University of Anbar
College of Engineering
Department of Electrical Engineering

LECTURE NOTES IN

Fundamentals of Digital Communications

[illegible]

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[illegible]

[illegible]

"Education is what remains after one has forgotten what one has learned in school." Albert Einstein

[illegible]

COURSE SYLLABUS

Week	Second Semester Class [illegible]
1	The Sampling Theory (Ideal & Natural), Natural Sampling Rate & Aliasing in [illegible]
2	PCM
3	Non-Uniform Quantization & Companding, DM & DPCM
4	[illegible]
5	[illegible]
6	Channel Capacity, Multi-level Baseband Signaling [illegible]
7	Probability of Error in Reception
8	Examples & Problems
9	Introduction to Digital Modulation, BASK (Mod/Demod/Pe)
10	BPSK (Mod/Demod/Pe)
11	BFSK (Mod/Demod/Pe)
12	Comparison Through [illegible], Spectral Efficiency [illegible]
13	M-ary Modulation, QPSK (Mod/Demod/Pe)
14	QAM (Mod/Demod/Pe)

REFERENCES

- Introduction to Communication Systems, Ferrel G. Stremler
- Instructor's Lectures

BIBLIOGRAPHY

[1] Modern Digital and Analog Communication Systems, B. P. Lathi.
[2] Communication Systems: an Introduction to Signals and Noise in Electrical Communication, Bruce Carlson.
[3] Communication Systems Engineering, John G. Proakis and Masoud Salehi.
[4] Digital and Analog Communication Systems, Leon W. Couch.

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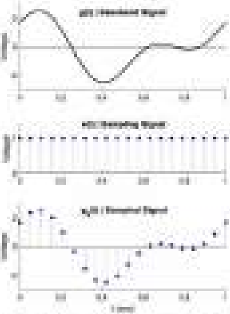
Part 1 Sampling & Pulse Modulation

1.1 INTRODUCTION

Experimental data and mathematical functions are frequently displayed as continuous curves, even though a finite number of discrete points were used to construct the graph. If these points or samples have sufficiently close spacing, we obtain a smooth curve drawn through them. Therefore, it can be said that a continuous curve is adequately described by its sample points alone.

Sampling, therefore, makes it possible to transmit messages in the form of pulse modulation rather than as continuous signals. Usually, the pulses are quite short compared to the time between them.



Pulse modulation offers two main advantages over continuous-wave modulation:

(1) The transmitted power can be concentrated into short bursts instead of being generated continuously.

(2) The time interval between pulses can be filled with sample values from other signals (TDM).

However, the main disadvantage is the requirement for large transmission bandwidth compared to the original message bandwidth.

1.2 Sampling Theorem

1.2.1 Ideal Sampling

Ideal sampling* is done by multiplying the baseband signal $g(t)$ by the impulse signal $\delta_T(t)$.

$$g_\delta(t) = g(t)\cdot\delta_T(t)$$

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The Fourier transform of $\delta_T(t)$ is

$$\Delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad \text{where } f_s = \frac{1}{T_s}$$

Let's now evaluate $G_\delta(f)$, the Fourier transform of the sampled signal $g_\delta(t)$. As

$$G_\delta(f) = \mathcal{F}[g_\delta(t)] = \mathcal{F}[g(t)\cdot\delta_T(t)]$$

Multiplication in the time domain is convolution in the frequency domain:

$$G_\delta(f) = G(f) * \Delta(f) = G(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Where * denotes convolution. Now, we apply simple properties of convolution to move $G(f)$ inside the sum, then $G_\delta(f)$ becomes

$$G_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(f) * \delta(f - nf_s)$$

By applying the shifting property of the delta function:

$$G_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(f - nf_s) = \frac{1}{T_s}\left[\begin{array}{l} G(f) + G(f - f_s) + G(f + f_s) + G(f - 2f_s) + \cdots \\ + G(f + 2f_s) + G(f - 3f_s) + G(f + 3f_s) + \cdots \end{array}\right]$$

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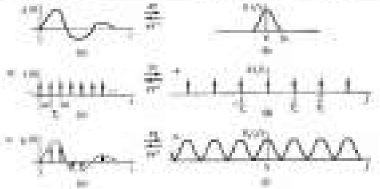
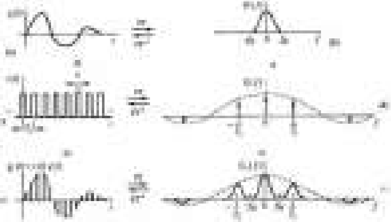
* [illegible]



1.2.2 Natural Sampling

The practical method of sampling is a naturally sampled signal which is produced by multiplying the baseband (with modern) signal $g(t)$ by the periodic pulse train $c(t)$.



Every real value from $g_s(t) = F[g_s(t)] = F[g(t) \cdot c(t)]$

And because $c(t)$ is a periodic signal

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t}$$

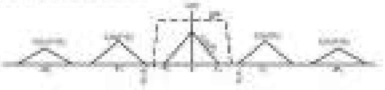
Where C_k is the Fourier series coefficient $C_k = \frac{1}{T_s}\,\text{sinc}\left(\frac{k\tau}{T_s}\right)$. So

$$G_s(f) = F\left\{g(t)\sum_{k=-\infty}^{\infty} C_k e^{j2\pi k f_s t}\right\} = \sum_{k=-\infty}^{\infty} C_k F\{g(t)e^{j2\pi k f_s t}\} = G_s(f) = \sum_{k=-\infty}^{\infty} C_k G(f - kf_s)$$

The spectrum $G_s(f)$ consists of many copies of $G(f)$ added together, where the k^{th} copy is shifted by kf_s and multiplied by C_k.

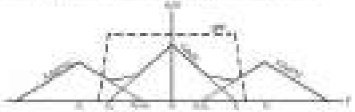
1.2.3 Reconstruction

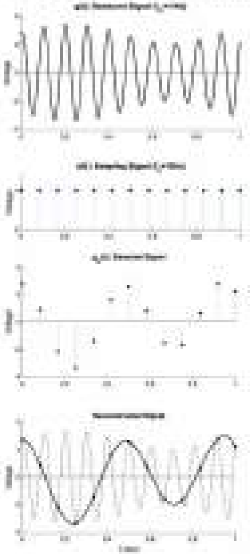
To perform successful sampling, we keep $f_s - f_m \geq f_m$, i.e., $f_s \geq 2f_m$. This ensures the signal $G(f)$ always contained perfectly in $G_s(f)$. As a result, $G(f)$ can be recovered exactly by simply passing $G_s(f)$ through a LPF that passes only $C_0G(f)$. Finally, we introduce a gain of $1/C_0$ in the LPF to restore the original signal.



1.2.4 Aliasing

When $f_s < 2f_m$, the copies overlap and hence the perfect reconstruction becomes impossible. If the Nyquist criterion is not considered, the folded back portion overlaps the original spectrum. This results a new shape of the reconstructed spectrum filtered by LPF. And this undoubtedly gives a signal different from $g(t)$. This problem is called "aliasing".





Two conditions are necessary to avoid the aliasing:

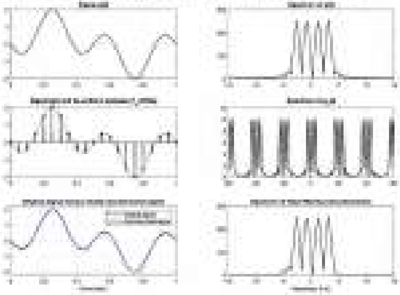
(1) The input signal must be limited according to Nyquist conditions i.e. $f_m \le \frac{f_s}{2}$.

(2) The sampling frequency must sufficiently greater than the maximum frequency component of the signal, i.e. $f_s \ge 2f_m$. This condition is called "Nyquist Rate".

The illustration below shows a bandlimited signal whose spectrum extends up to 150Hz. If we want to perform Nyquist sampling at the rate 200Hz, we must use an anti-aliasing filter whose cutoff frequency equals $\frac{f_s}{2}$ = 100Hz.

Another example from practical voice transmission systems. Although the average voice spectrum extends well up to 10kHz, most of the energy is concentrated in the range 300Hz–3.4kHz. For intelligibility, the bandwidth 4kHz is sufficient. A voice wave is pre-filtered by a 3.4kHz LPF and then sampled at 8kHz. These are the standard values for telephone sampling.

In the following illustration, we set $f_s = 20$Hz for $g(t) = \cos(2\pi t) + \cos(\pi t)$ [illegible]



Exercises:

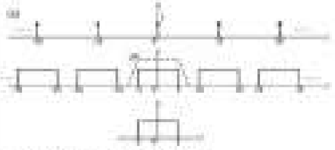
(1) The waveform $x(t) = 20 + 20\sin(500t + 30^\circ)$ is to be sampled periodically and later to be reproduced from these sample values. [a] Find the maximum allowable time interval between samples. [b] How many samples are needed to be stored to reproduce 1 second of this waveform?

(2) Determine the minimum (Nyquist) sampling rate for the signal $x(t) = x_1(t) + x_2(t)$ where $x_1(t)$ is band-limited to B_1 Hz and $x_2(t)$ is band limited to B_2 (Hz). [illegible]

(3) A signal $x(t)$ is ideally sampled at a sampling rate (a) 15Hz, (b) 9Hz, (c) 18Hz. If the sampled signal at the receiver is passed through an ideal LPF with cutoff frequency

[illegible] distorting.

Solution:



We see that there is no aliasing [illegible]

Now work for (b) and (c) and compare!

(c) Repeat the above example if the cutoff frequency of the filter is [illegible] and the analog signal is [illegible]

Solution: [illegible] can be written as:

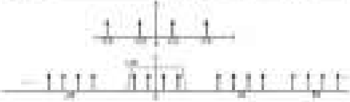
[illegible]

The spectrum of the ideally sampled signal is given by:

[illegible]

So, the spectrum of our sampled signal will be:

[illegible]



As shown above, there is no aliasing, and the cutoff frequency of the filter is suitable, and the output consists of the original signal only. We have no distortion.

The output of the LPF is:

$X(f) = 250[\delta(f - 15) + \delta(f + 15)] - [\delta(f - 75) + \delta(f + 75)]$

Taking the inverse Fourier transform of this equation, we get:

$x(t) = 50\cos(30\pi t) - 50\sin(150\pi t)$

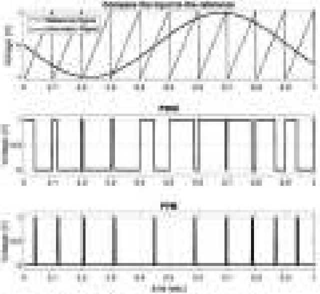
(5) Suppose $x(t)$ has the spectrum in the figure below, with $f_c = 25kHz$ and $B = 10kHz$. Sketch $X_\delta(f)$ for $f_s = 100kHz$, $50kHz$, $25kHz$ if ideal sampling is considered. Comment in each case on the possible reconstruction of $x(t)$ from $x_\delta(t)$.

(6) A signal $x(t)$ is band limited to 5kHz. It is sampled at a sampling rate of 10kHz using ideal sampling (impulses). At receiver location, the sampled signal is filtered by an ideal band pass filter that passes a band of 5Hz bandwidth centered on 10kHz. Discuss the form of the signal $y_r(t)$.

1.3 Pulse Modulation

Pulse modulation describes the process whereby the amplitude, the width or the position of individual pulses in a periodic pulse train are varied (i.e., modulated) in sympathy with the amplitude of a baseband information signal m(t).

The pulses of PAM have:	amplitude→variable,	width→fixed,	position→fixed
The pulses of PWM have:	amplitude→fixed,	width→variable,	position→fixed
The pulses of PPM have:	amplitude→fixed,	width→fixed,	position→variable

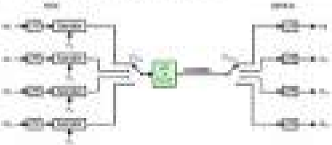


What modulation scheme could be used in a simple TV remote controller?

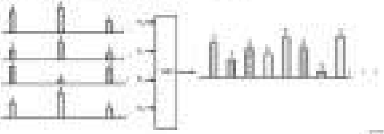
1.6 Time Division Multiplexing (TDM)

The use of short pulse width in PAM leaves sufficient spaces between samples for insertion of pulses other sampled signals. The method of combining several sampled signals in a defined sequence is called TDM.

The block diagram below demonstrates the PAM-TDM principle. In this system, 4 different signals are sampled at the same rate f_s (samples per seconds). The clock frequency f_{clk} must be fast enough to send the all the samples without missing a sample from a signal (in this case $f_{clk} = 4f_s$). The input signals are pre-filtered to prevent the aliasing. Complete synchronization between MUX and DEMUX is critical for correct reception.



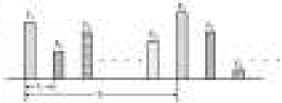
The diagram below illustrates the concept of multiplexing



Frequency), we need to understand the exact design for a TDM system. In the diagram below, let T_s be the time spacing between adjacent samples in a TDM signal. If all input signals are sampled equally at $f_s = 1/T_s$ then $T_s = T_s/N$, where N = number of multiplexed sampled signals. The output TDM pulse frequency (clock frequency) is:

$$R_s = \frac{1}{T_s} = \frac{N}{T_s} = Nf_s$$

Where in TDM



Examples:

(1) 10 analog signals (each is bandlimited to 5kHz) are transmitted simultaneously over a channel via the TDM-PAM system. Compute the required: (1) sampling frequency, (2) TDM clock frequency, (3) TDM output transmission rate.

(2) Several bandlimited signals each of a bandwidth of 10kHz are set to be sampled at the sampling frequency 40% over the Nyquist frequency then multiplexed through a channel with a bandwidth of 220kHz, what would be the maximum number of signals that can be multiplexed?

(3) Determine the minimum transmission bandwidth required to transmit the following signals: $g_1(t) = 20\cos(200\pi t)$, $g_2(t) = 10\cos(100\pi t)\cos(200\pi t)$, and $g_3(t) = 14\cos(150\pi t) + 25\cos(400\pi t)$ in a PAM/TDM system.

Solution: With TDM, $g_1(t) = 20\cos(200\pi t)$

$g_2(t) = 10\cos(100\pi t)\cos(200\pi t) = 5\cos(100\pi t) + 5\cos(300\pi t)$

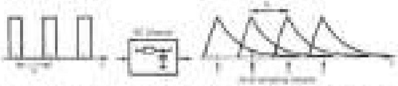
$g_3(t) = 14\cos(150\pi t) + 25\cos(400\pi t)$

Maximum frequency 200Hz (from the spectrum), and check

$f_s = 2f_{max} = 400Hz \to f_s = Nf_s = 3 \times 400 = 1200Hz \; ; \; B_T = \frac{R}{2} = 600Hz$

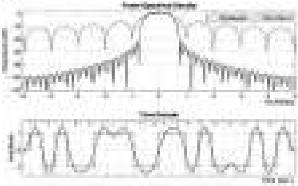
Crosstalk and Guard Times

In addition to rejecting the non-linearities, TDM systems must avoid inter-channel crosstalk. A TDM signal suffers from crosstalk if the transmission channel results in pulses whose tails overlap into the next time slot of the frame.



If we assume the transmission channel acts like a 1st order LPF, the response to a rectangular pulse is an exponential decay. To reduce the cross talk, the transmitted pulses must be separated by t_g. The guard time t_g is the minimum pulse spacing so that the tail of the pulse decays to a value less than A_{ct} by the time next pulse arrives.

Pulse Shaping



1.5 Pulse Code Modulation (PCM)

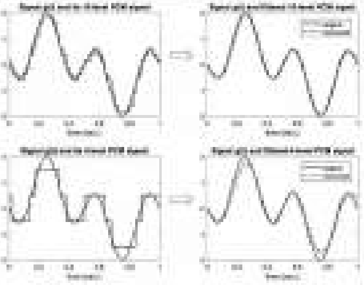
After sampling an analog signal it is possible to code the samples using discrete symbols, such as binary. The communication through binary alphabets is more efficient than the analog transmission. If the value of a sample is sent using only two possible elements, the reception becomes more reliable. It is easier for the receiver to discriminate the reception between two possible signals than estimating the value of a continuous signal. This significantly helps in reducing the effects of distortion and of additive noise.

In PCM, the sample values are rounded to the values of certain levels. The rounding off operation is known as "quantization". Then each sample is coded into a binary number which is equivalent to the value of the quantization level that is closest to the sample value.

1.5.1 Quantization

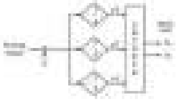
When an instantaneous signal is pulse amplitude modulated, it becomes discrete in time only. It remains analogue in amplitude since all the values within the specified range are allowed. PAM signal is said to be quantized when each pulse of the PAM signal is adjusted in amplitude to coincide with the nearest level within a finite set.

It is clear from the figure below, quantization error (noise) can be reduced by increasing the number of quantization levels (L), i.e. decreasing the step size (Δ) between the levels.



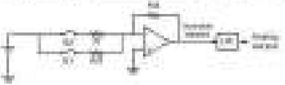
1.5.2 Encoding

[illegible]

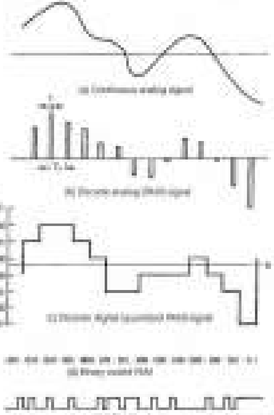


1.5.3 Decoding

[illegible]



The PCM technique is considered to be Analog to Digital Converter (ADC) at the transmitter and Digital to Analog Converter (DAC) at the receiver. The following figure illustrates the A/D



(a) Continuous analog signal

(b) Discrete analog (PAM) signal

(c) Discrete digital (quantized PAM) signal

(d) Binary coded PAM

(e) PCM signal

The complete block diagram of PCM system is:

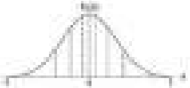
Modulator:

Demodulator:

1.3.4 Non-Uniform Quantization

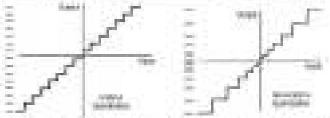
Uniform quantization assumes that the information signal has uniform PDF i.e. all quantization levels are used equally. For most signals it is not the case.

[illegible]



[illegible]

[illegible]



Practically, such optimization is a difficult procedure because it requires prior knowledge of the signal PDF. Problems arise if the information signal has an unknown PDF or if its PDF changes with time. However, they are similar in some signals. For example, in the case of voice signals, the PDF shape of different speakers is usually similar, but the power levels can vary widely between speakers, e.g. due to shouting and low softly spoken. Therefore, the approach taken in practice is to use uniform quantizing after nonlinear compression (i.e. companding).

COMPANDING (compression-expanding)

This is the process of compressing the information signal prior to linear quantization at transmission. The compression is achieved via non-linear amplitude characteristic circuit.

The receiver expands the reconstructed signal with the inverse characteristic to restore the original waveform.

So, PCM with companding system will be:

Companding Methods

There are two types of companders that are practically and widely used for speech coding:

(1) The μ-law compander (used in the US, Canada and Japan). The parameter μ controls the amount of compression and expansion. The standard compressor uses $\mu = 255$ followed by a uniform quantizer with 128 levels (7 bits per sample).

$$F(x) = \frac{\ln(1+\mu|x|)}{\ln(1+\mu)}$$

(2) The A-law compander (used in most countries). A is chosen to be 87.56.

$$F(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases}$$

[illegible] Signal to Quantization Noise Ratio

It is important to consider the quantization noise in the overall system quality. To calculate $(S/N)_q$ of a uniformly quantized signal, it is suitable to make the following assumptions:

(1) Linear quantization (i.e., equal increments between quantization levels).
(2) Zero mean signal (i.e., symmetrical of PDF around the 0 Volt).
(3) Uniform signal PDF (i.e., all signal levels equally likely).

Let L be the number of the levels of the quantizer, l be the number of bits per a PCM word ($L = 2^l$), and x_p be the peak (range) level of the quantizer. The quantization interval becomes:

$$q = \frac{2x_p}{L}$$

The PDF of the allowed levels is given by:

$$p(x) = \sum \frac{1}{L}\delta\left(x - \frac{iq}{2}\right)$$

The mean square signal after quantization is:

$$\overline{x^2} = \int x^2 p(x)\,dx = \frac{2}{L}\left[\int x^2\delta\left(x-\frac{q}{2}\right)dx + \int x^2\delta\left(x-\frac{3q}{2}\right)dx + \cdots\right]$$

$$= \frac{2}{L}\left(\frac{q}{2}\right)^2\left[1^2+3^2+5^2+\cdots+(L-1)^2\right] = \frac{2}{L}\left(\frac{q}{2}\right)^2\left[\frac{L(L-1)(L+1)}{6}\right]$$

$$\therefore \overline{x^2} = \frac{q^2}{12}\left(L^2-1\right)$$

Denoting the quantization error by e_q, the difference between the unquantized and quantized signal(s) as e_q, then the PDF of e_q is uniform:

$$p(e_q) = \begin{cases}\frac{1}{q} & -\frac{q}{2} \le e_q \le \frac{q}{2}\\ 0 & \text{elsewhere}\end{cases}$$

The mean square quantization error (noise) is:

[illegible]

Therefore, the average $(SQNR)$ will be:

[illegible]

Since the peak signal level is [illegible], then the peak $(SQNR)$ will be:

[illegible]

We may express it in decibels as [illegible]

Where α = [illegible] for the peak $(SQNR)$ and α = [illegible] for the average $(SQNR)$. This equation is called the [illegible] rule, and it points out that an additional [illegible] improvement in the $(SQNR)$ is obtained for each bit added to the PCM word.

Examples

(1) Consider the voltage of a bandlimited signal varies over [illegible]. This signal is sampled and quantized through a [illegible] bits per level quantizer. What is the quantization error for the samples whose voltages [illegible]?

Solution: [illegible] per sample so L = [illegible] levels in the quantizer. The maximum input voltage is [illegible]. The minimum input voltage is [illegible].

Then [illegible]. The step size [illegible].

The threshold voltages start from [illegible] step-by-step.

The voltage of a quantization level is the average of the two adjacent threshold voltages. So, the following table is constructed as below:

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Note: maximum [illegible]

Threshold Voltages	Q-Level Voltages	Q-Level Number
3		
	2.625	1
2.25		
	1.875	2
1.5		
	1.125	3
0.75		
	0.375	4
0		
	-0.375	5
-0.75		
	-1.125	6
-1.5		
	-1.875	7
-2.25		
	-2.625	8
-3		

(12) Repeat example [illegible] for L = 4 levels. Compare and Discuss.

(13) Consider the voltage of a baseband signal varies over the range [illegible]. This signal is sampled and quantized through an 8 levels quantizer. What is the quantization value results for each of the samples [illegible]?

(14) In a PCM system, [illegible] is to be held to minimum of [illegible]. Determine the number of required quantizing levels L and find the corresponding output [illegible].

Answer: [illegible]

(15) A binary channel with maximum allowable rate is [illegible] bps is available for PCM voice transmission. Find the appropriate values of the sampling frequency and quantization levels (assume voice signals are [illegible]).

Solution: [illegible]

(15) An analog signal with a bandwidth of 4.2k Hz is to be converted into binary PCM and transmitted over a channel. The peak signal to quantization ratio at the receiver output must be at least 55dB. (a) If we assume that [illegible] = 8 and that there is no ISI, what will be the word length and the number of quantization steps needed? (b) What will be the equivalent bit rate? (c) What will be the channel bandwidth required if rectangular pulse shapes are used?

Solution: (a) [illegible] levels. (b) [illegible] = 75.6kbps. (c) [illegible] = 37.8kHz.

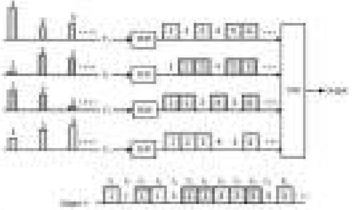
(17) A digital recording system samples each of two stereo input signals with a 16-bit ADC at 44.1k samples per second. (a) Determine the output data rate for a full-scale [illegible]. (b) The bit stream of digitized data is augmented by the addition of error-correcting, clock extraction, display and control bit fields. These additional bits represent 100 percent overhead. Determine the output bit rate of the system. (c) If we would like to store an hour's worth of music using this system, determine the memory size required. (d) For a comparison, a high-grade collegiate dictionary may contain 1500 pages, 2 columns per page, 100 lines per column, 8 words per line, 6 letters per word and 7 bits per letter (all on average). Determine the memory size required to store it.

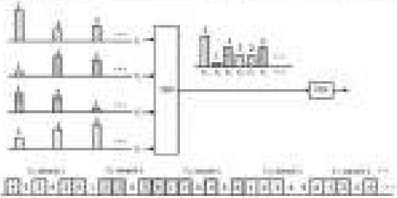
Solution: (a) [illegible] = 1.4112Mbps
And including the overhead bits rate = 2×1.4112 = 2.8224Mbps.
(c) [illegible] (d) 1500×2×100×8×6×7 = [illegible]
So, if we include the overhead bits for the book, we can store [illegible] of such huge book instead of one hour of music!!!

(18) An audio signal band limited to 300–3300Hz is sampled at the sampling rate 8000 samples per second. If the required average SNqR is 30dB: (a) what is the number of levels L needed and the corresponding code length? (b) Calculate the required system bandwidth.

Answer: (a) 5 bit per sample (b) 20kHz

1.5.6 PCM Multiplexing





[illegible]

Exercises

(1) A signal $m(t)$ band-limited to 3kHz is sampled at a rate $33\frac{1}{3}\%$ higher than the Nyquist rate. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak amplitude m_p. The quantized samples are binary coded. Find the minimum bandwidth of a channel required to transmit the encoded binary signal. If 24 such signals are time-division multiplexed, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

Solution: The Nyquist sampling rate $= 2 \times 3000 = 6000\text{Hz}$ (samples per second). The actual sampling rate $= 6000 \times \frac{4}{3} = 8000\text{Hz}$.

The quantization step is Δv, and the maximum quantization error is $\pm\Delta v/2$. Therefore $\frac{\Delta v}{2} = \frac{m_p}{L} = \frac{0.5}{100} m_p \Rightarrow L = 200$. For binary coding, L must be a power of 2. Hence, the next higher value of L that is a power of 2 is $L = 256$. We need $n = \log_2 256 = 8$ bits per sample. We require to transmit a total of $R_b = 8 \times 8000 = 64000\text{bps}$. Because we can transmit up to 2 bits per second per hertz of bandwidth, we require a minimum transmission bandwidth $B_T = R_b/2 = 32\text{kHz}$. The multiplexed signal has a total of $R_{total} = 24 \times 64000 = 1.536\text{Mbps}$, which requires a minimum of $1.536/2 = 0.768\text{MHz}$ of transmission channel bandwidth.

(2) Find the channel bandwidth required for transmission of a TDM binary signal consisting of 24 voice signals each with 4kHz bandwidth, 8k sample per second and 256 quantizing levels.

Solution: $n = \log_2 L = 8$ bits; one channel rate $= 8 \times 8k = 64$ kbps;
TDM rate $= 24 \times 64k = 1.536$ Mbps; $B = 768$ kHz

(3) Five telemetry signals, each of bandwidth 1kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 0.5% extra bits. Determine the minimum possible data rate that must be transmitted, and the minimum bandwidth required to transmit this signal.

(4) Twenty-three analog signals each with a bandwidth of 3.4kHz, are sampled at an 8kHz rate and multiplexed together with a synchronization channel (8kHz) into a TDM PAM signal. Calculate (a) commutator frequency, (b) the overall TDM PAM pulse rate, (c) The channel bandwidth required for the channel.

(5) Several high-fidelity audio channels having $f_m = 15$kHz are to be transmitted via binary PCM with $L = 32$. Determine how many of PCM signals can be multiplexed in a 1.4 Mbps channel. Then calculate the corresponding bandwidth efficiency.

Quality Factor Required Bits

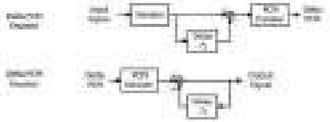
In addition to the channel effects, PCM performance depends primarily on the quantization noise. To make the reconstructed signal like the original baseband signal we must reduce the quantizing noise by increasing L. Now we learned that increasing the number of quantization levels requires more bits per sample to be transmitted. Large L is not preferred (costly and due to noise channel's limitation). So, beside the non-uniform quantization, several techniques are used to reduce the quantization noise at the same number of bits L.

1.6 BANDWIDTH REDUCTION TECHNIQUES

The channel bandwidth is limited, and it is a valuable resource. A frequent objective of the communications engineer is to transmit the maximum information rate via the minimum possible bandwidth. This is especially true for radio communications in which radio spectrum is a scarce and expensive valuable resource. The following systems are used to represent the same coding fidelity using fewer bits per sample.

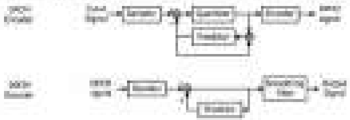
1.6.1 Delta PCM

Because the samples of most of the baseband signals are highly correlated, it is possible to transmit the information about the changes between samples instead of sending the sample values themselves. A simple way for such operation is the Delta PCM. This method transmits the difference between adjacent samples through code words. This difference is significantly less than the actual sample values (hence it is coded using fewer binary symbols per word than the conventional PCM). However, Delta PCM systems cannot accommodate rapidly varying baseband signals.



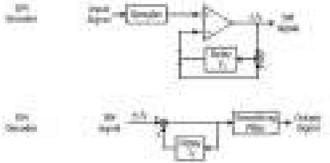
1.6.2 Differential PCM

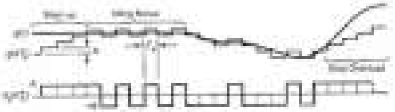
Since neighbor samples within many information signals are highly correlated, Differential PCM (DPCM) uses an algorithm to predict future values. Such algorithms monitor the trend of the baseband samples and use some models to predict the value of the incoming samples. Then DPCM waits until the actual value becomes available for examination and transmits the correction to the already predicted value. The correction signal represents the information signal's unpredictable part. By this means, DPCM reduces the redundancy in signal and allows the information to be transmitted using fewer symbols, less spectrum and shorter time.



1.7 Delta Modulation (DM)

If the quantizer of the DPCM system is constrained to two levels (i.e., the step levels $\pm\Delta$) and the predictor is one sample delay, then the resulting scheme is called DM. The information signal is represented by a stepped waveform. The resolution of this waveform depends on Δ & T_s values.

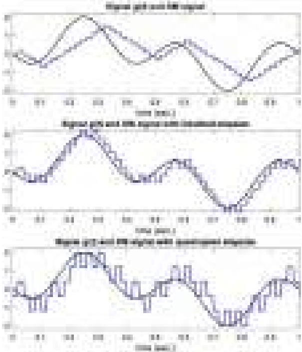




We observe from the figure above that:

1. The system might not be suitable (require a few time steps) to catch the required level of the input.
2. If the input is nearly constant, the reconstructed DM waveform exhibits ripple known as idling noise which almost filtered out at the receiver.
3. The case of slope overload problem occurs when the input changes too rapidly for the stepped waveform to follow.

To resolve these problems, we are clearly in a dilemma about how to choose the two parameters Δ and T_s. A small step size Δ is desirable for accuracy, but the bit rate ($f_s = 1/T_s$) should be fast to avoid slope overload, which is not recommended. Generally, as Δ get smaller, many small details of the information signal will be lost. This dilemma is illustrated in the figures below.



However, we can calculate the optimum values for these parameters that overcome the slope overload problem. As an example of the use of this condition for DM may be obtained with the sinusoidal modulation. Let the input be $g(t) = b\sin(\omega_m t)$, so that

$\max\left|\frac{dg(t)}{dt}\right| = b\omega_m$ = maximum slope of the signal

Since the maximum rate of rise $= \frac{\delta}{T_s} = \delta f_s$, then

$$\delta f_s \geq b\omega_m \;\Rightarrow\; \delta \geq \frac{2\pi b f_m}{f_s} \quad \text{or} \quad b \leq \frac{\delta f_s}{2\pi f_m}$$

The above δ condition may be applied to modulated signals by letting f_m be the highest frequency component, and $b = \max|g(t)|$.

Since the quantization noise $\overline{\sigma^2} = \int_{-\delta}^{\delta} \frac{x^2}{2\delta}\,dx = \frac{\delta^2}{3}$

And by filtering the noise to a bandwidth B, we get $N_q = \overline{\sigma^2} = \frac{\delta^2}{3}$

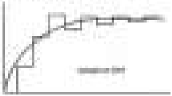
The mean square value of the information signal is

$S = \overline{g^2(t)} = \left(\frac{b}{\sqrt{2}}\right)^2 = \frac{b^2}{2} = \frac{1}{2}\left(\frac{\delta f_s}{2\pi f_m}\right)^2$

$$\therefore SNR = \frac{3 f_s^2}{8\pi^2 f_m^2}$$

1.7.3 Adaptive Delta Modulation

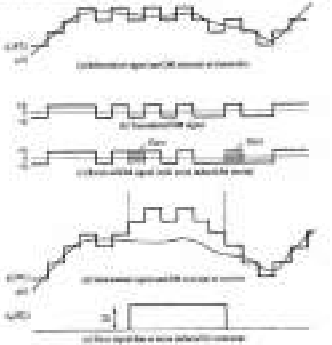
We have seen that: a large step size causes unacceptable quantization noise, and a small step size results in slope overload distortion. This means that a good choice for Δ is a "medium" value, but in some cases, the performance of the best "medium" value is not satisfactory. An approach that works well in these cases is to change the step size according to changes in the input. If the input tends to change rapidly, the step size is chosen to be large (and vice versa) for the output to follow the input quickly without distortion.



1.7.2 Errors in DM

If the SNR is not sufficiently high, then the DM receiver will occasionally [illegible] a received symbol in error (i.e. [illegible] instead of [illegible] or the reverse), and this is equivalent to the addition of an error of [illegible] to the accumulated signal at the DM receiver, as shown below. This [illegible]

DM is primarily used for [illegible] and speech transmission in telephony. It has been found that PCM is preferable for high quality speech transmission, whereas DM is easier to implement and yields transmission of acceptable quality.



Exercises

(1) What is the [illegible] amplitude for a [illegible] sinusoidal input for a DM system that [illegible] slope overload that has been sampled at [illegible] times the Nyquist rate with [illegible]?

Answer: [illegible]

(2) What is [illegible] step overload for DM system where the signal is a [illegible] sinusoidal given [illegible]?

(3) A DM system is tested with [illegible] sinusoidal [illegible] peak to peak, at the input. The signal is sampled at 10 times the Nyquist rate. (a) What step size required [illegible] slope overload? (b) Find the SNR.

(4) A delta modulator is designed to operate at four times the Nyquist rate for a signal with a [illegible] bandwidth. The quantizing step size is [illegible]. Determine: (a) the maximum amplitude of a [illegible] input sinusoid for which the delta modulator does not show slope overload. (b) the post filtered output signal-to-quantizing noise ratio for the signal in part (a).

Solution: (a) [illegible]

[illegible]

[illegible]

[illegible]

(b) [illegible]

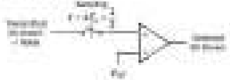
1.21 Probability of Error at Reception

The detection of digital signals involves two processes:

(1) Reduction of each received voltage pulse (i.e., symbol) to a single numerical value, (just like quantization).

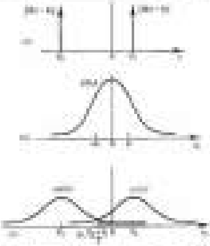
(2) Comparison of this value with a reference voltage to determine which symbol was transmitted.

The output of binary symbols (0 and 1) are represented by two voltage levels (e.g., 0V and 5V). Intuition tells us that a sensible strategy would be to set the reference V_{ref} half-way between the two voltage levels (i.e., at 2.5V).



In general, for equiprobable symbols the decision level is set at $\mu = \frac{V_1+V_0}{2}$. For unequal transmission of the symbols, we need to determine the optimum threshold using more advanced mathematics.

Since the noise with a Gaussian PDF is common and analytically tractable, the bit error rate (BER) of a communication system is often modeled assuming Gaussian noise above.



[illegible]

Let the probability of sending symbol 0 (the voltage level V_0) is:

$$p_0(V_0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(V_0)^2}{2\sigma^2}}$$

And the PDF of sending symbol 1 (the voltage level V_1) is:

$$p_1(V_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(V_1)^2}{2\sigma^2}}$$

Then if the symbol 0 is transmitted, let P_{e0} be the probability of the received signal plus noise that is above the threshold at the decision instant (i.e., seen as voltage level V_1), (shaded area under the curve $p_0(v_0)$ in Figure (c)). The equation of the probability of error is:

$$P_{e0} = \int_{V_T}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(V_0)^2}{2\sigma^2}} dV_0$$

Also, if the digital symbol 1 is transmitted let P_{e1} be the probability that the received signal plus noise is below the threshold at the decision instant (i.e., seen as voltage level V_0).

So:

$$P_{e1} = \int_{-\infty}^{A/2} \frac{1}{\sigma\sqrt{2\pi}} e^{-(v_0 - A)^2/2\sigma^2} dv_0$$

It is clear from the symmetry of this problem that P_{e0} is identical to P_{e1}, and for equiprobable symbols $p_0(s_0) = p_1(s_1) = \frac{1}{2}$ the mean probability of error P_e will be:

$$P_e = p_0 P_{e0} + p_1 P_{e1} = \frac{1}{2}(P_{e0} + P_{e1}) = \int_{A/2}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-v_0^2/2\sigma^2} dv_0$$

using $z = \frac{(v_0 - A/2)}{\sigma}$ $\Rightarrow P_e = \frac{1}{\sqrt{2\pi}} \int_{A/2\sigma}^{\infty} e^{-z^2/2} dz$

This integral cannot be evaluated analytically but it can be found as a complementary error function, which is defined by:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-z^2/2} dz$$

Thus: $$P_e = Q\left[\frac{A}{2\sigma}\right]$$

The advantage of using the Q-function in the expression for P_e is that this function has been extensively tabulated.

For unipolar binary ($s_0 = 0$, $s_1 = A$ $\rightarrow \rho = \frac{A}{2}$) the average signal power is $S = \frac{(0 + A^2)}{2}$ and the average noise power is $N = \sigma^2$, so that: $P_e = Q\left[\sqrt{\frac{S}{2N}}\right]$

For polar binary ($s_0 = -\frac{A}{2}$, $s_1 = \frac{A}{2}$ $\rightarrow \rho = \frac{A}{2}$) the average signal power is $S = \frac{(A/2)^2 + (A/2)^2}{2}$ and the average noise power is $N = \sigma^2$, we get: $P_e = Q\left[\sqrt{\frac{S}{N}}\right]$

Therefore, the averaged transmitted power for the unipolar signal must be twice that of the polar binary signal to achieve the same probability of error. For equiprobable signals, the polar binary signalling also has an advantage in the optimum decision threshold. It is simply set at the zero Volt, whereas the receiver for the ON-OFF binary signalling the threshold must be adjusted to half the amplitude of the received signal.

1.9 Channel Capacity

For any communication system, it is required to send data as fast as possible. But in the presence of noise and distortion along the channel, it is very hard to avoid errors at the receiver. The Shannon-Hartley theorem of channel capacity states that the maximum rate of information transmission R_{max} over a channel of the bandwidth B and the received signal to noise (SNR) is given by:

$$R_{max} = C = B\log_2\left(1+\frac{S}{N}\right) \quad bps$$

Where C = channel capacity = the maximum rate at which information can be transmitted across that channel without error. It is measured in bits per second (bps).

So if the bit signaling (transmission) rate:

- If $R \le C$, it is possible to transmit data with small probabilities of error, even with noise.
- If $R > C$, errors cannot be avoided regardless of the coding technique used.

If we would like to increase R_{max} in the above equation, we can increase B and/or the SNR.

- Larger SNR implies resorting to, for example, higher Tx powers or shorter distances. In some cases, this is not that easy to achieve. But in general, as the SNR increased, it would also increase without a limit the capacity of channel. However, you must note that, when $N \to 0$ then SNR $\to \infty$ and hence $R_{max} \to \infty$ regardless of B (is it possible?).
- As for increasing B, it requires changing the medium or buying a license for extra bandwidth. In general, as B increased, it can follow faster changes in the information signal, thereby increasing R. Nevertheless, when $B \to \infty$, C does not approach ∞. The noise is assumed to be white: the wider the bandwidth, the more the noise admitted to the system. This means, as B increases, SNR decreases [illegible].

Theoretical Capacity

Suppose that the noise is white noise PSD $\eta/2$ (W/Hz), and assume the received signal power is fixed at a value S (W), the channel capacity would be: $R_{max} = C = B\log_2\left(1+\frac{S}{\eta B}\right)$

Now, when $B \to \infty$ we get:

$$C = \lim_{B\to\infty} B\log_2\left(1+\frac{S}{\eta B}\right) = \lim_{B\to\infty}\left\{\frac{S}{\eta}\log_2\left(1+\frac{S}{\eta B}\right)^{\frac{\eta B}{S}}\right\} = \frac{S}{\eta}\log_2 e$$

$$\therefore C = 1.44\frac{S}{\eta}$$

This gives the maximum possible channel capacity as a function of the received signal power and the noise PSD. In actual systems design, the channel capacity might be compared to this value to decide whether a further increase in B is worthwhile.

Examples

(1) Let $S/\eta = 25dB$ at the receiver input. (a) What is the maximum possible rate of baseband data transmission without errors? Assume the channel bandwidth is 1kHz, 2kHz, 3kHz, 5kHz, and 1 MHz. (b) Repeat (a) if the channel bandwidth is increased to be very large (unlimited).

Solution: $\frac{S}{\eta} = 10^{2.3} = 199.526$, so $R_{max} = R_b$, $C = B_{ch}\log_2\left(1+\frac{S}{\eta B_{ch}}\right)$

At $B_{ch} = 1kHz$, $C = 252.0550bps$

At $B_{ch} = 2kHz$, $C = 271.2080bps$

At $B_{ch} = 3kHz$, $C = 279.0497bps$

At $B_{ch} = 5kHz$, $C = 282.2950bps$

At $B_{ch} = 1MHz$, $C = 285.032 bps$

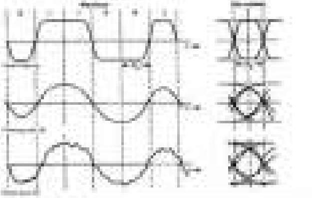
(b) we can assume $1/TB^{9}$ large number, which gives $C = 287.8026bps$. But it is more practical and easier to use the approximation when $B_{ch} \to \infty$:

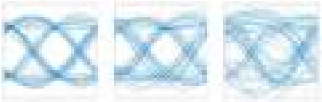
$C = 1.44\frac{S}{\eta} = 287.3129bps$ which is relatively close.

1.13 Inter-Symbol Interference (ISI)

Previously, we discussed the crosstalk problem (distortion caused by time dispersion). This results in spreading of time signals and overlapping among adjacent bit waveforms. This overlap is also known as ISI. It is caused not only by channel distortion but also by multi-path effects.

To illustrate the ISI problem, we should first introduce the eye pattern.





ISI: Good   ISI: Fair   ISI: Bad

To reduce the ISI, we need:

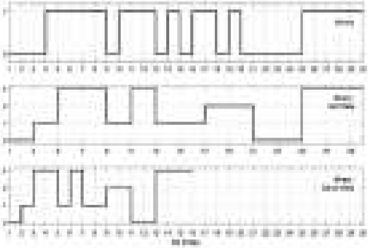
(a) Consider the Nyquist criterion for the maximum rate.

(b) Frequency limitation of the transmitted signal to B.

(c) Reduce the effects of the multi-path problem (Fading).

1.11 Multi-Level Baseband Signaling (M-ary)

Binary PAM keying means sending a single bit over the symbol interval T_b (as in the bit rate $1/T_b$ bits/sec). To increase the data transfer rate, it is possible to combine several bits in one symbol in this technique several are symbol per n data bits b_k are then transmitted symbols range between M levels (know that $M = 2^n$). The plots below illustrate the M-ary transmission.



In this example, we set 2 bits per symbol (hence $2^2 = 4$ levels). The information rate is still unchanged, but the symbol rate is halved (as in the second plot). In other words, we can send twice the information rate at the same symbol rate (as in the third plot). Practically, it is difficult to introduce the multi-level baseband signaling through noisy and distorting channels. The receiver now must distinguish the incoming symbols according to their levels. Thus, the probability of error increases as M increases (in gen.).

Advantage:

- A higher information transfer rate is possible for a given symbol rate and a channel bandwidth.

Disadvantages:

- M-ary baseband signaling results in reduced noise/interference immunity when it is compared to the binary signaling.
- It requires more complex symbol recovery mechanisms for receiver.
- It imposes a greater requirement for linearity and/or reduced distortion in the Tx/Rx hardware and in the channel.

Exercises

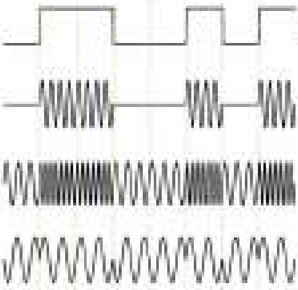
(1) A multilevel digital communication system sends one of 16 possible levels over the channel every 0.8ms. (a) What is the number of bits corresponding to each level? (b) What is the baud rate? (c) What is the bit rate?

(2) An analog waveform with $f_m = 1.5$kHz is to be quantized to $L \geq 200$ levels and transmitted via an M-ary PCM signal (having $M = 2^k$). Find the maximum allowed values of k and the corresponding value of m, if the available transmission bandwidth is 10kHz.

Part 2 Digital Modulation

2.1 Binary Digital Modulation

Since pulse-modulated signals consist of "low" frequencies, they cannot be efficiently transmitted through a channel with band-pass characteristics. Hence, for communication systems employing band-pass channels, it becomes advantageous to modulate a carrier signal with the digital data stream prior to transmission. Three basic forms of digital modulation corresponding to AM, FM & PM are known as Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK), and Phase Shift Keying (PSK).



2.2 Binary Amplitude Shift Keying (BASK)

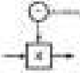
In BASK, the amplitude of a high-frequency carrier is switched between two values, ON-OFF keying (OOK).

$$s(t) = \begin{cases} A\cos(\omega_c t) & \text{for logic 1} \\ 0 & \text{for logic 0} \end{cases}$$

The bandwidth of BASK signal = ?

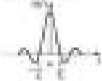
2.2.1 Generation

Baseband Data



OOK Modulator

Bandpass OOK signal



Baseband signal spectrum of a single symbol

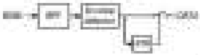
Bandpass signal spectrum

2.2.2 Demodulation

Coherent Correlation Receiver

Quasi-Coherent Squaring Receiver

Incoherent Envelope Receiver



[illegible] Probability of Error in BASK

The received signal model based on two possibilities:

$$r(t) = \begin{cases} [illegible] + n_o(t) & \text{for logic 1} \\ n_o(t) & \text{for logic 0} \end{cases}$$

Where $n_o(t)$ is the input noise to the decision maker.

At threshold [illegible], for $p_0(e_0) = p_1(e_1) = \frac{1}{2}$, the optimum decision threshold [illegible]

(E_1 = received signal energy for logic bit 1, where $E_0 = 0$), [illegible] with the Gaussian distributed noise, the probability of error is:

$$P_e = Q\left(\sqrt{\frac{E_{avg}}{\eta}}\right)$$

Since S = the average received signal power $= \frac{E_b}{T_b}$

E_{avg} = average bit received signal energy $= \frac{(E_1 + E_0)}{2} = \frac{E_1}{2}$

R_b = binary data rate $= \frac{1}{T_b}$, and T_b is the interval of binary bits.

η = [illegible], B = the bandwidth of the decision maker.

$$P_e = Q\left(\sqrt{\frac{S}{\eta B}}\right)$$

$$P_e = Q\left(\sqrt{T_b B \frac{S}{\eta B}}\right) \quad \text{Coherent Detection at } T_b B = 1$$

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{4\eta}\right) \quad \text{Noncoherent Detection}$$

3.3 Binary Frequency Shift Keying (BFSK)

In BFSK, the instantaneous frequency of the carrier is switched between two values in response to the binary code. We can consider the BFSK waveform as a combination of two ASK waveforms of different carrier frequencies.

$$s(t) = \begin{cases} A\cos(\omega_1 t) & \text{for logic 1} \\ A\cos(\omega_0 t) & \text{for logic 0} \end{cases}$$

3.3.1 Generation

As shown above, a BFSK carrier frequency is:

$$f_c = \frac{f_1 + f_0}{2}$$

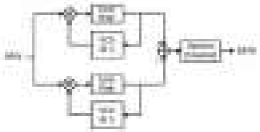
and the BFSK frequency deviation is:

$$\Delta f = \frac{f_1 - f_0}{2}$$

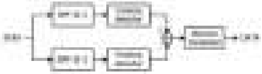
To define the bandwidth of a BFSK signal, we'll consider the bandwidth to be at the first zero crossing point in the BFSK spectrum. So, $B = 2\Delta f + 2R_b$ where $R_b = 1/T_b$ = the baud rate of the baseband data in bit/s, which is assumed here to be the nominal bandwidth of the binary

Demodulation

Coherent Detector using PLL



Noncoherent Detector



Probability of Error in BFSK

$$P_e = Q\left(\sqrt{\frac{E_{d}}{2\eta}}\right)$$

for $E_d = \frac{(E_1 + E_2)}{2} = \frac{2E_b}{2} = E_1 = E_2$

$$P_e = Q\left(\sqrt{\frac{E_b}{\eta}}\right)$$

$$P_e = Q\left(\sqrt{T_b B \frac{S}{N}}\right)$$ Coherent Detection ($T_b B = 1$)

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{2\eta}\right)$$ Noncoherent Detection

Compare your idea the BPSK results, is there any difference?

2.4 Binary Phase Shift Keying (BPSK)

BPSK converts the baseband binary to passband by changing the carrier's phase in sympathy with the baseband digital data.

$$s(t) = \begin{cases} A\cos(\omega_c t) & \text{for logic 1} \\ A\cos(\omega_c t + \phi) & \text{for logic 0} \end{cases}$$

In BPSK, $\phi = 180°$

2.4.1 Generation

2.4.2 Demodulation

Coherent Correlation Detection

2.4.3 Probability of Error in BPSK

$$P_e = Q\left(\sqrt{\frac{2E}{\eta}}\right)$$

Since $P_{BPSK} = P_s = E_b$. The coherent detection is at $T_b B = 1$.

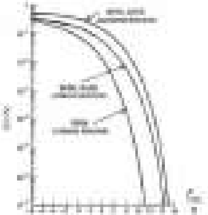
$$P_e = Q\left(\sqrt{\frac{2S}{\eta}}\right) \quad \text{and} \quad P_e = Q\left(\sqrt{T_b B\frac{2S}{\eta}}\right)$$

This means BPSK is better than the other schemes. What is the problem of detection?

2.5 Comparison of Binary Keying Techniques

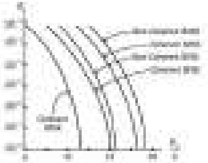
2.5.1 Through the Average Power

OOK & BFSK have equivalent $P_{e,b}$ for the same E_b; BPSK has better performance (less P_e). As shown:



2.5.2 Through the Peak Power

To get the same P_e, BFSK requires double the power of BPSK but also requires half the power of OOK, as all the energy of OOK concentrated in (transmitted with) only one type of data. As shown:



2.5.3 Through the Spectral Efficiency

If we define Γ as the spectral efficiency of data transmission as:

$$\Gamma = \frac{R_s}{B}\log_2 M \qquad (\text{bits/sec/Hz})$$

M = number of levels = 2^n (for binary $M = 2$, $n = 1$)

R_s = Symbol Rate rate (in Sps) = $r_s = \frac{1}{T_s}$

B = the required channel bandwidth of the transmitted signal

Then the comparison will be:

	Baseband	BASK	BFSK	BPSK
Data Rate (bps)	r_b	r_b	r_b	r_b
Bandwidth (B)	$\frac{r_b}{2}$	r_b	$2\Delta f + r_b$	r_b
Spectral efficiency (Γ)	2	1	$\frac{r_b}{2\Delta f + r_b}$	1

2.5.4 Through Systems

ASK:

- The transmitters of ASK are very easy to build.
- They have the advantage of transmitting no power at 0. Such systems find some applications in short range wireless telemetry systems.
- The receivers for non-coherent ASK are easy to build. The difference in performance between coherent & non-coherent detection is slight compared to the increase in complexity required. Therefore coherent detection of ASK is generally not common.
- The decision threshold in the receiver must be adjusted with changes in the levels of the received signal. Therefore it requires an Automatic Gain Control (AGC).

FSK:

- In contrast to ASK, the FSK systems operate symmetrically about a zero-decision threshold level regardless of carrier signal strength.
- Transmitters of FSK is slightly more complex than those for ASK.

- Non-coherent FSK receiver is relatively an easy and a popular choice for low to medium data transmission rates.
- FSK requires more bandwidth than ASK & PSK.

PSK:

- The performance of PSK systems is superior to both ASK & FSK systems.
- PSK systems require less transmitted power (for a given P_e).
- Synchronous detection is required, and carrier recovery systems are more difficult (therefore more expensive) to build.

The decision of which scheme of the digital modulation to be selected depends on the trade-off between (performance, cost, bandwidth, etc.). In addition, (propagation, distortion, fading, man-made noise (e.g., interference), non-Gaussian noise... etc.) may affect the choice.

Examples:

(1) A [illegible] BPSK signaling system uses pulse shaping to constrain its transmission to the ISR Nyquist bandwidth. The received signal power is [illegible] and the two-sided noise power spectral density is [illegible]. Find the bit error rate (BER) expected at the receiver.

> Solution: Nyquist ISR bandwidth [illegible]
>
> [illegible]
>
> [illegible]
>
> So the BER = [illegible] (bits per second)

(2) A rectangular pulse OOK signal has an average [illegible] power, at the input to an ideal correlation receiver, of [illegible]. The two-sided noise PSD is [illegible] W/Hz. What maximum bit rate can this system support [illegible] P_b = [illegible]

> Solution: [illegible]
>
> [illegible] kbps

(3) An OOK modulated signal is detected by a receiver; the [illegible] symbol at the receiver input is a rectangular pulse with amplitude [illegible] and duration of [illegible]. The noise at this point is known to be white and Gaussian with an r.m.s. value of [illegible] when measured in a noise bandwidth of [illegible]. Calculate P_b.

> Solution:
>
> [illegible]
>
> [illegible]
>
> [illegible]
>
> So P_b = [illegible]

(4) An ideal matched filter receiver detects an OOK signal. [illegible] pulse at the input of the receiver is a rectangular pulse [illegible] and duration of [illegible] The noise at the input of the receiver is AWGN and has a r.m.s. value of [illegible] when measured in a noise bandwidth of [illegible] What is the probability of bit error?

Answer: P_e = [illegible]

(5) Find the [illegible] needed for a [illegible] receiver to [illegible] if the following detectors are used (a) coherent (b) non coherent. Repeat for [illegible] and [illegible].

Solution [illegible]

[illegible]

Compare and explain these results.

(6) A [illegible] signal [illegible] with a peak voltage of [illegible] is [illegible] on [illegible] If the noise [illegible] what is the [illegible] Compare and discuss.

Answer: [illegible]

(7) A [illegible] high frequency transmitter [illegible] peak power [illegible] Path losses to the receiver are estimated to be [illegible] and the receiver input noise power is [illegible] Estimate the [illegible] probability of error [illegible] (a) OOK (b) PSK.

Solution: [illegible]

[illegible]

(8) A [illegible]

[illegible]

Answer: [illegible]

(9) [illegible]

- [illegible]
- [illegible]

(10) [illegible]

Solution: [illegible]

(11) [illegible]

Solution: [illegible]

(12) [illegible]

the bandwidth efficiency of this system? (c) As a complete system, is a solid or very large bandwidth valued to the [illegible] transmission rate possible?

(13) A digital baseband signal consisting of rectangular binary pulses occurring at a rate of 240 kbps is to be transmitted over a bandpass channel. For OOK signaling that is keyed by a baseband digital test pulse consisting of alternating 1's and 0's, sketch the magnitude spectrum and indicate the value of the null-to-null bandwidth. Assume a carrier frequency of 1200 kHz.

(14) Repeat the previous problem for the case of (a) BPSK signaling, (b) BFSK signaling assuming the Mark Frequency is 1200 kHz and the Space Frequency is [illegible] kHz.

(15) An ASK signal with non-coherent peak carrier levels are A_1 and A_0. Assuming message probabilities are $P_1 = P_0 = \frac{1}{2}$, plus assume $A_1 = 1$ mV, $A_0 = 0$ V, $T_b = 1$ ms, and $\eta = 0.01 \times 10^{-[illegible]}$ W/Hz. (a) Find the optimum threshold voltage. (b) Find the probability of error P_e.

Solution: [illegible] $= \frac{[illegible]}{2} =$ [illegible] $A_1 = [illegible]$, $T_b = 1$ ms, $P_e = [illegible]$

(16) PSK and coherent FSK systems use the same peak pulse amplitude and have the same values of P_e and η. How are the bit durations T_b related?

2.6 Modulation Techniques with Increased Spectral Efficiency

In addition to the reliability and the error P_e, the efficient usage of the bandwidth is an important aspect in the design of digital communication systems.

As we know, the basic target is to transmit the maximum information rate through the minimum possible bandwidth. We defined earlier the spectral efficiency Γ. To make Γ as large as possible, we can:

- Decrease B by filtering the transmitted signal prior to the transmission. How far is this method useful?

ii. Increase the number of transmitted bits per second (R), but we must consider the Shannon-Hartley limit of channel capacity, this is around R/B, E_b/N ≥ 1/2 for baseband signals and E_b/N ≥ 1 for Bandpass signals.

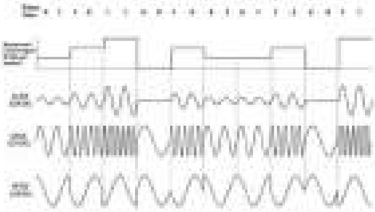
iii. The final option: increase M (i.e. increase the number of bits per a transmitted symbol). Practical systems currently exist with M of 4, 8, 16, 64, ... 1024.

The multi-level per symbol (M-symbol) signaling for the studied digital modulations will referred as M-ASK, MFSK and MPSK.

Note:

P_b : is the probability of bit errors.

P_s : is the probability of symbol errors.



In practice, only MPSK and QAM are used, because:

i. the increase of M in M-ASK results in level ambiguity at detection. The channel effects like the noise and distortion make the process of the detection very hard. Nevertheless, increasing M increases P_s. But it is still acceptable for small M through good channels.

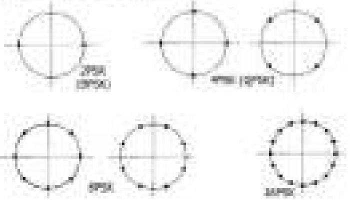
ii. as we studied before, the increase of M in MFSK requires larger bandwidth to accommodate the modulated signal, which decreases ?

M-Symbol Phase Shift Keying (MPSK)

MPSK employs the transmission of the number of the allocated phases states (from 0 to 2π), i.e., while the carrier amplitude being constant, the phase is changing according to the input taking one of the designed phase states.

$$MPSK = \cos(\omega_c t + \theta_i) \quad \text{where } i = 0, 1, 2, \ldots, M-1 \quad ; \quad \theta_i = \frac{2i\pi}{M} \quad \text{or} \quad \theta_i = \frac{(2i+1)\pi}{M}$$

Some examples on MPSK is shown in the following figure:

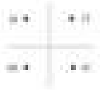


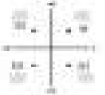
Quadrature Modulation

The phase and amplitude of the carrier at any given time determine the location on the constellation. The amplitudes of the I and Q channels are derived from the rectangular coordinates of the carrier's amplitude and phase.

Quadriphase-Shift Keying (QPSK)

Here we study the coherent QPSK as an example of MPSK. In QPSK, as with BPSK, information carried by the transmitted signal is contained in the phase. The phase of the carrier takes on one of four equally spaced values, as:





For this set of values, we may define the transmitted signal as

$$s_i(t) = \sqrt{2}\cos\left[2\pi f_c t + \frac{(2i-1)\pi}{4}\right] \qquad \text{where } i = 1, 2, 3, 4$$

$$= \sqrt{2}\cos\left[\frac{(2i-1)\pi}{4}\right]\cos\omega_c t - \sqrt{2}\sin\left[\frac{(2i-1)\pi}{4}\right]\sin\omega_c t$$

Based on this representation, we can make the following observations:

- There are two orthogonal basis functions, defined by a pair of quadrature carriers:

$$\phi_1 = \cos\omega_c t \quad \text{and} \quad \phi_2 = \sin\omega_c t$$

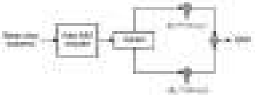
- There are four message points, and the associated signal vectors are defined by:

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} = \begin{bmatrix} \sqrt{2}\cos\left[\frac{(2i-1)\pi}{4}\right] \\ -\sqrt{2}\sin\left[\frac{(2i-1)\pi}{4}\right] \end{bmatrix} \qquad \text{where } i = 1, 2, 3, 4$$

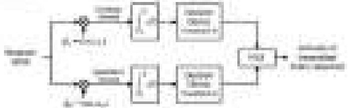
The elements of the signal vectors have their values summarized in the table below:

i	Gray-encoded input Dibit	Phase of QPSK signal (rad)	Coordinates of Message points	
			s_{i1}	s_{i2}
1	10	$\pi/4$	+1	+1
2	00	$3\pi/4$	−1	+1
3	01	$5\pi/4$	−1	−1
4	11	$7\pi/4$	+1	−1

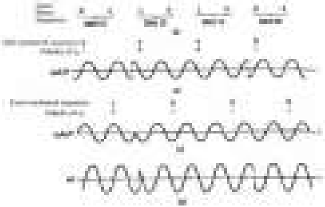
Modulator:



Demodulator:



The figure below illustrates the sequences and waveforms involved in the generation of a QPSK signal.



Probability of Error in MPSK

As M increases with E_s the probability of error would also be increased. So, the symbol probability of error is:

$$P_s = 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right) \qquad \text{or} \qquad P_s = 2Q\left(\sqrt{2\log_2 M\,\frac{E_b}{N_0}}\sin\frac{\pi}{M}\right)$$

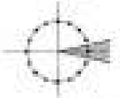
Note: these formulas are for $M \geq 4$ only, and if we use $M=2$ we'll get twice the correct result of BPSK.

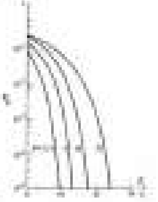
Now, if we use gray code to map binary symbols to phase state, the probability of bit error is:

$$P_b = \frac{P_s}{\log_2 M}$$

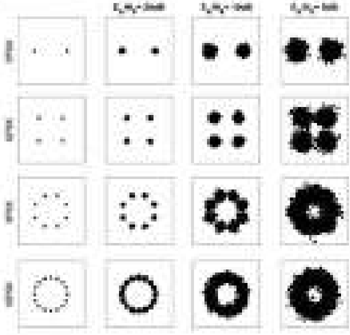
To compare the performance of MPSK through M, we should express P_s in terms of E_b, where:

$$E_b = \frac{E_s}{\log_2 M}$$





For 16PSK, 64PSK and higher M values, it will become so difficult (or sometimes impossible) to distinguish between close phases.



Bandwidth of MPSK Signals

The power spectra of M-PSK signals possess a main lobe bounded by well-defined spectral nulls (i.e., frequencies at which the power spectral density is zero). Accordingly, the spectral width of the main lobe provides a simple and popular measure for the bandwidth (null-to-null bandwidth).

So, the channel bandwidth required to pass M-PSK signals (more precisely, the main spectral lobe of M-ary signals) is given by:

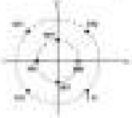
$$B = R_s = \frac{R_b}{\log_2 M}$$

Where R_s is the symbol rate (baud per second), R_b is the binary data rate (bits per second).

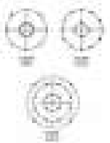
5.6.2 Hybrid Amplitude/Phase Modulation (QAM)

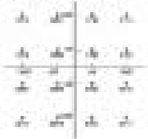
Also called Quadrature Amplitude Modulation. Here, it is possible to introduce amplitude as well as phase modulation to give an improved distribution of signal states in the phasor diagram. The following illustrates the 8QAM:

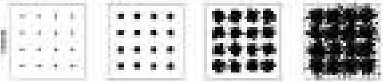
Bit Combination	Phase Shift [Deg]	Amplitude
000	0	Low
001	90	Low
010	45	High
011	135	High
100	270	Low
101	180	Low
110	225	High
111	315	High



The following figures illustrate the re-distribution of 16PSK to create 16QAM



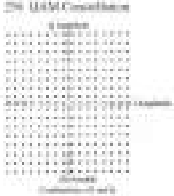




64 and 256 QAM Constellations

By adding more levels to the I and Q channels, higher data rates can be carried.

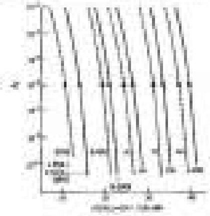
- The higher the number of levels, the more effect there will be if any noise or interference.
- 64 QAM uses 8 levels in the I direction and 8 levels in the Q direction for a total of 8 squared or 64 symbols.
- 256 QAM uses 16 levels in the I direction and 16 levels in the Q direction for a total of 16 squared or 256 symbols.



Probability of Error in MQAM

$$P_e \approx 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_{avg}}{(M-1)N_0}}\right)$$

The figure shows clearly that the MQAM is much better than the MPSK for the same M.



[3] An M-level system is to operate [illegible] PAM symbols over a 1200Hz channel. The [illegible] required for [illegible] 9000kbps. What [illegible] is required to maintain [illegible] P_e [illegible] $10^{-?}$

Solution: [illegible] band rate is $R_s = 1/T_s = B = 1200$ symbols per second. Therefore $R_s = 1200$ symbols per second. For the required [illegible] $R_b = 9000$bps we need $R_b = m \times R_s$ where $m = \log_2 M$.

$m \geq \frac{R_b}{R_s} = 7.5 \Rightarrow m = 8$ bits per symbol, $M = 256$ symbol levels.

[illegible]

[illegible]

[illegible] Thus

[illegible]

[illegible]

[illegible]

[4] An MPSK modulating system is used to transmit data at a rate [illegible] kbps over [illegible] channel. (a) Find the minimum number of M-ary levels of the MPSK that should be used, and [illegible] corresponding [illegible] rate. (b) Compute the BER of this system in the presence of additive white Gaussian noise with two-sided power spectral density [illegible] W/Hz. Assume [illegible] transmitter average power of [illegible] Watts, and expected net system loss of [illegible] dB.

Solution: (a) $R_s = B$, $m \geq \frac{R_b}{B}$ [illegible] bits per symbol $\therefore M = 64$ levels per symbol $\Rightarrow R_s = \frac{R_b}{6}$ [illegible] symbols per second.

(b) [illegible]

[illegible]

BER: [illegible] bits per second.

[13] A 128 QAM data link that operates at [illegible] Mbps. What is the underlying symbol rate on the channel? and what is the occupied bandwidth?

Solution: we know $n = \log_2 M = 7$ bits per symbol, the symbol rate is:

$R_s = \frac{R_b}{n} =$ [illegible] Mbps

The minimum occupied bandwidth occupied for ISI free channel is $B_T =$ [illegible]

[14] An MFSK format is used for transmitting data at a rate [illegible] kbps over channel with bandwidth extending from [illegible] Hz to [illegible] Hz. (a) Can an MASK satisfy the required information rate in the given channel? If yes, find the minimum number of M-ary levels of the MASK that should be used (b) Find the maximum information rate over the channel and the corresponding symbol rate if the received SNR is [illegible].

Solution: (a) $R_b =$ [illegible] kbps [illegible] $R_s \le$ Channel Bandwidth

$\therefore R_s =$ [illegible] Symbols per second, so the MASK can be used as R_s [illegible] R_b

So, we must use [illegible] MASK, with $n = \log_2 M$ [illegible] bits per symbol

As $R_b = nR_s \rightarrow R_b \le nR_s$ [illegible] $\rightarrow n =$ [illegible] bits per symbol, [illegible]

$\therefore R_s = \frac{R_b}{n} =$ [illegible]

(b) $C = B\log_2(1 + SNR) =$ [illegible] $\log_2(1 +$ [illegible]$) =$ [illegible]

$n \le$ [illegible] $=$ [illegible] $\rightarrow$ [illegible] bits per symbol

$\therefore R_s = \frac{R_b}{n} =$ [illegible] (still $\le$ Channel Bandwidth [illegible])

[15] It is required to transmit [illegible] kbps with $P_e \le 10^{-?}$. Three possible schemes are considered: (a) BPSK, (b) 16-ary PSK, (c) 32-ary FSK. If $\eta =$ [illegible] W/Hz determine the transmission bandwidth and the signal power required at the receiver input in each case.

Solution: (a) $B \ge R_s = R_b =$ [illegible] $S =$ [illegible]

(b) $B \ge R_s = \frac{R_b}{4} =$ [illegible] $S =$ [illegible]

(c) $B \ge R_s = \frac{R_b}{5} =$ [illegible] $S =$ [illegible]

(7) Find the following for the 16-QAM with a received carrier to noise ratio (CNR) of 25dB: (i) maximum spectral efficiency, (ii) probability of symbol error of the scheme under the condition of maximum spectral efficiency [illegible] (iii) the Gray coded probability of bit error.

Solution: [illegible]

[illegible]

[illegible]

[illegible]

(8) A 20Mbps bit stream is to be transmitted via QPSK. (a) What is the symbol rate? (b) What is the minimum required channel bandwidth? (c) What is the symbol error probability if received carrier power is [illegible] (d) What is symbol error rate?

Answer: [illegible] second.

(9) Determine the probability of error per symbol for a 10Mbps QPSK system when the received [illegible] at the receiver input is [illegible] additive white Gaussian noise and receiver noise bandwidth of [illegible].

(10) Show that two BPSK systems can operate simultaneously over the same channel by using orthogonal carriers (sine and cosine). Give a block diagram for the transmitters and receivers. What is the overall aggregate data rate for this orthogonal carrier multiplex system as a function of the channel and bandwidth? How does the aggregate data rate of the system compare to the data rate for a system that time division multiplexes the two sources and then transmits the TDM data via QPSK carrier?

(11) What is the minimum theoretical bandwidth of an 8-PSK signal carrying 6000bps?

(12) A baseband signal (band-limited to [illegible] kHz) is sampled at [illegible]% over Nyquist rate and coded using PCM system having [illegible] levels quantizer. The resulting NRZ [illegible] [illegible] through a [illegible] transmission channel after [illegible] with [illegible] PSK. The bandwidth of this channel was set to be [illegible] of the operating binary rate. If the channel is contaminated by a Gaussian additive white noise of PSD = [illegible] per km, calculate the expected minimum [illegible] power of the received signal [illegible] [illegible] error bits per second [illegible]

Solution: [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(13) Assume a channel has the following specifications: [illegible] W/Hz, [illegible] [illegible] Bandwidth [illegible] kHz. A transmitted [illegible] at a power of [illegible] [illegible] located at [illegible] km away from the transmitter. Using the MPSK scheme, [illegible] the maximum value of M to maintain probability of [illegible] [illegible] than [illegible]? What will be the spectral efficiency of this channel?

Answer: [illegible] on the given parameters, we get [illegible] per symbol. [illegible] [illegible] either [illegible] or [illegible] the [illegible] of [illegible] [illegible] increasing M [illegible] [illegible] [illegible], we must choose [illegible] [illegible] M = 8 levels per symbol.

(14) An MPSK modulating system is used to transmit data at a rate [illegible] over [illegible] channel, despite the presence of additive white Gaussian noise with one-sided power spectral density [illegible] W/Hz. Gray coding, a transmitter average power of [illegible], and expected bit [illegible] level of [illegible] dB. (a) Find the minimum number of M-ary levels of the MPSK that should be used, and the corresponding [illegible] rate. (b) Compute the [illegible] of this system.

Answer: (a) M = [illegible] levels per symbol (b) [illegible] [illegible] per second

(15) Given the input bits are [illegible], sketch the output of a BPSK, QPSK, DPSK and [illegible] modulations.

(16) A terminal-user telephone line has a usable bandwidth of [illegible]. Find the maximum data rate that can be supported on this line to produce a null-to-null bandwidth of [illegible] if QPSK signaling with a single carrier is used.

(17) Assume that R_b = [illegible]. For rectangular data pulses, calculate the second null-to-null bandwidth of BPSK, QPSK, 64PSK, 64QAM. Discuss the advantages and disadvantages of using each of these signaling methods.

(18) Rank the binary digital modulation systems if the primary criterion is (a) hardware simplicity, (b) lowest peak power requirement for a given error rate.

(19) If an MPSK system is used to send binary data at a rate [illegible] over a [illegible] channel. Calculate the required S/N ratio at receiver reception with bit error rate no more than [illegible].

(20) An MPSK system is used to send data through a [illegible] channel. If the minimum data rate required is [illegible], what is the lowest SNR (in dB) necessary to keep such data transfer with probability of error bits no worse than [illegible]?

(21) Assume a channel has the following specifications: η = [illegible] W/Hz, [illegible] bandwidth of [illegible], it is required to send a power of [illegible] to a receiver located at [illegible] away from the transmitter. Using the MPSK scheme, what should be the minimum value of M to maintain probability of **symbol** error no worse than [illegible]?

(22) It is required to transmit [illegible] with P_e [illegible] using the M-ary PSK scheme at noise PSD of [illegible] W/Hz, determine the transmission bandwidth and the signal power required at the receiver input.

Part 3 Appendix: Q-Function