Lecture Notes

on

Electric Power I- (First Semester)

(EE3317)

By

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References:

- Element of Power System Analysis / by W. Stevenson, McGraw- Hill Pub., 2005.
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Chapter One

General Background

1.1- Importance of Electrical Energy

1.2-Generation of Electrical Energy

1.3-Sources of Energy

1.4-Structure of Electric Power System

1.1- Importance of Electrical Energy:

- Energy is the basic necessity for the economic development of a country.
- The availability of huge amount of energy in the modern times has resulted in a shorter working day, higher agricultural and industrial production, a healthier and more balanced diet and better transportation facilities.
- Energy exists in different forms in nature but the most important form is the electrical energy.
- For example, if we want to convert electrical energy into heat, the only thing to be done is to pass electrical current through a wire of high resistance e.g., a heater. Similarly, electrical energy can be converted into light (e.g. electric bulb), mechanical energy (e.g. electric motors) etc.

1.2- Generation of Electrical Energy:

- Electrical energy is a manufactured commodity like clothing, furniture or tools. Just as the manufacture of a commodity involves the conversion of raw materials available in nature into the desired form, similarly electrical energy is produced from the forms of energy available in nature.
- So, the conversion of energy available in different forms in nature into electrical energy is known as *generation of electrical energy*.

- Energy is available in various forms from different natural sources such as pressure head of water, chemical energy of fuels, and nuclear energy of radioactive substances.
- Figure 1 shows the general way how to produce electrical energy:
  a- The prime mover is driven by the energy obtained from various sources such as burning of fuel, pressure of water, force of wind etc.
  b- For example, chemical energy of a fuel (e.g., coal) can be used to produce steam at high temperature and pressure.
  c- The steam is fed to a prime mover which may be a steam engine or a steam turbine.
  d- The turbine converts heat energy of steam into mechanical energy which is further converted into electrical energy by the alternator.

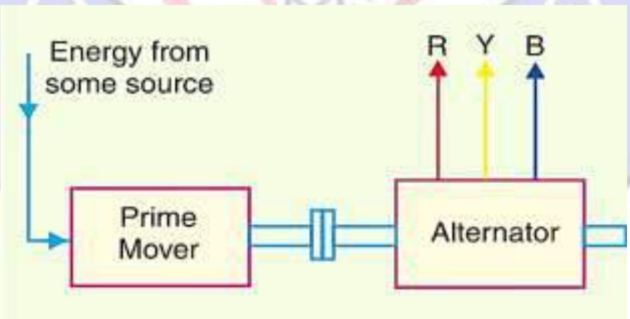


Fig. 1: General way to produce electrical energy

1.3- Sources of Energy:

Electrical energy is produced from energy available in various forms in nature, it is desirable to look into the various sources of energy. These sources of energy are:

1.3.1- The Sun
1.3.2- The Wind
1.3.3- Water
(renewable or non-conventional)

1.3.4- Fuels
1.3.5- Nuclear energy
(non-renewable or conventional)

1.3.1- The Sun:

- The Sun is the primary source of energy.
- The heat energy radiated by the Sun can be focused over a small area by means of reflectors.
- This heat can be used to raise steam and electrical energy can be produced with the help of turbine-alternator combination.
- However, this method has limited application because:
  - **a-** It requires a large area for the generation of even a small amount of electric power.
  - **b-** It cannot be used in cloudy days or at night.

1.3.2- The Wind:

- This method can be used where wind flows for a considerable length of time.
- The wind energy is used to run the wind mill which drives a small generator.
- In order to obtain the electrical energy from a wind mill continuously, the generator is arranged to charge the batteries.
- These batteries supply the energy when the wind stops.
- This method has the advantages that maintenance and generation costs are negligible.
- However, the drawbacks of this method are:

  a- Variable output.

  b- Unreliable because of uncertainty about wind pressure.

  c- Power generated is quite small.

1.3.3- Water:

- When water is stored at a suitable place, it possesses potential energy because of the head created.
- This water energy can be converted into mechanical energy with the help of water turbines.

- The water turbine drives the alternator which converts mechanical energy into electrical energy.
- This method of generation of electrical energy has become very popular because it has low production and maintenance costs.

1.3.4- Fuels:

- The main sources of energy are fuels viz., solid fuel as coal, liquid fuel as oil and gas fuel as natural gas.
- The heat energy of these fuels is converted into mechanical energy by suitable prime movers such as steam engines, steam turbines, internal combustion engines etc.
- The prime mover drives the alternator which converts mechanical energy into electrical energy.

Advantages of Liquid Fuels Over Solid Fuels:

1- The handling of liquid fuels is easier and they require less storage space.
2- The combustion of liquid fuels is uniform.
3- The solid fuels have higher percentage of moisture.
4- The waste product of solid fuels is a large quantity of ash and its disposal becomes a problem.
5- The firing of liquid fuels can be easily controlled.

Advantages of Solid Fuels Over Liquid Fuels:

1- In case of liquid fuels, there is a danger of explosion.
2- Liquid fuels require special types of burners for burning.
3- Liquid fuels pose problems in cold climates since the oil stored in the tanks is to be heated in order to avoid the stoppage of oil flow.

1.3.5- Nuclear energy:

- Towards the end of Second World War, it was discovered that large amount of heat energy is liberated by the fission of uranium and other fissionable materials.
- It is estimated that heat produced by 1 kg of nuclear fuel is equal to that produced by 4500 tons of coal.
- The heat produced due to nuclear fission can be utilized to raise steam with suitable arrangements.
- The steam can run the steam turbine which in turn can drive the alternator to produce electrical energy.
- However, there are some difficulties in the use of nuclear energy:
  - **a**- high cost of nuclear plant.
  - **b**- Problem of disposal of radioactive waste.

1.4- Structure of Electric Power System:

- The function of a power station is to deliver power to a large number of consumers.
- However, the power demands of different consumers vary in accordance with their activities.
- The result of this variation in demand is that load on a power station is never constant, rather it varies from time to time.
- Unfortunately, electrical power cannot be stored and, therefore, the power station must produce power as and when demanded to meet the requirements of the consumers.
- The power demanded by the consumers is supplied by the power station through the transmission and distribution networks.
- As the consumers' load demand changes, the power supply by the power station changes accordingly.
- Figure 2 shows the typical representation of a transmission-distribution scheme.

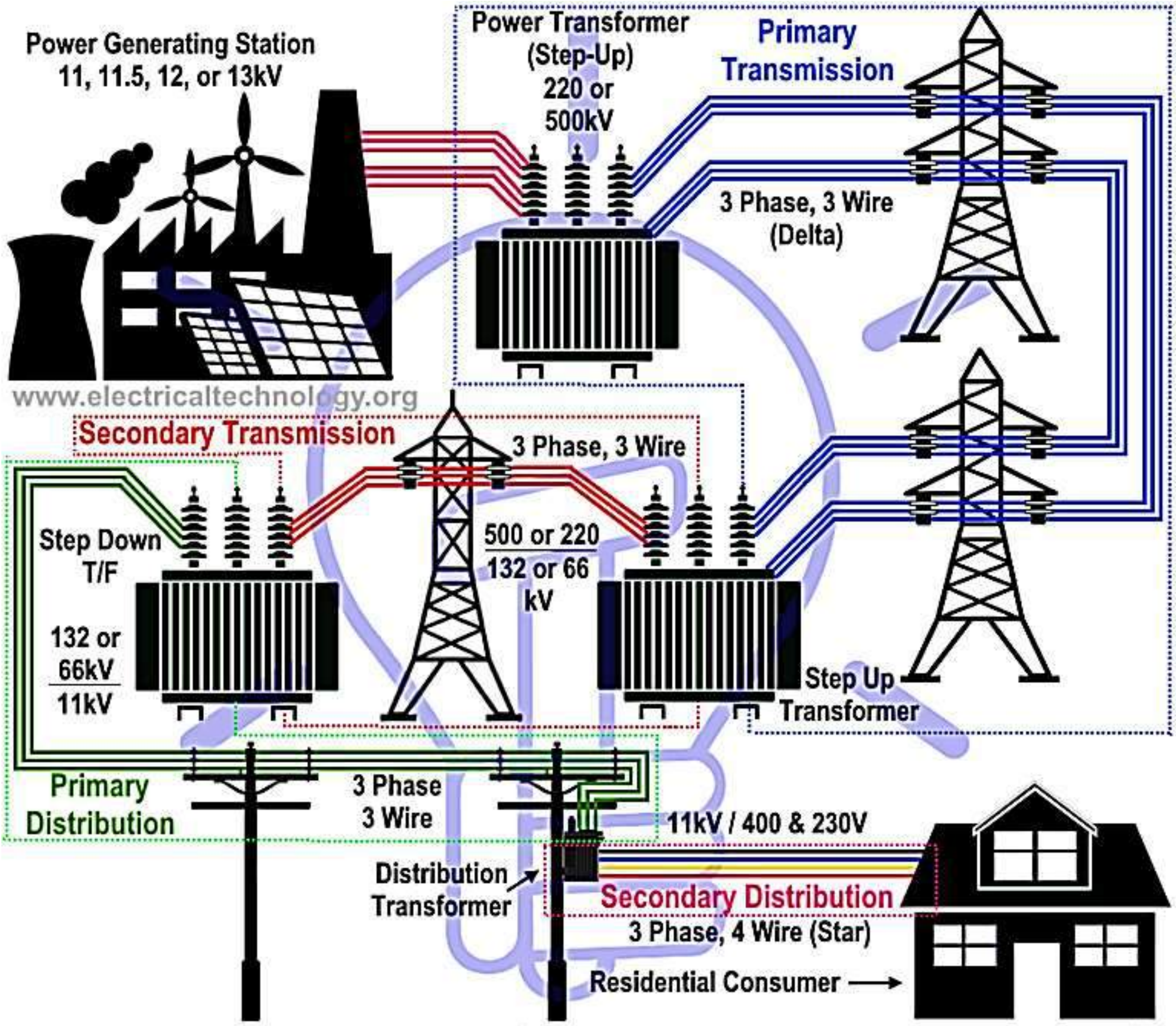


Fig. 2: Typical scheme of generation, transmission, and distribution (from google)

Chapter Two

Generating Stations

2.1- Introduction to Generating Stations

2.2- Steam Power Station (Thermal Station)

2.3- Hydro-electric Power Station

2.4- Diesel Power Station

2.5- Nuclear Power Station

2.6- Variable Load on Power Station

2.1- Introduction to Generating Stations:

- Bulk electric power is produced by special plants known as generating stations or power plants.
- The prime mover (e.g., steam turbine, water turbine etc.) converts energy from some other form into mechanical energy.
- The alternator converts mechanical energy of the prime mover into electrical energy.
- The electrical energy produced by the generating station is transmitted and distributed with the help of conductors to various consumers.

2.2- Steam Power Station (Thermal Station):

- A generating station which converts heat energy of coal combustion into electrical energy is known as a steam power station.
- A steam power station basically works on the **Rankine cycle**.
- Steam is produced in the boiler by utilizing the heat of coal combustion.
- The steam is then expanded in the prime mover (i.e., steam turbine) and is condensed in a condenser to be fed into the boiler again.
- The steam turbine drives the alternator which converts mechanical energy of the turbine into electrical energy.
- This type of power station is suitable where ***coal and water*** are available.

Advantages:

- Fuel (i.e., coal) used is cheap.
- Less initial cost as compared to other generating stations.
- Coal can be transported to the site of the plant by rail or road.
- It requires less space as compared to the hydroelectric power station.
- Cost of generation is lesser than that of the diesel power station.

Disadvantages:

- It pollutes the atmosphere due to the production of large amount of smoke and fumes.
- Its running cost is high as compared to hydroelectric plant.

2.2.1- Schematic Arrangement of Steam Power Station:

The schematic arrangement of a modern steam power station is shown in Figure 3. The whole arrangement can be divided into the following stages:

a- Coal and ash handling arrangement.

b- Steam generating plant.

c- Steam turbine.

d- Alternator.

e- Feed water.

f- Cooling arrangement.

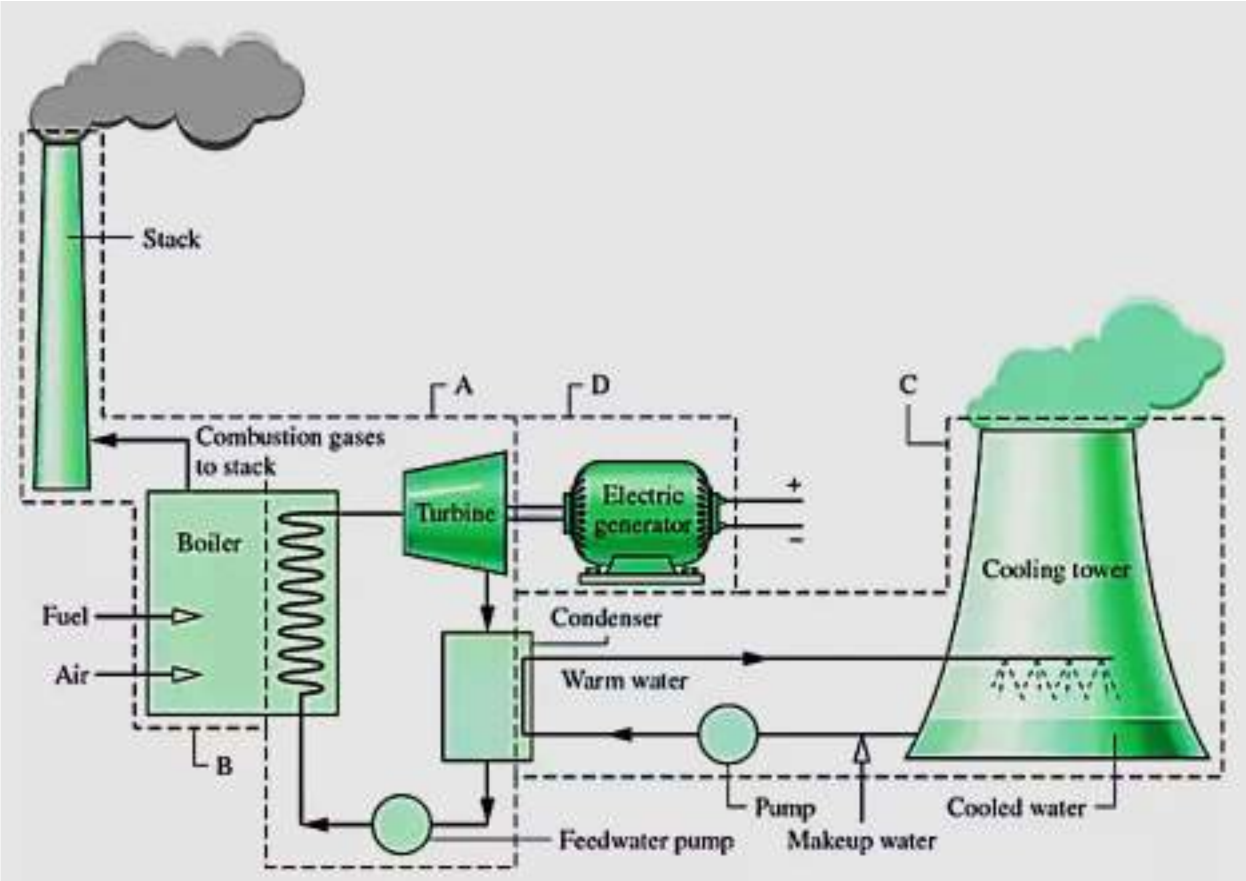


Fig. 3: Steam power plant (from google)

a- Coal and ash handling arrangement:

- From the coal storage plant, coal is crushed into small pieces and then delivered to the coal handling plant.
- The coal is burnt in the boiler and the ash produced after the complete combustion of coal is removed to the ash handling plant and then delivered to the ash storage plant for disposal.

- In fact, in a thermal station, about 50 % to 60 % of the total operating cost consists of fuel purchasing and its handling.

b- Steam generating plant:

The main part of the steam generating plant is boiler which is used to convert the water into high pressure steam.

c- Steam turbine:

- The dry steam is fed to the steam turbine through main valve.
- The heat energy of steam when passing over the blades of turbine is converted into mechanical energy.
- After giving heat energy to the turbine, the steam is exhausted to the condenser which condenses the exhausted steam by means of cold-water circulation.

d- Alternator:

- The alternator converts mechanical energy of turbine into electrical energy.
- The electrical output from the alternator is delivered to the bus bars through transformer, circuit breakers and isolators.

e- Feed water (Condenser):

- It condensate the steam that leaves out turbine.
- It converts the low-pressure steam to water.

f- Cooling arrangement:

- The circulating water takes up the heat of the exhausted steam and itself becomes hot.
- The hot water from the condenser is passed on to the cooling towers where it is cooled.
- The cold water from the cooling tower is reused in the condenser.

2.2.2- Choice of Site for Steam Power Stations:

- **Supply of fuel:** The steam power station should be located near the coal mines so that transportation cost of fuel is minimum.
- **Availability of water:** As huge amount of water is required for the condenser.
- **Transportation facilities:** The transportation facilities must exist such as rail, road, and etc.
- **Cost and type of land**: The steam power station should be located at a place where land is cheap.

- **Nearness to load centers:** In order to reduce the transmission cost, the plant should be located near the center of the load.
- **Distance from populated area:** As huge amount of coal is burnt in a steam power station; therefore, smoke and fumes pollute the surrounding area.

2.3- Hydro-electric Power Station:

- A generating station which utilizes the potential energy of water at a high level for the generation of electrical energy is known as a hydro-electric power station.
- In a hydro-electric power station, water head is created by constructing a dam across a river or lake.
- From the dam, water is led to a water turbine.
- The water turbine captures the energy in the falling water and changes the hydraulic energy (i.e., product of head and flow of water) into mechanical energy at the turbine shaft.
- The turbine drives the alternator which converts mechanical energy into electrical energy.
- Hydro-electric power stations are becoming very popular because the reserves of fuels (i.e., coal and oil) are depleting day by day.

Advantages:

- It requires no fuel; water is used for the generation of electrical energy.
- It is clean as no smoke or ash is produced.
- It requires small running charges because water is the source of energy which is available free of cost.
- It is simple in construction and requires less maintenance.
- It is robust and has a longer life

Disadvantages:

- It involves high capital cost due to construction of dam.
- There is uncertainty about the availability of huge amount of water due to dependence on weather conditions.
- Skilled and experienced hands are required to build the plant.
- It requires high cost of transmission lines as the plant is located in hilly areas which are away from the consumers.

2.3.1- Schematic Arrangement of Hydro-electric Power Station

- The schematic arrangement of a hydro-electric plant is shown in Figure 4.
- The dam is constructed across a river or lake.
- The water collects at the back of the dam to form a reservoir.

- A pressure tunnel is taken off from the reservoir and water brought to the valve house at the start of the penstock.
- The valve house contains automatic isolating valves which controls the water flow to the power house and cuts off supply of water when the penstock bursts.
- The penstock is a huge steel pipe which is used to take water from the valve house to the turbine.
- The water turbine converts hydraulic energy into mechanical energy.
- The turbine drives the alternator which converts mechanical energy into electrical energy.
- A surge tank (open from top) is built to protect the penstock from bursting in case the turbine gates suddenly close due to electrical load being thrown off.
- When the gates close, there is a sudden stopping of water at the lower end of the penstock and the penstock can burst like a paper log.
- The surge tank absorbs this pressure swing by increase in its level of water.

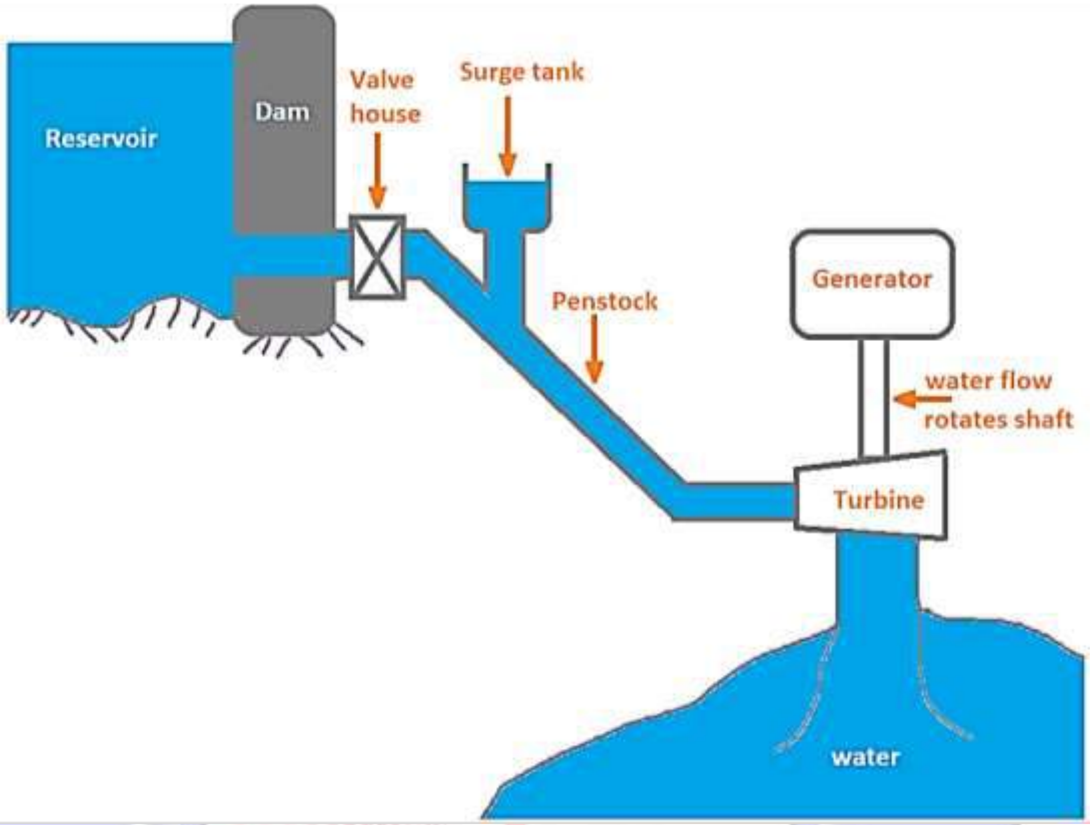


Fig. 4: Hydroelectric power plant (from google)

2.3.2- Choice of Site for Hydro-electric Power Stations

The following points should be taken into account while selecting the site for a hydro-electric power station:

1- **Availability of water:** The primary requirement of a hydro-electric power station is the availability of huge quantity of water.

2- **Storage of water:** There are wide variations in water supply from a river during the year. This makes it necessary to store water by constructing a dam in order to ensure the generation of power throughout the year.

3- **Cost and type of land:** The land for the construction of the plant should be available at a reasonable price. Further, the bearing capacity of the ground should be adequate to withstand the weight of heavy equipment to be installed.

4- **Transportation facilities:** The site selected should be accessible by rail and road so that necessary equipment and machinery could be easily transported.

2.4- Diesel Power Station

- A generating station in which diesel engine is used as the prime mover for the generation of electrical energy is known as diesel power station.
- In a diesel power station, diesel engine is used as the prime mover.
- The diesel burns inside the engine and the products of this combustion act as the "working fluid" to produce mechanical energy.
- The diesel engine drives the alternator which converts mechanical energy into electrical energy.
- As the generation cost is considerable due to high price of diesel, therefore, such power stations are only used to produce small power.
- Steam power stations and hydro-electric plants are used to generate bulk power at cheaper cost, yet diesel power stations are finding at places where demand of power is less.

عدد محطات إنتاج الطاقة الكهربائية والكمية المنتجة ونسبة المشاركة الفعلية حسب نوع المحطات في العراق عدا إقليم كردستان لسنة 2014

NUMBER OF ELECRICAL ENERGY PRODUCTION STATIONS,THE BRODUCED AMOUNT AND THE PERCENTAGE OF ACTUAL PARTICIPATION BY TYPE OF STATIONS IN IRAQ EXCLUDING KURDISTAN REGION FOR 2014

Table (17/16) A — جدول (16/17) أ

Production stations	نسبة المشاركة الفعلية في الإنتاج % Percentage of actual participation in production %	كمية الإنتاج (م و.س) Amount of production (Mw.H)	عدد المحطات Number of stations	محطات الإنتاج
Steam stations	30.9	20838527	8	المحطات البخارية
Gaseous stations	54.9	37049525	27	المحطات الغازية
Barges *	0.0	0	6	المحطات المتنقلة *
Diesel stations	9.8	6623187	11	محطات الديزل
Hydroelectric stations	4.3	2930797	8	المحطات الكهرومائية
Total	99.5	67442036	60	إجمالي المحطات
Supporting diesel	0.0	0	..	ديزلات ساندة
Ministry of Oil Diesel	0.5	325959	..	ديزلات وزارة النفط
Total	0.5	325959	..	إجمالي الديزلات
Iraq total of electrical energy production	100.0	67767995	60	إجمالي العراق لإنتاج الطاقة الكهربائية
Imported energy + Barges	15.3	12250551	..	الطاقة المستوردة + البارجات
Iraq total of electrical energy		80018546	60	إجمالي العراق للطاقة الكهربائية

.. Data are not available — .. بيانات غير متوفرة

* there is no amount of electricity production for the transferred stations because they are out of work — * لاتوجد كميات إنتاج كهرباء للمحطات المتنقلة بسبب عطل المحطات

Source : Ministry of Electricity / Information Center / Statistics Division — المصدر : وزارة الكهرباء / مركز المعلوماتية / قسم الإحصاء

- Diesel power plants are also used as standby sets for continuity of supply to important points such as hospitals, radio stations, houses and telephone exchanges.

Advantages:

1- It requires less space as compared with other power stations.

2- It can be located at any place.

3- It can be started quickly and can pick up load in a short time.

4- It requires less quantity of water for cooling.

5- The thermal efficiency of the plant is higher than that of a steam power station.

6- It requires fewer operating staff.

Disadvantages:

1- The plant has high running charges as the fuel (i.e., diesel) used is costly.

2- The plant does not work satisfactorily under overload conditions for a longer period.

3- The plant can only generate small power.

4- The cost of lubrication is generally high.

5- The maintenance charges are generally high.

2.5- Nuclear Power Station

- A generating station in which nuclear energy is converted into electrical energy is known as a nuclear power station.
- In nuclear power station, Uranium (U^{235}) is subjected to nuclear fission in a special apparatus known as a reactor.
- The heat energy thus released is utilized in raising steam at high temperature and pressure.

- The steam runs the steam turbine which converts steam energy into mechanical energy.
- The turbine drives the alternator which converts mechanical energy into electrical energy.
- The most important feature of a nuclear power station is that huge amount of electrical energy can be produced from a relatively small amount of nuclear fuel as compared to other conventional types of power stations.
- It has been found that complete fission of 1 kg of Uranium (U^{235}) can produce as much energy as can be produced by the burning of 4,500 tons of high-grade coal.

Advantages:

1. The amount of fuel required is small but it is used for producing bulk electrical energy which is considered economical.
2. A nuclear power plant requires less space as compared to any other type of the same size.
3. It can be located near the load centers because it does not require large quantities of water and no need to be near coal mines.
4. It ensures reliability of operation.

Disadvantages:

1. The erection and commissioning of the plant requires greater technical know-how.
2. The fission by-products are generally radioactive and may cause a dangerous amount of radioactive pollution.
3. Maintenance charges are high due to lack of standardization.
4. Nuclear power plants are not well suited for varying loads as the reactor does not respond to the load fluctuations efficiently.

2.5.1- Schematic Arrangement of Nuclear Power Station

- The schematic arrangement of a nuclear power station is shown in Figure 6.
- This arrangement can be divided into the following main stages:
    1. Nuclear reactor.
    2. Heat exchanger.
    3. Steam turbine.
    4. Alternator.

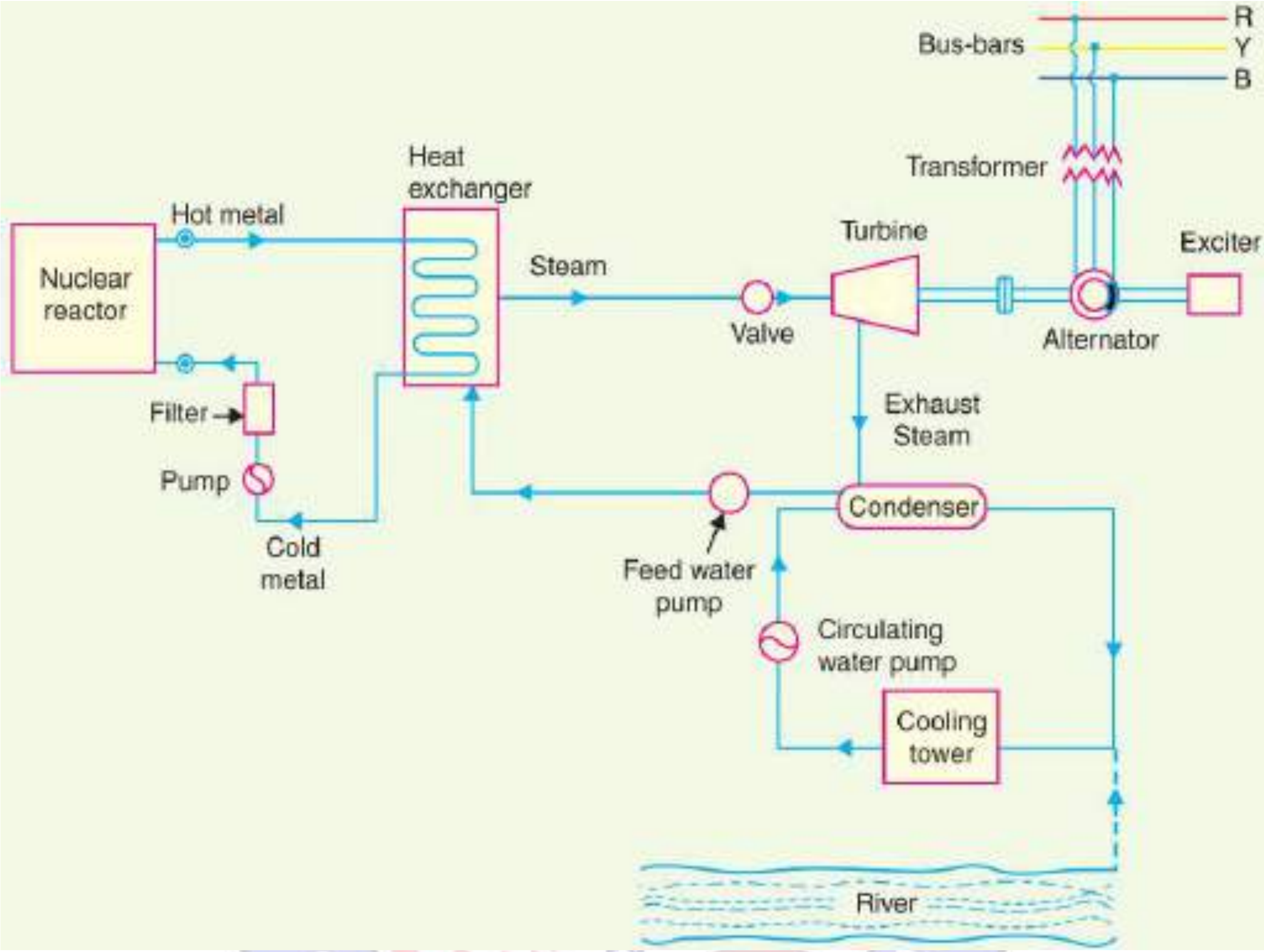


Fig. 6: Nuclear Power Station

1. Nuclear reactor:

- Nuclear fuel (U^{235}) is subjected to nuclear fission.
- A nuclear reactor is a cylindrical pressure vessel and contains fuel rods of Uranium, moderator and control rods (See Fig.7).

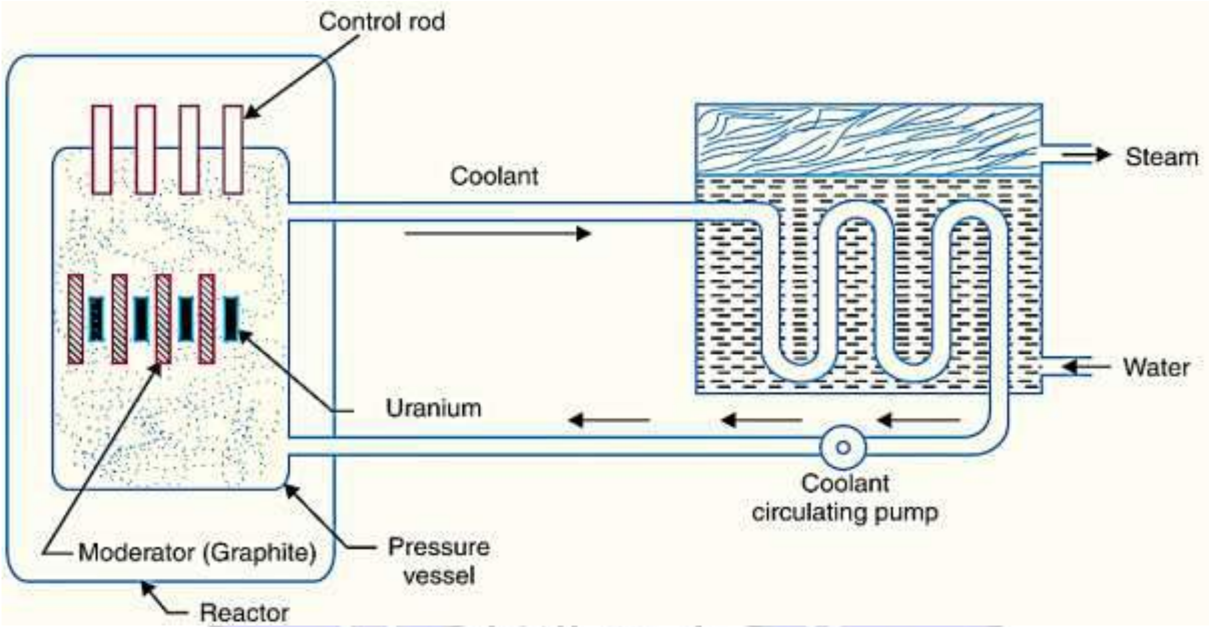


Fig. 7: A nuclear reactor

- The fuel rods constitute the fission material and release huge amount of energy when bombarded with slow moving neutrons.
- The moderator consists of graphite rods which enclose the fuel rods.
- The moderator slows down the neutrons before they bombard the fuel rods.
- The control rods are of cadmium and are inserted into the reactor.
- Cadmium is strong neutron absorber and thus regulates the supply of neutrons for fission.
- When the control rods are pushed in deep enough, they absorb most of fission neutrons and hence few are available for chain reaction which, therefore, stops.
- By pulling out the control rods, power of the nuclear reactor is increased, whereas by pushing them in, it is reduced.

- In practice, the lowering or raising of control rods is accomplished automatically according to the requirement of load.
- The heat produced in the reactor is removed by the coolant, generally a sodium metal.
- The coolant carries the heat to the heat exchanger.

2. **Heat exchanger:**

- The coolant gives up heat to the heat exchanger which is utilized in raising the steam.
- After giving up heat, the coolant is again fed to the reactor.

3. **Steam turbine:**

- The steam produced in the heat exchanger is led to the steam turbine through a valve.

4. **Alternator:**

- The steam turbine drives the alternator which converts mechanical energy into electrical energy.
- The output from the alternator is delivered to the bus-bars through transformer, circuit breakers and isolators.

2.5.2- Selection of Site for Nuclear Power Station

- **Availability of water:** the plant site should be located where ample quantity of water is available.
- **Disposal of waste:** The waste produced by fission in a nuclear power station is generally radioactive which must be disposed off properly to avoid health hazards.
- **Distance from populated areas:** The site should be away from the populated areas as there is a danger of presence of radioactivity in the atmosphere near.
- **Transportation facilities:** To transport the heavy equipment during erection and to facilitate the movement of the workers employed in the plant.

2.6- Variable Load on Power Station

- The function of a power station is to deliver power to a large number of consumers.
- However, the power demands of different consumers vary in accordance with their activities.
- The result of this variation in demand is that load on a power station is never constant, rather it varies from time to time.
- The load on a power station varies from time to time due to uncertain demands of the consumers is known as variable load on the station.

2.6.1- Load Curve:

- The curve showing the variation of load on the power station with respect to time is known as a load curve.
- **Daily load curve:** It shows the variations of load with respect to time during the day (i.e., 24 hours) as shown in figure 8.

Fig. 8: daily load curve of a power station.

- The area under the daily load curve gives the number of units generated in the day.

Units generated/day = Area (in kWh) under daily load curve

- The highest point on the daily load curve represents the maximum demand on the station on that day (It is clear that from fig. 8, being maximum at 6 P.M. in this case).
- The area under the daily load curve divided by the total number of hours gives the average load on the station in the day.

$$\text{Average load} = \frac{\text{Area (in kWh) under daily load curve}}{24 \text{ hours}}$$

- The ratio of the area under the load curve to the total area of rectangle in which it is contained gives the load factor.

$$\text{Load factor} = \frac{\text{Average load}}{\text{Max. demand}} = \frac{\text{Average load} \times 24}{\text{Max. demand} \times 24}$$
$$= \frac{\text{Area (in kWh) under daily load curve}}{\text{Total area of rectangle in which the load curve is contained}}$$

- **Monthly load curve:** It can be obtained from the daily load curves of that month (average values of power over a month at different times of the day are calculated and then plotted on the graph).
- The monthly load curve is generally used to fix the rates of energy.
- **Yearly load curve:** It can be obtained by considering the monthly load curves of that particular year.
- The yearly load curve is generally used to determine the annual load factor.

2.6.2- Load Duration Curve:

- When the load elements of a load curve are arranged in the order of descending magnitudes, the curve thus obtained is called a load duration curve as shown in figure 9.

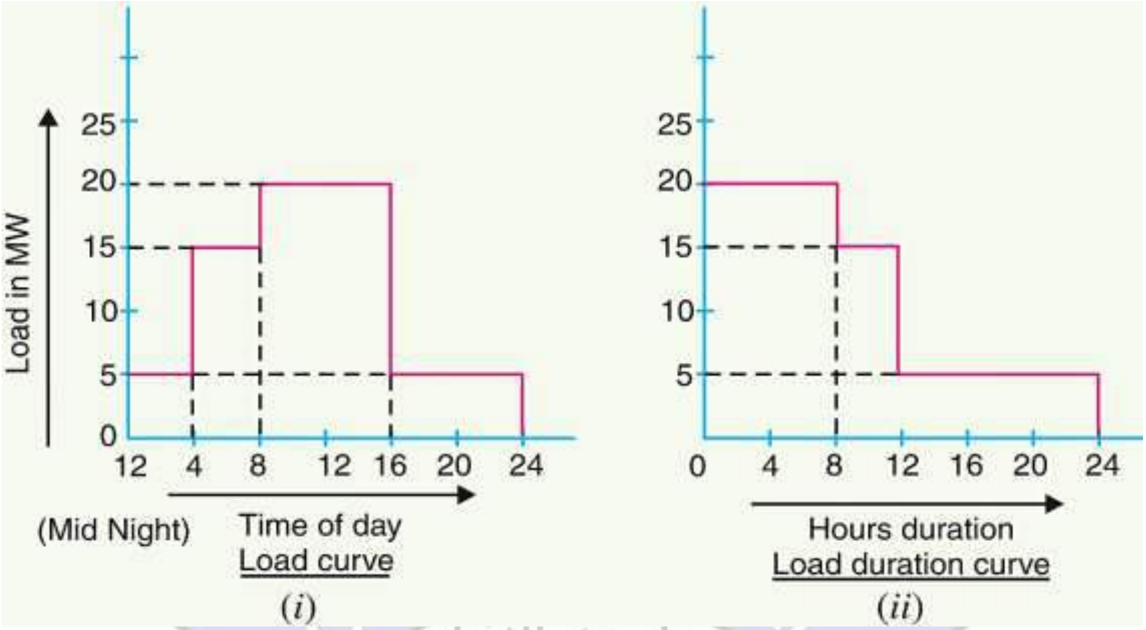


Fig. 9: load duration curve

- The load duration curve is obtained from the same data as the load curve but the ordinates are arranged in the order of descending magnitudes.
- In other words, the maximum load is represented to the left and decreasing loads are represented to the right in the descending order.
- Hence the area under the load duration curve and the area under the load curve are equal. For example, 20 MW for 8 hours; 15 MW for 4 hours and 5 MW for 12 hours.

2.6.3- Base Load and Peak Load:

- The changing load on the power station makes its load curve of variable nature.

- However, the load curve on a power station can be considered in two parts, namely;
- **Base load:** The unvarying load which occurs almost the whole day on the station is known as base load as shown in figure 10.

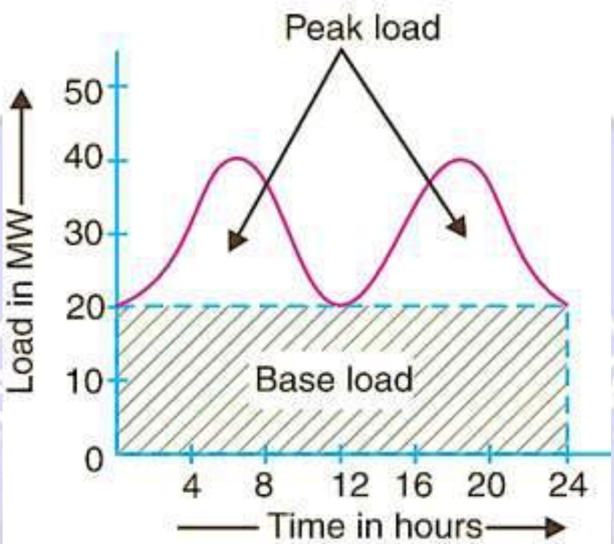


Fig. 10 beak and base loads

- It is clear that 20 MW of load has to be supplied by the station at all times of day and night i.e. throughout 24 hours.
- **Peak load:** The various peak demands of load over and above the base load of the station is known as peak load.
- It is clear that there are two peak demands of load in figure 10 excluding base load.
- These peak demands of the station generally form a small part of the total load.

2.6.4- Types of Loads:

- The load may be resistive (e.g., electric lamp), inductive (e.g., induction motor), capacitive or some combination of them.
- The various types of loads on the power system are:

1-Residential Load:

- consists of lights, fans, refrigerators, heaters, television, small motors for pumping water etc.
- Most of the residential load occurs only for some hours during the day (i.e., 24 hours) e.g., lighting load occurs during night time and load occurs for few hours.
- For this reason, the load factor is low (10% to 12%).

2-Commercial load:

- It consists of lighting for shops, fans and electric equipment's used in restaurants etc.
- This class of load occurs for more hours during the day as compared to the residential load.
- The commercial load has seasonal variations due to the extensive use of air-conditioners and space heaters.

3-Industrial load:

- The magnitude of industrial load depends upon the type of industry.

- Thus, small scale industry requires load up to 25 kW, medium scale industry between 25kW and 100 kW and large-scale industry requires load above 500 kW.

2.6.5-Important Terms and Factors:

1- Connected load:

It is the sum of continuous ratings of all the equipments connected to supply system.

- A power station supplies load to thousands of consumers.
- Each consumer has certain equipment installed in his building.
- The sum of the continuous ratings of all the equipments in the consumer's building is the "connected load" of the consumer.
- For instance, if a consumer has connections of five 100-watt lamps and a power point of 500 watts, then connected load of the consumer is $5 \times 100 + 500 = 1000$ watts.
- The sum of the connected loads of all the consumers is the connected load to the power station.

2- Maximum demand:

It is the greatest demand of load on the power station during a given period.

- The load on the power station varies from time to time.
- The maximum of all the demands that have occurred during a given period (say a day) is the maximum demand.
- Referring back to the load curve of Fig. 8, the maximum demand on the power station during the day is 6 MW and it occurs at 6 P.M.

- Maximum demand is generally less than the connected load because all the consumers do not switch on their connected load to the system at a time.
- The knowledge of maximum demand is very important as it helps in determining the installed capacity of the station.
- The station must be capable of meeting the maximum demand.

3- Demand factor:

It is the ratio of maximum demand on the power station to its connected load i.e.,

$$\text{Demand factor} = \frac{\text{Maximum demand}}{\text{Connected load}}$$

- The value of demand factor is usually less than 1.
- It is expected because maximum demand on the power station is less than the connected load.
- If the maximum demand on the power station is 80 MW and the connected load is 100 MW, then demand factor = 80/100 = 0·8.
- The knowledge of demand factor is vital in determining the capacity of the plant equipment

4- Average load (Average demand):

The average of loads occurring on the power station in a given period (day or month or year).

$$\text{Daily average load} = \frac{\text{No. of units (kWh) generated in a day}}{\text{24 hours}}$$

$$\text{Monthly average load} = \frac{\text{No. of units (kWh) generated in a month}}{\text{Number of hours in a month}}$$

$$\text{Yearly average load} = \frac{\text{No. of units (kWh) generated in a year}}{\text{8760 hours}}$$

5- Load factor:

The ratio of average load to the maximum demand during a given period i.e.,

$$\text{Load factor} = \frac{\text{Average load}}{\text{Max. demand}}$$

If the plant is in operation for T hours,

$$\text{Load factor} = \frac{\text{Average load} \times \text{T}}{\text{Max. demand} \times \text{T}}$$

$$= \frac{\text{Units generated in T hours}}{\text{Max. demand} \times \text{T hours}}$$

- The load factor may be daily load factor, monthly load factor or annual load factor if the time period considered is a day or month or year.
- Load factor is always less than 1 because average load is smaller than the maximum demand.
- The load factor plays key role in determining the overall cost
- per unit generated.
- Higher the load factor of the power station, lesser will be the cost per unit generated.

6- Diversity factor:

The ratio of the sum of individual maximum demands to the maximum demand on power station, i.e.,

$$\text{Diversity factor} = \frac{\text{Sum of individual max. demands}}{\text{Max. demand on power station}}$$

- A power station supplies load to various types of consumers whose maximum demands generally do not occur at the same time.
- Therefore, the maximum demand on the power station is always less than the sum of individual maximum demands of the consumers.
- Obviously, diversity† factor will always be greater than 1.
- The greater the diversity factor, the lesser is the cost of generation of power.

7- Plant capacity factor:

It is the ratio of actual energy produced to the maximum possible energy that could have been produced during a given period i.e.,

$$\text{Plant capacity factor} = \frac{\text{Actual energy produced}}{\text{Max. energy that could have been produced}}$$

$$= \frac{\text{Average demand} \times T^{**}}{\text{Plant capacity} \times T}$$

$$= \frac{\text{Average demand}}{\text{Plant capacity}}$$

- Thus, if the considered period is one year,

$$\text{Annual plant capacity factor} = \frac{\text{Annual kWh output}}{\text{Plant capacity} \times 8760}$$

- The plant capacity factor is an indication of the reserve capacity of the plant.
- A power station is so designed that it has some reserve capacity for meeting the increased load demand in future.
- Therefore, the installed capacity of the plant is always somewhat greater than the maximum demand on the plant.

$$\text{Reserve capacity} = \text{Plant capacity} - \text{Max. demand}$$

7- Plant use factor:

It is ratio of kWh generated to the product of plant capacity and the number of hours for which the plant was in operation i.e.

$$\text{Plant use factor} = \frac{\text{Station output in kWh}}{\text{Plant capacity} \times \text{Hours of use}}$$

- Suppose a plant having installed capacity of 20 MW produces annual output of $7{\cdot}35 \times 10^6$ kWh and remains in operation for 2190 hours in a year. Then,

$$\text{Plant use factor} = \frac{7{\cdot}35 \times 10^6}{\left(20 \times 10^3\right) \times 2190} = 0{\cdot}167 = 16{\cdot}7\%$$

Example 3.2:

A generating station has a connected load of 43MW and a maximum demand of 20 MW; the units generated being $61{\cdot}5 \times 10^6$ per annum. Calculate (i) the demand factor and (ii) load factor.

Solution.

(*i*) $$\text{Demand factor} = \frac{\text{Max. demand}}{\text{Connected load}} = \frac{20}{43} = 0{\cdot}465$$

(*ii*) $$\text{Average demand} = \frac{\text{Units generated / annum}}{\text{Hours in a year}} = \frac{61 \cdot 5 \times 10^6}{8760} = 7020 \text{ kW}$$

$$\therefore \quad \text{Load factor} = \frac{\text{Average demand}}{\text{Max. demand}} = \frac{7020}{20 \times 10^3} = 0{\cdot}351 \text{ or } 35{\cdot}1\%$$

Example 3.4:

A generating station has a maximum demand of 25MW, a load factor of 60%, a plant capacity factor of 50% and a plant use factor of 72%. Find (i) the reserve capacity of the plant (ii) the daily energy produced and (iii) maximum energy that could be produced daily if the plant while running as per schedule, were fully loaded.

$$\text{Load factor} = \frac{\text{Average demand}}{\text{Maximum demand}}$$

$$0{\cdot}60 = \frac{\text{Average demand}}{25}$$

$$\text{Average demand} = 25 \times 0{\cdot}60 = 15 \text{ MW}$$

$$\text{Plant capacity factor} = \frac{\text{Average demand}}{\text{Plant capacity}}$$

$$\text{Plant capacity} = \frac{\text{Average demand}}{\text{Plant capacity factor}} = \frac{15}{0 \cdot 5} = 30 \text{ MW}$$

$$\therefore \quad \text{Reserve capacity of plant} = \text{Plant capacity} - \text{maximum demand} = 30 - 25 = \mathbf{5\ MW}$$

(*ii*) Daily energy produced

$$= \text{Average demand} \times 24 = 15 \times 24 = \mathbf{360\ MWh}$$

(*iii*) Maximum energy that could be produced

$$= \frac{\text{Actual energy produced in a day}}{\text{Plant use factor}}$$

$$= \frac{360}{0 \cdot 72} = \mathbf{500\ MWh/day}$$

Example:

Example: A power station is to supply three consumers . The daily demand of three consumers are given below :

Time (hours)	Consumer (1)	Consumer 2)	Consumer (3)
0 – 6	200 KW	100 KW	No - load
6 – 14	600 KW	1000 KW	400 KW
14 – 18	No - load	600 KW	400 KW
18 – 24	800 KW	No - load	600 KW

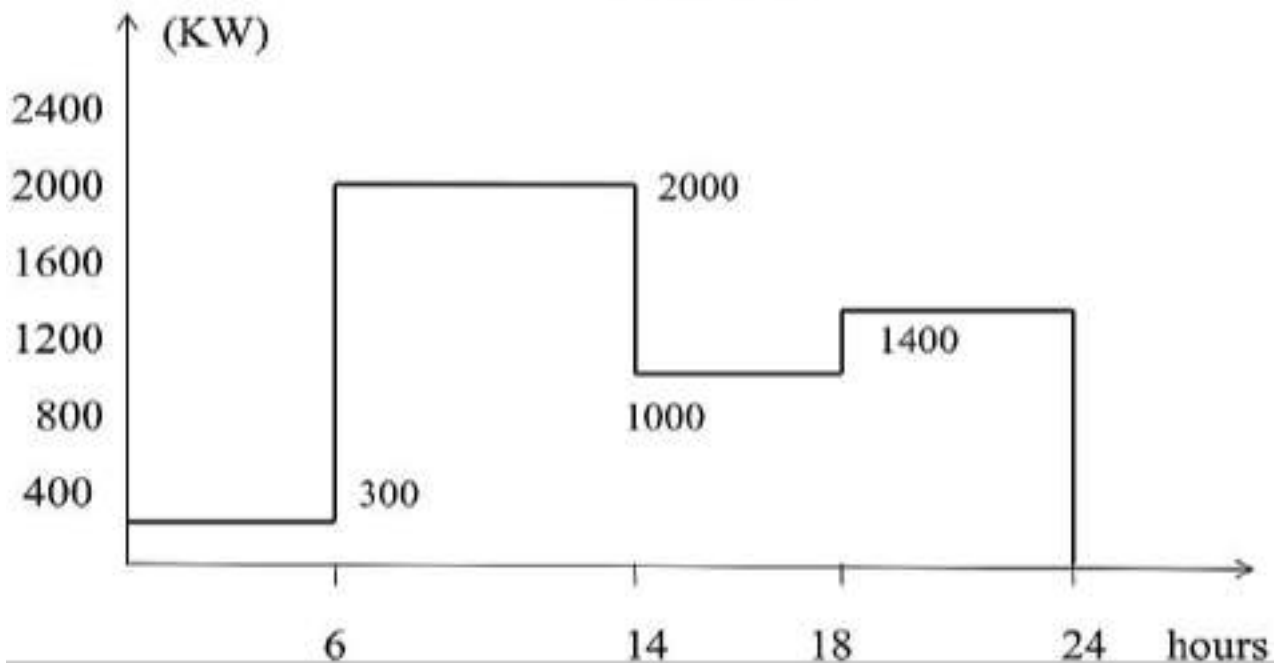
Plot the load curve of power station and , find :

1- Load factor of individual consumer.

2- Diversity factor of power station .

3- Load factor of power station .

Solution :



1- load factor of consumer $= \dfrac{Energy\ consumed\ /\ day}{Max.\ demand \times hours\ in\ day} \times 100$

load factor of consumer(1) =

$$\frac{200 \times 6 + 600 \times 8 + 0 \times 4 + 800 \times 6}{800 \times 24} \times 100 = 56.25\ \%$$

load factor of consumer(2) =

$$\frac{100 \times 6 + 1000 \times 8 + 600 \times 4 + 0 \times 6}{1000 \times 24} \times 100 = 45.8\ \%$$

load factor of consumer(3) =

$$\frac{0 \times 6 + 400 \times 8 + 400 \times 4 + 600 \times 6}{600 \times 24} \times 100 = 58.3\ \%$$

2- Diversity factor $= \dfrac{Sum\ of\ indiviual\ \max.\ demands}{\max.\ demand\ on\ power\ station}$

From load curve , the Max. demand on power station is 2000 KW

$$\text{Diversity factor} = \frac{800 + 1000 + 600}{2000} = 1.2$$

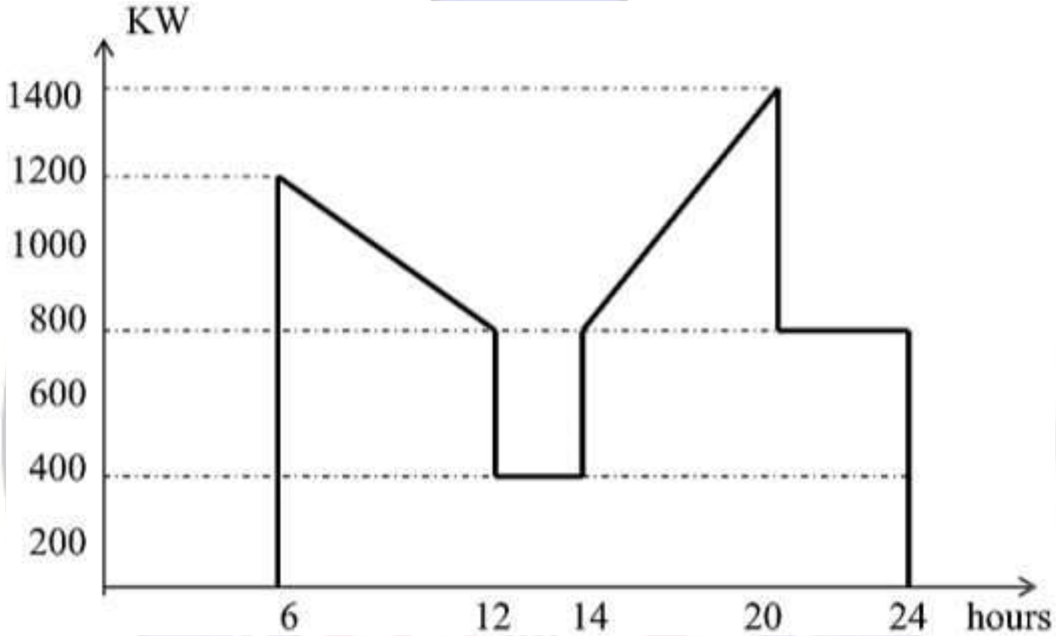
3- Load factor of power station =

$$\frac{300 \times 6 + 2000 \times 8 + 1000 \times 4 + 1400 \times 6}{2000 \times 24} \times 100 = 62.9\ \%$$

Example: A power station is to supply two loads . The daily load curve of the station is as shown in figure below . If the load factor of the two loads are 0.416 and 0.458 respectively , and the diversity factor of the power station is 1.142 , find :

1- The load factor of power station .

2- The energy consumed per day and the maximum demand of each load .



Solution :

1- $\text{load factor } (L.F) = \dfrac{\textit{Energy consumed (generated) / day}}{\textit{Max. demand} \times \textit{hours in day}} \times 100$

Load factor of power station =

$$\frac{18 \times 400 + 6 \times 400 + 10 \times 400 + \frac{1}{2}(6 \times 400) + \frac{1}{2}(6 \times 600)}{1400 \times 24}$$

$$= \frac{16600}{1400 \times 24} = 0.49$$

2- Let : *E1* – energy consumed by load 1

E2 – energy consumed by load 2

M1 – Max. demand of load 1

M2 – Max. demand of load 2

L.F1 , *L.F2* - load factor of loads 1 and 2

$$L.F1 = \frac{E1}{24 \times M1} \quad ; \quad 0.416 = \frac{E1}{24 \times M1}$$

$10\,M1 = E1$ ……….. (1)

$$0.458 = \frac{E2}{24 \times M2}$$

$$11\,M2 = E2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\text{Diversity factor} = \frac{Sum\ of\ indiviual\ \max.\ demands}{\max.\ demand\ on\ power\ station}$$

$$1.142 = \frac{M1 + M2}{1400}$$

$$1600 = M1 + M2 \quad \ldots\ldots\ldots\ldots(3)$$

$$\text{Energy generated from power station} = E1 + E1$$

$$16600 = E1 + E2 \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Put (3) and (4) in (2)

$$11\,(1600 - M1) = 16600 - E1$$

$$17600 - 16600 - 11\,M1 + E1 = 0 \quad \ldots\ldots\ldots(5)$$

Put (1) in (5)

$$1000 - 11\,M1 + 10\,M1 = 0$$

$M1$ = 1000 KW

From (3) **$M2$ = 1600 – 1000 = 600 KW**

From (1) **$E1$ = 10 x 1000 = 10000 KWh**

From (2) **$E2$ = 11x 600 = 6600 KWh**

Chapter Three

Overhead Transmission Lines

(Electrical Design)

3.1- Constants of a Transmission Line
3.2- Resistance of a Transmission Line
3.3- Inductance of a Transmission line
3.4- Capacitance of Transmission Line
3.5- Examples.

3.1- Constants of a Transmission Line:

- A transmission line has:
  - ✓ Series resistance (accounts for ohmic (I^2R) line losses).
  - ✓ Series inductance-including resistance and inductive (gives rise to series – voltage drops along the line).
  - ✓ Shunt capacitance (gives rise to line – charging currents).

uniformly distributed along the whole length of the line as shown in figure 11.

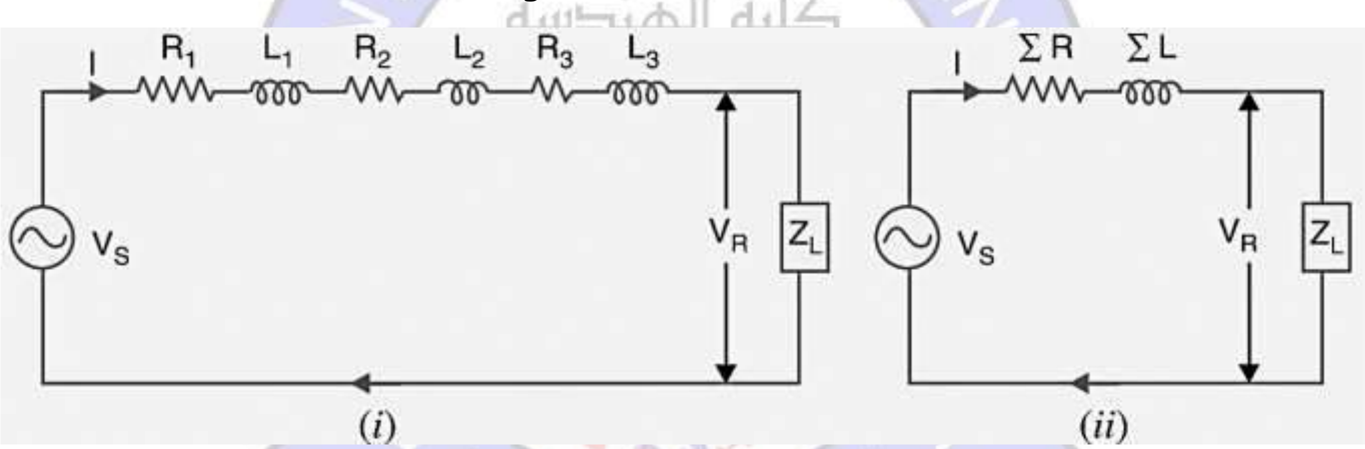


Fig. 11:

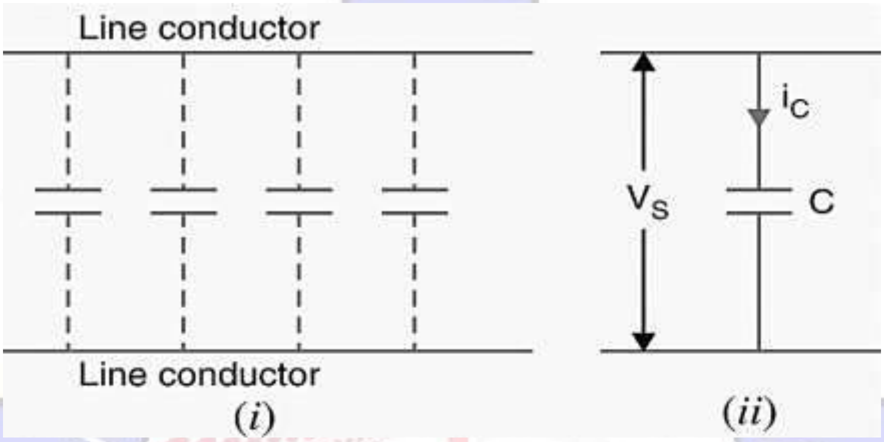
- If each section of the line is of equal length corresponding to unit length (say one meter) of the line we will have:

$R_1 = R_2 = R_3 .... = R$, resistance per unit length of the line i.e. ohms/loop meter.

Similarly,

$L_1 = L_2 = L_3 ..... = L$ henrys/loop meter,

- Therefore, the resistance and inductance are considered as lumped parameters.
- As any two conductors of an overhead transmission line are separated by air which acts as an insulation (capacitor) as shown below



- Therefore, capacitance exists between any two overhead line conductors.
- The capacitance is uniformly distributed along the whole length of the line.

$$C_1 = C_2 = C_3 ...... = C \text{ farads/loop meter}$$

- When an alternating voltage is impressed on a transmission line, the charge on the conductors at any point increases and decreases with the increase and decrease of the instantaneous value of the voltage between conductors at that point.

$$\text{Capacitance,} \quad C = \frac{q}{v} \text{ farad}$$

- The current flows between the conductors known as a charging current.
- The charging current will flow in the line even when it is open-circuited i.e., supplying no load.
- It affects the voltage drop along the line as well as the efficiency and power factor of the line.

3.2- Resistance of a Transmission Line:

- The resistance of transmission line conductors is the most important cause of power loss in a transmission line.

$$\boldsymbol{R} = \frac{\textbf{\textit{power loss in conductor}}}{|\boldsymbol{I}|^2}\ \Omega$$

- Where the power is in watts and I is the rms current in the conductor in amperes.
- The d.c resistance of various type of conductors at specified temperature T is found by:

$$R_{dc.T} = \frac{\rho_T l}{A} \qquad \Omega$$

Where ρ_T - conductor resistivity at temperature T $(\Omega . m)$.

l – conductor length (m).

A – conductor cross- sectional area (m^2).

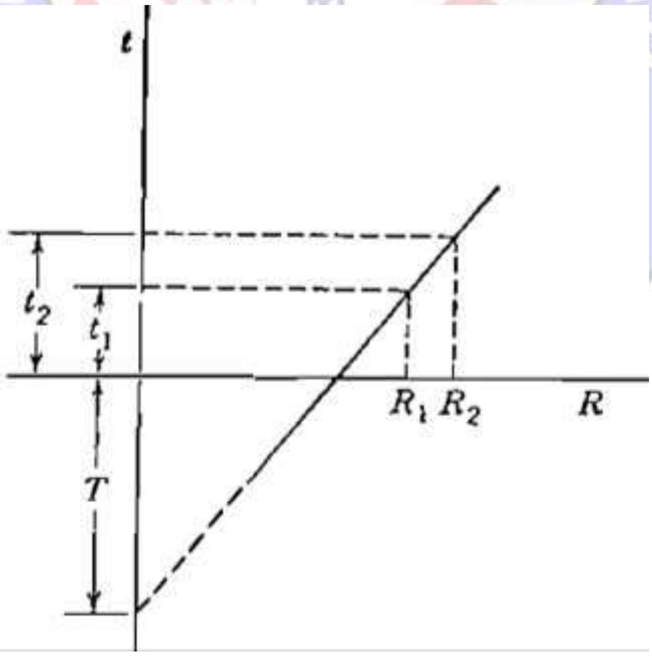
- The resistance of T.L conductor is affected by the following factors:

  - ✓ **Spiraling of a stranded conductor:**

    Spiraling makes the strands length 2% longer than the actual conductor length. As a result, the dc resistance of a stranded is 1 or 2% larger than that calculated from eq. above for a specified conductor length.

  - ✓ **Temperature:**

    The variation of resistance of conductors with temperature is practically linear over the normal range of operation. If temperature is plotted on the vertical axis and resistance on the horizontal axis, as shown below.



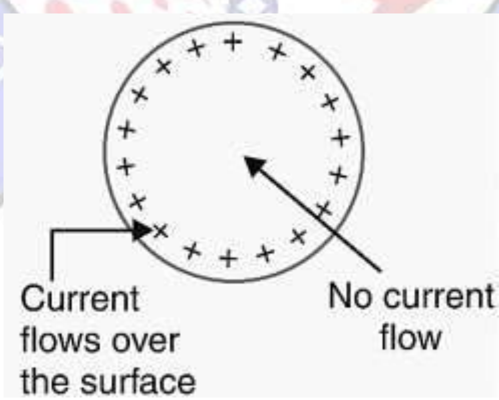
$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

where R_1 and R_2 are the resistances of the conductor at temperatures t_1 and t_2, respectively, in degrees Celsius and T is the constant determined from the graph.

The point of intersection of the extended line with the temperature axis at zero resistance is a constant of the material.

- ✓ **Skin Effect:**

For dc, the current distribution (current density) is uniform throughout the conductor cross-section. However, for ac, the current flowing through the conductor does not distribute uniformly, rather it has the tendency to concentrate near the surface of the conductor (it is known a skin effect) as shown below.



Due to skin effect, the effective area of cross-section of the conductor through which current flows is reduced. Consequently, the resistance of the conductor is slightly increased when carrying an alternating current. Thus, the inductance near the center of conductor is larger than the surface due to it is surrounded by a greater magnetic flux. The high reactance at the center causes alternating current to flow near the surface of conductor. The skin effect depends upon the following factors:

- Nature of material:
- Diameter of wire: increases with the diameter of wire.
- Frequency: increases with the increase in frequency.
- Shape of wire: less for stranded conductor than the solid conductor.

The skin effect is negligible when the supply frequency is less than 50 Hz.

3.3- Inductance of a Transmission line:

- When alternating current flows through a conductor, a magnetic field is produced according to Oersted's law.
- Due to the varying of magnetic field, induced EMF is produced according to Faraday's law.
- The direction of induced current (due to EMF) opposes the direction of applied current to the inductor according to Lenz's law.
- Mathematically, inductance of a circuit is defined as the flux linkages per unit current.

Inductance, $L = \frac{\psi}{I}$ henry

where ψ = flux linkages in weber-turns

I = current in amperes

- In order to find the inductance of a circuit, the following determinations are important:
  - ✓ Magnetic field intensity ($H = \frac{I}{2\pi r}\ A/m$, Ampere's law: emf around any closed path equals to the current enclosed by the same path)
  - ✓ Magnetic flux density ($B = \mu H\ Tesla$, μ is permeability of medium (conductor) in which the magnetic field is measured). Where; $\mu = \mu_r \mu_o$, μ_r is relative permeability and μ_o is permeability of free space ($\mu_o = 4\pi * 10^{-7}$.

- ✓ Magnetic flux ($\phi = BA\cos\theta$ weber, wb). It is the measured of magnetic field density through a closed surface.
- ✓ Magnetic flux linkages (Ψ or λ =Nϕ, wb-turn). Where N is the number of turns for surface of area.
- ✓ Inductance from flux linkages per ampere is $L = \frac{\Psi}{I}$ henry (H).

3.3.1- Inductance of a solid cylindrical conductor due to internal flux:

- Figure 12 shows a 1-meter section of a solid cylindrical conductor with radius r, carrying current I.

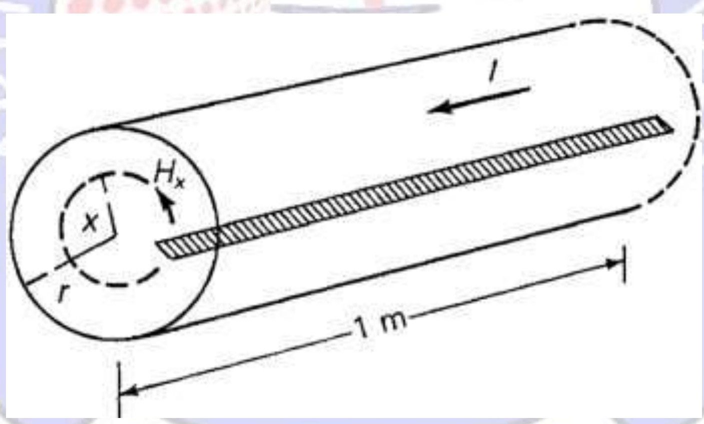


Fig. 12:

- For simplicity, assume that the conductor: (1) is nonmagnetic ($\mu = \mu_o = 4\pi * 10^{-7}$ henry/meter), (2) has a uniform current density (skin effect is neglected).

- To determine the magnetic field intensity (H_x) inside the conductor, select the dashed circle of radius x < r shown in Fig. 12 as a closed path.

Key points :-

① Assumption :- Current density, J is constant

مهملة ← (skin effect) ← بـ(J) تسبب ثابتة ولكن

للترددات (50 Hz, 60 Hz) ← نعتبرها ثابتة.

∴ for High frequencies → J is not constant.

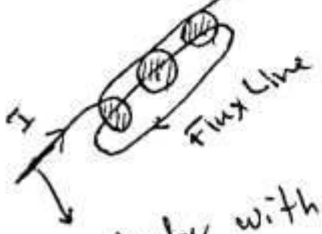
(50, 60 Hz) Low Frequencies → J is constant.

$$J_x = \frac{I_x}{\pi x^2} \quad ; \quad J = \frac{I}{\pi r^2} \Rightarrow \frac{I_x}{\pi x^2} = \frac{I}{\pi r^2}$$

(J_x ← لنقطة تبعد x عن centre الموصل. ; J ← لكل الموصل ; πx^2, πr^2 ← مساحة)

$$\Rightarrow \boxed{I_x = \left(\frac{x}{r}\right)^2 I} \Rightarrow ①$$

② No. of turns (N) ⇒ $\lambda = N\Phi$ ↳ $N = 3$



السبب في ذلك لان الموصل (conductor) تكررت لفته 3 مرات خلال مساحة المقطع العرضي

conductor with 3 turns

$I_{linked} = 3\,I$

ثلاث مرات للتيار المار في الموصل.

∴ $N = \frac{I_{linked}}{I}$ ⇒ (Ratio of the current that be linked to the current passing that conductor)

* في حالتنا ← $I_x = I_{linked}$

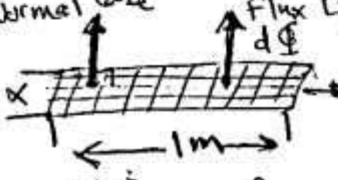
$$\boxed{N = \frac{I_x}{I} = \frac{x^2}{r^2}} \Rightarrow ②$$

* من معادلة رقم (1) نعوض عن $\left(\frac{I_x}{I}\right)$

(3) $\mu = \mu_r \mu_o \Rightarrow \mu = \mu_o = 4\pi \times 10^{-7}$

↳ Non magnetic material, μ_r for AL, Cu = 1

(4) width = dx



Normal case ⇒ flux ∥ surface of area ⇒ $\theta = 0$

$\theta = 0 \Rightarrow \cos(0) = 1 \Rightarrow \cos\theta = \cos(0) = 1$

$A = dx \cdot 1 \Rightarrow A = dx$

$\Phi = BA\cos\theta$

Derivations:- From eq (3) & (1) [I_x]

(1) $H_x = \dfrac{I_x}{2\pi x} \longrightarrow (3)$

$H_x = \dfrac{\frac{x^2}{r^2} I}{2\pi x} \Rightarrow \boxed{H_x = \dfrac{x I}{2\pi r^2}}$ A/m

(2) $B_x = \mu H_x = \dfrac{\mu_o x I}{2\pi r^2}$ Wb/m² $\{ \because \mu = \mu_r \mu_o, \; \& \; \mu_r = 1 \}$

(3) $d\Phi_x = BA\cos\theta = B_x \cdot dx \cdot 1 = \dfrac{\mu_o x I}{2\pi r^2} \cdot dx$ Wb/m $\{ \because A = dx \cdot 1 \; \& \; \theta = 0, \; \cos(0) = 1 \}$

(4) $d\lambda_x = N d\Phi_x = \dfrac{x^2}{r^2} \cdot \dfrac{\mu_o I}{2\pi r^2} \cdot x\, dx$ Wb-t/m $\{ \because N = \dfrac{x^2}{r^2} \}$

(5) $\lambda_{int} = \int_0^r d\lambda_x$ (for $0 \le x \le r$)

$= \dfrac{\mu_o I}{2\pi r^2} \int_0^r x^3 dx = \dfrac{\mu_o}{2\pi r^2} \dfrac{r^4}{4}$ $\quad \{ \because \int_0^r x^3 dx = \dfrac{x^4}{4}\Big]_0^r = \dfrac{r^4}{4} \}$

$\lambda_{int} = \dfrac{\mu_o I}{8\pi}$ $\{ \mu_o = 4\pi \times 10^{-7} \}$

$\boxed{\lambda_{int} = \left(\tfrac{1}{2} \times 10^{-7}\right) I \; (\text{Wb-t/m})}$

(6) $L_{Int} = \frac{\lambda_{Int}}{I} = \frac{(\frac{1}{2} \times 10^{-7}) I}{I}$ (4)

$$L_{Int} = \frac{1}{2} \times 10^{-7} \ (H/m)$$ ⇒ it is a constant, it is independent of the radius of a conductor

3.3.2- Inductance of a solid cylindrical conductor due to external flux (flux linkages between two points):

- Figure 13 shows a solid cylindrical conductor with radius r, carrying current I.

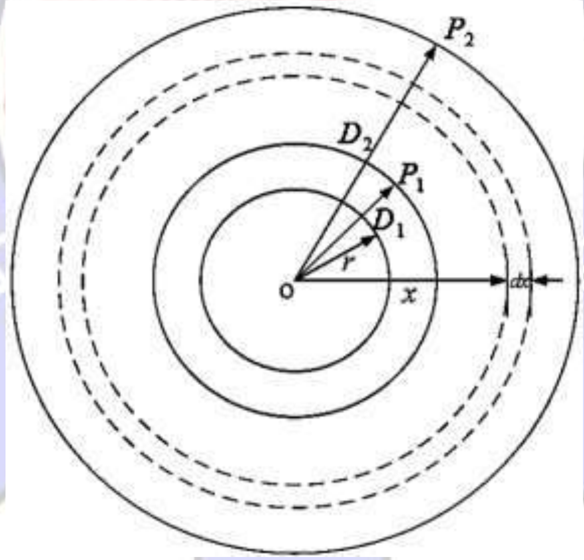


Fig. 13:

- In order to determine the magnetic field intensity (H_x) outside the conductor, select the dashed circle of radius x > r shown in Fig. 13 as a closed path.

Derivation:- same because the current cassing I_x all the current inside conductor $\boxed{I_x = I}$

① $H_x = \frac{I_x}{2\pi x} = \frac{I}{2\pi x}$

② $B_x = \mu H_x = \frac{\mu_0 I}{2\pi x}$ $\left\{ \mu_r = 1 \Rightarrow \mu = \mu_r \mu_0 \Rightarrow \mu = \mu_0 \right\}$

③ $d\Phi_x = B_x A \cos\theta = B_x \cdot dx \cdot 1$ $\left\{ \because A = dx \ \& \ \cos\theta = 1 \right\}$

$= \frac{\mu_0 I}{2\pi} \cdot \frac{1}{x} dx$

④ $d\lambda_x = N d\Phi_x = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{x} dx$ $\left\{ \because N = \frac{I_{linked}}{I} = \frac{I}{I} = 1 \right\}$

⑤ $\lambda_{ext} = \int_{D_1}^{D_2} d\lambda_x$ $\left\{ \text{for } D_1 < x < D_2 ; \ x > r \right\}$

$\Rightarrow \lambda_{ext} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx$ $\left\{ \because \int_{D_1}^{D_2} \frac{1}{x} dx = \ln x \Big]_{D_1}^{D_2} = \ln D_2 - \ln D_1 = \ln\left(\frac{D_2}{D_1}\right) \right.$

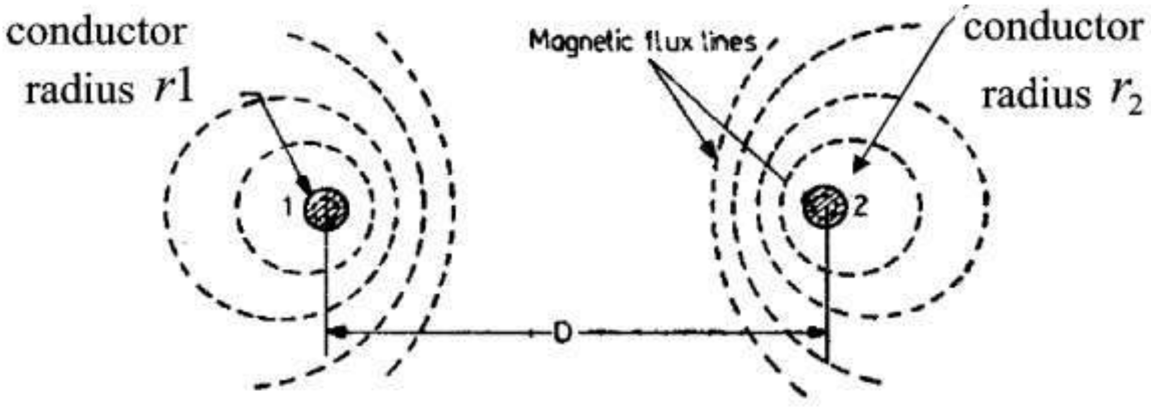
$$\boxed{\lambda_{ext} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)} \ (\text{wb.t/m})$$

$$\boxed{(2\times10^{-7}) I \ln\left(\frac{D_2}{D_1}\right)}$$

⑥ $$\boxed{L_{ext} = \frac{\lambda_{ext}}{I} = (2\times10^{-7}) \ln\left[\frac{D_2}{D_1}\right] \ (H)}$$

3.3.3- Inductance of single – phase Two – wire line:

- Figure 14 shows a single-phase two-wire line consisting of two solid cylindrical conductors 1 and 2.



The magnetic field associated with a single phase line.

Fig. 14:

- Conductor 1 with radius r_1 and conductor 2 with radius r_2. So, the total inductance of the circuit due to current in conductor 1 is:

$$L_1 = L_{\text{int}} + L_{ext}$$

$$L_1 = \frac{1}{2}10^{-7} + 2\times10^{-7}\ln\frac{D}{r_1}$$

$$= 2\times10^{-7}\left(\frac{1}{4} + \ln\frac{D}{r_1}\right) = 2\times10^{-7}\left(\ln e^{1/4} + \ln\frac{D}{r_1}\right)$$

$$= 2\times10^{-7}\ln\left(\frac{D}{r_1}\times e^{1/4}\right) = 2\times10^{-7}\ln\left(\frac{D}{e^{-1/4}\times r_1}\right)$$

$$\text{But}, \quad e^{-1/4} = 0.7788$$

$$L_1 = 2\times10^{-7}\ln\left(\frac{D}{0.7788\,r_1}\right)$$

$$\boxed{L_1 = 2\times10^{-7}\ln\left(\frac{D}{r_1'}\right) \qquad henrys\,/\,m}$$

Where , $r_1' = 0.7788$

r_1' is called Geometric mean radius (G.M.R)

- The inductance due to current in conductor 2 is:

$$L_2 = 2\times10^{-7}\ln\left(\frac{D}{r_2'}\right) \qquad henrys\,/\,m$$

Total inductance for the complete circuit :

$$L = L_1 + L_2 = 2\times10^{-7}\left(\ln\frac{D}{r_1'} + \ln\frac{D}{r_2'}\right)$$

$$= 2\times10^{-7}\ln\left(\frac{D^2}{r_1' r_2'}\right)$$

$$= 4\times10^{-7}\,\frac{1}{2}\ln\left(\frac{D^2}{r_1' r_2'}\right) = 4\times10^{-7}\ln\left(\frac{D^2}{r_1' r_2'}\right)^{1/2}$$

$$= 4\times10^{-7}\ln\left(\frac{D}{\sqrt{r_1' r_2'}}\right)$$

If the radius of the two conductor is same , i.e :

$r_1' = r_2' = r'$, therefore $\sqrt{r_1' r_2'} = r'$

$$\therefore \quad L = 4\times10^{-7}\ln\left(\frac{D}{r'}\right) \qquad henrys\ /\ m \quad (H\ /\ m)$$

And ,

$$L = 0.4\ln\left(\frac{D}{r'}\right) \qquad mH\ /\ Km$$

3.3.4- Flux linkages of one conductor in a group:

- Consider the array of N solid cylindrical conductors shown in figure 15. Assume that each conductor n carries current I_n.

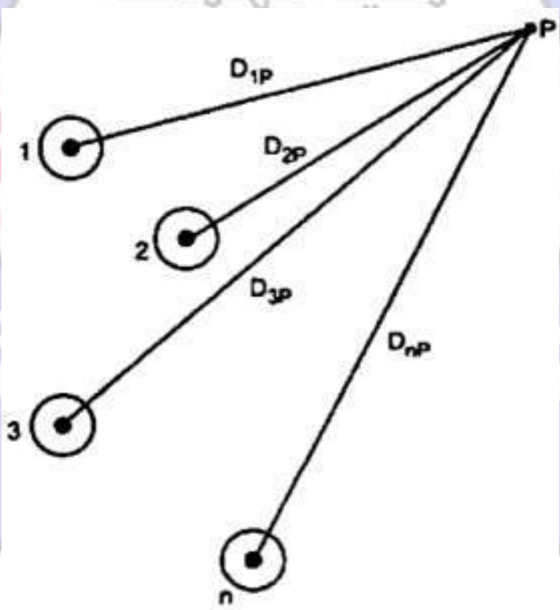


Fig. 15:

- Assume that the sum of the conductor currents is zero.

$$I_1 + I_2 + I_3 + ... + I_n = 0$$

If ψ_{1p1} denotes all the flux linkages of conductor 1 due to its own current I_1, internal and external, upto point P.

$$\psi_{1p1} = \left(\frac{I_1}{2} + 2I_1 \ln\frac{D_1P}{r_1}\right) \times 10^{-7}$$

$$= 2\times10^{-7} I_1 \ln\frac{D_1P}{r_1'} \qquad Wb.T/m$$

where $r_1' = 0.7788\ r_1$

Also, ψ_{1p2} - is flux linkages with conductor 1 due to current in conductor 2 (I_2), but excluding flux beyond point (P) is equal to the flux produced by I_2 between the point P and conductor 1 (i.e. the flux linkages due to I_2 with in limiting distance D_{2p} and D_{12} from conductor 2).

$$\psi_{1p2} = 2\times10^{-7} I_2 \ln\frac{D_{2p}}{D_{12}}$$

Similarly for $\psi_{1p3}, \ldots\ldots \psi_{1pn}$

The flux linkages ψ_{1p} with conductor 1 due to $I_1, I_2, I_3, \ldots I_n$, but excluding flux beyond point P is:

$$\psi_{1p} = 2\times10^{-7}\left(I_1 \ln\frac{D_{1p}}{r_1'} + I_2 \ln\frac{D_{2p}}{D_{12}} + I_3 \ln\frac{D_{3p}}{D_{13}} + \ldots + I_n \ln\frac{D_{np}}{D_{1n}}\right)$$

$$= 2\times10^{-7}\begin{pmatrix} I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}} + \\ I_1 \ln D_{1p} + I_2 \ln D_{2p} + I_3 \ln D_{3p} + ... + I_n \ln D_{np} \end{pmatrix}$$

But , $I_n = -\left(I_1 + I_2 + I_3 + ... + I_{n-1}\right)$

$$\therefore \psi_{1p} = 2\times10^{-7}\begin{pmatrix} I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}} + \\ I_1 \ln D_{1p} + I_2 \ln D_{2p} + I_3 \ln D_{3p} + ... - (I_1 + \\ I_2 + I_3 + ... + I_{n-1}) \ln D_{np} \end{pmatrix}$$

$$\psi_{1p} = 2\times10^{-7}\begin{pmatrix} I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}} + \\ I_1 \ln\frac{D_{1p}}{D_{np}} + I_2 \ln\frac{D_{2p}}{D_{np}} + ... + I_{n-1} \ln\frac{D_{(n-1)p}}{D_{np}} \end{pmatrix}$$

Now if point (P) moves infinity, terms such as :

$\frac{D_{1p}}{D_{np}} ; \frac{D_{2p}}{D_{np}} ..... \frac{D_{(n-1)p}}{D_{np}}$ approach the value 1 and ,

$\ln(1) = 0$

$$\therefore\ \psi_1 = 2\times10^{-7}\left(I_1 \ln\frac{1}{r_1'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}} \right) \quad Wb.T/m$$

Also denoting r_1' as D_{11} we have :

$$\boxed{\psi_1 = 2\times10^{-7}\left(I_1 \ln\frac{1}{D_{11}} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}} \right)}$$

$$Wb.T/m$$

3.3.5- Inductance of composite conductor lines:

- Stranded conductors come under the general classification of composite conductors.
- This means conductors composed of two or more strands electrically in parallel, the strands are identical and share the current equally.
- Figure 16 shows a single-phase line composed of two composite conductors A and B.

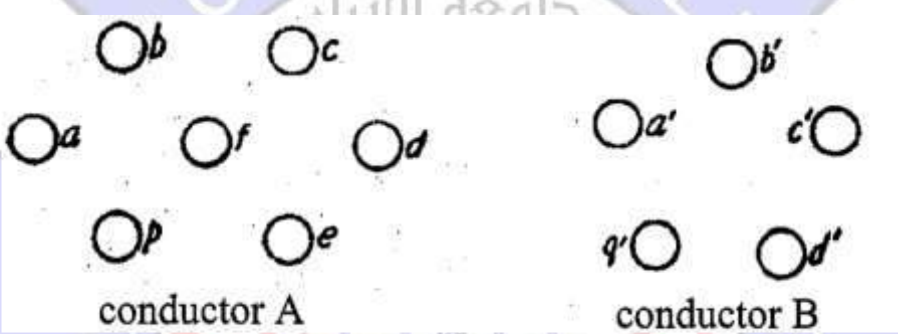


Fig. 16:

- Conductor A is composed of p identical parallel filaments (strands) while conductor B is composed of q' identical strands.
- The current in conductor A is I and the return current in conductor B is $-I$.
- Assume that the current is equally divided among the strands in both conductors A and B.
- Therefore, each strand of conductor A carries the current $\frac{I}{p}$ while each strand of conductor B carries the current $-\frac{I}{q'}$.

- The letter D represents the distances between the strands.
- By applying the following equation; flux linkages for one conductor in a group.

$$\psi_1 = 2\times10^{-7}\left(I_1 \ln\frac{1}{D_{11}} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}}\right)$$

$$Wb.T/m$$

- Based on above equation, the flux linkages of strand a in conductor A can be obtained as;

$$\psi_a = 2\times10^{-7}\left(\frac{I}{P}\ln\frac{1}{D_{aa}} + \frac{I}{P}\ln\frac{1}{D_{ab}} + ... + \frac{I}{P}\ln\frac{1}{D_{ap}}\right) +$$

$$2\times10^{-7}\left(\frac{-I}{q'}\ln\frac{1}{D_{aa'}} + \frac{-I}{q'}\ln\frac{1}{D_{ab'}} + ... + \frac{-I}{q'}\ln\frac{1}{D_{aq'}}\right)$$

$$= 2\times10^{-7} I \begin{bmatrix} \ln\left(\frac{1}{D_{aa}}\right)^{1/p} + \ln\left(\frac{1}{D_{ab}}\right)^{1/p} + ... + \ln\left(\frac{1}{D_{ap}}\right)^{1/p} + \\ \ln(D_{aa'})^{1/q} + \ln(D_{ab'})^{1/q} + ... + \ln(D_{aq'})^{1/q} \end{bmatrix}$$

$$\text{Where} \quad \ln\left(\frac{1}{D_{aa'}}\right)^{-1/q} = \ln\frac{1}{(D_{aa'})^{-1/q}} = \ln(D_{aa'})^{1/q}$$

$$\psi_a = 2\times10^{-7} I \ln\left[\frac{\sqrt[q]{D_{aa'}D_{ab'}D_{ac'}....D_{aq'}}}{\sqrt[p]{D_{aa}D_{ab}D_{ac}....D_{ap}}}\right] \quad Wb.T/m$$

- To find the inductance of strand a in conductor A, the above equation is divided by $\frac{I}{p}$.

$$L_a = \frac{\psi_a}{I/p} = 2p \times 10^{-7} \ln\left[\frac{\sqrt[q]{D_{aa'}D_{ab'}D_{ac'}\ldots.D_{aq'}}}{\sqrt[p]{D_{aa}D_{ab}D_{ac}\ldots.D_{ap}}}\right] \quad H/m$$

- In the same way for above, the inductance of strand b in conductor A can be determined as;

$$L_b = \frac{\psi_b}{I/p} = 2p \times 10^{-7} \ln\left[\frac{\sqrt[q]{D_{ba'}D_{bb'}D_{bc'}\ldots.D_{bq'}}}{\sqrt[p]{D_{ba}D_{bb}D_{bc}\ldots.D_{bp}}}\right] \quad H/m$$

- The average inductance of the strands of conductor A is

$$L_{average} = \frac{L_a + L_b + L_c + \ldots.. + L_p}{p}$$

- Conductor A is composed of p strands electrically in parallel;

1- If , $L_a = L_b = L_c = L_p = L_{average}$

$$\therefore \quad L_A = \frac{L_a}{p}$$

Where L_A , inductance of conductor A .

2 - If $L_a \neq L_b \neq L_c \neq L_p$

$$\therefore \quad L_A = \frac{L_{average}}{P} = \frac{L_a + L_b + L_c + \ldots.. L_p}{P^2}$$

$$L_A = 2\times10^{-7}\ \ln\left[\frac{\sqrt[pq]{(D_{aa'}D_{ab'}D_{ac'}\ldots.D_{aq'})(D_{ba'}D_{bb'}D_{bc'}\ldots.D_{bq'})\ldots}}{\sqrt[p^2]{(D_{aa}D_{ab}D_{ac}\ldots.D_{ap})(D_{ba}D_{bb}D_{bc}\ldots.D_{bp})\ldots}}\ \frac{\ldots.(D_{pa'}D_{pb'}D_{pc'}\ldots.D_{pq'})}{\ldots.(D_{pa}D_{pb}D_{pc}\ldots.D_{pp})}\right] H/m$$

- The numerator is called geometric mean distance (G.M.D) between conductors A and B which denoted D_m. But denominator is called geometric mean radius (G.M.R) and denoted D_s.

$$\therefore\ L_A = 2\times10^{-7}\ \ln\frac{D_m}{D_{SA}} \quad henrys/meter$$

$$L_B = 2\times10^{-7}\ \ln\frac{D_m}{D_{SB}} \quad henrys/meter$$

If conductors A and B are identical i.e. $D_{SA} = D_{SB} = D_S$

$$\therefore\ L = L_A + L_B = 4\times10^{-7}\ \ln\frac{D_m}{D_S} \quad H/m$$

$$= 0.4\times10^{-7}\ln\frac{D_m}{D_S} \quad mH/Km$$

Example 4.2:

One circuit of a single-phase transmission line is composed of three solid 0.25-cm-radius wires. The return circuit is composed of two 0.5-cm radius wires. The arrangement of conductors is shown in Fig. 17. Find the inductance due to the current in each side of the line and the inductance of the complete line in henrys per meter (and in millihenrys per mile).

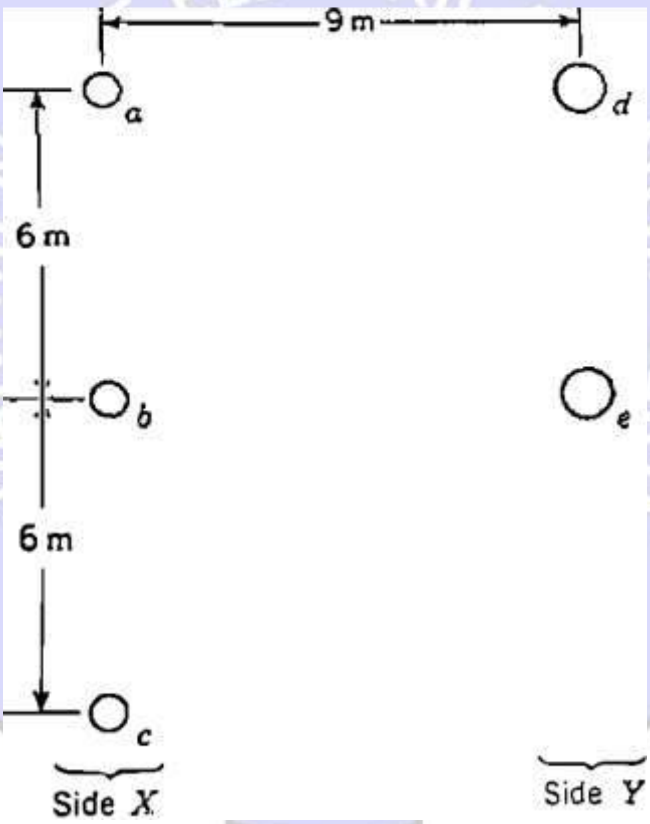


Fig. 17:

Solution. Find the GMD between sides X and Y:

$$D_m = \sqrt[6]{D_{ad} D_{ae} D_{bd} D_{be} D_{cd} D_{ce}}$$

$$D_{ad} = D_{be} = 9\ \text{m}$$

$$D_{ae} = D_{bd} = D_{ce} = \sqrt{6^2 + 9^2} = \sqrt{117}$$

$$D_{cd} = \sqrt{9^2 + 12^2} = 15\ \text{m}$$

$$D_m = \sqrt[6]{9^2 \times 15 \times 117^{3/2}} = 10.743\ \text{m}$$

Then, find the GMR for side X

$$D_s = \sqrt[9]{D_{aa} D_{ab} D_{ac} D_{ba} D_{bb} D_{bc} D_{ca} D_{cb} D_{cc}}$$

$$= \sqrt[9]{(0.25 \times 0.7788 \times 10^{-2})^3 \times 6^4 \times 12^2} = 0.481\ \text{m}$$

and for side Y

$$D_s = \sqrt[4]{(0.5 \times 0.7788 \times 10^{-2})^2 \times 6^2} = 0.153\ \text{m}$$

$$L_X = 2 \times 10^{-7} \ln \frac{10.743}{0.481} = 6.212 \times 10^{-7}\ \text{H/m}$$

$$L_Y = 2 \times 10^{-7} \ln \frac{10.743}{0.153} = 8.503 \times 10^{-7}\ \text{H/m}$$

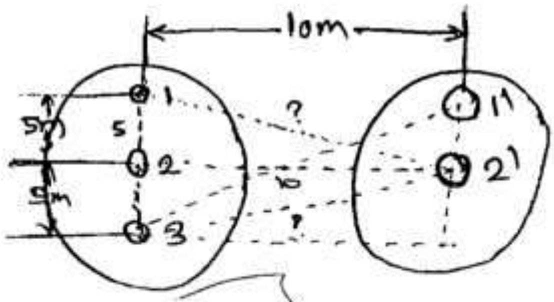
$$L = L_X + L_Y = 14.715 \times 10^{-7}\ \text{H/m}$$

Example:

$$L_x = 2\times10^{-7} \ln \frac{GMD_{xy}}{GMR_x} \; ; \; L_Y = 2\times10^{-7} \ln \frac{GMD_{Yx}}{GMR_Y}$$

Ex) We have composite conductor, conductor X has 3 strands (1,2,3)

Find Lx, LY, and L = ?



Cond. X: $r_x = 0.1cm$ of each strand, $n = 3$

Cond. Y: $r_y = 0.5cm$, $M = 2$

Sol.

$GMD_{xy} = GMD_{yx}$

$$= \left[D_{11'} \cdot D_{12'} \cdot D_{21'} \cdot D_{22'} \cdot D_{31'} \cdot D_{32'}\right]^{\frac{1}{mn}(2)(3)}$$

$\rightarrow D_{11'} = D_{22'} = 10m$

$\rightarrow D_{12'} = D_{21'} = D_{32'} = \sqrt{(10)^2 + (5)^2} = 11.18\,m$

$\rightarrow D_{31'} = \sqrt{(10)^2 + (10)^2} = 14.14\,m$

$$\therefore GMD_{xy} = GMD_{yx} = \left[(10)^2 \cdot (11.18)^3 \cdot (14.14)\right]^{1/6\,(m\times n)}$$

$$= 11.2\,m$$

$GMR_x = \left[D_{12} \cdot D_{13} \cdot D_{21} \cdot D_{23} \cdot D_{31} \cdot D_{32}\right]^{\frac{1}{n^2}}$ ✗

$\rightarrow D_{12} = D_{21} = D_{23} = D_{32} = 5m$

$$GMR_x = \left[D_{11} \cdot D_{12} \cdot D_{13} \cdot D_{22} \cdot D_{21} \cdot D_{23} \cdot D_{33} \cdot D_{31} \cdot D_{32}\right]^{\frac{1}{(3)^2}}$$

(self distance: D_{11})

where; $D_{11} = r_1' = 0.7788r$

$\rightarrow D_{13} = D_{31} = 10\,m$

$\rightarrow D_{11} = D_{22} = D_{33} = 0.7788\,(0.1\times10^{-2})$ (r_x)

$= 0.7788 \times 10^{-3}$

Note:
GMD → mn values
GMR_x → n^2 values
GMR_y → m^2 values

$$\therefore GMR_x = \left[(0.7788\times10^{-3})^3 \cdot (5)^4 \cdot (10)^2\right]^{1/9}$$

$$= 0.31\, m$$

$$GMR_y = \left[D_{1'1'} \cdot D_{1'2'} \cdot D_{2'2'} \cdot D_{2'1'}\right]^{\frac{1}{m^2}}$$

$$\hookrightarrow D_{1'2'} = D_{2'1'} = 5\, m$$

$$\hookrightarrow D_{1'1'} = D_{2'2'} = 0.7788(\overbrace{0.5\times10^{-2}}^{r_y})$$

$$= 3.894\times10^{-3}$$

$$\therefore GMR_y = \left[(5)^2 \cdot (3.894\times10^{-3})^2\right]^{\frac{1}{4}}$$

$$= 0.139 \simeq 0.14\, m$$

$$\therefore L_x = 2\times10^{-7} \ln \frac{GMD}{GMR_x}$$

$$L_x = 2\times10^{-7} \ln\left(\frac{11.2}{0.31}\right)$$

$$L_x = 7.17\times10^{-7}\ H/m$$

$$\therefore L_y = 2\times10^{-7} \ln \frac{GMD}{GMR_y}$$

$$= 2\times10^{-7} \ln\left(\frac{11.2}{0.14}\right)$$

$$L_y = 8.76\times10^{-7}\ (H/m)$$

$$\therefore L = L_x + L_y = 7.17\times10^{-7} + 8.76\times10^{-7}$$

$$L = 15.93\times10^{-7}\ (H/m)$$

3.3.6- Inductance of three-phase lines with symmetrical spacing:

- By applying the following equation; flux linkages for one conductor in a group.

$$\psi_1 = 2\times10^{-7}\left(I_1 \ln\frac{1}{D_{11}} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + ... + I_n \ln\frac{1}{D_{1n}}\right)$$

$Wb.T/m$

- Figure 18 shows the conductors of a three-phase transmission line (a, b, and c) with symmetrical spacing (D). Radius of each conductor is r.

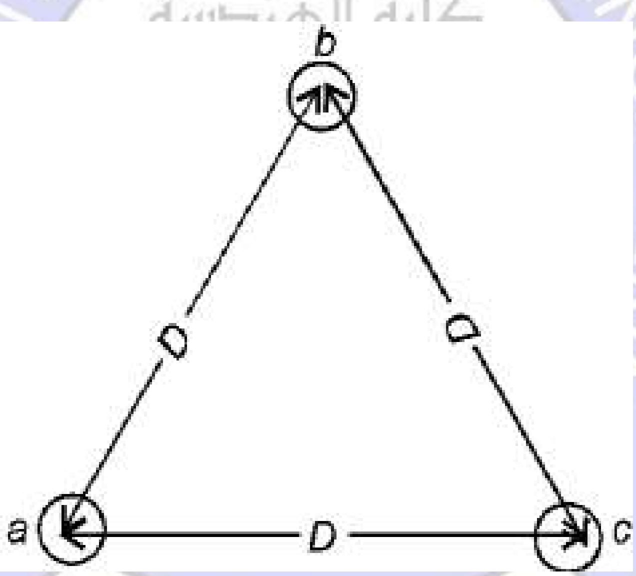


Fig. 18:

- Based on above equation, the total flux linkage of conductor in phase a due to its own current (I_a) and also due to the mutual inductance effects of I_b and I_c is given by:

$$\psi_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D}\right) \qquad Wb.T/m$$

- Assume balanced three phase currents;

$$I_a + I_b + I_c = 0$$

$$I_a = -(I_b + I_c)$$

$$\therefore \quad \psi_a = 2\times10^{-7} I_a \ln\frac{D}{r'} \qquad WbT/m$$

$$r' = 0.7788 \times r$$

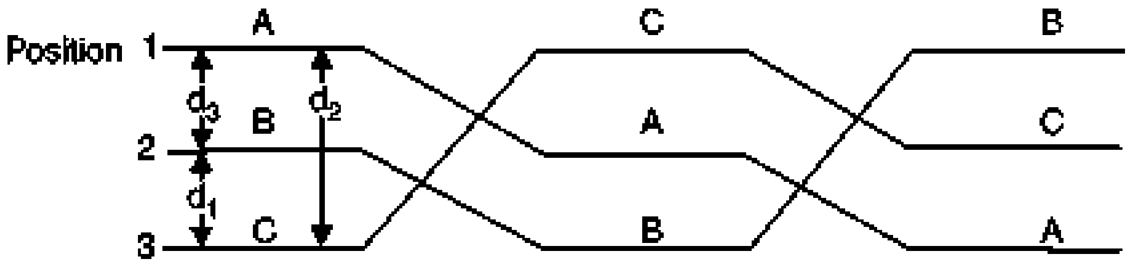
L_a - Inductance per phase of 3-phase line .

IF conductor is stranded , $r' \Rightarrow D_S$ *, therefore:*

$$L_a = 2\times10^{-7} \ln\frac{D}{D_S} \qquad H/m$$

3.3.7- Inductance of three-phase lines with unsymmetrical spacing:

- When the distances among conductors for 3-phase lines are not equal (unsymmetrical). Then, the flux linkages and inductance of each phase are not the same.
- A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced.
- Therefore, the voltage at the receiving end will not be the same for all phases.
- To make the voltage drops in all conductors are equal, the positions of the conductors will be exchanged at regular intervals along the line. This is knowns as transposition as shown below.



- The phase conductors are designated as A, B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.
- However, the transposed conductors are spaced unsymmetrical as shown in figure 19.

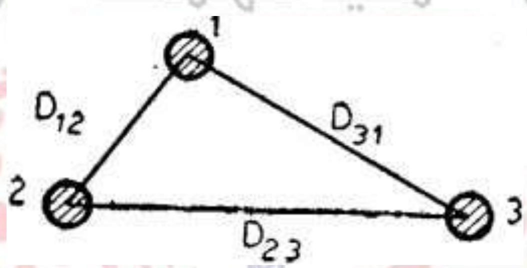


Fig. 19:

- To find the inductance of three phase line;

Assume $\psi_{a1}, \psi_{a2}, \psi_{a3}$ - are the flux linkages of conductor (a) in positions 1 , 2 , and 3 respectively and by using eq. (15) :

$$\psi_{a1} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{31}}\right)$$

$$\psi_{a2} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right)$$

$$\psi_{a3} = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{31}} + I_c \ln\frac{1}{D_{23}}\right)$$

Then average flux linkages of conductor (a) :

$$\psi_a = \frac{\psi_{a1} + \psi_{a2} + \psi_{a3}}{3}$$

$$\psi_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + \frac{1}{3} I_b \ln\frac{1}{D_{12}D_{23}D_{31}} + \frac{1}{3} I_c \ln\frac{1}{D_{12}D_{23}D_{31}} \right)$$

$$= 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{\sqrt[3]{D_{12}D_{23}D_{31}}} + I_c \ln\frac{1}{\sqrt[3]{D_{12}D_{23}D_{31}}} \right)$$

But , $-I_a = (I_b + I_c)$

$$\therefore \quad \psi_a = 2\times10^{-7}\left(I_a \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'} \right) \quad WbT/m$$

$$L_a = \frac{\psi_a}{I_a} = 2\times10^{-7}\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'} \right) \quad H/m$$

$$L_a = 2\times10^{-7} \ln\frac{D_{eq}}{D_s} \quad H/m$$

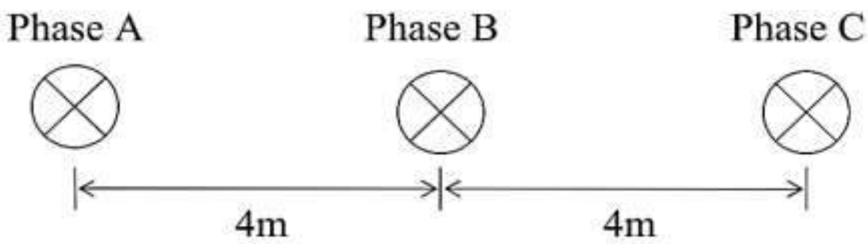
Where , $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$; $D_s = r'$

$L_a = L_b = L_c$ (because of the circuit is balance)

$$\text{Average phase inductance} = \frac{1}{3}\left(L_a + L_b + L_c\right)$$
$$= \frac{3L_a}{3} = L_a$$
$$= 2\times10^{-7}\ln\frac{D_{eq}}{D_s} \qquad H/m$$

Example:

A 3-phase , 50Hz , 132 KV overhead transmission lines has conductor diameter of 4 cm each , are arranged in a horizontal plane as shown in fig . supplies a balanced load , assume the line is completely transposed . Find the inductance per Km per phase .



Solution :

$$D_m = D_{eq} = \sqrt[3]{D_{AB}D_{BC}D_{CA}}$$
$$= \sqrt[3]{4\times4\times8} = 5.04\ m$$
$$D_S = r' = 0.7788\times r$$
$$= 0.7788\times\frac{4}{2} = 1.5576\ cm$$
$$L = 2\times10^{-7}\ln\frac{D_m}{D_s} = 2\times10^{-7}\ln\frac{5.04}{1.5576\times10^{-2}}$$

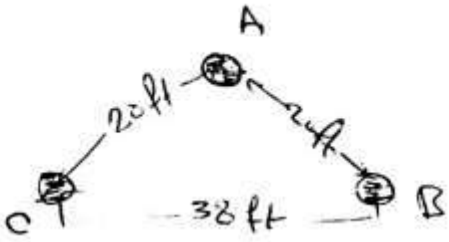
$$= 11.53\times10^{-7} \quad H/m$$
$$= 1.153 \quad mH/Km$$

Example:

A single circuit 3-phase is operated at 6.. is arranged as follows. The conductors are ACSR Drake, Find XL per phase (Ω/mile)

$D_s = 0.0373$ ft.

$L = 2\times10^{-7} \ln\left(\frac{D_m}{D_s}\right)$



$D_m = \sqrt[3]{D_{AB} \cdot A_{BC} \cdot A_{CA}}$

$= \sqrt[3]{20 \times 38 \times 20} = 24.77$ ft

$L = 2\times10^{-7} \ln\left(\frac{24.77}{0.0373}\right) \Rightarrow L = 12.997\times10^{-7}$ H/m

$X_L = 2\pi f L \Rightarrow 2\pi \times 60 \times 12.997\times10^{-7}$ Ω/m

$X_L = 0.788$ Ω/mile per phase

3.3.8- Inductance of bundled conductors:

- In extra high voltage (EHV) transmission line, bundle conductors (use more than one conductor per phase) are used to reduce the effect of corona.
- Figure 20 below shows common EHV bundles consisting of two, three, or four conductors.

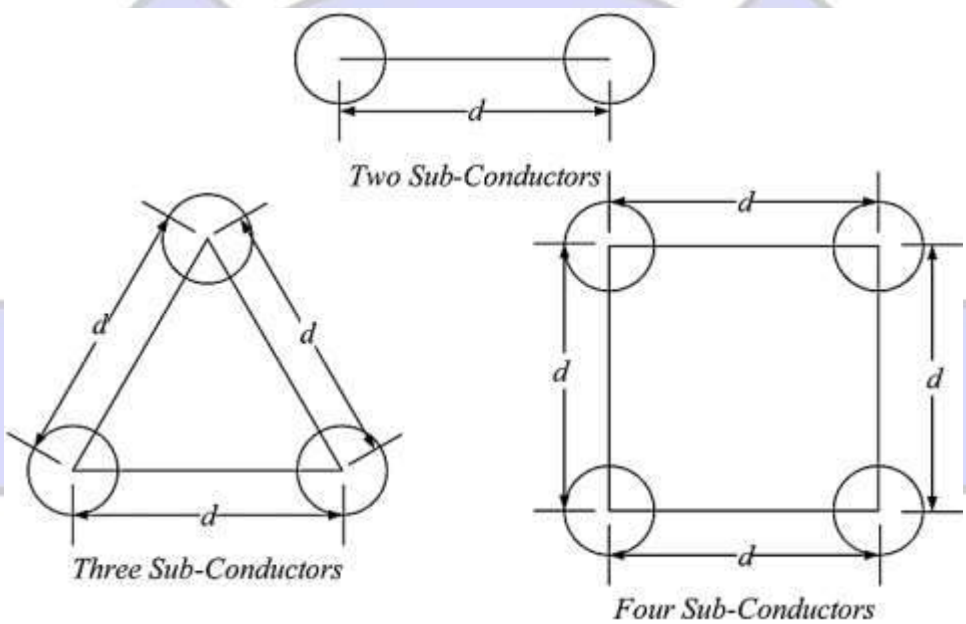


Fig. 20:

- Where the small letter of d is the bundle spacing.
- To calculate inductance of bundle conductor, the equation below is considered;

$$L_a = \frac{\psi_a}{I_a} = 2\times10^{-7}\left(\ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r'}\right) \quad H/m$$

$$L_a = 2\times10^{-7}\ln\frac{D_{eq}}{D_s} \quad H/m$$

Where , $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$; $D_s = r'$

- D_s in above equation is replaced by the GMR of the bundle, which is denoted by D_{SL}^b;

1- For two – conductor bundle :

$$D_{SL}^b = \sqrt[2^2]{(D_S\times d)^2} = \sqrt[4]{(D_S\times d)^2} = \sqrt{(D_S\times d)}$$

2- For three – conductor bundle :

$$D_{SL}^b = \sqrt[3^2]{(D_S\times d\times d)^3} = \sqrt[9]{(D_S\times d^2)^3} = \sqrt[3]{D_S\times d^2}$$

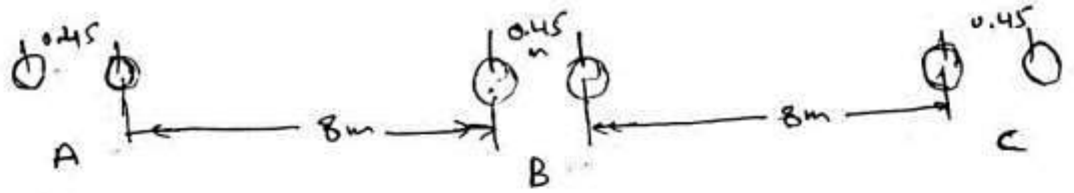
3- For four – conductor bundle :

$$D_{SL}^b = \sqrt[4^2]{(D_S\times d\times d\times d\sqrt{2})^4} = \sqrt[16]{(D_S\times d^3\sqrt{2})^4} = \sqrt[4]{D_S\times d^3\sqrt{2}}$$
$$= 1.09\times\sqrt[4]{D_S\times d^3}$$

IF conductor is solid , $D_S \Rightarrow r'$ *in above three condition*

Example:

Inductance 3-φ single double bundle circuit, Each conductor of bundled conductor line shown below. is ACSR pheasant, find XL per phase in (Ω/mile) and (Ω/km), when Ds = 0.0466 ft and f = 60 Hz.



$$L = 2\times10^{-7}\ln\left(\frac{D_M}{D_s^b}\right)$$

$$D_M = \sqrt[3]{D_{AB}\cdot D_{BC}\cdot D_{CA}} = \sqrt[3]{8 * 8 * 16} = 10.08m$$

$$D_s^b = \sqrt{D_s\cdot d} = \sqrt{(0.0466\times0.3048)\times0.45} = 0.08\,m$$

التحويل من ft ← m

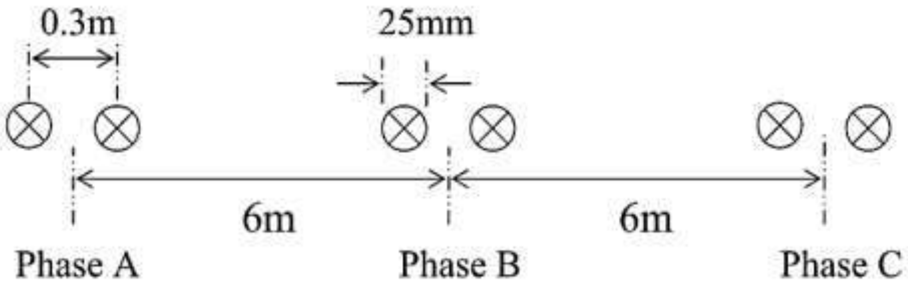
$$\therefore L = 2\times10^{-7}\ln\left(\frac{10.08}{0.08}\right) \Rightarrow L = 9.673\times10^{-7}\ H/m$$

$$X_L = 2\pi fL \Rightarrow 2\pi\times60\times9.673\times10^{-7}\ \Omega/m = 3.647\times10^{-4}\ \Omega/m$$

$$X_L = 0.365\ \Omega/km \ \text{OR}\ X_L = 0.587\ \Omega/mile$$

Example:

A 3-phase , 50Hz , 400 KV overhead transmission lines are arranged in a horizontal plane , each phase has two – strand bundle conductors , the diameter of each strand is 25mm , as shown in the fig. below. Find the inductance per Km per phase .



Solution :

$$D_m = D_{eq} = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$
$$= \sqrt[3]{6 \times 6 \times 12} = 7.56\ m$$
$$D_S = r' = 0.7788 \times r$$
$$= 0.7788 \times \frac{25}{2} = 9.735\ mm$$
$$D_{SL}^b = \sqrt{D_s \times d}$$
$$= \sqrt{9.735 \times 10^{-3} \times 0.3} = 0.054\ m$$
$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s} = 2 \times 10^{-7} \ln \frac{7.56}{0.054}$$
$$= 9.88 \times 10^{-7}\ H/m = 0.988\ \ mH/Km$$

3.3.9- Inductance of parallel-circuit three-phase lines:

- A three-phase double circuit line consists of two parallel conductors for each phase.
- To enhance the maximum transmission capability, it is desirable to have a configuration which results in minimum inductance per phase.
- This is possible if mutual GMD (D_m) is low and self GMD (D_s or $GMR = 0.7788\, r$) is high as shown in figure 21 below.

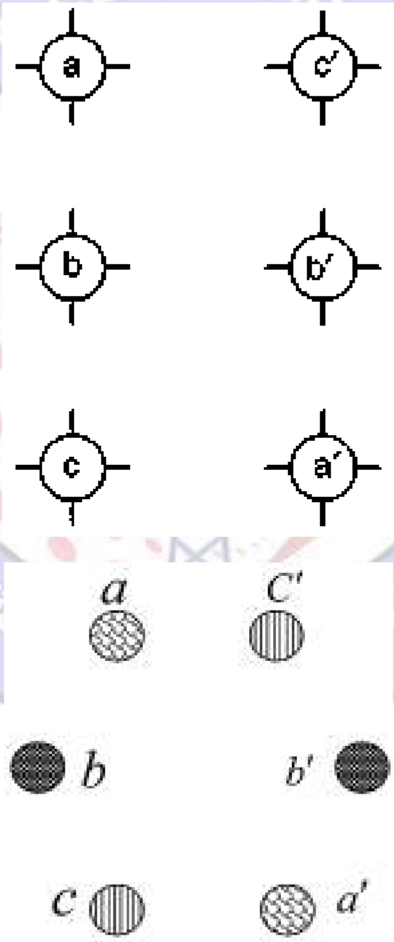


Fig. 21:

- To calculate the inductance, it is necessary to determine mutual GMD (D_m) and self GMD (D_s). Suppose the radius of each conductor is r.

Self-GMD of conductor = 0·7788 r

Self-GMD of combination aa' is

$$D_{s1} = (**D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$$

Self-GMD of combination bb' is

$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$$

Self-GMD of combination cc' is

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4}$$

Equivalent self-GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$

The value of D_s is the same for all the phases as each conductor has the same radius.

Mutual-GMD between phases A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual-GMD between phases B and C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual-GMD between phases C and A is

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

Equivalent mutual-GMD, $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

- The self-GMD (GMR) of a conductor depends upon the size and shape of the conductor and is independent of the spacing between the conductors.
- The mutual GMD depends only upon the spacing and is independent of the exact size, shape and orientation of the conductor.

- The inductance formulas can be expressed in terms of GMD as listed below.

(i) Single phase line

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and D_m = Spacing between conductors = d

(ii) Single circuit 3-ϕ line

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and $D_m = (d_1\, d_2\, d_3)^{1/3}$

(iii) Double circuit 3-ϕ line

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

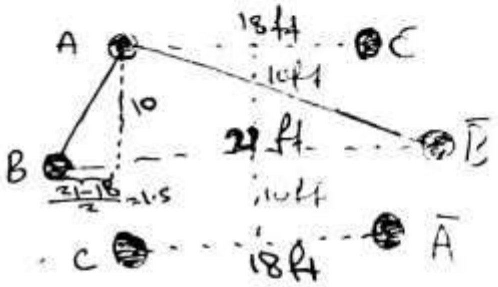
where $D_s = (D_{s1}\, D_{s2}\, D_{s3})^{1/3}$ and $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

Example:

Ex1:- A 3-ph double circuit line is composed of ACSR Ostrich, Find L and XL per phase in (Ω/mile) and (Ω/km), $D_s = 0.0229$ ft .

$X_L = \omega . L = 2\pi f . L$

$L = 2\times10^{-7} \ln\left(\frac{D_m^P}{D_s^P}\right)$



$$D_m^p = \sqrt[3]{D_{ab}^p \cdot D_{ac}^p \cdot D_{bc}^p}$$

$$D_{ab}^p = \sqrt[4]{D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'}}$$

$$D_{ac}^p = \sqrt[4]{D_{ac} \cdot D_{ac'} \cdot D_{a'c} \cdot D_{a'c'}}$$

$$D_{bc}^p = \sqrt[4]{D_{bc} \cdot D_{bc'} \cdot D_{b'c} \cdot D_{b'c'}}$$

$$D_s^p = \sqrt[3]{GMR_a \cdot GMR_b \cdot GMR_c}$$

a لموصلين self distance a, a' — للموصلين b, b' — للموصلين c, c'

$$GMR_a = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}}$$

($D_{aa} = D_s$, $D_{a'a'} = D_s$)

$$= \sqrt[4]{(D_s \cdot D_{aa'})^2}$$

$$GMR_b = \sqrt[4]{D_{bb} \cdot D_{bb'} \cdot D_{b'b'} \cdot D_{b'b}}$$

($D_{bb} = D_s$, $D_{b'b'} = D_s$)

$$= \sqrt[4]{(D_s \cdot D_{bb'})^2}$$

$$GMR_c = \sqrt[4]{D_{cc} \cdot D_{cc'} \cdot D_{c'c'} \cdot D_{c'c}}$$

$$= \sqrt[4]{(D_s \cdot D_{cc'})^2}$$

$$D_{ab} = \sqrt{(10)^2 + (1.5)^2} = 10.11 \text{ ft} = D_{a'b'}$$

$$D_{ab'} = \sqrt{(19.5)^2 + (10)^2} = D_{a'b} = 21.91 \text{ ft}$$

$$\therefore D_{ab}^p = \sqrt[4]{(10.11)^2 \cdot (21.91)^2} = 14.88 \text{ ft} = D_{bc}^p$$

$D_{ac}^p = ?$

$D_{ac} = 20 \text{ ft} = D_{a'c'}$

$D_{ac'} = D_{a'c} = 18 \text{ ft}$.

$$D_{ac}^p = \sqrt[4]{(20)^2 \cdot (18)^2} = 18.97 \text{ ft}$$

$$\therefore D_m^p = \sqrt[3]{(14.88)(18.97)(14.88)} = 16.14 \text{ ft}.$$

$GMR_a =$

$$D_{aa'} = \sqrt{(20)^2 + (18)^2} = 26.9 \text{ ft}.$$

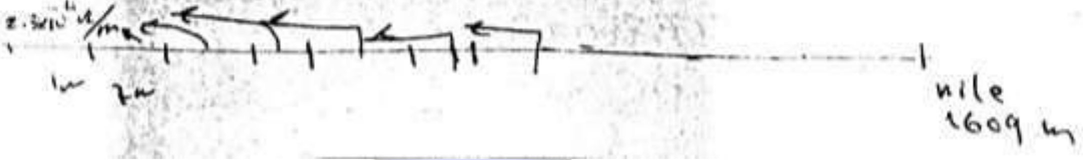
$$L = 6.13 \times 10^{-7} \text{ H/m}$$

$$X_L = 2\pi f L = 2\pi \times 60 \times 6.13\times10^{-7}\ \Omega/m = 2.3\times10^{-4}\ \Omega/m$$

Inductive Reactance

$$\Rightarrow 0.372\ \Omega/mile.$$

1 mile = 1609 m



- Note, if there is bundle with three-phase double circuit.

ما الذي متغير اذا تم اضافة Bundle مثلا
فيز مثلاً Phase A
الذي يتغير هو حساب الـ self distance لكل فيز

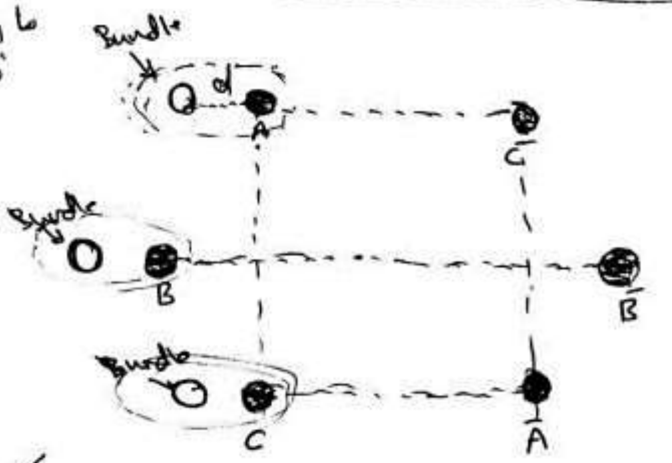
مثال

$$GMR_a = \sqrt{D_s \cdot D_{aa'}}$$

$$= \sqrt{D_s^b \cdot D_{aa'}}$$

$$D_s^b = \sqrt{D_s \cdot d}$$

وهكذا

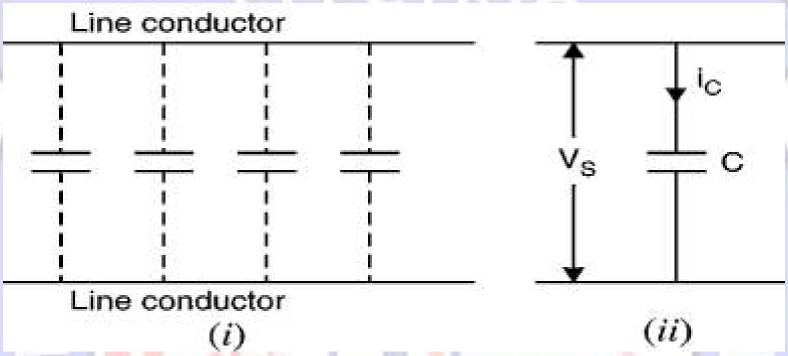


3.4- Capacitance of Transmission Line

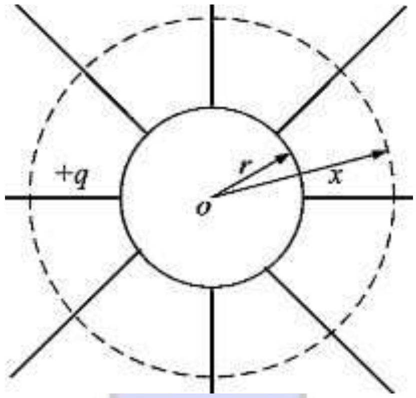
- When alternating voltage is applied to the transmission line conductors, the line capacitance draws a leading current.

$$C = \frac{q}{V} \qquad F/m$$

- The flow of charge is a current, and this current is called the charging current of the line.



- The charging current will flow in the line even when it is open-circuited i.e., supplying no load.
- The line capacitance is proportional to the length of the transmission line and may be neglected for a line less than 80 km of length.
- Figure 22 shows a solid cylindrical conductor has a uniform charge (assumed positive charge) throughout its length and is isolated from other charges.
- So that the charge is uniformly distributed around its periphery, the electric flux lines are radial.



- From Gauss theorem, it is known that the electric field density at distance X is;

$$D = \frac{q}{2\pi x} \qquad coulomb/m^2$$

Where q - charge on the conductor coulomb /m
x - distance , from the conductor to the point where the electric flux density is computed.
D - the electric flux density .

$$\text{The electric field intensity} = \frac{\text{The electric flux density}}{\text{The permittivity of the medium}}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi x \varepsilon} \qquad V/m$$

Where ε - actual permittivity of material .

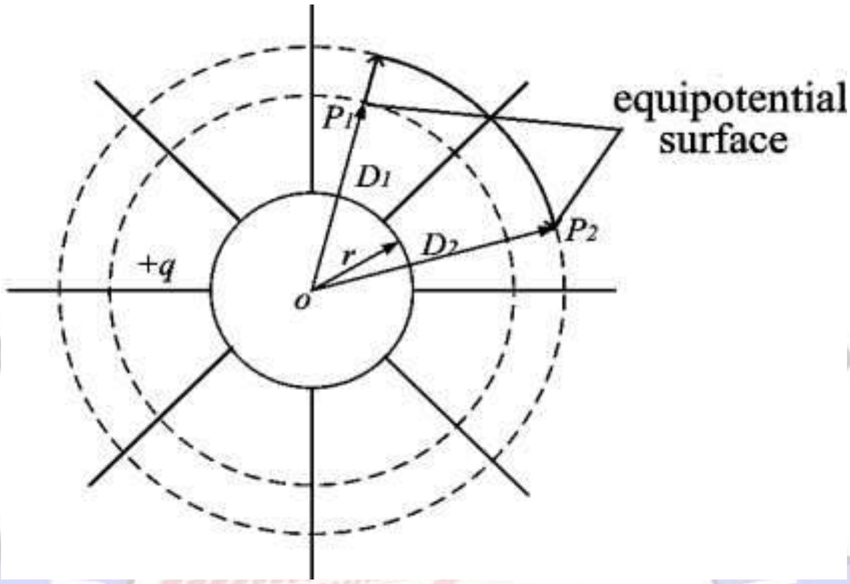
$$\varepsilon_r = \varepsilon / \varepsilon_0$$

ε_r - relative permittivity ; ε_0 - permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12} = \frac{1}{4\pi \times 9 \times 10^9} \qquad F/m$$

- The amount of capacitance between conductors is a function of conductor radius, spacing and height above the ground.

- Since the equipotential surface is orthogonal to electric flux lines, the equipotential surfaces are concentric cylinders surrounding the conductors.



- By definition, the capacitance between the conductors is the ratio of charge on the conductors to the potential difference between them.
- The work done to move a unit charge of one coulomb from P_2 to P_1 is numerically equal to the potential difference.

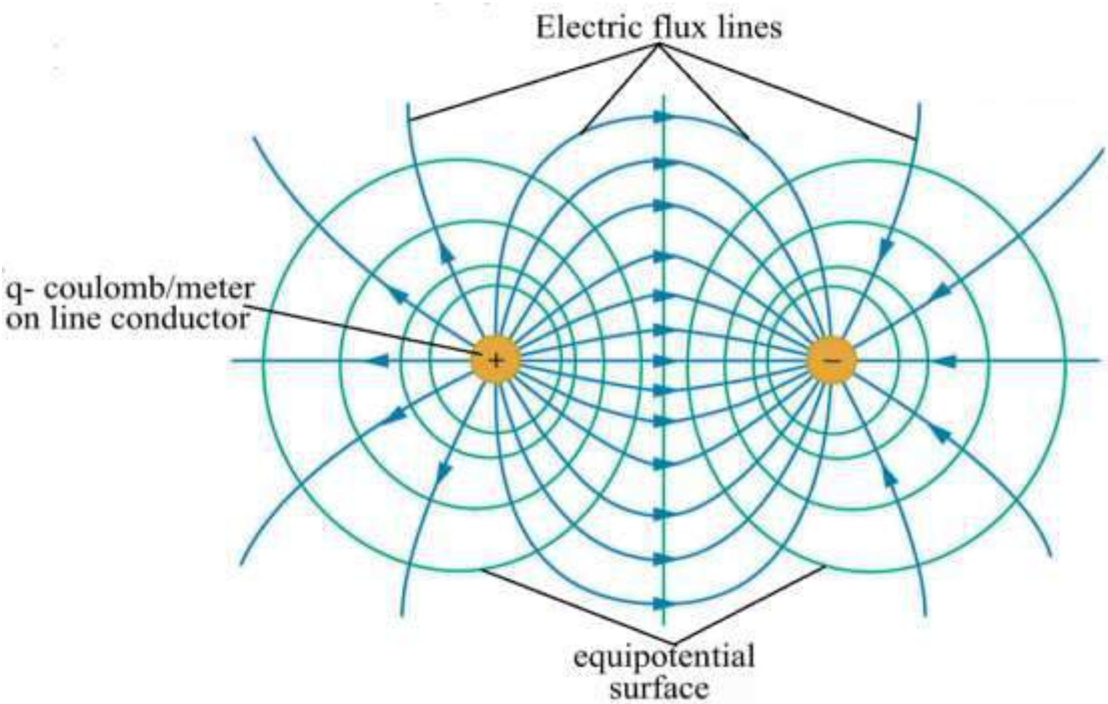
Therefore , instantaneous voltage drop between P_1 and P_2 is :

$$v_{12} = \int_{D_1}^{D_2} E\, dx$$

$$= \int_{D_1}^{D_2} \frac{q}{2\pi x \varepsilon} dx = \frac{q}{2\pi \varepsilon} \ln \frac{D_2}{D_1} \quad volts$$

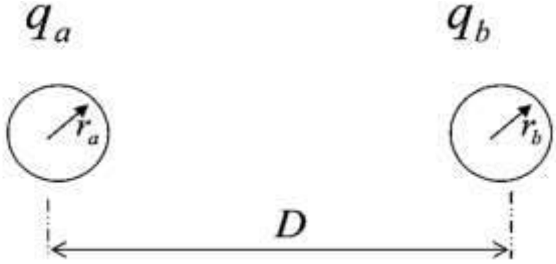
3.4.1- Capacitance of a two-wire line

- Electric charge is a source of electric fields. Electric field lines originate from positive charges and terminate at negative charges as shown below.



Electric field between two-line conductors

- Consider a single-phase overhead transmission line consisting of two parallel conductors A and B spaced D meters apart in air. Let us consider the two conductors A and B having charges q_a and q_b as shown below.



- The potential difference between two conductors (V_{ab}) is given by;

$$v_{ab} \ due \ \ to \ q_a = \frac{q_a}{2\pi\varepsilon} \ln \frac{D}{r_a} \quad volt$$

And voltage due to q_b, *calculated by :*

$$v_{ba} = \frac{q_b}{2\pi\varepsilon} \ln \frac{D}{r_b} \quad volt$$

$$v_{ab} = -v_{ba}$$

$$\therefore \quad v_{ab} = -\frac{q_b}{2\pi\varepsilon} \ln \frac{D}{r_b} = \frac{q_b}{2\pi\varepsilon} \ln \left(\frac{D}{r_b} \right)^{-1}$$

$$\therefore \quad v_{ab} \ due \ \ to \ q_b = \frac{q_b}{2\pi\varepsilon} \ln \frac{r_b}{D} \quad volt$$

By the principle of superposition the voltage drop from conductor (a) to conductor (b) due to charges on both conductors is the sum of the voltage drop caused by each charge alone .

$$\therefore \quad v_{ab} = \frac{1}{2\pi\varepsilon} \left(q_a \ln \frac{D}{r_a} + q_b \ln \frac{r_b}{D} \right) volt$$

For two-wire line $q_a = - \ q_b$, so that :

$$v_{ab} = \frac{q_a}{2\pi\varepsilon} \ln \frac{D^2}{r_a r_b} = \frac{q_a}{2\pi\varepsilon} \ln \left(\frac{D}{\sqrt{r_a r_b}} \right)^2$$

$$= \frac{q_a}{\pi\varepsilon} \ln\left(\frac{D}{\sqrt{r_a r_b}}\right) \quad volts$$

$$C_{ab} = \frac{q_a}{v_{ab}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_a r_b}}\right)} \quad Farads \quad / meter$$

If $r_a = r_b = r$ (radius of two conductors are same)

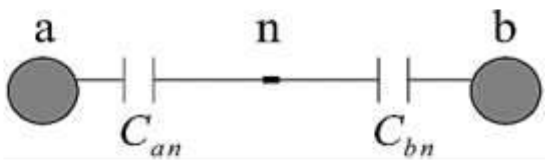
$$\therefore \quad C_{ab} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F / m$$

$$If, \varepsilon_r = 1, and\ \varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \quad F/m$$

$$\therefore \quad C_{ab} = \frac{10^{-9}}{36 \ln\left(\frac{D}{r}\right)} \quad F / m$$

$$= \frac{1}{36 \ln \frac{D}{r}} \quad \mu F / Km$$

- Capacitance to neutral, if we want to find the capacitance between one of the conductors and a neutral point as shown below.



C_{an} - capacitance between conductor (a) and neutral

C_{bn} - capacitance between conductor (b) and neutral

$$C_{an} = C_{bn} = C_n = 2\ C_{ab}$$

$$\therefore \quad C_n = 2 \times \frac{1}{36\ \ln \frac{D}{r}} = \frac{1}{18\ \ln \frac{D}{r}}$$

$$\therefore \quad C_n = \frac{0.0555}{\ln \frac{D}{r}} \quad \mu F\ /\ Km$$

Capacitive reactance between one conductor and neutral is :

$$X_c = \frac{1}{2\pi f\ C_n}$$

Example:

A single-phase transmission line has two parallel conductors 3 metres apart, radius of each conductor being 1 cm. Calculate the capacitance of the line per km. Given that $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ F/m.

Solution.

Conductor radius, $r = 1$ cm

Spacing of conductors, $d = 3$ m $= 300$ cm

Capacitance of the line $= \dfrac{\pi\,\varepsilon_0}{\log_e d/r}$ F/m $= \dfrac{\pi \times 8{\cdot}854 \times 10^{-12}}{\log_e 300/1}$ F/m

$= 0{\cdot}4875 \times 10^{-11}$ F/m $= 0{\cdot}4875 \times 10^{-8}$ F/km

$=$ **$0{\cdot}4875 \times 10^{-2}$ μF/km**

3.4.2- Capacitance of three-phase line with symmetrical spacing

- Figure 23 shows the three conductors A, B and C of the 3-phase overhead transmission line having charges q_a, q_b and q_c per meter length respectively.

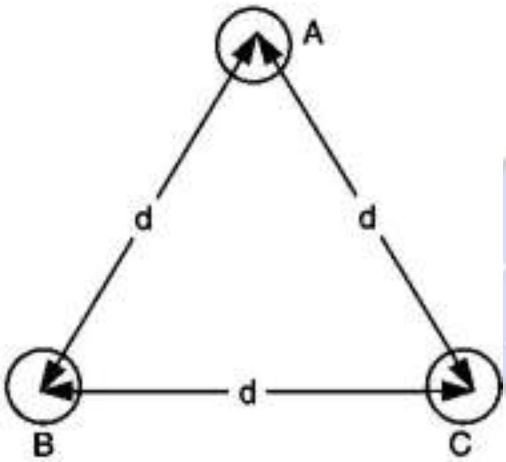


Fig. 23:

- Let the distance d among the conductors is the same for all. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line.
- The potential differences of V_{ab} and V_{ac} are given by;

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{D}{D}\right)$$

$$v_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D}\right)$$

$$v_{ab} + v_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{r} + (q_b + q_c)\ln\frac{r}{D}\right) \quad Volts$$

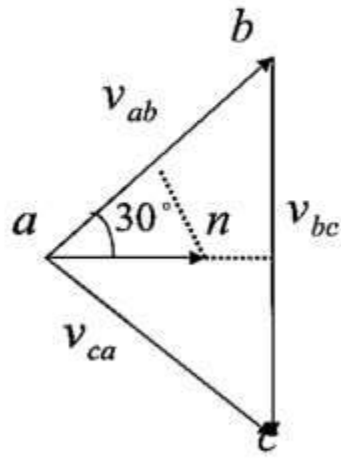
But $q_b + q_c = -q_a$

$$\therefore\ v_{ab} + v_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln\frac{D}{r} - q_a \ln\frac{r}{D}\right)$$

$$= \frac{1}{2\pi\varepsilon}\left(q_a \ln\left(\frac{D}{r}\right)^2 - q_a \ln\frac{r}{D}\right) = \frac{1}{2\pi\varepsilon}\left(q_a \ln\left(\frac{D}{r}\right)^3\right)$$

$$= \frac{3q_a}{2\pi\varepsilon}\ln\frac{D}{r} \quad volts$$

Phasor diagram of voltages v_{ab}, v_{bc} *and* v_{ca} :



$$\vec{v}_{ab} = |v_{ab}|\angle 30^\circ$$

$$\frac{|v_{ab}|}{2} = V_{an}\cos 30^\circ$$

$$= \frac{\sqrt{3}}{2}V_{an}$$

$$\therefore\quad |v_{ab}| = \sqrt{3}\,V_{an}$$

$$\vec{v}_{ab} = \sqrt{3}V_{an}\angle 30^\circ$$

$$\vec{v}_{ac} = -\vec{v}_{ca} = \sqrt{3}V_{an}\angle -30^\circ$$

$$\therefore\quad \vec{v}_{ab} + \vec{v}_{ac} = \sqrt{3}V_{an}\angle 30^\circ + \sqrt{3}V_{an}\angle -30^\circ$$

$$= \sqrt{3}V_{an} \times 2 \times \cos 30^\circ = \sqrt{3}V_{an} \times \sqrt{3}$$

$$= 3V_{an}$$

$$\therefore \quad 3V_{an} = \frac{3q_a}{2\pi\varepsilon} \ln \frac{D}{r}$$

$$V_{an} = \frac{q_a}{2\pi\varepsilon} \ln \frac{D}{r} \quad volt$$

$$C_n = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon}{\ln \frac{D}{r}} \quad F/m \quad \text{to neutral}$$

$$For \ \varepsilon_r = 1$$

$$\therefore \quad C_n = \frac{1}{18 \ln \frac{D}{r}} = \frac{0.0555}{\ln \frac{D}{r}} \quad \mu F/Km$$

Example:

A 3-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2 m side. Calculate the capacitance of each line conductor per km. Given that diameter of each conductor is 1·25 cm.

Solution.

Conductor radius, $r = 1{\cdot}25/2 = 0{\cdot}625$ cm

Spacing of conductors, $d = 2$ m $= 200$ cm

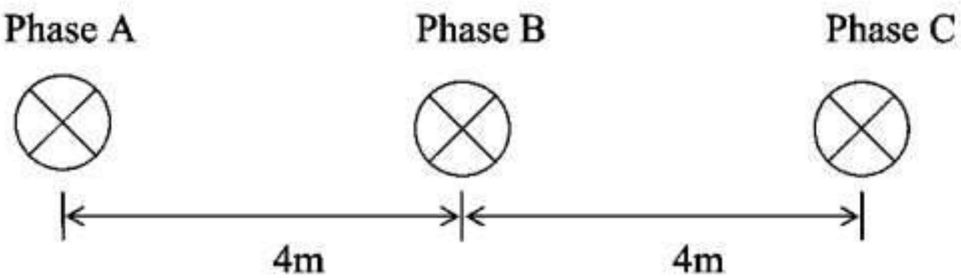
Capacitance of each line conductor

$$= \frac{2\pi\varepsilon_0}{\log_e d/r} \text{ F/m} = \frac{2\pi \times 8{\cdot}854 \times 10^{-12}}{\log_e 200/0{\cdot}625} \text{ F/m}$$

$$= 0{\cdot}0096 \times 10^{-9} \text{ F/m} = 0{\cdot}0096 \times 10^{-6} \text{ F/km} = \mathbf{0{\cdot}0096\ \mu F/km}$$

Example:

A 3-phase , 50Hz , 132 KV overhead transmission lines has conductor diameter of 4 cm each , are arranged in a horizontal plane as shown in fig . supplies a balanced load , assume the line is completely transposed . Find the capacitance to neutral per phase per Km.



Solution :

$$D_m = D_{eq} = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

$$= \sqrt[3]{4 \times 4 \times 8} = 5.04\ m$$

$$D_S = r = 2\ cm$$

$$C = \frac{0.0555}{\ln \frac{D_m}{D_S}} = \frac{0.0555}{\ln \frac{5.04}{2 \times 10^{-2}}} = \frac{0.0555}{5.529} = 0.01\ \mu F / Km$$

3.4.3- Capacitance of three-phase line with unsymmetrical spacing

- Figure 24 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions i.e. QA + QB + QC = 0

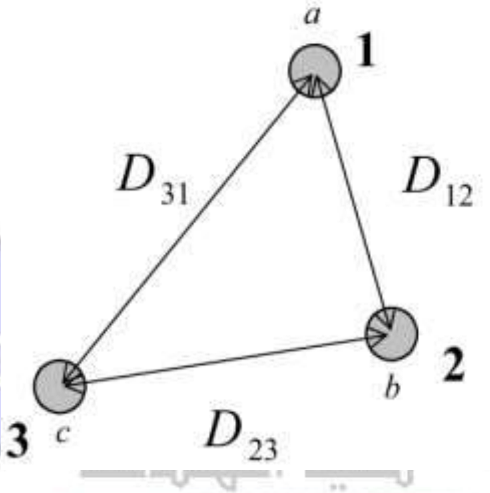


Fig. 24:

- Considering all the three sections of the transposed line for phase a.

$$v_{ab1} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right]$$

$$v_{ab2} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right]$$

$$v_{ab3} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right]$$

$$v_{ab} = \frac{v_{ab1} + v_{ab2} + v_{ab3}}{3}$$

$$v_{ab} = \frac{1}{6\pi\varepsilon}\left[q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + + q_c \ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}} \right]$$

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{r} + q_b \ln\frac{r}{\sqrt[3]{D_{12}D_{23}D_{31}}} \right]$$

If, $D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$

$$\therefore \quad v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{eq}}{r} + q_b \ln\frac{r}{D_{eq}} \right]$$

Similarly, $v_{ac} = \dfrac{1}{2\pi\varepsilon}\left[q_a \ln\dfrac{D_{eq}}{r} + q_c \ln\dfrac{r}{D_{eq}} \right]$

As in section (a) :

$$v_{ab} + v_{ac} = 3V_{an} \quad and \quad q_b + q_c = -q_a$$

$$\therefore \quad v_{an} = \frac{1}{3\times 2\pi\varepsilon}\left[2\times q_a \ln\frac{D_{eq}}{r} - q_a \ln\frac{r}{D_{eq}} \right]$$

$$v_{an} = \frac{1}{3 \times 2\pi\varepsilon}\left[q_a \ln\frac{D_{eq}^2}{r^2} - q_a \ln\frac{r}{D_{eq}}\right]$$

$$v_{an} = \frac{1}{3 \times 2\pi\varepsilon}\left[q_a \ln\frac{D_{eq}^3}{r^3}\right]$$

$$v_{an} = \frac{q_a}{2\pi\varepsilon}\ln\frac{D_{eq}}{r} \quad volts$$

$$C_n = \frac{q_a}{v_{an}} = \frac{2\pi\varepsilon}{\ln\frac{D_{eq}}{r}} \quad F/m$$

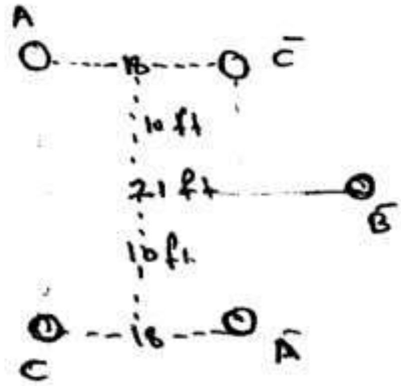
$$\therefore \quad C_n = \frac{1}{18 \times \ln\frac{D_{eq}}{r}} = \frac{0.0555}{\ln\frac{D_{eq}}{r}} \quad \mu F/Km$$

Example:

Ex :- A 3-ph double circuit line is composed of ACSR Ostrich, Find C & Xc per phase in (Ω-mile) and (Ω.km), outside diameter is 0.68 inch). f = 60 Hz

$$C_{an} = \frac{2\pi\epsilon}{\ln\frac{D_{eq}^{P}}{D_{sc}^{P}}} \quad \epsilon \rightarrow 8.85\times10^{-12}$$

$D_{sc}^{P} \rightarrow$ [illegible]



$$D_{eq}^{P} = \sqrt[3]{D_{AB}^{P}\, D_{AC}^{P}\, D_{BC}^{P}} = 16.1\ ft$$

تحسب من خلال ال (conductors)

$$D_{AB}^{P} = \sqrt[4]{D_{AB}\cdot D_{AB'}\cdot D_{A'B}\cdot D_{A'B'}} \;,\; D_{AC}^{P} \,,\; D_{BC}^{P} \leftarrow$$

$$D_{sc}^{P} = \sqrt[3]{GMR_{Ac}\cdot GMR_{Bc}\cdot GMR_{cc}}$$

$$GMR_{A_c} = \sqrt[4]{r\cdot D_{AA'}\cdot r\cdot D_{A'A}} = \sqrt{r\,D_{AA'}}$$

مساوي

$$GMR_{B_c} = \sqrt{r\,D_{BB'}} \qquad GMR_{c_c} = \sqrt{r\,D_{CC'}}$$

$$r = \frac{0.68}{2\times12} \Rightarrow r = 0.0283\ ft$$

للتحويل من (inch ← ft) ← 2×12 ← للحصول على r من القطر.

$$\therefore D_{sc}^{P} = \sqrt[3]{\sqrt{0.0283\times26.9}\cdot\sqrt{0.0283\times21}\cdot\sqrt{0.0283\times26.9}}$$

$$D_{sc}^{P} = 0.873\ ft$$

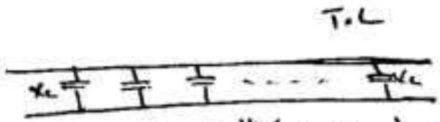
$$C_{an} = 18.807\times10^{-12}\ F/m$$

$$X_c = \frac{1}{2\pi f c}\ \Omega.m \Rightarrow X_c =$$

(3.14) (22/7)

$$X_c(\Omega.km) = \frac{X_c(\Omega.m)}{1000}$$

$$X_c(\Omega.mile) = \frac{X_c(\Omega.m)}{1609}$$



كلما زاد ال T.L ← كلما قل ال (Xc)

$\frac{1}{X_{eq}}$ = X_c توازي

[illegible] ال X_L ← تعزيز في

3.4.4- Capacitance of bundle conductors

- The bundle conductor comprises two, three, and four conductors. Geometric mean radius GMR (self-GMD) of the bundle conductor cab be calculated;

Instead of (r), put D_{sc}^{b}, where :

D_{sc}^{b} - is the GMR of a bundle conductor, and can be calculated as in inductance

1- For two – conductor bundle :

$$D_{SC}^{b} = \sqrt[2^2]{(D_S \times d)^2} = \sqrt[4]{(D_S \times d)^2} = \sqrt{(D_S \times d)}$$

2- For three – conductor bundle :

$$D_{SC}^{b} = \sqrt[3^2]{(D_S \times d \times d)^3} = \sqrt[9]{(D_S \times d^2)^3} = \sqrt[3]{D_S \times d^2}$$

3- For four – conductor bundle :

$$D_{SC}^{b} = \sqrt[4^2]{(D_S \times d \times d \times d\sqrt{2})^4} = \sqrt[16]{(D_S \times d^3\sqrt{2})^4} = \sqrt[4]{D_S \times d^3\sqrt{2}}$$
$$= 1.09 \times \sqrt[4]{D_S \times d^3}$$

IF conductor is solid , $D_S \Rightarrow r$ *in above three condition*

∴ Capacitance of 3-phase line , bundle conductor with equilateral spacing :

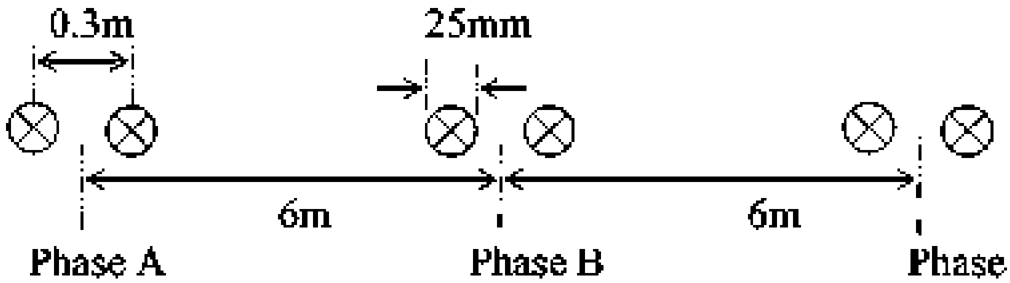
$$C_{nb} = \frac{0.0555}{\ln \frac{D}{D_{sc}^{b}}} \quad \mu F / Km$$

And for unsymmetrical spacing but transposed , the capacitance is :

$$C_{nb} = \frac{0.0555}{\ln \frac{D_{eq}}{D_{sc}^{b}}} \quad \mu F / Km$$

Example:

A 3-phase , 50Hz , 400 KV overhead transmission lines are arranged in a horizontal plane , each phase has two – strand bundle conductors , the diameter of each strand is 25mm , as shown in the fig. below . Find the capacitance to neutral per phase per Km .



Solution :

$$D_m = D_{eq} = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$
$$= \sqrt[3]{6 \times 6 \times 12} = 7.56\ m$$

$$D_S = r = \frac{25}{2} = 12.5\ mm$$

$$D_{SC}^{b} = \sqrt{D_S \times d}$$
$$= \sqrt{12.5 \times 10^{-3} \times 0.3} = 0.0612\ m$$

$$C_{nb} = \frac{0.0555}{\ln \frac{D_{eq}}{D_{SC}^{b}}} = \frac{0.0555}{\ln \frac{7.56}{0.0612}} = \frac{0.0555}{4.816} = 0.0115\ \mu F / Km$$

3.4.5- Effect of earth on the capacitance of a single-phase line

- Figure 25 shows a single conductor with uniform charge distribution and with height h above earth plane.
- Consider that the conductor has a positive charge q coulomb/m, an equal amount of negative charge $-q$ coulomb/m is induced on the earth.
- The electric field lines will originate from positive charge on the conductor and terminate at the negative charge on the earth.
- Also, the electric field lines are perpendicular to the surfaces of earth and conductor.

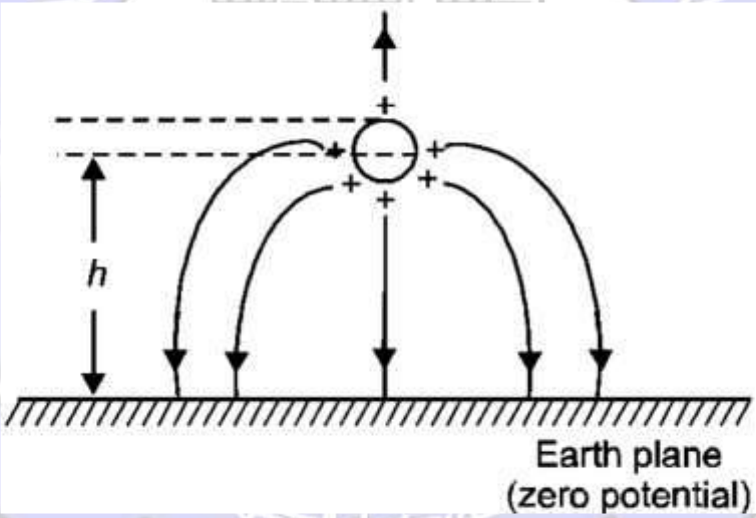


Fig. 25:

- Figure 26 shows that the earth is replaced by image conductor, lies directly below the original conductor.
- The electric field above the plane is the same as it is when the ground is present instead of image conductors.

- The voltage between any two points above the earth is the same in figure 25 and 26.

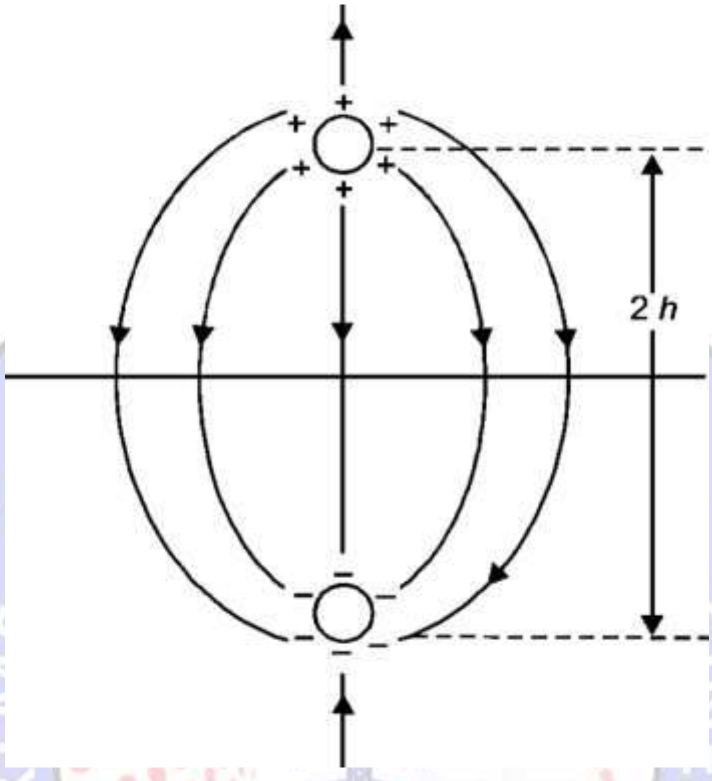


Fig. 26:

- Figure 27 shows a single-phase line with flat horizontal spacing. The earth plane is replaced by separate image conductor for each overhead conductor.

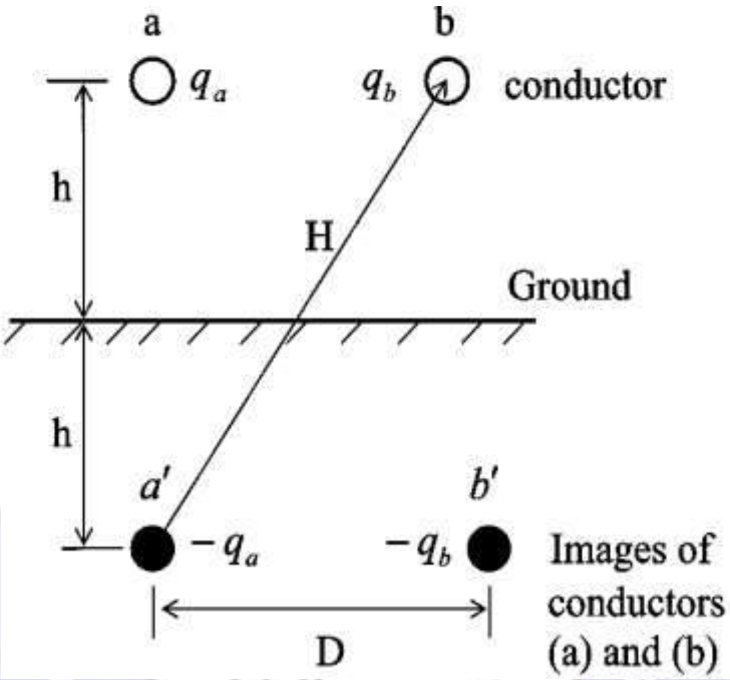


Fig. 27:

- Potential difference between conductor a and b can be easily obtained as follows;

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right] + \frac{1}{2\pi\varepsilon}\left[-q_a \ln\frac{H}{2h} - q_b \ln\frac{2h}{H}\right]$$

$$but \quad H = \sqrt{D^2 + (2h)^2} = \sqrt{D^2 + 4h^2}$$

$$\therefore v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right] + \frac{1}{2\pi\varepsilon}\left[-q_a \ln\frac{\sqrt{D^2+4h^2}}{2h} - q_b \ln\frac{2h}{\sqrt{D^2+4h^2}}\right]$$

$$v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{2hD}{r\sqrt{D^2+4h^2}} + q_b \ln\frac{r\sqrt{D^2+4h^2}}{2hD}\right]$$

$$but \quad q_a = -q_b$$

$$\therefore\ v_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{4h^2D^2}{r^2(D^2+4h^2)}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\left(\frac{D}{r\sqrt{1+(D^2/4h^2)}}\right)^2\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[2q_a \ln\frac{D}{r\sqrt{1+(D^2/4h^2)}}\right]$$

$$= \frac{q_a}{\pi\varepsilon}\left[\ln\frac{D}{r} + \ln\frac{1}{\sqrt{1+(D^2/4h^2)}}\right] \quad volts$$

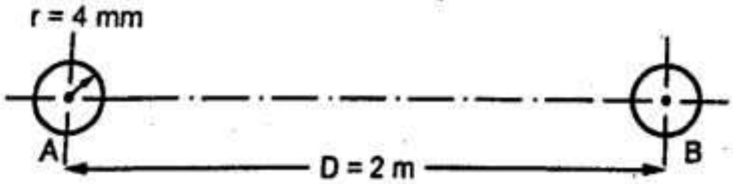
$$\therefore C_{ab} = \frac{q_a}{v_{ab}} = \frac{\pi\varepsilon}{\ln\frac{D}{r} + \left(\ln\frac{1}{\sqrt{1+(D^2/4h^2)}}\right)} \quad F/m$$

effect of earth term

Example:

A single-phase line 40-kilometer-long consisting of two wires, the diameter of each is four millimeters and the distance between wires is two meters. Find the capacitance with and without effect of ground, consider the height above ground is five meters.

Neglecting the presence of ground



$$\text{Line Length} = 40 \text{ km}$$

$$r = 4 \text{ mm} = 4\times10^{-3} \text{ m}$$

$$D = 2 \text{ m}$$

$$\varepsilon_0 = 8.854\times10^{-12} \text{ F/m}$$

$$C_{AB} = \frac{\pi\varepsilon_0}{ln(D/r)} = \frac{\pi\times 8.854\times10^{-12}}{ln\left(2/4\times10^{-3}\right)}$$

$$= 4.47585\times10^{-12} \text{ F/m}$$

For line of 40 km length

$$C_{AB} = 4.47585\times10^{-10}\times40\times10^{3}$$

$$= 1.7903\times10^{-7} \text{ F}$$

$$\therefore \quad C_{AB} = 0.17903 \text{ μF}$$

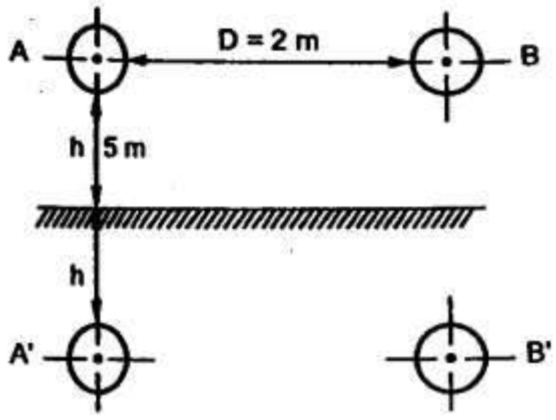


Fig. 3.54 (a)

Considering the presence of ground

Here,

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

h = 5 m

D = 2 m

$r = 4 \text{ mm} = 4 \times 10^{-3}$ m

We have,

$$C_{AB} = \frac{\pi \varepsilon_0}{ln\left[\frac{D}{r\sqrt{1+(D^2/4h^2)}}\right]} \text{ F/m}$$

$$= \frac{\pi \times 8.854 \times 10^{-12}}{ln\left[\frac{2}{4\times10^{-3}\sqrt{1+[(2)^2/4(5)^2]}}\right]}$$

$$= 4.49 \times 10^{-12} \text{ F/m}$$

For a line of length of 40 km

$$C_{AB} = 4.49 \times 10^{-12} \times 40 \times 10^3 = 1.7960 \times 10^{-7} \text{ F}$$

$$= \mathbf{0.17960\ \mu F}$$

Chapter Four

Performance of Overhead Transmission Lines

4.1- Introduction

4.2- Equivalent circuit of Short Transmission line

4.3- Equivalent circuit of Medium Transmission line

4.4- Equivalent circuit of Long Transmission line

4.5- Examples

4.1- Introduction

- The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission.
- These values are influenced by the line constants R, L and C of the transmission line.
- The overhead transmission lines are classified as:
  - ✓ **Short transmission lines:**

    The length of an overhead transmission line is upto about 80 km and the line voltage is less than 20 kV. Therefore, the capacitance effects are small and hence can be neglected. To study the transmission line performance, the only resistance and inductance are taken into.
  - ✓ **Medium transmission lines:**

    The length of an overhead transmission line is between 80 to 250 km and the line voltage is between 20 kV to 100 kV. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points

- **Long transmission lines:**

  The length of an overhead transmission line is more than 250 km and line voltage is greater than 100 kV.

4.2- Equivalent circuit of Short Transmission line:

- The equivalent circuit of a single-phase short transmission line is shown in figure 28. The total line resistance and inductance are lumped.

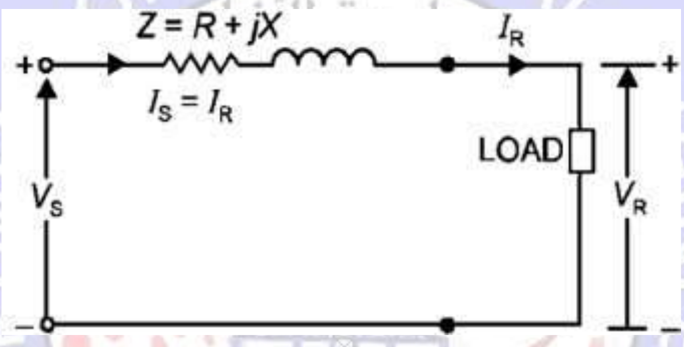


Fig. 28:

- The circuit is a simple A.C series circuit, some terms need to be defined;

I = load current
R = loop resistance *i.e.*, resistance of both conductors
X_L = loop reactance
V_R = receiving end voltage
$\cos\phi_R$ = receiving end power factor (lagging)
V_S = sending end voltage
$\cos\phi_S$ = sending end power factor

V_s & V_r - phase voltage (rms value)

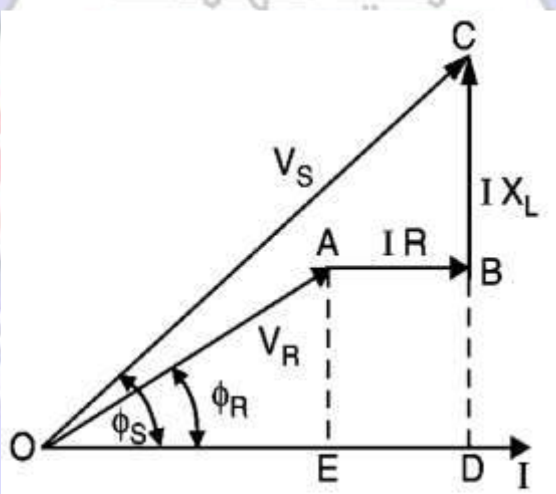
$$\vec{I}_S = \vec{I}_r = \vec{I}$$

$$\vec{V}_S = \vec{V}_r + Z\vec{I}_r$$

Eqs. can be written in matrix form as:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

- The phasor diagram of the line for lagging load power factor is shown below.



$$V_S = V_R + IR\cos\phi_R + IX_L\sin\phi_R$$

$$\text{Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

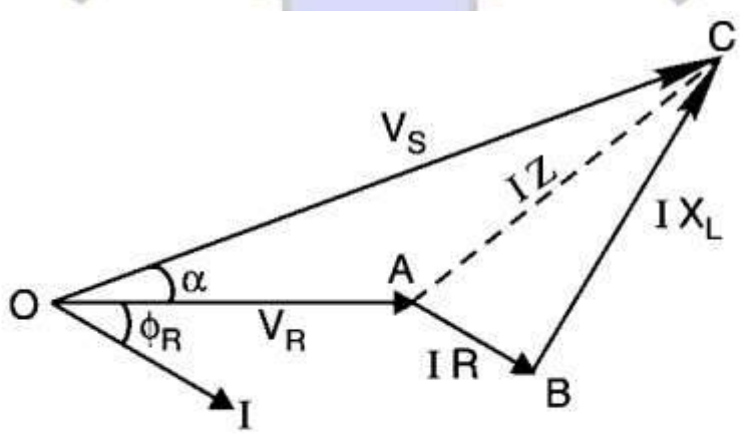
$$\text{Sending end } p.f., \cos\phi_S = \frac{OD}{OC} = \frac{V_R\cos\phi_R + IR}{V_S}$$

$$\text{Power delivered} = V_R I_R \cos \phi_R$$
$$\text{Line losses} = I^2 R$$
$$\text{Power sent out} = V_R I_R \cos \phi_R + I^2 R$$
$$\text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent out}} \times 100$$
$$= \frac{V_R\, I_R \cos \phi_R}{V_R\, I_R \cos \phi_R + I^2 R} \times 100$$

- **Effect of load power factor ($p.f$) on regulation and efficiency:**
  - ✓ Note, the receiving voltage (V_R) will be taken as voltage reference for inductive load (lagging $p.f$);

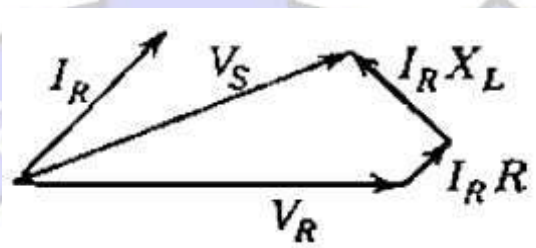
Taking $\vec{V_R}$ as the reference phasor, draw the phasor diagram as shown. It is clear that $\vec{V_S}$ is the phasor sum of $\vec{V_R}$ and $\vec{I}\,\vec{Z}$.

$$*\vec{V_R} = V_R + j\,0$$
$$\vec{I} = \vec{I}\ \angle -\phi_R = I(\cos \phi_R - j \sin \phi_R)$$
$$\vec{Z} = R + jX_L$$
$$\therefore \quad \vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$$
$$= (V_R + j\,0) + I(\cos \phi_R - j \sin \phi_R)(R + j\,X_L)$$



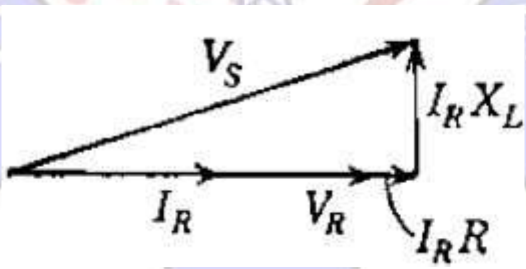
Voltage regulation. When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

- ✓ Note, the receiving voltage (V_R) will be taken as voltage reference for capacitive load (leading $p.f$);



When the load p.f. is leading to this extent that $I X_L \sin\phi_R > I R \cos\phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S.

- ✓ Note, the receiving voltage (V_R) will be taken as voltage reference for resistive load (unity $p.f$);



When the load p.f. is unity or such leading that $I R \cos\phi_R > I X_L \sin\phi_R$, then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S.

transmission efficiency. The power delivered to the load depends upon the power factor.

$$P = V_R * I \cos \phi_R \quad \text{(For 1-phase line)}$$

$$\therefore \quad I = \frac{P}{V_R \cos \phi_R}$$

$$P = 3 V_R I \cos \phi_R \quad \text{(For 3-phase line)}$$

$$\therefore \quad I = \frac{P}{3V_R \cos \phi_R}$$

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage (V_R), the load current I is inversely proportional to the load p.f. $\cos \phi_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load p.f. and *vice-versa*,

Example:

A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (*i*) *sending end voltage* (*ii*) *sending end power factor and* (*iii*) *transmission efficiency.*

Solution.

Load power factor, $\cos \phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + j X_L = 10 + j\,15$

Receiving end voltage, $V_R = 33 \text{ kV} = 33{,}000 \text{ V}$

$$\therefore \quad \text{Line current, } I = \frac{kW \times 10^3}{V_R \cos \phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0 \cdot 8} = 41{\cdot}67 \text{ A}$$

$$\text{As } \cos \phi_R = 0{\cdot}8 \quad \therefore \quad \sin \phi_R = 0{\cdot}6$$

Taking receiving end voltage $\vec{V_R}$ as the reference phasor,

$$\vec{V_R} = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

∴ Magnitude of $V_S = \sqrt{(33{,}708\cdot 3)^2 + (250)^2} = \mathbf{33{,}709\ V}$

$$\alpha = \tan^{-1}\frac{250}{33{,}708\cdot 3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^\circ$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^\circ + 0{\cdot}42^\circ = 37{\cdot}29^\circ$$

∴ Sending end p.f., $\cos\phi_S = \cos 37{\cdot}29^\circ = \mathbf{0{\cdot}7956\ lagging}$

(*iii*) Line losses $= I^2R = (41{\cdot}67)^2 \times 10 = 17{,}364 \text{ W} = 17{\cdot}364 \text{ kW}$

Output delivered $= 1100$ kW

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364$ kW

∴ Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent}} \times 100 = \dfrac{1100}{1117\cdot 364} \times 100 = \mathbf{98{\cdot}44\%}$

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R \text{ (approximately)}$$

$$= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6$$

$$= 33{,}000 + 333{\cdot}36 + 375{\cdot}03$$

$= 33708{\cdot}39$ V which is approximately the same as above

$$\cos\phi_S = \frac{V_R\cos\phi_R + I\,R}{V_S} = \frac{33{,}000 \times 0\cdot 8 + 41\cdot 67 \times 10}{33{,}708\cdot 39} = \frac{26{,}816\cdot 7}{33{,}708\cdot 39}$$

$$= 0{\cdot}7958$$

4.3- Equivalent Circuit of Medium Transmission Line:

- As the length and voltage of the line are increased, the capacitance gradually becomes importance.
- Since the length of transmission line is between 80 to 250 km and usually operate at voltages greater than 20 kV until 100 kV, the effects of capacitance cannot be neglected.
- Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.
- The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points.
- The most commonly used methods for the solution of medium transmissions lines are: (i) Nominal T method (ii) Nominal π method.

4.3.1- Nominal T method:

- In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Figure 29.

- Therefore, in this arrangement, full charging current flows over half the line.

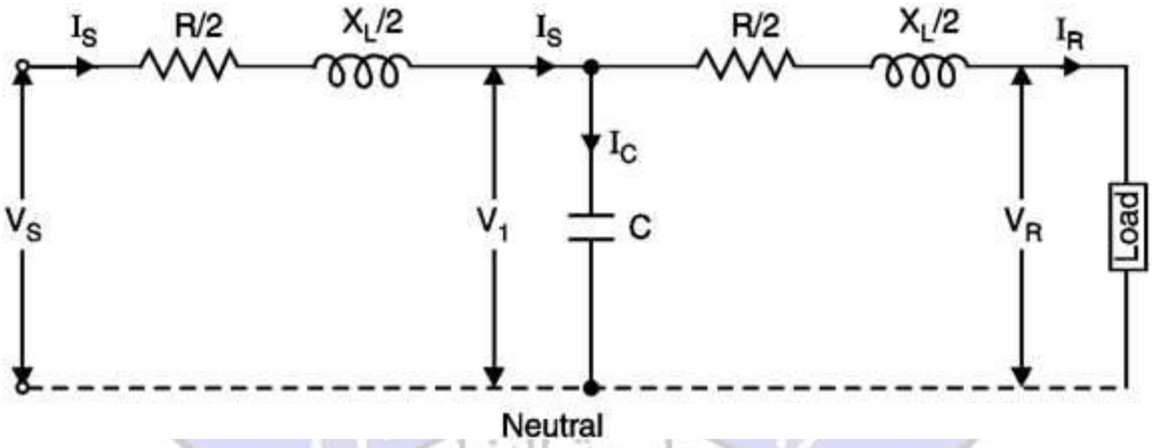


Fig. 29:

Let

I_R = load current per phase ;

X_L = inductive reactance per phase ;

$\cos\phi_R$ = receiving end power factor (*lagging*) ;

V_1 = voltage across capacitor C

R = resistance per phase

C = capacitance per phase

V_S = sending end voltage/phase

- The phasor diagram for the nominal T method is shown in figure 30. Taking the receiving end voltage $\overrightarrow{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\overrightarrow{V_R} = V_R + j\,0$

Load current, $\overrightarrow{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$

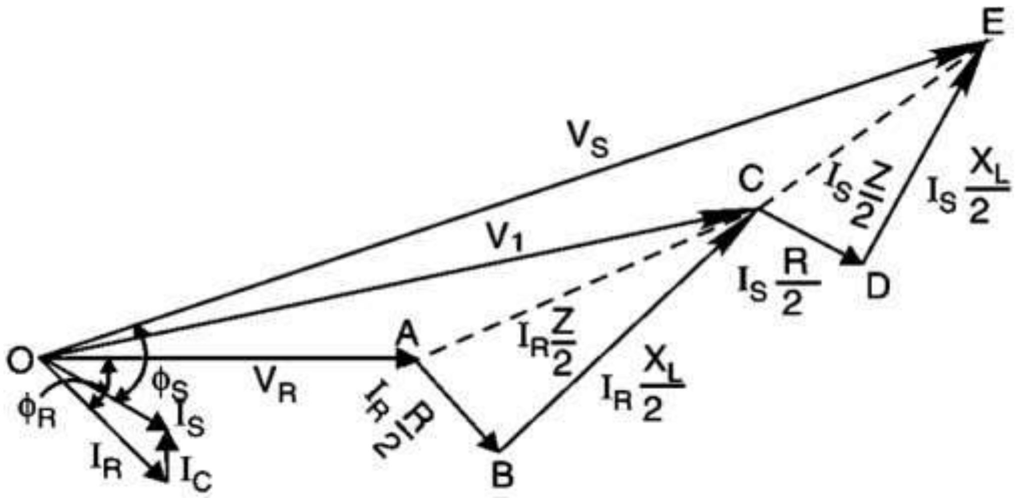


Fig. 30:

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\,\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi f\,C\,\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\frac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\frac{R}{2} + j\,\frac{X_L}{2}\right)$

Example:

A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :

Resistance/km/phase	= $0.1\,\Omega$
Inductive reactance/km/phase	= $0{\cdot}2\,\Omega$
Capacitive susceptance/km/phase	= $0{\cdot}04 \times 10^{-4}$ *siemen*

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0·8 lagging. Use nominal T method.

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase. $X_L = 0{\cdot}2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0{\cdot}04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $I_R = \dfrac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8} = 109$ A

$\cos\phi_R = 0{\cdot}8;\quad \sin\phi_R = 0{\cdot}6$

Impedance per phase, $\vec{Z} = R + jX_L = 10 + j\,20$

Receiving end voltage, $\vec{V_R} = V_R + j\,0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\,(0{\cdot}8 - j\,0{\cdot}6) = 87{\cdot}2 - j\,65{\cdot}4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 38,\,105 + (87{\cdot}2 - j\,65{\cdot}4)\,(5 + j\,10)$

$= 38{,}105 + 436 + j\,872 - j\,327 + 654 = 39{,}195 + j\,545$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87{\cdot}2 - j\,65{\cdot}4) + (-0{\cdot}218 + j\,15{\cdot}6)$

$= 87{\cdot}0 - j\,49{\cdot}8 = 100\ \angle -29°47'$ A

∴ Sending end current $=$ **100 A**

(ii) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87{\cdot}0 - j\,49{\cdot}8)\,(5 + j\,10)$

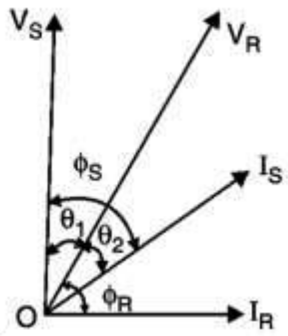
$= 39{,}195 + j\,545 + 434{\cdot}9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\ \angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145 \times \sqrt{3} = 69\ 533\ \text{V} = \mathbf{69{\cdot}533\ kV}$

(iii) Referring to phasor diagram,



$$\theta_1 = \text{angle between } \vec{V_R} \text{ and } \vec{V_S} = 1°40'$$

$$\theta_2 = \text{angle between } \vec{V_R} \text{ and } \vec{I_S} = 29°\,47'$$

$$\therefore \quad \phi_S = \text{angle between } \vec{V_S} \text{ and } \vec{I_S}$$

$$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$$

$\therefore$ Sending end power factor, $\cos\phi_S = \cos 31°27' = \mathbf{0{\cdot}853\ lag}$

(iv) Sending end power $= 3\,V_S I_S \cos\phi_S = 3 \times 40{,}145 \times 100 \times 0{\cdot}853$

$= 10273105 \text{ W} = 10273{\cdot}105 \text{ kW}$

Power delivered $= 10{,}000$ kW

$\therefore$ Transmission efficiency $= \dfrac{10{,}000}{10273{\cdot}105} \times 100 = \mathbf{97{\cdot}34\%}$

Example:

A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is $2{\cdot}5 \times 10^{-6}$ siemen/km/phase. Calculate : (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T method.

4.3.2- Nominal π method:

- In this method, capacitance of each conductor (i.e., line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in figure 31.
- It is obvious that capacitance at the sending end has no effect on the line drop.
- However, its charging current must be added to line current in order to obtain the total sending end current.

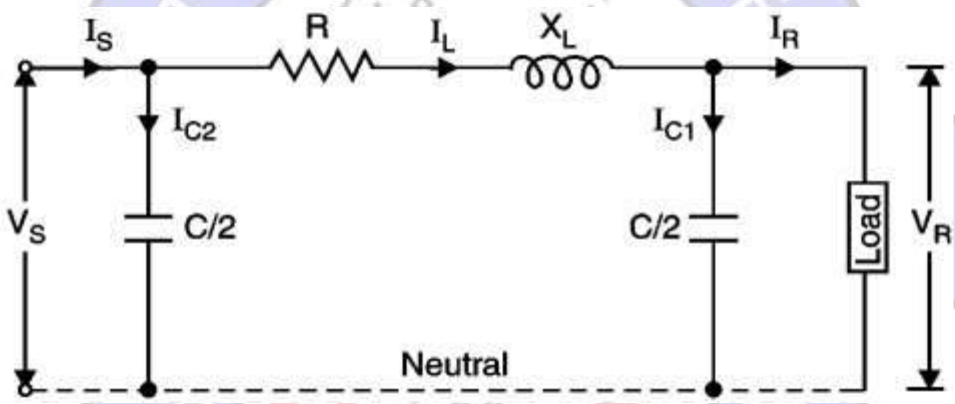


Fig. 31:

Let I_R = load current per phase
R = resistance per phase
X_L = inductive reactance per phase
C = capacitance per phase
$\cos \phi_R$ = receiving end power factor (*lagging*)
V_S = sending end voltage per phase

- The phasor diagram for the nominal π method is shown in figure 32.

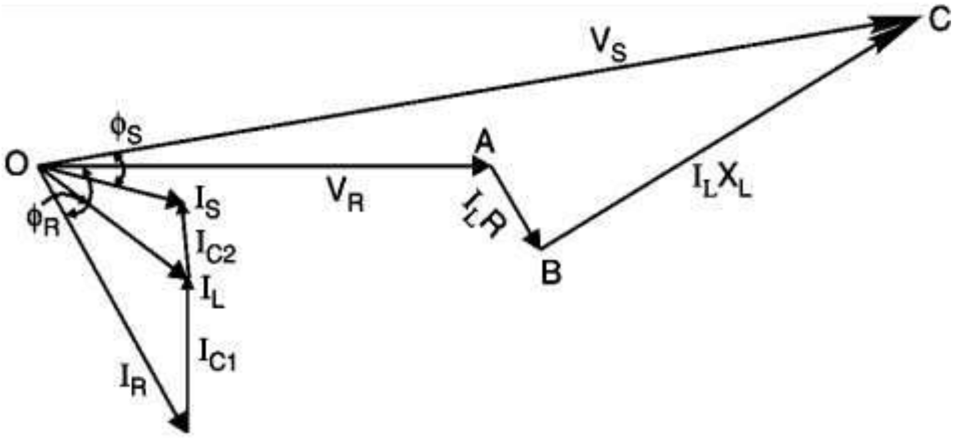


Fig. 32:

- Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0$$

Load current, $$\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R)$$

Charging current at load end is

$$\vec{I_{C1}} = j\,\omega\,(C/2)\,\vec{V_R} = j\,\pi\,f\,C\,\vec{V_R}$$

Line current, $$\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$$

Sending end voltage, $$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + jX_L)$$

Charging current at the sending end is

$$\vec{I_{C2}} = j\,\omega\,(C/2)\,\vec{V_S} = j\,\pi\,f\,C\,\vec{V_S}$$

∴ Sending end current, $$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$$

Example:

A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3×10^{-6} S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.

Solution. Fig. shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 150 = 15\ \Omega$

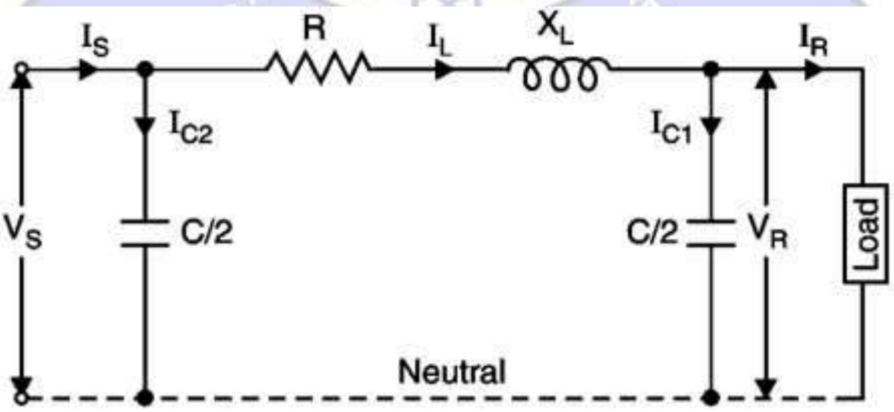
Total reactance/phase, $X_L = 0{\cdot}5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 328 \text{ A}$$

$\cos\phi_R = 0{\cdot}8$; $\sin\phi_R = 0{\cdot}6$



Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 328\,(0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$

Charging current at the load end is

$$\vec{I_{C1}} = \vec{V_R}\, j\,\frac{Y}{2} = 63{,}508 \times j\,\frac{45\times10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$

Sending end voltage,
$$\begin{aligned}\vec{V_S} &= \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)\\ &= 63{,}508 + (262{\cdot}4 - j\,182{\cdot}5)\,(15 + j\,75)\\ &= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687\\ &= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881 \angle 11°47' \text{ V}\end{aligned}$$

∴ Line to line sending end voltage = $82{,}881 \times \sqrt{3} = 1{,}43{,}550$ V = **143·55 kV**

Charging current at the sending end is

$$\begin{aligned}I_{C2} &= j\vec{V_S}\,Y/2 = (81{,}131 + j\,16{,}942\cdot5)\,j\,\frac{45\times10^{-5}}{2}\\ &= -3{\cdot}81 + j\,18{\cdot}25\end{aligned}$$

Sending end current,
$$\begin{aligned}\vec{I_S} &= \vec{I_L} + \vec{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-3{\cdot}81 + j\,18{\cdot}25)\\ &= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4 \angle -32{\cdot}4° \text{ A}\end{aligned}$$

∴ Sending end current = **306·4 A**

Example:

A 100-km long, 3-phase, 50-Hz transmission line has following line constants:

Resistance/phase/km = 0·1 Ω

Reactance/phase/km = 0·5 Ω

Susceptance/phase/km = 10×10^{-6} *S*

If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method :

(*i*) *sending end power factor*

(*ii*) *regulation*

(*iii*) *transmission efficiency*

4.4- Equivalent circuit of Long Transmission line:

- Figure 33 shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed.

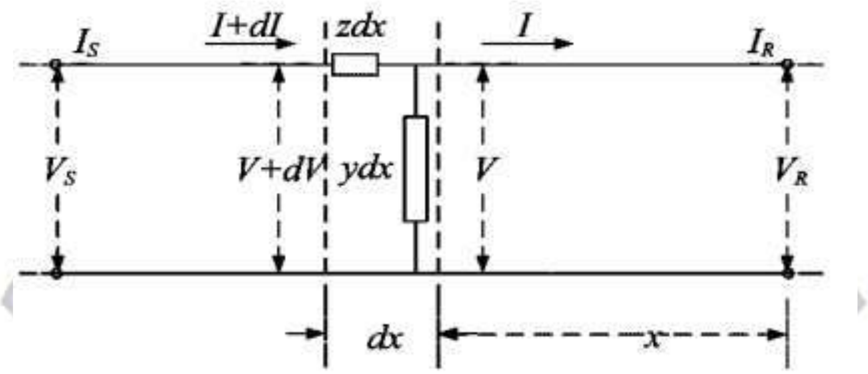


Fig. 33:

- Consider a small element in the line of length 'dx' situated at a distance 'x' from the receiving end.

Let
z = series impedance of the line per unit length
y = shunt admittance of the line per unit length
V = voltage at the end of element towards receiving end
$V + dV$ = voltage at the end of element towards sending end
$I + dI$ = current entering the element dx
I = current leaving the element dx

Then for the small element dx,

$z\,dx$ = series impedance
$y\,dx$ = shunt admittance

Obviously, $dV = I\,z\,dx$

or
$$\frac{dV}{dx} = I\,z \qquad ...(i)$$

- Now, the current entering the element 'dx' is $(I + dI)$ whereas the current leaving the same element is I.
- The difference in the currents flows through shunt admittance of the element i.e.,

$$dI = \text{Current through shunt admittance of element} = V\,y\,dx$$

or $$\frac{dI}{dx} = Vy \qquad \text{...(ii)}$$

- For elemental section under consideration, we can write the circuit equations as follows:

$$\frac{dV(x)}{dx} = z.I(x) \qquad \text{...(6.20)}$$

$$\frac{dI(x)}{dx} = y.V(x) \qquad \text{...(6.22)}$$

Differentiating eqn. (6.20) and substituting from eqn. (6.22), we get,

$$\frac{d^2V(x)}{dx^2} = z \cdot \frac{dI(x)}{dx} = z.yV(x)$$

$\therefore$ $$\frac{d^2V(x)}{dx^2} - zy\,V(x) = 0 \qquad \text{...(6.23)}$$

Let $$\gamma^2 = zy \qquad \text{...(6.24)}$$

Therefore, $$\frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0 \qquad \text{...(6.25)}$$

The solution of the above equation is

$$V(x) = C_1 e^{\gamma x} + C_2\, e^{-\gamma x} \qquad \text{...(6.26)}$$

where, γ, known as the propagation constant and is given by,

$$\gamma = \alpha + j\beta = \sqrt{zy} \qquad \text{...(6.27)}$$

The real part α is known as the attenuation constant, and the imaginary part β is known as the phase constant. β is measured in radian per unit length. From eqn. (6.20), the current is,

$$I(x) = \frac{1}{Z} \cdot \frac{dV(x)}{dx}$$

$$\therefore \quad I(x) = \frac{\gamma}{z}\,(C_1 e^{\gamma x} - C_2 e^{-\gamma x})$$

$$\therefore \quad I(x) = \sqrt{\frac{y}{z}}\,(C_1 e^{\gamma x} - C_2 e^{-\gamma x})$$

$$\therefore \quad I(x) = \frac{1}{Z_C}(C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \qquad ...(6.28)$$

where, Z_C is known as the characteristic impedance, given by

$$Z_C = \sqrt{\frac{z}{y}} \qquad ...(6.29)$$

For Over Head Transmission Line , $Z_c \approx 400 \to 600\Omega$, and for underground cable $Z_c \approx 40 \to 60\Omega$

Now note that, when $x = 0$, $V(x) = V_R$ and from eqn. (6.26), we get

$$V_R = C_1 + C_2 \qquad ...(6.30)$$

also when $x = 0$, $I(x) = I_R$ and from eqn. (6.28), we get,

$$I_R = \frac{1}{Z_C}\,(C_1 - C_2) \qquad ...(6.31)$$

Solving eqns. (6.30) and (6.31), we obtain,

$$C_1 = \frac{V_R + Z_C I_R}{2} \qquad ...(6.32)$$

$$C_2 = \frac{(V_R - Z_C I_R)}{2} \qquad ...(6.33)$$

Substituting the values of C_1 and C_2 from eqns. (6.32) and (6.33) into eqns. (6.26) and (6.28), we get

$$V(x) = \frac{(V_R + Z_C I_R)}{2} e^{\gamma x} + \frac{(V_R - Z_C I_R)}{2} e^{-\gamma x} \qquad ...(6.34)$$

$$I(x) = \frac{(V_R + Z_C I_R)}{2Z_C} e^{\gamma x} - \frac{(V_R - Z_C I_R)}{2Z_C} e^{-\gamma x} \qquad ...(6.35)$$

The equations for voltage and currents can be rearranged as follows:

$$V(x) = \frac{(e^{\gamma x} + e^{-\gamma x})}{2} V_R + Z_C \frac{(e^{\gamma x} - e^{-\gamma x})}{2} I_R \qquad ...(6.36)$$

$$I(x) = \frac{(e^{\gamma x} - e^{-\gamma x})}{2Z_C} V_R + \frac{(e^{\gamma x} + e^{-\gamma x})}{2} I_R \qquad ...(6.37)$$

or

$$V(x) = \cosh(\gamma x) V_R + Z_C \sinh(\gamma x)\, I_R \qquad ...(6.38)$$

$$I(x) = \frac{1}{Z_C} \sinh(\gamma x) V_R + \cosh(\gamma x) I_R \qquad ...(6.39)$$

Our interest is in the relation between the sending end and the receiving end of the line. Therefore, when $x = l$, $V(l) = V_S$ and $I(l) = I_S$. The result is

$$V_S = \cosh(\gamma l) V_R + Z_C \sinh(\gamma l) I_R \qquad ...(6.40)$$

$$I_S = \frac{1}{Z_C} \sinh(\gamma l)\, V_R + \cosh(\gamma l) I_R \qquad ...(6.41)$$

Therefore, *ABCD* constants are:

$$A = \cosh(\gamma l) \quad ; \quad B = Z_C \sinh(\gamma l) \qquad ...(6.42)$$

$$C = \frac{1}{Z_C} \sinh(\gamma l) \quad ; \quad D = \cosh(\gamma l) \qquad ...(6.43)$$

Ex: Calculation of Z_c & γl

consider a 500 km long line for which the per kilometer line impedance and admittance are given respectively by

$z = 0.1 + j0.5145\ \Omega$ and $y = j3.1734 \times 10^{-6}\ \mho$

Sol

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.1 + j0.5145}{j3.1734\times10^{-6}}} = \sqrt{\frac{0.5241\angle 79}{3.1734\times10^{-6}\angle 90}}$$

$$\sqrt{\angle\alpha} = \frac{\alpha}{2}$$

$$= \sqrt{\frac{0.5241}{3.1734\times10^{-6}}}\ \angle\left(\frac{79-90}{2}\right) \qquad Z_c = 406.4024\angle -5.5^\circ\ \Omega$$

$$\gamma l = \sqrt{zy}\cdot l = \sqrt{0.5241\times3.1734\times10^{-6}} \times 500 \angle\left(\frac{79+90}{2}\right)$$

$$= 0.6448\angle 84.5^\circ = \underbrace{0.0618}_{\alpha l} + j\underbrace{0.6419}_{\beta l}$$

$\cosh\gamma l = 0.8025 + j0.037$ and $\sinh\gamma l = 0.0495 + j0.5998$

$\therefore A = D = 0.8025 + j0.037$

$B = 43.4 + j240.72\ \Omega$; $C = -2.01\times10^{-5} + j0.0015$

Example:

A 3-ϕ transmission line 200 km long has the following constants :

Resistance/phase/km = 0·16 Ω

Reactance/phase/km = 0·25 Ω

Shunt admittance/phase/km = $1{\cdot}5 \times 10^{-6}$ S

Calculate the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

$l = 200\,km$; Resistance/phase/km = $0.16\,\Omega$; Reactance/phase/km = $0.25\,\Omega$; shunt admittance/phase/km $(Y) = 1.5 \times 10^{-6}\,S$; $P_r = 20\,MW$; $P.f = 0.8$ lag ; $V_r = 110\,kV$.

Sol.:-

Total resistance/phase $(R) = 0.16 \times 200 = 32\,\Omega$

Total reactance/phase $(X_L) = 0.25 \times 200 = 50\,\Omega$

Total admittance/phase $(Y) = j\,1.5 \times 10^{-6} \times 200 = 0.0003\angle 90°$

series Impedance/phase $(Z) = R + jX_L = 32 + j50$

$= 59.4\angle 58°$

The sending-end Voltage/phase (V_s) is

$$V_s = \cosh(\gamma l)\,V_r + Z_c \sinh(\gamma l)\,I_r$$

where,

$V_r = 110\,kV = 110000\,V$; $I_r = \dfrac{P_r}{\sqrt{3}\,V_r \cos\phi}$

$$I_r = \frac{20 \times 10^{6}}{\sqrt{3} \times 110 \times 10^{3} \times 0.8} = 131.22\,A$$

$$V_r \text{ per phase} = \frac{110 \times 10^{3}}{\sqrt{3}} = 63508.53\,V$$

$$\gamma l = \sqrt{ZY} \Rightarrow \gamma l = \sqrt{59.4\angle 58 \times 0.0003\angle 90}$$

$$\Rightarrow \gamma l = \sqrt{59.4 \times 0.0003} \times \left(\frac{58+90}{2}\right)$$

$$\Rightarrow \gamma l = 0.133\angle 74° = 0.037 + j0.128$$

$$Z_c = \sqrt{\frac{Z}{Y}} \Rightarrow Z_c = \sqrt{\frac{59.4\angle 58}{0.0003\angle 90}}$$

$$\Rightarrow Z_c = \sqrt{\frac{59.4}{0.0003}} \times \left(\frac{58-90}{2}\right) \Rightarrow Z_c = 445\angle -16$$

$\therefore \sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$ → $0.128 * \frac{180}{\pi}$

$e^{\gamma l} = e^{0.037} e^{j0.128} \Rightarrow e^{\gamma l} = 1.038\angle 7.334^\circ$

$\Rightarrow e^{\gamma l} = 1.029 + j0.132$

$e^{-\gamma l} = e^{-0.037} e^{-j0.128} \Rightarrow e^{-\gamma l} = 0.964\angle -7.334^\circ$

$\Rightarrow e^{-\gamma l} = 0.956 - j0.123$

$\therefore \sinh(\gamma l) = \frac{1.029 + j0.132 - (0.956 - j0.123)}{2}$

$\Rightarrow \sinh(\gamma l) = \frac{0.073 + j0.255}{2} = 0.0365 + j0.1275$

$= 0.1326\angle 74.03^\circ$

$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$

$\Rightarrow = \frac{1.029 + j0.132 + 0.956 - j0.123}{2}$

$\Rightarrow \cosh(\gamma l) = \frac{1.985 + j0.009}{2} = 0.9925 + j0.0045$

$= 0.9925\angle 0.259^\circ$

$\therefore V_s = 0.9925\angle 0.259 * 63508.53\angle 0 +$

$445\angle -16 * 0.1326\angle 74.03 * 131.22\angle -36.87$

$\Rightarrow V_s = 63032.22\angle 0.259 + 7742.89\angle 21.16$

$\Rightarrow V_s = 63031.58 + j284.93 + 7220.83 + j2794.98$

$\Rightarrow V_s = 70252.41 + j3079.91 = 70319.89\angle 2.51$

$\Rightarrow V_{S\,\text{line to line}} = 70319.89 * \sqrt{3}$

$= 121.798\ KV$

The sending-end current /phase (I_s) is

$$I_s = \frac{1}{Z_c}\sinh(\gamma l)\,V_r + \cosh(\gamma l)\,I_r$$

$$\Rightarrow I_s = \frac{1}{445\angle -16} * 0.1326\angle 74.03^\circ * 63508.53\angle 0^\circ$$

$$+ 0.9925\angle 0.259 * 131.22\angle -36.87$$

$$\Rightarrow I_s = 18.92\angle 90.03 + 130.24\angle -36.611$$

$$\Rightarrow I_s = -0.0099 + j18.92 + 104.54 - j77.67$$

$$\Rightarrow I_s = 104.53 - j59.67 = 120.36\angle 29.72$$