Mechanical Vibrations

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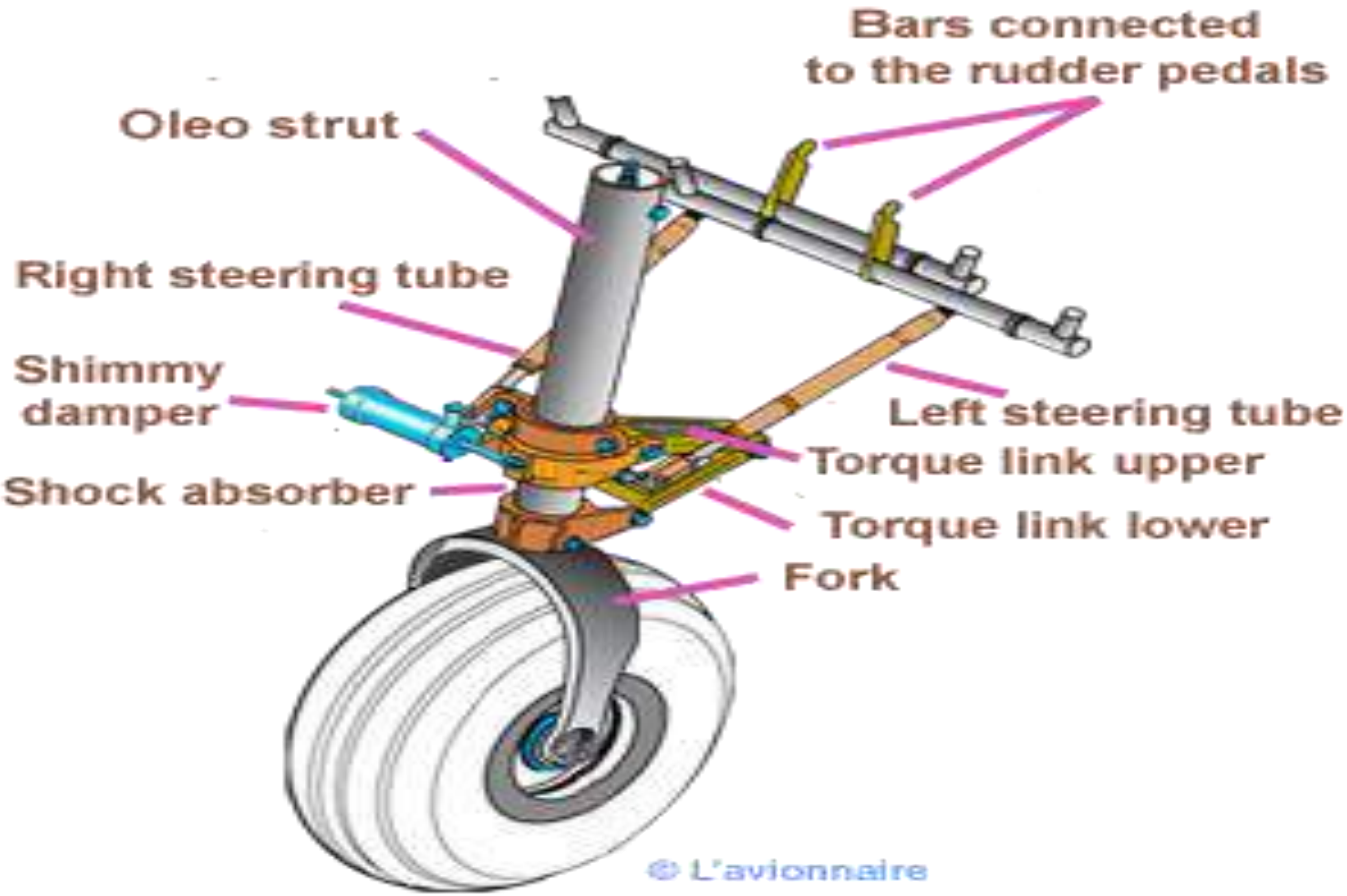


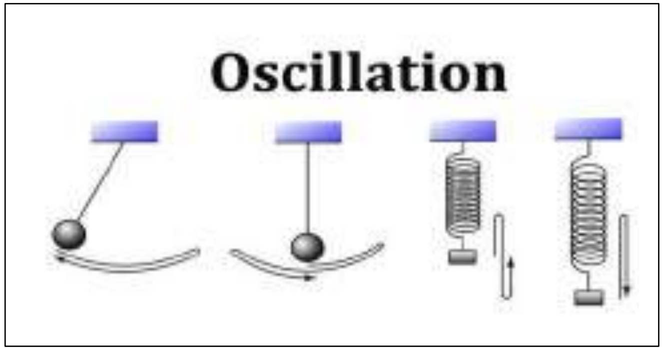
Bars connected
to the rudder pedals
Oleo strut
Right steering tube
Shimmy
damper
Left steering tube
Torque link upper
Shock absorber
Torque link lower
Fork
© L'avionnaire

CHAPTER ONE

Oscillatory Motion

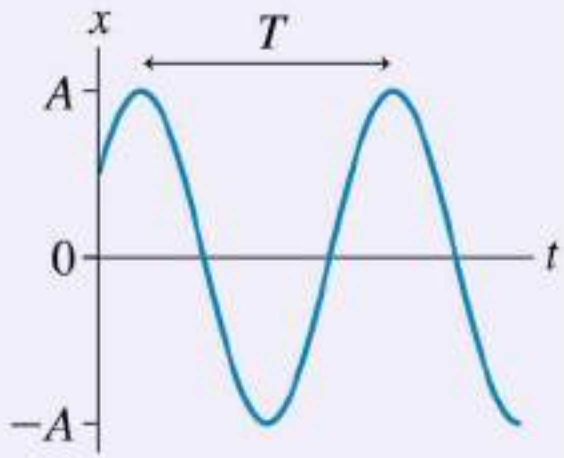
Chapter Goal: To understand systems that oscillate with simple harmonic motion.

Oscillation: In general, it can be defined as a periodic variation of a matter between two values or about its central value.



What are oscillations?

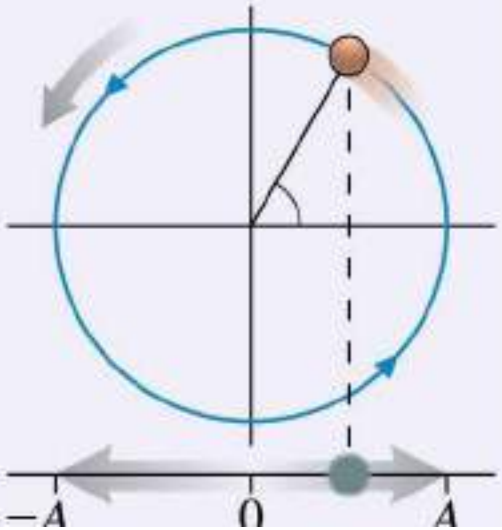
Oscillatory motion is a repetitive motion back and forth around an equilibrium position. We'll describe oscillations in terms of their amplitude, period, and frequency. The most important oscillation is simple harmonic motion (SHM), where the position and velocity graphs are sinusoidal.



How is SHM related to circular motion?

The projection of uniform circular motion onto a line oscillates back and forth in SHM.

- This link to circular motion will help us develop the mathematics of SHM.
- A phase constant, based on the angle on a circle, will describe the initial conditions.



« LOOKING BACK Section 4.4 Circular motion

Simple Harmonic Motion

Some abbreviations that I use all the time:

SHM = **S**imple **H**armonic **M**otion

SHO = **S**imple **H**armonic **O**scillator

Important Question: In this context, what does **Harmonic** mean?

Mathematically, it means that the motion is describable in terms of **Harmonic Functions** which are **sines** and **cosines**.

In the first three sections, your author introduces SHM from an empirical point of view. He defines things like **period, frequency, angular frequency, amplitude**, the relation to uniform circular motion, and energy considerations.

Read these sections carefully. I'm going to jump ahead to the dynamics treatment of SHM – since we know how to do Newtonian dynamics. After that, we'll come back to some of the basic ideas.

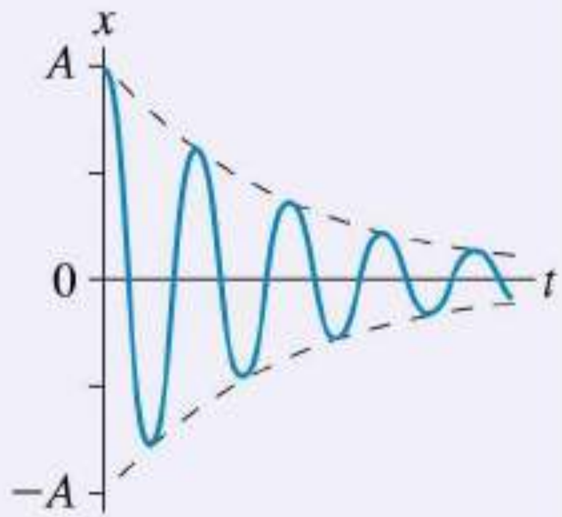
Why sine-cosine oscillations are important…

Fourier Theorem: You can build any arbitrary oscillation by superposing (i.e., adding and subtracting) sine and cosine oscillations of different frequencies. Go to https://en.wikipedia.org/wiki/Fourier_series, and look at:

- Square-wave oscillation
- Triangle-wave oscillation

What if there's friction?

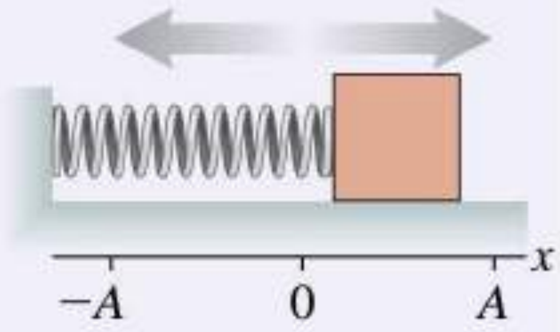
If there's dissipation, the system "runs down." This is called a damped oscillation. The oscillation amplitude undergoes exponential decay. But the amplitude can grow very large when an oscillatory system is *driven* at its natural frequency. This is called resonance.



What things undergo SHM?

The prototype of SHM is a mass oscillating on a spring. Lessons learned from this system apply to all SHM.

- A pendulum is a classic example of SHM.
- Any system with a linear restoring force undergoes SHM.



Why is SHM important?

Simple harmonic motion is one of the most common and important motions in science and engineering.

- Oscillations and vibrations occur in mechanical, electrical, chemical, and atomic systems. Understanding how a system might oscillate is an important part of engineering design.
- More complex oscillations can be understood in terms of SHM.
- Oscillations are the sources of waves, which we'll study in the next two chapters.

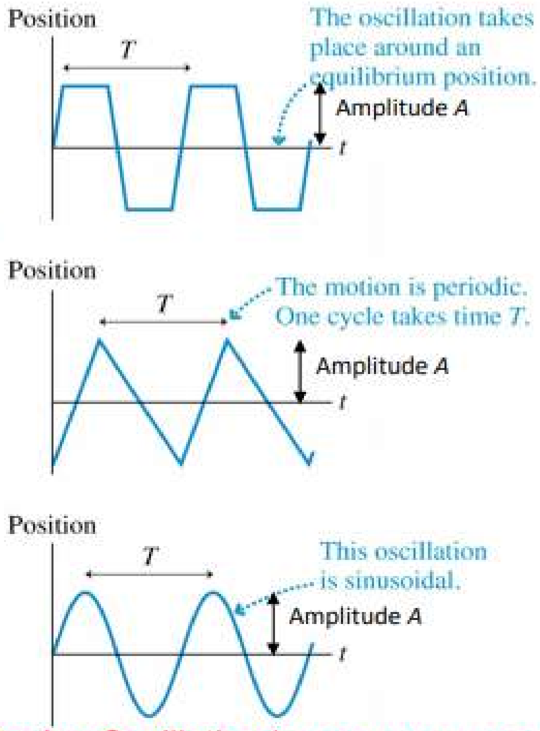
Oscillatory Motion

- Objects that undergo a repetitive motion back and forth around an equilibrium position are called oscillators.
- The time to complete one full cycle, or one oscillation, is called the **period** *T*.
- The number of cycles per second is called the **frequency** *f*, measured in Hz:

$$f = \frac{1}{T} \qquad \text{or} \qquad T = \frac{1}{f}$$

1 Hz = 1 cycle per second = 1 s^{-1}

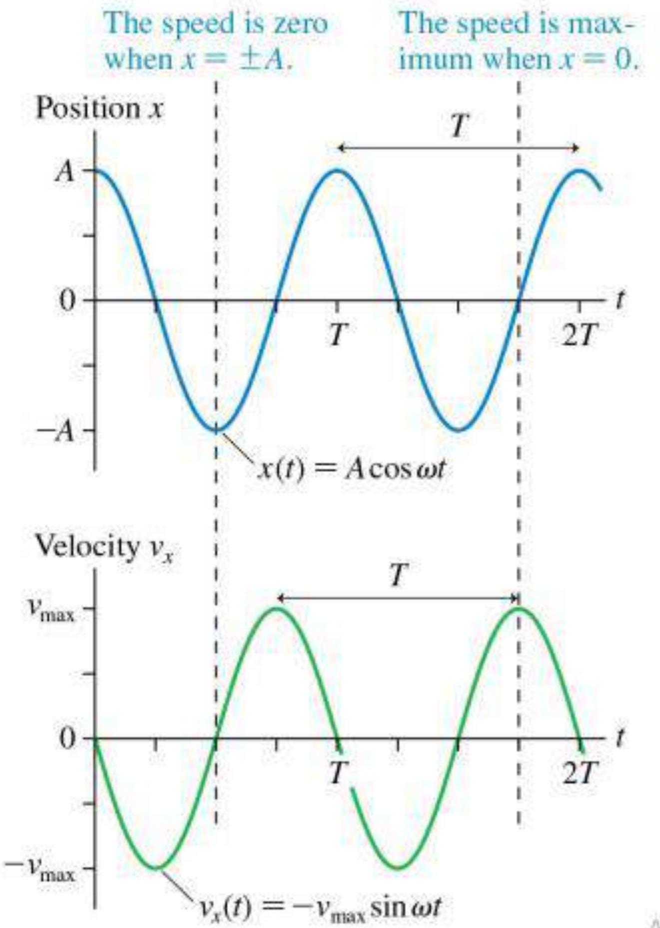
- Maximum displacement from the equilibrium position is called the **Amplitude** *A*.



Sine-Cosine Oscillation is very, very, very, very Important for Music and Electronic Industry b/c of Fourier Theorem.
Go to Wikipedia "Fourier Series".

Simple Harmonic Motion

- The top image shows position versus time for an object undergoing simple harmonic motion.
- The bottom image shows the velocity versus time graph for the same object.
- The velocity is zero at the times when $x = \pm A$; these are the *turning points* of the motion.
- The maximum speed v_{max} is reached at the times when $x = 0$.



Simple Harmonic Motion

- If the object is released from rest at time $t = 0$, we can model the motion with the cosine function:

$$x(t) = A\cos(\omega t)$$

- Cosine is a *sinusoidal* function.
- ω is called the angular frequency, defined as

$$\omega = 2\pi/T$$

- The units of ω are rad/s:

$$\omega = 2\pi f$$

A

Simple Harmonic Motion

- The position of the oscillator is

$$x(t) = A\cos(\omega t)$$

- Using the derivative of the position function, we find the velocity:

$$v_x(t) = \frac{dx}{dt} = -\frac{2\pi A}{T}\sin\left(\frac{2\pi t}{T}\right) = -2\pi f A\sin(2\pi f t) = -\omega A\sin\omega t$$

- The maximum speed is

$$v_{\max} = \omega A$$

EXAMPLE 15.1 | A system in simple harmonic motion

An air-track glider is attached to a spring, pulled 20.0 cm to the right, and released at $t = 0$ s. It makes 15 oscillations in 10.0 s.

a. What is the period of oscillation?

b. What is the object's maximum speed?

c. What are the position and velocity at $t = 0.800$ s?

MODEL An object oscillating on a spring is in SHM.

EXAMPLE 15.1 | A system in simple harmonic motion

SOLVE a. The oscillation frequency is

$$f = \frac{15 \text{ oscillations}}{10.0 \text{ s}} = 1.50 \text{ oscillations/s} = 1.50 \text{ Hz}$$

Thus the period is $T = 1/f = 0.667$ s.

b. The oscillation amplitude is $A = 0.200$ m. Thus

$$v_{\text{max}} = \frac{2\pi A}{T} = \frac{2\pi(0.200 \text{ m})}{0.667 \text{ s}} = 1.88 \text{ m/s}$$

EXAMPLE 15.1 | **A system in simple harmonic motion**

SOLVE c. The object starts at $x = +A$ at $t = 0$ s. This is exactly the oscillation described by Equations 15.2 and 15.6. The position at $t = 0.800$ s is

$$x = A\cos\left(\frac{2\pi t}{T}\right) = (0.200\text{ m})\cos\left(\frac{2\pi(0.800\text{ s})}{0.667\text{ s}}\right)$$

$$= (0.200\text{ m})\cos(7.54\text{ rad}) = 0.0625\text{ m} = 6.25\text{ cm}$$

The velocity at this instant of time is

$$v_x = -v_{\max}\sin\left(\frac{2\pi t}{T}\right) = -(1.88\text{ m/s})\sin\left(\frac{2\pi(0.800\text{ s})}{0.667\text{ s}}\right)$$

$$= -(1.88\text{ m/s})\sin(7.54\text{ rad}) = -1.79\text{ m/s} = -179\text{ cm/s}$$

At $t = 0.800$ s, which is slightly more than one period, the object is 6.25 cm to the right of equilibrium and moving to the *left* at 179 cm/s. Notice the use of radians in the calculations.