequencies below some given B, then the signal is said to be bandlimited to B. Some bandlimited signals are:

- Telephone quality speech: frequency extends up to ~4kHz.
- Audible music: the frequency extends up to ~20kHz.
- An ideal square pulse: the frequency extends up to infinity.

Several definitions of the bandwidth are commonly used:

- Absolute bandwidth: it contains all the frequency components of the signal, i.e. the spectrum is not available.
- 3dB bandwidth (or the half-power bandwidth): it contains the frequency components whose values are at least $1/\sqrt{2}$ times the maximum component.
- Null-to-null bandwidth (or zero-crossing bandwidth).
- Power bandwidth: it contains the frequency components that cover ≈99% of the total power.

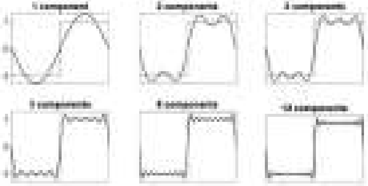
Limiting the frequency response of a system will channel (remove) some of the frequency components of these signals and causes the time-domain representation to be distorted.

Although such (bandlimited) ideal systems have a solid theoretical basis, they also involve many practical considerations. There is always a trade-off between fidelity to the original signal (that is, the absence of any distortion of its waveform) and such factors as bandwidth, [illegible]

Increasing the bandwidth often increases the cost of a communication system. Not only is the hardware likely to be more expensive, but bandwidth itself may be in short supply.

In communications over cables, the total bandwidth of a given cable is fixed by the technology employed. The more bandwidth used by each signal, the fewer signals can be carried by the cable.

$$x(t) = \frac{4}{\pi}\left[\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots + \frac{1}{2k-1}\sin(2k-1)t\right]$$



Generally, the bandwidth of a system (B) is defined as the interval of positive frequencies over which the magnitude $|H(f)|/|F(f)|$ remains within -3 dB (that is $\frac{1}{\sqrt{2}}$ in voltage or ½ in power).

Note: We consider the required B for transmitting a baseband signal = Symbol Rate ÷ 2

2.8 FILTERS

A filter is a device that passes electric signals at certain frequency ranges while preventing the passage of others. For illustration, let the input signal be:

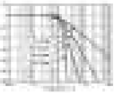
$$x(t) = 2 + 3\cos(2\pi 500t) + [illegible]$$

This is plotted in time domain (T.D.) as:

Plot it in frequency domain (F.D.)

2.8.1 Low Pass Filter (LPF)

[illegible]

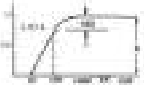


$x(t) = 2 + 3\cos(2\pi 500t)$. This is plotted in T.D. as:

Plot it in frequency domain (F.D.)

2.4.2 High Pass Filter (HPF)

[illegible]

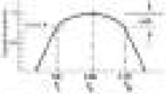


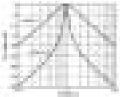
$x(t) =$ [illegible], plotted in T.D. as:

Plot it in frequency domain (F.D.)

2.4.3 Band Pass Filter (BPF)

[illegible]



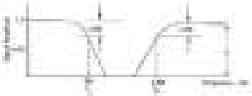


$x(t) =$ [illegible]

Plot it in frequency domain (F.D.)

2.4.4 Band Reject Filter (BRF)

[illegible]



[illegible]

2.4.5 All Pass Filter (APF)

[illegible]

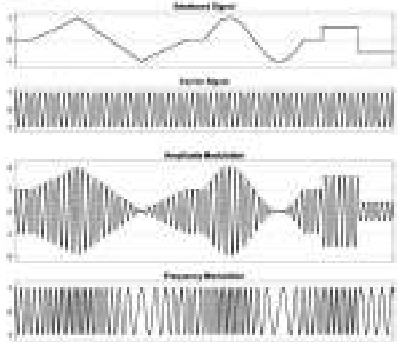
[illegible]

Part 3 Analog Modulation

3.1 Introduction

Baseband signals produced by various sources are not always suitable for direct transmission over a given channel. These signals are further modified to facilitate transmission using 'modulation', which is simply is a process that causes a shift in the range of frequencies of a signal.

In this part, we study the principles of continuous-wave (CW) modulation. This configuration of modulation uses a sinusoidal carrier whose amplitude or angle is varied in accordance with a message signal.



A carrier is a sinusoid of high frequency and one of its parameters (amplitude, frequency or phase) is varied in proportion to the baseband signal. Accordingly, we have Amplitude Modulation (AM), Frequency Modulation (FM) and Phase Modulation (PM).

$$y(t) = a(t)\cos(\theta(t)) + \dots + n(t)$$

Changing only [$a(t)$ for AM] or [$\theta(t)$ for FM] or [$\phi(t)$ for PM].

At the receiver, the modulated signal must pass through the reverse process called Demodulation, to reconstruct the Baseband signal.

FM and PM are very close relatives (in fact you can't have one without the other). Hence, we will consider AM and FM only. Both are used in ordinary (analog) broadcasting. Commercial radio stations are licensed to use carrier frequencies between about 500kHz to 1600kHz using AM, and frequencies between 88MHz and 108MHz using FM.

Among the most important reasons for using modulation:

- Ease of Radiation: For efficient radiation of electromagnetic energy, the antenna length must be $\geq \lambda/10$. For many baseband signals, the wavelength is too large for reasonable antenna dimensions. For example, the frequency band of speech can be extended up to 3kHz or $\lambda = 100$km; i.e., the antenna length = 10km!!. But if we modulate such audio signal using a 1MHz carrier, the required length for the antenna becomes ≈ 30m which is a reasonable size.
- Simultaneous Transmission of Several Signals: Most baseband signals occupy the same frequency band, so they cannot be transmitted over the same channel at the same time. So, via modulation, it is possible to send several signals over the same channel at the same time.
- Overcome the Channel Problems.

3.2 Amplitude Modulation

3.2.1 DSB-LC (Double Side Band – Large Carrier)

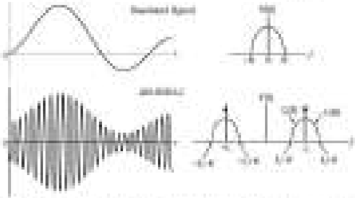
Modulation

Let $m(t)$ be the information signal (the message signal, or the baseband signal). It is bandlimited to B Hz. This signal can be transmitted through a channel by modulating the carrier signal $c(t) = A_c \cos(\omega_c t)$ where $f_c \gg B$. The amplitude-modulated wave becomes:

$$\varphi(t) = A_c \cos(\omega_c t) + m(t) \cos(\omega_c t)$$

$$= (A_c + m(t)) \cos(\omega_c t)$$

Where A_c = the peak value of the unmodulated carrier.



AM shifts the frequency spectrum of a signal from the zero to $\pm f_c$ without changing its shape. Since the bandwidth of the AM signal is double of $m(t)$, so it is called DSB.

Positive frequency of $m(t)$ → Upper Side Band (USB)

Negative frequency of $m(t)$ → Lower Side Band (LSB)

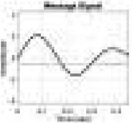
So, the general AM-DSB-LC wave can then be described as

$$s(t) = A_c[1 + \mu \times m(t)]\cos(\omega_c t)$$

Where μ is the modulation index, and $\mu = \frac{\text{maximum amplitude of modulating signal}}{\text{amplitude of unmodulated carrier}} = \frac{A_m}{A_c}$

Generally, according to the values of A_c and $|\mu|$:

- $|\mu| \le 1$, under-modulation, $s(t)$ contains $m(t)$, hence correctly demodulation
- $|\mu| > 1$, over-modulation, distortion in demodulation. Why?



To ensure proper modulation, we must prevent the phase reversal of the modulated signal i.e., the term $[1 + \mu \times m(t)]$ must be always positive.

For illustration, let $m(t) = A_m\cos(2\pi Bt)$, where in general, A_m can be considered as the maximum value of $m(t)$, and B is the maximum frequency component of the baseband signal. This gives,

$$s(t) = A_c\cos(\omega_c t) + A_m\cos(2\pi Bt)\cos(\omega_c t)$$
$$= [A_c + A_m\cos(2\pi Bt)]\cos(\omega_c t)$$
$$= A_c\left[1 + \frac{A_m}{A_c}\cos(2\pi Bt)\right]\cos(\omega_c t)$$
$$= A_c[1 + \mu\cos(2\pi Bt)]\cos(\omega_c t)$$

For $m(t) = A_m \cos(2\pi f_m t)$, undistorted modulation can be achieved by considering the following:

$$A_c - |A_m| \geq 0 \rightarrow A_c \geq |A_m|$$

the μ can be computed from the time domain signal as:

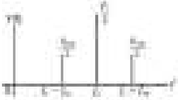
$\mu =$ [illegible]

Spectrum of DSB-LC

For illustration, we can consider $m(t) = A_m \cos(\omega_m t)$. So the voltage in an AM equation is:

$$y(t) = A_c[1 + \mu \cos(\omega_m t)]\cos(\omega_c t)$$

$$= A_c \cos(\omega_c t) + \frac{\mu A_c}{2}\cos[(\omega_c - \omega_m)t] + \frac{\mu A_c}{2}\cos[(\omega_c + \omega_m)t]$$



Power Calculations of DSB-LC

The carrier does not contain any information about $m(t)$, but it is the price to make cheap receivers available.

If $y(t) = A_c \cos(\omega_c t) + A_c \mu\, m(t) \cos(\omega_c t)$, using $m(t) = \cos(\omega_m t)$, the mean power (mean square value) is:

$$P_t = \frac{\overline{y^2(t)}}{R} = \frac{A_c^2}{R}\overline{\cos^2(\omega_c t)} + \frac{\mu^2 A_c^2}{4R}\overline{\cos^2[(\omega_c - \omega_m)t]} + \frac{\mu^2 A_c^2}{4R}\overline{\cos^2[(\omega_c + \omega_m)t]}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = P_c + P_{LSB} + P_{USB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{4R} = P_c + P_{SB} = P_c + \frac{\mu^2}{2}P_c$$

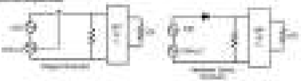
$$\therefore P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

Next, let the information power to the total power ratio (Transmission Efficiency):

$$\eta = \frac{P_m}{P_t} = \frac{P_{s(avg)}}{P_{t(avg)}} = \frac{\mu^2}{2+\mu^2}$$

Since the best demodulation occurs at $\mu \le 1$, so best η for DSB-LC system is 33%, i.e., 67% of the total power is spent in the carrier with (no information power).

Generation of DSB-LC:

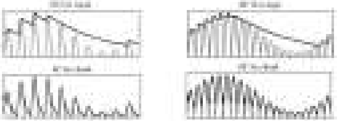


Demodulation:

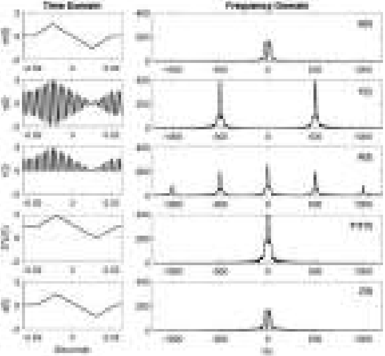
(1) Synchronous (coherent) detection (via PLL)

EX: Given $s(t) = A_c \cos(\omega_c t) + \frac{A_c\mu}{2}\cos(\omega_c - \omega_m)t + \frac{A_c\mu}{2}\cos(\omega_c + \omega_m)t$ and $\omega_c \gg \omega_m$, find $s(t)$?

(2) Asynchronous (Using an envelope detector), as $m(t)$ is available in the envelope of DSB-LC signal, it is possible to use simple envelope detector (ED) as a demodulator. Why we use asynchronous detection?



The following is an example that illustrates the [illegible]



[illegible]

[1] A transmitter radiates 9kW without modulation and 10.125kW after amplitude modulation. Determine the depth of the modulation.

Solution: $P_t = P_c\left(1+\frac{\mu^2}{2}\right) \Rightarrow 10.125 = 9\left(1+\frac{\mu^2}{2}\right) \Rightarrow \mu = 0.5$

[2] The antenna current of an AM transmitter is 8A when only the carrier is sent, but it increases to 8.93A when the carrier is sinusoidally modulated. Find the percentage depth of modulation and determine the antenna current when the $\mu = 0.8$.

Solution: $P_t = P_c\left(1+\frac{\mu^2}{2}\right), P = I^2R \Rightarrow I_t = I_c\sqrt{1+\frac{\mu^2}{2}}$

$\Rightarrow I_t = 8.93A \Rightarrow \mu = 70.1\%$. At $\mu = 0.8$, $I_t = I_c\sqrt{1+\frac{\mu^2}{2}} = 9.19A$

[3] The r.m.s. antenna current of an AM transmitter increased by 15% over the unmodulated value and has sideband power of 1kW. Find the AM transmitter output power and the corresponding modulation index.

Solution: $I_t = (1+0.15)I_c = 1.15I_c \Rightarrow P_t = (1.15)^2P_c \Rightarrow P_c = (1.15)^{-2}P_t$

$P_t = P_c + P_{SB} \Rightarrow P_t = (1.15)^{-2}P_t + P_{SB} \Rightarrow P_t = \frac{P_{SB}}{1-(1.15)^{-2}}$

$P_t = P_c\left(1+\frac{\mu^2}{2}\right) \Rightarrow 1+\frac{\mu^2}{2} = \frac{P_t}{P_c} = (1.15)^2 \Rightarrow \mu = 80.3\%$

[4] An AM transmitter gives carrier output of 5kW at $\mu = 80\%$. Calculate P_t and P_{SB}.

Solution: $P_t = P_c\left(1+\frac{\mu^2}{2}\right) = 6.6kW$, $P_t = P_c + P_{SB} \Rightarrow P_{SB} = 1.6kW$

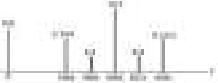
Also $P_{SB} = \frac{\mu^2 P_c}{2}$, $P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = 800W$

[5] Determine the saving, in signal power, in the case of 100% modulated AM signal, if the carrier is suppressed before transmission.

Solution: [illegible]

If the carrier is suppressed, $P_t = P_{SB} = \frac{\mu^2 P_c}{2}$

Power Saving $= \frac{[illegible]}{[illegible]} = 66.7\%$



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The last term can be ignored (low pass) yields:

[illegible]

[illegible]

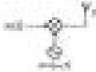
[illegible]

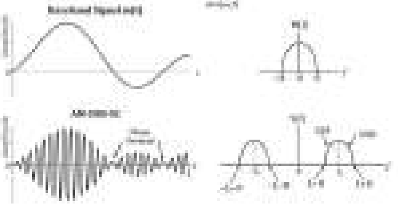
Hence the given system could be used as a demodulator.

3.2.2 DSB-SC (Double Side Band – Suppressed Carrier)

Modulation

The amplitude modulation can be achieved without the large carrier component in its spectrum. This saves the transmission energy and hence improves efficiency. The information signal can be transmitted through a channel by modulating the carrier signal $\cos\omega_c t$ via simple multiplication or coupling. The AM-DSB-SC waveform is $y(t) = m(t)\cos(\omega_c t)$.





Demodulation

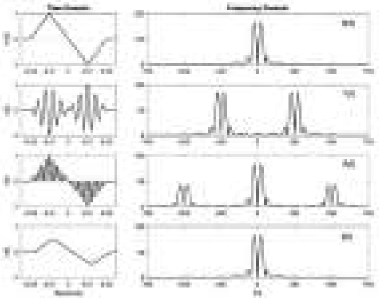
Demodulation is retrieving the shifted spectrum of m(t) in y(t) back to its original position.

$$x(t) = y(t) \times \cos(\omega_c t) = m(t)\cos^2(\omega_c t)$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(2\omega_c t)$$

The last term is eliminated by LPF. So the output becomes $z(t) = \frac{1}{2}m(t)$

Illustration:



Imperfection in Synchronization in DSB-SC

Due to several reasons, the locally generated carrier signal at the receiver end has some differences in frequency and/or phase. If the signal $s(t) = m(t)\cos\omega_c t$ is received, and the receiver carrier is $\cos[(\omega_c + \Delta\omega)t + \theta]$, then:

$$r(t) = s(t)\cos[(\omega_c + \Delta\omega)t + \theta]$$

$$= m(t)\cos(\omega_c t)\cos[(\omega_c + \Delta\omega)t + \theta]$$

$$= \frac{1}{2}m(t)\cos(\Delta\omega t + \theta) + \frac{1}{2}m(t)\cos[(2\omega_c + \Delta\omega)t + \theta]$$

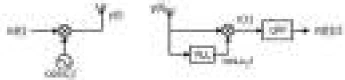
The second term will be removed by LPF, yields $m(t)/2$ multiplied by a factor ≤ 1, or

$$y(t) = \frac{1}{2}m(t)\cos(\Delta\omega t + \theta)$$

The value of this undesired factor will be governed by $\Delta\omega$ and θ. So:

- $\cos(\Delta\omega t + \theta) = 1$ when $\Delta\omega = \theta = 0$ (Synchronous or Coherent Reception)
- $\cos(\Delta\omega t + \theta) < 1$ when $\Delta\omega \neq 0$

So, it is important to perform the synchronous detection of DSB to recover the output. This can be done using the PLL when the modulator and the demodulator are remotely located, all synchronous receivers must involve a PLL to re-generate a fixed and in phase carrier.



Power Calculations of DSB-SC

Let modulating signal or the baseband signal is $m(t) = A_m\cos(\omega_m t)$, and the carrier is $c(t) = A_c\cos(\omega_c t)$, then modulated signal is

$s(t) = m(t) \times c(t)$

$= A_m A_c \cos(2\pi f_m t)\cos(2\pi f_c t)$

$= \frac{A_m A_c}{2}\cos(2\pi(f_c + f_m)t) + \frac{A_m A_c}{2}\cos(2\pi(f_c - f_m)t)$

Here the USB frequency is $(f_c + f_m)$ and the LSB frequency is $(f_c - f_m)$. And the bandwidth of $s(t)$ is $f_{USB} - f_{LSB} = 2f_m$.

The power of DSBSC is the sum of powers of the USB and the LSB components

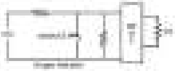
$P_t = P_{USB} + P_{LSB}$

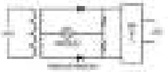
The formula for power of cosine signal is $P = \frac{V_{rms}^2}{R} = \frac{(V_m/\sqrt{2})^2}{R}$, so

$P_{USB} = \frac{(A_m A_c/2\sqrt{2})^2}{R} = \frac{A_m^2 A_c^2}{8R} = P_{LSB}$

$\therefore P_t = \frac{A_m^2 A_c^2}{4R}$

Generation of DSB-SC



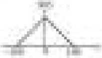


Exercises

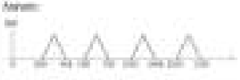
(1) The modulating signal [illegible] is applied at the input of [illegible] modulator operating at a carrier frequency of [illegible]. Sketch the spectral density of [illegible] and the resulting [illegible] waveform identifying the upper and lower sidebands.

Answer:

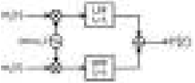
(2) For the [illegible] input signal to the following system, draw the spectrum of the output signal.



Answer:

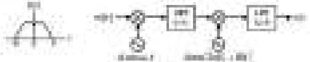


(3) [illegible] derive an expression for the output [illegible] of the following system:



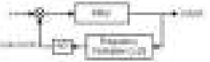
(4) For each of the signals: (1) $m(t) = [illegible]$, (2) $m(t) = [illegible]$: (a) Sketch the spectrum of $m(t)$, (b) sketch the spectrum of the signal [illegible], (c) identify USB and LSB spectrum and equations.

(5) The message signal $m(t)$ has a Fourier transform shown in the figure below. The signal is applied to the system shown to generate the signal $y(t)$. [illegible] Show that if $y(t)$ is transmitted, the receiver can pass it through a replica of this system to obtain $m(t)$ back. "Scrambling for communication privacy".



(6) System shown below is used for scrambling audio signals. Output $y(t)$ is the scrambled version of input $m(t)$. Find the spectrum of $y(t)$ and suggest a method of descrambling $y(t)$ to obtain $m(t)$.

(7) If $x = [illegible]$ is applied to the following system. Find the filter specifications and the output signal.



(8) A [illegible] signal whose frequency is [illegible] is used to modulate a [illegible] carrier as AM-DSB-SC. Assuming no losses occurred in the channel, determine the output signal of the demodulator when the local carrier of the receiver is: (a) [illegible], (b) [illegible].

(9) An AM receiver is shown. (a) write expression for $y(t)$ if the cutoff frequency of the LPF is f_c. (b) determine minimum phase θ if 87% of the maximum possible value of the signal is receivable.

Solution: (a) $v(t) = x(t)\cos(\omega_c t + \theta) = \frac{m(t)}{2}[\cos\theta + \cos(2\omega_c t + \theta)]$

$\to y(t) = \frac{m(t)}{2}\cos\theta$

(b) $\cos\theta = 0.87 \to \theta = 0.52$ rad $= 29.5$ deg

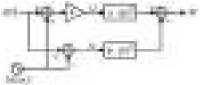
(10) When the audio input to a given system is [illegible], the measured frequency component at 1kHz in the output is [illegible], and the frequency component at [illegible] is [illegible]. Represent the amplifier output-input characteristics by $e_o = a_1 e_i + a_2 e_i^2$ and calculate the numerical values of a_1, a_2 from the data given.

Solution:

$e_o = $ [illegible]

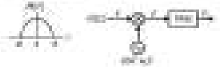
[illegible]

(11) Show that an appropriate choice of the gain k produces SSB modulation without filtering.



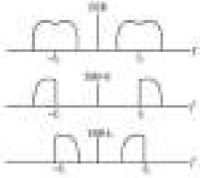
(12) Design a DSB-SC modulator that generates a modulated signal $km(t)\cos(\omega_c t)$ with the carrier frequency $f_c = 300$kHz. The following equipments are available in the stockroom: a signal generator of frequency 100kHz, multipliers, and a BPF tuned to 300kHz. Show how you (?) generate the desired signal.

[illegible] Design a DSB-SC modulator that generates the signal [illegible], where $m(t)$ is bandlimited to B Hz. The figure below shows a DSB-SC modulator available at the stock room. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like. (a) What kind of filter is required? (b) Draw [illegible] the signal spectra at points y and z and indicate the frequency bands occupied by these spectra. (c) What is the minimum value of ω_c? (d) Would this scheme work if the carrier generator output were [illegible]? Explain.



1.1.3 SSB (Single Side Band)

[illegible]



Modulation:

(1) Filtering method

SSB can be generated by filtering the DSB signal. [illegible]

(2) Phase shift method

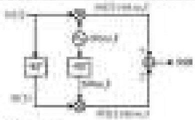
[illegible]

$\phi(t) = \cos(\omega_m t)\cos(\omega_c t) = \frac{1}{2}\cos(\omega_c + \omega_m)t + \frac{1}{2}\cos(\omega_c - \omega_m)t = DSB$

Solely:

$\cos(\omega_c + \omega_m)t = \cos(\omega_c t)\cos(\omega_m t) - \sin(\omega_c t)\sin(\omega_m t) \Rightarrow$ Upper SSB

$\cos(\omega_c - \omega_m)t = \cos(\omega_c t)\cos(\omega_m t) + \sin(\omega_c t)\sin(\omega_m t) \Rightarrow$ Lower SSB



But $\sin\theta = \cos(\theta \mp 90°)$ hence:

Upper SSB is $y(t) = m(t)\cos(\omega_c t) - \hat{m}(t)\sin(\omega_c t)$

Lower SSB is $y(t) = m(t)\cos(\omega_c t) + \hat{m}(t)\sin(\omega_c t)$

Where $\hat{m}(t)$ is shifting the phase of $m(t)$ by 90°

The main problem of SSB systems is the practical realization of the 90° phase shifter.

<u>Demodulation</u>

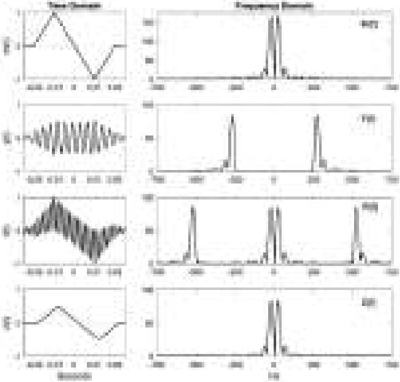
$y(t) = m(t)\cos(\omega_c t) \pm \hat{m}(t)\sin(\omega_c t)$

$z(t) = y(t)\cos(\omega_c t)$

$= m(t)\cos^2(\omega_c t) \pm \hat{m}(t)\sin(\omega_c t)\cos(\omega_c t)$

$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos(2\omega_c t) \pm \frac{1}{2}\hat{m}(t)\sin(2\omega_c t)$

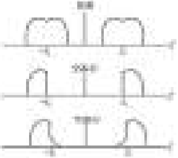
The high frequency parts are removed by LPF yields: $m(t)/2$.



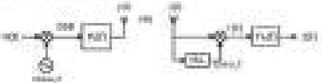
3.2.4 VSB (Vestigial Side Band)

Because of chaotic BW usage in DSB method and the selective filtering and phase shifter limitations in SSB method, VSB is a compromise between DSB and SSB.

Instead of rejecting one sideband completely as in SSB, a gradual cutoff of one sideband is accepted. (VSB BW = one sideband + 25% of the other sideband)



Modulation and Demodulation



$\Phi_{VSB}(f) = [M(f+f_c) + M(f-f_c)]\,H_v(f)$

$e(t) = \varphi_{VSB}(t)\cos\omega_c t \Leftrightarrow E(f) = \Phi_{VSB}(f+f_c) + \Phi_{VSB}(f-f_c)$

$$E(f) = M(f)[H_v(f+f_c) + H_v(f-f_c)] + \ldots$$

And to obtain $x(t) = m(t)$, $H_o(f) = \frac{1}{H_v(f+f_c) + H_v(f-f_c)}$

1.2.5. AM Summary

(1) Full amplitude modulation (DSB-LC), in which the upper and lower sidebands are transmitted in full, accompanied by the carrier wave. Accordingly, demodulation of an AM signal is done rather simply in the receiver by using an envelope detector, for example. It is for this reason we find that full AM is commonly used in commercial AM radio broadcasting, which involves a single powerful transmitter and numerous receivers that are relatively inexpensive to build.

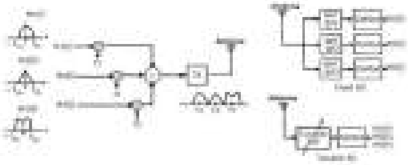
(2) DSB-SC in which only the upper and lower sidebands are transmitted. The suppression of the carrier wave means that DSB-SC modulation requires much less power than full AM to transmit the same message signal; this advantage is, however, attained at the expense of increased receiver complexity. Suppressed carrier signals require complicated circuits at receivers for synchronization. DSB-SC is therefore well suited for point-to-point communication involving one transmitter and one receiver.

(3) SSB, in which only the upper sideband or lower sideband is transmitted. This is the optimum form of CW modulation: it requires the minimum transmitted power and the minimum channel bandwidth for conveying a message signal from one point to another. However, its use is limited to message signals with an energy gap centered on zero frequency (useful in complex systems).

(4) VSB, in which almost all of one sideband and a vestige of the other sideband are transmitted. It requires a channel bandwidth that is between that required for SSB and DSB-SC systems, and the saving in bandwidth can be significant if modulating signals with large bandwidths are being handled, as in the case of television signals and high-speed data.

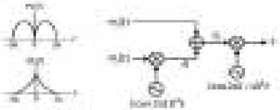
3.3 Frequency Division Multiplexing (FDM)

It is possible to send several signals simultaneously by choosing different carrier frequency for each signal. These carriers must be chosen so that the signals spectra do not overlapping. This technique is used basically in the commercial radio and TV stations. In practical AM radio broadcasting stations, they are allocated 10KHz per station i.e., 5kHz per sideband. It is sufficient?

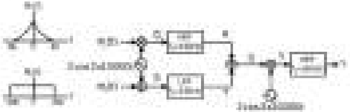


Exercise:

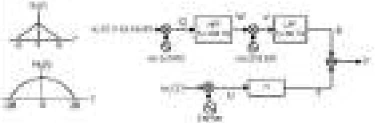
Q1 Two baseband signals $m_1(t)$ and $m_2(t)$ are to be transmitted simultaneously over a channel by multiplexing scheme shown below. (a) Sketch signal spectra at points B, C, D. (b) What must be the bandwidth of the channel to handle it without distortion? (c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ back from the modulated signal.



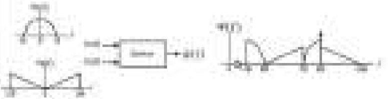
(2) Two baseband signals $m_1(t)$ and $m_2(t)$ are to be transmitted simultaneously over a channel by multiplexing scheme shown below. (a) Sketch signal spectra at points [illegible] (b) How much the minimum bandwidth of the channel need to handle Y without distortion? (c) Design the required receiver to recover $m_1(t)$ and $m_2(t)$.



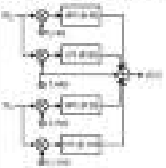
(3) The two baseband signals $m_1(t)$ and $m_2(t)$ are to be transmitted simultaneously over a channel by multiplexing scheme shown below. Sketch signal spectra at each indicated point, and what value of the carrier and filter specification required to [illegible] $m_2(t)$ [illegible] the spectrum of the signal at point [illegible]

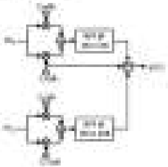


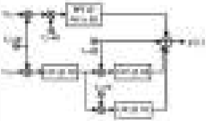
(10) Design a system scheme that should multiplex the two baseband signals $m_1(t)$ and $m_2(t)$ in the manner of the output $y(t)$. Then you can use any component you may need.



Solution: Apparently, we can get the requested signal using 'many' different system schemes and below are three different solutions:







But From 'the engineering's point of view', we must consider the complexity and the cost in the design of a systems.

3.8 Carrier Frequency Conversion

Which sometimes referred to as frequency translating, changing, mixing or heterodyning. Suppose that we have a modulated wave $s(t)$ whose spectrum is centered on a carrier frequency f_1 and the requirement is to translate it upward/downward in frequency such that its carrier frequency is changed from f_1 to a new value f_2. This requirement may be accomplished using the mixer shown in figure below:

Where the local oscillator frequency $f_L = f_1 \pm f_2$

If $s(t) = m(t)\cos(\omega_1 t)$ then

$x(t) = s(t)\cos(\omega_L t)$

$= m(t)\cos(\omega_1 t)\cos[(\omega_1 \pm \omega_2)t]$

$= \frac{1}{2}m(t)\cos(\omega_2 t) + \frac{1}{2}m(t)\cos[(2\omega_1 \pm \omega_2)t]$, the last term is removed by the BPF [illegible]

$\therefore y(t) = \frac{1}{2}m(t)\cos(\omega_2 t)$

Exercises

(1) What are the frequency options required at the receiver's local oscillator to change the carrier of the incoming signal from 1MHz to 2.25MHz? Show your results in algebraic computations.

(2) Draw the complete diagram with all the required details to convert the carrier of the signal $m(t)\cos(2\pi 1000 t)$ to be 1.25kHz.

(3) A DSB-SC signal is given by $m(t)\cos(2\pi 10^6 t)$, the carrier frequency of this signal, 1MHz, is to be changed to 400kHz. The only available equipments are: one multiplier, a BPF tuned to 400kHz, and one sine wave generator whose frequency can be varied from 1MHz to 2MHz. Show how you can obtain the desired signal.

3.5 Super-Heterodyne Receiver

For commercial radio receivers, it is difficult to design a good demodulator for wide range of frequencies. In this type, the carrier of the incoming signal is translated to a fixed frequency value (called intermediate frequency f_{IF} = 455 kHz) and finally demodulated by well designed system.

If $x(t) = m(t)\cos\omega_c t$ then for the moment, let $e(t) = y(t)$, the frequency conversion results:

$$e(t) = \frac{1}{2} m(t)\cos(\omega_{IF} t)$$

Which represents the main message signal $m(t)$ is modulated by new carrier of the value f_{IF}.

<u>Image Station Problem</u>

Now suppose there is another station with a carrier $f_c + 2f_{IF}$ is available at the receiving input, i.e.

$$x(t) = y_1(t) + y_2(t)$$
$$= m_1(t)\cos(\omega_c t) + m_2(t)\cos[(\omega_c + 2\omega_{IF})t]$$
$$e(t) = \frac{1}{2}m_1(t)[\cos(\omega_{IF} t) + \cos((2\omega_c - \omega_{IF})t)] + \frac{1}{2}m_2(t)[\cos(\omega_{IF} t) + \cos((2\omega_c + 3\omega_{IF})t)]$$
$$\Rightarrow e(t) = \frac{1}{2}[m_1(t) + m_2(t)]\cos(\omega_{IF} t)$$

Which means both m_1 and m_2 will pass to the demodulator (overlapping). So, in the super-heterodyne receivers, for each desired station frequency f_c there is an image station at frequency $= f_c + 2f_{IF}$ which called the image frequency. The RF-Amplifier stage of the super-heterodyne receivers prevents an image station from passing.

Exercises

(1) Example: It is desired to receive the AM radio station at 1000kHz with f_{IF} = 455kHz. Calculate (a) the local oscillator frequency required, (b) the image station frequency.

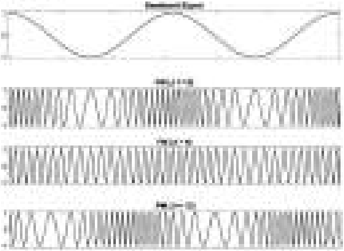
Solution: (a) $f_{osc} = f_s + f_{IF}$ = 1455kHz, (b) $f_{image} = f_s + 2f_{IF}$ = 1910kHz.

(2) Consider a Super-heterodyne receiver designed to receive the frequency band of 1MHz to 30MHz with IF frequency 450kHz, what is the range of frequencies generated by the local oscillator for this receiver?

3.6 FREQUENCY MODULATION

We saw in AM that the information contents of m(t) is transmitted through changing the amplitude of the carrier in proportion to the amplitude of m(t). In the angle modulation, the information content is transmitted through changing the frequency of the carrier instead. The instantaneous frequency of the carrier is determined by the value of the baseband signal.

In this modulation type, the carrier frequency varies in a proportion to the amplitude of the message signal m(t).



3.6.1 Definitions

Let us consider the carrier function as:

$$y(t) = A_c \cos\theta(t)$$

where $\theta(t)$ is the angle. Since the instantaneous frequency $\omega_i(t)$ is the slope of $\theta(t)$, so

$$\omega_i(t) = \frac{d\theta(t)}{dt}, \theta(t) = \int \omega_i(t) dt$$

In the case of the Frequency Modulation (FM), $\omega_i(t)$ is varied linearly with the modulating signal i.e.,

$$\omega_i(t) = \omega_c + k_f m(t)$$

where k_f is the modulator sensitivity, and ω_c is the carrier frequency at $m(t) = 0$. The angle of the carrier becomes

$$\theta(t) = \int_0^t [\omega_c + k_f m(\tau)]d\tau + \theta_0$$

If we assume $\theta_0 = 0$, the FM signal becomes

$$s_{FM}(t) = A_c \cos\left(\omega_c t + k_f \int_0^t m(\tau)d\tau\right)$$

In words: the instantaneous frequency varies linearly with the integral of the modulating signal.

FM and PM are closely related to each other; if we know the properties of the one, we can determine those of the other. For this reason, the material on angle modulation hereafter is devoted to FM.

To simplify the analysis of this equation, we consider $m(t) = A_m \cos(\omega_m t)$. So,

$$\omega_i(t) = \omega_c + k_f A_m \cos\omega_m t$$

$$= \omega_c + \Delta\omega \cos\omega_m t$$

Where $\Delta\omega = k_f A_m$ is the peak frequency deviation or the maximum frequency shift away from f_c. Hence $\Delta\omega = \Delta f$. The instantaneous angle of the carrier is

$$\theta(t) = \int_0^t \omega_i(\tau)d\tau = \omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t, \quad \text{at } \theta_0 = 0$$

$$\Rightarrow \theta(t) = \omega_c t + \beta \sin\omega_m t$$

Where $\beta = \frac{\Delta\omega}{\omega_m}$ = deviation ratio/index of the FM signal.

Hence, the formula of the FM waveform will be:

$$s_{FM}(t) = A_c \cos(\omega_c t + \beta \sin\omega_m t)$$

Since the peak deviation is given by Δf, the carrier swing $= 2\Delta f$

Remarks:

- There is no "over-modulation" situation with FM signal.
- β can take on any value from 0 to infinity. Its range is not limited as it is for AM.
- As β is increased, the signal becomes more resistant to interfering noise however occupies more bandwidth.

Examples:

Q1. A 100 kHz FM transmitter has a modulation sensitivity of 10 kHz/V. If a 5kHz sine wave of $1V_{pk}$ is applied to this transmitter, determine the maximum frequency deviation that occurs and the swing range of the carrier.

Solution: $\Delta f = k_f \times A_m = 1 \times 10^4 = 10kHz$, carrier swing $= 2\Delta f = 20kHz$.

Q2. A certain FM signal is represented by $x(t) = 10\cos(10^6 t + 15\sin(2000t))$, what information can we get from this expression?

Solution: (a) carrier amplitude = 10V, (b) $f_c = \frac{10^6}{2\pi} = 159.15$ kHz, [illegible]
(c) $f_m = \frac{2000}{2\pi} = 318Hz$, (d) $\Delta f = \beta f_m = 4770Hz$.

Q3. Determine the instantaneous frequency of the signal $A\cos(100t + \pi t^2)$ at $t = 0$.

Solution: $\theta(t) = 100t + \pi t^2$, $\omega(t) = \frac{d\theta(t)}{dt} = 100 + 2\pi t$, at $t = 0 \rightarrow \omega(0) = 100$
$\rightarrow f_i = 16Hz$

Q4. A frequency-modulated signal is described by the equation $x_{FM} = 10\cos(2\pi 10^6 t + 5\sin(1000t) + 10\cos(2000\pi t))$. Find: (a) The total power of the modulated signal (b) Frequency deviation Δf (c) β.

Solution: the bandwidth of the baseband signal is the highest frequency component available, so $f_m = \frac{2000\pi}{2\pi} = 1000Hz$.

(a) Carrier amplitude 10, the power is $P = (10)^2/2 = 50W$.

(b) $\omega_i = \frac{d\theta(t)}{dt} = \omega_c + 5000\cos(1000t) - 20000\pi\sin(2000\pi t)$

the carrier deviation is $\Delta\omega = 5000\cos(1000t) - 20000\pi\sin(2000\pi t)$, which has maximum value of $5000 + 20000\pi$, so $\Delta f = 10.8kHz$.

(c) $\beta = \Delta f / f_m = 10.8$

(3) Repeat the previous example but for $s_{FM} = 12\cos(2\pi 10^6 t + 4\sin(2\pi 2000 t))$ [illegible]

[illegible]

Solution: P_{ave} = [illegible]

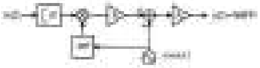
3.4.2 Spectrum of FM Signals

The spectrum of FM signals is rather 'messy' as it has very different sidebands spaced at multiples of f_m from the carrier. As a result, the bandwidth needed to accommodate an FM signal is greater than that for an AM signal having the same modulating frequency (Except for NBFM). According to the value of β, we have the following FM formats.

Narrowband FM

In the FM system where β is small (< 0.2), several approximations lead to the following method of FM generation:

$$s(t) = A_c\left[\cos(\omega_c t) - k_f \sin(\omega_c t) \times \int_0^t m(\tau)\, d\tau\right]$$



We can easily notice in the above chart that the spectrum and BW occupied by this NBFM waveform is $2f_m$ (just like the AM DSB).

Wide Band FM

To determine the spectrum of FM signal with values of $\beta > 0.5$ (which is typical), we may simplify matters by using the complex representation of band-pass signals when $m(t) = A_m \cos\omega_m t$:

$$s(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$$

$$= \mathrm{Re}[A_c \exp(j\omega_c t + j\beta \sin\omega_m t)]$$

$$= \mathrm{Re}[\tilde{s}(t) \exp(j\omega_c t)]$$

where $\tilde{s}(t)$ is the complex envelope of the FM signal $s(t)$, defined by:

$$\tilde{s}(t) = A_c \exp[j\beta \sin\omega_m t]$$

Thus, unlike the original FM signal $s(t)$, the complex envelope $\tilde{s}(t)$ is a periodic function of time with a fundamental frequency equal to the modulation frequency f_m. We may therefore expand $\tilde{s}(t)$ in the form of a complex Fourier series as follows:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t)$$

where the complex Fourier coefficient c_n is defined by:

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt$$

$$= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt$$

Define a new variable $x = 2\pi f_m t$.

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx$$

Here the integral on the right-hand side is recognized as the n^{th} order Bessel function of the first kind and argument β. This function is commonly denoted by the symbol $J_n(\beta)$, so:

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx$$

Accordingly, we may reduce c_n (equation) to $c_n = A_c J_n(\beta)$.

We substitute to get the complex envelope of the FM signal in terms of the $J_n(\beta)$

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t)$$

And

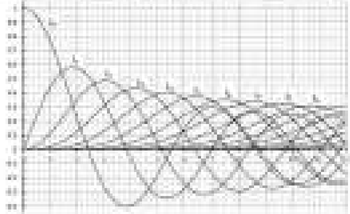
$$s(t) = A_c \operatorname{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + n f_m)t]\right]$$

$$\rightarrow s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] \qquad \text{Eq.[illegible]}$$

$$= A_c \left\{ \begin{array}{l} J_0(\beta)\cos(2\pi f_c t) \\ + J_1(\beta)\cos[2\pi(f_c + f_m)t] + J_{-1}(\beta)\cos[2\pi(f_c - f_m)t] \\ + J_2(\beta)\cos[2\pi(f_c + 2f_m)t] + J_{-2}(\beta)\cos[2\pi(f_c - 2f_m)t] \\ + J_3(\beta)\cos[2\pi(f_c + 3f_m)t] + J_{-3}(\beta)\cos[2\pi(f_c - 3f_m)t] \\ \ldots \end{array} \right\}$$

Knowing that: $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

In the figure below, we plot the Bessel function $J_n(\beta)$ versus the modulation index β for different positive integer values of n. (see the properties of Bessel functions [illegible] Page 29[illegible])



Thus using the equations and the curves above, we may observe:

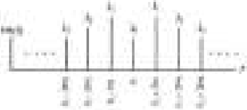
(1) The spectrum of an FM signal contains a carrier component and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations of $f_m, 2f_m, 3f_m, \ldots$

(2) For the special case of $\beta < 0.3$, only the Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values, so that the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$ (NBFM)

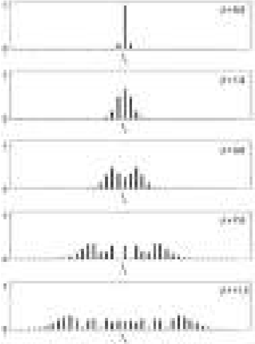
(3) The amplitude of the carrier component varies with β according to $J_0(\beta)$. That is, unlike an AM signal, the amplitude of the carrier component of an FM signal is dependent on β. As the envelope of an FM signal is constant, so that the average power of such a signal is:

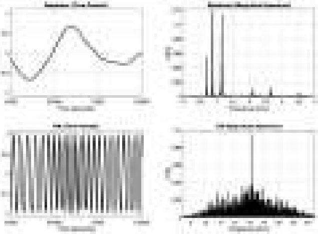
$$P = \overline{s^2(t)} = \frac{A_c^2}{2R} \qquad P_{total} = \frac{A_c^2}{2R}\sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

To sketch the spectrum of an FM signal at a certain β, we must get the values of $J_n(\beta)$ from a plot or a table of Bessel functions (see Appendix B: Bessel Functions). So, the plot may look like the following diagram, which is plotted using Eq.? (Note: we assume $\phi_c = 0$)



It is evident that, the frequency modulation of $m(t) = \cos\omega_m t$ has infinite number of sidebands. However the magnitude of the spectral components of the higher order sidebands becomes negligible.





3.6.3 Bandwidth of an FM Signal

If B_m = the bandwidth of the baseband (message) signal, the bandwidth of an FM signal is calculated as:

$$B_{FM} = 2\beta B_m$$

where β = the highest index of the significant sideband.

In general, the bandwidth of a WBFM signal, which contains at least 98% of the signal power, can be approximated as (Carson's Rule):

$$B_{FM} = 2(B_m + \Delta f) = 2(1+\beta)B_m$$

3.6.4 Power in FM

For sinusoidal input signal, the FM formula is $\varphi(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t))$

Mean square power of each sideband of an FM signal $P_n = \frac{A_c^2}{2} J_n^2$

Mean square power of an **unmodulated** FM carrier is $P_c = \frac{A_c^2}{2} = P_{total}$

Mean square power of total FM signal (equal to that delivered to the load):

$$P_t = \sum_{n=-\infty}^{\infty} P_n = P_c\left[J_0^2 + 2\sum_{n=1}^{\infty} J_n^2\right] = P_c\left[J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \cdots)\right]$$

where $J_0^2 + 2\sum J_n^2 = 1$ (i.e., all sidebands are included), $P_t = P_c = P_{total}$

[illegible]

[illegible]

(1) [illegible] carrier is frequency modulated by a sinusoidal signal such that the peak frequency deviation is [illegible]. Find the bandwidth of the resulting FM signal [illegible]

Solution:

[illegible]

[illegible]

[illegible]

(2) [illegible] carrier is frequency modulated [illegible] frequency deviation of [illegible] (i) carrier swing, (ii) highest and lowest frequencies [illegible]

Solution: [illegible]

[illegible]

[illegible]

(3) Consider the FM signal [illegible]. Determine the channel bandwidth occupied by this signal, the total load power.

Solution: [illegible]

[illegible]

[illegible]

(4) [illegible] sinusoidal wave with frequency [illegible] frequency modulate a carrier with [illegible] peak voltage. The [illegible] frequency deviation [illegible] Calculate the carrier power and the total power delivered to a load of [illegible]. Also find the bandwidth of the FM signal. Answer [illegible] following:

(a) [illegible]

[illegible] and

[illegible]

(b) [illegible] Bessel [illegible]

[illegible]
[illegible]

(c) [illegible]

[illegible]
[illegible]

(5) [illegible] FM [illegible] the bandwidth of the FM signal, [illegible]

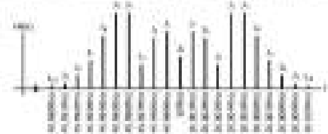
Solution: [illegible]
[illegible]
[illegible]

(6) FM [illegible]

Solution: From Bessel function graph, we see that [illegible]
[illegible]
[illegible]

(7) [illegible] AM [illegible] FM system. [illegible] AM system. [illegible] of the (a) FM, (b) AM systems.

Solution: [illegible]
[illegible]



Since the center frequency of the ideal BPF is located at f_c, we can calculate n as:

$$n = \left\lfloor \frac{f_{BPF}}{f_m} \right\rfloor$$

(a) The BW of the [illegible] filter is $B =$ [illegible], so only the carrier would pass through this filter. Hence the power delivered to the load is

$P_c = P_t \times J_0^2 =$ [illegible]

(b) Here the $B =$ [illegible], at each side of the center frequency, there are

$n =$ [illegible], so only the carrier and the first order sidebands would pass through.

Hence $P_t =$ [illegible]

(c) $n =$ [illegible]. The carrier and the first [illegible] pairs of sidebands would pass. The power will be:

$P_t =$ [illegible]

(d) $n =$ [illegible]

(e) $n =$ [illegible]

When does $P_t = P_c =$ [illegible]?

(11) Repeat the previous example with the center frequency of the ideal BPF is [illegible].

Sol: The following expression need not apply in this case.

$$n = \left\lfloor \frac{f_{BPF}}{f_m} \right\rfloor$$

(12) An analog signal with a bandwidth of 5kHz is to be transmitted through an FM system of a bandwidth of 30kHz, find the maximum value of β, for high fidelity reproduction. (Hint: use the FM bandwidth approximation)

(13) For a modulating signal $m(t) = 2\cos(1000t) + [illegible]\cos(2000\pi t)$, (i) Write the expression for φ_{FM} when $A_c = 10$, $\omega_c = 10^6$, $k_f = 1000\pi$. Use [illegible] integral of $m(t)$ [illegible], (ii) Estimate the bandwidth of φ_{FM}.

(14) An FM system with $\Delta f = 10kHz$ has been designed for $f_m = 10kHz$. Approximately what percentage of system bandwidth occupancy when the modulating signal is a unit amplitude sine at $f_m = 1kHz$, 5kHz, 15kHz.

3.4.3 Generation of WBFM

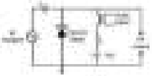
Direct Method

Using the Voltage-Controlled Oscillator (VCO), in which the output signal frequency varies linearly with the input control voltage. The principle of such circuit is changing L and/or C in a transmitted form as

$$f_i = \frac{1}{2\pi\sqrt{LC}}$$

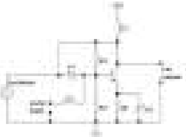
The following are examples of a VCO:

(1) The varactor diode modulator: The reverse diode capacitance varies with the bias V_b and the audio input signal.



(2) The Transistor Reactance Modulator: Here the g_m of the transistor is caused to vary by changing the operating point of the transistor according to the varying in audio signal. The equivalent C is obtained as

$$C_{eq} = \frac{g_m R_1 C_1}{R_1 + R_2}$$



Indirect Method:

Also called Armstrong's method. First, it generates NBFM, then it changes the signal to WBFM via frequency multipliers and converters.

2.4.6 Demodulation of FM Signals

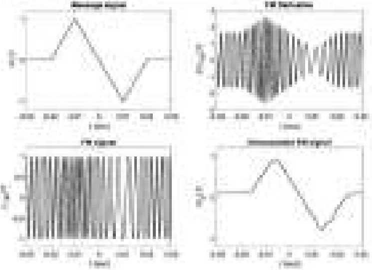
(1) Differentiator: the equation of the FM (below) can be demodulated using a differentiating circuit.

$$s_{FM}(t) = A_c \cos\left(\omega_c t + k_f \int m(\lambda) d\lambda\right)$$

$$\frac{d}{dt}\left[s_{FM}(t)\right] = \frac{d}{dt}\left[A_c \cos\left(\omega_c t + k_f \int m(\lambda) d\lambda\right)\right]$$

$$= A_c(\omega_c + k_f m(t)) \sin\left(\omega_c t + k_f \int m(\lambda) d\lambda - \pi\right)$$

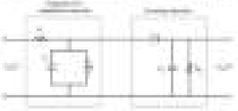
The following example illustrates the decoding procedure.



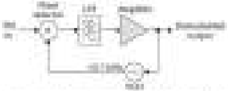
(2) Zero Crossing Detector: because the information is contained in the zero crossings of the FM waveform, it is possible to clip (limit) the amplitude of the FM waveform, which results in a square wave. Counting the zero crossings in each time interval is an indication of the amplitude of the baseband signal.

(3) Discriminator: The purpose of the discriminator is to convert the variation of frequency to a variation of amplitude (inverse of VCO). The simplest circuit of discriminator is a tuned RLC circuit that has a rapid change of amplitude with frequency on both sides of the resonant frequency, especially when the Q factor of the circuit is high.

FM $\rightarrow$ $\boxed{\text{Discriminator}}$ $\rightarrow$ m(t)



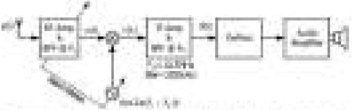
(d) PLL: A typical modern FM receiver uses a PLL (integrated) as the detector. The PLL is insensitive to amplitude variations and can perform the frequency-to-voltage conversion; it can therefore be used as an FM detector.



The PLL detector works in the following manner. The free VCO runs at the intermediate frequency 10.7 MHz. The incoming signal, if unmodulated, locks up with the VCO signal, causing there to be no signal output from the LPF stage. Now let us assume that the incoming signal has been modulated by a single audio tone. The phase detector will output an error voltage to the VCO to drive the VCO into lockup with the incoming signal. Because the incoming signal frequency is deviating both above and below the 10.7 MHz rest frequency at a certain number of cycles per second, the VCO will do the same, following the input signal frequency variations. The error voltage from the LPF, which drives the VCO, will be identical to the original modulating signal, and hence is taken as the demodulated output.

3.6.? Super-Heterodyne Receiver in FM

It is the typical receiver of the commercial FM broadcast stations. This system is like the AM super-heterodyne receiver but with $f_{IF} = 10.7$MHz.



The frequency band of FM radio broadcasting is 88MHz to 108MHz, with $\beta \geq 5$ ($\Delta f \leq$ 75kHz, W=15kHz); $\Delta f \leq$ 25kHz for the sound portion of TV broadcasting. Practically B_T = 200kHz for both FM radio and TV.

Each commercial FM radio broadcast station is allocated a 150kHz channel plus 50kHz guard band. The bandwidth given to FM radio stations is very large compared to AM radio stations; therefore the sound resolution is significantly enhanced.

Summary

- Generally, AM techniques are simpler than those for FM in terms of the required electronic circuits and the system structure (Tx and Rx). Hence AM systems are cheaper. However, one significant drawback of AM techniques is that they tend to be rather sensitive to impulsive interference which can be caused by e.g. lightening or car ignition noise, since the information is contained in the instantaneous amplitude of the signal.
- Unlike AM, FM is a nonlinear modulation process. Accordingly, spectral analysis of FM is more difficult than for AM.
- For constant Δf, the bandwidth of FM is controlled by [illegible]

Part 4 Noise

4.1 Introduction

Noise in communication systems is caused by unwanted signals. Basically, the sources of these signals are:

(1) External Noise:

- Atmospheric Noise: It is caused by naturally occurring disturbances in the earth's atmosphere such as: lightning discharges, thunderstorms and other natural electric disturbances. At 20MHz and above atmospheric noise is less severe.
- Industrial Noise: this includes sources like ignition of cars and aircrafts, electric motors, switching equipment, leakage from high voltage lines, etc.
- Extra-terrestrial Noise:
  - Solar Noise: It originates from the sun which radiates a broad spectrum of frequencies, including those which are used for broadcasting.
  - Cosmic Noise: Distant stars also radiate noise in much the same way as the sun. Noise also comes from distant galaxies in much the same way as they come from the Milky Way.

(2) Internal Noise: This type is generated by any of the active or passive devices found in the receiver. It is proportional to the bandwidth over which it is measured.

By careful engineering, the effects of many unwanted signals could be reduced or eliminated, however, other signals could not be removed. But one relatively basic source of electrical noise is the thermal effects [illegible] in conducting media, wiring, resistors, etc.

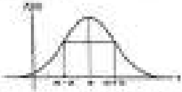
4.2 Definitions

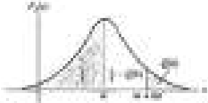
4.2.1 Power Spectral Density

Because we deal with random signals, we need to review the PSD. The Power Spectral density (PSD) describes the distribution of power versus the frequency for energy signals:

$$S(f) = \lim_{T \to \infty} \frac{|X_T(f)|^2}{T}$$

where $X_T(f)$ is the Fourier transform of the truncated segment from the random signal.





4.3 Thermal Noise

Among the considered noise sources, we can focus on the first and most is the case of the most commonly considered sources. Thermal noise is an important noise source of white noise. Thermal Noise is produced because of the thermally excited random motion of free electrons in a conducting medium, such as resistor. The path of each electron in motion is randomly interrupted due to collisions. The net effect of the motion of the electrons is an electric current in the resistor which is random with a mean of zero. From thermodynamic and quantum mechanical considerations, the PSD of thermal noise is:

$$S_n(f) = \frac{2h|f|}{e^{\frac{h|f|}{kT}} - 1} \approx 2kT \ \text{Watts/Hz} \quad \text{for} \quad |f| \ll \frac{kT}{h}$$

T = temperature of conducting medium in Kelvin

k = Boltzmann's constant = 1.38×10^{-23} Joule/Kelvin

h = Planck's constant = 6.625×10^{-34} Joule.sec

$$P_n = \int_{-B}^{B} S_n(f)\,df = 4kTB \ \text{Watt}$$

So, the mean-square voltage and current generated by a resistor R within the band width B is

$$\overline{v^2(t)} = 4RP_n = 4kTBR \quad \text{Volt}^2$$

$$\overline{i^2(t)} = \frac{P_n}{R} = \frac{4kTB}{R} \quad \text{Ampere}^2$$

For band-limited thermal noise, the models for voltage and current equivalent circuits are shown below, assuming R is noise-free and bandwidth B.

5.6 White Noise and Filtered Noise

Most noise sources in the electrical systems are Gaussian. They also have a flat spectral density over a wide frequency range. This means the noise is almost the same at all frequencies in equal proportion. Therefore, it is called "white noise" by analogy to white light. White noise is a convenient model (and often an accurate one) in communications. The assumption of a Gaussian process allows us to model all the noise properties; however, some applications (beyond our scope) may need a more advanced model for the noise.

We'll write the spectral density of the white noise in general as:

$$S_n(f) = \frac{\eta}{2} \quad \text{for all } f \text{ (two-sided PSD)}$$

Where η (in Watt per Hertz) is the power spectral density of the white noise. The PSD $S_n(f) = \eta$ when it is associated with the positive frequency only, and hence it is referred to as one-sided PSD. For the two-sided PSD, $S_n(f) = \frac{\eta}{2}$ where it is used for all the frequencies. The value of η depends upon two factors: the type of noise, and the type of spectral density. If the source is a thermal (resistor, then) the mean square voltage and mean square current spectral densities are

$$\eta_v = 4kTR \ \text{Volts}^2, \quad \eta_i = 4kT/R \ \text{Ampere}^2$$

We can find noise power P_n as:

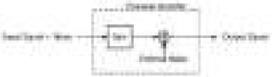
$$P_n = \int_{-\infty}^{\infty} \frac{\eta}{2} df \to \infty$$

Practically, the communication systems are band-limited to B Hz:

$$P_n = \sigma^2 = \int_{-B}^{B} \frac{\eta}{2} df = \eta B \ \text{Watt}$$

4.5 Amplifier Noise

Since the received signal is very weak, it is crucial to have an amplifier at an early stage of the reception. Because the received signal has already been corrupted by the noise from the external noise, it is important to make the internal noise within the receiver itself as small as possible. Since we cannot practically implement a noiseless amplifier, we may consider theoretically an amplifier like:



Let the received signal S_i comprises S as the message without noise and N_i as the additive noise. So, if fed input to the amplifier of the receiver is $S_i = S + N_i$, the output will be:

$$S_o = A(S + N_{in}) = AS + (AN_i + N_{int}) = AS + N_o$$

Where A is the gain of the ideal amplifier A_{id} ... It is obvious that the final noise component represents the addition of the internal and the amplified noise of the external noise. To discuss the noise within the receiver, we need to use and define the following measures and terms.

4.6 Noise Computations

4.6.1 SNR

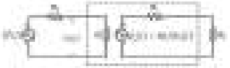
The Signal-to-Noise Ratio (SNR) is an important measure for the performance of a system. It refers to the amount of noise that has been induced to the system in proportion to the desired signal:

$$\frac{S}{N} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\overline{s^2(t)}}{\overline{n^2(t)}} \Rightarrow \left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{S}{N}\right)$$

4.4.2 Available Power and T_N

The figure below depicts the circuit model of a noiseless amplifier inserted between a source R_s and a load R_L. This model has an input resistance R_i, an output resistance R_o, and a voltage transfer function $H(f)$. The source generates a mean square voltage density $G_s(f)$ representing noise or an information signal or both. The available power density from the source is $g_s(f) = G_s(f)/4R_s$. The available power density at the output of the amplifier is

$$g_a(f) = \frac{G_o(f)}{4R_o} = \frac{|H(f)|^2 G_i(f)}{4R_o} = \frac{|H(f)|^2}{4R_o}\left(\frac{R_i}{R_s + R_i}\right)^2 G_s(f)$$



We define $G_a(f)$ as the gain of the amplifier's available power:

$$G_a(f) = \frac{g_a(f)}{g_s(f)} = \frac{G_o(f)/4R_o}{G_s(f)/4R_s} = \left(\frac{|H(f)|R_i}{R_s + R_i}\right)^2 \frac{R_s}{R_o}$$

Assuming that the source generates thermal white noise at the temperature T_s, then $g_s = kT_s/2$. The available noise power at the output of a noiseless amplifier is

$$g_a(f) = G_a(f)g_s(f) = G_a(f)kT_s/2$$

Since the internal noise is independent of the source noise, we write

$$g_a(f) = G_a(f)kT_s/2 + g_{int}(f)$$

Integration then yields the total available output noise power

$$N_o = \int_0^\infty g_a(f)df = kT_s\int_0^\infty G_a(f)df + \int_0^\infty g_{int}(f)df$$

Most amplifiers in a communication system have a frequency-selective response, with maximum power gain g and noise equivalent bandwidth B_N, so:

$$gB_N = \int_0^\infty G_a(f)df$$

We define the effective noise temperature of the amplifier to be:

$$T_e = \frac{1}{kGB_N}\int_0^{\infty} S_{no}(f)\,df$$

Hence, the total output noise power then becomes:

$$N_o = kGB_N(T_i + T_e)$$

$$N_i = kT_i \longrightarrow \oplus \longrightarrow \boxed{G(f)} \longrightarrow \boxed{B_N} \longrightarrow N_o$$

kT_e

Now let the figure below represent a noisy amplifier with signal plus white noise at the input.

$$S_i \longrightarrow \oplus \longrightarrow \boxed{T_e, B_N, g} \longrightarrow (S/N)_o$$

$N_i = kT_s$

As the available signal power at the output is $S_o = gS_i$. Then

$$\left(\frac{S}{N}\right)_o = \frac{gS_i}{N_o} = \frac{S_i}{k(T_s + T_e)B_N}$$

Since $\left(\frac{S}{N}\right)_i = \frac{S_i}{kT_sB_N}$ then

$$\left(\frac{S}{N}\right)_o = \frac{1}{1 + T_e/T_s}\left(\frac{S}{N}\right)_i$$

we see that in general $(S/N)_o < (S/N)_i$, however we may get $(S/N)_o \approx (S/N)_i$ when $T_e \ll T_s$. At (lower) frequencies below ~1000 MHz, the noise has little effect, and the amplifier appears to be noiseless. At higher frequencies, T_e becomes significant which often affect the design of the receivers and the antennas. Some very low noise amplifiers have $T_e \approx 30°C \approx 300$ K while it is $T_e \approx 1000$ K for noisy systems. Knowing T_e is not measured by the ambient temperature of the amplifier, sometimes cryogenic cooling is employed to lower the noise temperature. In this class we set $T_s = 290$K.

4.6.3 Noise Figure

Another measure of amplifier noise is the noise figure F (also called 'figure of merit'). It is expressed as $F = SNR_i / SNR_o$ (normally in dB). Then for the case where $T_s = T_0 = 290K$,

$$F = 1 + \frac{T_e}{T_0} \quad \text{or} \quad F_{dB} = 10\log_{10}(F)$$

A very noisy amplifier has $T_e \gg T_0 \Rightarrow F \gg 1$

At room temperature, $F = 2$.

A low-noise amplifier has $T_e < T_0 \Rightarrow 1 < F < 2$ (in this case someone usually should work in dB). What is the value of F of a perfect system?

4.6.4 Noise in Multi-Stage Systems

Sometimes complex systems contain different stages to process the incoming signal. Since each stage generates internal noise differently, they must be carefully designed to perform the optimum reception.

To develop expressions for the overall performance of the system in terms of the parameters of the individual stages, let's consider the following cascade of two noisy two-port sub-systems. In this figure, the subscripts identify the effective noise temperature, the maximum power gain, and the noise bandwidth per stage.

$$n_i = kT_s \rightarrow \boxed{T_1,\ g_1,\ B_1} \rightarrow \boxed{T_2,\ g_2,\ B_2} \rightarrow n_o$$

The overall power gain therefore equals the product, i.e., $g = g_1 g_2$.

The total output noise power consists of three terms:

(1) Source noise amplified by both stages.

(2) Internal noise from the first stage, amplified by the second stage.

(3) Internal noise from the second stage.

Assuming $B_2 \le B_1$ and $B_N = B_2$, then:

$$N_o = g_1 g_2 k T_s B_N + g_2 (g_1 k T_1 B_N) + g_2 k T_2 B_N = g k \left(T_s + T_1 + \frac{T_2}{g_1}\right) B_N$$

The overall effective noise temperature and noise figure are:

$$T_e = T_1 + \frac{T_2}{g_1} \qquad \text{and} \qquad F = 1 + \frac{T_1}{T_0} + \frac{T_2}{g_1 T_0} = F_1 + \frac{F_2 - 1}{g_1}$$

The foregoing analysis is readily generalized to:

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots \qquad \text{and} \qquad F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

[illegible]

[illegible]

4.6.5 Time Representation of Bandpass Noise

[illegible]

$n(t) = n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$

where f_c = carrier frequency. Note that $\overline{n^2(t)} = \overline{n_c^2(t)} = \overline{n_s^2(t)}$

4.7 Noise in AM Systems

4.7.1 Synchronous DSB-SC

We assume synchronous demodulation. So, if the input is $x(t) = m(t)\cos(\omega_c t)$, then

$S_i = \overline{x^2(t)} = \frac{1}{2}\overline{m^2(t)}$, and as the output is $\frac{1}{2}m(t)$ then $S_o = \overline{\left[\frac{1}{2}m(t)\right]^2} = \frac{1}{4}\overline{m^2(t)} = \frac{1}{2}S_i$

If the input noise is $n_i(t)$ then $N_i = \overline{n_i^2(t)}$. The multiplier output:

$r(t) = n_i(t)\cos\omega_c t = [n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t]\cos\omega_c t$

$$= \frac{1}{2}n_c(t) + \frac{1}{2}n_c(t)\cos(2\omega_c t) - \frac{1}{2}n_s(t)\sin(2\omega_c t)$$

The output from the LPF will be $n_o(t) = \frac{1}{2}n_c(t)$ so

$N_o = \overline{n_o^2(t)} = \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \frac{1}{4}N_i$

As a result:

$$\frac{S_o}{N_o} = 2\frac{S_i}{N_i}$$

This means that the detector improves the SNR in DSB-SC system by factor of two. This improvement is achieved because the coherent (synchronous) detector rejects $n_s(t)$ and halves $n_c(t)$.

4.7.2 Synchronous DSB-LC

The use of synchronous detector for DSB-LC provides the same advantages mentioned above. The same computations can be used here as well by using $m(t)$ as $[A + m(t)]$ instead. So:

$S_i = \frac{1}{2}A^2 + \frac{1}{2}\overline{m^2(t)}$ assuming $\overline{m(t)} = 0$

Hence $\dfrac{S_o}{N_o} = \dfrac{2\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \cdot \dfrac{S_i}{N}$

Note: for $m(t) = A\cos\omega_m t$ (tone):

$$\frac{S_o}{N_o} = \frac{2}{3} \cdot \frac{S_i}{N}$$

4.7.3 Envelope Detector DSB-LC

The input signal to the envelope detector is:

$R(t) = s(t) + n_i(t) = [A + m(t)]\cos(\omega_c t) + n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$

For high input SNR, it is just like synchronous, i.e.,

$$\frac{S_o}{N_o} = \frac{2\overline{m^2(t)}}{A^2 + \overline{m^2(t)}} \cdot \frac{S_i}{N}$$

For input SNR < [illegible]

$$\frac{S_o}{N_o} = \frac{S_i}{[illegible]}$$

In fact, for input SNR < 7 [illegible], the noise takes over the control of the envelope.

4.7.4 SSB-SC

Since the SSB-SC signal is: $s(t) = m(t)\cos(\omega_c t) \mp m_h(t)\sin(\omega_c t)$, we have:

$S_i = \overline{s^2(t)} = \frac{1}{2}\overline{m^2(t)} + \frac{1}{2}\overline{m_h^2(t)}$

But $m_h(t)$ is only $90°$ shifting of $m(t)$, so $S_i = \overline{m^2(t)}$, and $S_o = \overline{\left[\frac{1}{2}m(t)\right]^2} = \frac{1}{4}\overline{m^2(t)} = \frac{1}{4}S_i$. This results in:

$$\frac{S_o}{N_o} = \frac{S_i}{N}$$

4.3 Noise in FM Systems

In FM signals $s(t) = A\cos(\omega_c t + k_f\int_0^t m(\alpha)d\alpha)$ and $\omega_i(t) = \frac{d\theta}{dt} - \omega_c = k_f m(t) \Rightarrow$ $S_o = \overline{y_o^2(t)} = k_f^2\overline{m^2(t)}$. The PSD of output noise of FM demodulator is $S_{n_o} = \frac{\eta}{A^2} f^2$.

Since S_{n_o} is a parabolic spectrum, therefore, the effect of noise in FM for higher frequency components is much higher than the effect of noise on lower frequency.

Assuming LPF cutoff frequency (in Hz) = f_m, the output noise power from the LPF is:

$$N_o = \int_{-f_m}^{f_m} S_{n_o}(f)df = \int_{-f_m}^{f_m} \frac{\eta}{A^2} f^2 df = \frac{2\eta f_m^3}{3A^2}$$

The output signal power $S_o = k_f^2 P_m$, where P_m is the average power of $m(t)$. So,

$$\frac{S_o}{N_o} = \frac{3A^2 k_f^2 P_m}{2\eta f_m^3}$$

But $\beta = \frac{k_f \max|m(t)|}{f_m}$ and $S_i = \frac{A^2}{2}$ and $N_i = \eta f_m$.

This yields

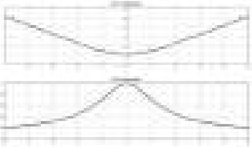
$$\frac{S_o}{N_o} = \frac{3\beta^2 P_m}{[\max|m(t)|]^2}\frac{S_i}{N_i}$$

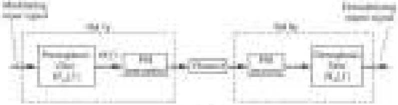
and in case of $m(t) = \cos(\omega_m t)$ we get

$$\frac{S_o}{N_o} = \frac{3\beta^2}{2}\frac{S_i}{N_i}$$

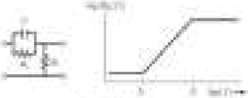
SNR Improvement in FM using Pre-emphasis/De-emphasis

The audio signals have most of its energy at the lower frequency (300 – 3,000 Hz), and as at the output of the FM demodulator the noise PSD rises parabolically with the frequency, that means the noise PSD is largest in the frequency range where the signal PSD is smallest. To remedy this situation, we emphasize the high-frequency components in the input signal at the transmitter. At the output of the FM demodulator in the receiver, the inverse operation is performed. The signal spectrum is restored to its original shape but the noise which was added after the pre-emphasis is now reduced.





Preemphasis Filter



Deemphasis Filter

Exercises

Remember: Temperature conversion: [illegible] and [illegible]

(1) Determine the noise voltage produced by a 1MΩ resistor at room temperature (17°C) over a 1MHz bandwidth.

Solution: $v = \sqrt{4kTRB} = $ [illegible]

(2) An Envelope detector is used for detection of an AM signal with 10% sinusoidal modulation. The received signal power is 10^{-8} W and the highest modulating frequency is 5kHz, while the input noise spectral density is 10^{-14} W/Hz. Find the SNR and SNR_o of the system.

Solution: [illegible]

[illegible]

(3) A received modulating signal (10kHz bandwidth) has an input signal-to-noise ratio of 30dB. Find SNR_o at the demodulator output if the signal was (i) FM with 75kHz deviation; (ii) AM with $\mu = 0.5$. Compare!

Solution: (i) FM: [illegible]

(ii) [illegible] = [illegible] (which one is higher? So, which is the better?)

(4) In a receiver, the input signal is 10μV while the internal noise at the input is 2μV. With amplification, the output signal is 2V, while the output noise is [illegible]. Calculate the noise figure.

Solution: $F = \frac{SNR_i}{SNR_o} = $ [illegible] $= 4$, what is the unit?

(5) A resistor 1kΩ is maintained at 290°K and is shunted by [illegible] capacitor. Determine the r.m.s noise voltage across the resistor over a frequency bandwidth of [illegible].

Solution: The resultant impedance: [illegible] and the resistive part is [illegible]

[illegible]

[6] Two resistors connected in series and at different temperatures are shown below. (a) derive relations for the noise equivalent resistor and temperature R_{eq}, T_{eq}. (b) calculate R_{eq}, T_{eq} for the case of [illegible]

Solution: [illegible]

[7] Estimate the [illegible] noise of a radio receiver with the first stage having a power gain of [illegible] If the received signal power is [illegible], for the noise temperature is [illegible]

Solution: [illegible]

[8] In a DSB system, the signal has a frequency range [illegible] and amplitude [illegible] If the input noise power is considered in the entire signal bandwidth, find: (a) SNR at the receiver input, (b) SNR at the receiver output.

Solution: [illegible]

[9] A [illegible] sinusoidal message signal modulates the carrier [illegible] The modulation scheme is [illegible] The channel noise is additive white with PSD of [illegible] at the receiver [illegible] (a) Find the signal power and the noise power at the output of [illegible] (b) SNR.

(14) Determine the effective input noise temperature of a receiver has average noise figure of 6dB.

(15) A carrier signal given by [illegible] is FM modulated by the signal [illegible] and sent through channel with noise [illegible]. If the modulation constant is [illegible]. (1) write the equation of the modulated signal, (2) sketch the spectrum, (3) calculate the bandwidth, (4) calculate the number of the significant sidebands, (5) calculate the input SNR at the receiver if the path loss [illegible] (6) [illegible] at the transmitter output, (7) calculate the output SNR of the receiver, (8) calculate the instantaneous frequency at [illegible], (9) calculate the instantaneous amplitude at [illegible].

(16) An amplifier operating over the frequency range from [illegible] has a [illegible] input resistor. What is the r.m.s noise voltage at the input to the amplifier if the ambient temperature is 27°C?

(17) Calculate the r.m.s noise voltage generated in a bandwidth of [illegible] by resistor [illegible] maintained at 27°C. Find the available noise power over that bandwidth. Re-calculate these values if [illegible].

(18) Ex. Example 4.7.1

(19) Ex. Problem 4.8.2

Part 5 Appendices

5.1 Appendix A: Basic Constants and Definitions

Pi = the ratio of a circle's circumference to its diameter: $\pi = 3.14159265 \ldots \left(\approx \frac{22}{7}\right)$

Planck's constant: $h = 6.62606896 \times 10^{-34}$ J·s

Boltzmann's constant: $k = 1.3806503 \times 10^{-23}$ J/K

Speed of light in a vacuum: $c = 299792458$ m/s

Temperature conversions: $°F = 1.8 \times °C + 32$ and $°K = °C + 273.15$

Greek Alphabets

Name	Lower Case	Upper Case
Alpha	α	A
Beta	β	B
Chi	χ	X
Delta	δ	Δ
Epsilon	ϵ	E
Eta	η	H
Gamma	γ	Γ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M
Nu	ν	N

Name	Lower Case	Upper Case
Omega	ω	Ω
Omicron	o	O
Phi	ϕ	Φ
Pi	π	Π
Psi	ψ	Ψ
Rho	ρ	P
Sigma	σ and ς	Σ
Tau	τ	T
Theta	θ	Θ
Upsilon	υ	Υ
Xi	ξ	Ξ
Zeta	ζ	Z

Unit scaling

Symbol	Prefix	Factor by which unit is multiplied
T	Tera	10^{12}
G	Giga	10^{9}
M	Mega	10^{6}
k	Kilo	10^{3}
h	Hecto	10^{2}
da	Deca	10

Symbol	Prefix	Factor by which unit is multiplied
d	Deci	10^{-1}
c	Centi	10^{-2}
m	Milli	10^{-3}
μ	Micro	10^{-6}
n	Nano	10^{-9}
p	Pico	10^{-12}

5.2 Appendix B: Some Identities

Quadratic formula: $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

Properties of Logarithms

$$\log_b 1 = 0$$
$$\log_b b = 1$$
$$\log_b(xy) = \log_b x + \log_b y$$
$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$
$$\log_b x^n = n\log_b x$$
$$\log_b x = \frac{1}{\log_x b}$$
$$x = b^{\log_b x}$$
$$x^n = b^{n\log_b x}$$
$$\log_a x = \log_a b \times \log_b x = \frac{\log_b x}{\log_b a}$$

Laws of Exponents

$$x^a x^b = x^{a+b}$$
$$x^a y^a = (xy)^a$$
$$(x^a)^b = x^{ab}$$
$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$
$$x^{-a} = \frac{1}{x^a}$$
$$x^{a-b} = \frac{x^a}{x^b}$$

Euler Identities

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

5.3 Appendix C: Phase-Locked Loop (PLL)

Synchronous (coherent) demodulation, using the PLL is the most widely method because of their low cost and superior performance, as it can track the phase and the frequency of the carrier component of an input signal.

The Phase-locked Loop (PLL) is a system where a locally generated signal is forced to follow the phase and frequency of input signal.

The phase detector compares the phase of the input signal to the feedback signal (locally generated), due to a signal $E(t)$ proportional to the phase difference between them. The loop filter produces a control voltage (V_c) and applies it to the VCO input in such a way as to force the VCO's output to follow the input signal.

The simplest type of phase detector is the multiplier.

So: $e(t) = AB\sin(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o)$

$$= \frac{AB}{2}[\sin(\theta_i - \theta_o) + \sin(2\omega_c t + \theta_i + \theta_o)]$$ the last term is removed by LF.

$v_o = \frac{AB}{2}\sin(\theta_i - \theta_o)$ is applied to VCO to control the frequency of the output.

The acquisition and locking performance of PLL is described by:

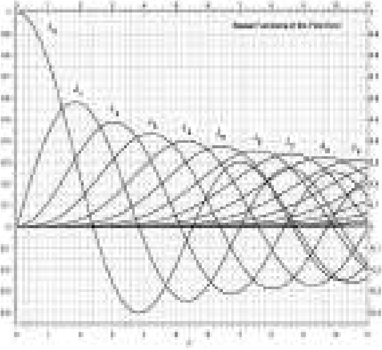
Lock-In Range: the frequency range over which the PLL acquires phase without cycle slipping.

Hold-In Range: the input frequency range over which the PLL can hold lock.

3.8 APPENDIX D: BESSEL FUNCTION

$J_n(\beta)$ = n^{th} order Bessel function of the first kind at argument β

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin\theta - n\theta)}\,d\theta$$



Good Luck