## Electromagnetic Fields

## 5- Conductor dielectrics and capacitance:-

The subject of this chapter is to consider the effect of electric field on different materials such as conductors, dielectrics (insulators) then the depend of resistor and capacitors.

The conductors : materials have very large number of free charges so when applied potential across this conductor, there will be current and current density. The dielectric (insulator), material with no any free charges so no currents and good insulator for electricity.

## Current and current density:

Electric charges in motion constitute a current (Ampere)
$\mathrm{I}=\frac{d Q}{d t} \quad($ Ampere or $\mathrm{C} / \mathrm{sec})$
Current density $(\bar{J})$, measured in amperes per $m^{2}$ or $\left(A / m^{2}\right)$. It is a vector can be given by many relations:
$\Delta I=J_{n} \Delta s(\Delta I$ crossing surface $\Delta s$ normally $)$
Then $J_{n}=\frac{\Delta I}{\Delta s}$
In case where $\Delta I$ is not normal to $\Delta s$ then
$\Delta I=\bar{J} . \Delta \bar{s}$
Or $\mathrm{I}=\iint \bar{J} . d \bar{s}=\int_{s} \bar{J} . d \bar{s} A$
where
I (Total current) and Current density $(\bar{J})$ may be related to the velocity of volume charge density at point.

For
$\Delta Q=\rho_{v} \Delta s \Delta l$ as shown, if this charge moving a distance $\Delta x$ in time
$\Delta t$ then the current produced is $\Delta I=\frac{\Delta Q}{\Delta t}=\rho_{v} \Delta s \frac{\Delta x}{\Delta t}$
Or $\Delta I=\rho_{v} \Delta s v_{x}$
$v_{x}$ : Velocity in x-direction
And
$J_{x}=\rho_{v} v_{x}$
And in general:
$\bar{J}=\rho_{v} \bar{v} \quad A / m^{2}$ where $\bar{v}=v_{x} \bar{a}_{x}+v_{y} \bar{a}_{y}+v_{z} \bar{a}_{z}$

## Continuity of current:-

The principle of conservation of charge states that charges can be neither created nor destroyed although equal amount of positive and negative charge may be simultaneously created, obtained by separation destroyed or lost by recombination.

The continuity equation follows from this principle when we consider any region bounded by a closed surface. The current through the closed surface is
$\mathrm{I}=\oint_{S} \bar{J} . d \bar{s}$
If Q : is the charge inside
Then the rate of decrease is $-\frac{d Q i}{d t}$ and the principle of conservation of charge requires.
$\mathrm{I}=\oint_{S} \bar{J} . d \bar{s}=-\frac{\mathrm{dQi}}{\mathrm{dt}}(-$ ve for out ward flowing current $)$
But $\oint_{s} \bar{J} \cdot d \bar{s}=\iiint(\nabla . \bar{J}) d v$
then:
$\iiint(\nabla \cdot \bar{J}) d v=\frac{-d}{d t} \iiint \rho_{v} d v$
Or $\int_{v}(\nabla . \bar{J}) d v=\int_{v}-\frac{\partial \rho_{v}}{\partial t} d v \quad$ (continuity equ.)
$\therefore \nabla . \bar{J}=-\frac{\partial \rho_{v}}{\partial t} \quad$ (Point form)

## Metallic conductors:-

The energy- band structure in three different types of materials at 0 k can be shown below for conductor exhibits no energy gap between that valence and conduction bands, the valence band is the band of electron orbitals that electrons can jump out of, moving into the conduction band when excited. The insulator shows a large energy gap the semiconductor has only a small energy gap.

Energy | Conduction band | cond band | cond .band |
| :---: | :---: | :---: |
| energy gap | energy gap |  |
| Valence band | valence band | valence band |
| Conductor | insulator | semiconductor |

If $\mathrm{Q}=-\mathrm{e}$ (for electron) then

$$
\bar{F}=-e E \text { and the drift velocity of electron is given by } \bar{V} d=-\mu_{e} \bar{E}
$$ where $\mu_{e}=$ mobility of electron ( 0.0012 for aluminum, 0.0032 for copper

$$
\rho_{v}=\rho_{e}(\text { free electron charge density })
$$

$$
\bar{J}=-\rho_{v} \mu_{e} \bar{E}
$$

$$
\bar{J}=\rho_{h} \mu_{e} \bar{E} \text { for holes }
$$

For metallic conductor the relation between $\bar{J}$ and $\bar{E}$ can be :
$\bar{J}=\sigma \bar{E}$
This can be proved as:

$\mathrm{I}=\iint \bar{J} \cdot d \bar{s}=J s$
$\mathrm{Vab}=-\int_{b}^{a} \bar{E} \cdot d \bar{l}=E l=I R=J S R$
$\therefore J=\frac{I}{S}=\frac{E l}{S R}$
$\therefore J=\sigma \bar{E} \quad$ Where $\frac{l}{S R}=\sigma=\frac{1}{\text { resistivity }}$
$\mathrm{J}=\sigma \frac{V}{l}$

$$
R=\frac{l}{\sigma S}
$$

$\mathrm{R}=\frac{V a b}{I}=\frac{-\int_{b}^{a} \bar{E} \cdot d \bar{l}}{\iint \sigma \bar{E} \cdot d \bar{s}}$ (inside the conductor)
Therefor the conductivity is
$\sigma=-\rho_{e} \mu_{e} \quad$ For electrons
For semiconductors, we have electron and holes then conductivity is
$\sigma=-\rho_{e} \mu_{e}+\rho_{h} \mu_{h}$
For pure Germanium $\mu_{e}=0.36, \mu_{h}=0.17\left(\mathrm{~m}^{2} / v\right.$. sec $)$
For silicon: $\mu_{e}=0.12, \mu_{e}=0.025$
There at 300 k
Ex1:-
Let $\bar{J}=10 y^{2} z \overline{a x}-2 x^{2} y \overline{a y}+2 x^{2} z \overline{a z} \mathrm{~A} / m \leq^{2}$ find:
a) The total current crossing the surface $\mathrm{x}=3,2 \leq y \leq 3$,
$3.8 \leq z \leq 5.2$ in the $\bar{a} x$ direction.
b) The magnitude of the current density at the center of this area .
c) The average value of $J_{X}$ over the surface.

Solution:

$$
\begin{aligned}
& \mathrm{a}-\mathrm{I}=\bar{J} \cdot \bar{d} \mathrm{~s} \quad \text { but } \bar{d} \mathrm{~s}=\mathrm{dy} \mathrm{dz} \bar{a} x, \text { then } \\
& I_{x}=\iint(\bar{J}) \cdot \mathrm{dy} \mathrm{dz} \bar{a} x=\int_{3.8}^{5.2} \int_{y=2}^{3} 10 y^{2} z d y d z \\
& \therefore I_{x}=10\left[\frac{y^{3}}{3}\right]_{2}^{3}\left[\frac{z^{2}}{2}\right]_{3.8}^{5.2}=399 \mathrm{~A}
\end{aligned}
$$

b- The center of this area is $x=3$
$y_{0}=\frac{3-2}{2}=0.5, z_{0}=\frac{5.2-3.8}{2}=0.7$ then $p(3,2.5,4.5)$
$\therefore \bar{J}=10(2.5)^{2}(4.5) \bar{a} x-2(9)(2.5) \bar{a} y+2(9)(4.5) \bar{a} z$
$|\bar{J}|=\sqrt{J_{x}^{2}+J_{y}^{2}+J_{z}^{2}}=296 \mathrm{~A} / \mathrm{m}^{2}$
c- $J_{x}=\frac{I_{x}}{\text { area }}=\frac{399}{\iint d y d z}=\frac{399}{[y]_{2}^{3}[z]_{3.8}^{5.2}}=\frac{399}{(1)(1.4)}=$
$285 A / m^{2}$
Ex2:- In a region near the origin current density is in the radial (out word) direction with a value of $10 r^{-1.5} \mathrm{~A} / m^{2}$
a) How much current is crossing the spherical surface $r=1 \mathrm{~mm}$
b) Repeat for $r=2 \mathrm{~mm}$
c) At what rate is $\rho_{v}$ increasing at apoint where $\mathrm{r}=1 \mathrm{~mm}$
d) At what rate is the total charge increasing with in the sphere $r=1 \mathrm{~mm}$ Solution:

$$
\begin{gathered}
\mathrm{a}-\bar{J}=10 r^{-1.5} \bar{a} r A / m^{2} \\
\mathrm{I}=\iint \bar{J} . d s \quad \text { but } \quad \mathrm{ds}=r^{2} \sin \theta d \theta d \emptyset \bar{a} \mathrm{r}
\end{gathered}
$$

$$
\begin{gathered}
I=\int_{0}^{2 \pi} \int_{0}^{\pi}\left(10 r^{-1.5}\right)\left(r^{2} \sin \theta d \theta d \emptyset\right)=10 \sqrt{r}[4 \pi] \\
\therefore I=10 \sqrt{10^{-3}}[4 \pi]=\frac{4 \pi}{\sqrt{10}}=3.97 A \\
\text { b- } I=10 \sqrt{2 * 10^{-3}}[4 \pi]=5.62 A \\
\text { c- } \nabla \cdot \bar{J}=\frac{-\partial \rho_{v}}{\partial t} \quad \therefore \frac{\partial \rho_{v}}{\partial t}=-\nabla \cdot \bar{J} \\
\therefore \nabla \cdot \bar{J}=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} J_{r}\right)=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(10 r^{\frac{1}{2}}\right)=\frac{5}{r^{2} \sqrt{r}} \\
\quad \frac{\partial \rho_{v}}{\partial t}=\frac{-5}{10^{-6} \sqrt{10^{-3}}}=-5 \sqrt{10} \times 10^{7}=-1.581 \times 10^{8} \frac{C}{m^{2}} \cdot s \\
=-\int_{0}^{10^{-3}} \int_{0}^{2 \pi} \int_{0}^{\pi} \frac{5}{r^{2 \sqrt{r}}}\left(r^{2} \sin \theta d \theta d \emptyset d r\right) \\
\quad \mathrm{d} \frac{\partial Q}{\partial t}=\frac{\partial\left(\rho_{v \Delta v)}\right.}{\partial t}=\iiint^{\frac{\partial \rho_{v}}{\partial t} d v} \\
=-5[4 \pi] \int_{0}^{10^{-3}} r^{-1 / 2} d r \\
=-40 \pi[\sqrt{r}]_{0}^{10^{-3}}=\frac{-4 \pi}{\sqrt{10}}=-3.97 \\
\therefore \frac{\partial Q}{\partial t}=-3.97 C / s
\end{gathered}
$$

## Dielectric material and polarization :

The dielectric materials are insulators which has no any free charges then all electrons are bounded to the protons so no free charges even when electric field is applied. when $\bar{E}$ is applied these atoms becomes as atomic dipoles in response to external fields, these charges are called bound charges the dielectric material have permittivities different from the permittivity of free space these can be denoted by the relative permittivity $\varepsilon_{r}$ where it is greater than one :
$\varepsilon_{r} \geq 1$ for dielectric materials and
$\varepsilon=\varepsilon_{0} \varepsilon_{r} \quad$ where $\varepsilon_{0}$ for free space
$\varepsilon_{r}=5.4$ (mica), $\varepsilon_{r}=3$ (paper), $\varepsilon_{r}=16$ (germanium), $\varepsilon_{r}=3.5$ (nylon)
$\varepsilon_{r}=2.1($ Teflnon $), \varepsilon_{r}=3.8\left(\right.$ Quartz), $\varepsilon_{r}=4$ (glass).......
The characteristic which all dielectric materials have in common (solid ,liquid ,or gas) is their ability to store energy. This storage takes place by means of a shift in the relative positions of the internal bound charges against the normal molecular and atomic forces.

When any dielectric atom facing applied electric field then this atom should be charged to becomes an atomic dipole with dipole moment $\bar{p}$ $=\mathrm{Q} \bar{d} \mathrm{c} . \mathrm{m}$ and $\bar{p}_{\text {total }}=\sum_{i=1}^{n} \bar{p} i$ in volume of $\Delta v$.

Where Q is the positive of the two bound charges composing the dipole, and $d$ is the vector from the negative to the positive charge .

And the polarization is defined as the dipole moment per unit volume
$\bar{p}=\lim _{\Delta v \rightarrow 0} \frac{1}{\Delta v} \sum_{i=1}^{n} \bar{p} i \mathrm{C} / m^{2}$


If a dielectric in volume $\Delta v$ placed in electric field then all atoms should become atomic dipoles and there is surface bound charges on either faces of the dielectric these bound charges induces more free charges on the original electrodes so total charge increased and flux density increased .This process can be explained in steps and the polarization can be calculated:

electrads
$\bar{D}_{e}=E_{0} \cdot ._{.}$
The charges on the electrodes $Q_{0}$ but when we placed the dielectrics all the atoms become dipoles which induced new charges on the electrods then
$Q_{t}=Q_{0}+Q_{b}$ and the surfact charge density is
$\sigma_{t}=\sigma_{0}+\sigma_{b} \mathrm{C} / \mathrm{m}^{2}$
$\frac{Q t}{\Delta s}=\frac{Q 0}{\Delta s}+\frac{Q b}{\Delta s} \quad$ and $\quad \sigma_{b}=|\bar{p}|=\frac{Q b}{\Delta s} \mathrm{C} / \mathrm{m}^{2}$
And $\quad \mathrm{D}=D_{0}+\sigma_{b}$
or

$$
\bar{D}=\overline{D_{0}}+\bar{P}
$$

$\varepsilon_{0} \varepsilon_{r} \bar{E}=\varepsilon_{0} \bar{E}+\bar{P} \quad$ then $\bar{p}=\left(\varepsilon_{r}-1\right) \varepsilon_{0} \bar{E} \mathrm{C} / \mathrm{m}^{2}$
$\bar{p}$ is the polarization in the dielectric
or
$\bar{p}=X_{e} \varepsilon_{0} \bar{E} \quad$ where $X_{e}=\left(\varepsilon_{r}-1\right)$
If $\varepsilon_{r}=1$ there is no polarization.
EX: find the polarization within a material which :
a- Has an electric flux density of $1.5 \mu_{c} / m^{2}$ in an electric field intensity of $15 K_{v} / \mathrm{m}$.
b- Has $\mathrm{D}=2.8 \mu \mathrm{c} / \mathrm{m}^{2}$ and $X_{e}=1.7$.
c- Has $10^{20}$ molecules $/ m^{2}$, each with a dipole moment of $1.5 \times 10^{-26} \mathrm{c} . \mathrm{m}$ when $\mathrm{E}=10^{5} \mathrm{v} / \mathrm{m}$.
d- Has $\mathrm{E}=50 k_{v} / \mathrm{m}$ and the relative permittivity is 4.4
solution:
a- $\bar{D} \varepsilon_{0} \bar{E}+\bar{P}$
$\therefore \bar{P}=\bar{D}-\varepsilon_{0} \bar{E}=1.5 \times 10^{-6}-8.85 \times 10^{-12} \times 15 \times 10^{3}$

$$
=(1.5-0.133) \times 10^{-6}=1.367 \mu c / \mathrm{m}^{2}
$$

b- $X_{e}=\varepsilon_{r}-1$
$\mathrm{E}=\frac{D}{\varepsilon_{0} \varepsilon_{r}}=\frac{2.8}{2.7 \varepsilon_{0}} \mu \nu / \mathrm{m}$
$\therefore \mathrm{P}=\mathrm{D}-\varepsilon_{0} \mathrm{E}=2.8 \frac{2.8}{2.7}=1.763 \mathrm{Mc} / \mathrm{m}^{2}$
c- $\mathrm{P}=\frac{\Sigma p i}{\Delta v}$ but $\Sigma p i=10^{20} \times 1.5 \times 10^{-26}=1.5 \mu c . m$
$\therefore P \frac{1.5 \times 10^{-6}}{1}=1.5 \mu c / \mathrm{m}^{2}$ where $\Delta v=1 \mathrm{~m}^{3}$
d- $\mathrm{P}=\left(\varepsilon_{r}-1\right) \varepsilon_{0} \bar{E}$
$=3.4 \times 8.85 \times 10^{-12}\left(50 \times 10^{3}\right)$
$\therefore \mathrm{P}=1.505 \mu \mathrm{c} / \mathrm{m}^{2}$

## Boundary conditions:

When $\bar{E}, \bar{D}$ passing through acertain region (1) and then into another region (2), so we have $\bar{E}_{1}, \bar{D}_{1}$ in region (1) and $\bar{E}_{2}, \bar{D}_{2}$ in region (2) these are different in both region and have different directions with the normal to the boundary. There are boundary conditions at the interface between the two region (two conditions). Using these two boundary conditions and the characteristic of the two media then we can calculate the electric field in either region as:
$\bar{E}_{1}=\bar{E}_{t 1}+\bar{E}_{n 1}$ and $\bar{E}_{2}=\bar{E}_{t 2}+\bar{E}_{n 2}$
$\bar{D}_{1}=\bar{D}_{t 1}+\bar{D}_{n 1}$ and $\bar{D}_{2}=\bar{D}_{t 2}+\bar{D}_{n 2}$




For the electric field $\bar{E}$, use a closed path enclosing the boundary then $\oint \bar{E} \cdot \bar{d} \mathrm{l}=0$ then
$E_{t 2}-E_{t 1}=0$
$E_{t 1}=E_{t 2} \ldots \ldots$.(1) the first boundary condition *
For the flux density, using gauss s law at the boundary :
$\therefore D_{n 2}-D_{n 1}=\rho s$.
if $\rho s=0$ on the boundary then the $2^{\text {nd }}$ boundary condition
$D_{n 1}=D_{n 2}$ *
A) if both region are dielectrics $((\rho s=0))$ then
$E_{t 1}=E_{t 2}$ and $D_{n 1}=D_{n 2}$ *
but $D_{n 1}=\varepsilon_{1} E_{n 1}$ and $D_{n 2}=\varepsilon_{2} E_{n 2}$
Divide both condition to give :
$\therefore \frac{E_{t 1}}{\varepsilon_{1} E_{n 1}}=\frac{E_{t 2}}{\varepsilon_{2} E_{n 2}}$
$\therefore \frac{\tan \theta 1}{\varepsilon 1}=\frac{\tan \theta 2}{\varepsilon 2}$ or $\frac{\tan \theta 1}{\tan \theta 2}=\frac{\varepsilon 1}{\varepsilon 2}=\frac{\varepsilon r 1}{\varepsilon r 2}$ *
Where $\varepsilon_{1}=\varepsilon_{0} \varepsilon_{r 1}$ and $\varepsilon_{2}=\varepsilon_{0} \varepsilon_{r 2}$
B) if medium (1) dielectric and medium (2) conductor then in the conductor $E_{t 2}=0, D_{n 2}=0$ so
$E_{t 1}=E_{t 2}=0$
And $D_{n 2}-D_{n 1}=\rho s$ so $-D_{n 1}=\rho s$
$\therefore D_{n 1}=\rho s$

## if (1) is conductor then $D_{n 2}=\rho s$ for (2)dielectric

Also $\bar{E} n=\left(\frac{\bar{E} \cdot \bar{a} \mathrm{n}}{|\bar{a} \mathrm{n}|}\right) \bar{a} \mathrm{n}$ and $\bar{D} \mathrm{n}=\left(\frac{\bar{E} \cdot \bar{a} \mathrm{n}}{|\bar{a} \mathrm{n}|}\right) \bar{a} \mathrm{n}$

$$
\bar{E} \mathrm{t}=\bar{E}-\bar{E} \mathrm{n}, \quad \bar{D} \mathrm{t}=\bar{D}-\bar{D} \mathrm{n}
$$

Using the angle $\theta$ with the normal to the boundary then
$\mathrm{En}=\mathrm{E} \cos \theta$
And $\mathrm{Et}=\mathrm{E} \sin \theta$
$\theta=\tan ^{-1}\left(\frac{E t}{E n}\right)$
EX: The region $\mathrm{z}<0$ contains a dielectric material for which $\varepsilon_{r 1}=$ 2.5 while the region $\mathrm{z}>0$ is characterized by $\varepsilon_{r 2}=4$ let
$\bar{E} 1=-30 \bar{a} x+50 \bar{a} y+70 \bar{a} z$. find
a- $\bar{E}_{n 1}, \bar{E}_{n 1}, \bar{D}_{n 2}, \bar{D}_{n 2}, \theta_{1}, \theta_{2}$
b- $\quad \bar{E}_{2}, \bar{D}_{2}, \bar{p}_{1}, \bar{p}_{2}$
Solution :
a- The xy - plane is the boundary surface so $\bar{a} z$ normal to the interface .
$\therefore \bar{a} n=\bar{a} \mathrm{z}$
$\bar{E}_{1}=\bar{E}_{n 1}+\bar{E}_{t 1} \quad$ but $\quad \bar{E}_{n 1}=\left(\frac{\bar{E}_{1} \bar{a} \mathrm{z} .}{|\bar{a} \mathrm{z}|}\right) \bar{a} \mathrm{z}$
$\therefore \bar{E}_{n 1}=70 \bar{a} \mathrm{z}$ then $\bar{E}_{t 1}=\bar{E}_{1}-\bar{E}_{n 1}$
$\therefore \bar{E}_{t 1}=-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y} \mathrm{v} / \mathrm{m}$
$\left|\bar{E}_{t 1}\right|=\sqrt{900+2500}=58.309$
$\bar{D}=\epsilon \bar{E}$ then $D_{n 2}=\epsilon_{2} \bar{E}_{n 2}$ and, $\bar{D}_{t 2}=\epsilon_{2} \bar{E}_{t 2}$
From the boundary conditions for the two media are dielectrics then $\bar{E}_{t 1=} \bar{E}_{t 2}$ then $\bar{E}_{t 2}=-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y}$
$\bar{D}_{t 2=} \epsilon_{2} \bar{E}_{t 2}$ and $\bar{D}_{n 1=} \bar{D}_{n 2}$ then
$\bar{D}_{n 1=} \epsilon_{1} \bar{E}_{n 1}=2.5 \varepsilon_{0}(70 \bar{a} \mathrm{z})$
$\bar{D}_{n 2}=175 \varepsilon_{0} \bar{a} \mathrm{zc} / m^{2}, \bar{E}_{n 2}=\frac{175 \varepsilon_{0}}{4 \varepsilon_{0}}=43.75$
$\bar{D}_{t 2}=4 \varepsilon_{0}(-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y})=(-120 \bar{a} \bar{x}+200 \bar{a} \mathrm{y}) \varepsilon_{0} \mathrm{c} / m^{2}$
$\theta_{1}=\tan ^{-1}\left(\frac{\bar{E}_{t 1}}{\bar{E}_{n 1}}\right)=\tan ^{-1}\left(\frac{58.309}{70}\right)=39.8^{\circ}$
$\theta_{2}=\tan ^{-1}\left(\frac{\bar{E}_{t 2}}{\bar{E}_{n 2}}\right)=\tan ^{-1}\left(\frac{58.309}{43.75}\right)=53.1^{\circ}$
$\mathrm{b}-\bar{E}_{2}=\bar{E}_{t 2}+\bar{E}_{n 2}=-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y}+43.75 \bar{a} \mathrm{z} \mathrm{v} / \mathrm{m}$
$\bar{D}_{2}=\epsilon_{2} \bar{E}_{2}=4 \varepsilon_{0}(-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y}+43.75 \bar{a} \mathrm{z})$
$\therefore \bar{D}_{2}=1.062 \bar{a} \mathrm{x}+1.771 \bar{a} \mathrm{y}+1.549 \bar{a} \mathrm{znc} / m^{2}$
$\bar{p}_{1}=\left(\varepsilon_{r 1}-1\right) \varepsilon_{0} \bar{E}_{1}=1.5 \varepsilon_{0}(-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y}+70 \bar{a} \mathrm{z}) \mathrm{v} / \mathrm{m}$
$\bar{p}_{2}=\left(\varepsilon_{r 2}-1\right) \varepsilon_{0} \bar{E}_{2}=3 \varepsilon_{0}(-30 \bar{a} \mathrm{x}+50 \bar{a} \mathrm{y}+47.75 \bar{a} \mathrm{z}) \mathrm{v} / \mathrm{m}$
Capacitance: any two surfaces with potential different v and total charge on either one is Q (coulomb)then the Capacitance between these surfaces is:
$\mathrm{C}=\frac{Q}{V}$ farad for any shape of capacitor
The total can be given by any distribution ((line, surfaces, volume)) and the potential different may be calculated from the electric field. For surfaces charge $\rho s \mathrm{c} / \mathrm{m}^{2}$ and parallel plates with separation d then
$\mathrm{Q}=\iint \bar{D} \cdot d \bar{s}=\iint \epsilon \bar{E} \cdot \mathrm{~d} \bar{s}$
and $\mathrm{v}=\int_{d}^{o} \bar{E} . \mathrm{d} \bar{l}$


But as given before $\quad \bar{D} \mathrm{n}=\rho s \bar{a} \mathrm{Z} \quad$ or $\quad \bar{E} \mathrm{n}=\frac{\rho s}{\epsilon} \quad$ two plates then:
$\mathrm{Q}=\rho_{s} \mathrm{~S} \quad$ and $\quad \mathrm{V}=\int_{d}^{o} \frac{\rho s}{\epsilon} \mathrm{dz}=\frac{\rho s d}{\epsilon}$
$\therefore \mathrm{C}=\frac{Q}{V}=\frac{\epsilon s}{d}$
Or C $=\frac{\varepsilon_{0 \varepsilon_{r} s}}{d} \quad$ farad for parallel plant only
The energy stored in such capacitor is:
$\mathrm{We}=\frac{1}{2} \iiint \epsilon E^{2} \mathrm{~d} v=\frac{1}{2} \epsilon \iiint\left(\frac{\rho s}{\epsilon}\right)^{2} \mathrm{dx} \mathrm{dy} \mathrm{dz}=\frac{1}{2} \frac{\rho s^{2}}{\epsilon} \mathrm{sd} *\left(\frac{\epsilon d}{\epsilon d}\right)$
$\therefore$ we $=\frac{1}{2} \frac{\rho s^{2} d^{2}}{\epsilon^{2}}\left(\frac{\epsilon S}{d}\right)=\frac{1}{2} c v^{2}=\frac{1}{2} \frac{Q^{2}}{c}$ joule
The capacitance of the parallel plate capacitor depends on S (plate area), d (separation), and $\varepsilon_{r}$ for the dielectric between the plates.

Series, parallel ,and compound capacitors can be forward :

a- for series: $\quad \mathrm{V}=V_{1+} V_{2}, \quad \mathrm{~d}=d_{1+} d_{2}$
$\therefore \mathrm{C}=\frac{1}{\frac{d 1}{\epsilon 1 s}+\frac{d 2}{\epsilon 2 s}}=\frac{1}{\frac{1}{c 1}+\frac{1}{c 2}}=\frac{c 1 c 2}{c 1+c 2}=\mathrm{c}$ eq.
$\therefore \frac{1}{C}=\frac{1}{C 1}+\frac{1}{C 2}+\frac{1}{C 3}+\ldots \ldots$.
b- for parallel capacitor : $V_{1}=V_{2}=\mathrm{V}, d_{1}=d_{2}=\mathrm{d}$
$\mathrm{E}=E_{1}=E_{2}=\frac{v}{d}, \quad Q_{1}=\rho_{s 1} \mathrm{~s} 1$ and $Q_{2}=\rho_{s 2} \mathrm{~s} 2=D_{2} S_{2}$
$\mathrm{Q}=Q_{1+} Q_{2}=D_{1} S_{1}+D_{2} S_{2}=\epsilon_{1} E S_{1}+\epsilon_{2} E S_{2}$
$\therefore \mathrm{C}=\frac{Q}{V}=\frac{\epsilon_{1} E S_{1}+\epsilon_{2} E S_{2}}{E d}=\frac{\epsilon_{1} E S_{1}+\epsilon_{2} E S_{2}}{d}=\mathrm{C} 1+\mathrm{C} 2$
$\therefore \mathrm{C}=\mathrm{C} 1+\mathrm{C} 2=\mathrm{ceq}$
c- the compound is combination of both series and parallel $\epsilon_{0}$ get equivalent capacitance :

EX1: point charge of $10 \mu c$ at $(0,0,0)$, calculate the capacitance between the surface $\mathrm{r}=2$ and $\mathrm{r}=6$ with dielectric of $\varepsilon_{r}=3.5$ between these two surface .

Solution:
$\mathrm{C}=\frac{Q}{V a b} \quad$ but $\quad \mathrm{Vab}=-\int \bar{E} . \mathrm{d} \bar{l}$
And

$$
\bar{E}=\frac{Q \quad \overline{a r}}{4 \pi \epsilon_{0 r^{2}}}
$$

Then Vab $=-\int_{6}^{2} \frac{Q}{4 \pi \epsilon_{0 \epsilon_{r} r^{2}}}=\frac{Q}{4 \pi \epsilon_{0} \epsilon_{r}}\left[\frac{1}{r}\right]_{6}^{2}$
$\therefore \mathrm{vab}=\frac{Q}{4 \pi \epsilon_{0} \epsilon_{r}}\left[\frac{1}{2}-\frac{1}{6}\right]$
$\therefore \mathrm{C}=\frac{4 \pi \epsilon_{0} \epsilon_{r}}{\left[\frac{1}{2}-\frac{1}{6}\right]}=\frac{4 \pi \epsilon_{0}(3.5) 6}{2}=1.17 \mathrm{nF}$.
EX2: infinite line along the z-axis with $20 \mathrm{nc} / \mathrm{m}$.
Calculate the capacitance between $\rho=10$ and $\rho=4$ for 5 m length.
Solution:
$\mathrm{C}=\frac{Q}{V a b} \quad$ but $\quad \mathrm{Q}=\int \rho_{l} d l=\rho_{l} l$
$\mathrm{Vab}=-\int \bar{E} . \mathrm{d} \bar{l}$ but $\bar{E}=\frac{\rho_{l}}{2 \pi \epsilon_{0} \rho} \overline{a \rho}$
And $\mathrm{d} \bar{l}=\mathrm{d} \rho \bar{a} \rho+\rho d \emptyset \bar{a} \emptyset+\mathrm{dz} \bar{a} \mathrm{z}$
$\therefore \mathrm{Vab}=\int_{10}^{4}\left(\frac{\rho_{l}}{2 \pi \epsilon_{0} \rho}\right) \mathrm{d} \rho=-\frac{\rho_{l}}{2 \pi \epsilon_{0}} \int_{10}^{4} \frac{d \rho}{\rho}$
$\left.\therefore \mathrm{vab}=-\frac{\rho_{l}}{2 \pi \epsilon_{0}} \ln \rho\right]_{10}^{4}=-\frac{\rho_{l}}{2 \pi \epsilon_{0}} \ln \frac{10}{4}=-\frac{\rho_{l}(0.916)}{2 \pi \epsilon_{0}}$
$\therefore \mathrm{C}=\frac{\rho_{l} l}{0.916 \rho_{l}}\left(2 \pi \epsilon_{0}\right)=\frac{2 \pi \epsilon_{0}(5)}{0.916}=0.3 \mathrm{nf}$.

Ex3: for the figure shown let $\varepsilon_{r 1}=4, \varepsilon_{r 2}=6, d_{1}=3 \mathrm{~mm}, d_{2}=2 \mathrm{~mm}$ and $\mathrm{s}=12 \mathrm{~cm}^{2}$ if $\rho_{s}$ on the lower plate be $240 \mathrm{nc} / \mathrm{m}^{2}$ find E in each region the voltage between the two plates and the capacitance between the two plates.

Solution:-
On the lower plate
$D_{n}=\rho_{s}$ then $D_{n 1}=D_{1}$ and $E_{1}=\frac{D_{1}}{\varepsilon_{0 \varepsilon_{r 1}}}$

$E_{1}=\frac{240 \times 10^{-9}}{(4)\left(8.85 \times 10^{-12}\right)}=6780 \mathrm{v} / \mathrm{m}$
At the boundary between medium (1) and (2) we have :
$D_{n 1}=D_{n 2}$ then $E_{2}=\frac{D_{n 2}}{\epsilon_{2}}=\frac{240 \times 10^{-9}}{(6)\left(8.85 \times 10^{-12}\right.}=4520 \mathrm{~V} / \mathrm{m}$
$V_{1}=E_{1} d_{1}=6780 \times 3 \times 10^{-3}=20.34$ volts
$V_{2}=E_{2} d_{2}=4520 \times 2 \times 10^{-3}=9.04$ volts.
$\mathrm{V}=V_{1+} V_{1}=20.34+9.04=29.39$ volts
(the voltage between the two plates).
There are many methods to calculate the capacitance.

$$
\begin{aligned}
& C=\frac{Q}{V}, \text { or } \frac{1}{C}=\frac{1}{C 1}+\frac{1}{C 2} \text { and use } C 1=\frac{\varepsilon_{1} s}{d_{1}}, C 2=\frac{\varepsilon_{2} s}{d_{2}} \\
& \therefore C 1=\frac{4\left(8,85 \times 10^{-12}\right)\left(12 \times 10^{-4}\right)}{3 \times 10^{-3}}=0.0146 \mathrm{nF} \\
& C 2=\frac{6\left(8,85 \times 10^{-12}\right)\left(12 \times 10^{-4}\right)}{2 \times 10^{-3}}=0.03186 n F \\
& \therefore C=\frac{C 1 C 2}{C 1+C 2}=\frac{(0.0146)(0.03186)}{0.0146+0.03186} \times 10^{-9}=9.8 \mathrm{PF}
\end{aligned}
$$

Or $\mathrm{C}=\frac{Q}{V}$

$$
=\frac{0.288 \times 10^{-9}}{29.34}=9.8 \mathrm{PF}
$$

Electro-magnetics II Dr. Ahmed A. Abbas
Lecture: 4

## Poisson's and laplace's equations:

These equations are partial differential equations can be obtained and used to get the electrostatic quantities in free space or through volume charge. These equations may be represented as single, two or three variables and these are of $2^{\text {nd }}$ order to be solved using a certain conditions. To derive these equation using the divergence then:
$\nabla \cdot \bar{D}=\left[\begin{array}{ll}\rho & \text { inside the volume charge. } \\ 0 & \text { outside the }\end{array}\right.$
utside the volume charge.

But $\bar{D}=\in \bar{E}$
$\therefore \nabla \cdot \bar{E}=\left[\begin{array}{c}\frac{\rho V}{\epsilon} \\ { }_{0}\end{array}\right.$
and $\bar{E}=-\nabla \mathrm{V}$
then
$\nabla \cdot(\nabla \mathrm{V})=\left[_{0}^{-\frac{\rho v}{\epsilon}}\right.$
$\nabla \cdot(\nabla \mathrm{V})=\frac{-\rho v}{\epsilon} \quad$ poison equation inside the volume charge
$\nabla \cdot(\nabla \mathrm{V})=0 \quad$ laplace equation outside the volume charge
$\nabla$. ( $\mathrm{\nabla V}$ ) can be used as $\nabla^{2} V$ so
$\nabla^{2} V=\left[_{0}^{-\frac{\rho v}{\epsilon}}\right.$ using $\nabla \mathrm{V}$ and $\nabla$. ( $\nabla \mathrm{V}$ ) as given before to get the final relation
$\nabla \cdot(\nabla \mathrm{V})=\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)$
$\therefore \nabla^{2} V=\frac{\partial^{2} V}{\partial x^{2}}+\frac{\partial^{2} V}{\partial y^{2}}+\frac{\partial^{2} V}{\partial z^{2}}=\frac{-\rho v}{\epsilon} \quad($ (Poison $\left.)\right)$
Or
$\nabla^{2} V=\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{\partial y^{2}}+\frac{\partial^{2} v}{\partial z^{2}}=0 \quad(($ Laplace $))$

In cylindrical we have :
$\nabla^{2} V=\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial v}{\partial \rho}\right)+\frac{1}{\rho^{2}}\left(\frac{\partial^{2} v}{\partial \emptyset^{2}}\right)+\frac{\partial^{2} v}{\partial z^{2}}$
In spherical:
$\nabla^{2} V=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial v}{\partial r}\right)+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial v}{\partial \theta}\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} v}{\partial \Phi^{2}}$
If the potential V is a function of X then these equation reduced to
$\frac{\partial^{2} v}{\partial x^{2}}=0 \quad$ or $\frac{\partial^{2} v}{\partial x^{2}}=\frac{-\rho v}{\epsilon}$ to be solve
If V is a function of $\rho$ only then
$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial v}{\partial \rho}\right)=\left[-\frac{\rho v}{\epsilon} \quad\right.$ and so on for others
$\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{\partial y^{2}}=0$ or $\frac{-\rho v}{\epsilon}$
To get $\mathrm{V}, \mathrm{E}, \mathrm{D} . . . .$. .then a solution should be given using the boundary conditions , this solution gives the relation $V$ then $E, D, \ldots$ can be estimated

## Solution of Poisson's and laplace's equations:

If the potential V is function of single variable ( $\mathrm{x}, \mathrm{y}, \mathrm{z}, \rho, \emptyset, \mathrm{r}, \theta$ ) the solution of these equation can be deduced by integrating twice as show in the following examples:

Ex1: Assume that V is a function of x and at $\mathrm{x}=4, \mathrm{~V}=50$ volt at $\mathrm{x}=6, \mathrm{~V}=150$ volt. find $\mathrm{V}, \bar{E}$ and draw these with respect to $X$ _axis.

Solution :-
$\nabla^{2} V=0$ and $\frac{\partial^{2} v}{\partial x^{2}}=0 \quad($ Integrate twice $\left.)\right)$
$\therefore \frac{\partial v}{\partial x}=A \quad$ then the final solution is :
$V=A x+B$ use the boundary conditions
at $x=4, v=50$ then
$50=4 \mathrm{~A}+\mathrm{B}$ $\qquad$ 1
at $\mathrm{x}=6, \mathrm{~V}=150$ then
$150=6 A+B$ $\qquad$ 2

Solve eq. 1 and eq. 2 to get the constant $A, B$
$100=2 A$ then $A=50$ and $B=50-200=-150$
$\therefore \mathrm{V}=50 \mathrm{x}-150 \quad$ final solution with constant values
At $x=0, V=-150$, at $x=1, V=-100$, at $x=3, V=0$,
At $x=6, V=150$, at $x=-1, V=-200$
$\bar{E}=-\nabla \mathrm{V}=\frac{-\partial V}{\partial X} \bar{a} x=-50 \bar{a} x$

Ex2:- in a dielectric region for wich $\epsilon r=2.5$, the $\mathrm{V}=20$ volt at $\rho=2$ and $\mathrm{V}=50 \mathrm{v}$ at $\rho=1$ give the general solution for $\mathrm{V},|\bar{E}|$, every where

Solution :-
$V=\mathrm{f}(\rho)$ then using Laplace eq.
we have :
$\frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial v}{\partial \rho}\right)=0(($ Integrate twice $))$ then
$\rho \frac{\partial v}{\partial \rho}=A$
And $\frac{\partial v}{\partial \rho}=\frac{A}{\rho}$ then $\partial v=\mathrm{A} \frac{\partial \rho}{\rho}$
$\therefore \mathrm{V}=\mathrm{A} \operatorname{Ln} \rho+B$ solution to get $\mathrm{A}, \mathrm{B}$ then
$\mathrm{V}=20$ at $\rho=2, \mathrm{~V}=50$ at $\rho=1$.

$$
\bar{E}=-\nabla \mathrm{V}=\frac{-\partial V}{\partial \rho} \bar{a} \rho
$$

$\therefore 20=\mathrm{A} \mathrm{Ln} 2+\mathrm{B}$ $\qquad$ 1
$50=\mathrm{A} \mathrm{Ln} 1+\mathrm{B} \_\ldots 2$ solve to get $\mathrm{A}, \mathrm{B}$
$30=\mathrm{A} \operatorname{Ln}\left(\frac{1}{2}\right)$
$\therefore \mathrm{A}=\frac{30}{\operatorname{Ln}\left(\frac{1}{2}\right)}=-43.3$
$\therefore \mathrm{B}=50$ then $\mathrm{V}=-43.3$

Ex3:- $\rho v=25 \mathrm{mc} \backslash m^{2}$ is distributed uniformly through a certain region if $\mathrm{V}=40$ volt at $\mathrm{y}=3$ and $\mathrm{V}=30$ volt at $\mathrm{y}=5$. Give the solution and calculate V at $\mathrm{y}=0,1,-2$ use $\epsilon r=3$.

Solution:-
$\nabla^{2} V=\frac{-\rho v}{\epsilon}$ then $V=f(y)$, so
$\frac{\partial^{2} v}{\partial y^{2}}=\frac{-\rho v}{\epsilon_{0} \epsilon_{r}}=\frac{-\rho v}{3 \epsilon_{0}}$
$\therefore \frac{\partial v}{\partial y}=\frac{-\rho v}{3 \epsilon_{0}} y+A$
$V=\frac{-\rho v}{6 \epsilon_{0}} y^{2}+A y+B$
To get $A, B$ use the condition given:
$\therefore 40=\frac{-\rho v}{6 \epsilon_{0}}(9)+3 A+B$ $-1$
$30=\frac{-\rho v}{6 \epsilon_{0}}(25)+5 A+B--------2$

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 4
$\therefore 10=\frac{\rho v}{6 \epsilon_{0}}(6)-2 A$
$\therefore \mathrm{A}=\frac{1}{2}\left(\frac{\rho v}{\epsilon_{0}}-10\right)=\left(\frac{25 \times 10^{-6}}{17.7 \times 10^{-12}}-5\right)$
$B=40+\frac{3 \rho v}{2 \epsilon_{0}}-3\left(\frac{\rho v}{2 \epsilon_{0}}-5\right)=55$ use these then
$\mathrm{V}=\frac{-\rho v}{6 \epsilon_{0}} y^{2}+\left(\frac{\rho v}{2 \epsilon_{0}}-5\right) y+55$
Use $y=0,1,-2$ to calculate $V$ at these positions

## The steady magnetic field:-

In this part, the magnetic field produce by current sources is introduce. This include the basic relations that can be used to develop the magnetic field study for d.c current sources. These sources may be line current $(\bar{I})$ surface current $(\bar{K})$ or volume current $(\bar{J})$ for any type of current, there should be magnetic field (intensity, density ,flux ,force, potential and so on).

Other formula as also included using Laplace's and Poisson's equations
Biot-savart law:-
It states that at any point Q the magnetic field intensity produced by the current source is proportional to the current, dimension and the sine of the angle lying between the current and a line connecting the filament to the point P where the field is desired. This can be given as:-
$\mathrm{d} \bar{H}=\frac{I \bar{d} \times \bar{a} R}{4 \pi|\bar{R}|^{2}}=\frac{I d l \sin \theta}{4 \pi|\bar{R}|^{2}}$
for total length then
$\bar{H}=\int_{l 1}^{l 2} \frac{I d \bar{l} \times \bar{a} R}{4 \pi|\bar{R}|^{2}} \quad \mathrm{~A} / \mathrm{m}$
$\bar{H}:$ magenetic filed intensity
or
$\bar{H}=\int_{l 1}^{l 2} \frac{I d \bar{l} \times \bar{R}}{4 \pi|\bar{R}|^{3}} \quad$ and $\bar{H}=\int_{l 1}^{l 2} \frac{I d l \sin \theta}{4 \pi|\bar{R}|^{2}}$
For surface current with $\bar{K} A / m$ then
$\bar{H}=\iint \frac{\bar{K} d s \times \overline{a R}}{4 \pi|\bar{R}|^{2}}$
F. Alm then
where $\bar{K}$ : surface current density $\left(\mathrm{K}=\frac{I}{b}\right) \mathrm{A} / \mathrm{m}$
for current density $\bar{J} \mathrm{~A} \backslash m^{2}$ then


Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 5
$\bar{H}=\iiint \frac{\bar{J} d v \times \bar{a} R}{4 \pi|\bar{R}|^{2}}$

Use any of these three relations to simplify the final formula for the magnetic field intensity ( $\bar{H}$ )

Ex1:- line current $\bar{I}$ along the Z - axis and the point $\mathrm{p}(0, \mathrm{y}, 0)$. Determine the magnetic field at p using the Biot-Savart law. Use $\bar{I}=10 \bar{a} z A$ and $\mathrm{y}=3$ for calculation of $\bar{H}$ at p using finite length ( $\left.1_{1}=l_{2}=10 \mathrm{~m}\right)$ and infinite length.

Solution:-
$\bar{H}=\int_{l 1}^{l 2} \frac{\bar{I} d l \times \bar{a} R}{4 \pi R^{2}}$
but $\bar{I} d l=I d z \bar{a} z$
$\bar{R}=y \bar{a} y \pm z \bar{a} z=\rho \bar{a} \rho+z \bar{a} z$
$|\bar{R}|=\sqrt{y^{2}+z^{2}}=\sqrt{\rho^{2}+z^{2}}$
$\bar{a} R=\frac{\rho}{\sqrt{\rho^{2}+z^{2}}} \bar{a} \rho \pm \frac{z}{\sqrt{\rho^{2}+z^{2}}} \bar{a} z$
$\bar{H}=\int_{l 1}^{l 2} \frac{I d z \bar{a} z \times(\rho \bar{a} \rho \pm z \bar{a} z)}{4 \pi\left(\rho^{2}+z^{2}\right)^{3 / 2}}$
But

$$
\bar{a} z \times \bar{a} \rho=\bar{a} \emptyset
$$

And
$\bar{a} z \times \bar{a} z=0$
$\therefore \bar{H}=\frac{I}{4 \pi} \int_{l 1}^{l 2} \frac{\rho d z}{\left(\rho^{2}+z^{2}\right)^{3 / 2}} \bar{a} \emptyset$

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 5

$$
=\frac{I \rho}{4 \pi}\left[\frac{z}{\rho^{2} \sqrt{\rho^{2}+z^{2}}}\right]_{l_{1}}^{l_{2}} \bar{a} \emptyset
$$

Or
$\bar{H}=\frac{I}{4 \pi \rho}\left[\frac{l_{2}}{\sqrt{\rho^{2}+l_{2}^{2}}}-\frac{l_{1}}{\sqrt{\rho^{2}+l_{2}^{2}}}\right] \bar{a} \emptyset$
$\therefore \bar{H}=\frac{I}{4 \pi \rho}\left[\sin \propto_{2}-\sin \propto_{1}\right] \bar{a} \emptyset \quad$ for finite length $1_{1}=l_{2}$

For infinite length when $l_{2}=\infty$ then $\alpha_{2}=90^{\circ}$ and if $l_{1}=-\infty$ then $\alpha_{1}=-90^{\circ}$ then
$\bar{H}=\frac{I}{4 \pi \rho}[\sin 90-(\sin -90)] \bar{a} \emptyset$
$\bar{H}=\frac{I}{2 \pi \rho} \bar{a} \emptyset \quad \mathrm{~A} / \mathrm{m}$
where $\bar{a} \emptyset=\bar{a} I \times \bar{a} \rho$
Or $\bar{a} H=\overline{a I} \times \bar{a} n$
For $l_{1}=l_{2=} 10$ then $\alpha_{1}=\sin ^{-1}\left(\frac{10}{\sqrt{10^{2}}+3^{2}}\right)=73.3=-\alpha_{2}$
$\therefore \bar{H}=\frac{I}{4 \pi \rho}[\sin 73.3-\sin (-73.3)] \bar{a} \emptyset=\frac{10}{2 \pi(3)}(0.958) \bar{a} \emptyset$
$\bar{H}=0.508 \bar{a} \emptyset \quad \mathrm{~A} / \mathrm{m}$
For infinite length then $\bar{H}=\frac{10}{2 \pi(3)} \bar{a} \emptyset=0.53 \bar{a} \emptyset A / m$
$\bar{a} \emptyset=-\sin \emptyset \bar{a} x+\cos \emptyset \bar{a} y=-\sin 90 \bar{a} x+\cos 90 \bar{a} y$
$\therefore \bar{a} \emptyset=-\bar{a} x$ then
$\bar{H}=-0.53 \bar{a} x A / m$

Ex2:-Two infinite line current $\overline{\mathrm{I}}_{1}=2 \overline{\mathrm{a}} \mathrm{y} \mathrm{A}$ at $\mathrm{x}=\mathrm{z}=4$ and $\overline{\mathrm{I}}_{2}=4 \overline{\mathrm{a}} \mathrm{x}$ at $\mathrm{y}=3, \mathrm{z}=-6$. Calculate $\overline{\mathrm{H}}$ at $(0,0,0)$.

Solution :-
At $(0,0,0)$ then use
$\bar{H}=\frac{I_{1}}{2 \pi \rho_{1}} \bar{a} \emptyset$
$I_{1}=2 A$
$\bar{\rho}_{1}=-4 \bar{a} x-4 \bar{a} z,\left|\bar{\rho}_{1}\right|=\sqrt{32}$

$$
\bar{a} \rho_{1}=\frac{-4}{\sqrt{32}} \bar{a} x-\frac{4}{\sqrt{32}} \bar{a} z
$$

$\therefore \bar{a} \emptyset_{1}=\bar{a} I_{1} \times \bar{a} \rho_{1}=\bar{a} y \times\left(\frac{-4}{\sqrt{32}} \bar{a} x-\frac{4}{\sqrt{32}} \bar{a} z\right)$
$\bar{a} \emptyset_{1}=\frac{4}{\sqrt{32}}(\bar{a} z-\bar{a} x)$
$\therefore \bar{H}_{1}=\frac{2}{2 \pi \sqrt{32}}\left[\frac{4}{\sqrt{32}}(\bar{a} z-\bar{a} x)\right]=\frac{1}{8 \pi}(\bar{a} z-\bar{a} x)$
$\bar{H}_{2}=\frac{I_{2}}{2 \pi \rho_{2}} \bar{a} \emptyset_{2}$
$\bar{\rho}_{2}=-3 \bar{a} y+6 \bar{a} z,\left|\bar{\rho}_{2}\right|=\sqrt{45}, \bar{a} \rho_{2}=\frac{-3}{\sqrt{45}} \bar{a} y+\frac{6}{\sqrt{45}} \bar{a} z$
$\therefore \bar{a} \emptyset_{2}=a x \times\left(\frac{-3}{\sqrt{45}} \bar{a} y+\frac{6}{\sqrt{45}} \bar{a} z\right)=\frac{3}{\sqrt{45}}(-\bar{a} z-2 \bar{a} y)$
$\therefore \bar{H}_{2}=\frac{4}{2 \pi \sqrt{45}}\left[\frac{-3}{\sqrt{45}}(2 \bar{a} y+\bar{a} z)\right]=\frac{6}{45 \pi}[-2 \bar{a} y-\bar{a} z]$
$\therefore \bar{H}=\bar{H}_{1}+\bar{H}_{2}=\frac{-\bar{a} x}{\pi}-\frac{12}{45 \pi} \bar{a} y+\left(\frac{1}{8 \pi}-\frac{6}{45 \pi}\right) \bar{a} z$

$$
\bar{H}=-0.04 \bar{a} x-0.085 \bar{a} y-0.002 \bar{a} z \quad A / m
$$

Electro-magnetics II Dr. Ahmed A. Abbas
Lecture: 6

## Ampere's circuital law:-

This is a law that helps in solving problems more easily and it is derived from the Biot Savart law. It states that the line integral of $\bar{H}$ about any closed path is exactly equal to the direct current enclosed by that path.

$$
\oint \bar{H} . d \bar{l}=I
$$

To prove this let consider infinite line current along the z -
axis then $\bar{H}=\frac{I}{2 \pi \rho} \bar{a} \emptyset$ and choose a closed path circular path $(d l=\rho d \emptyset)$ then
$\oint \bar{H} \cdot d \bar{l}=\int_{0}^{2 \pi}\left(\frac{I}{2 \pi \rho} a \emptyset\right) \cdot(\rho d \varnothing \bar{a} \emptyset)$

$$
=\frac{I}{2 \pi} \int_{0}^{2 \pi} d \emptyset=\mathrm{I}
$$

Then $\oint \bar{H} . d \bar{l}=I$
Ampere's law applications:-
Infinity long coaxial transmission line: if this line is carrying a uniformly distributed total current (I) in the center conductor and (-I)in the outer conductor. By symmetry, H is not a function of $\varnothing$ or $\mathbf{Z}$, but it depends on $\rho$. Use Ampere's law in all region to get $\bar{H}$ in each one we have four regions :
$0 \leq \rho \leq \mathrm{a}, \mathrm{a} \leq \rho \leq \mathrm{b}, \mathrm{b} \leq \rho \leq \mathrm{c}$, and $\rho \geq \mathrm{c}$ :


Region $1:$ - for $0 \leq \rho \leq$ a then $\oint \bar{H} . d \bar{l}=I ̀ I ~ a n d ~$
$\grave{I}=\frac{J \pi \rho^{2}}{J \pi a^{2}} I=I\left(\frac{\rho}{a}\right)^{2}$
$\therefore \oint \bar{H} \cdot d \bar{l}=I \frac{\rho^{2}}{a^{2}}$

Electro-magnetics II Dr. Ahmed A. Abbas
Lecture: 6
$\mathrm{H}(2 \pi \rho)=I \frac{\rho^{2}}{a^{2}} \quad$ then $\quad \bar{H}=\frac{I \rho}{2 \pi a^{2}} \bar{a} \emptyset \mathrm{~A} / \mathrm{m}$
Region 2:- for $\mathrm{a} \leq \rho \leq \mathrm{b}$ then $\oint \bar{H} . d \bar{l}=I$
$\mathrm{H}(2 \pi \rho)=\mathrm{I}$ then $\bar{H}=\frac{I}{2 \pi \rho} \bar{a} \emptyset$
Region 3:-for $\mathrm{b} \leq \rho \leq \mathrm{c}$ then $\therefore \oint \bar{H} . d \bar{l}=I-\bar{I}$
But $\left.\frac{\grave{I}}{I}=\frac{J \pi\left(\rho^{2}-a^{2}\right)}{J \pi\left(c^{2}-b^{2}\right.}\right)$
$\therefore \grave{I}=I\left(\frac{\rho^{2}-b^{2}}{c^{2}-b^{2}}\right)$
$H(2 \pi \rho)=I-I\left(\frac{\rho^{2}-b^{2}}{c^{2}-b^{2}}\right)$ this gives $:-$
$\bar{H}=\frac{I}{2 \pi \rho} \frac{c^{2}-\rho^{2}}{c^{2}-b^{2}} \bar{a} \emptyset$
Region 4:- for $\mathrm{c} \leq \rho \leq \infty$ then $\oint \bar{H} . d \bar{l}=I-I=0$

$\therefore \mathrm{H}(2 \pi \rho)=0 \quad$ then $\mathrm{H}=0$
b- Consider a sheet of current :- If a current sheet in the xy-plane $(\mathrm{z}=0)$ with current in y direction with $\bar{K}=k y \bar{a} y$ as shown for a closed path enclosed part of the sheet then :
$\oint \bar{H} \cdot d \bar{l}=\int_{0}^{l} k d k$
$\therefore H_{x 1} l+H_{x 2}(-l)=k l$
or $H_{x 1} l-H_{x 2}=k y$
but $H_{x}$ is the same for all positive z and is the same for all negative z . The magnetic field H on side of the current sheet is the negative of that on the other : above the sheet:
$H_{x}=\frac{1}{2} k y \quad(\mathrm{z}>0)$
While below it,
$H_{x}=-\frac{1}{2} k y(\mathrm{z}<0)$

Electro-magnetics II Dr. Ahmed A. Abbas
Lecture: 6

In general for any sheet of current
$\bar{H}=\frac{1}{2} \bar{K} \times \bar{a} n$
Ex1:- Two infinite current sheet $\bar{K}_{1}=20 \bar{a} z$ at $\mathrm{y}=4$ and
$\bar{K}_{2}=-20 \bar{a} z$ at $\mathrm{y}=1$ find $\bar{H}$ every where
Solution:-
Region 1: $\quad-\infty \leq y \leq 1$

$\bar{H}_{1}=\frac{1}{2} \bar{K}_{1} \times(-\bar{a} y)=\frac{1}{2} 20 \bar{a} z \times(-\bar{a} y)$
$\therefore \bar{H}_{1}=10 \bar{a} x$
$\bar{H}_{2}=\frac{1}{2} \bar{K}_{2} \times(-\bar{a} y)=10(-a z) \times(-a y)$
$\therefore \bar{H}_{2}=-10 \bar{a} x$
$\bar{H}=\bar{H}_{1}+\bar{H}_{2}=0$
Region 2: $4 \leq \mathrm{y} \leq \infty$ similar $\bar{H}=0$ for region $1 \leq \mathrm{y} \leq 4, \bar{H}=20 \bar{a} x$

## c- Solenoid and Toroid :-

For infinite length solenoid with N-turn or circular current sheet then using Ampere's law, the magnetic field inside and outside these :

$\mathrm{H}=0 \quad(\rho>a)$

$$
\mathrm{H}=0 \quad(\rho>a)
$$

$\bar{H}=K a \bar{a} \emptyset(\rho<a)$
$\bar{H}=\frac{N I}{d} \bar{a} z(\rho<a)$

Toroids:

$\mathrm{H}=0$ (outside)

$$
\begin{aligned}
& \mathrm{H}=0 \text { (outside) } \\
& \bar{H}=\frac{N I}{2 \pi \rho} \bar{a} \emptyset(\text { inside })
\end{aligned}
$$

Ex2:-For $\mathrm{I}=2 \mathrm{~A}$ in a toroide of 500 turns with dimension cross section radius is $1 \mathrm{~cm}, \rho_{0}=$ 20 cm (center to center). Find $\bar{H}$ at $\mathrm{y}=0,10 \mathrm{~cm}, 20,22 \mathrm{~cm}$

Solution :-
At $\mathrm{y}=0$ then $\mathrm{H}=0$ (outside)
$\mathrm{y}=10$ then $\mathrm{H}=0$ (outside)
$\mathrm{y}=20 \mathrm{~cm}$ then $\mathrm{H}=\frac{N I}{2 \pi \rho}=\frac{500 \times 2}{2 \pi\left(20 \times 10^{-2}\right)}=\frac{500}{\pi(0.2)} \mathrm{A} / \mathrm{m}$
$\mathrm{y}=22$ then $\mathrm{H}=0$ (outside).

Magnetic flux and magnetic flux-density:-
Each current source produce a magnetic flux ( $\psi \mathrm{m}$ ), measured in weber ( wb ). When this flux crosses acertain area then there is magnetic flux density $(\bar{B})$ in $\mathrm{wb} / \mathrm{m}^{2}$ or (Tesla). Also $\bar{B}$ can be calculated from:
$\bar{B}=\mu \bar{H}=\mu_{0} \mu_{r} \bar{H} \quad \mathrm{wb} / m^{2}$ or Tesla
$\mu_{r}$ : is the relative permeability and $\mu_{0}$ : is the permeability for free space

$$
\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}
$$

All magnetic materials have $\mu_{r} \gg 1$ and for air (free space) $\mu_{r}=1$
The magnetic flux is
$\Psi_{m}=\iint \bar{B} \cdot d \bar{s}$
wb
Or

$$
\Psi_{m}=\mu \iint \bar{H} \cdot d \bar{s}
$$

Then by calculating $\bar{H}$, we can get $\bar{B}$ through the material for a certain $\mu_{r}$.

Also $\psi_{m}=\iint B d s \cos \theta$

$\Psi_{m}=0$ when $\theta=90^{\circ}$ and maximum when $\theta=0$
Ex3:- Infinite current $\bar{I}=20 \bar{a} z$ A. Along the z -axis calculate $\bar{B}$ at $(3,4,5)$ then find magnetic flux crosses the surface:
a- $\rho=4,0 \leq \emptyset \leq \pi, 1 \leq z \leq 10$.

b- $\varnothing=90^{\circ}, 1 \leq \rho \leq 4,1 \leq z \leq 10$.

Solution :-

$$
\bar{H}=\frac{I}{2 \pi \rho} \bar{a} \phi=\frac{20}{2 \pi \rho} \bar{a} \varnothing
$$

But $\bar{\rho}=3 \bar{a} x+4 \bar{a} y$ then
$|\bar{\rho}|=\sqrt{9+16}=5$
$\therefore \bar{H}=\frac{20}{10 \pi} \mathrm{a} \emptyset=\frac{2}{\pi} \bar{a} \emptyset$
$\bar{B}=\mu_{0} \bar{H}=4 \pi \times 10^{-7}\left(\frac{2}{\pi}\right) \bar{a} \emptyset=8 \times 10^{-7} \bar{a} \emptyset$ tesla
$\mathrm{a}-\bar{B}=\mu_{0} \frac{I}{2 \pi \rho} \bar{a} \emptyset$ then $\Psi_{m}=\iint \bar{B} \cdot d \bar{s}$
But $d \bar{s}=\rho d \emptyset d z \bar{a} \rho$

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 6
$\therefore \Psi_{m}=\iint \frac{\mu_{0} I}{2 \pi \rho} \bar{a} \emptyset \cdot(\rho d \emptyset d z \bar{a} \rho)=0$ weber
$\mathrm{b}-\mathrm{d} \bar{s}=d \rho d z \bar{a} \emptyset$

$$
\begin{gathered}
\therefore \psi_{m}=\iint \frac{\mu_{0} I}{2 \pi \rho} \bar{a} \emptyset \cdot(d \rho d z \bar{a} \emptyset)=\frac{\mu_{0} I}{2 \pi} \iint \frac{d \rho}{\rho} d z \\
\psi_{m}=\frac{\mu_{0} I}{2 \pi}[z]_{1}^{10}[\ln \rho]_{1}^{4}
\end{gathered}
$$

$=\frac{4 \pi \times 10^{-7}(20)}{2 \pi}(10-1) \ln 4$
$=36 \times 10^{-6} \ln 4$ webers
$=49.9 \mu$ webers

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 7

## Curl:-

Choose a small rectangle with sides $\Delta x$ and $\Delta y$ and current produces $\bar{H}$ at the center on this area. where
$H_{0=} H_{x 0} \bar{a} x+H_{y 0} \bar{a} y+H_{z 0} \bar{a} z$
the closed line integral of $\bar{H}$ about this path is the sum of the four values of $\bar{H} . \Delta l$ on each side .
$\therefore(\bar{H} . \Delta \bar{l})_{1 \rightarrow 2}=\left(H_{y 0}+\frac{1}{2} \frac{\partial H y}{\partial x} \Delta x\right) \Delta y$
$(\bar{H} . \Delta \bar{l})_{2 \rightarrow 3}=-\left(H_{x 0}+\frac{1}{2} \frac{\partial H x}{\partial y} \Delta y\right) \Delta x$
Continuing for the remaining two segments and adding results gives:
$\oint \bar{H} \cdot d \bar{l}=\left(\frac{\partial H y}{\partial x}-\frac{\partial H x}{\partial y}\right) \Delta x \Delta y=$ current enclosed

$$
=\mathrm{Jz} \Delta x \Delta y
$$

$\therefore \frac{\phi \bar{H} . d \bar{l}}{\Delta x \Delta y}=\left(\frac{\partial H y}{\partial x}-\frac{\partial H x}{\partial y}\right)=J Z$
For $\Delta x \Delta y=\Delta s$ then

$$
\lim _{\Delta s \rightarrow 0}=\frac{\oint \bar{H} \cdot d \vec{l}}{\Delta s}=\frac{\partial H y}{\partial x}-\frac{\partial H x}{\partial y}=J z
$$

In similar way we can find that :
$\lim _{\Delta y, \Delta z \rightarrow 0}=\frac{\oint \bar{H} . \Delta \hat{l}}{\Delta y \Delta z}=\frac{\partial H z}{\partial y}-\frac{\partial H y}{\partial z}=J x$
And also $\frac{\partial H x}{\partial z}-\frac{\partial H z}{\partial x}=J y$

In general

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 7
$(\operatorname{curl} \overline{\bar{H}})_{n}=\lim \frac{\phi \hat{\bar{H}} \cdot d \hat{l}}{\Delta s n}=J n$
This can be expressed in the from :

$$
\operatorname{curl} \bar{H}=\nabla \times \bar{H}=\left|\begin{array}{ccc}
\bar{a} x & \bar{a} y & \bar{a} z \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
H x & H y & H z
\end{array}\right|=\bar{J} A / m^{2}
$$

Or $\nabla \times \bar{H}=\bar{\jmath} \quad$ where $\bar{\jmath}=j_{x} \bar{a} x+j_{y} \bar{a} y+j_{z} \bar{a} z$
And $\nabla \times \bar{B}=\mu \bar{J}$
$\therefore$ curl $\bar{H}=\nabla \times \bar{H}=\left(\frac{\partial H z}{\partial y}-\frac{\partial H y}{\partial z}\right) \bar{a} x+\left(\frac{\partial H x}{\partial z}-\frac{\partial H z}{\partial x}\right) \overline{a y}+\left(\frac{\partial H y}{\partial x}-\frac{\partial H x}{\partial y}\right) \bar{a} z$
In cylindrical :-

$$
\nabla \times \bar{H}=\left(\frac{1}{\rho} \frac{\partial H z}{\partial \emptyset}-\frac{\partial H \emptyset}{\partial z}\right) \bar{a} \rho+\left(\frac{\partial H \rho}{\partial z}-\frac{\partial H z}{\partial \rho}\right) a \emptyset+\left[\frac{1}{\rho} \frac{\partial(\rho H \varnothing)}{\partial \rho}-\frac{1}{\rho} \frac{\partial H \rho}{\partial \emptyset}\right] \bar{a} z
$$

In spherical :-

$$
\begin{aligned}
& \nabla \times \bar{H}= \frac{1}{r \sin \theta}\left[\frac{\partial(H \sin \theta)}{\partial \theta}-\frac{\partial H \theta}{\partial \emptyset}\right] \bar{a} r+\frac{1}{r}\left[\frac{1}{\sin \theta} \frac{\partial H r}{\partial \phi}-\frac{\partial(r H \varnothing)}{\partial r}\right] \bar{a} \theta+\frac{1}{r}\left[\frac{\partial(r H \theta)}{\partial r}\right. \\
&\left.-\frac{\partial H r}{\partial \theta}\right] \bar{a} \varnothing
\end{aligned}
$$

In this case we can get the current density by using the expression of $\bar{H}$ or $\bar{B}$ then the current can be calculate
$I=\iint \bar{J} . d \bar{s} \quad$ but $\bar{\jmath}=\nabla \times \bar{H}$
$\therefore I=\iint(\nabla \times \bar{H}) . d \bar{s}$
Ex1:- given the magnetic field intensity $\bar{H}=y^{2} z \bar{a} x+2(x+1) y z \bar{a} y-(x+1) z^{2} \bar{a} z$
a- Find curl $\bar{H}$ at $(2,4,5)$.
b- Calculate the current through $1 \leq \mathrm{x} \leq 3, \mathrm{y}=2,1 \leq \mathrm{Z} \leq 2$.

Solution :-
a- $\quad \operatorname{curl} \bar{H}=\nabla \times \bar{H}=\left|\begin{array}{ccc}\bar{a} x & \bar{a} y & \bar{a} z \\ \frac{\partial}{\partial X} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^{2} z & 2(x+1) y z & -(x+1) z^{2}\end{array}\right|$
$=\left[\frac{\partial}{\partial y}\left(-(x+1) z^{2}\right)-\frac{\partial}{\partial z}[2(x+1) y z] \bar{a} x-\left[\frac{\partial}{\partial x}-\left[(x+1) z^{2}\right]-\frac{\partial}{\partial z}\left(y^{2} z\right)\right] \bar{a} y+\left[\frac{\partial}{\partial x}(2(x+\right.\right.$

1) $\left.y z-\frac{\partial}{\partial y} y^{2} z\right] \bar{a} z$
$\therefore \nabla \times \bar{H}=-2(x+1) y \bar{a} x+\left(y^{2}+z^{2}\right) \bar{a} y=J_{x} \bar{a} x+J_{y} \bar{a} y$
at $(2,4,5)$ then $\nabla \times \bar{H}=-24 \bar{a} x+41 \bar{a} y$
b- for $\mathrm{y}=2$ then $\mathrm{d} \bar{s}=d x d z \bar{a} y$
$\mathrm{I}=\iint \bar{J} \cdot d \bar{s}=\iint J y \bar{a} y \cdot d x d z a y=\iint J y d x d z$
but $J y=(\nabla \times \bar{H}) \mathrm{y}=\left(y^{2}+z^{2}\right)$
$\therefore I=\int_{z=1}^{2} \int_{x=1}^{3}\left(y^{2}+z^{2}\right) d x d z$
$=\int_{1}^{2} \int_{1}^{3} 4 d x d z+\int_{1}^{2} \int_{1}^{3} z^{2} d x d z$
$=(4)(1)(2)+\frac{1}{3}(7)(2)=8+\frac{14}{3}=12.66 \mathrm{~A}$

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 7

## Stokes' theorem:-

From Ampere law:
$I=\oint \bar{H} \cdot d l \quad$ but $I=\iint \bar{J} \cdot d \bar{s}=\iint(\nabla \times \bar{H}) \cdot d \bar{s}$
$\therefore I=\oint \bar{H} \cdot d \bar{l}=\iint(\nabla \times \bar{H}) \cdot d \bar{s}$
This is stokes theorem for calculating the current by using both side
Ex2:- for $\bar{H}=y^{2} z \bar{a} x+2(x+1) y z \bar{a} y-(x+1) z^{2} \bar{a} z$ and the region $\mathrm{x}=2,1 \leq \mathrm{y} \leq 2,1 \leq \mathrm{z}$
$\leq 3$. Evaluate the total current through this region using both sides of Stokes' theorem

Solution:-
$I x=\oint \bar{H} \cdot d \bar{l}=\iint(\nabla \times \bar{H}) \cdot d \bar{s}$
but $\nabla \times \bar{H}=-2(x+1) y \bar{a} x+\left(y^{2}+z^{2}\right) \bar{a} y$
and for $\mathrm{x}=2$ then $\mathrm{d} \bar{s} x=d y d z \bar{a} x$
$\therefore$ R.H.side is $\int_{z=1}^{3} \int_{y=1}^{2}-2(x+1) y d y d z$
$\therefore I x=-\int_{1}^{3} \int_{1}^{2} 4 y d y d z-\int_{1}^{3} \int_{1}^{2} 2 y d y d z$
$=-4\left(\frac{3}{2}\right) 2-2\left(\frac{3}{2}\right) 2=-18 A$ or $\bar{I}=-18 \bar{a} x A$

Electro-magnetics II Dr. Ahmed A. Abbas Lecture: 7

$$
\begin{aligned}
I x=\oint \bar{H} \cdot d \bar{l} & =\int_{1}^{2} H y d y+\int_{1}^{3} H z d z-\int_{1}^{2} H y d y \\
& -\int_{1}^{3} H z d z \\
& =\int_{1}^{1} 2(x+1) y(1) d y \\
& +\int_{1}^{3}-(x+1) z^{2} d z-\int_{1}^{2} 2(x+1) y(3) d y-\int_{1}^{3}-(x+1) z^{2} d z
\end{aligned}
$$

$\therefore I x=6 \int_{1}^{2} y d y-18 \int_{1}^{2} y d y=-12\left[\frac{y^{2}}{2}\right]_{1}^{2}=-18 A$
$I_{x}=\oint \bar{H} \cdot d \bar{l}=\iint(\nabla \times \bar{H}) \cdot d \bar{s}=-18 A$
or $\bar{I}=-18 \bar{a} x A$

The scalar and vector magnetic potential:-
The scalar electrostatic potential v is related to the electric filed intensity by integral and differential forms. In magnetic field there are scalar potential $\mathrm{V}_{\mathrm{m}}$ can be related to the magnetic field intensity $\bar{H}$ by integral and differential forms and there is vector magnetic potential $\bar{A}$ can be given directly from Biot-savart law and also can be related to $\bar{B}$ for region contains $\bar{H}$ then the magnetic scalar potential is :-
$\mathrm{V}_{\mathrm{mab}}=-\int_{b}^{a} \bar{H} \cdot d \bar{l} \quad$ outside the current region
If $\mathrm{V}_{\mathrm{m}}$ is known then
$\mathrm{H}=-\nabla V m \quad$ for $\mathrm{J}=0$ ((outside the current region $))$
Where
$\nabla \times \bar{H}=\bar{\jmath}$ (inside) then $\nabla \times(-\nabla V m)=\mathrm{J}$
But $\nabla \times(-\nabla \mathrm{Vm})=0$
So $\bar{H}=-\nabla \mathrm{Vm}$ is valide for region free of current.
also $\nabla \cdot \bar{B}=0$ then $\mu_{0} \nabla \cdot \bar{H}=0$
$\therefore \nabla \cdot(-\nabla V m)=0$
Or $\nabla^{2} V m=0 \quad$ for $(J=0)$ only
To find the magnetic potential inside the conductor then, we should introduce the vector magnetic potential $\bar{A}$ as given in the general forms:
for line current
$\bar{A}=\int_{e} \frac{\mu I d \bar{l}}{4 \pi R} \quad w b / m$


For surface current

$$
\bar{A}=\iint \frac{\mu \bar{K} d s}{4 \pi R}
$$

and for volume current then
$\bar{A}=\iiint \frac{\mu \bar{J} d v}{4 \pi R}$
The magnetic flux-density can be calculated using :
$\bar{B}=\nabla \times \bar{A} \quad$ or $\bar{H}=\frac{1}{\mu}(\nabla \times \bar{A})$
But $\nabla \times \bar{H}=$ ((outside the conductor inside the conductor ))
$\nabla \times \bar{H}=\frac{1}{\mu} \quad \nabla \times \nabla \times \bar{A}=\left[_{\bar{J}}^{0}\right.$
And $\nabla \times \nabla \times \bar{A}=\nabla(\nabla . \bar{A})-\nabla^{2} A=-\nabla^{2} A$
$\therefore \nabla^{2} A=0$ for outside the conductor because $(\nabla(\nabla \cdot \bar{A})=0$
And
$\nabla^{2} A=-\mu \bar{J} \quad$ inside the conductor
These are Poisson's and Laplace's Equations for the magnetic field.
Magnetic forces:-
If there is a moving charge in magnetic field $\bar{B}$ then this field exerted magnetic force on the charge given by :-
$\mathrm{d} \bar{F}=d Q \bar{V} \times \bar{B}$
but
$\bar{J}=\rho v \bar{V}$ and $d Q=\rho v d v$
$\therefore d \bar{F}=\rho v d v \bar{V} \times \bar{B}$
Or
$d \bar{F}=\bar{J} \times \bar{B} d v \quad$ for current density

Electro-magnetics II Dr. Ahmed A. Abbas
Lecture: 8

For other distributions $\bar{J} d v=\bar{K} d s=I d l$
Then
$d \overline{\bar{F}}=\bar{K} \times \bar{B} d s \quad(($ surface current $))$
$d \bar{F}=I d \bar{l} \times \bar{B} \quad(($ Line current $))$
average force is
$\bar{F}=\int_{l 1}^{l 2} I d \bar{l} \times \bar{B} \quad$ Newton on $\bar{F}=B I L \sin \theta$
$\bar{F}=\iint(\bar{K} \times \bar{B}) d s$
$\bar{F}=\iiint(\bar{J} \times \bar{B}) d v$
Ex1:- Two infinite line current $\mathrm{I}_{1}=\mathrm{I}_{2}=10 \overline{\mathrm{a} z}$ A located parallet to the Z -axis at $\mathrm{x}=2, \mathrm{y}= \pm 3$ calculate the force exerted on 5 m length of $\mathrm{I}_{2}$.

## Solution :-

$\overline{\mathrm{B}}_{1}=\frac{\mu_{0} \mathrm{I}_{1}}{2 \pi \mathrm{r}} \overline{\mathrm{a}} \varnothing$
$\overline{\mathrm{r}}=6 \overline{\mathrm{a}} \mathrm{y},|\overline{\mathrm{r}}|=6$

$$
\overline{\mathrm{a}} \mathrm{r}=\overline{\mathrm{a}} \mathrm{y}
$$

$\bar{a} \emptyset=\bar{a} z \times \bar{a} y=-\bar{a} x$
$\therefore \overline{\mathrm{B}}_{1}$ at the position of $\mathrm{I}_{2}$ is
$\overline{\mathrm{B}}_{1}=\frac{\mu_{0} \mathrm{I}_{1}}{2 \pi(6)}(-\overline{\mathrm{ax}})$
$\therefore$ force on $\mathrm{I}_{2}$ is
$\overline{\mathrm{F}}_{2}=\int_{0}^{5} \mathrm{I}_{2} \quad \mathrm{~d} \overline{\mathrm{l}}_{2} \times \mathrm{B}_{1}$
$=\frac{\mu_{0} \mathrm{I}_{1} \mathrm{I}_{2}}{12 \pi}(5)[\overline{\mathrm{a}} \mathrm{z} \times(-\overline{\mathrm{a}} \mathrm{x})]$
$=\frac{500 \mu_{0}}{12 \pi}(-\bar{a} y)$
$\therefore \overline{\mathrm{F}}_{2}=\frac{-500}{3} * 10^{-7} \overline{\mathrm{a} y}$
$\therefore \overline{\mathrm{F}}_{2}=-16.6$ āy $\mu \mathrm{N}$
$\overline{\mathrm{F}}_{2}=-\overline{\mathrm{F}}_{1}$ so $\overline{\mathrm{F}}_{1}=16.6$ à $\mu \mathrm{N}$

