Al-Anbar University
Electrical Engineering Department
fundemantal of Electric Engineering
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LECTURE 01

IDEAL VOLTAGE AND CURRENT SOURCES
OPEN AND SHORT CIRCUITS

Topics

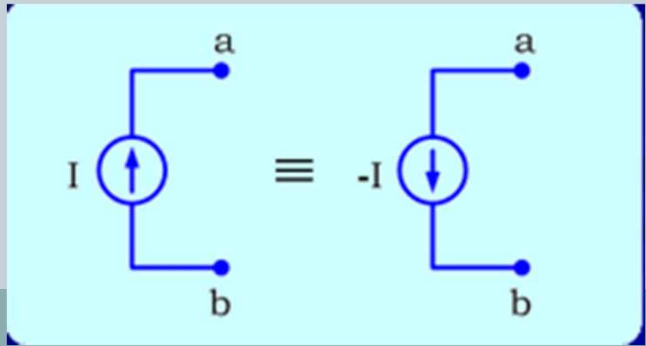
- Ideal voltage source
- Ideal current source
- Open circuit
- Short circuit

Objectives

- Recognize the symbols of ideal voltage and current sources
- Find voltage polarity
- Find current direction
- Calculate voltage and current in simple resistive circuits
- Recognize invalid connections to the ideal voltage and current sources

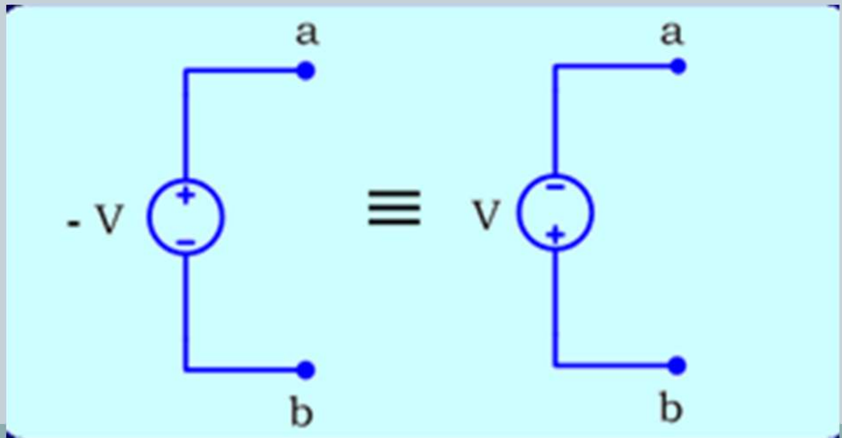
Review of electrical fundemantal

- Charge : its samples (q) its unite (coulomb)
- q_{e}=-1.602×10^{-19} (مقدار شحنة الالكترون)
- q_{p}=+1.602×10^{-19} (شحنة البروتون)
- Current :its symbol (i) its unite (Ampere(A)) or (milli ampere(mA)).
- i=$\frac{dq}{dt}$ its unite = $\frac{\text{coulomb}}{sec}$= Ampere
- Voltage source : its symbol (V) its unite (volt(v)).
- The direction of current source as
- Shown in figure below



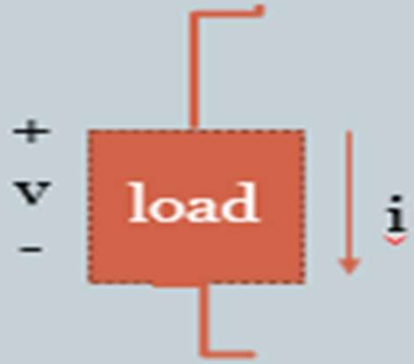
Review of electrical fundamental

- Voltage source : its symbol (V) its unite (volt(v)).
- $v=\frac{dw}{dq}$ its unite = $\frac{\text{joule}}{coulomb}$= volt
- Where w represent the energy .
- The polarity of the voltage source as shown in figure below

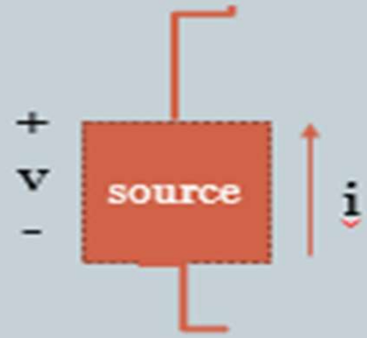


Review of electrical fundamental

- The passive element: is the element dissipated energy like (resistance (load)).

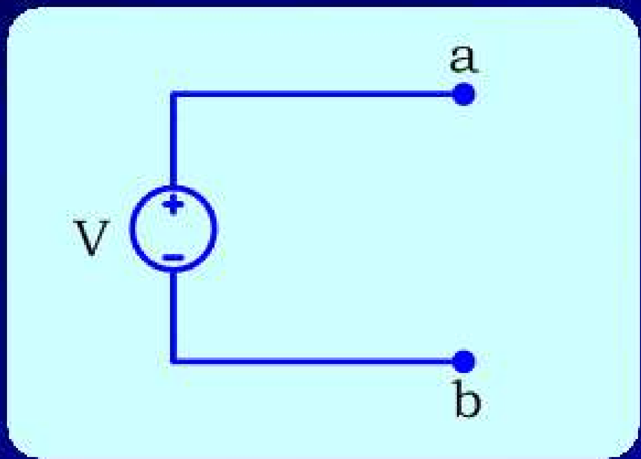


- The active element: is the element produced energy like (d.c supply (voltage source).



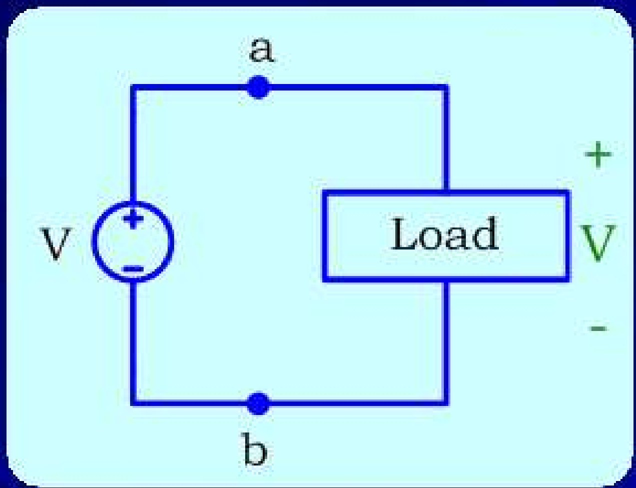
Ideal Voltage Source

The symbol of an ideal voltage source is shown. The value of the voltage source is V volts and the terminals a and b are used to connect the ideal voltage source to other elements.



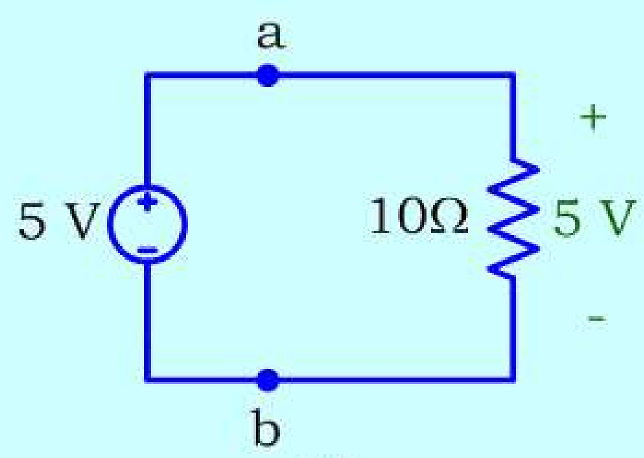
Ideal Voltage Source

When any load is connected across the terminals of an ideal voltage source of voltage V, the same voltage V appears across the load, irrespective of the load.

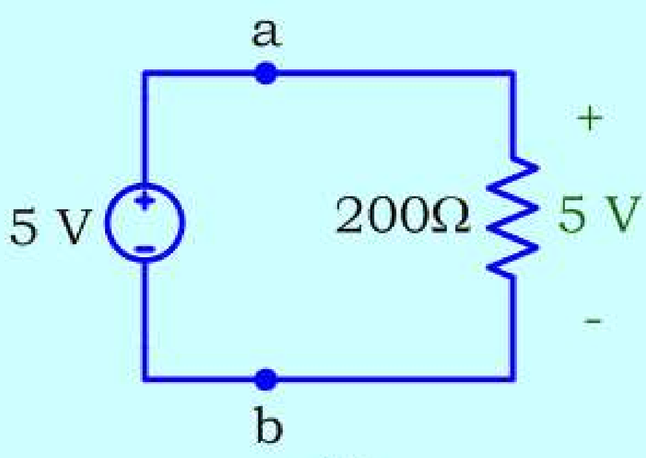


Example1

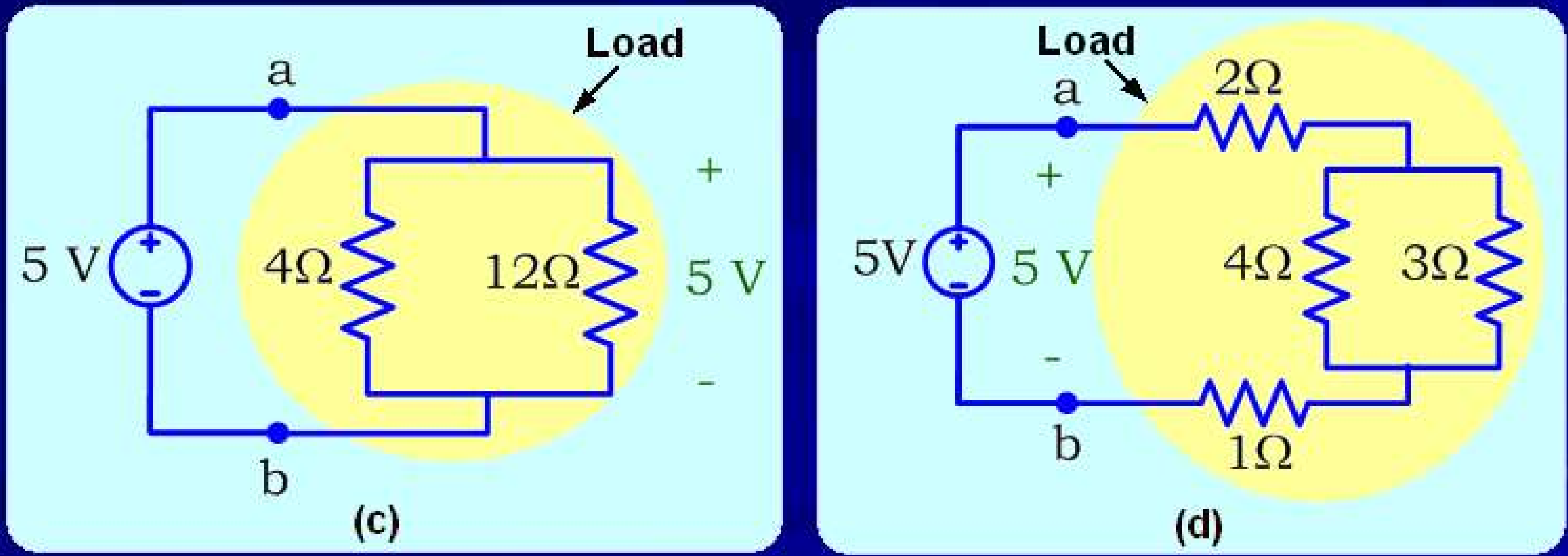
Various resistive loads are connected to the 5V ideal voltage source as shown in the figure below. In each case, the voltage across the load is 5V.



(a)



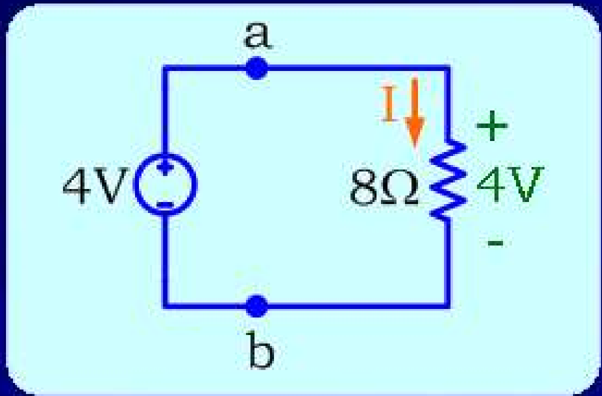
(b)



Note that the equivalent resistance of the resistive load shown in circuit (c) and circuit (d) is considered to be the load

Example 2

Calculate the current I in the following circuit

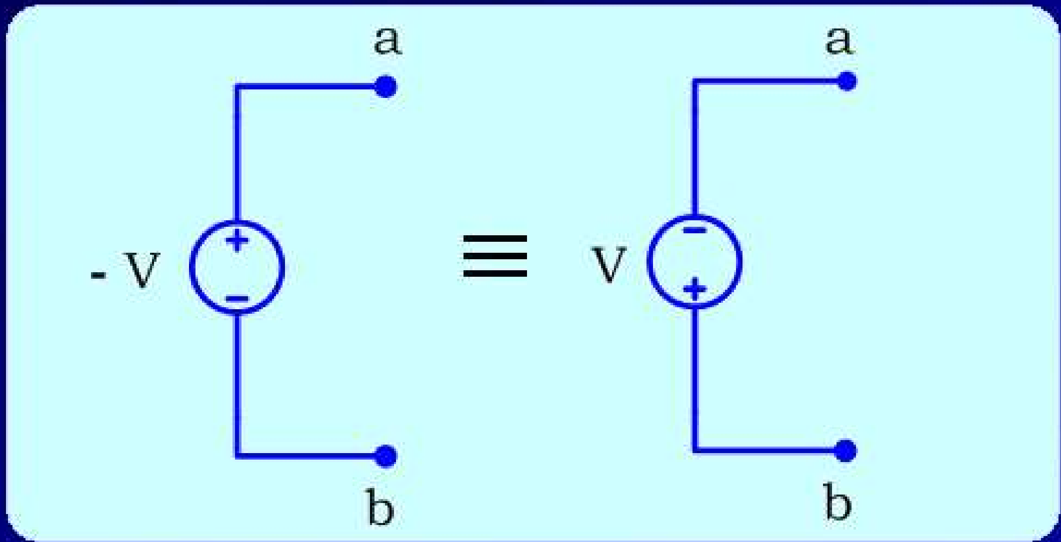


Using Ohm's law

$$I = \frac{V}{R} = \frac{4}{8} = 0.5\text{A}$$

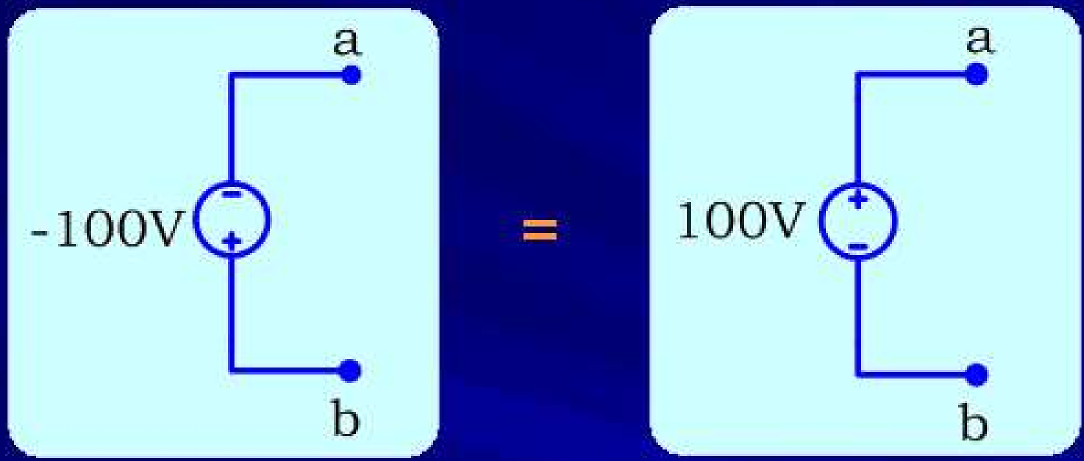
Equivalent source

The following ideal voltage sources are equivalent. If you invert the algebraic sign of the voltage V, you must also reverse the polarity. Otherwise, the sources are not equivalent.



Example 3

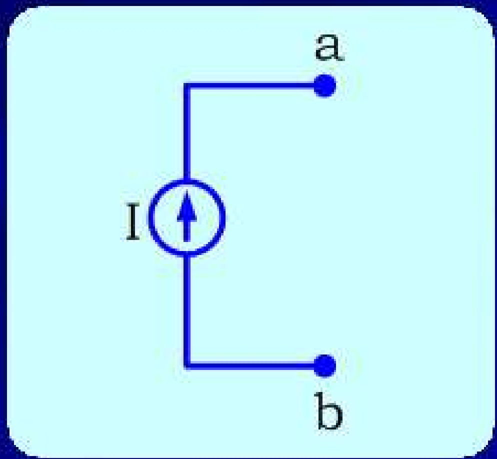
Is the actual polarity of terminal a positive or negative?



By inverting the sign of the ideal voltage source from -100V to +100V and reversing the polarity of the voltage, we conclude that the actual polarity of terminal a is (+) or positive polarity. This means that terminal a is actually at a higher potential than terminal b.

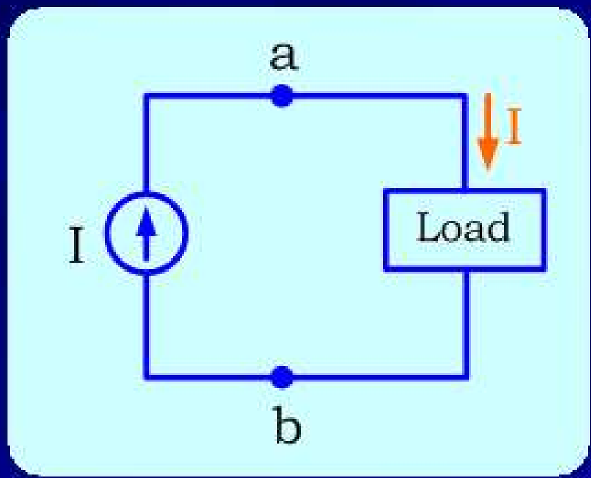
Ideal Current Source

The symbol of an ideal current source is shown. The value of the current source is I amperes and the terminals a and b are used to connect the ideal current source to other circuit elements.



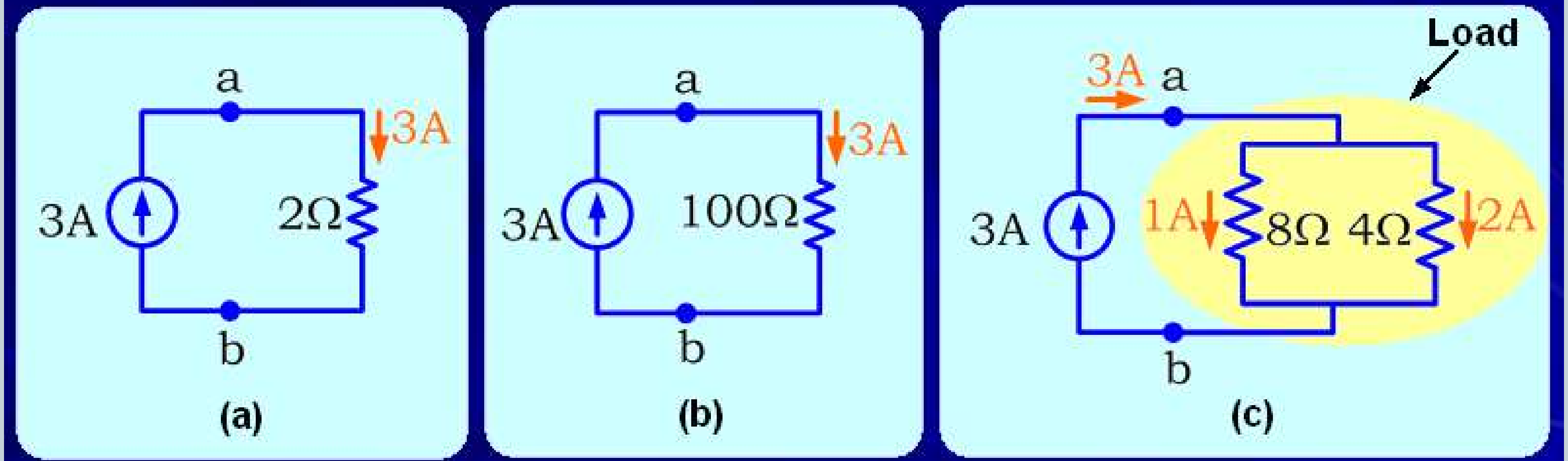
Ideal Current Source

When any load is connected across the terminals of an ideal current source of current I, the same current I flows through the load, irrespective of the load.



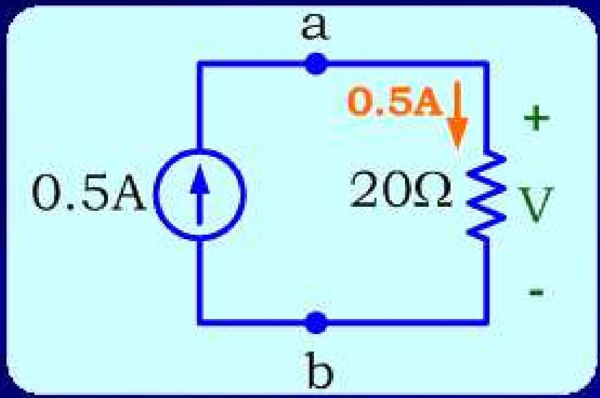
Example1

The 3A ideal current source shown below is connected to different resistive loads. In each case, the current that flows across the load is also 3A. Note that in circuit (c), the current through the resistive load is 3A, but the current that flow into the individual resistances that make up the load are each less than 3A.



Example 2

Calculate the voltage V in the following circuit

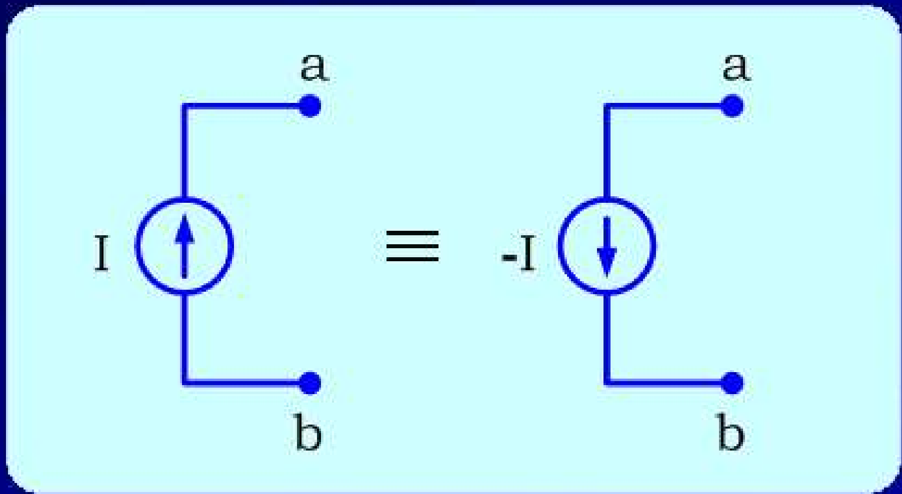


Using Ohm's law

$$V = IR = 0.5 \times 20 = 10\text{V}$$

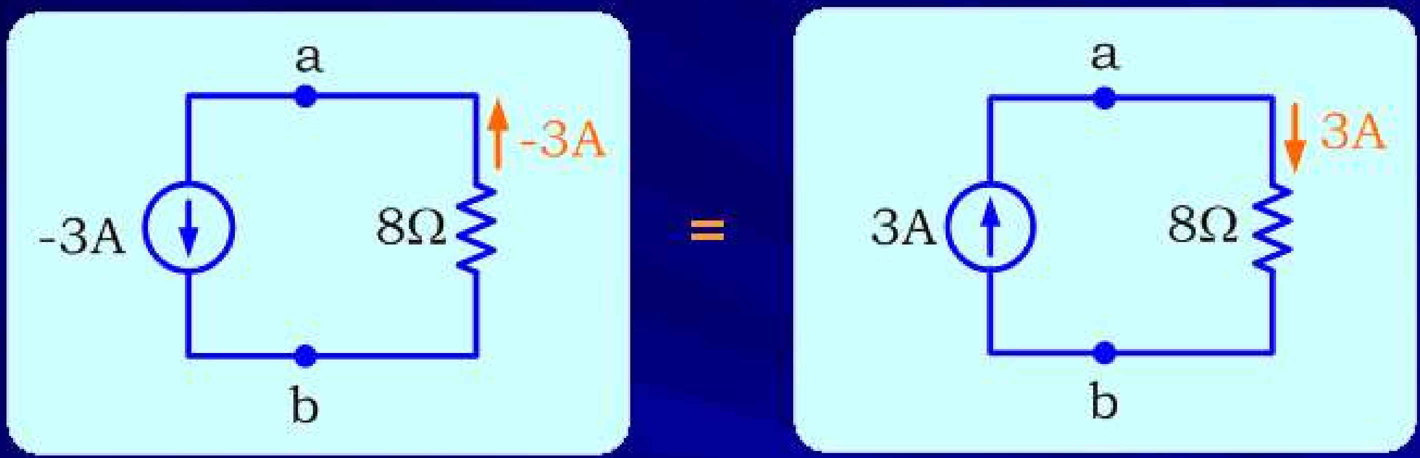
Equivalent source

The following ideal current sources are equivalent. If you invert the algebraic sign of the current I, you must also reverse the direction of current flow. Otherwise, the sources are not equivalent.



Example 3

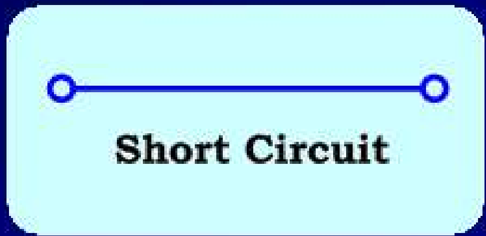
What is the actual direction of the current in the 8Ω resistor?



By inverting the sign of the ideal current source from -3A to +3A and reversing the direction of current, we conclude that the actual direction of current through 8Ω resistor is from terminal a down to terminal b.

The Short Circuit

When a resistor has zero resistance (i.e. R = 0Ω) we call it a short circuit.

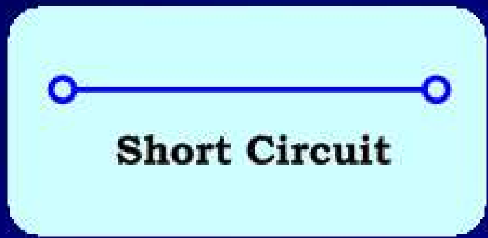


The current through a short circuit is generally not equal to zero. However, the voltage across a short circuit is always equal to zero, because:

$$V = IR = I \times 0 = 0$$

The Short Circuit

When a resistor has zero resistance (i.e. $R = 0\Omega$) we call it a short circuit.

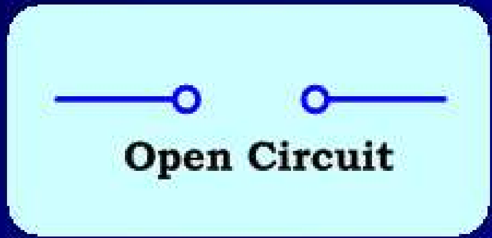


The current through a short circuit is generally not equal to zero. However, the voltage across a short circuit is always equal to zero, because:

$$V = IR = I \times 0 = 0$$

The Open Circuit

When a resistor has an infinite resistance (i.e. $R = \infty$) we call it an open circuit.



The voltage across an open circuit is generally not equal to zero. However, the current through an open circuit is always equal to zero, because:

$$I = \frac{V}{R} = \frac{V}{\infty} = 0$$

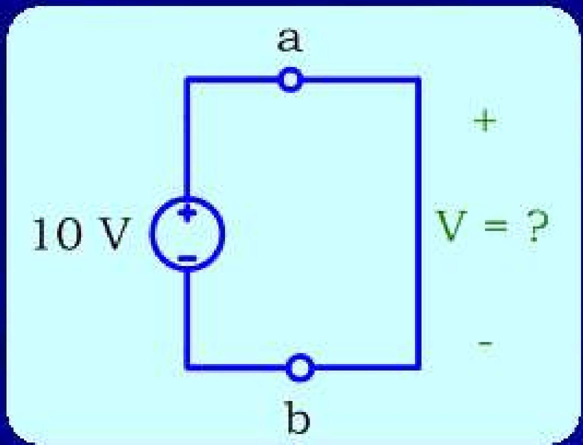
The Open and Short Circuit

When the 10V ideal voltage source is connected to a short circuit as shown below, we immediately face a problem.

What is the voltage across the load in this case ? Is the voltage 'V' 10V or 0V?

This is an ambiguous question which cannot be answered.

It is invalid in this course to connect a short circuit across the terminals of an ideal voltage source. However, as we shall see later, we are allowed to connect a short circuit across the terminals of a realistic voltage source.

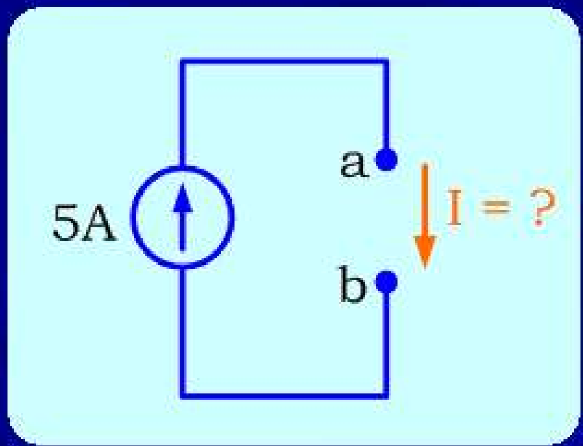


The Open and Short Circuit

The same type of problem faces us, when we connect a 5A current source to an open circuit load, as shown in the figure below,

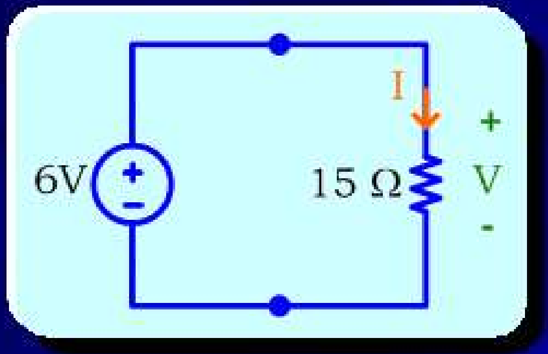
What is the current 'I' through the load in this case? Is it 5A or 0A?

There is no answer to this question also. Thus, it is also invalid in this course to connect an open circuit to the terminals of an ideal current source.



Self Test

Calculate the unknown quantities in the following circuit



Circuit (a)

A $V = -6V, \; I = 0.4A$

B $V = 6V, \; I = 0.4A$

C $V = 6V, \; I = 90A$

D $V = 6V, \; I = 2.5A$

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LECTURE 02
ENERGY AND POWER-
OHM'S LAW

- Topics
- Energy and power in electric circuit
- Ohm’s law

After finishing this lecture, you should be able to:

- Understand the relation between power and energy
- Understand the passive sign convention
- Use the passive sign convention in power calculation
- Determine if the power is actually absorbed or delivered
- Verify power conservation
- Use the passive sign convention in Ohm's Law

Electric Energy and Power

The power ***p(t)*** *absorbed by* an electric element and the energy ***w(t)*** in the *same* element are related by

$$p(t) = \frac{dw(t)}{dt}$$

Unit of w is Joule (J)

Unit of p is Watt (W)

Unit of t is second (s)

Direction of Power Flow

If the energy $w(t)$ *increases* with time [$w(t)$ has a +ve slope], then

$dw(t)/dt > 0$ => $p(t) > 0$ => power is being actually absorbed by the element

If the energy $w(t)$ *decreases* with time [$w(t)$ has a -ve slope], then

$dw(t)/dt < 0$ => $p(t) < 0$ => power is being actually delivered by the element

$w(t)$ increases <=> power being absorbed

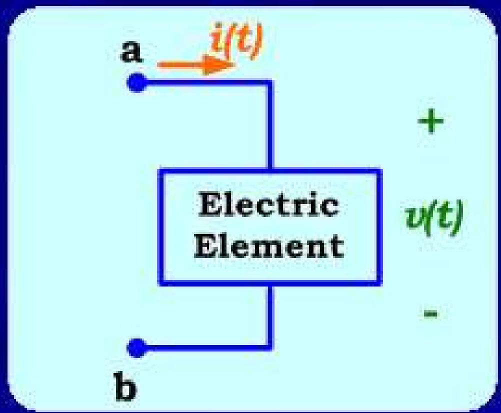
$w(t)$ decreases <=> power being delivered

Relation with v-i

The power ***p(t)*** can be expressed in terms of ***v(t)*** and ***i(t)***

$$p(t) = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t)$$

The above relation applies *only* when the current enters the element from the (+) terminal and leaves the (-) terminal, as shown below

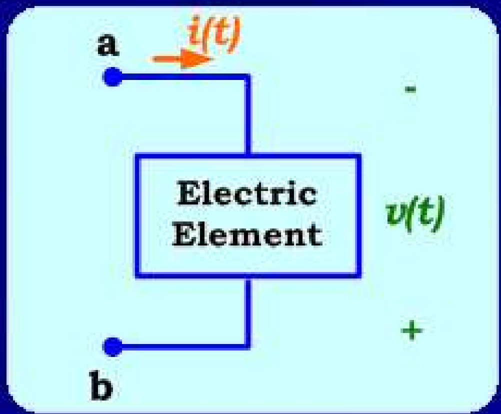


Relation with v-i

If the current enters the element from the (-) terminal and leaves the (+) terminal, as shown below, then we have

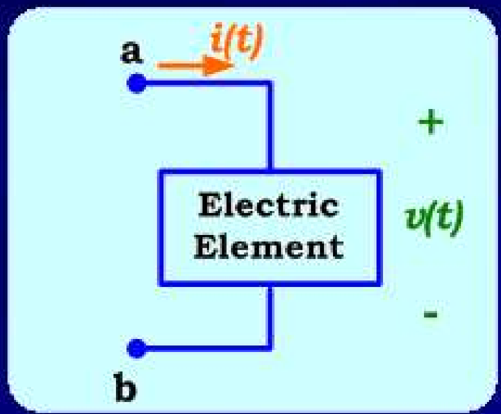
$$p(t) = -v(t)\,i(t)$$

In this case it is *necessary* to insert a minus sign in the power expression, in order to have consistent results.

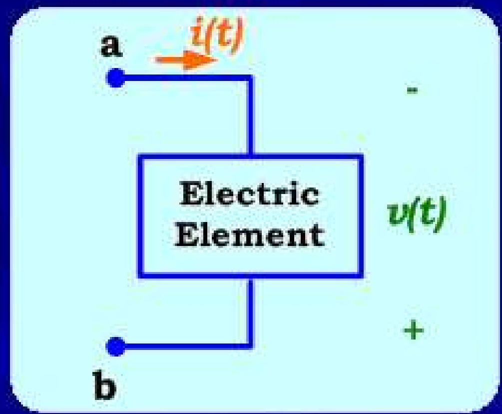


Relation with v-i

Summary



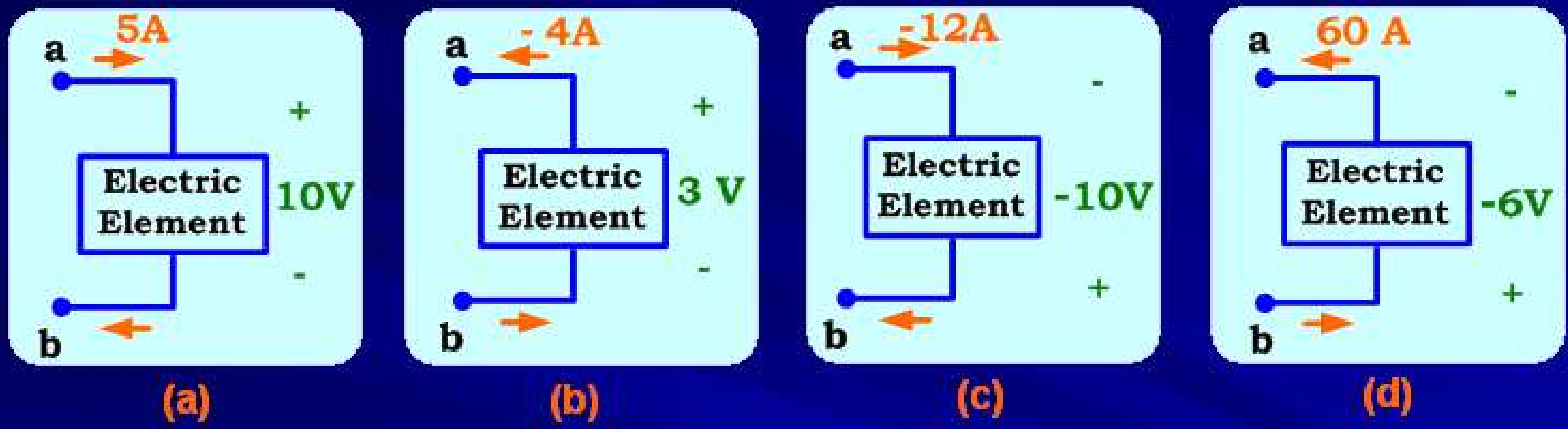
$$p = +v\ i$$



$$p = -v\ i$$

Example 1

Calculate the power absorbed by each element in the given circuits. State whether the power is *actually absorbed* or *delivered* by the element.



Solution:

(a) $p = +i\,v = +(5)(10) = +50\ W$ => $p > 0$ => power actually absorbed

(b) $p = -i\,v = -(-4)(3) = +12\ W$ => $p > 0$ => power actually absorbed

(c) $p = -i\,v = -(-12)(-10) = -120\ W$ => $p < 0$ => power actually delivered

(d) $p = +i\,v = +(60)(-6) = -360\ W$ => $p < 0$ => power actually delivered

Equivalent Statements

The following statements are equivalent

Power *absorbed by* the element

Power *delivered to* the element

Power *dissipated by* the element

Power *consumed by* the element

The following statements are also equivalent

Power *delivered by* the element

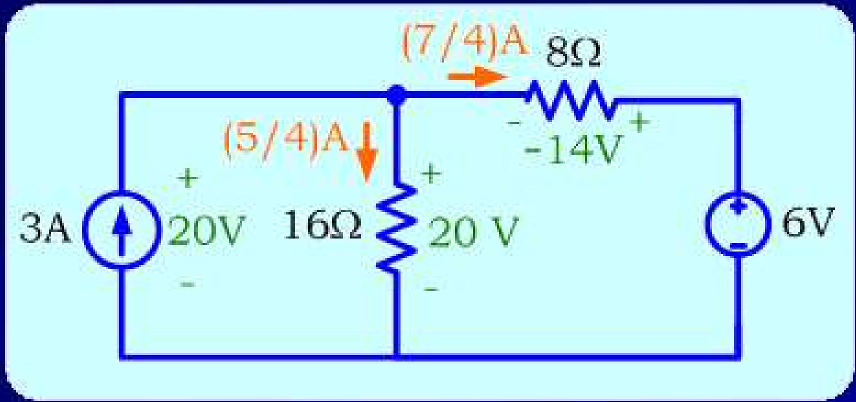
Power *generated by* the element

The symbol p will be reserved for the power absorbed by the element

Example 2

(i) Calculate the power absorbed by each element in the given circuit.

(ii) Show that the total power dissipated is equal to the total power generated



Solution:

(i) $p_{6V} = +i\,v = +(7/4)(6) = 10.5W$ => dissipated

$p_{3A} = -iv = -(3)(20) = -60W$ => generated

$p_{16W} = +iv = +(5/4)(20) = 25W$ => dissipated

$p_{8W} = -iv = -(7/4)(-14) = 24.5W$ => dissipated

(ii) $\Sigma\ p_{dis} = 10.5 + 25 + 24.5 = 60W$

$\Sigma\ p_{gen} = 60W$

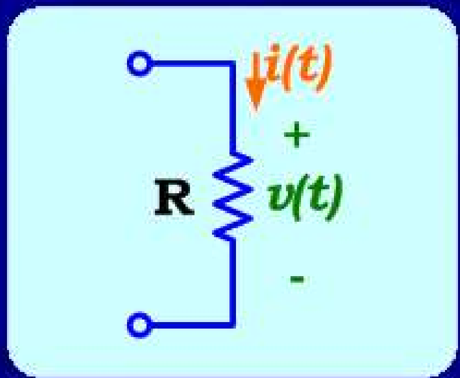
$\Sigma p_{dis} = \Sigma\ p_{gen}$

Ohm's Law

The voltage ***v(t)*** and current ***i(t)*** in a resistor ***R*** are related by

$$v = iR$$

The above relation is valid *only if* current ***i(t)*** enters the resistor from the (+) terminal and leaves the (-) terminal, as shown below

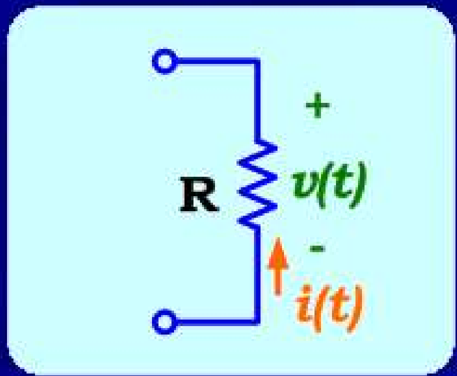


Ohm's Law

If the current ***i(t)*** enters the resistor from the (-) terminal and leaves the (+) terminal, as shown below, then Ohm's law *must be* change to

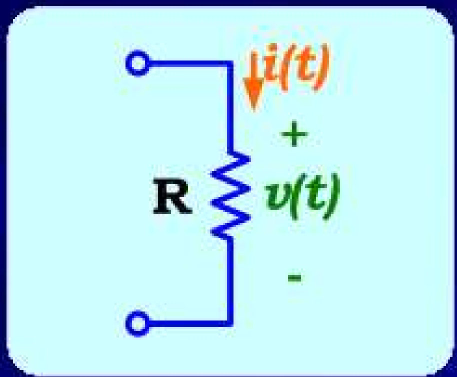
$$v = -iR$$

In this case it is necessary to insert a minus sign in the expression, in order to have consistent results.

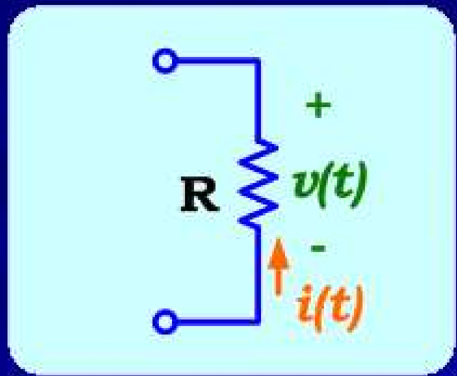


Ohm's Law

Summary



$$v = iR$$



$$v = -iR$$

The Passive Sign Convention

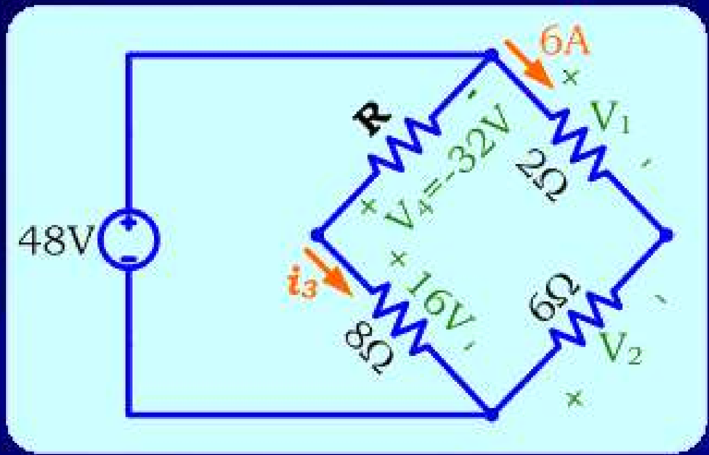
The use of the ± signs in the Ohm's law and the power expression is known as the *passive sign convention*

$i(t)$ enters the (+) terminal $\Rightarrow$ $p = +vi$ and $v = +iR$

$i(t)$ enters the (-) terminal $\Rightarrow$ $p = -vi$ and $v = -iR$

Example

Calculate the unknown quantities in the following circuit



Solution:

$$v_1 = +(6)(2) = 12\text{V}$$

$$i_3 = +\frac{16}{8} = 2\text{A}$$

$$v_2 = -(6)(6) = -36\text{V}$$

$$R = -\frac{v_4}{i_3} = -\frac{(-32)}{2} = 16\Omega$$

Al-Anbar University
College of engineering
Electrical Engineering Department

fundamental of Electric Circuit 1
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Stage 1
2021-2022

LECTURE 03
KIRCHHOFF'S CURRENT LAWS
KIRCHHOFF'S VOLTAGE LAWS-
DEPENDENT VOLTAGE SOURCES
DEPENDENT CURRENT SOURCES

Topics

- Kirchhoff's Voltage Law
- Fundamental Laws of Electric Circuits
- Dependent Voltage Source
- Dependent Current Source

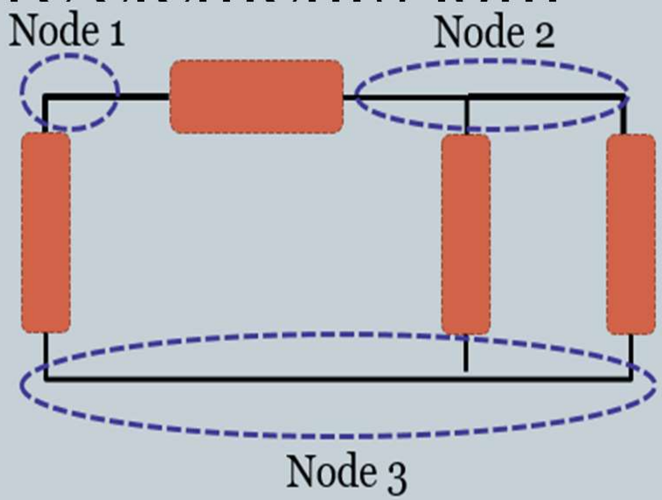
Objectives

- Apply Kirchhoff's voltage law
- Recognize invalid circuits
- Use the fundamental laws to analyze electric circuits
- Recognize the symbol of a dependent source
- Distinguish between the four possible types of dependent sources
- Analyze circuits that contain dependent sources

What is the terms node ?

A nodes is a point where two or more elements join

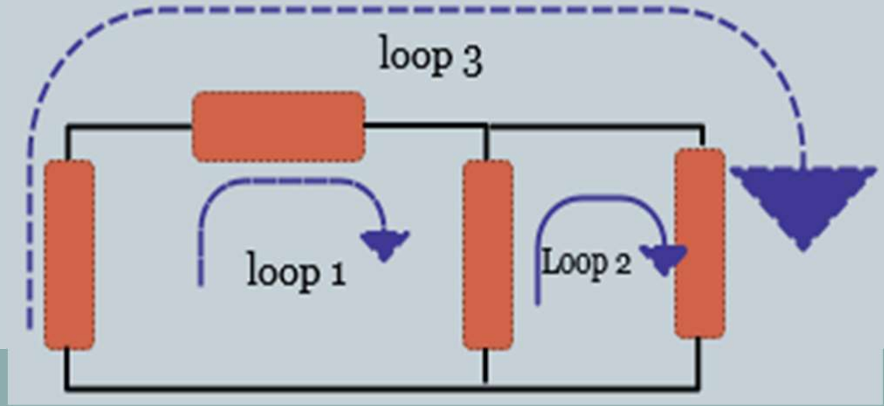
3- nodes shown in the figure



What is the terms loop?

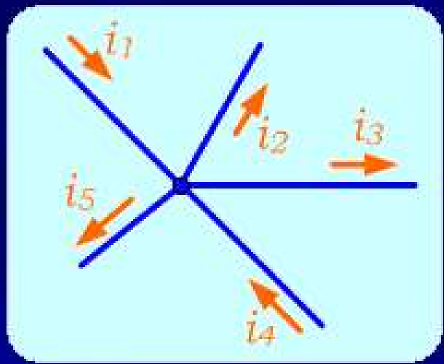
Any closed path in the circuit.

3- loops shown in figure



Kirchhoff's Current Law (KCL)

The sum of currents *entering* a node (interconnection of two or more branches) is equal to the sum of currents *leaving* that node



$$i_1 + i_4 = i_2 + i_3 + i_5$$

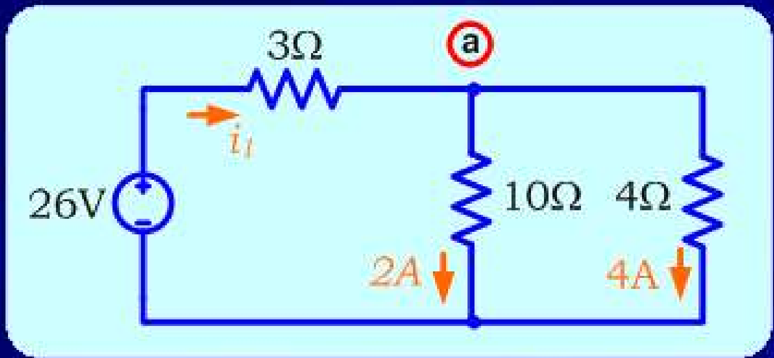
Equivalent Statement of KCL:

The algebraic sum of currents entering a node (currents entering the node is taken as positive) is equal to zero

$$i_1 - i_2 - i_3 + i_4 - i_5 = 0$$

Example 1

Calculate the unknown current in the following circuit

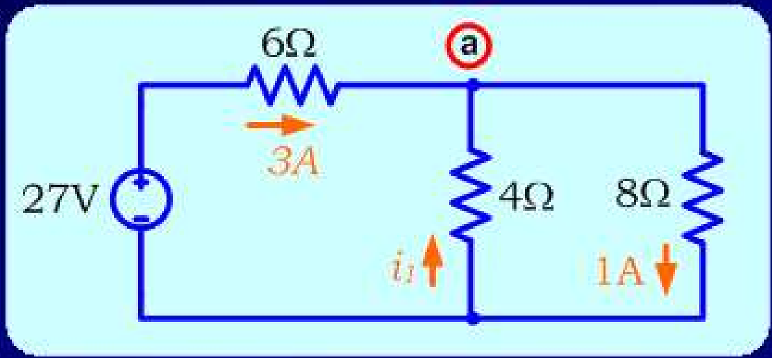


Solution:

KCL at node a $\Rightarrow$ $i_1 = 2 + 4 = 6\,A$

Example 2

Calculate the unknown current in the following circuit



Solution:

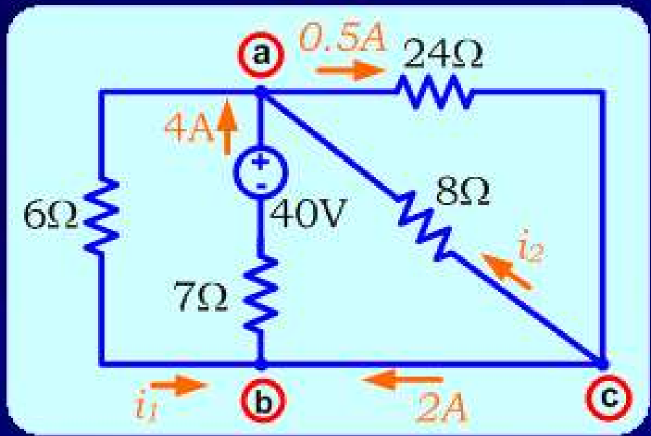
KCL at node a $\Rightarrow 3 + i_1 = 1 \Rightarrow i_1 = -2\,A$

Alternatively,

KCL at node a $\Rightarrow 3 + i_1 - 1 = 0 \Rightarrow i_1 = -2\,A$

Example 3

Calculate the unknown currents in the following circuit



Solution:

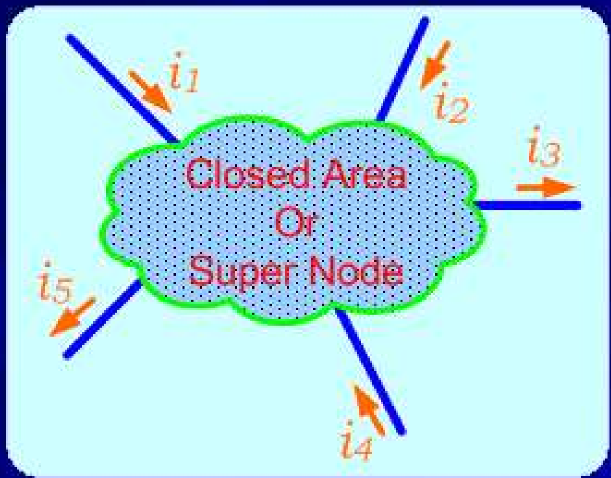
KCL at node b $\Rightarrow i_1 - 4 + 2 = 0 \Rightarrow i_1 = 2A$

KCL at node c $\Rightarrow 0.5 - i_2 - 2 = 0 \Rightarrow i_2 = -1.5A$

Check KCL at node a $\Rightarrow -i_1 + 4 + i_2 - 0.5 = -(2) + 4 + (-1.5) - 0.5 = -4 + 4 = 0$

Supernode

KCL is also applicable to a *closed area* (super node)

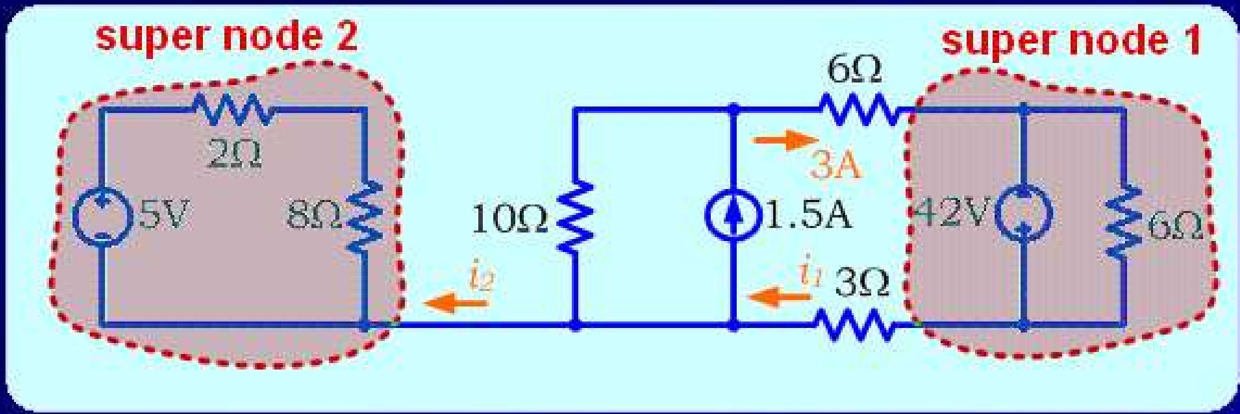


The algebraic sum of currents entering a super node is equal to zero

$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

Example 4

Calculate the unknown currents in the circuit shown below



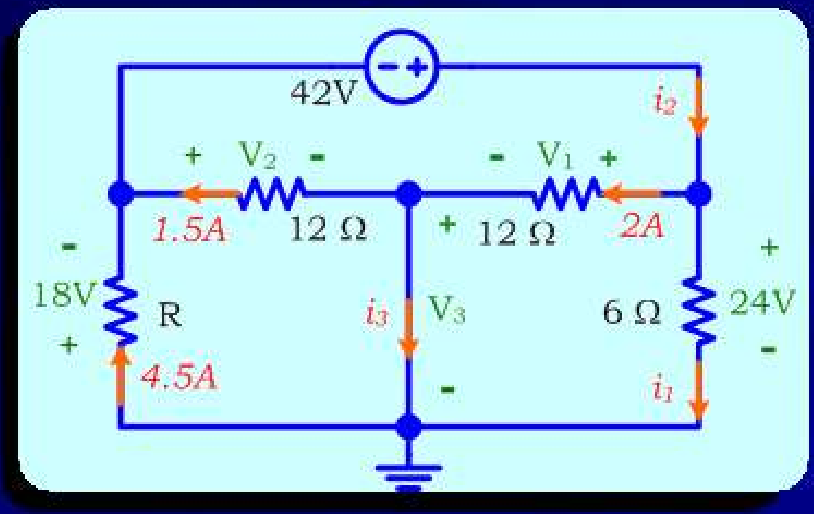
Solution:

KCL at super node 1 $\Rightarrow$ $3 - i_1 = 0$ $\Rightarrow$ $i_1 = 3A$

KCL at super node 2 $\Rightarrow$ $i_2 = 0$ $\Rightarrow$ $i_2 = 0A$

Self Test

In the circuit shown below, calculate:



(a) i_1

A $i_1 = 144A$

B $i_1 = -4A$

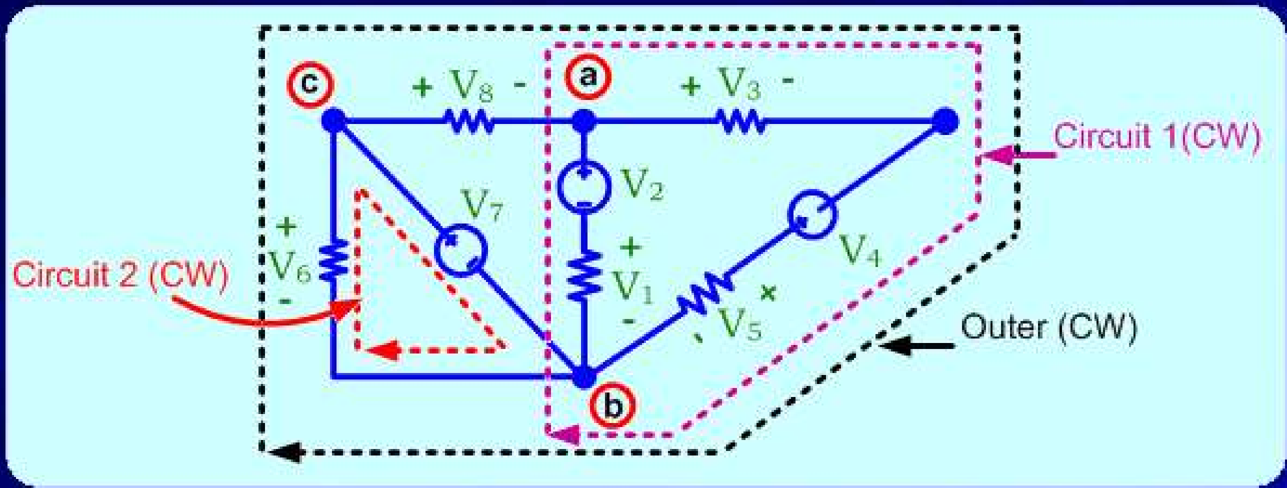
C $i_1 = 4A$

D $i_1 = 1/4\ A$

E $i_1 = -\ 1/4\ A$

Kirchhoff's Voltage Law (KVL)

The ***algebraic*** sum of voltages around ***any closed*** circuit is equal to zero



KVL around circuit 1 (CW) $\Rightarrow$ $-v_1 - v_2 + v_3 - v_4 + v_5 = 0 \cdots\cdots\cdots (1)$

KVL around circuit 1 (CCW) $\Rightarrow$ $-v_5 + v_4 - v_3 + v_2 + v_1 = 0 \cdots\cdots\cdots (2)$ [Same as (1)]

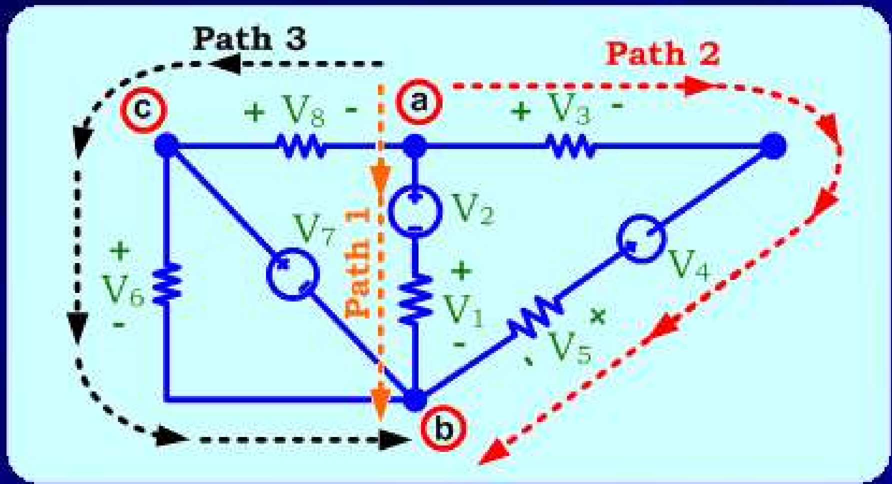
CW = Clockwise direction & CCW = Counter clockwise direction

KVL around outer circuit (CW) $\Rightarrow$ $-v_6 + v_8 + v_3 - v_4 + v_5 = 0 \cdots\cdots\cdots (3)$

KVL around circuit 2 (CW) $\Rightarrow$ $-v_6 + v_7 = 0$ $\Rightarrow$ $v_6 = v_7$ [Parallel elements]

Alternate statement for KVL

The *algebraic* sum of voltages between two nodes is *independent* of the path taken from the first node to the second node

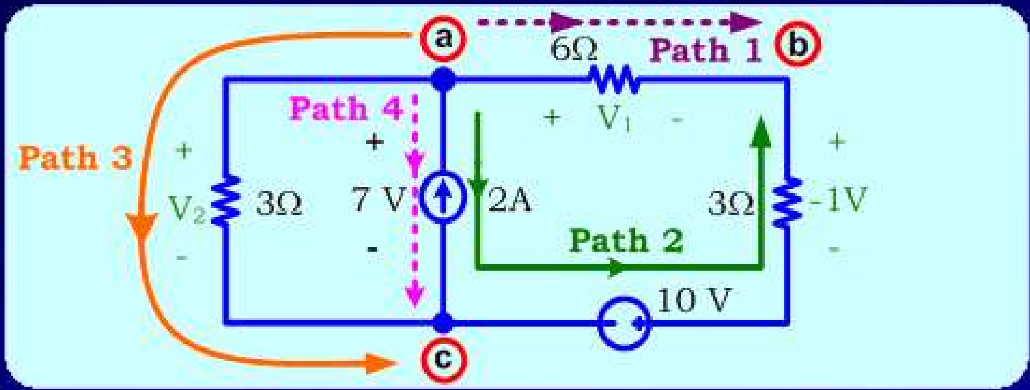


KVL $\text{Node } a \xrightarrow{path1\&2} \text{Node } b \Rightarrow +v_2 + v_1 = +v_3 - v_4 + v_5 \cdots\cdots\cdots (4)$ Same as (1) in the previous slide

KVL $\text{Node } a \xrightarrow{path2\&3} \text{Node } b \Rightarrow +v_3 - v_4 + v_5 = -v_8 + v_6 \cdots\cdots\cdots (5)$ Same as (3) in the previous slide

Example 1

Calculate the unknown voltages in the given circuit



Solution:

Applying KVL

Right-hand circuit (CW) $\Rightarrow -(7)+v_1+(-1)+10=0 \Rightarrow v_1=-2V$

Right-hand circuit (CCW) $\Rightarrow +(7)-(10)-(-1)-v_1=0 \Rightarrow v_1=-2V$

$Node\ a \xrightarrow{path1\&2} Node\ b \Rightarrow +v_1=+(7)-(10)-(-1) \Rightarrow v_1=-2V$

Same answer in all cases

Left-hand circuit (CW) $\Rightarrow +(7)-(v_2)=0 \Rightarrow v_2=7V$

$Node\ a \xrightarrow{path3\&4} Node\ c \Rightarrow +v_2=+7 \Rightarrow v_2=7V$

Same answer in both cases

Fundamental Laws of Electric Circuits

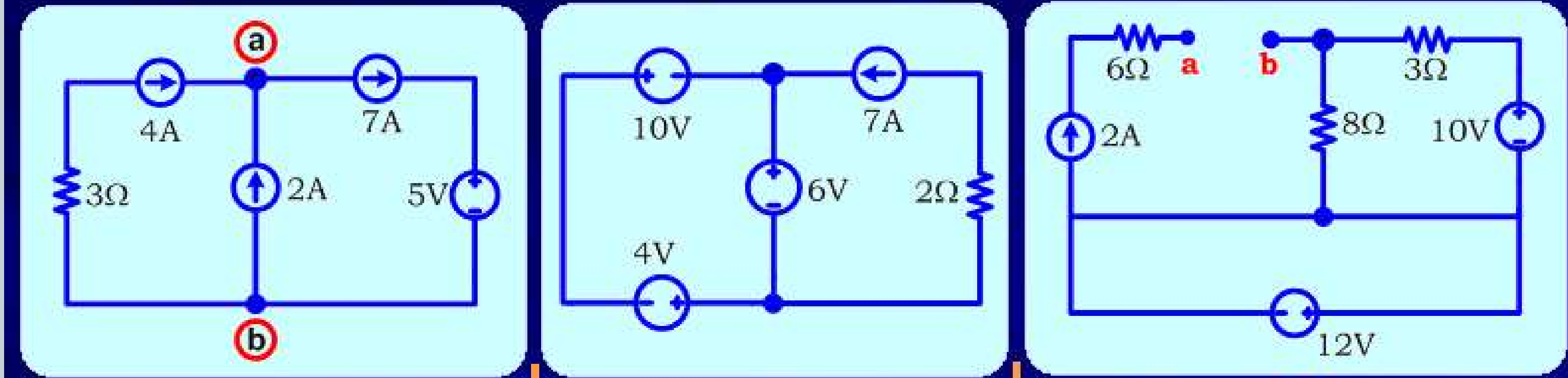
1. Ohm's Law, KCL and KVL are the fundamental laws of electric circuits

2. *All* the fundamental laws of electric circuits *must be satisfied*

3. If a given circuit violates *at least one* of the fundamental laws, the circuit is *not valid*

Example 2

All the given circuits below are invalid, why?



(a) (b) (c)

(a) KCL at node a

$\Rightarrow \quad 4+2=7$

$\Rightarrow$ KCL not satisfied

(b) KVL around left hand

$\Rightarrow \quad +10+6+4=0$

$\Rightarrow \quad 20=0$

$\Rightarrow$ KVL not satisfied

(c) KCL at node a

$\Rightarrow \quad 2=0$

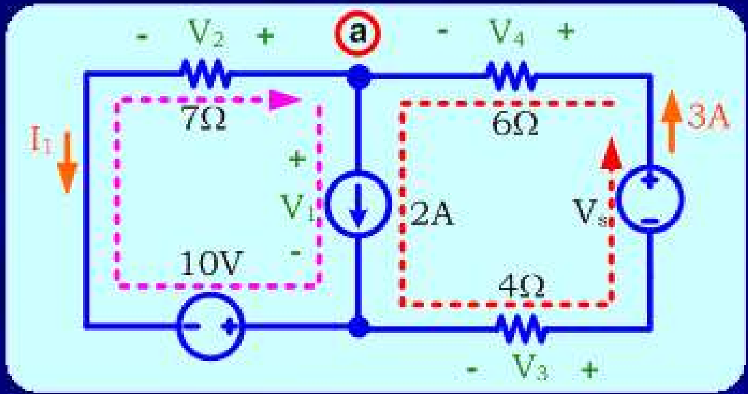
$\Rightarrow$ KCL not satisfied

KVL around lower part of the circuit $\Rightarrow \quad 12=0$

$\Rightarrow$ KVL not satisfied

Example 3

In the given circuit calculate the unknown quantities



Solution:

KCL at node a $\Rightarrow 3 = 2 + I_1 \Rightarrow I_1 = 1A$

Ohm's Law $\Rightarrow v_2 = +7I_1 = 7\times 1 = 7V$

KVL around left hand circuit $\Rightarrow v_1 + 10 - v_2 = 0$

$\Rightarrow v_1 + 10 - 7 = 0$

$\Rightarrow v_1 = -3V$

Ohm's Law $\Rightarrow v_3 = -3\times 4 = -12V$

KVL around right hand circuit $\Rightarrow +v_4 + v_1 - v_3 - v_s = 0$

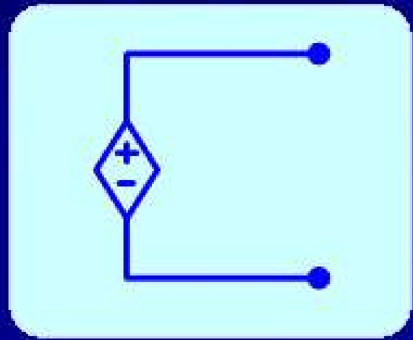
$\Rightarrow +(3\times 6) + v_1 - v_3 - v_s = 0$ (Ohm's Law $v_4 = 18V$)

$\Rightarrow +18 + (-3) - (-12) - v_s = 0$

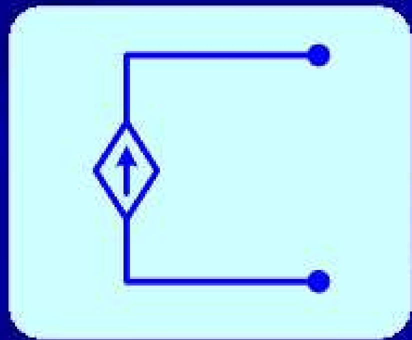
$\Rightarrow v_s = 27V$

Ideal Dependent Sources

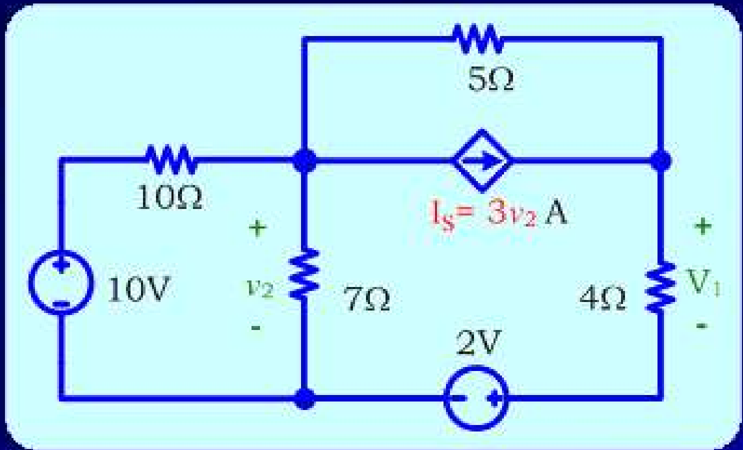
A voltage source whose voltage depends on another voltage or current is called a *dependent voltage source*



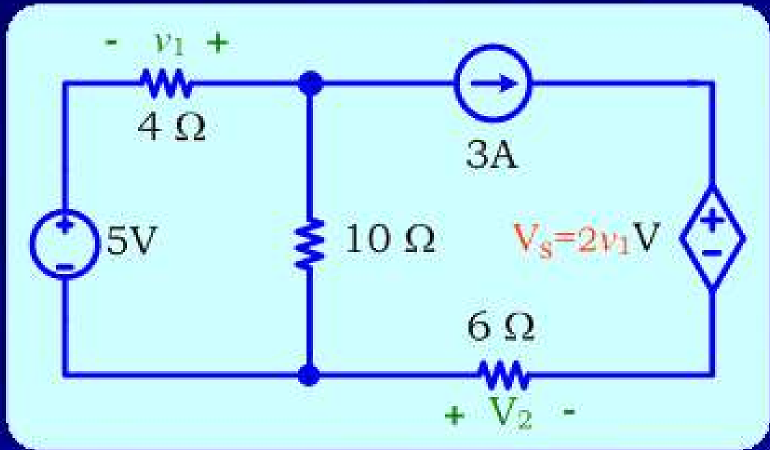
A current source whose current depends on another voltage or current is called a *dependent current source*



Example 4



(a)



(b)

Circuit (a) I_S depends on v_2

⇒ I_S is *voltage dependent* current source

Circuit (b) V_S depends on v_1

⇒ V_S is *voltage dependent* voltage source

Types of Dependent Sources

Four possible types of dependent sources

1. Voltage dependent voltage source (it is a *voltage source* that depends on another voltage)
2. Current dependent voltage source (it is a *voltage source* that depends on another current)
3. Voltage dependent current source (it is a *current source* that depends on another voltage)
4. Current dependent current source (it is a *current source* that depends on another current)

Example 5

(a) Calculate the value of the dependent current source

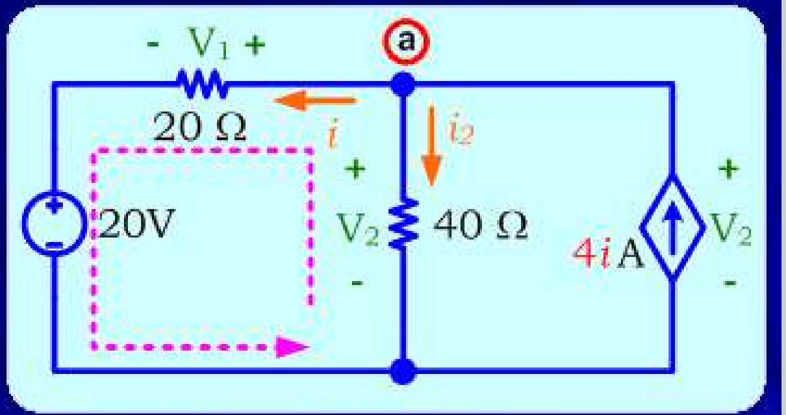
(b) Show that the power generated is equal to the power dissipated

Solution: (a)

KCL at node a

$$\Rightarrow -i - i_2 + 4i = 0$$

$$\Rightarrow i_2 = 3i \qquad (1)$$



KVL around left hand circuit

$$\Rightarrow -v_2 + v_1 + 20 = 0$$

$$\Rightarrow -(40i_2) + (20i) + 20 = 0 \quad \text{(Ohm's Law)}$$

$$\Rightarrow -(40 \times 3i) + 20i + 20 = 0 \quad \text{[using (1)]}$$

$$\Rightarrow -120i + 20i + 20 = 0$$

$$\Rightarrow i = \frac{20}{100} = \frac{1}{5}\text{A}$$

Value of dependent current source $\Rightarrow 4i = 4 \times \frac{1}{5} = \frac{4}{5}\text{A}$

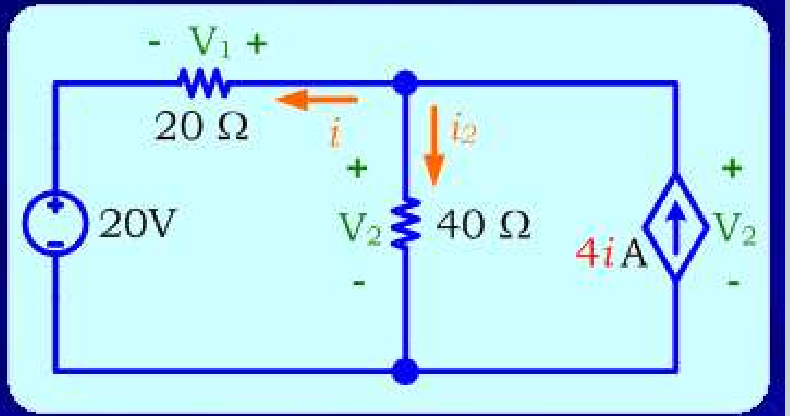
Example 5 (Contd...)

(a) Calculate the value of the dependent current source

(b) Show that the power generated is equal to the power dissipated

Solution: (b)

From previous slide part (a) $\Rightarrow$ $i = \frac{1}{5}\text{A}$



$$p_{20V} = +20i = +20 \times \frac{1}{5} = \frac{20}{5} = 4\,\text{W}$$

$$p_{20\Omega} = +iv_1 = +(\tfrac{1}{5})(20 \times \tfrac{1}{5}) = +(\tfrac{1}{5})(4) = \frac{4}{5}\,\text{W}$$

$$i_2 = 3i = \frac{3}{5}\text{A} \quad \& \quad v_2 = +40i_2 = +40 \times \frac{3}{5} = 24\,\text{V}$$

$$p_{40\Omega} = +i_2 v_2 = +\frac{3}{5} \times 24 = \frac{72}{5}\,\text{W}$$

$$p_{4iA} = -(4i)v_2 = -(\tfrac{4}{5}) \times 24 = -\frac{96}{5} = -19.2\,\text{W}$$

$$\sum p_{dis} = 4 + \frac{4}{5} + \frac{72}{5} = \frac{96}{5} = 19.2\,\text{W}$$

$$\sum p_{gen} = 19.2\,\text{W}$$

$$\therefore \quad \sum p_{dis} = \sum p_{gen} = 19.2\,\text{W}$$

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2021-2022

LECTURE 04

SERIES AND PARALLEL CONNECTIONS
EQUIVALENT RESISTANCE-CONDUCTANCE

Topics

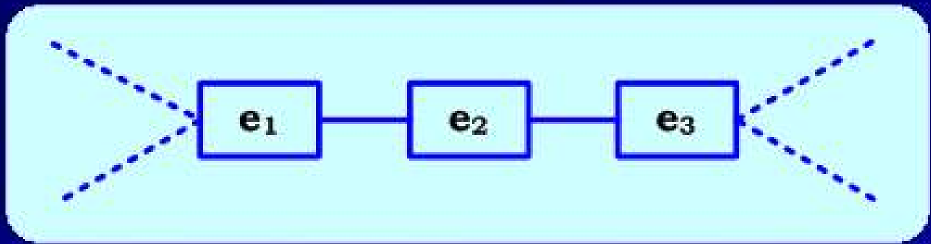
- Series connection
- Parallel connection
- Equivalent resistance
- Conductance
- Power absorbed by a resistor

Objectives

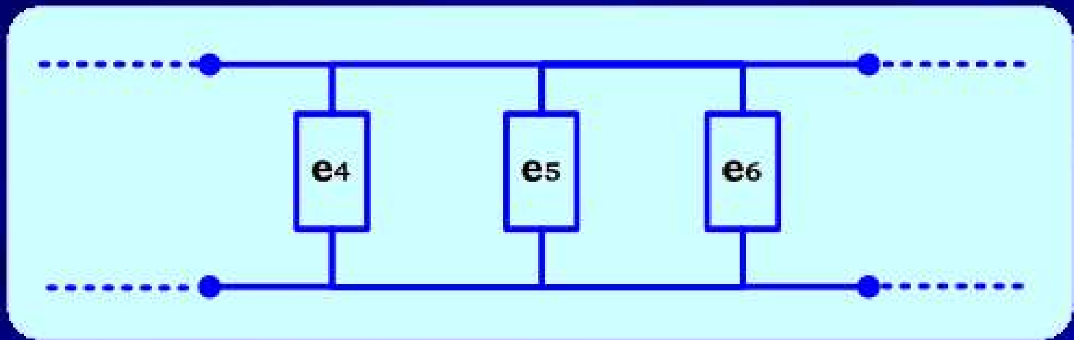
- Recognize series connections
- Recognize parallel connections
- Understand the meaning of series and parallel connections
- Calculate the equivalent resistance
- Relate conductance to resistance
- Understand power absorption by a resistor

Series and Parallel Connections

The electric elements e_1, e_2 *and* e_3 are connected in series



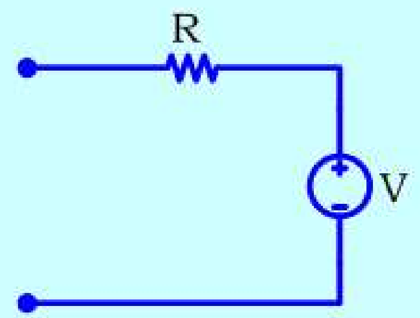
The electric elements e_4, e_5 *and* e_6 are connected in parallel



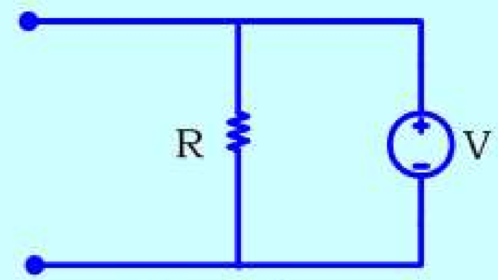
Example

Any two terminal electric element can be connected in series or in parallel to *any other* element

For example, a voltage source can be connected in series or in parallel to a resistor



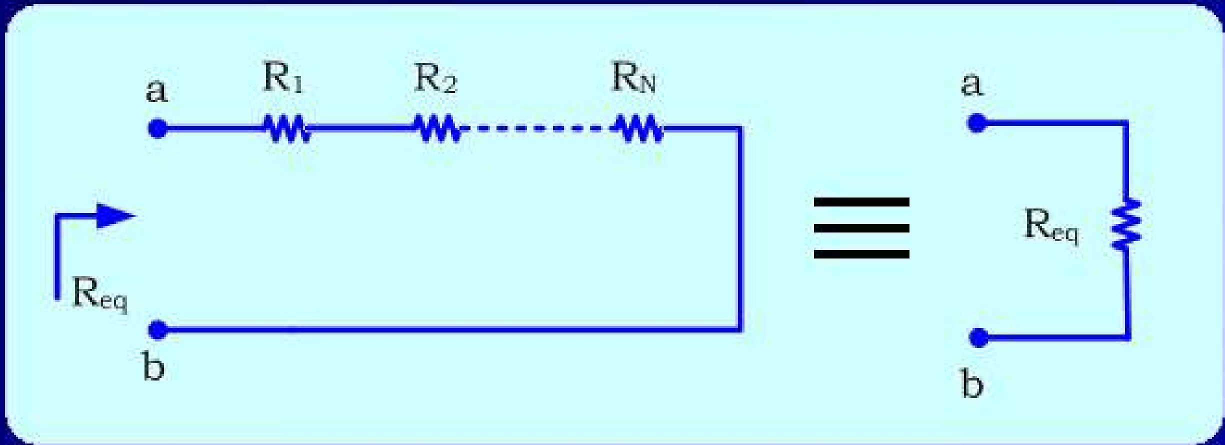
Series



Parallel

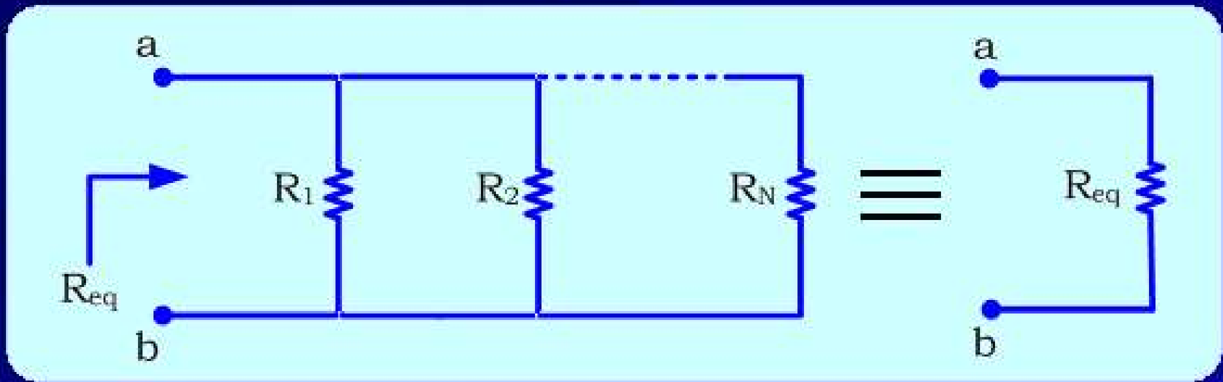
Equivalent Resistance of N Resistors in Series

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{i=1}^{N} R_i$$



Equivalent Resistance of N Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

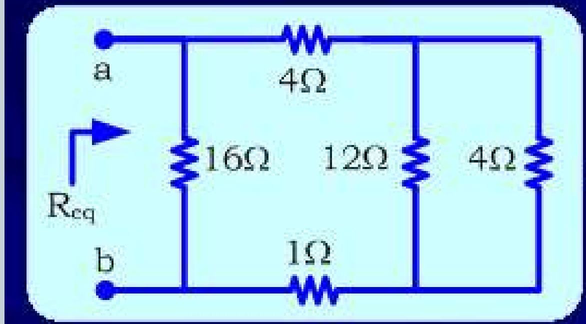


Special Case: If two resistors R_1 and R_2 are in parallel

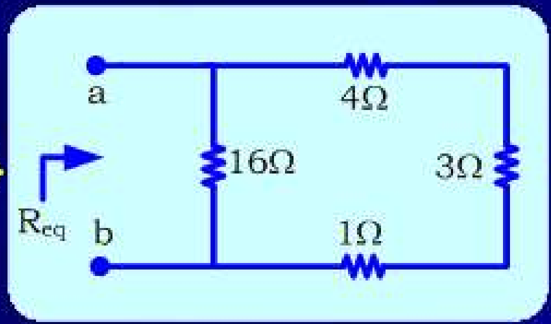
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow R_{eq} = \frac{\text{Product}}{\text{Sum}}$$

Example 1

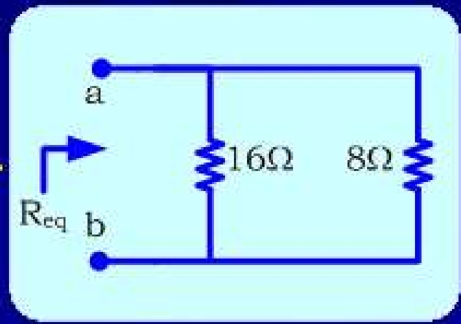
Calculate the equivalent resistance seen to the right of a-b



⇒



⇒



12Ω and 4Ω in parallel

$$\frac{12\times 4}{12+4}=\frac{48}{16}=3\Omega$$

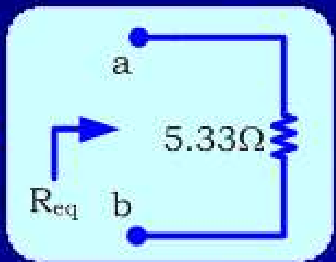
4Ω, 3Ω and 1Ω in series

$$4+3+1=8\Omega$$

16Ω and 8Ω in parallel

$$\frac{16\times 8}{16+8}=\frac{16\times 8}{24}=\frac{16}{3}=5.33\Omega$$

⇒



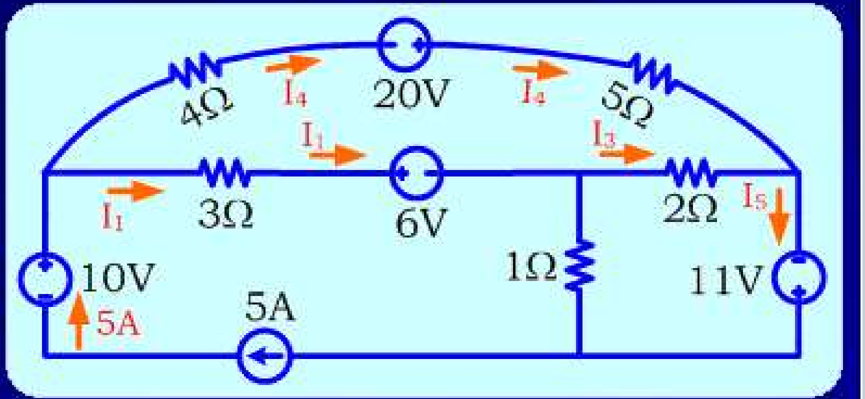
$$\therefore R_{eq}=5.33\Omega$$

Series and Parallel Connections

Why?

(a) 3Ω and 6V source are in series
10V and 5A sources are in series
4Ω, 20V source and 5Ω are in series

(b) 6V source and 2Ω are not in series
2Ω and 11V source are not in series



Solution

(a) 3Ω and 6V are in series ⟹ the *same current* I_1 passes through them
10V and 5A sources are in series ⟹ the *same current* 5A passes through them
4Ω, 20V source and 5Ω are in series ⟹ the *same current* I_4 passes through them

(b) 6V source and 2Ω are not in series ⟹ *different* currents I_1 and I_3 passes through them
2Ω and 11V source are not in series ⟹ *different* currents I_3 and I_5 passes through them

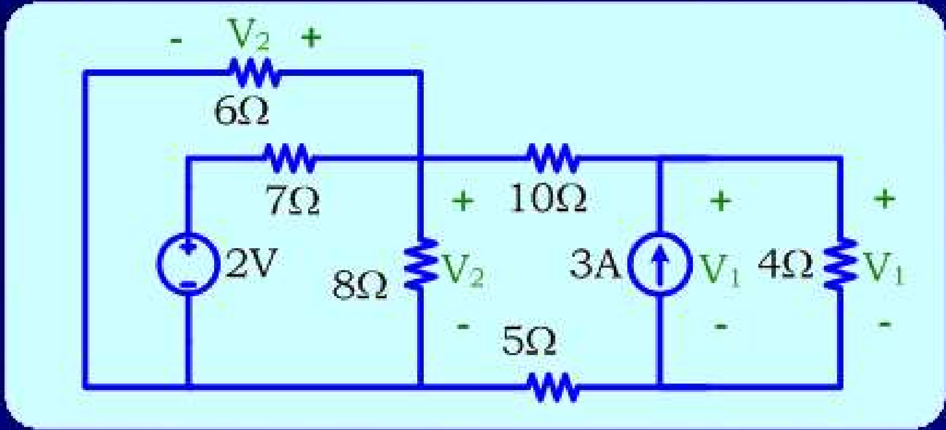
Series and Parallel Connections

3A source and 4Ω are in parallel

6Ω and 8Ω are in parallel

2V source and 8Ω are not in parallel

Why?



Solution

the *same voltage* V_1 appears across 3A and 4Ω ⟹ they are in parallel

the *same voltage* V_2 appears across 6Ω and 8Ω ⟹ they are in parallel

different voltages appear across 2V and 8Ω ⟹ they are not in parallel

Conductance

The conductance *G* of a resistor is the reciprocal of the resistance R

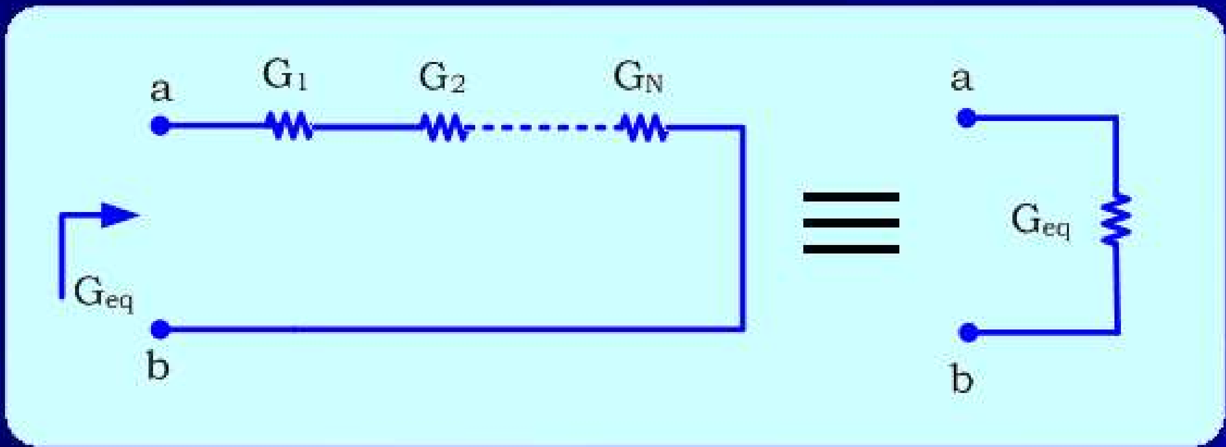
$$G = \frac{1}{R}$$

Unit of G is $\frac{1}{\Omega}$ or *Siemens* $[S]$ $\Rightarrow$ $\frac{1}{\Omega} \equiv S$

Conductance

N conductances in series

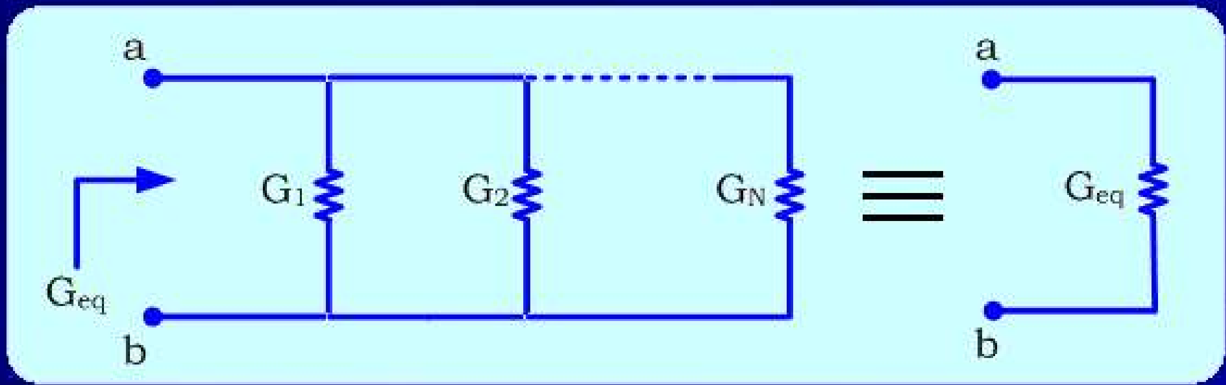
$$\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \ldots + \frac{1}{G_N} = \sum_{i=1}^{N} \frac{1}{G_i}$$



Conductance

N conductances in parallel

$$G_{eq} = G_1 + G_2 + \ldots + G_N = \sum_{i=1}^{N} G_i$$



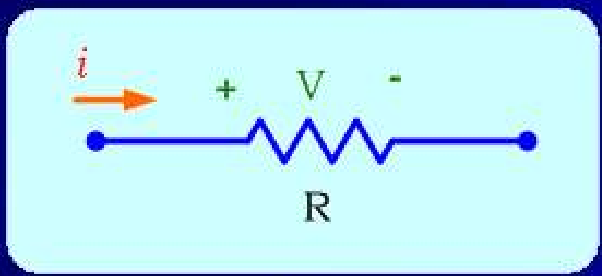
Power Absorbed by a Resistor

Using circuit (a) $p_R = +iv = +i(iR) = i^2R = \dfrac{v^2}{R}$

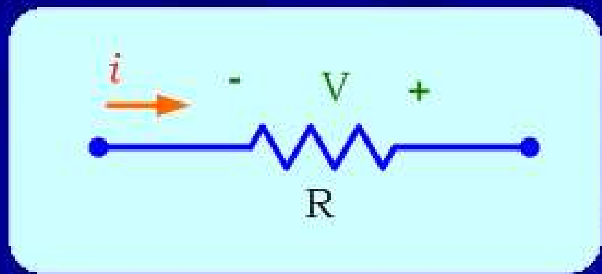
Using circuit (b) $p_R = -iv = -i(-iR) = i^2R = \dfrac{v^2}{R}$

$\therefore\ p_R = \dfrac{v^2}{R} = i^2R$ (regardless of the direction of i and polarity of v)

$\therefore\ p_R \geq 0 \Longrightarrow$ a resistor does not generate electric power, it always absorbs it



(a)



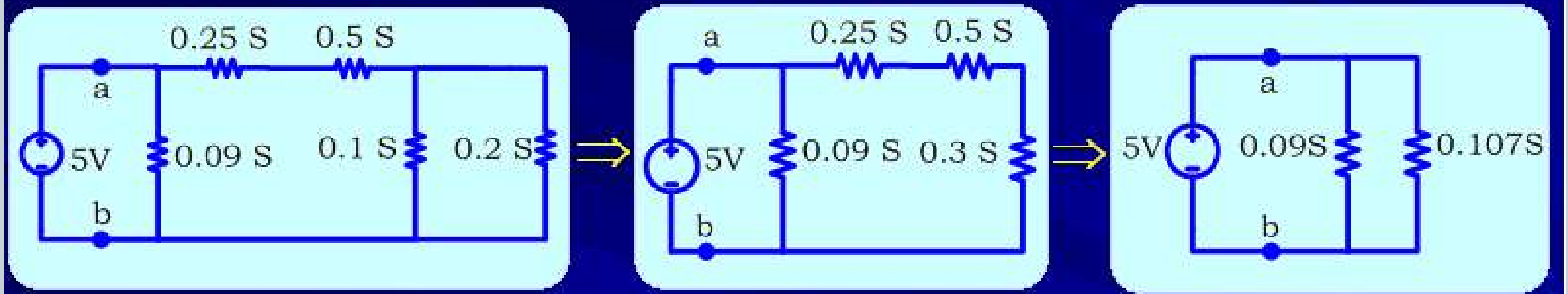
(b)

Example 2

In the given circuit calculate

(a) Geq seen by the voltage source

(b) Req (c) the power absorbed by the load



(a) 0.1S and 0.2S in parallel

$0.1 + 0.2 = 0.3S$

0.25S, 0.5S, 0.3S in series

$$\frac{1}{0.25} + \frac{1}{0.5} + \frac{1}{0.3} = 4 + 2 + 3.33 = 9.33$$

$$\Rightarrow \frac{1}{9.33} = 0.107S$$

0.107 & 0.09 in parallel

$0.107 + 0.09 = 0.197S$

$\therefore G_{eq} = 0.197S$

(b) $R_{eq} = \frac{1}{Geq} = \frac{1}{0.197} = 5.08\Omega$

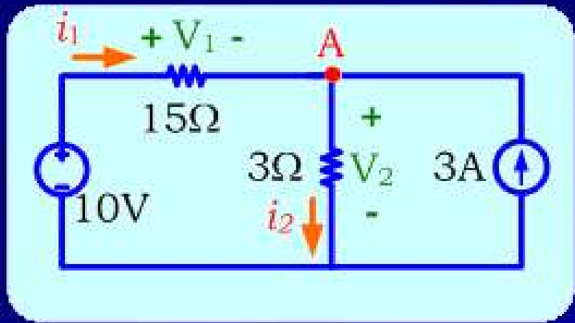
(c) $p_{5.08\Omega} = \frac{v^2}{R} = \frac{(5)^2}{5.08} = 4.97W$

Example 3

Calculate

(a) the power absorbed by the 3Ω resistor

(b) the equivalent resistance seen by the 10V source



(a) KVL $\Rightarrow -10 + v_1 + v_2 = 0$

Ohm's Law $\Rightarrow -10 + 15i_1 + 3i_2 = 0 \cdots\cdots (1)$

KCL $\Rightarrow i_1 + 3 = i_2 \cdots\cdots\cdots\cdots (2)$

Solving (1) and (2) $\Rightarrow -10 + 15(i_2 - 3) + 3i_2 = 0$

$\Rightarrow 18i_2 = 55 \;\Rightarrow\; i_2 = \frac{55}{18} = 3.056\text{A}$

$\therefore p_{3\Omega} = 3i_2^2 = 3(3.056)^2 = 28.02\text{W}$

(b) Using (2) $\Rightarrow i_1 = i_2 - 3 = 3.056 - 3 = 0.056\text{A}$

$\therefore R_{eq} = +\frac{v}{i_1} = +\frac{10}{0.056} = 178.57\Omega$

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LECTURE 05

SOURCES IN SERIES AND PARALLEL
VOLTAGE DIVIDER RULE-CURRENT DIVIDER RULE

Topics

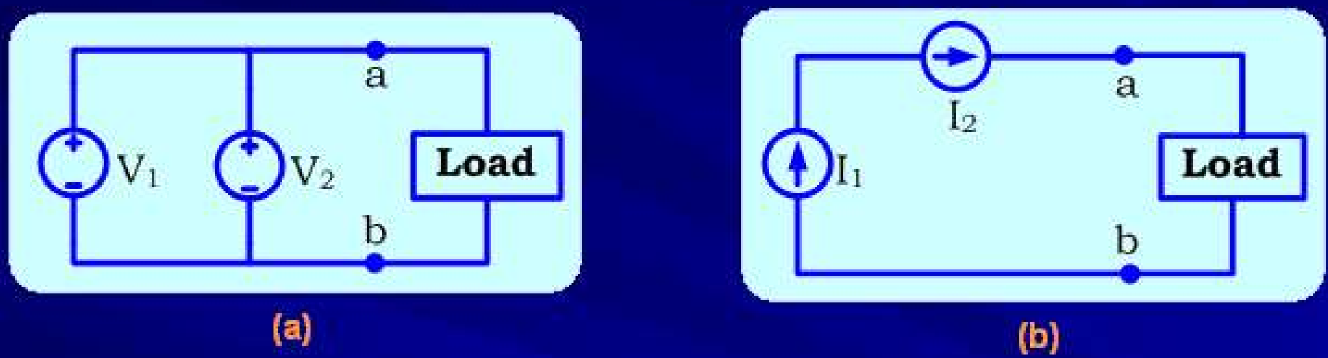
- Voltage Sources in Series and Parallel
- Current Sources in Series and Parallel
- Combining KVL and Ohm's Law
- Voltage Divider Rule
- Current Divider Rule

Objectives

- Recognize invalid series and parallel source connections
- Combine voltage sources in series
- Combine current sources in parallel
- Directly incorporate Ohm's Law in KVL
- Use the Voltage Divider Rule to simplify circuit analysis
- Use the Current Divider Rule to simplify circuit analysis

Series and Parallel Connection of Sources

Both circuits are invalid, why?



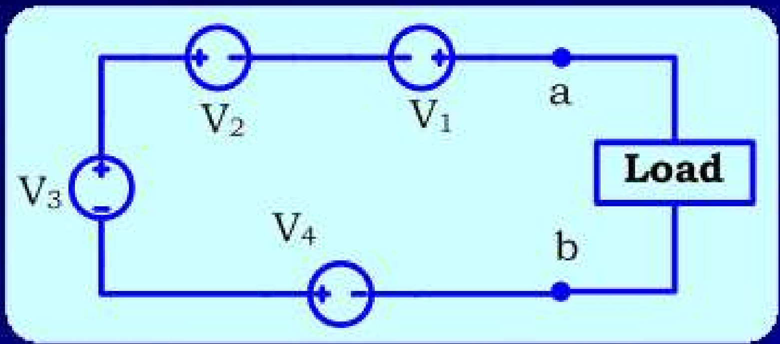
Circuit (a) violates KVL ⇒ Ideal voltage sources cannot be combined in parallel
(unless they have the same voltage)

Circuit (b) violates KCL ⇒ Ideal current sources cannot be combined in series
(unless they have the same current)

Voltage sources in series

We can connect ideal voltage sources in series

Voltage sources in series can be reduced to a single voltage source



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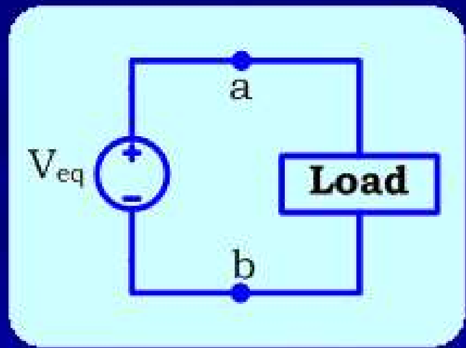
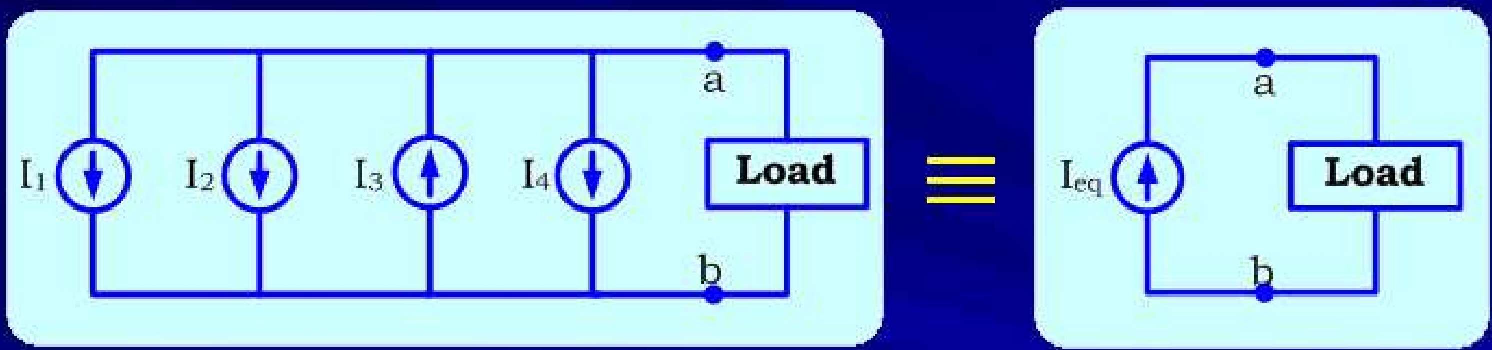


$$V_{eq} = V_1 - V_2 + V_3 + V_4$$

Current sources in parallel

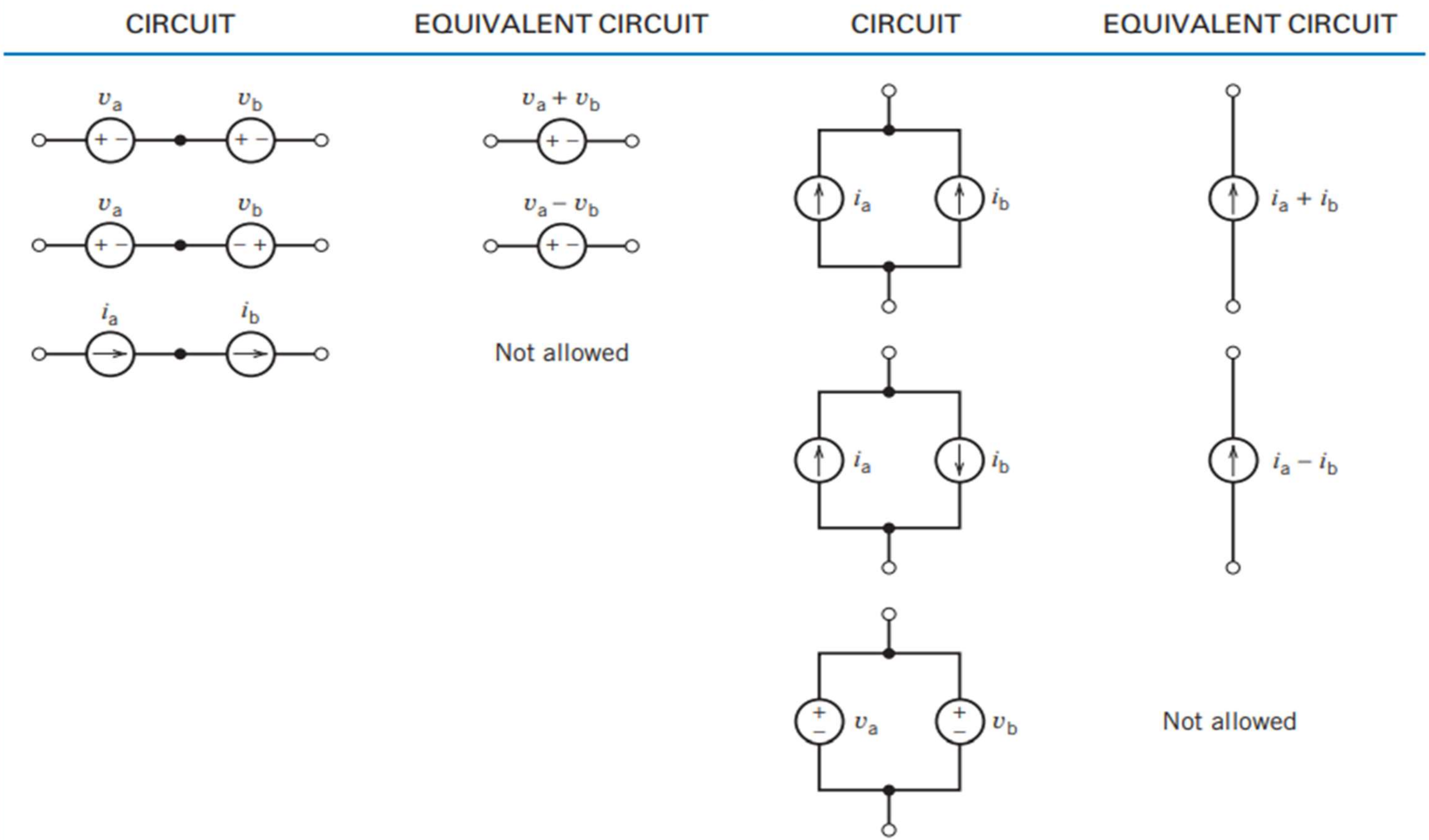
We can connect ideal current sources in parallel

Current sources in parallel can be combined as a single current source



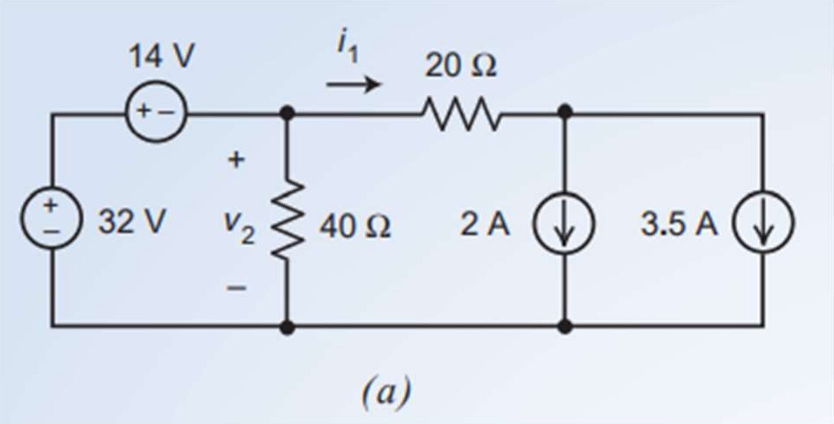
$$I_{eq} = -I_1 - I_2 + I_3 - I_4$$

Parallel and serious voltage and current sources



Example

Figures 3.5-3*a* and *c* show two similar circuits. Both contain series voltage sources and parallel current sources. In each circuit, replace the series voltage sources with an equivalent voltage source and the parallel current sources with an equivalent current source.



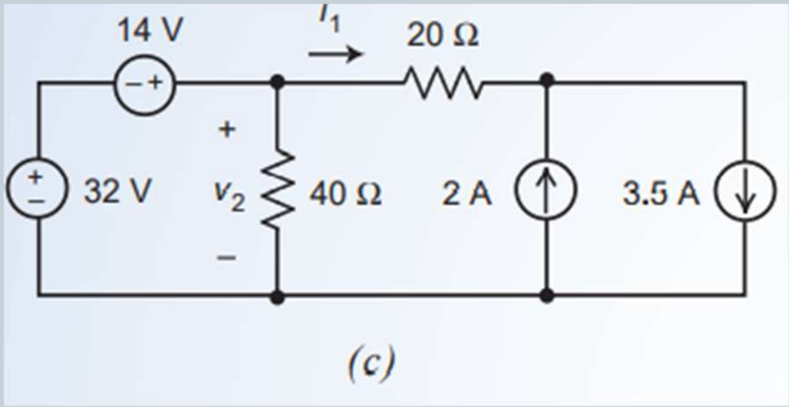
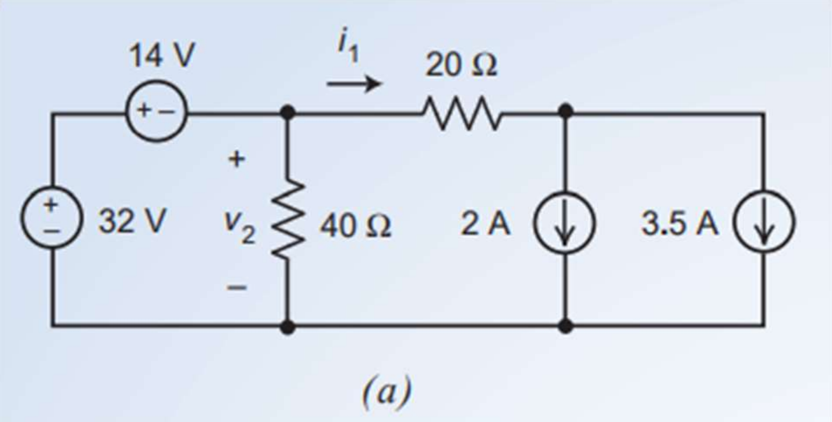


Figure 3.5-3

Solution



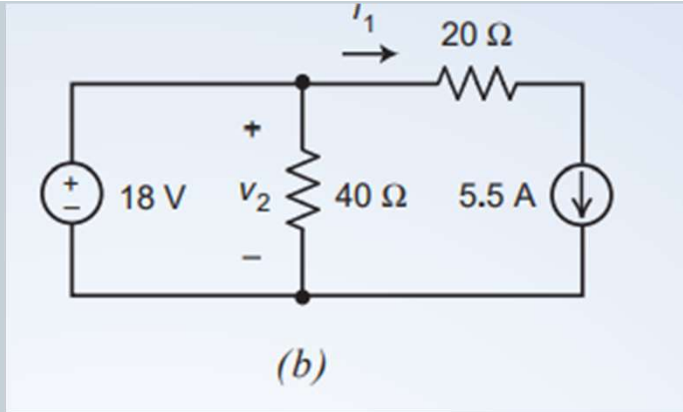


Figure 3.5-3

Consider first the circuit in Figure 3.5-3*a*. Apply KVL to the left mesh to get

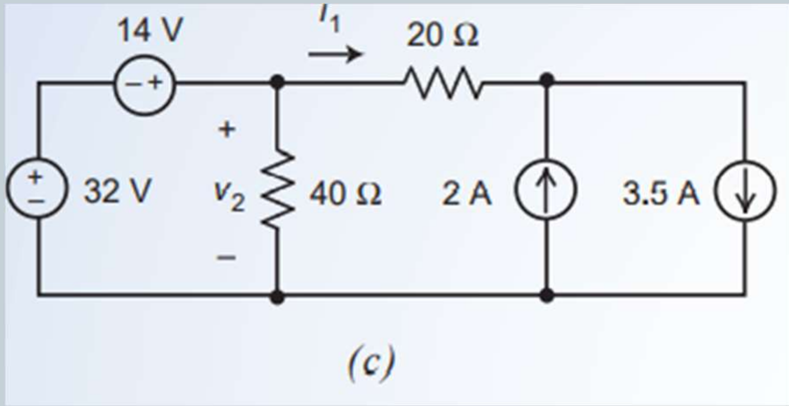
$$14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 18 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 = 2 + 3.5 \quad \Rightarrow \quad i_1 = 5.5$$

These equations suggest that we replace the series voltage sources by a single 18-V source and replace the parallel current sources by a single 5.5-A source. Figure 3.5-3*b* shows the result.

Solution



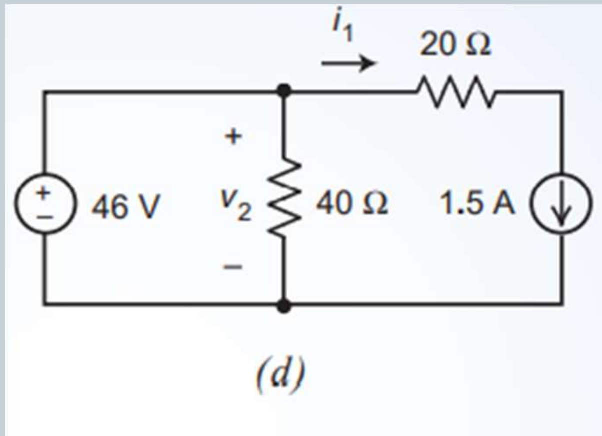


Figure 3.5-3

Next, consider first the circuit in Figure 3.5-3*c*. Apply KVL to the left mesh to get

$$-14 + v_2 - 32 = 0 \quad \Rightarrow \quad v_2 - 46 = 0$$

Next apply KCL at the right node of the 20 Ω to get

$$i_1 + 2 = 3.5 \quad \Rightarrow \quad i_1 = 1.5$$

Combining Ohm's Law and KVL

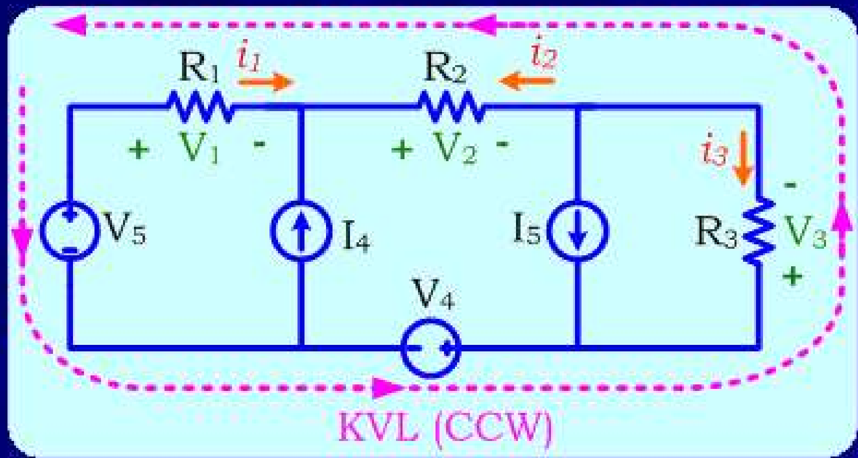
KVL around outer circuit (CW)

$$-v_5 + v_1 + v_2 - v_3 + v_4 = 0$$

Using Ohm's Law

$$-v_5 + (i_1 R_1) + (-i_2 R_2) - (-i_3 R_3) + v_4 = 0$$

$$\Rightarrow -v_5 + i_1 R_1 - i_2 R_2 + i_3 R_3 + v_4 = 0 \cdots\cdots (1)$$



KVL equation can be written directly in terms of the resistor currents i_1, i_2 and i_3

i (through R) same as KVL direction	$\Rightarrow$	$+iR$
i (through R) opposite to KVL direction	$\Rightarrow$	$-iR$

Using this rule,

KVL around outer circuit (CCW) $\Rightarrow +v_5 - v_4 - i_3 R_3 + i_2 R_2 - i_1 R_1 = 0$ [The same as (1)]

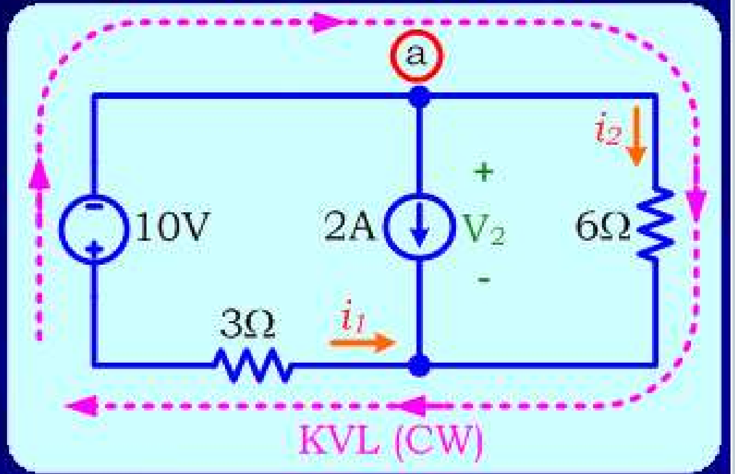
Ohm's Law can also be combined with KCL. This case will be covered in later lectures

Example 2

In the given circuit calculate

(a) i_1 and i_2

(b) the power absorbed by the current source



Solution

(a) KVL around outer circuit (CW)

$$10 + 6i_2 - 3i_1 = 0 \cdots\cdots (1)$$

KCL at node 'a'

$$i_1 + 2 + i_2 = 0 \cdots\cdots (2)$$

Solving (1) and (2) $\Rightarrow 10 + 6(-i_1 - 2) - 3i_1 = 0 \Rightarrow \boxed{i_1 = -\frac{2}{9}\text{A}}$

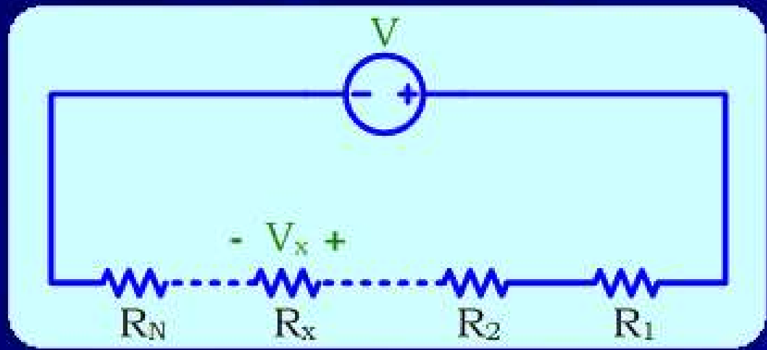
Substituting in (2) $\Rightarrow -\frac{2}{9} + 2 + i_2 = 0 \Rightarrow \boxed{i_2 = -2 + \frac{2}{9} = -\frac{16}{9}\text{A}}$

(b) Ohm's Law $\Rightarrow v_2 = 6i_2 \Rightarrow v_2 = 6(-\frac{16}{9}) = -\frac{32}{3}\text{V}$

$$\boxed{p_{2A} = +iv = +(2)(-\frac{32}{3}) = -21.33\text{W}}$$

The Voltage Divider Rule

The total voltage across the *series* resistors $R_1, R_2, \ldots, R_N$ is V



$$i = \frac{V}{R_{eq}} = \frac{V}{\sum_{i=1}^{N} R_i}$$

$$v_x = iR_x = \frac{V}{\sum_{i=1}^{N} R_i} R_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

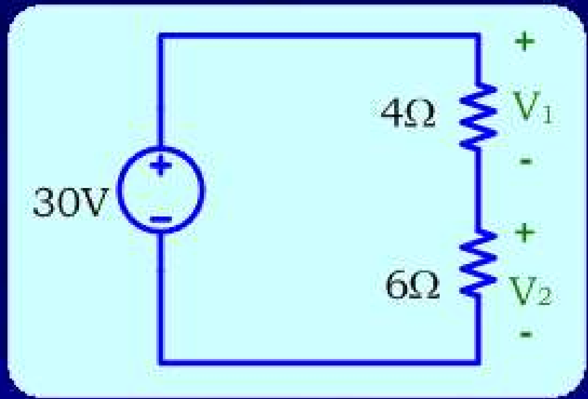
$$\text{VDR} \Rightarrow v_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

$$\therefore v_{resistor} = \frac{\text{resistor}}{\text{sum}} \times (\text{total voltage})$$

The VDR is valid for *any* number of resistors in *series*

Example 3

Calculate the unknown voltages



Solution

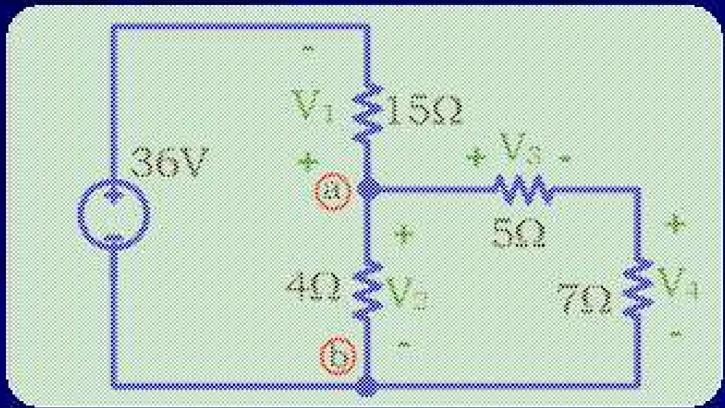
$$\text{VDR} \Rightarrow v_1 = \frac{4}{4+6} \times 30 = 12\text{V}$$

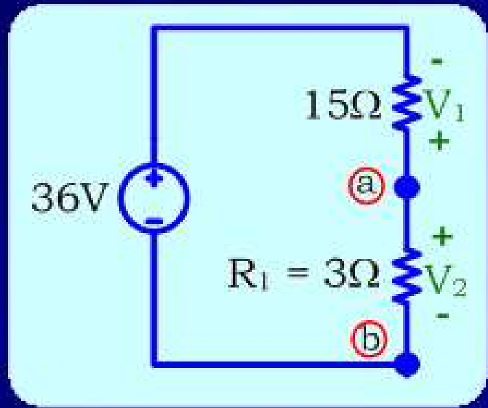
$$\text{VDR} \Rightarrow v_2 = \frac{6}{4+6} \times 30 = 18\text{V}$$

VDR $\Rightarrow$ Higher voltage drop across the higher resistance

Example 4

Calculate the unknown voltages





Solution

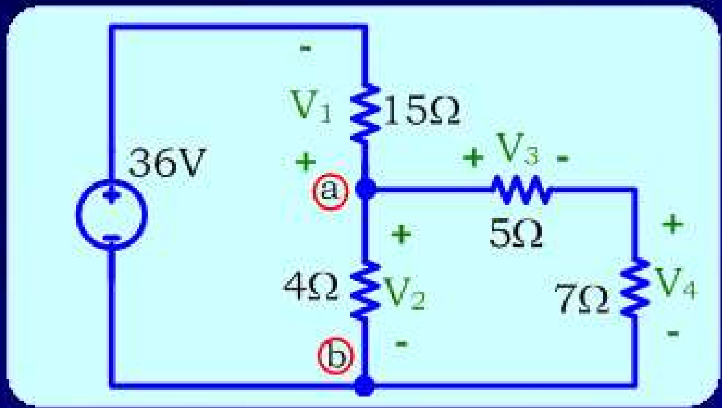
$$5+7=12\Omega \Rightarrow R_1=\frac{4\times 12}{4+12}=3\Omega$$

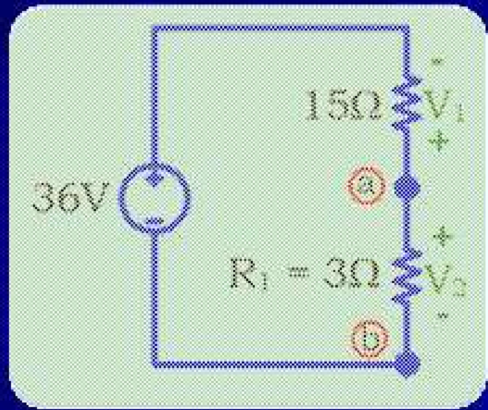
VDR $\Rightarrow v_1=-\frac{15}{15+3}\times 36$ (a minus sign is required here. Why?) $\Rightarrow \boxed{v_1=-30\text{V}}$

VDR $\Rightarrow v_2=\frac{3}{15+3}\times 36 \Rightarrow \boxed{v_2 = 6\text{ V}}$

Check: KVL $\Rightarrow -36-v_1+v_2=-36-(-30)+(6)=-36+30+6=0$

Example 4 (Contd...)





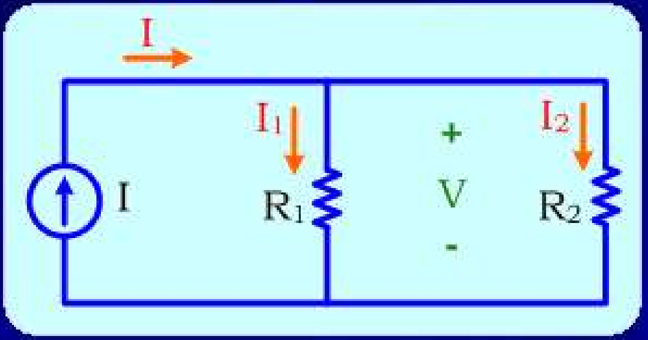
From previous slide $v_1 = -30\text{V}$ $v_2 = 6\text{ V}$

VDR $\Rightarrow v_3 = \frac{5}{5+7} \times v_2 = \frac{5}{12} \times 6 \Rightarrow v_3 = 2.5\text{V}$

VDR $\Rightarrow v_4 = \frac{7}{5+7} \times v_2 = \frac{7}{12} \times 6 \Rightarrow v_4 = 3.5\text{V}$

The Current Divider Rule

The total current entering into the *parallel* combination of resistors R_1 & R_2 is I



$$V = IR_{eq} = I\frac{R_1R_2}{R_1+R_2}$$

$$I_1 = \frac{V}{R_1} \quad \& \quad I_2 = \frac{V}{R_2}$$

$$I_1 = \frac{1}{R_1}\times\frac{R_1R_2}{R_1+R_2}I \Rightarrow I_1 = \frac{R_2}{R_1+R_2}I \cdots\cdots (1)$$

Similarly $$I_2 = \frac{R_1}{R_1+R_2}I \cdots\cdots (2)$$

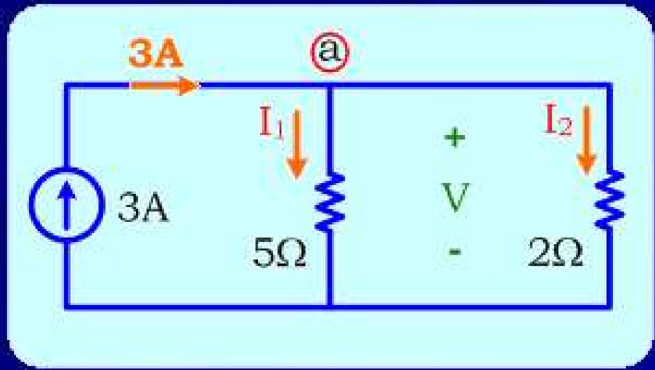
CDR $\Rightarrow$ $I = \frac{\text{other resistor}}{\text{sum}} \times \text{total current}$

CDR applies to *only two* resistors in *parallel*

Example 5

(a) Use CDR to calculate I_1 and I_2

(b) Verify your resluts by checking KCL



Solution

(a) CDR $\Rightarrow I_1 = \frac{2}{2+5} \times 3 \Rightarrow I_1 = \frac{6}{7}\text{A}$

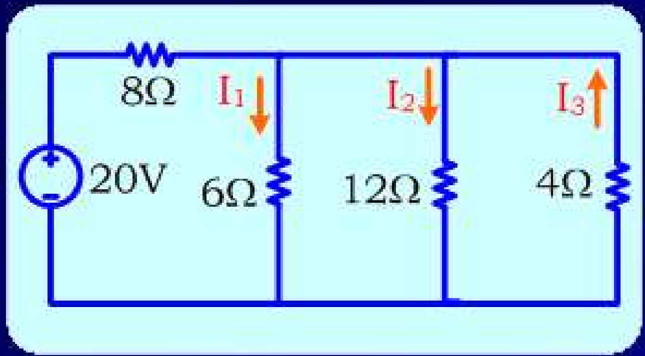
CDR $\Rightarrow I_2 = \frac{5}{2+5} \times 3 \Rightarrow I_2 = \frac{15}{7}\text{A}$

(b) KCL at node a $\Rightarrow I_s - I_1 - I_2 = 3 - \frac{6}{7} - \frac{15}{7} = 3 - \frac{21}{7} = 0$ (KCL verified)

CDR $\Rightarrow$ Higher current passes through the lower resistance

Example 6

Use CDR to calculate I_1, I_2 and I_3



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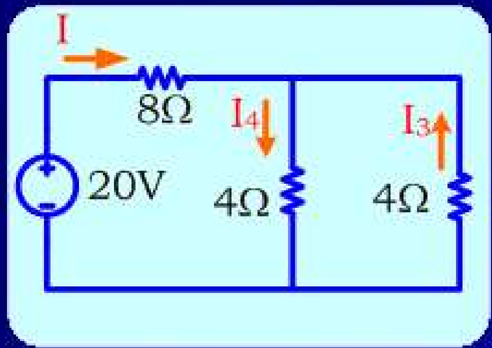

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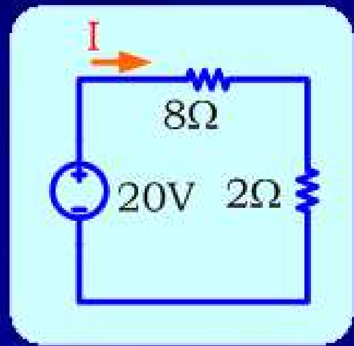


6Ω & 12Ω are in parallel $\Rightarrow \frac{6\times12}{6+12}=\frac{72}{18}=4\Omega$

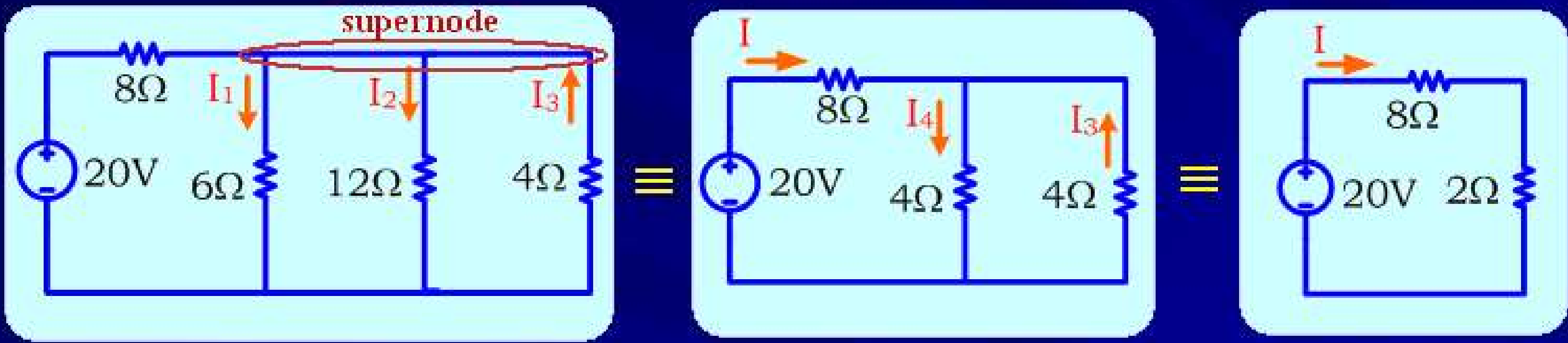
4Ω & 4Ω are in parallel $\Rightarrow \frac{4\times4}{4+4}=\frac{16}{8}=2\Omega$

$\therefore R_{eq}=8+2=10\Omega$

$\Rightarrow I=\frac{20}{R_{eq}}=\frac{20}{10}=2A$

Example 6

Use CDR to calculate I_1, I_2 and I_3



$$I = \frac{20}{R_{eq}} = \frac{20}{10} = 2\text{A}$$

CDR $\Rightarrow$ $I_4 = \frac{4}{4+4} \times 2 = 1\text{A}$, $I_3 = -\frac{4}{4+4} \times 2 = -1\text{A}$ (the minus sign is necessary for I_3, why?)

CDR $\Rightarrow$ $I_1 = \frac{12}{6+12} \times I_4 = \frac{2}{3}\text{A}$, $I_2 = \frac{6}{6+12} \times I_4 = \frac{1}{3}\text{A}$ (I_4 is the total current through 6Ω and 12Ω)

Check KCL at supernode $\Rightarrow I - I_1 - I_2 + I_3 = 2 - \frac{2}{3} - \frac{1}{3} + (-1) = 1 - 1 = 0$ (KCL is verified)

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fundamental of Electric Circuit 1
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2021-2022

LECTURE 07
WYE-DELTA TRANSFORMATION

Topics

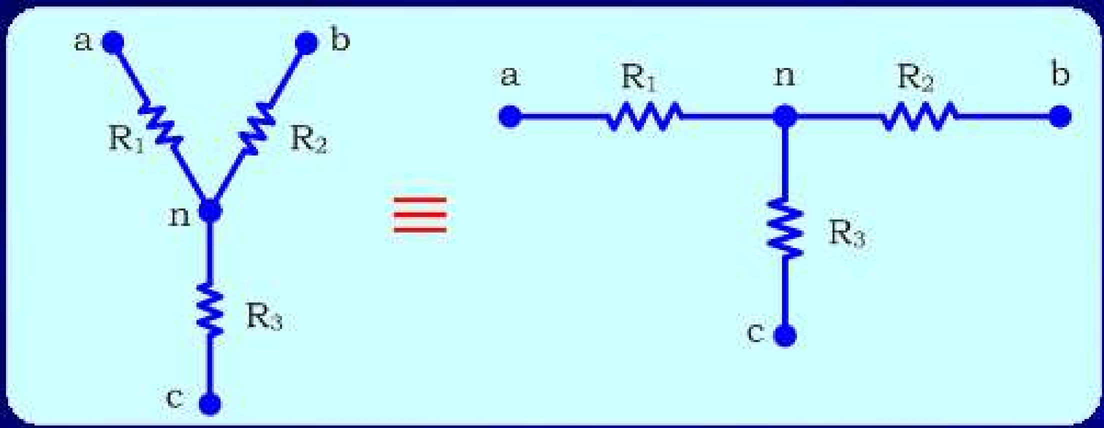
- △ - Connection
- Y - Connection
- △-Y Transformation

Objectives

- Recognize Y and △ connections
- Redraw the circuit to make it easier to identify Y and △ connections
- Use the transformation relations to perform Y-△ transformations
- Use Y-△ transformation to simplify analysis of certain circuits

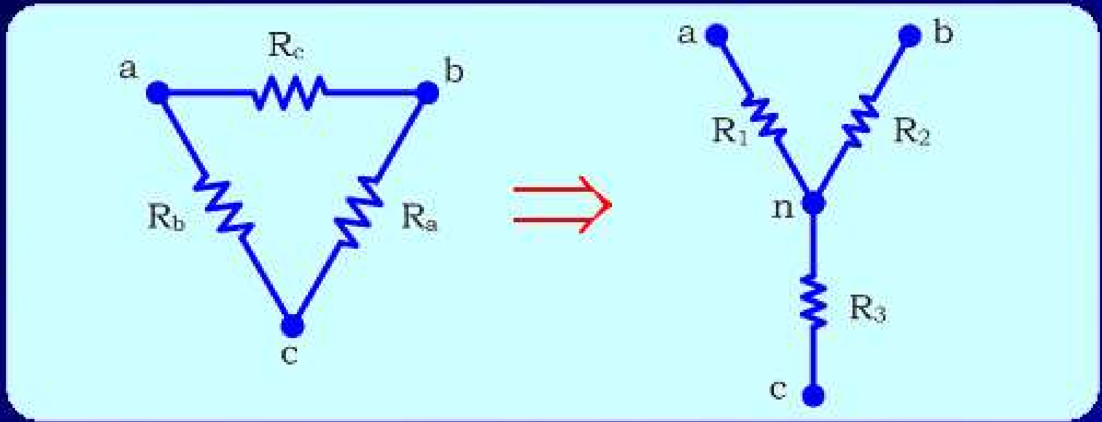
Y - Connection

R_1, R_2 and R_3 form a Y connection



The ***terminals*** of the Y connection are also labeled as *a*, *b* and *c*.

Δ-Y Transformation



Relations for Δ-Y Transformation

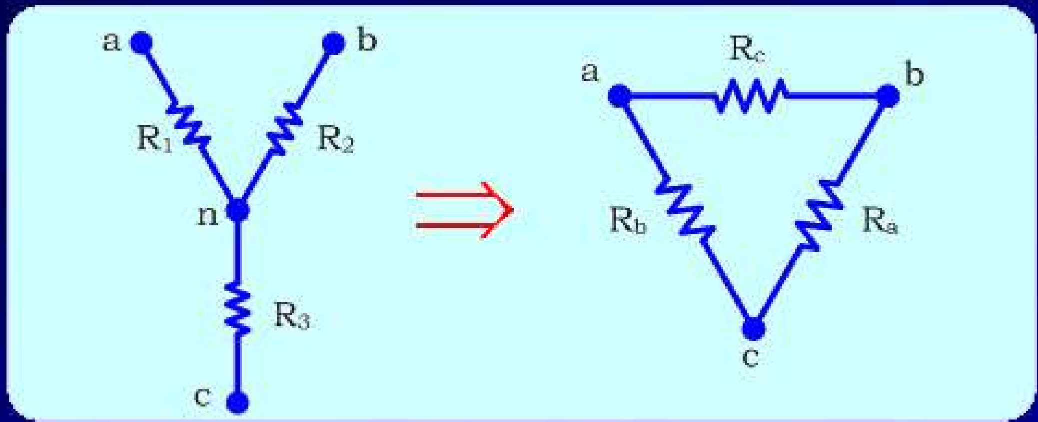
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

You need only to remember one of the above relations, since the other two are similar

Y-△ Transformation



Relations for Y-△ Transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

The numerator is the same making the above relations easy to recall

Notice that the nodes *a*, *b* and *c are kept the same in both circuits*

Example 1

Use △→Y transformation to calculate

(a) R_{eq} seen by the voltage source (b) i_s

Solution

(a) We cannot calculate R_{eq} using the series parallel approach

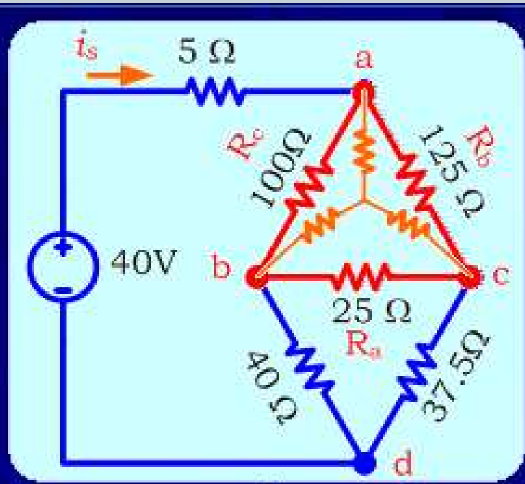
We can transform the (upper or lower) △ to Y

Transforming the upper △ to Y

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{125 \times 100}{25 + 125 + 100} = \frac{125 \times 100}{250} = 50\Omega$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{25 \times 100}{250} = 10\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{25 \times 125}{250} = 12.5\Omega$$



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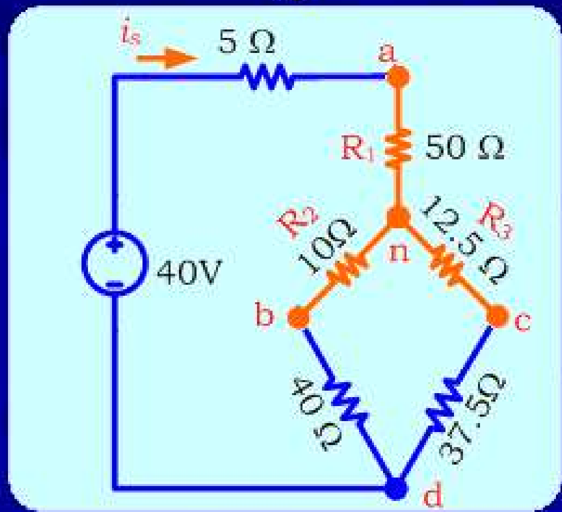
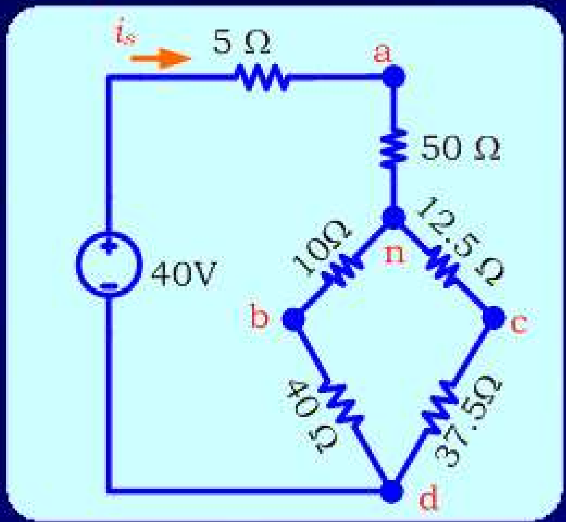
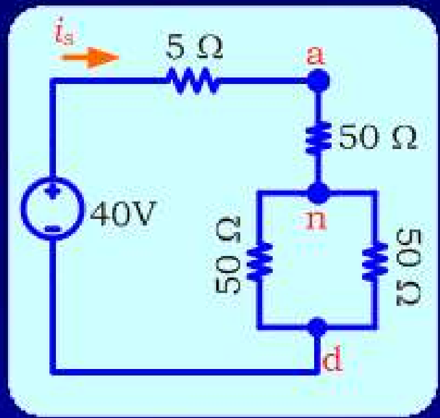
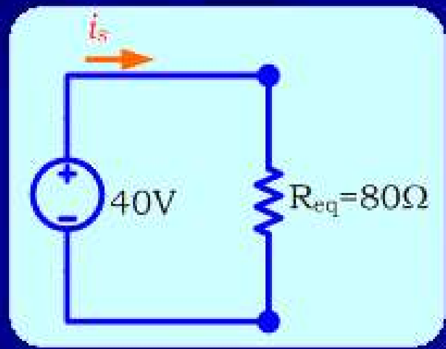


Example 1 (Contd...)

Use △→Y transformation to calculate (a) R_{eq} seen by the voltage source (b) i_s







12.5Ω & 37.5Ω (in series)

$\Rightarrow 12.5+37.5=50\Omega$

10Ω & 40Ω (in series)

$\Rightarrow 10+40=50\Omega$

$\therefore R_{eq}=5+50+(50/2)$

$\Rightarrow R_{eq}=80\Omega$

(b) $i_s=\dfrac{40}{R_{eq}}=\dfrac{40}{80}=0.5A$

Example 2

Let us explore some other possibilities for solving the previous problem. Repeat the previous example using Y→△ transformation

(a) R_{eq} seen by the voltage source (b) i_s

Solution

(a) Redraw the circuit

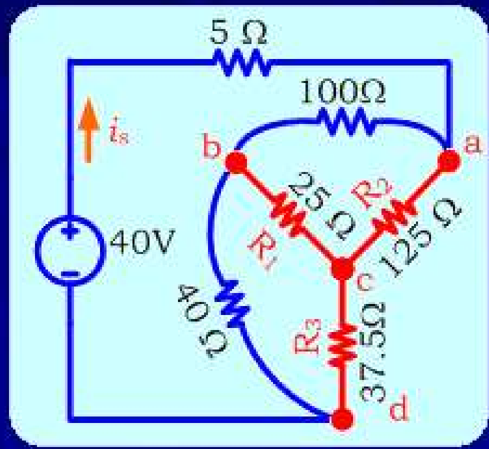
25Ω, 125Ω and 37.5Ω form a Y connection

Using Y to △ transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{25\times125 + 125\times37.5 + 37.5\times25}{25} = 350\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{8750}{125} = 70\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{8750}{37.5} = 233.33\Omega$$



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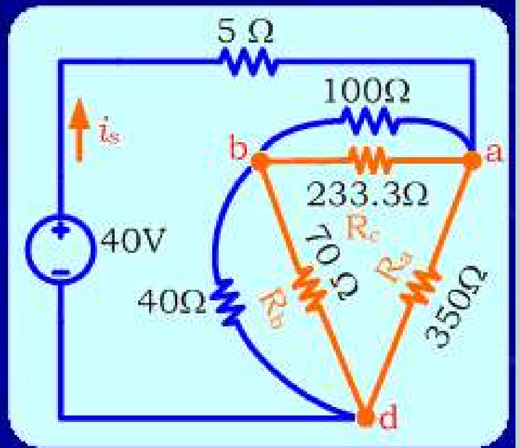
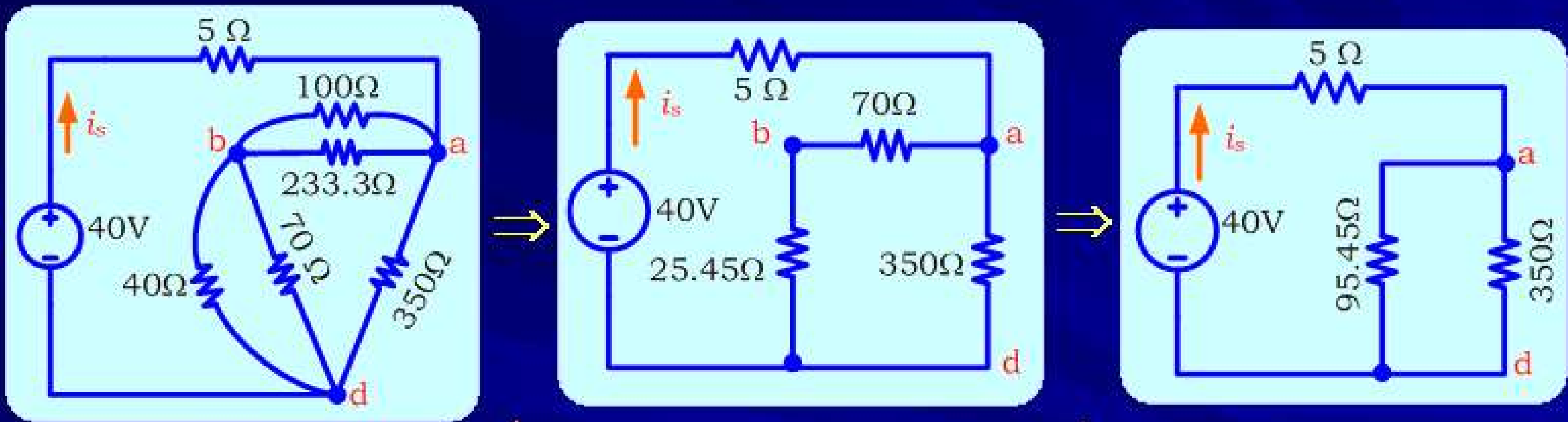


Example 2 (Contd...)

Use Y→△ transformation to calculate (a) R_{eq} seen by the voltage source (b) i_s



$40\Omega \| 70\Omega$

$$\Rightarrow \frac{40\times 70}{40+70} = 25.455\Omega$$

$100\Omega \| 233.33\Omega$

$$\Rightarrow \frac{100\times 233.33}{100+233.33} = 70\Omega$$

70Ω & 25.45Ω (in series)

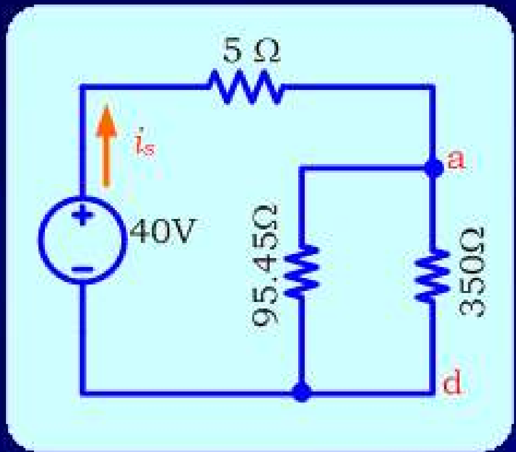
$$\Rightarrow 70 + 25.45 = 95.45\Omega$$

$95.45\Omega \| 350\Omega$

$$\Rightarrow \frac{95.45\times 350}{95.45+350} = 75\Omega$$

Example 2 (Contd...)

Use Y→△ transformation to calculate (a) R_{eq} seen by the voltage source (b) i_s



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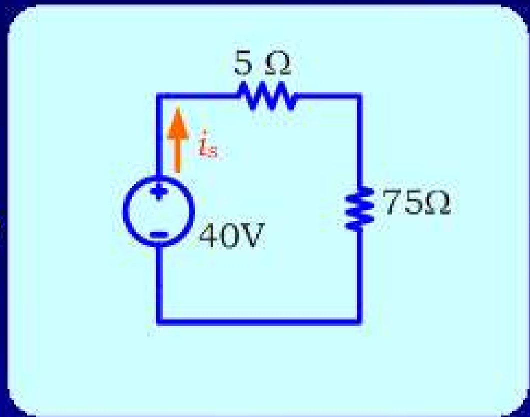

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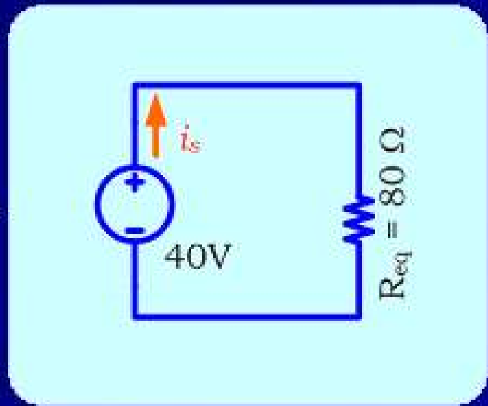

$95.45\Omega \| 350\Omega$

$$\Rightarrow \frac{95.45 \times 350}{95.45 + 350} = 75\Omega$$

5Ω & 75Ω (in series)

$$\Rightarrow 5 + 75 = 80\Omega$$

$$\therefore R_{eq} = 80\Omega$$

same answer as before

(b) $i_s = \dfrac{40}{R_{eq}} = \dfrac{40}{80} = 0.5\text{A}$

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LECTURE 08
CIRCUIT ANALYSIS TECHNIQUE
NODAL ANALYSIS (INTRODUCTION)

Topics

- Definition of Nodal Voltages
- Nodal Analysis in the absence of Voltage Sources

Objectives

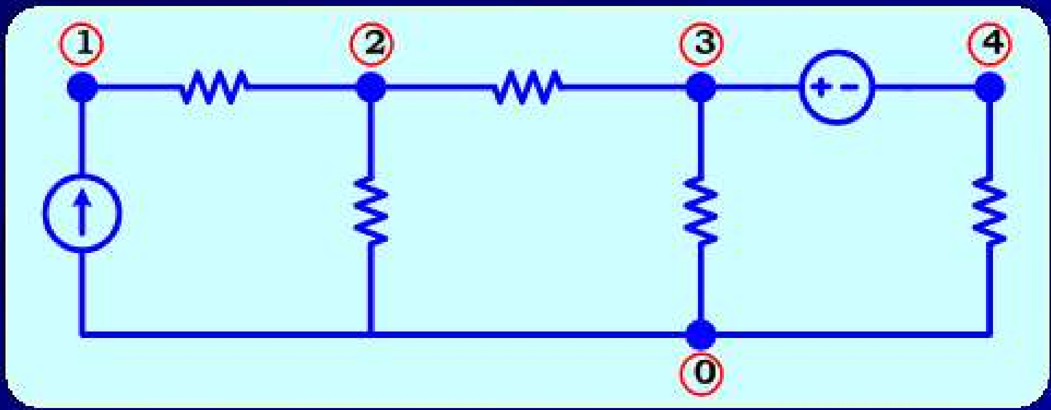
- Understand the meaning of a nodal voltage
- Understand the meaning of a reference node
- Differentiate between voltages across elements and nodal voltages
- Relate nodal voltages to voltages across elements
- Determine the number of unknown nodal voltages
- Apply the Nodal Analysis Procedure in the absence of voltage sources
- Apply the Nodal Analysis Procedure directly

Definition of Essential Nodes

The essential nodes of the circuit are labelled '0', '1', '2', '3', '4', etc.

All points that are connected by a short circuit belong to the same essential node.

All points in the lower part of the circuit are connected by a short circuit, they all belong to node 0. The same applies to nodes "1", "2", "3", "4".

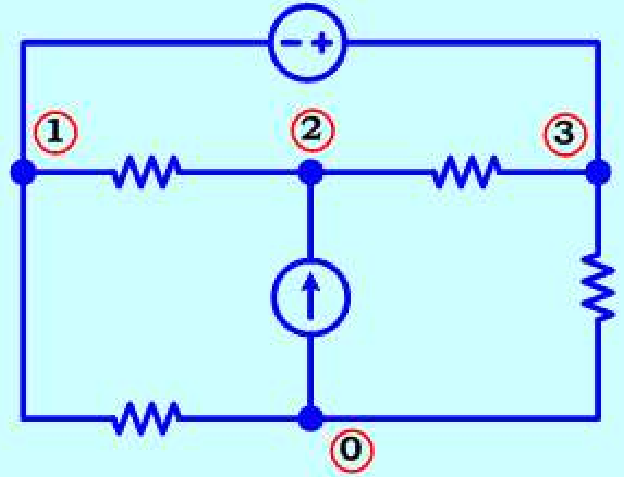


In this circuit, there are five essential nodes.

Example: Label the essential nodes starting from node 0.

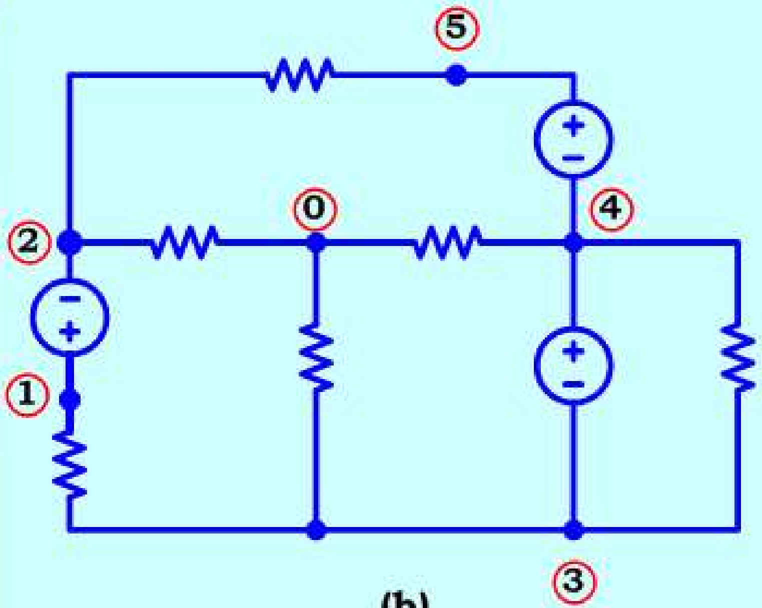
Solution:

Circuit (a) has 4 essential nodes.



(a)

Circuit (b) has 6 essential nodes.



(b)

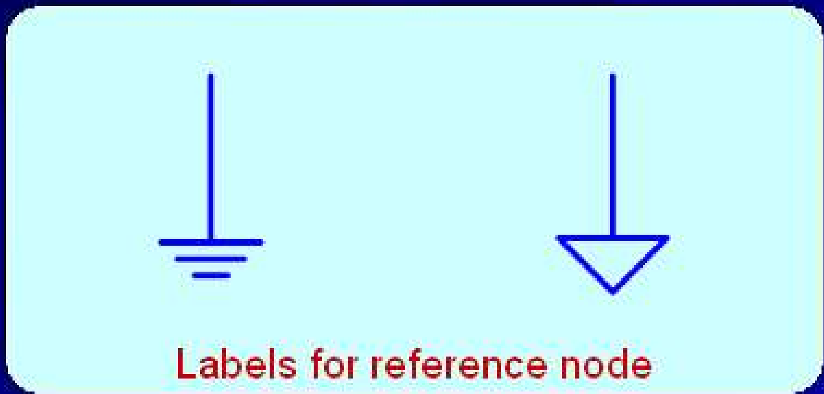
The essential nodes can be labelled as we like.

Reference Node

The node labelled '0' is called a *reference node*.

We will see later that the *reference* node always has a *zero Nodal Voltage*.

Possible labels for the reference node are shown below:



Nodal Voltages

We associate a voltage with every essential node. These voltages are called ***Nodal Voltages.***

V_0, V_1, V_2, V_3 $\Leftrightarrow$ Nodal voltages of essential nodes 0, 1, 2, 3.

The nodal voltage V_i is the voltage drop *from* node "i" *to* the reference node "0".

$$V_i \equiv V_{i,0}$$

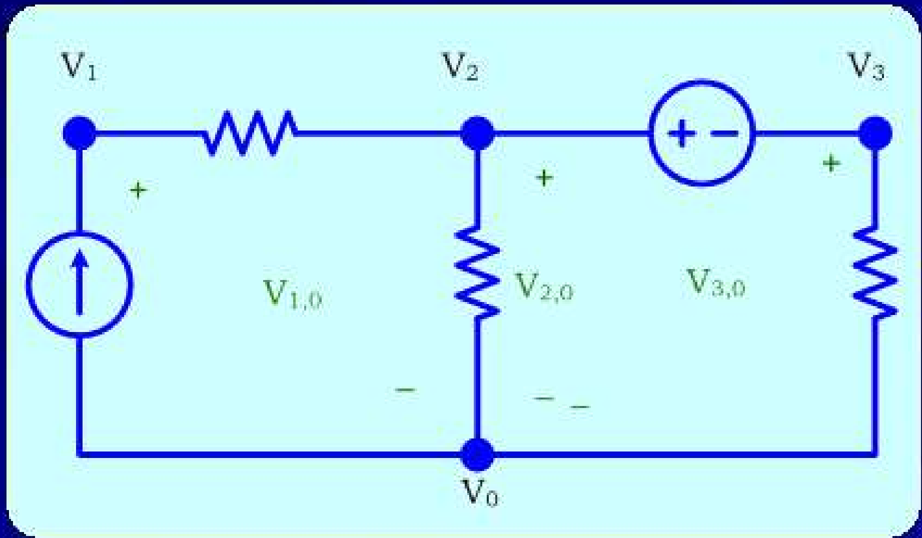
$\Rightarrow$

$$V_0 \equiv V_{0,0} \qquad V_1 \equiv V_{1,0}$$

$$V_2 \equiv V_{2,0} \qquad V_3 \equiv V_{3,0}$$

Reference node: $V_0 \equiv V_{0,0} = 0$

$\Rightarrow$ Reference node always has zero nodal voltage.

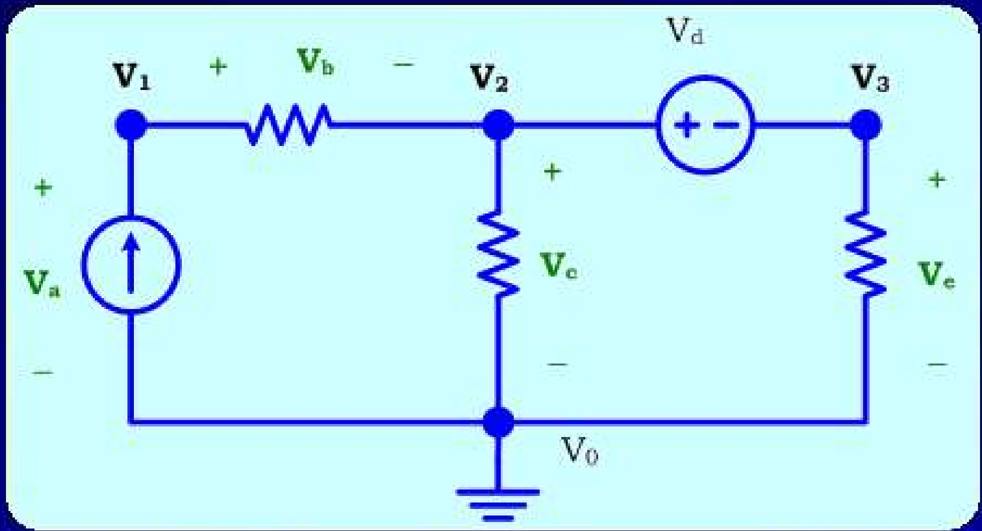


Relation between Nodal Voltages and Voltages across Elements

Nodal Voltages (NV) $\Longrightarrow$ V_1, V_2, V_3

The reference nodal voltage $V_0 = 0$.

Voltage across elements (VAE) $\Longrightarrow$ V_a, V_b, V_c, V_d, V_e



Another label is used to mark the reference node in this case.

Relation between Nodal Voltages and Voltage across Elements

Applying KVL:

$-V_a + V_{10} = 0 \Rightarrow -V_a + V_1 = 0 \Rightarrow V_a = V_1$

$V_a = V_1 \Rightarrow \boxed{V_a = V_1 - V_0}$

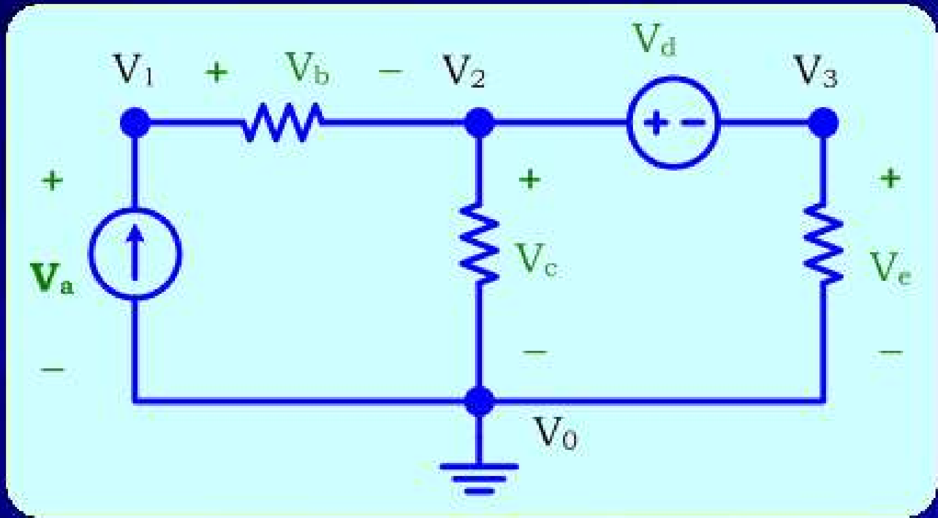
$-V_1 + V_b + V_2 = 0$

$\Rightarrow \boxed{V_b = V_1 - V_2}$

$V_c = V_2 \Rightarrow \boxed{V_c = V_2 - V_0}$

$-V_2 + V_d + V_3 = 0$

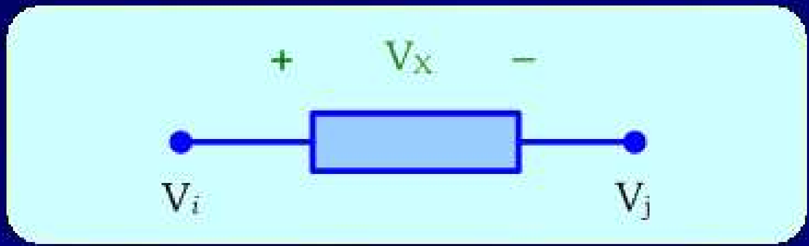
$\Rightarrow \boxed{V_d = V_2 - V_3}$



$\boxed{V_e = ?}$

Voltage across Elements: General Equation

$$\Rightarrow \quad V_x = V_i - V_j$$



where:

V_x = *Voltage Across Element*

V_i = *Nodal Voltage on the '+' side of* V_x

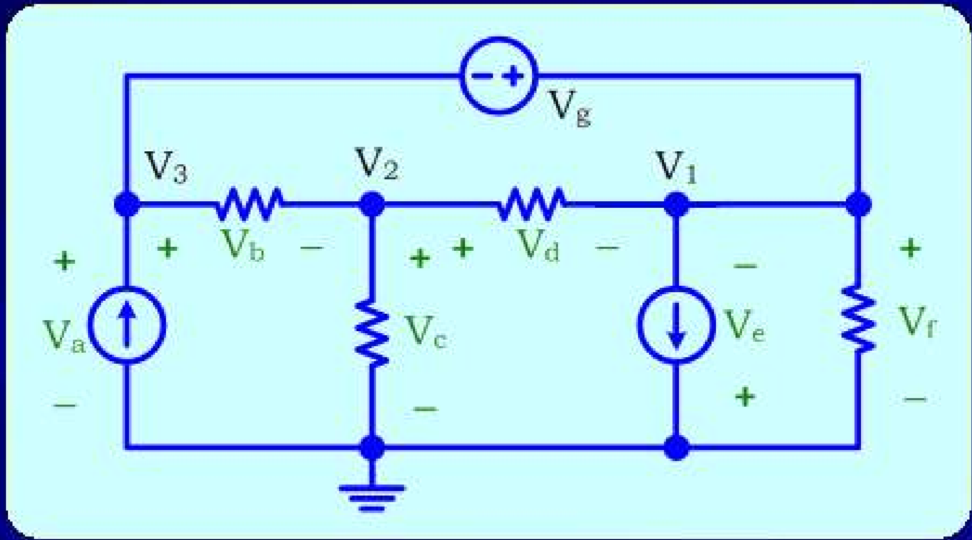
V_j = *Nodal Voltage on the '-' side of* V_x

Example

Express the VAE V_a, V_b, V_c, V_d, V_e, V_f, V_g in terms of the NV V_1, V_2, V_3

$V_x = V_i - V_j$

$V_a = V_3 - V_0 = V_3 - 0 = V_3$
$V_b = V_3 - V_2$
$V_c = V_2 - 0 = V_2$
$V_d = V_2 - V_1$
$V_e = 0 - V_1 = -V_1$ *(why?)*
$V_f = V_1 - 0 = V_1$
$V_g = V_1 - V_3$



If we know all NV implies we know all VAE.

- Nodal analysis procedure

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $v_1, v_2, \ldots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n-1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 1

Derive the nodal equations. (do not simplify and do not solve).

Solution:

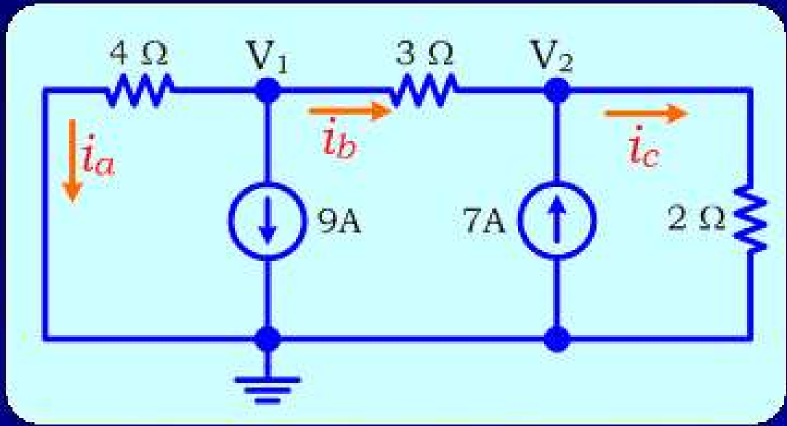
This time, we will combine steps 2 & 3 (Ohm's law and KVL) into a single step.
The voltage across resistances will not be shown explicitly.

Node 1:

KCL $\Rightarrow$ $i_a + 9 + i_b = 0$

Ohms' Law then KVL

$$\Rightarrow \frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \qquad (1)$$



Node 2:

KCL $\Rightarrow$ $-i_b - 7 + i_c = 0$

Ohms' Law then KVL

$$\Rightarrow -\frac{V_1 - V_2}{3} - 7 + \frac{V_2 - 0}{2} = 0 \qquad (2)$$

Example 2

Repeat the previous example by combining steps 1, 2, and 3 (KCL, Ohm's law, and KVL) into a single step.

Solution:

This time we will not show current through resistances or voltages across resistances.

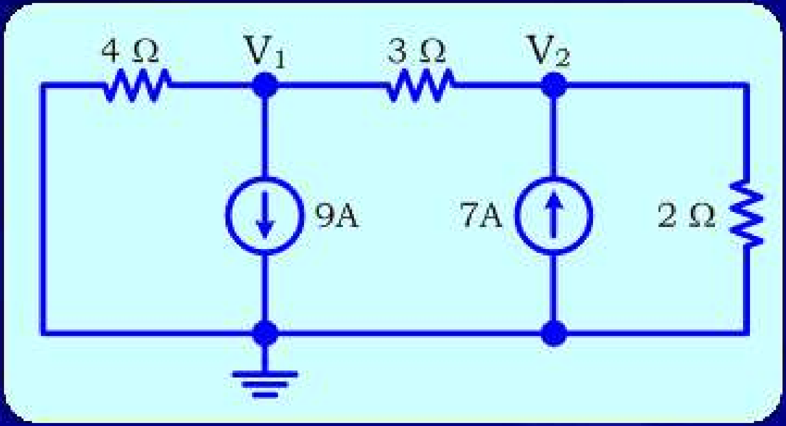
Important: We will imagine the currents through resistors to be leaving the node under consideration.

Node 1:

$$\Rightarrow \frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \qquad (1)$$

Node 2:

$$\Rightarrow \frac{V_2 - V_1}{3} - 7 + \frac{V_2 - 0}{2} = 0 \qquad (2)$$



which are the same equations obtained in the previous example.

Nodal Analysis: Some Conclusions

From the examples shown in this lecture, it is easy to conclude that:

1. $N_u = N_{ess} - 1$

   N_u = number of unknown nodal voltages

   N_{ess} = number of essential nodes

2. $N_u \leq N_{ele}$

   N_{ele} = number of unknown voltages across elements

Thus, nodal analysis is efficient because the number of unknown voltages is reduced.

Voltage Sources connected to the Reference node

The case of voltage sources connected to the reference node is taken up first and it is illustrated with the help of an example.

Example3 Calculate the nodal voltages V_1, V_2, V_3.

Solution:

Nodes 1 & 2

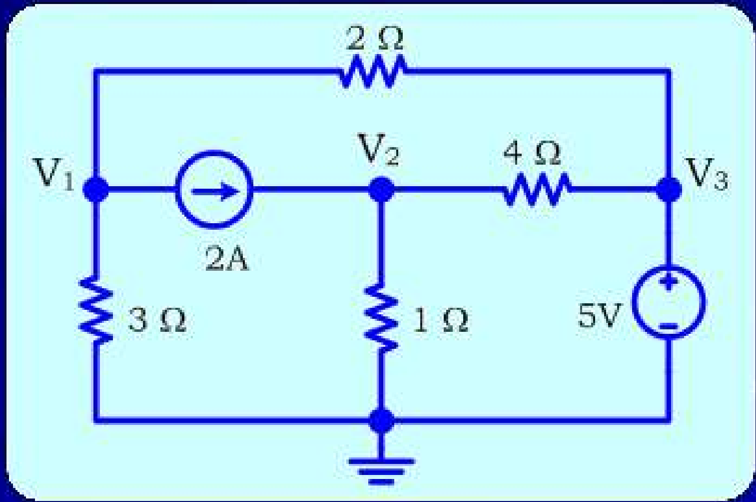
⇒ No voltage sources connected

⇒ No special treatment required

Node 3

⇒ Voltage source connected

⇒ Needs special treatment



Solution: Applying KCL

KCL at node 1:

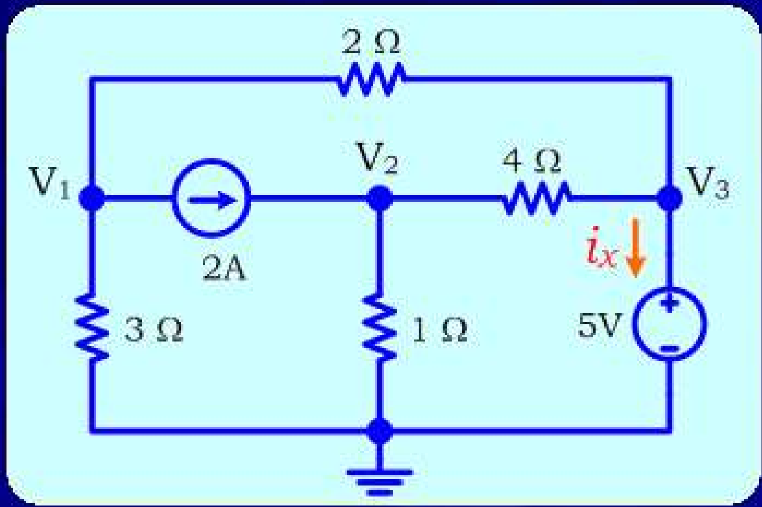
$$\Rightarrow \frac{V_1-0}{3}+2+\frac{V_1-V_3}{2}=0$$

$$\Rightarrow 5V_1-3V_3=-6 \quad (1)$$

KCL at node 2:

$$\Rightarrow -2+\frac{V_2-0}{1}+\frac{V_2-V_3}{4}=0$$

$$\Rightarrow 5V_2-V_3=8 \quad (2)$$



KCL at node 3:

$$\Rightarrow \frac{V_3-V_2}{4}+\frac{V_3-V_1}{2}+i_x=0$$ (problem!)

i_x cannot directly be replaced with nodal voltages, because Ohm's law does not apply to voltages sources

Solution: Solve equations

We have 3 unknowns $\Longrightarrow$ We need 3 equations $\Longrightarrow$ one equation is missing

For node 3, the basic Nodal Analysis procedure must be revised.

The 5V source is connected to the reference node.

Apply KVL: Node 3 and reference node

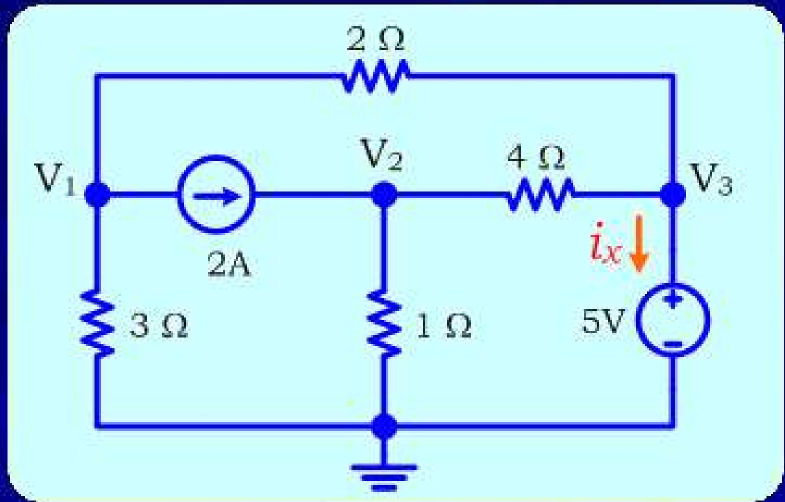
$\Longrightarrow V_3 - 0 = 5$

$\Longrightarrow V_3 = 5 \quad (3)$

From the previous slide,

$5V_1 - 3V_3 = -6 \quad (1)$

$5V_2 - V_3 = 8 \quad (2)$



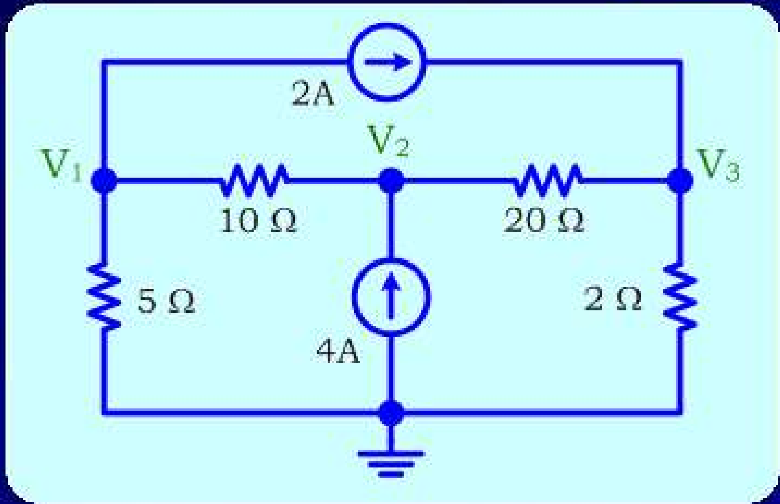
Solving the above set of equations, we get:

$V_1 = 1.8V$ & $V_2 = 2.6V$ & $V_3 = 5V$

Voltage source connected to reference $\Longrightarrow$ Use KVL only (do not use KCL)

Practice Problem

In the circuit shown, calculate the nodal voltages V_1, V_2 & V_3



Answer:

$V_1 = 6.67V \quad V_2 = 40V \quad V_3 = 26.67V$

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LECTURE 9
MESH ANALYSIS

Topics

- Mesh Analysis without Current Sources
- Mesh Analysis with Current Sources

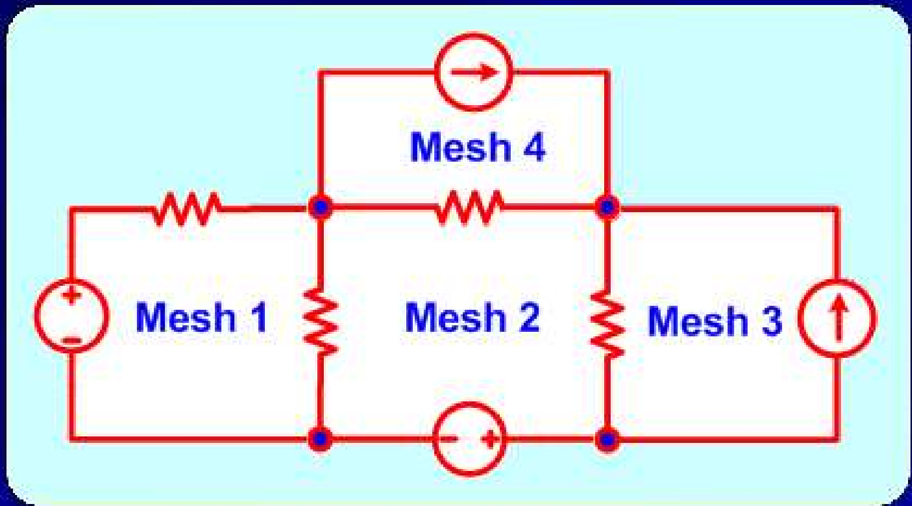
Objectives

- Understand mesh currents
- Relate currents through elements to mesh currents
- Apply Mesh Analysis in the absence of current sources
- Understand the concept of a super mesh
- Apply Mesh Analysis in the presence of current sources

Definition of a Mesh

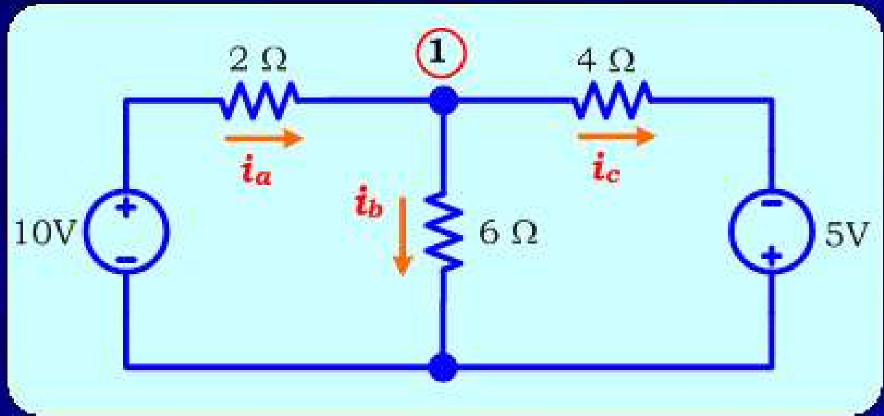
A mesh is simply a window in an electric circuit.

This circuit contains four windows (meshes).



Currents through Elements and Mesh Currents

The currents i_a, i_b, and i_c are currents through elements.



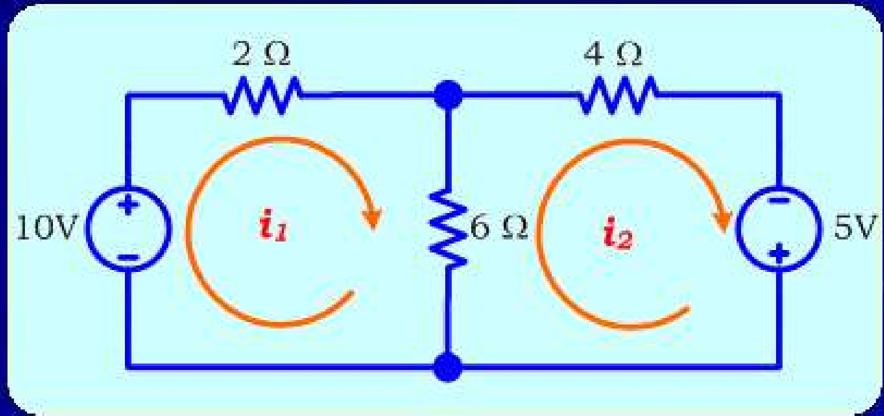
KCL at node 1:

$$\Rightarrow \; i_a = i_b + i_c \quad \Rightarrow \; i_b = i_a - i_c$$

The imaginary currents i_1, and i_2 are mesh currents.

We imagine i_1 to circulate around mesh 1 (Clockwise).

We imagine i_2 to circulate around mesh 2 (also Clockwise).



$$i_a = i_1 \qquad i_c = i_2$$

$$i_b = i_a - i_b = i_1 - i_2$$

Example:

Express the currents through elements (CTE) i_w , i_x , i_y , and i_z in terms of mesh currents (MC) currents i_1 , i_2 , and i_3

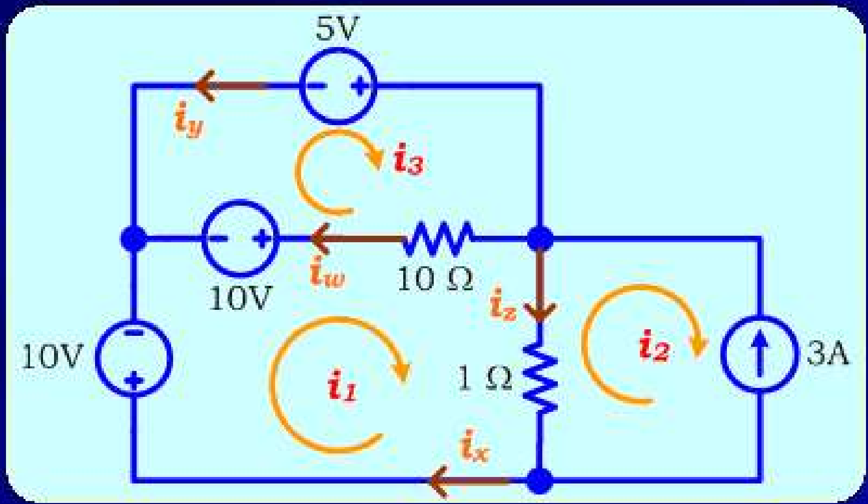
Solution:

$i_x = i_1$

$i_y = -i_3$

$i_z = i_1 - i_2$

$i_w = i_3 - i_1$



Number of MC $\leq$ Number of CTE

We know all MC $\Rightarrow$ We know all CTE

Mesh Analysis without Current Sources

The Mesh Analysis procedure for circuits without current sources will be considered first. This procedure is illustrated below:

Mesh Analysis

Step 1: KVL

↓

Step 2: Ohm's Law

↓

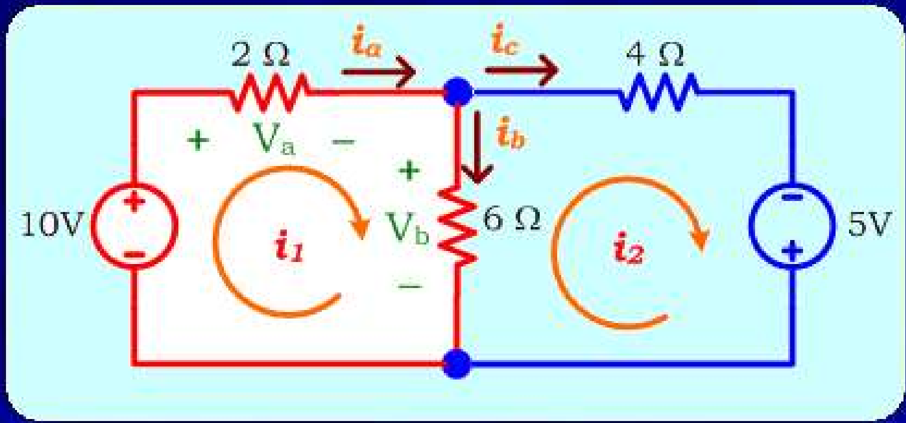
Step 3: KCL

Mesh Analysis ⇒ VOC

Example 1: Calculate the mesh currents i_1 and i_2

Solution:

Procedure:
First we will deal with Mesh 1



1. KVL: $\Rightarrow -10 + V_a + V_b = 0$

2. Ohm's Law:

$$\Rightarrow -10 + 2i_a + 6i_b = 0$$

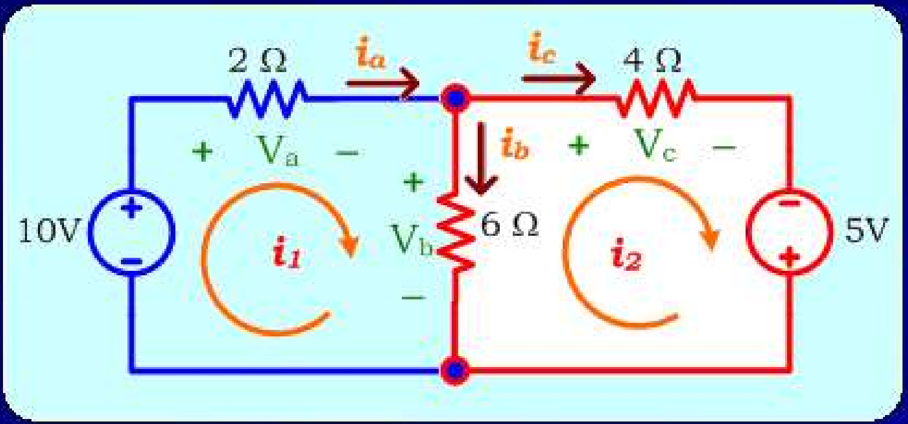
3. KCL:

$$\Rightarrow -10 + 2i_1 + 6(i_1 - i_2) = 0$$ (CTE are expressed in terms of MC)

Simplify: $\Rightarrow$ $8i_1 - 6i_2 = 10 \qquad (1)$

Solution (contd):

We will repeat the procedure for Mesh 2



1. KVL: $\Rightarrow -V_b + V_c - 5 = 0$

2. Ohm's Law:

$$\Rightarrow -6i_b + 4i_c - 5 = 0$$

3. KCL: $\Rightarrow -6(i_1 - i_2) + 4i_2 - 5 = 0$ (CTE are expressed in terms of MC)

Simplify: $\Rightarrow -6i_1 + 10i_2 = 5$ *(2)*

From the previous slide

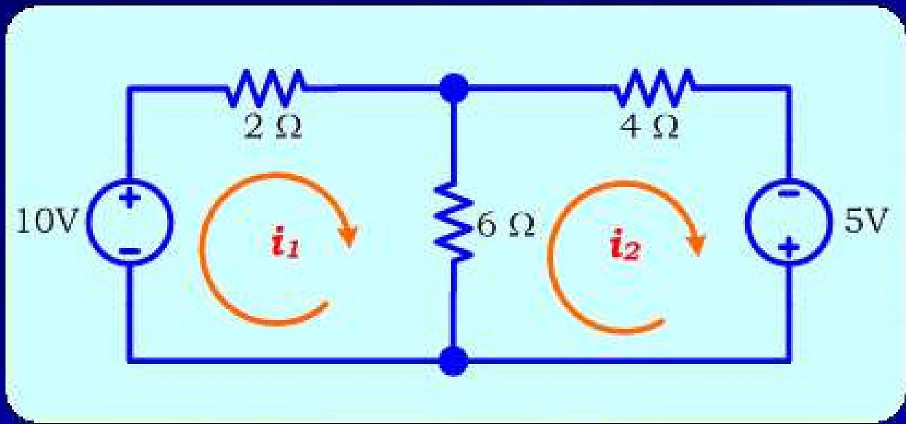
$8i_1 - 6i_2 = 10$ *(1)*

Equations (1) and (2) contain only mesh unknowns i_1 and i_2

Solving (1) and (2), we get: $\Rightarrow i_1 = 2.955A \quad i_2 = 2.273A$

Example 2:

Calculate the mesh currents i_1 and i_2. Repeat the previous example by combining steps 1, 2, and 3.



Solution:

Mesh 1: KVL, & Ohm's Law, & KCL

$\Rightarrow -10 + 2i_1 + 6(i_1 - i_2) = 0$

$8i_1 - 6i_2 = 10$ (1)

Mesh 2: KVL, & Ohm's Law, & KCL

$\Rightarrow 6(i_2 - i_1) + 4i_2 - 5 = 0$

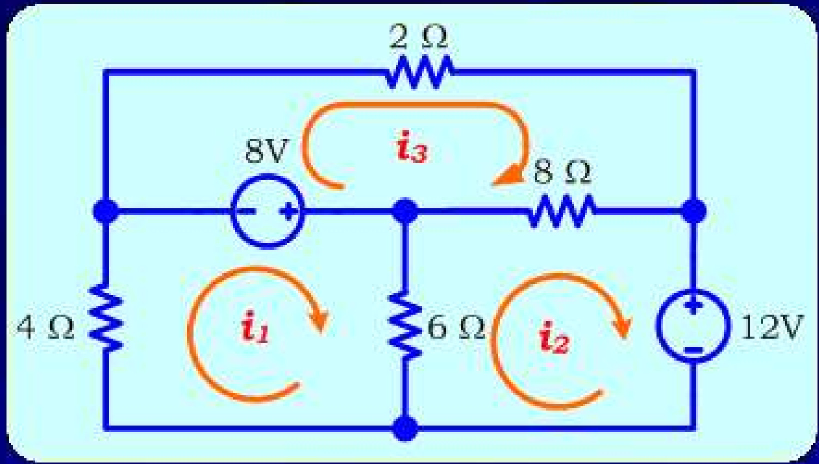
$-6i_1 + 10i_2 = 5$ (2)

Current through resistors = CTR

Always imagine CTR to be in the same direction as KVL.

Express the imagined CTR in terms of MC (Mesh Currents).

Example 3: Calculate the mesh currents i_1, i_2 and i_3.



Solution:

Mesh 1: $\Rightarrow 4i_1 - 8 + 6(i_1 - i_2) = 0 \quad \Rightarrow 10i_1 - 6i_2 = 8 \quad (1)$

Mesh 2: $\Rightarrow 6(i_2 - i_1) + 8(i_2 - i_3) + 12 = 0 \quad \Rightarrow -6i_1 + 14i_2 - 8i_3 = -12 \quad (2)$

Mesh 3: $\Rightarrow 2i_3 + 8(i_3 - i_2) + 8 = 0 \quad \Rightarrow -8i_2 + 10i_3 = -8 \quad (3)$

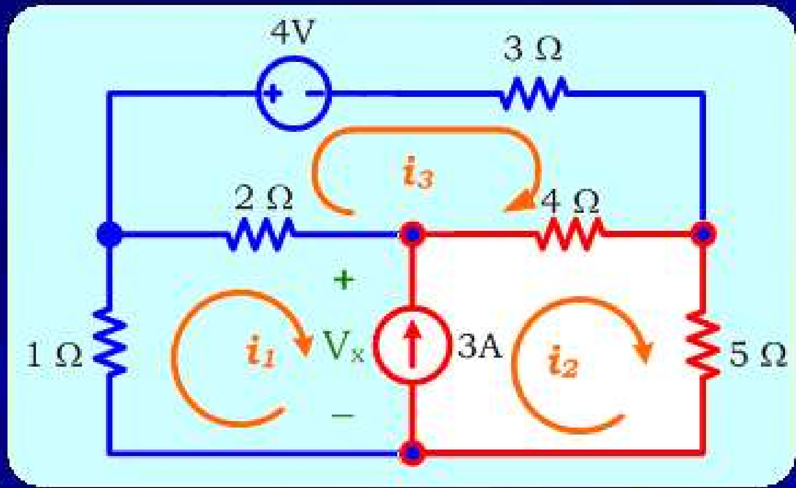
Solving (1), (2), and (3), we get: $\Rightarrow i_1 = -1.24\text{A} \quad i_2 = -3.40\text{A} \quad i_3 = -3.52\text{A}$

Mesh Analysis with Current Sources

When the circuit contains current sources, the previous procedure is modified.

Example 1: Calculate the mesh currents i_1 , i_2 and i_3

Solution:



KVL around Mesh 1:

$$1i_1 + 2(i_1 - i_3) + V_x = 0$$ (problem!)

We cannot directly replace V_x by mesh currents, because Ohm's law does not apply to current sources.

KVL around Mesh 2:

$$-V_x + 4(i_2 - i_3) + 5i_2 = 0$$ (similar problem!)

Solution (contd):

Mesh 1 & 2 contain a current source (they share the *3A* source)

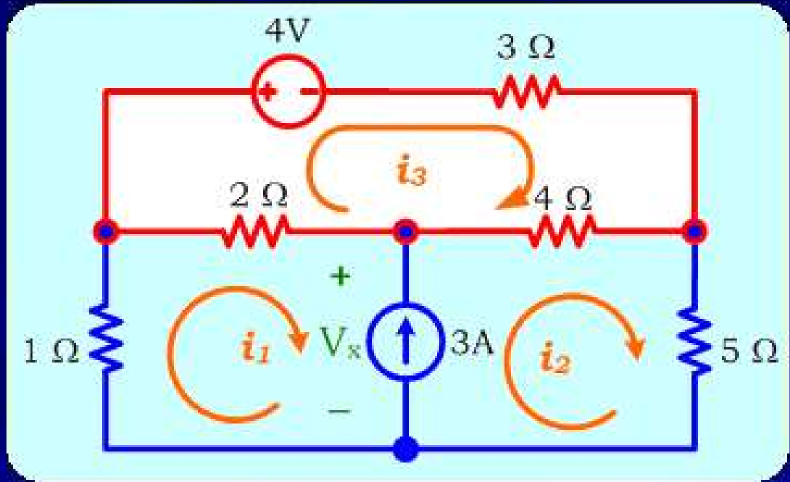
What to do in this case?

Combine Mesh 1 & Mesh 2 into a Super Mesh (SM).

To avoid V_x, apply KVL around SM

$$\Rightarrow 1i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 5i_2 = 0$$

$$3i_1 + 9i_2 - 6i_3 = 0 \qquad (1)$$



We need one more equation.

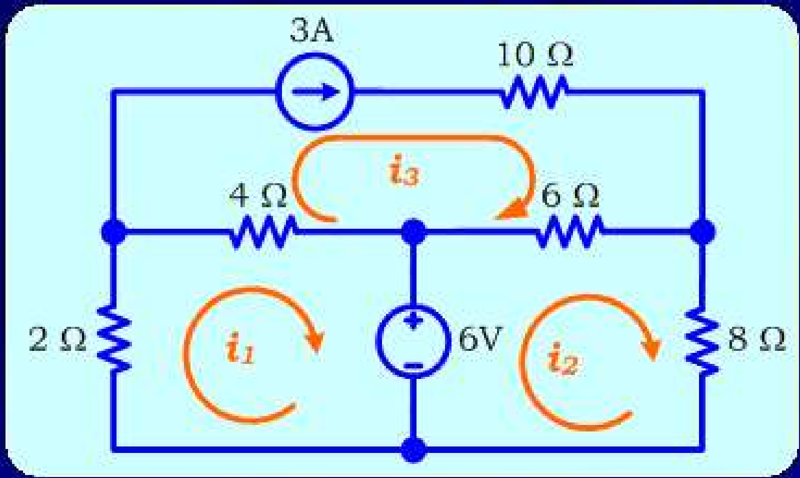
Apply KCL $\Rightarrow i_2 - i_1 = 3 \qquad (2)$

Mesh 3 does not contain a current source $\Rightarrow$ No special treatment

KVL around Mesh 3 $\Rightarrow 4 + 3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0 \Rightarrow -2i_1 - 4i_2 + 9i_3 = -4 \qquad (3)$

Solving (1), (2) and (3), we get: $\Rightarrow i_1 = -2.708A \quad i_2 = 0.292A \quad i_3 = -0.917A$

Example 2: Calculate the mesh currents i_1 , i_2 and i_3 .



Solution: Mesh 1 & 2 does not contain current sources.

$\Rightarrow$ Just apply KVL around Mesh 1 & 2

KVL around Mesh 1:

$\Rightarrow \quad 2i_1 + 4(i_1 - i_3) + 6 = 0$

$\Rightarrow \quad 6i_1 - 4i_3 = -6 \qquad (1)$

KVL around Mesh 2:

$\Rightarrow \quad -6 + 6(i_2 - i_3) + 8i_2 = 0$

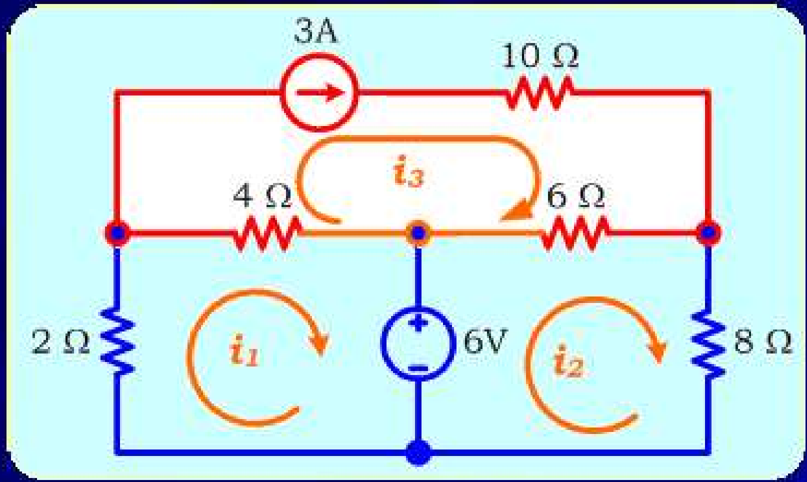
$\Rightarrow \quad 14i_2 - 6i_3 = 6 \qquad (2)$

Solution (contd):

From the previous slide,

$$6i_1 - 4i_3 = -6 \qquad (1)$$

$$14i_2 - 6i_3 = 6 \qquad (2)$$



Mesh 3 contains a 3A current source (not shared by another mesh)

Do not apply KVL (because KVL involves voltage across the current source).

Apply only KCL $\Rightarrow$ $i_3 = 3 \qquad (3)$

[Note: Since we need only one equation from mesh 3, KCL provides it]

Solving (1), (2), and (3), we get: $\Rightarrow$ $i_1 = 1.000A \quad i_2 = 1.714A \quad i_3 = 3.000A$

Mesh Analysis with Current Sources : Summary

If a current source is shared by two meshes, then follow the procedure described below:

1. Combine the two meshes into a Super Mesh

2. Apply KVL around the Super Mesh

3. Apply KCL

If a current source is in one mesh only (not shared), then:

$\Longrightarrow$ Apply KCL only (do NOT apply KVL)