Introduction
Prestressed Concrete Design

Learning Objectives

- *Explain* what prestressed concrete is and why we prestress concrete.
- *Describe* the difference between prestressed and non-prestressed concrete beam behavior.
- *Identify* the main limitation to early prestressed concrete and *explain* what changed to make prestressed concrete feasible.
- *Describe* the construction procedure for and *differentiate* between the behavior of pretensioned and post-tensioned concrete beams
- *Explain* why the capacity of prestressed and non-prestressed members is approximately the same.

1.1 – Prestressed Concrete

Prestressed concrete is a type of reinforced concrete in which the steel reinforcement has been tensioned against the concrete.

Tensioning operation results in self-equilibrating system of internal forces.

Strain differential exists between concrete and reinforcement.

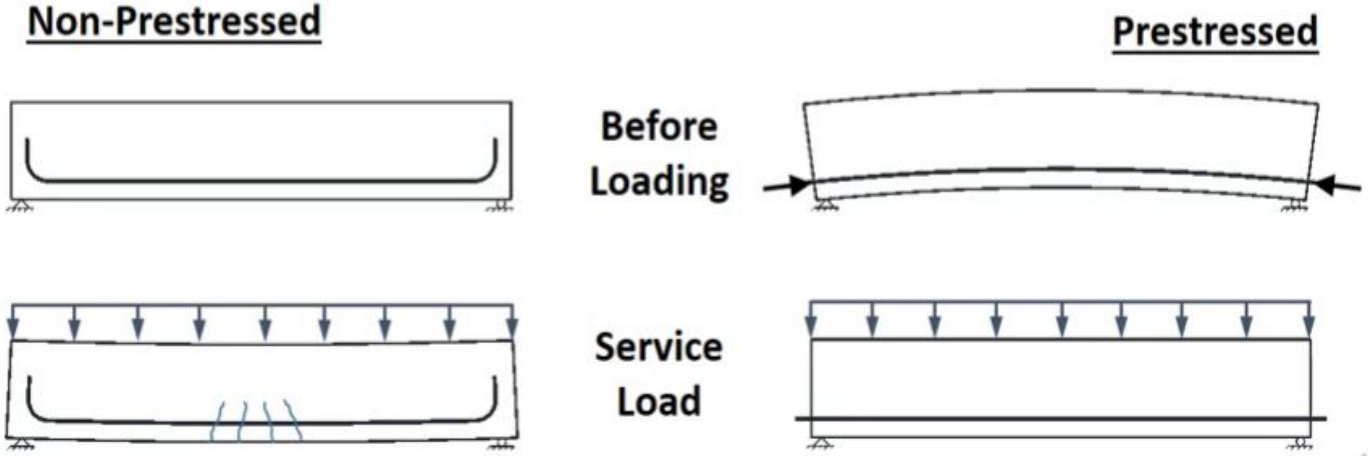
$$N = f_c A_c + f_s A_s + f_p A_p$$

Total strain = ε
Concrete strain = $\varepsilon = \varepsilon_{cf} + \varepsilon_{cr} + \varepsilon_{sh}$
Prestressing strain = $\varepsilon = \varepsilon_{pf} - \Delta\varepsilon_p$ A_c

1.1 – Prestressed Concrete

Why do we prestress?

- Concrete weak in tensile strength ($f'_t \ll f'_c$)
- After cracking, considerable loss in stiffness
- Precompression substantially increases the external load required to crack the concrete resulting in a member that is strong, tough and stiff



Non-Prestressed

Prestressed

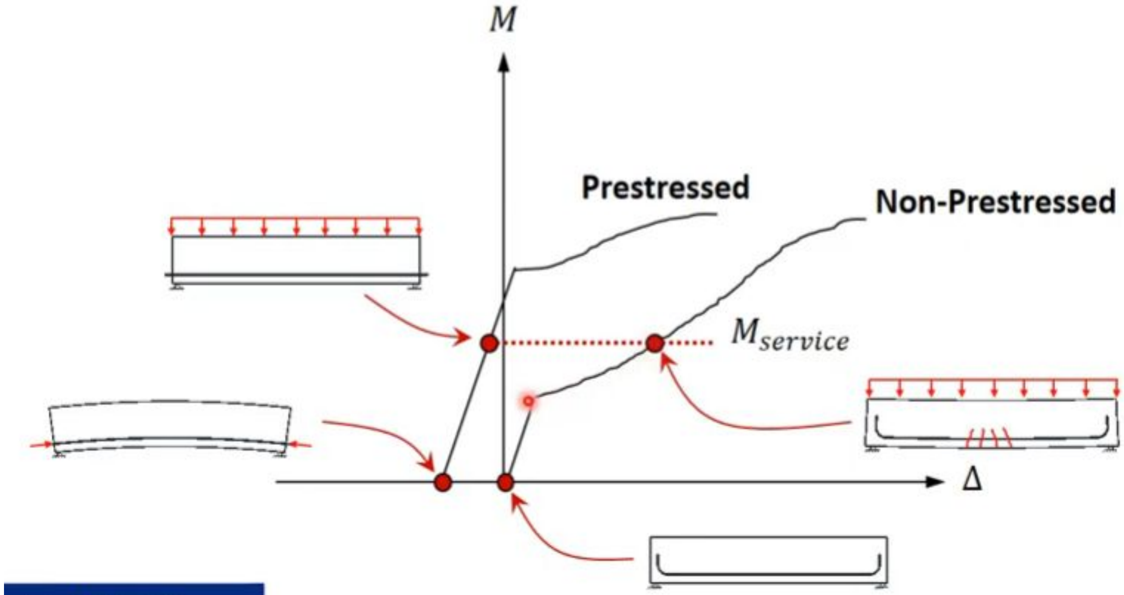
Before Loading

Service Load

1.1 – Prestressed Concrete

Why do we prestress?

- Does not increase strength, increases serviceability and stiffness

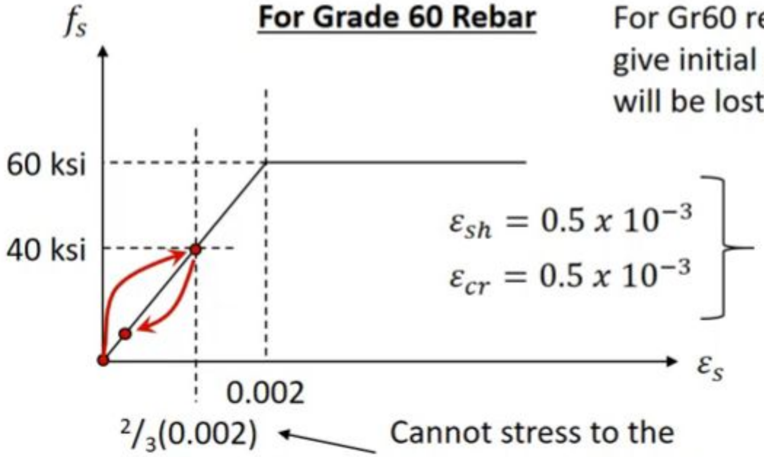


1.1 – Prestressed Concrete

Early Limitations

For conventional strength steel (f_y = 60 ksi), nearly all prestressing is lost due to prestress losses (e.g., creep and shrinkage)

For Grade 60 Rebar



$$\varepsilon_{sh} = 0.5\ x\ 10^{-3}$$
$$\varepsilon_{cr} = 0.5\ x\ 10^{-3}$$

Will lose 30 ksi of the 40 ksi due to creep and shrinkage

For Gr60 rebar, stressing to 2/3 of yield will give initial prestress of 40 ksi. About 30 ksi will be lost due to creep and shrinkage.

Cannot stress to the maximum yield strength

Prestressing was not possible before the creation of high-strength steel

1.1 – Prestressed Concrete

Origins of Prestressed Concrete

Eugene Freyssinet (1879-1962)

- French engineer considered the father of prestressed concrete
- Idea came to him during series of lectures given by Charles Rabut (French engineer and lecturer who built a 23’ prestressed concrete cantilever) in 1904
- His initial recommendations for practical use of prestressing in 1933:
  - Use metals with very high elastic limits
  - Submit them to very strong initial tensions (much greater than 70 ksi)
  - Use stiff concrete

Freyssinet built several long-span concrete arch bridges early in his career. Some had prestressed components, but there were limitations because of creep.

Fig. 1. Le Veurdre Bridge across the Allier River, France (1910-1911). Spans were 222 – 238 – 223 ft (67.5 – 72 – 67.5 m). This bridge incorporated the first use of thrust by jacks at midspan for decentering and also compensating for concrete creep and shrinkage. (Designed and built by Eugene Freyssinet.)

Fig. 4. Luzancy Bridge across the Marne River, France (1946). This elegant bridge (designed and built by Freyssinet) was the first

1.1 – Prestressed Concrete

Origins of Prestressed Concrete

Gustave Magnel (1889-1955)

- Belgian professor who brought prestressed concrete to the English-speaking world
- Spent WW2 exploring Freyssinet's ideas and carrying out some research on prestressed concrete

Fig. 6. Walnut Lane Bridge as it appeared in 1976.

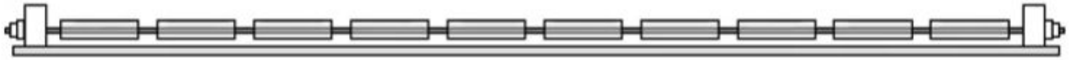
- Magnel had unique ability to communicate in English and teach
- He was known as an excellent teacher. His goal in teaching was to simplify complex problems.
- Helped to design the Walnut Lane Bridge in Philadelphia, which was the first prestressed concrete bridge in the US

1.1 – Prestressed Concrete

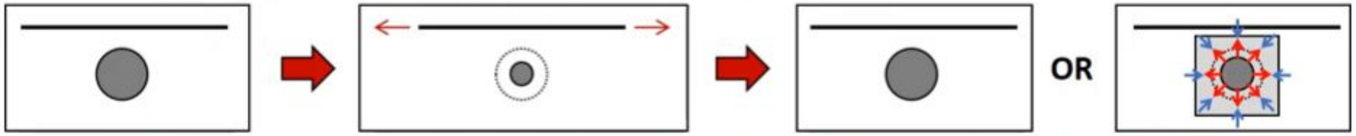
Origins of Prestressed Concrete

Ewald Hoyer

- German engineer; first to use pretensioned concrete (between 1935 and 1939)
- Cast thin flat slabs (2" x 4') pretensioned with 0.08" diameter wire between two buttresses several hundred feet apart



- Only a small diameter wire could be used to ensure adequate bond between wire and concrete. Bond was based on Hoyer's Effect



- Pretensioned concrete was not widely used until the invention of 7-wire strand, which improved the bond with concrete and allowed for larger diameter strands.

1.1 – Prestressed Concrete

Origins of Prestressed Concrete

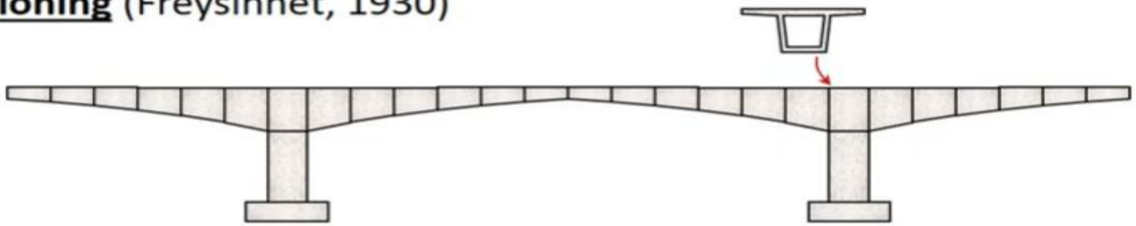
Ulrich Finsterwalder (1987-1988)

- German engineering who developed the double cantilever idea of prestressing construction
- Progressed idea that prestressed concrete can be a safe, economical, and elegant solution to almost any major structural problem

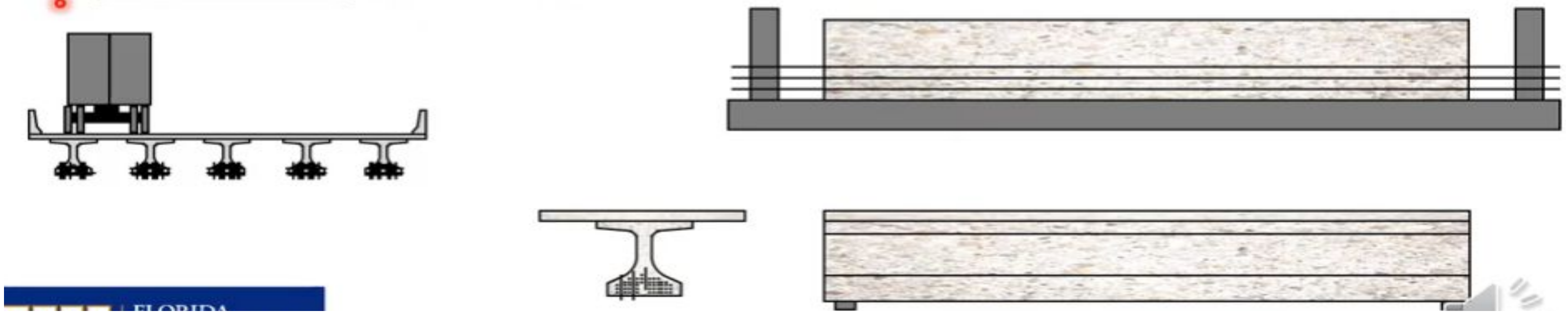
1.2 – Prestressing Systems

Types of Prestressing

Post-Tensioning (Freysinnet, 1930)



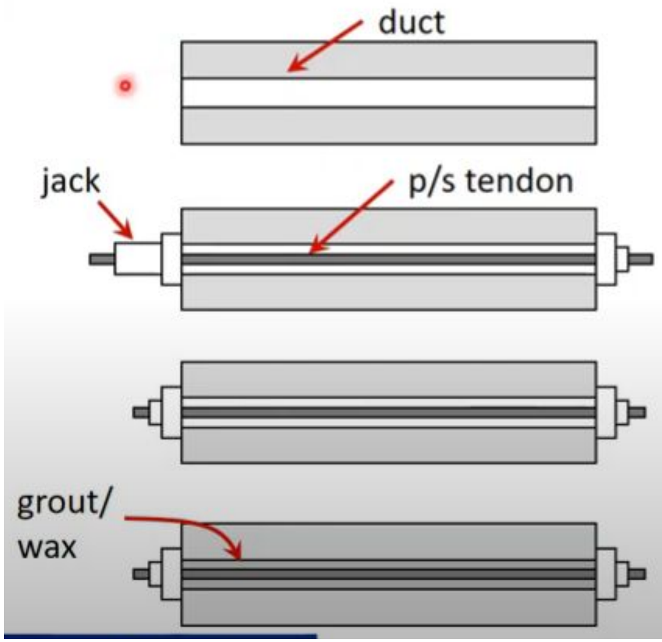
Pretensioning (Hoyer, 1938)



1.2 – Prestressing Systems

Types of Prestressing

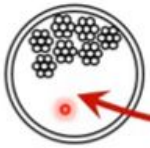
Post-Tensioning



1. Cast member with duct
2. Tension p/s tendon using jack after concrete has hardened
3. Anchor p/s tendon (lock in strain differential)
4. Pump grout or wax into duct

1.2 – Prestressing Systems

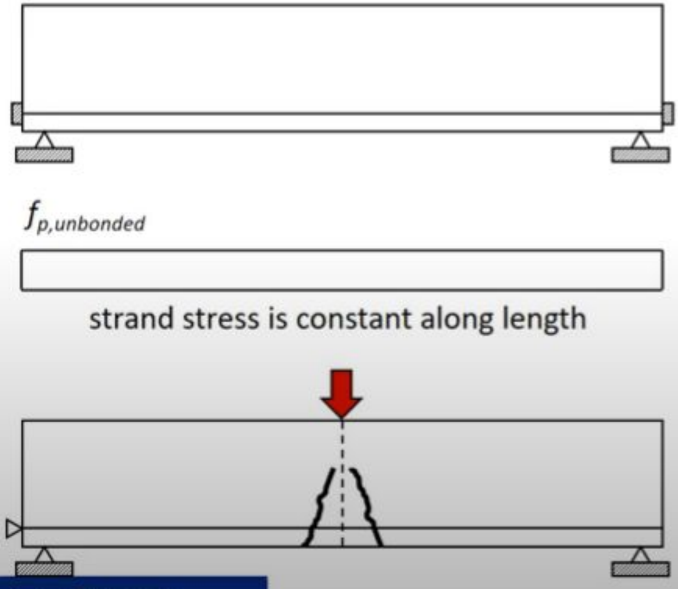
Types of Prestressing



need to fill empty duct with some material for durability

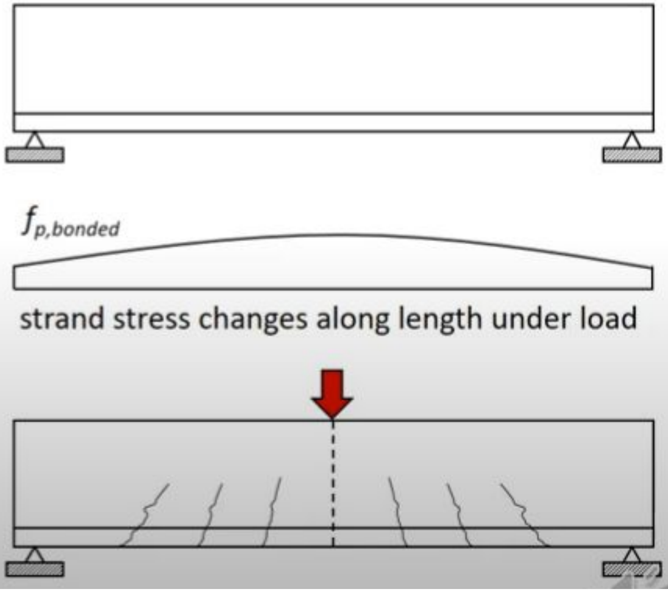
Post-Tensioning

Unbonded: wax or grease

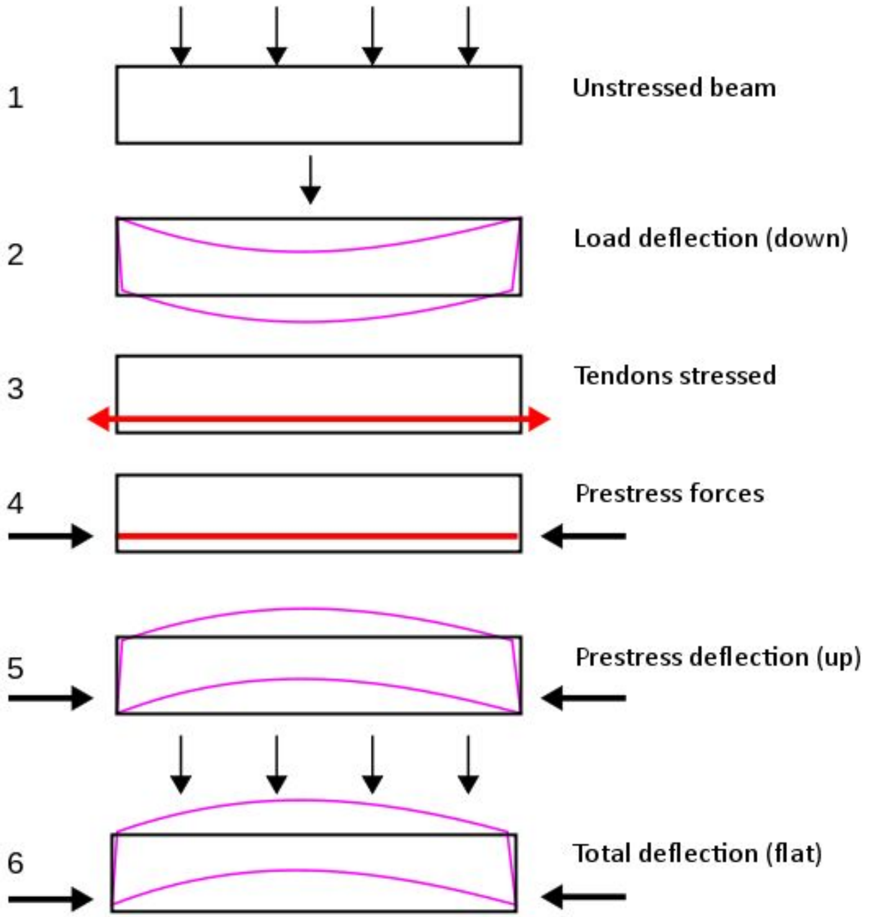


strand stress is constant along length

Bonded: grout



strand stress changes along length under load



1 Unstressed beam

2 Load deflection (down)

3 Tendons stressed

4 Prestress forces

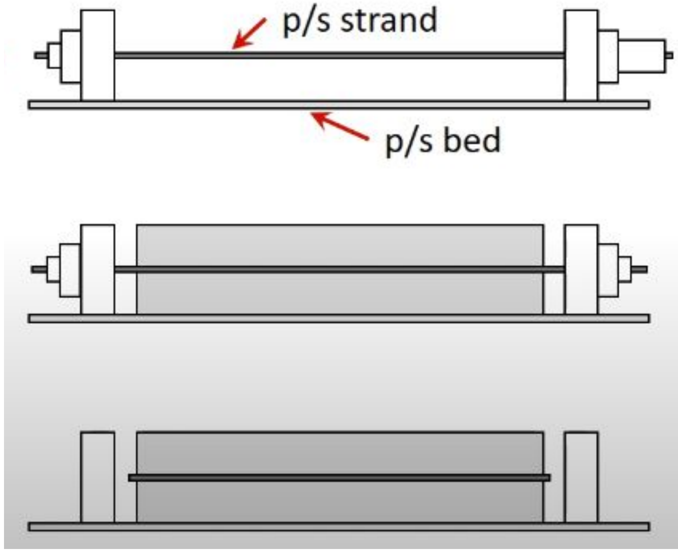
5 Prestress deflection (up)

6 Total deflection (flat)

1.2 – Prestressing Systems

Types of Prestressing

Pretensioning



1. Tension wire in p/s bed

2. Cast concrete member (Note: no duct; concrete must bond)

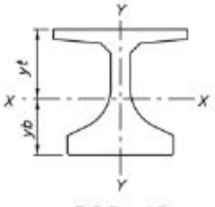
3. Release strands from bed member shortens (transfer)

1.2 – Prestressing Systems

Types of Prestressing

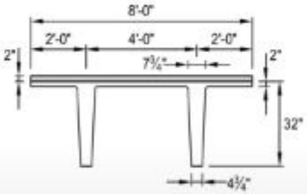
Pretensioning

- Process can be expensive because of cost of bed, end blocks, and formwork
- Repetition reduces cost → standardized sections

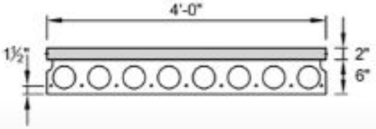


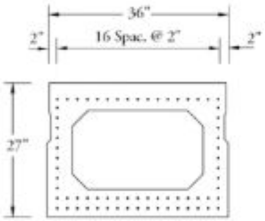
F.I.B.-45
(INDEX 20045)

8'-0" in. × 32"
Normalweight Concrete

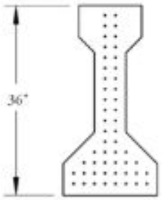


4'-0" × 6"
Normalweight Concrete





Type BI-36

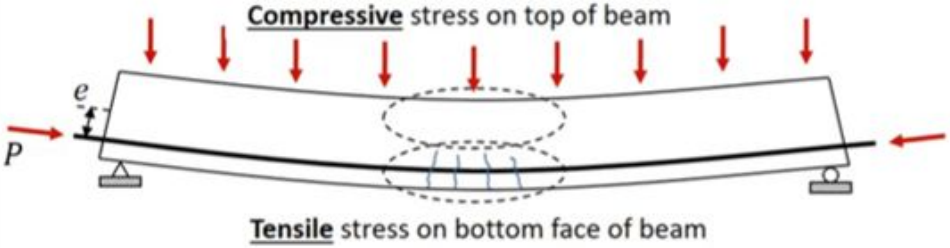


Type II

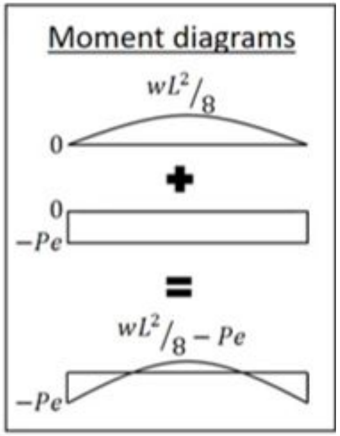
- Speed of construction → precasters want to turn beds over every day
  - Type III cement is used to give higher early strengths
- Designer will specify two concrete strengths
  - Release strength – concrete strength required before transfer
  - Ultimate strength – strength needed to prevent cracking under service loads and provide sufficient strength for ultimate loads

1.3 – Design Concept

- Prestressing placed where tensile stresses develop


Compressive stress on top of beam

Tensile stress on bottom face of beam


Moment diagrams

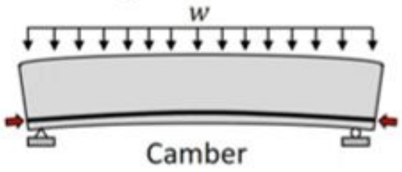
$$e_{min} = \frac{M_{max} - s_b f_b}{P_f} - k_t$$

keep strands between kern points to meet stress checks

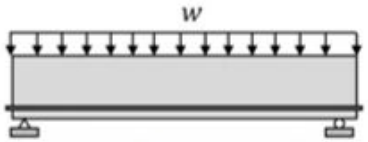
$$e_{max} = k_b + \frac{M_{min} + s_t f_t}{P_i}$$

1.3 – Design Concept

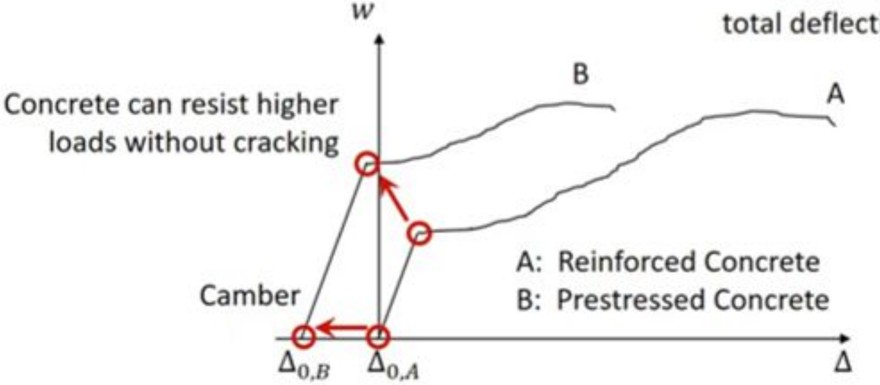
- Prestressing placed where tensile stresses develop
- Tensioning of prestressed reinforcement pre-compresses surrounding concrete giving it ability to resist higher loads prior to cracking



$$A_p f_p = A_s f_s$$



Beam will have close to 0" total deflection under load

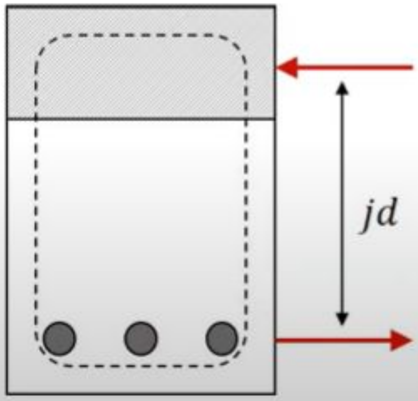


Concrete can resist higher loads without cracking

A: Reinforced Concrete
B: Prestressed Concrete

1.3 – Design Concept

- Prestressing placed where tensile stresses develop
- Tensioning of prestressed reinforcement pre-compresses surrounding concrete giving it ability to resist higher loads prior to cracking
- Prestressing will not greatly impact ultimate strength



$$C_c = 0.85f'_c ba$$

$$T = A_s f_y = A_p f_p$$

$$M_n = A_s f_y (jd) = A_p f_p (jd)$$

If: $A_p f_p = A_s f_s$

1.4 – Typical Prestressed Concrete Structures

- **Bridges** – approximately 50% of bridges are constructed with prestressed concrete
- **Parking garages**
- **Office buildings**

Lake Erie College of Osteopathic Medicine Building, Bradenton, FL (Coreslab)

- **Other**: water towers, nuclear containment structures, storage tanks, towers, offshore structures

1.4 – Typical Prestressed Concrete Structures

Speed of Construction

Burdines Parking Structure

- 1552 precast pieces
- 1521 parking spaces
- 469,087 square feet
- Precast erection beginning to end → 11 weeks

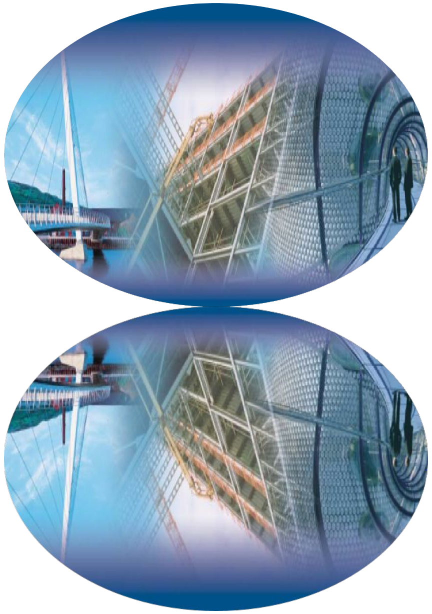
February 24

March 24

April 23

May 21

Open for Business (11 weeks after



UNIVERSITY OF ANBAR

Lecturer
Dr. Mohammed Mohammed

This course is directed towards seniors and first-year graduate students in civil engineering who have taken a class in reinforced concrete design, but who have not yet taken a class in prestressed concrete design.

At the end of this course,you should have a good understanding of the concepts and theory underlying the use of prestressed concrete, and the ability to analyze and design prestressed-concrete structures. This course will focus on statically determinate prestressed concrete beams, because these are commonly used in bridges and buildings. As time permits, we will also discuss the design of statically indeterminate (continuous) structures.

1. **Basic Concepts**
   Lecture 1: Section 1.3
   Lecture 2: Section 1.4 – 1.6

2. **Materials and Systems for Prestressing**
   Lecture 3: Section 2.3.2, 2.4
   Lecture 4: Section 2.5 – 2.7
   Lecture 5: Section 2.5 – 2.7
   Lecture 6: Section 2.10.1 – 2.10.3

3. **Partial Loss of Prestress**
   Lecture 7: Section 3.2 – 3.4
   Lecture 8: Section 3.5 – 3.7
   Lecture 9: Section 3.8 – 3.10

4. **Flexural Design of Prestressed Concrete Elements**
   Lecture 10: Section 4.2
   Lecture 11: Section 4.3
   Lecture 12: Section 4.4
   Lecture 13: Section 4.4
   Lecture 14: Section 4.5.1 – 4.5.2
   Lecture 15: Section 4.5.3 – 4.5.4
   Lecture 16: Section 4.6.1 – 4.6.3
   Lecture 17: Section 4.8
   Lecture 18: Exam 1
   Lecture 19: Section 4.9
   Lecture 20: Section 4.9
   Lecture 21: Section 4.12
   Lecture 22: Section 4.14 – 4.15
   Lecture 23: Section 4.16

5. Shear and torsional Strength Design

Lecture 24: Section 5.5
Lecture 25: Section 5.7
Lecture 26: Section 5.8 – 5.11
Lecture 27: Section 5.12 – 5.14
Lecture 28: Section 5.16, 5.17.1 – 5.17.5
Lecture 29: Section 5.16, 5.17.1 – 5.17.5
Lecture 30: Exam 2

6. Indeterminate Prestressed Concrete Structures

Lecture 31: Section 6.4
Lecture 32: Section 6.5
Lecture 33: Section 6.6
Lecture 34: Section 6.7
Lecture 35: Section 6.9
Lecture 36: Section 6.12
Lecture 37: Section 6.12

7. Camber, Deflection, and Crack Control

Lecture 38: Section 7.4 – 7.5
Lecture 39: Section 7.7, 7.10

8. Prestressed Compression and Tension Members

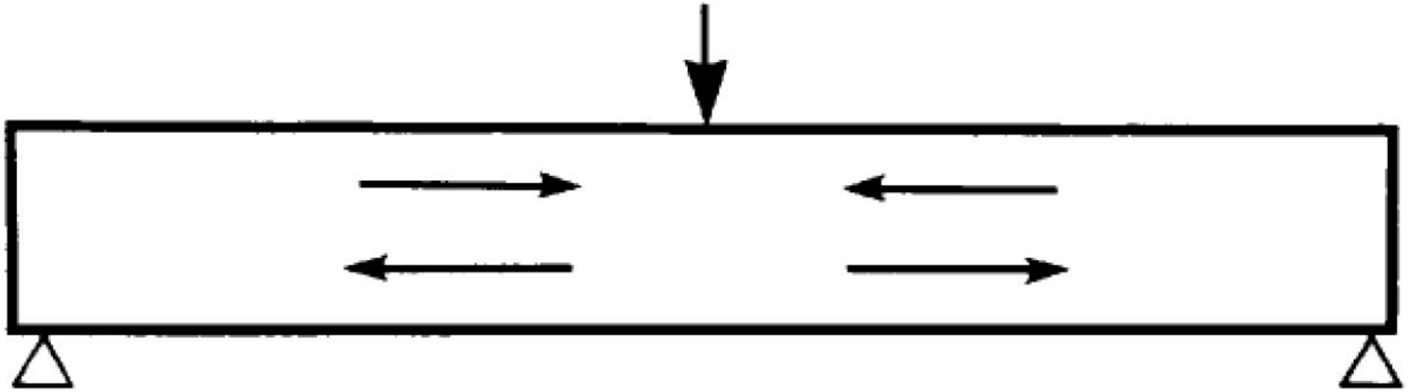
Lecture 40: Section 8.4 – 8.5
Lecture 41: Section 8.6 – 8.8, 8.10
Lecture 42: Section 8.6 – 8.9, 8.10

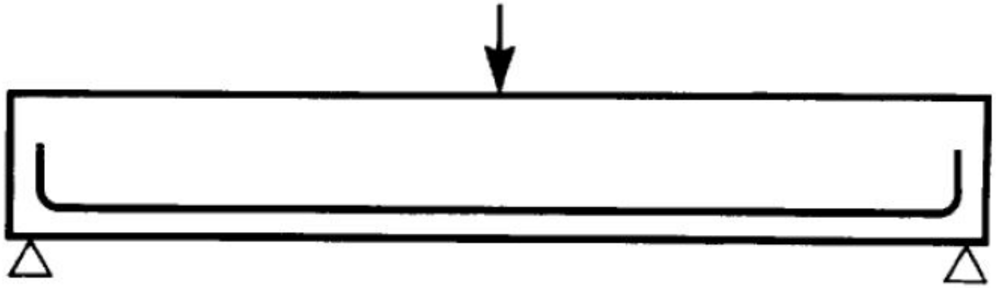
9. Two-Way Prestressed Concrete Floor Systems

Lecture 43: Section 9.1 – 9.2
Lecture 44: Section 9.3 – 9.4

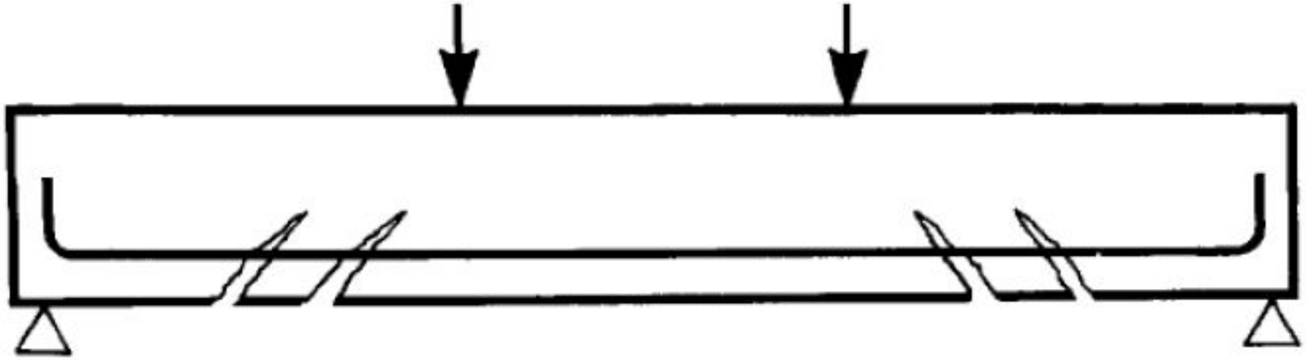
Concrete

Concrete is very strong in compression but weak in tension. In an ordinary concrete beam the tensile stress at the bottom:

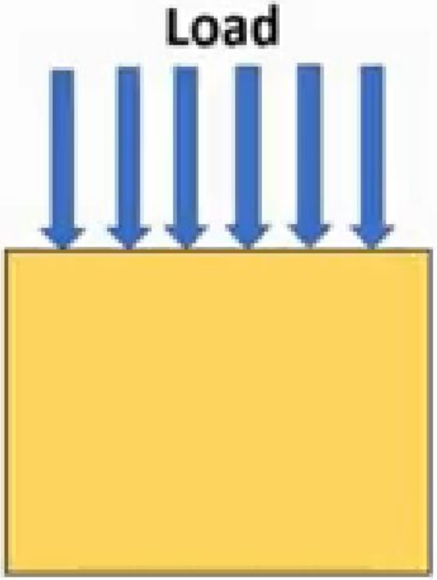


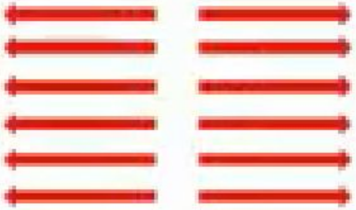


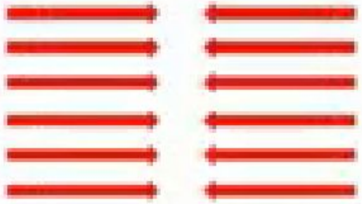
But we still get cracking, which is due to both bending and shear:

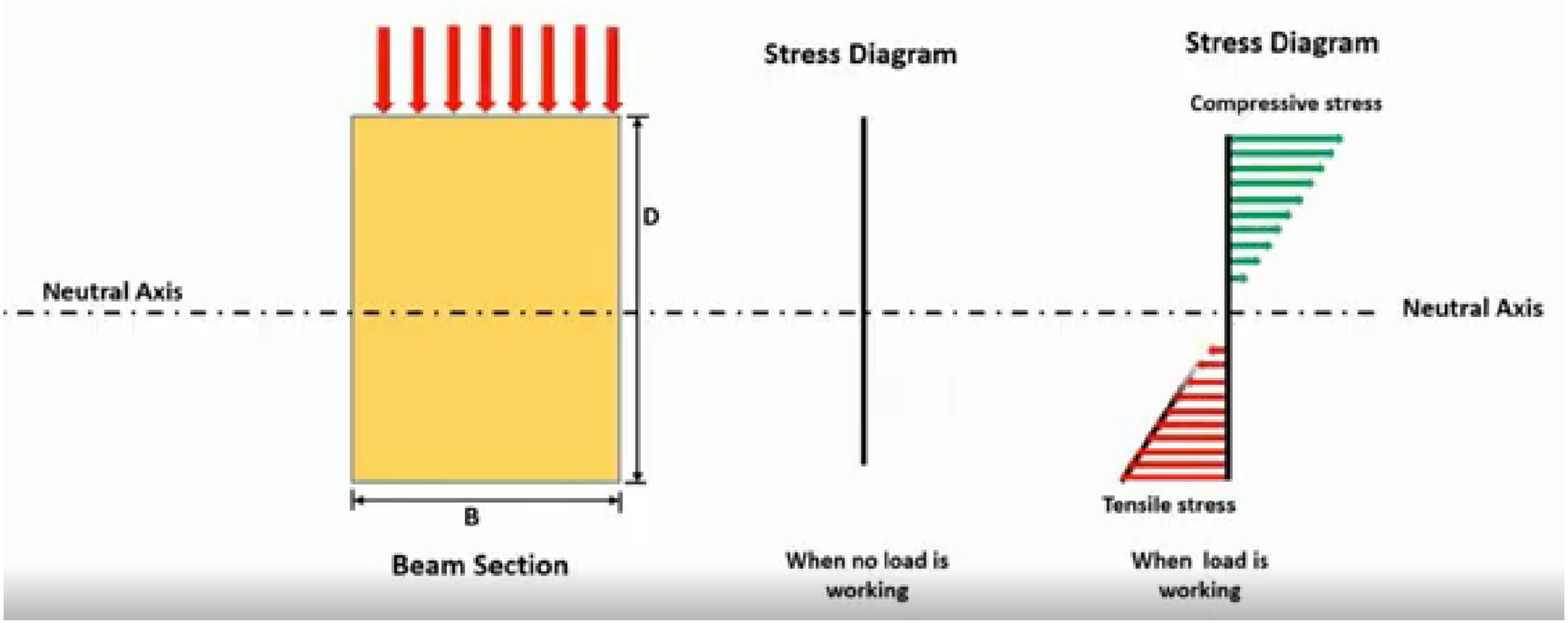


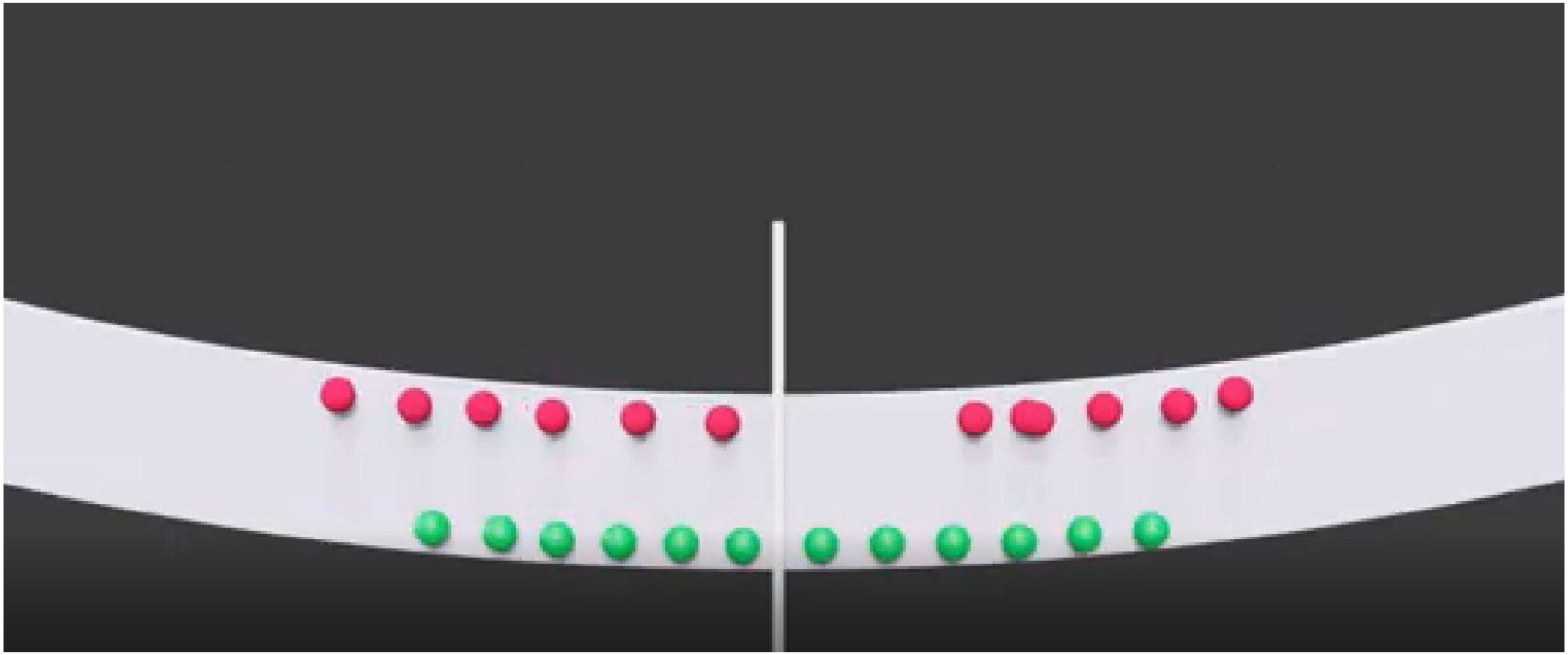
In prestressed concrete, because the prestressing keeps the concrete in compression, no cracking occurs. This is often preferable where durability is a concern.

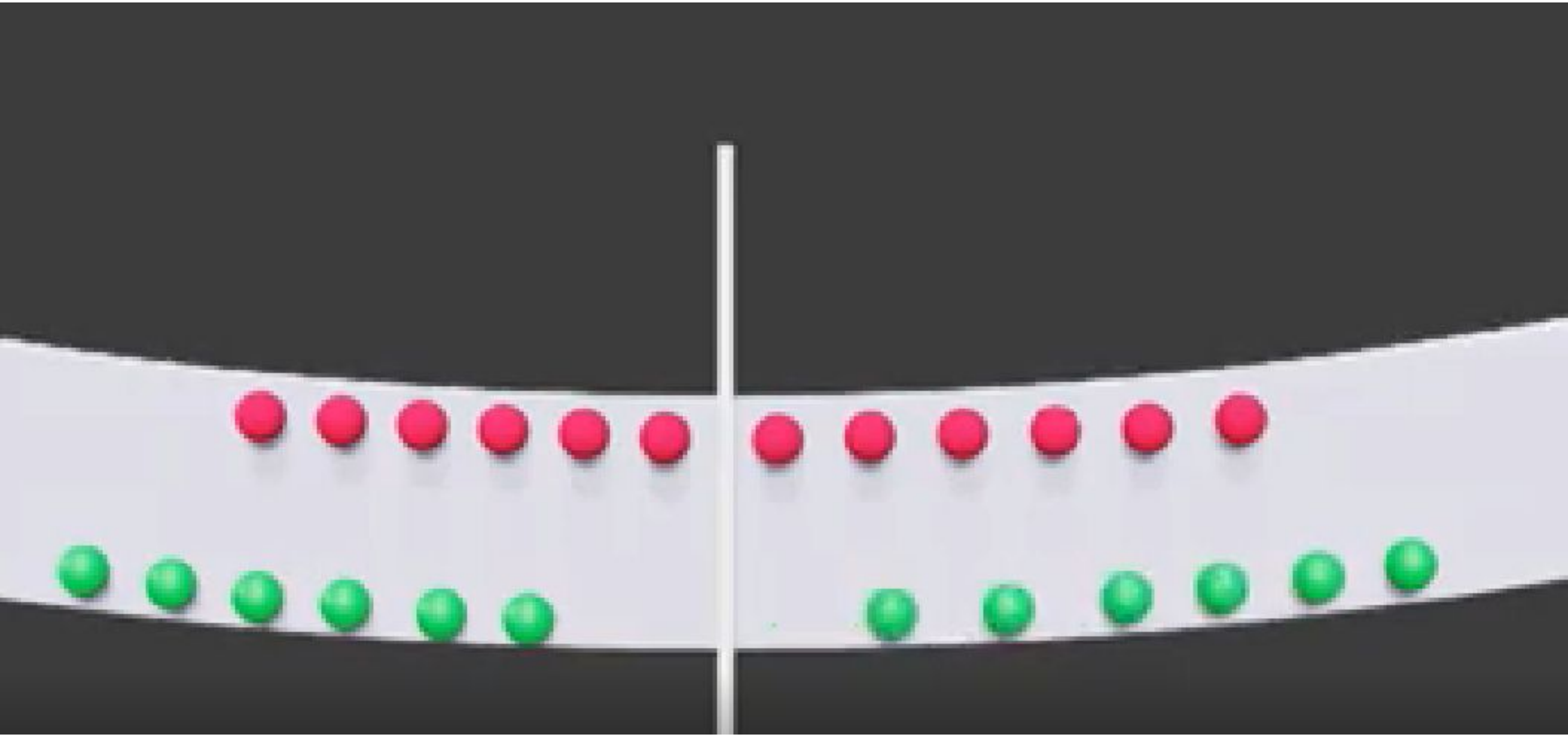


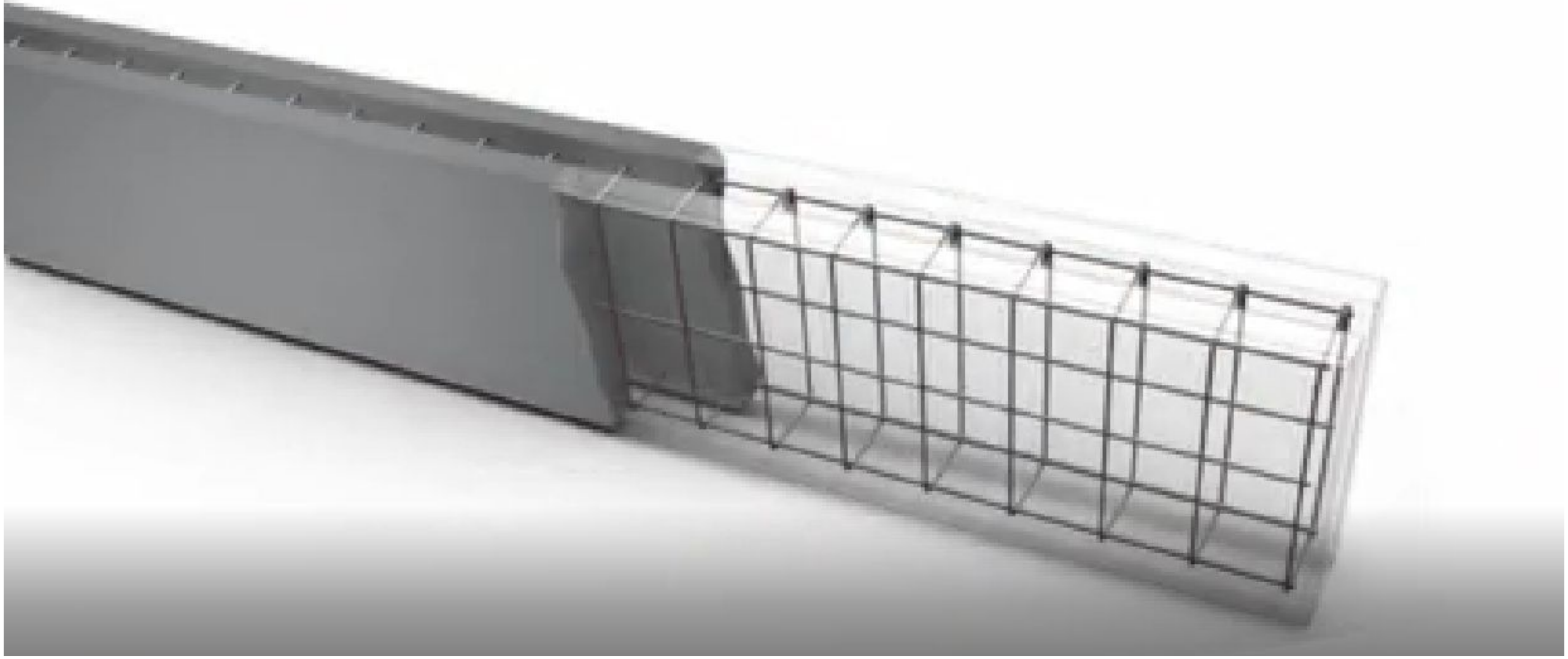

Tensile Stress


Compressive Stress

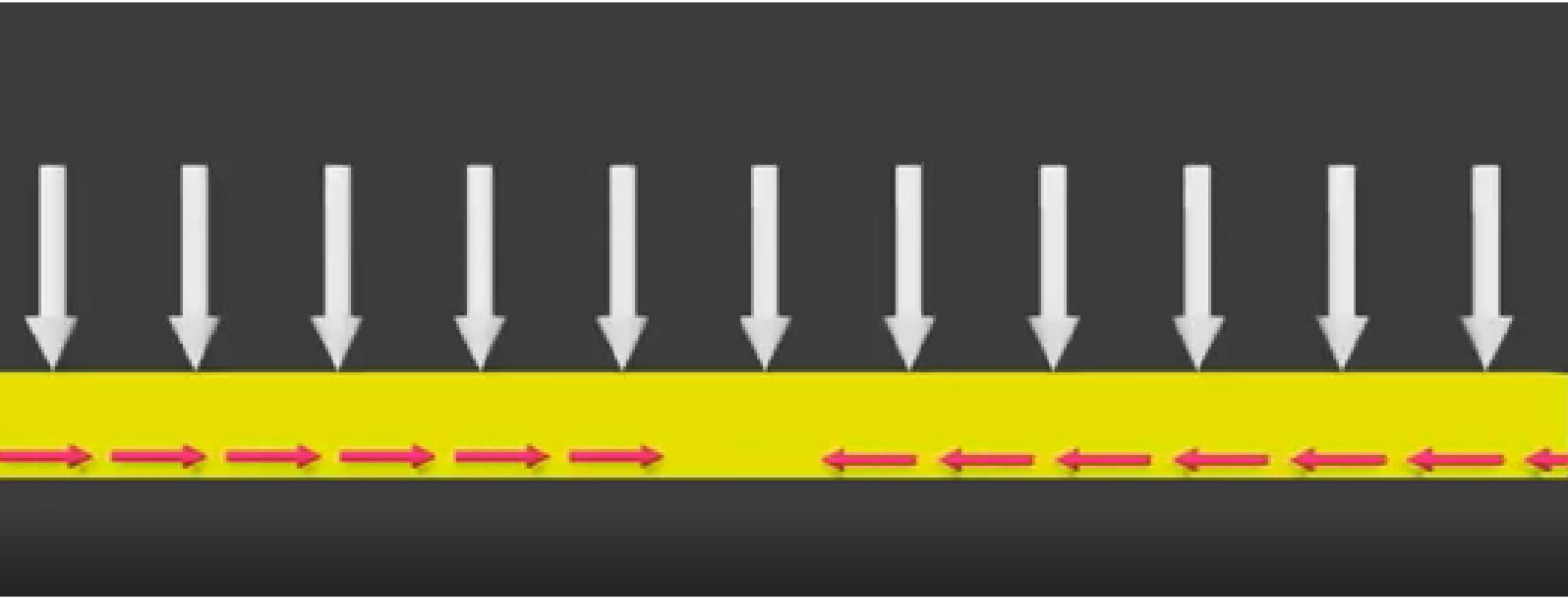

Stress Diagram
Stress Diagram
Compressive stress
D
Neutral Axis
Neutral Axis
Tensile stress
B
Beam Section
When no load is working
When load is working

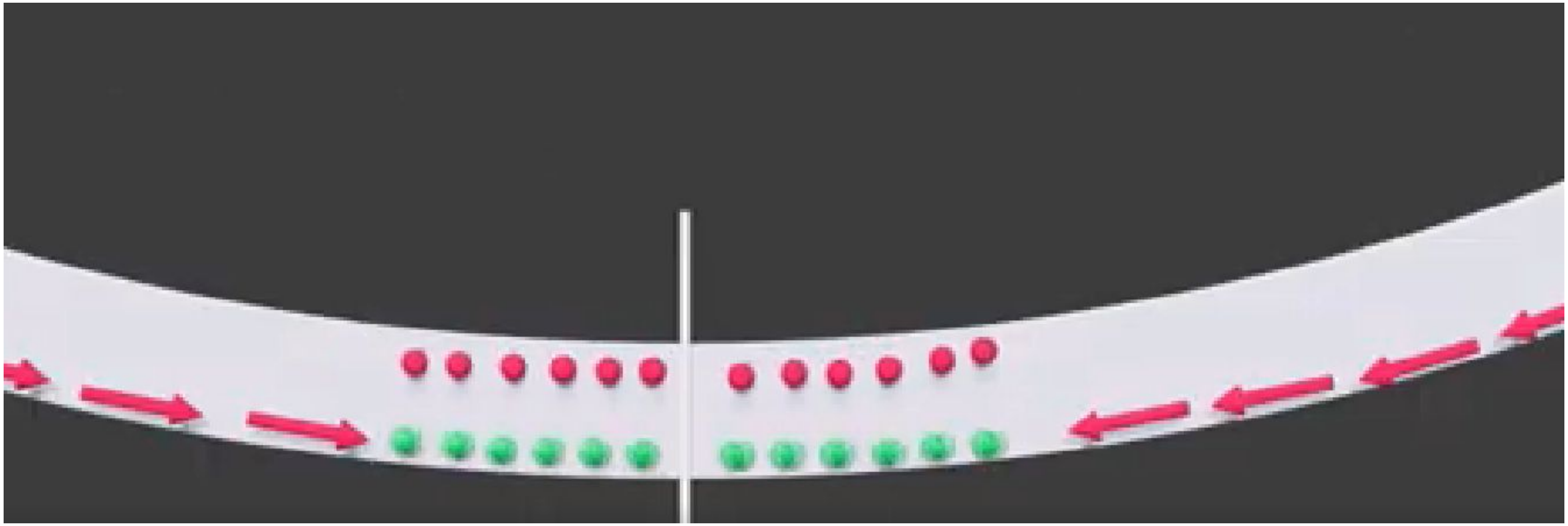






Prestress before applying the Load



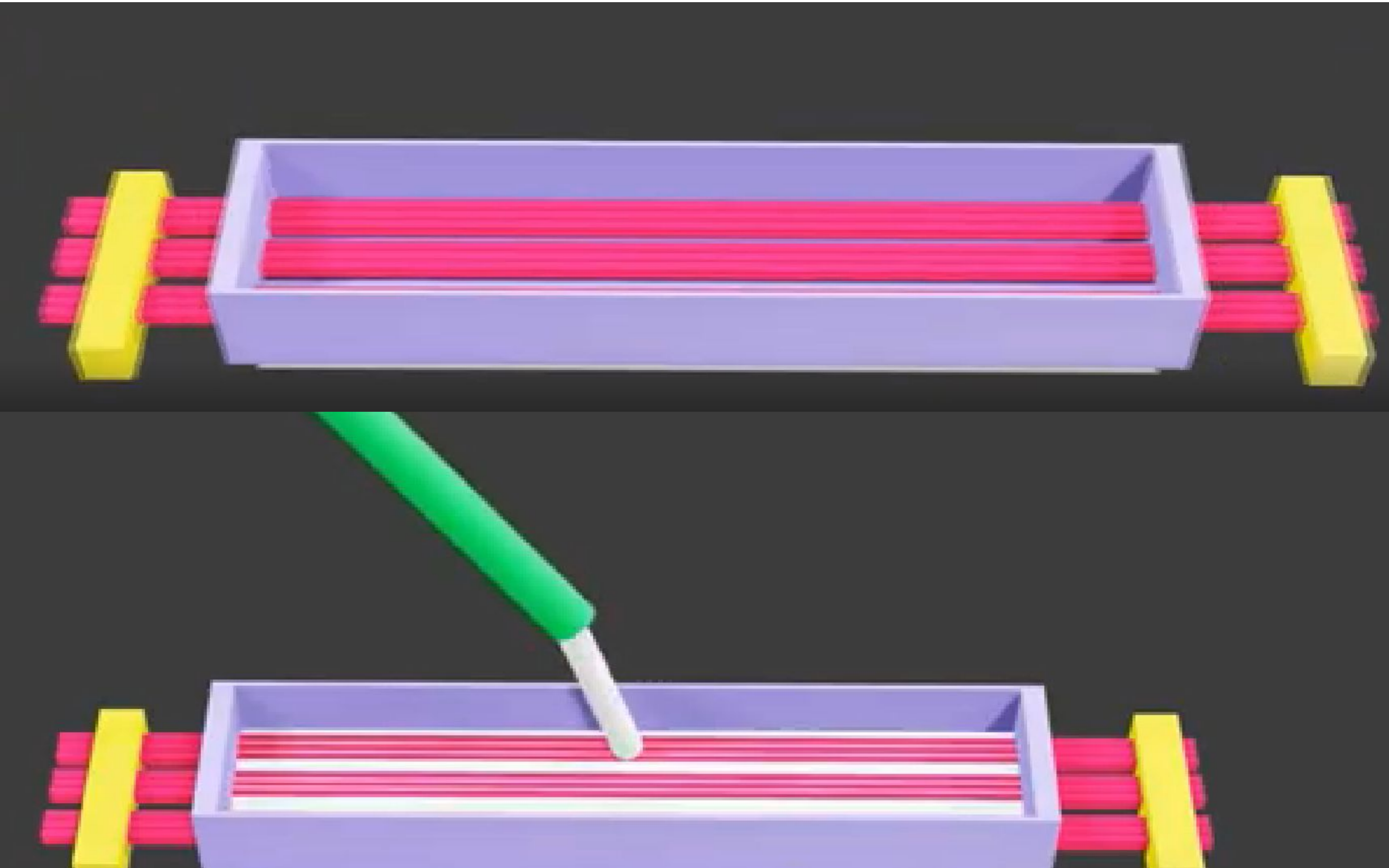


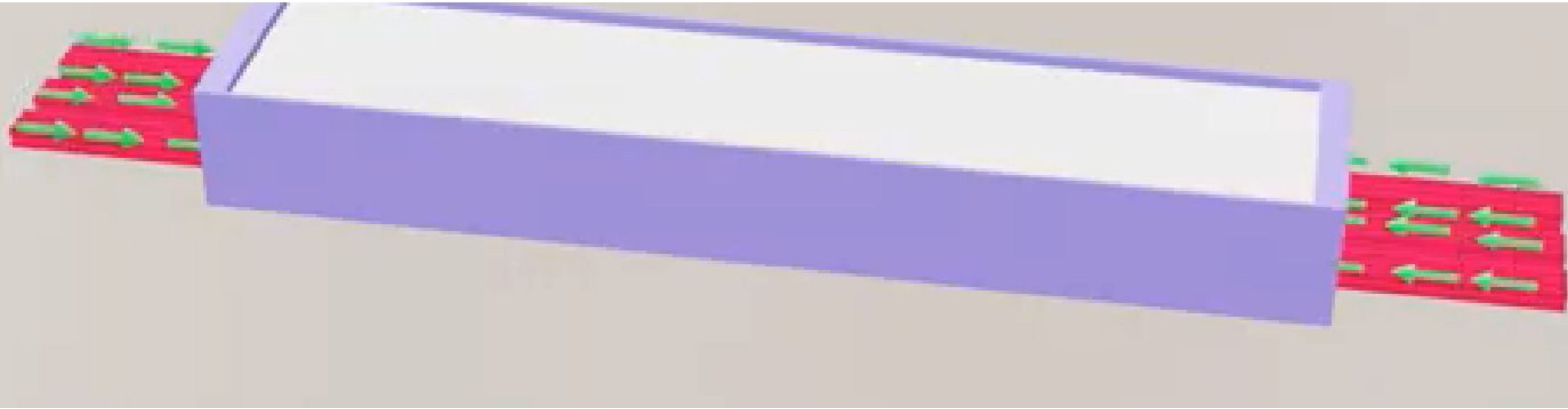
METHODS OF PREPARING PRE-STRESSED CONCRETE

PRE TENSIONING

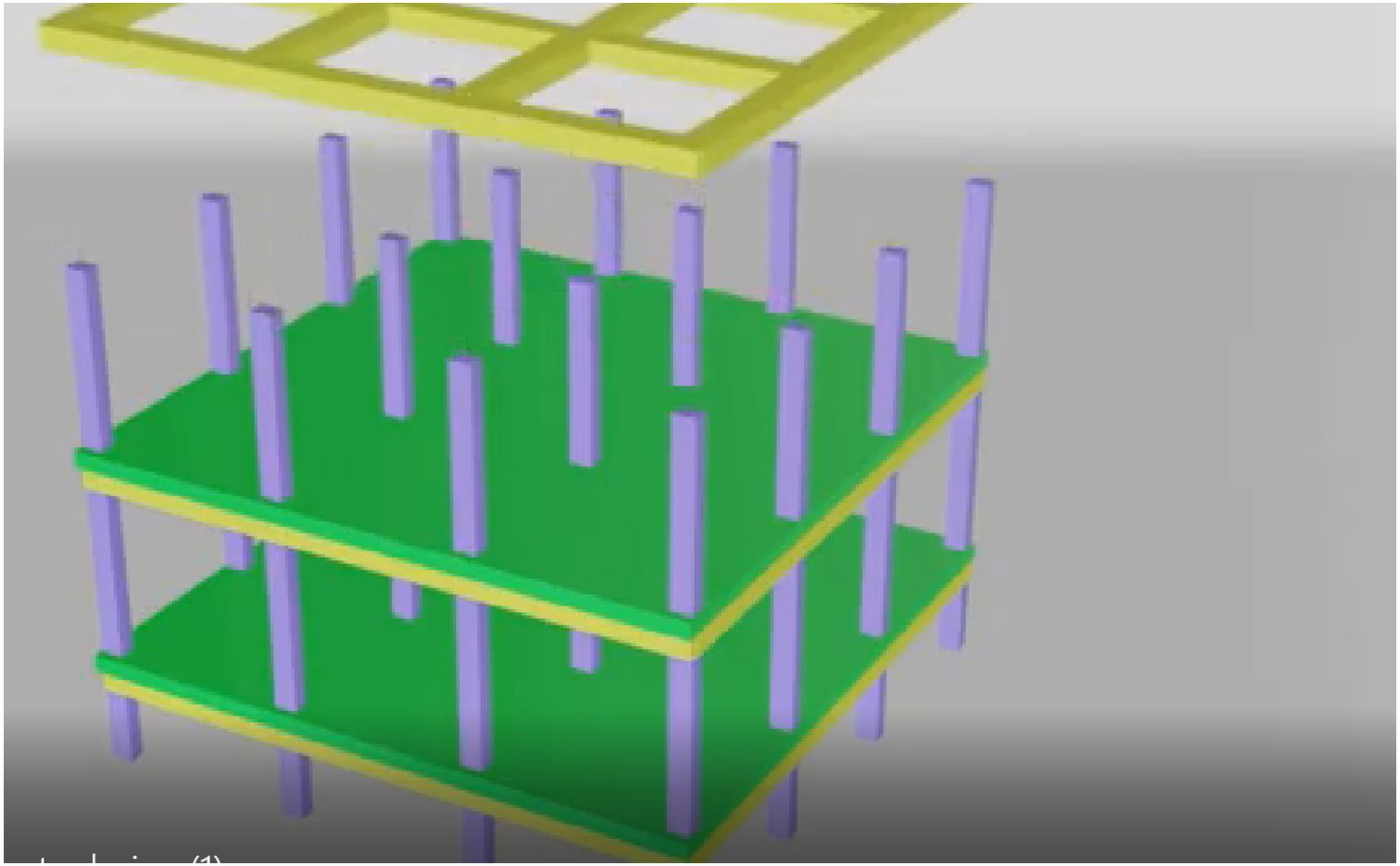
POST TENSIONING

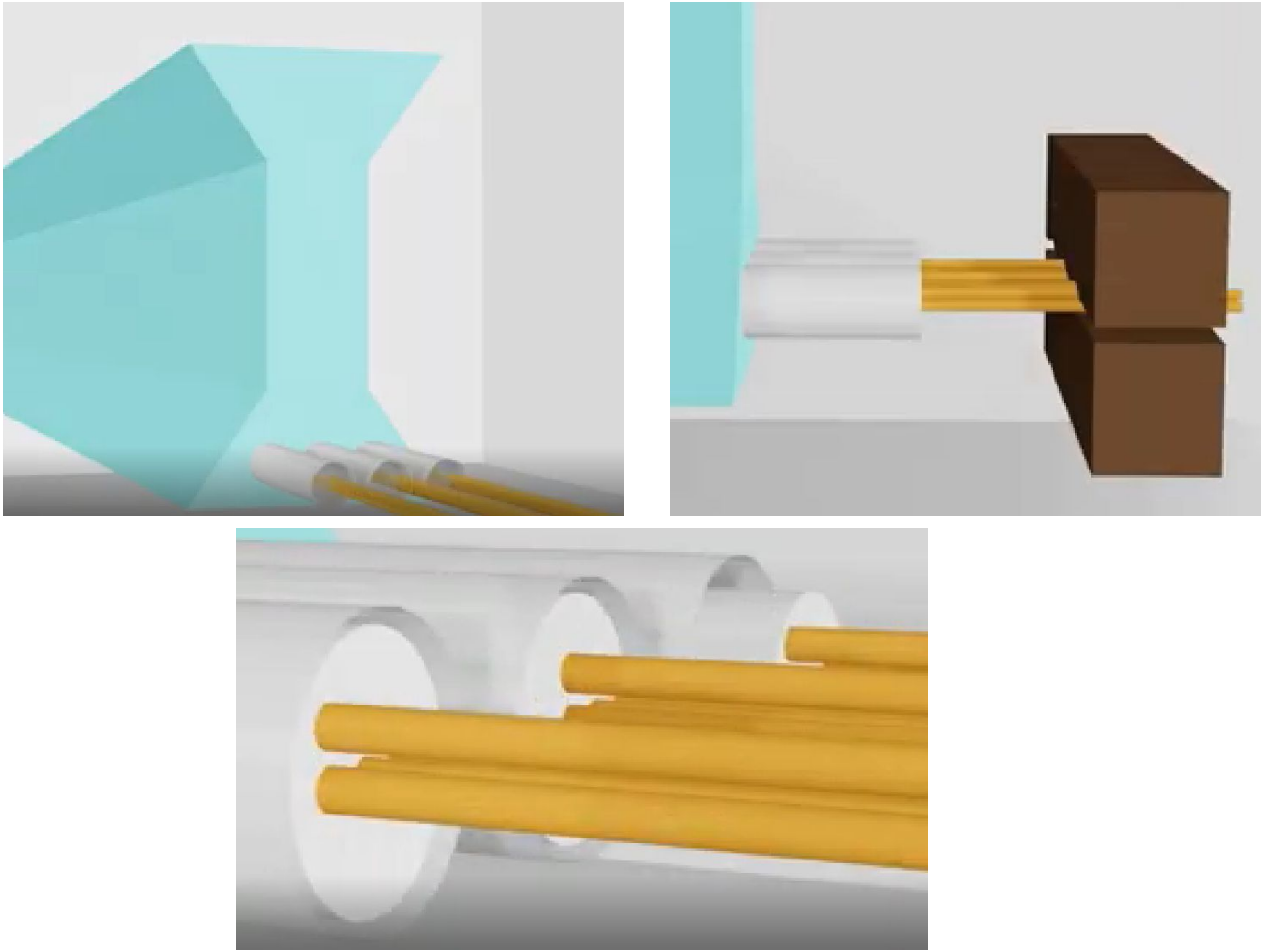
- Pre-tensioning





Post-Tensioning





One of the main advantages of prestressed over reinforced concrete is that, for a given span and loading, a smaller prestressed concrete member is required. This saving of the dead load of the structure is particularly important in long-span structures such as bridges, where the dead load is a large proportion of the total load. As well as a saving in concrete material for members, there is also a saving in foundation costs, and this can be a significant factor in areas of poor foundation material.

Another important advantage of prestressed concrete is that by suitable prestressing the structure can be rendered crack-free, which has important implications for durability, especially for liquid-retaining structures.

A third advantage is that prestressing offers a means of controlling deflections. A prestress force eccentric to the centroid of a member will cause a vertical deflection, usually in the opposite direction to that caused by the applied load. By suitable choice of prestress force, the deflections under applied load can be reduced or eliminated entirely.

Against the advantages listed above must be listed some disadvantages of using prestressed concrete. The fact that most, if not all, of the concrete cross-section is in compression under all load conditions means that any inherent problems due to long-term creep movements are increased. From the point of view of construction, a high level of quality control is required, both for material production and for locating the tendons within the structure.

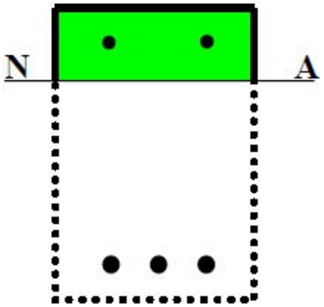
The technology required for prestressing concrete may not be available in many developing countries, and, if specified, may prove to be uneconomical since all equipment and personnel would have to be imported.

1.3 *Advantages of Prestressed Concrete*

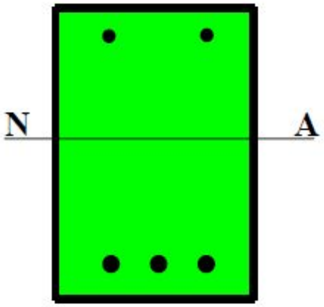
The main advantages of prestressed concrete (PSC) are:

Smaller Section Sizes

Since PSC uses the whole concrete section, the second moment of area is bigger and so the section is stiffer:



RC



PSC

Smaller Deflections

The larger second moment of area greatly reduces deflections for a given section size.

Increased Spans

The smaller section size reduces self weight. Hence a given section can span further with prestressed concrete than it can with ordinary reinforced concrete.

Durability

Since the entire section remains in compression, no cracking of the concrete can occur and hence there is little penetration of the cover. This greatly improves the long-term durability of structures, especially bridges and also means that concrete tanks can be made as watertight as steel tanks, with far greater durability.

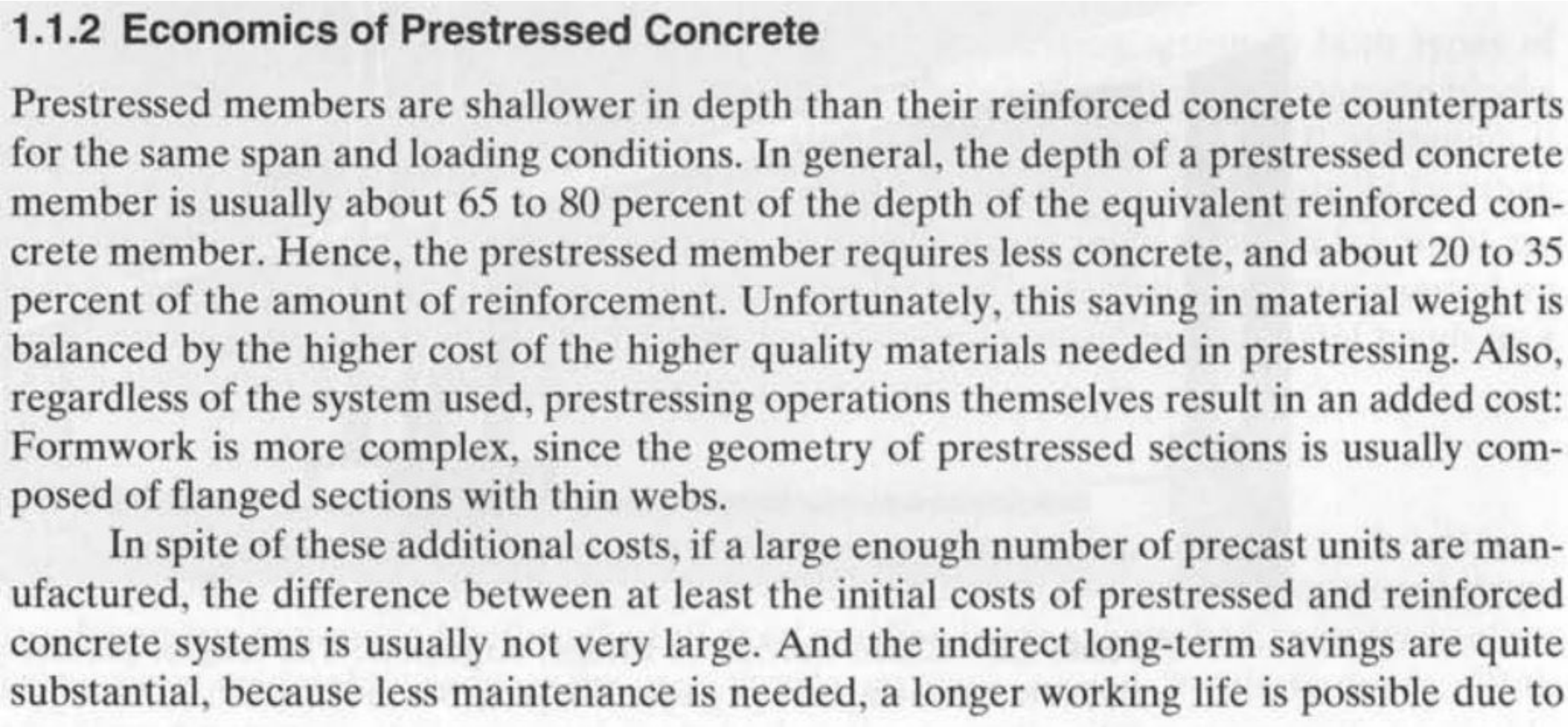
1.1.1 Comparison with Reinforced Concrete

From the preceding discussion, it is plain that permanent stresses in the prestressed structural member are created before the full dead and live loads are applied, in order to eliminate or considerably reduce the net tensile stresses caused by these loads. With reinforced concrete, it is assumed that the tensile strength of the concrete is negligible and disregarded. This is because the tensile forces resulting from the bending moments are resisted by the bond created in the reinforcement process. Cracking and deflection are therefore essentially irrecoverable in reinforced concrete once the member has reached its limit state at service load.

The reinforcement in the reinforced concrete member does not exert any force of its own on the member, contrary to the action of prestressing steel. The steel required to produce the prestressing force in the prestressed member actively preloads the member, permitting a relatively high controlled recovery of cracking and deflection. Once the flexural tensile strength of the concrete is exceeded, the prestressed member starts to act like a reinforced concrete element.

By controlling the amount of prestress, a structural system can be made either flexible or rigid without influencing its strength. In reinforced concrete, such a flexibility in behavior is considerably more difficult to achieve if considerations of economy are to be observed in the design. Flexible structures such as fender piles in wharves have to be highly energy absorbent, and prestressed concrete can provide the required resiliency. Structures designed to withstand heavy vibrations, such as machine foundations, can easily be made rigid through the contribution of the prestressing force to the reduction of their otherwise flexible deformation behavior.

الخرسانة سابقة الإجهاد هي شكل من أشكال الخرسانة المستخدمة في الانشاءات والتي يتم وضعها تحت ضغط قبل تحميلها بأحمال تتجاوز وزنها ويتم إنتاج هذا الضغط من خلال تحزيمها "بأوتار عالية القوة" من الداخل بحجم الخرسانة ويتم ذلك لتحسين أداء الخرسانة في التشغيل ويمكن أن تتكون الأوتار من أسلاك مفردة أو متعددة أو أشرطة مترابطة وتكون أكثر استخداماً من الحديد عالي الشد أو ألياف الكربون وأساس الخرسانة السابقة الإجهاد هو أنه بمجرد الضغط الأولي تكون الخرسانة لها خصائص مثل الخرسانة عالية التحمل لقوى الضغط والصلب المرن عالي المقاومة لقوى الشدوهذا يمكن أن يؤدي إلى تحسين القدرة الإنشائية والخدمية مقارنة بالخرسانة المسلحة التقليدية وفي كثير من الحالات يتم استخدام الخرسانة الجاهزة لمجموعة واسعة من المباني والهياكل الخرسانية حيث يمكن تحسين أدائها عن طريق تكبير بحور(span) الكمرات، وتقليل سمك البلاطات ، وتوفير فى المواد بالمقارنة مع الخرسانة المسلحة العادية. وتشمل تطبيقاتها النموذجية المباني الشاهقة، والابراج السكنية، والأساسات، وهياكل الجسور والسدود، والصوامع والصهاريج، والأرصفة الصناعية، والهياكل النووية ومن خصائص الخرسانة انها تقاوم قوى الضغط بقدر أكبر من قوى الشد لذا تم ابتكار طريقة سبق الإجهاد للتغلب على ضعف الشد. وللتمكن من إنتاج كمرات أو بلاطات أو جسور ذات بحور أكبر من البحور التي يمكن ان تنتجها الخرسانة المسلحة



1.1.2 Economics of Prestressed Concrete

Prestressed members are shallower in depth than their reinforced concrete counterparts for the same span and loading conditions. In general, the depth of a prestressed concrete member is usually about 65 to 80 percent of the depth of the equivalent reinforced concrete member. Hence, the prestressed member requires less concrete, and about 20 to 35 percent of the amount of reinforcement. Unfortunately, this saving in material weight is balanced by the higher cost of the higher quality materials needed in prestressing. Also, regardless of the system used, prestressing operations themselves result in an added cost: Formwork is more complex, since the geometry of prestressed sections is usually composed of flanged sections with thin webs.

In spite of these additional costs, if a large enough number of precast units are manufactured, the difference between at least the initial costs of prestressed and reinforced concrete systems is usually not very large. And the indirect long-term savings are quite substantial, because less maintenance is needed, a longer working life is possible due to

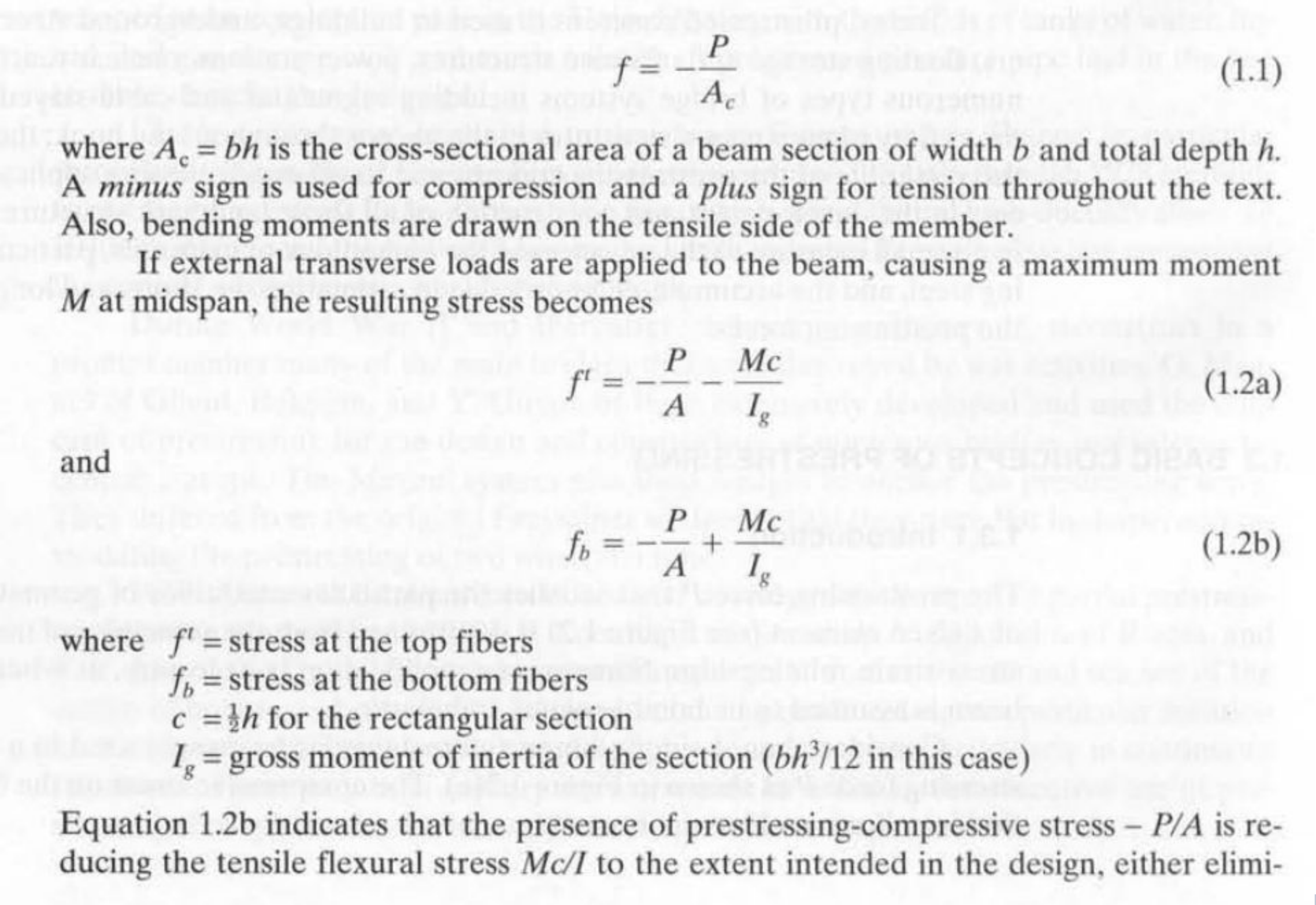
better quality control of the concrete, and lighter foundations are achieved due to the smaller cumulative weight of the superstructure.

Once the beam span of reinforced concrete exceeds 70 to 90 feet, the dead weight of the beam becomes excessive, resulting in heavier members and, consequently, greater long-term deflection and cracking. Thus, for larger spans, prestressed concrete becomes mandatory since arches are expensive to construct and do not perform as well due to the severe long-term shrinkage and creep they undergo. Very large spans such as segmental bridges or cable-stayed bridges can *only* be constructed through the use of prestressing.

1.3.1 Introduction

The prestressing force P that satisfies the particular conditions of geometry and loading of a given element (see Figure 1.2) is determined from the principles of mechanics and of stress-strain relationships. Sometimes simplification is necessary, as when a prestressed beam is assumed to be homogeneous and elastic.

Consider, then, a simply supported rectangular beam subjected to a *concentric* prestressing force P as shown in Figure 1.2(a). The compressive stress on the beam cross section is uniform and has an intensity



$$f = -\frac{P}{A_c} \tag{1.1}$$

where $A_c = bh$ is the cross-sectional area of a beam section of width b and total depth h. A *minus* sign is used for compression and a *plus* sign for tension throughout the text. Also, bending moments are drawn on the tensile side of the member.

If external transverse loads are applied to the beam, causing a maximum moment M at midspan, the resulting stress becomes

$$f^t = -\frac{P}{A} - \frac{Mc}{I_g} \tag{1.2a}$$

and

$$f_b = -\frac{P}{A} + \frac{Mc}{I_g} \tag{1.2b}$$

where f^t = stress at the top fibers
f_b = stress at the bottom fibers
$c = \frac{1}{2}h$ for the rectangular section
I_g = gross moment of inertia of the section ($bh^3/12$ in this case)

Equation 1.2b indicates that the presence of prestressing-compressive stress $-P/A$ is reducing the tensile flexural stress Mc/I to the extent intended in the design, either elimi-

Basic Concepts of Prestressing

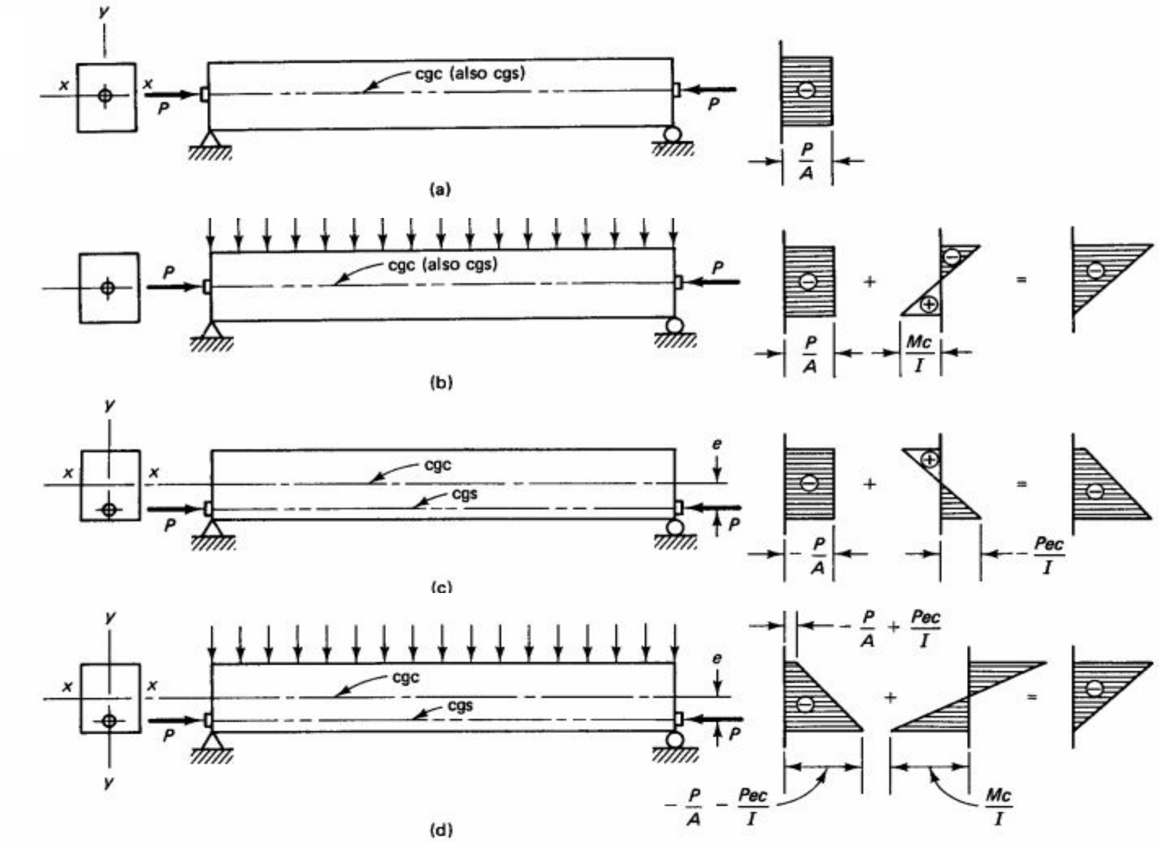
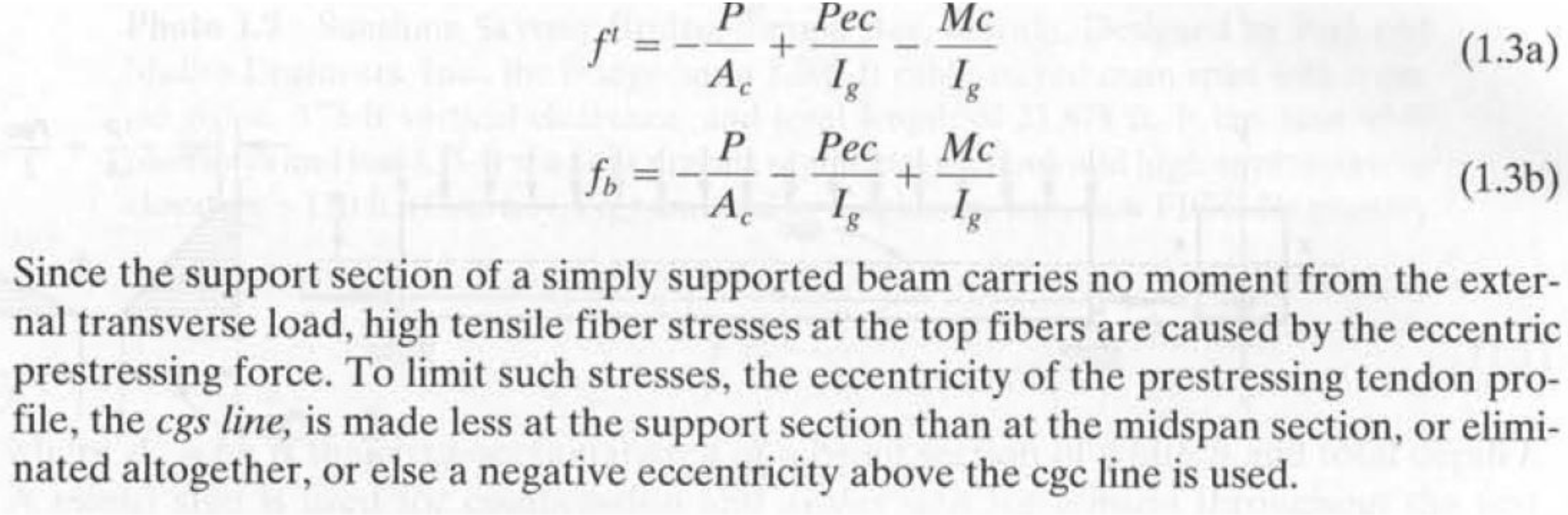


Figure 1.2 Concrete fiber stress distribution in a rectangular beam with straight tendon. (a) Concentric tendon, prestress only. (b) Concentric tendon, self-weight added. (c) Eccentric tendon, prestress only. (d) Eccentric tendon, self-weight added.

nating tension totally (even inducing compression), or permitting a level of tensile stress within allowable code limits. The section is then considered uncracked and behaves elastically: the concrete's inability to withstand tensile stresses is effectively compensated for by the compressive force of the prestressing tendon.

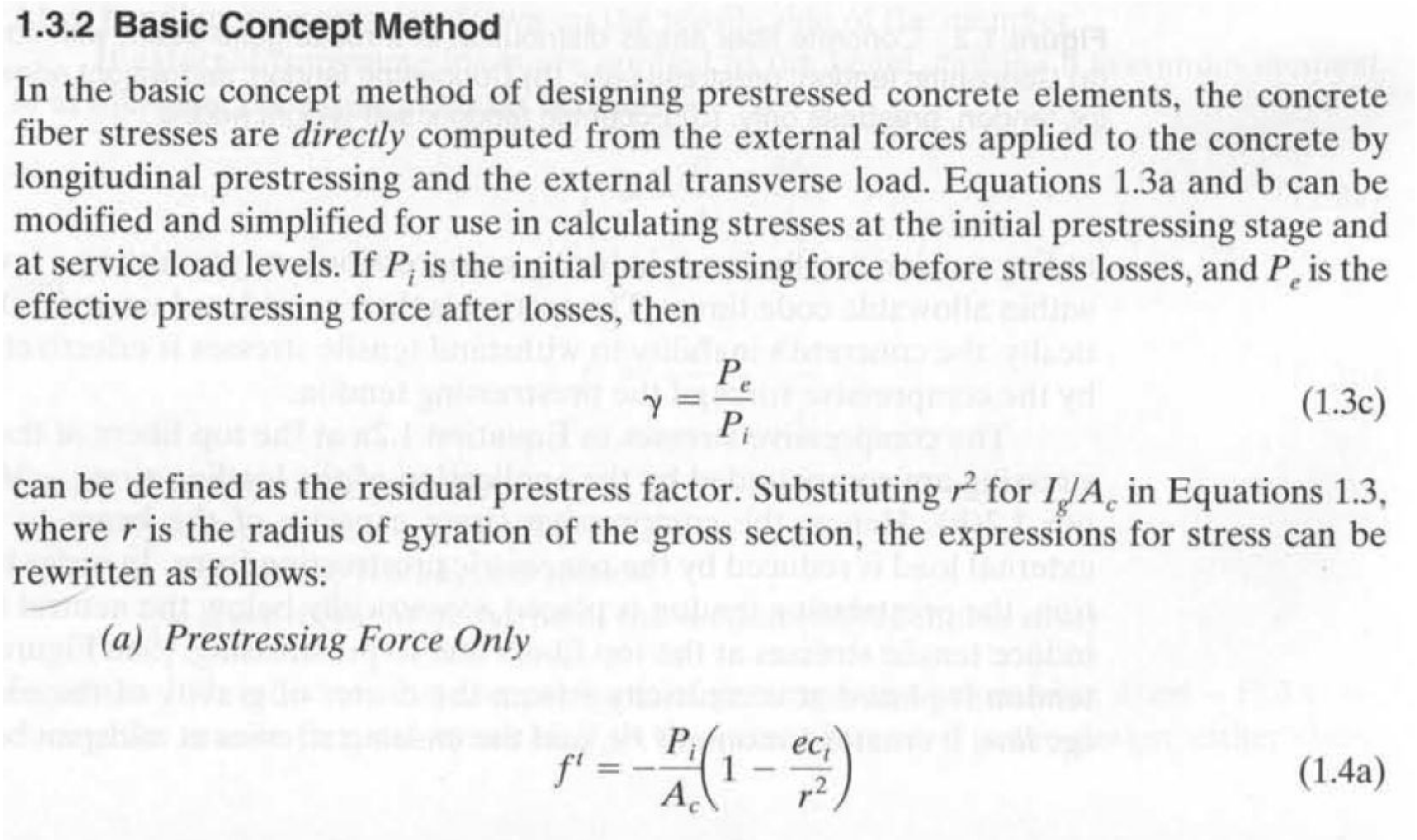
The compressive stresses in Equation 1.2a at the top fibers of the beam due to prestressing are compounded by the application of the loading stress $-Mc/I$, as seen in Figure 1.2(b). Hence, the compressive stress capacity of the beam to take a substantial external load is reduced by the *concentric* prestressing force. In order to avoid this limitation, the prestressing tendon is placed *eccentrically* below the neutral axis at midspan, to induce tensile stresses at the top fibers due to prestressing. [See Figure 1.2(c), (d).] If the tendon is placed at eccentricity e from the center of gravity of the concrete, termed the *cgc line,* it creates a moment Pe, and the ensuing stresses at midspan become



$$f^t = -\frac{P}{A_c} + \frac{Pec}{I_g} - \frac{Mc}{I_g} \tag{1.3a}$$

$$f_b = -\frac{P}{A_c} - \frac{Pec}{I_g} + \frac{Mc}{I_g} \tag{1.3b}$$

Since the support section of a simply supported beam carries no moment from the external transverse load, high tensile fiber stresses at the top fibers are caused by the eccentric prestressing force. To limit such stresses, the eccentricity of the prestressing tendon profile, the *cgs line,* is made less at the support section than at the midspan section, or eliminated altogether, or else a negative eccentricity above the cgc line is used.



1.3.2 Basic Concept Method

In the basic concept method of designing prestressed concrete elements, the concrete fiber stresses are *directly* computed from the external forces applied to the concrete by longitudinal prestressing and the external transverse load. Equations 1.3a and b can be modified and simplified for use in calculating stresses at the initial prestressing stage and at service load levels. If P_i is the initial prestressing force before stress losses, and P_e is the effective prestressing force after losses, then

$$\gamma = \frac{P_e}{P_i} \qquad (1.3c)$$

can be defined as the residual prestress factor. Substituting r^2 for I_g/A_c in Equations 1.3, where r is the radius of gyration of the gross section, the expressions for stress can be rewritten as follows:

(a) *Prestressing Force Only*

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) \qquad (1.4a)$$

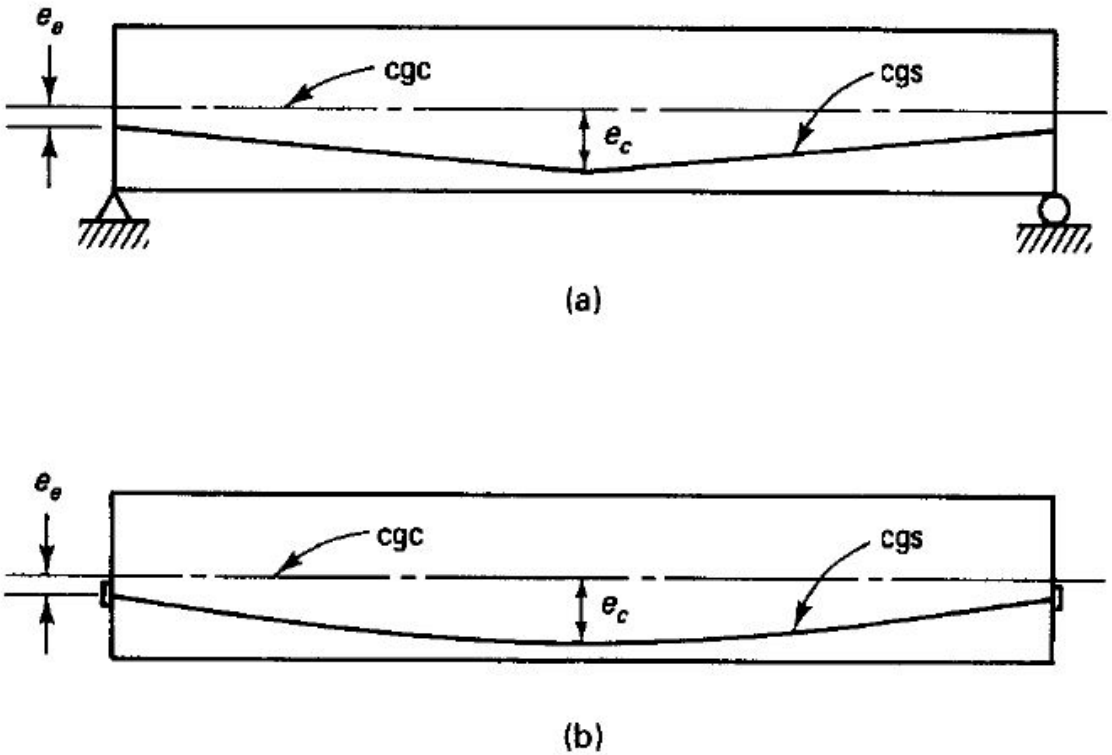


Figure 1.3 Prestressing tendon profile. (a) Harped tendon. (b) Draped tendon.

where S^t and S_b are the moduli of the sections for the top and bottom fibers, respectively.

The change in eccentricity from the midspan to the support section is obtained by raising the prestressing tendon either abruptly from the midspan to the support, a process called harping, or gradually in a parabolic form, a process called draping. Figure 1.3(a) shows a harped profile usually used for pretensioned beams and for concentrated transverse loads. Figure 1.3(b) shows a draped tendon usually used in post-tensioning.

Subsequent to erection and installation of the floor or deck, live loads act on the structure, causing a superimposed moment M_s. The full intensity of such loads normally occurs after the building is completed and some time-dependent losses in prestress have already taken place. Hence, the prestressing force used in the stress equations would have to be the effective prestressing force P_e. If the total moment due to gravity loads is M_T, then

$$M_T = M_D + M_{SD} + M_L \tag{1.6}$$

where M_D = moment due to self-weight
M_{SD} = moment due to superimposed dead load, such as flooring
M_L = moment due to live load, including impact and seismic loads if any

Equations 1.5 then become

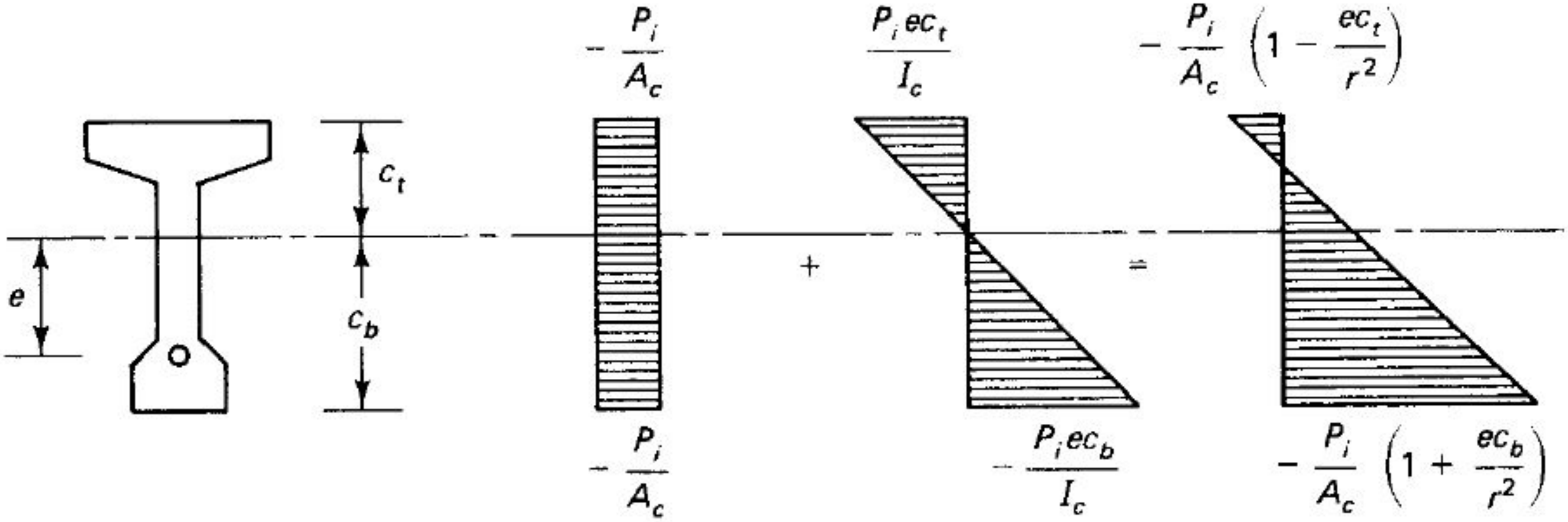
$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \tag{1.7a}$$

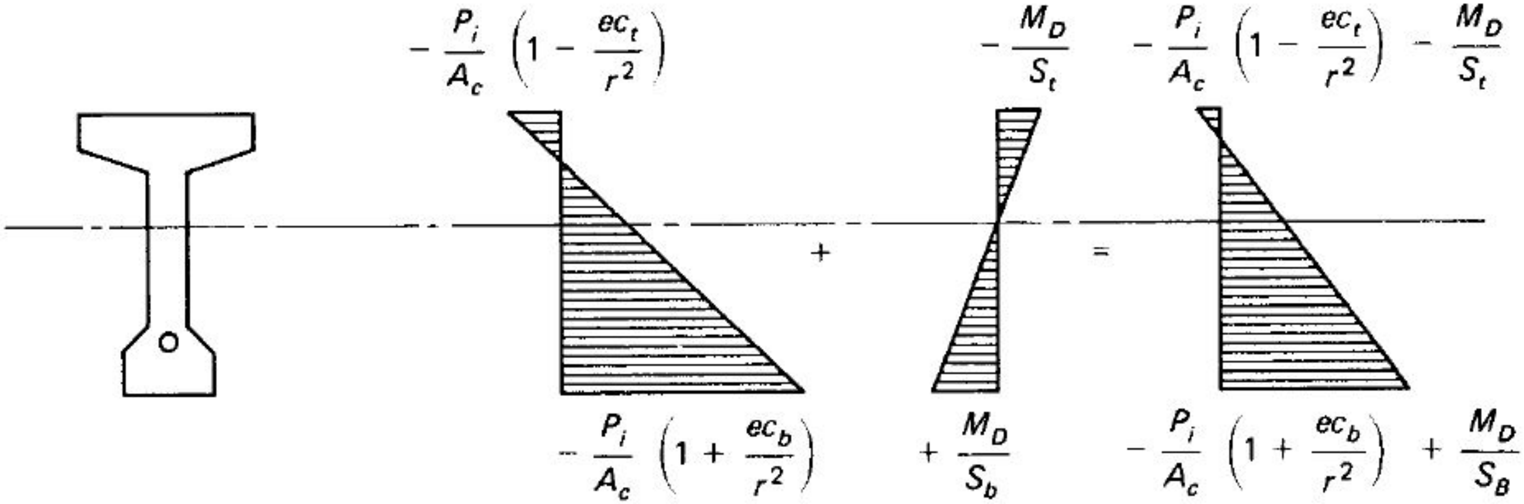
$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \tag{1.7b}$$

Some typical elastic concrete stress distributions at the critical section of a prestressed flanged section are shown in Figure 1.4. The tensile stress in the concrete in part (c) permitted at the extreme fibers of the section cannot exceed the maximum permissible in the code, e.g., $f_t = 6\sqrt{f_c'}$ at midspan in the ACI code. If it is exceeded, bonded nonprestressed reinforcement proportioned to resist the total tensile force has to be provided to control cracking at service loads.

1.3.3 C-Line Method

In this line-of-pressure or thrust concept, the beam is analyzed as if it were a plain concrete elastic beam using the basic principles of statics. The prestressing force is considered an ex-


$-\frac{P_i}{A_c}$
$\frac{P_i e c_t}{I_c}$
$-\frac{P_i}{A_c}\left(1-\frac{e c_t}{r^2}\right)$
c_t
e
c_b
+
=
$-\frac{P_i}{A_c}$
$-\frac{P_i e c_b}{I_c}$
$-\frac{P_i}{A_c}\left(1+\frac{e c_b}{r^2}\right)$


$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)$
$-\frac{M_D}{S_t}$
$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_D}{S_t}$
+
=
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)$
$+\frac{M_D}{S_b}$
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_D}{S_B}$

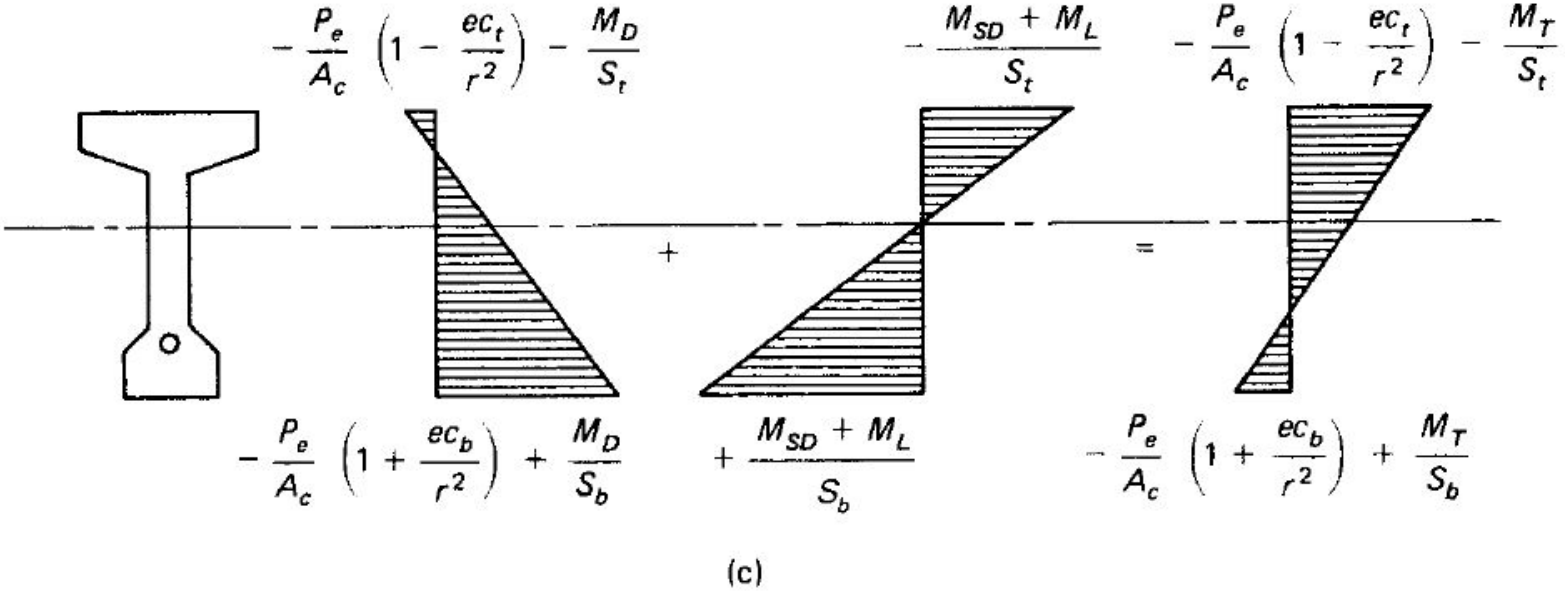
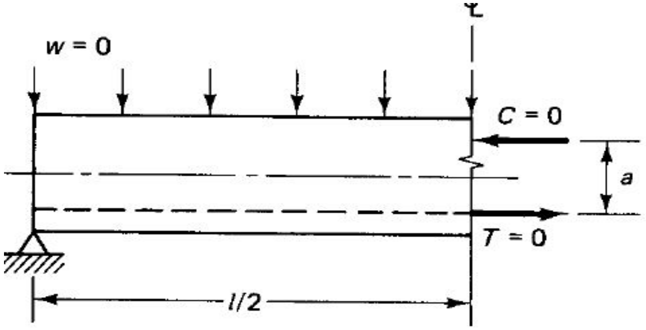


Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

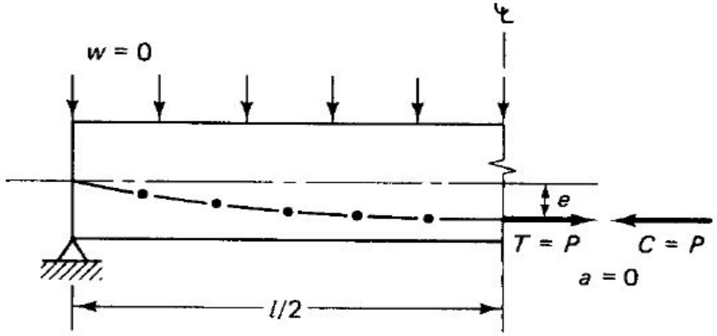
ternal compressive force, with a constant tensile force T in the tendon throughout the span. In this manner, the effects of external gravity loads are disregarded. Equilibrium equations $\Sigma H = 0$ and $\Sigma M = 0$ are applied to maintain equilibrium in the section.

Figure 1.5 shows the relative line of action of the compressive force C and the tensile force T in a reinforced concrete beam as compared to that in a prestressed concrete beam. It is plain that in a reinforced concrete beam, T can have a finite value only when transverse and other external loads act. The moment arm a remains basically constant throughout the elastic loading history of the reinforced concrete beam while it changes from a value $a = 0$ at prestressing to a maximum at full superimposed load.

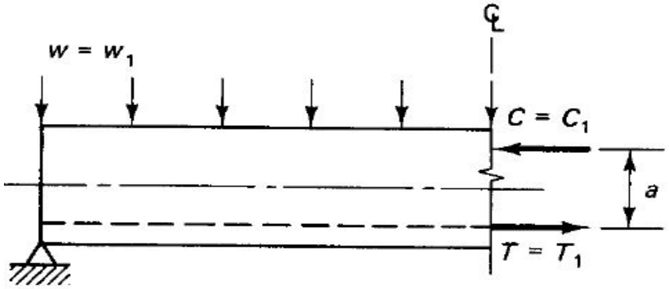
Taking a free-body diagram of a segment of a beam as in Figure 1.6, it is evident that the C-line, or center-of-pressure line, is at a varying distance a from the T-line. The moment is given by



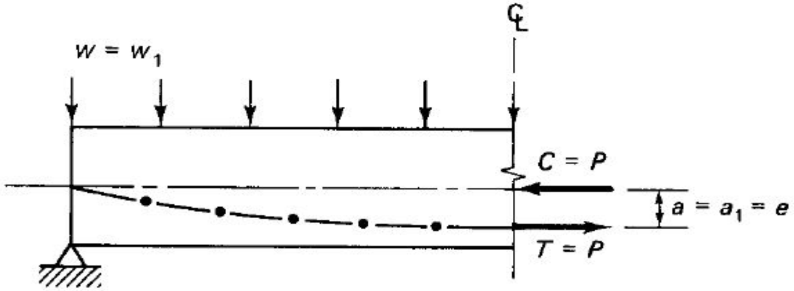
(a)



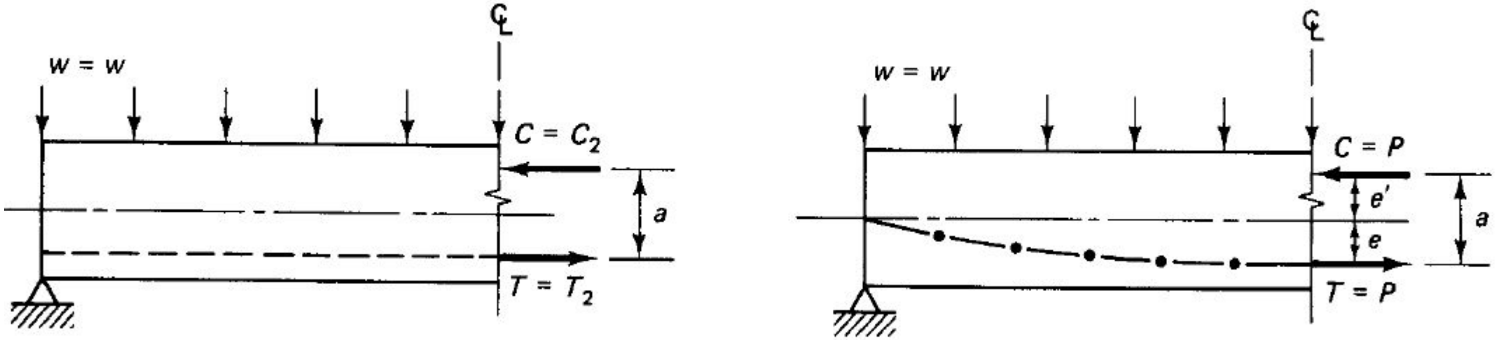
(b)



(c)



(d)



(e) (f)

Figure 1.5 Comparative free-body diagrams of a reinforced concrete (R.C.) beam and a prestressed concrete (P.C.) beam. (a) R.C. beam with no load. (b) P.C. beam with no load. (c) R.C. beam with load w_1. (d) P.C. beam with load w_1. (e) R.C. beam with typical load w. (f) P.C. beam with typical load w.

$$M = Ca = Ta \tag{1.8}$$

and the eccentricity e is known or predetermined, so that in Figure 1.6,

$$e' = a - e \tag{1.9a}$$

Since $C = T$, $a = M/T$, giving

$$e' = \frac{M}{T} - e \tag{1.9b}$$

From the figure,

$$f^t = -\frac{C}{A_c} - \frac{Ce'c_t}{I_c} \tag{1.10a}$$

$$f_b = -\frac{C}{A_c} + \frac{Ce'c_b}{I_c} \tag{1.10b}$$

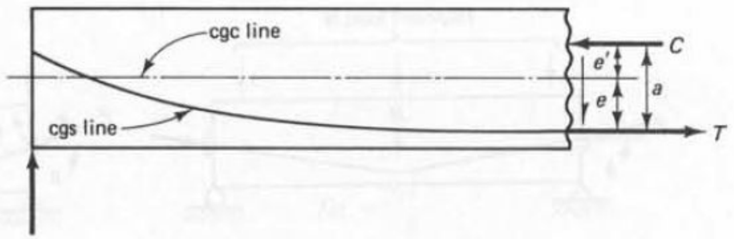


Figure 1.6 Free-body diagram for the C-line (center of pressure).

But in the tendon the force T equals the prestressing force P_e; so

$$f^t = -\frac{P_e}{A_c} - \frac{P_e e' c_t}{I_c} \tag{1.11a}$$

$$f_b = -\frac{P_e}{A_c} + \frac{P_e e' c_b}{I_c} \tag{1.11b}$$

Since $I_c = A_c r^2$, Equations 1.11a and b can be rewritten as

$$f^t = -\frac{P_e}{A_c}\left(1 + \frac{e' c_t}{r^2}\right) \tag{1.12a}$$

$$f_b = -\frac{P_e}{A_c}\left(1 - \frac{e' c_b}{r^2}\right) \tag{1.12b}$$

Equations 1.12a and b and Equations 1.7a and b should yield identical values for the fiber stresses.

Creep is indicated when strain in a solid increases with time while the stress producing the strain is kept constant. In more practical terms, creep is the increased strain or deformation of a structural element under a constant load. Depending on the construction material, structural design, and service conditions, creep can result in significant displacements in a structure. Severe creep strains can result in serviceability problems, stress redistribution, prestress loss, and even failure of structural elements.

Creep and shrinkage of concrete are two physical properties of concrete. The creep of concrete, which originates from the calcium silicate hydrates (C-S-H) in the hardened Portland cement paste (which is the binder of mineral aggregates), is fundamentally different from the creep of metals and polymers.

Unlike the creep of metals, it occurs at all stress levels and, within the service stress range, is linearly dependent on the stress if the pore water content is constant. Unlike the creep of polymers and metals, it exhibits multi-months aging, caused by chemical hardening due to hydration which stiffens the microstructure, and multi-year aging, caused by long-term relaxation of self-equilibrated micro-stresses in the nano-po $J(t,t')$, icrostructure of the C-S-H. If concrete is fullϵ dried, it does not creep, but it is next to impossible to dry concrete fully without severe $\sigma = 1$ applied at age t',

Changes of pore water content due to drying or wetting processes cause significant volume changes of concrete in load-free specimens. They are called the shrinkage (typically causing strains between 0.0002 and 0.0005, and in low strength concretes even 0.0012) or swelling (< 0.00005 in normal concretes, < 0.00020 in high strength concretes).
To separate shrinkage from creep, the compliance function defined as the stress-produced strain (i.e., the total strain minus shrinkage) caused at time t by a unit sustained uniaxial stress is measured as the strain difference between the loaded and load-free specimens.

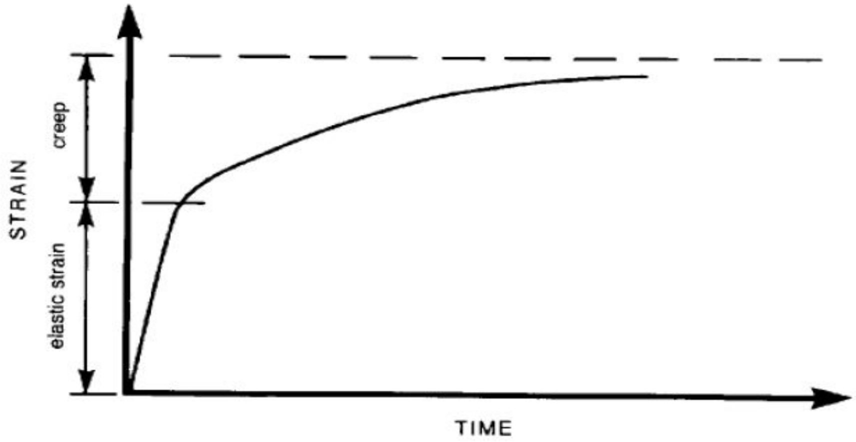
1.4 Materials

Concrete

The main factors for concrete used in PSC are:

- Ordinary portland cement-based concrete is used but strength usually greater than 50 N/mm^2;
- A high early strength is required to enable quicker application of prestress;
- A larger elastic modulus is needed to reduce the shortening of the member;
- A mix that reduces creep of the concrete to minimize losses of prestress;

You can see the importance creep has in PSC from this graph:

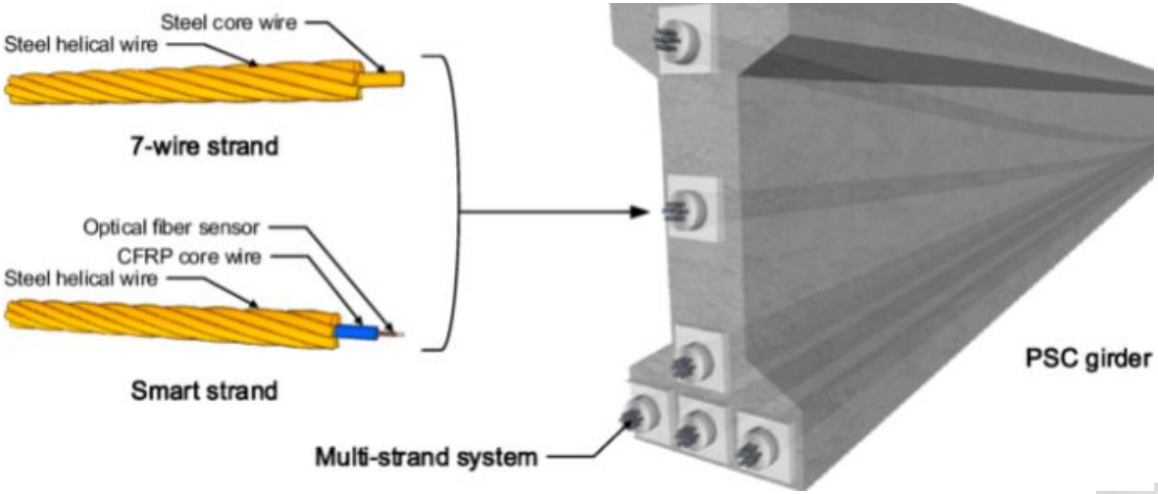


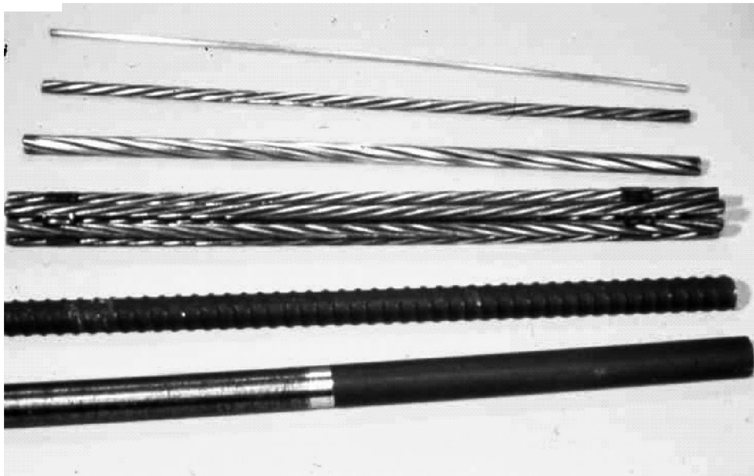
Steel

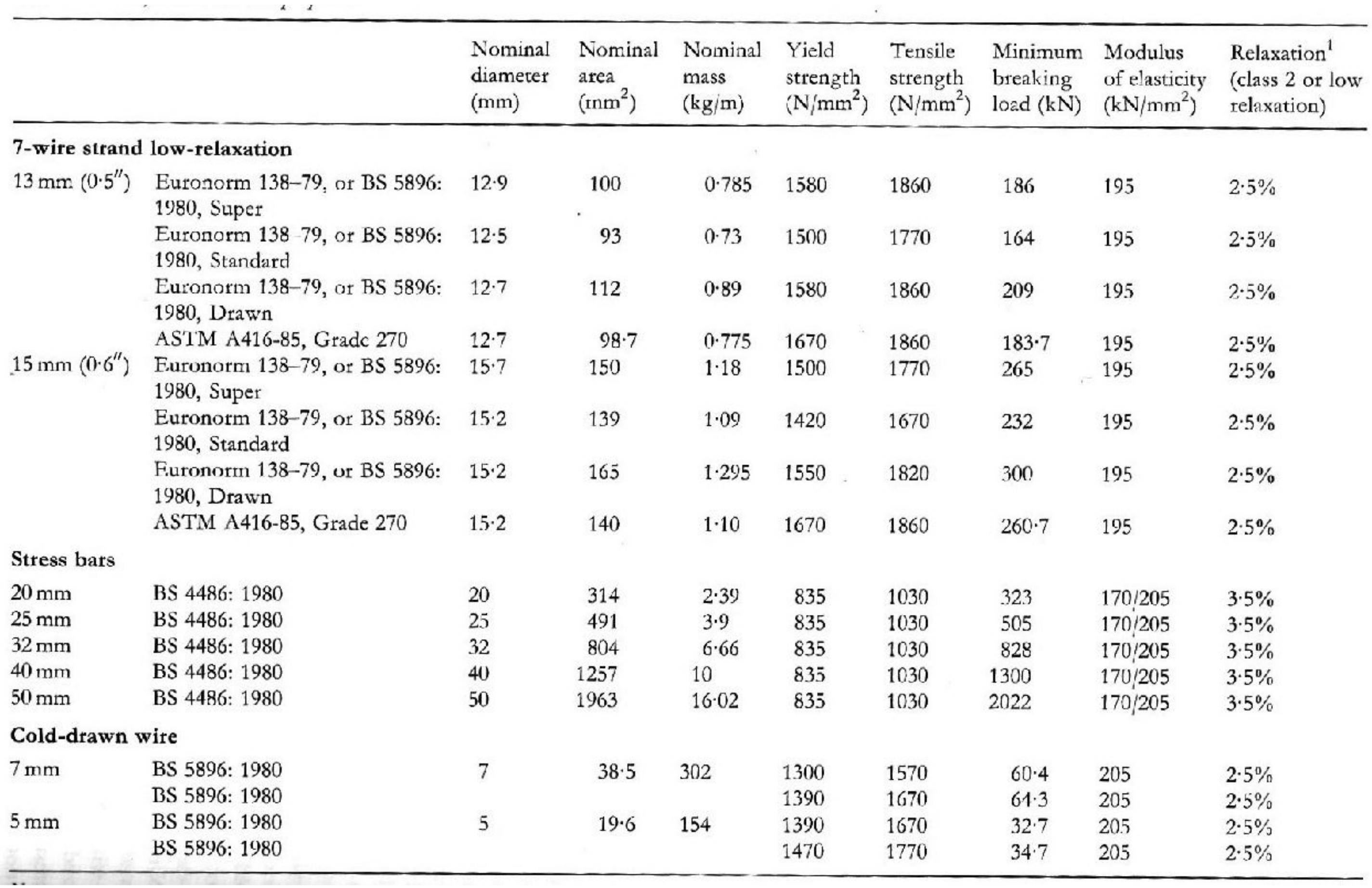
The steel used for prestressing has a nominal yield strength of between 1550 to 1800 N/mm^2. The different forms the steel may take are:

- Wires: individually drawn wires of 7 mm diameter;
- Strands: a collection of wires (usually 7) wound together and thus having a diameter that is different to its area;
- Tendon: A collection of strands encased in a duct – only used in post-tensioning;
- Bar: a specially formed bar of high strength steel of greater than 20 mm diameter.

Prestressed concrete bridge beams typically use 15.7 mm diameter (but with an area of 150 mm^2)7-wire super strand which has a breaking load of 265 kN.


Steel core wire
Steel helical wire
7-wire strand
Optical fiber sensor
CFRP core wire
Steel helical wire
Smart strand
PSC girder
Multi-strand system





		Nominal diameter (mm)	Nominal area (mm^2)	Nominal mass (kg/m)	Yield strength (N/mm^2)	Tensile strength (N/mm^2)	Minimum breaking load (kN)	Modulus of elasticity (kN/mm^2)	Relaxation[1] (class 2 or low relaxation)
7-wire strand low-relaxation									
13 mm (0·5″)	Euronorm 138–79, or BS 5896: 1980, Super	12·9	100	0·785	1580	1860	186	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Standard	12·5	93	0·73	1500	1770	164	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Drawn	12·7	112	0·89	1580	1860	209	195	2·5%
	ASTM A416-85, Grade 270	12·7	98·7	0·775	1670	1860	183·7	195	2·5%
15 mm (0·6″)	Euronorm 138–79, or BS 5896: 1980, Super	15·7	150	1·18	1500	1770	265	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Standard	15·2	139	1·09	1420	1670	232	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Drawn	15·2	165	1·295	1550	1820	300	195	2·5%
	ASTM A416-85, Grade 270	15·2	140	1·10	1670	1860	260·7	195	2·5%
Stress bars									
20 mm	BS 4486: 1980	20	314	2·39	835	1030	323	170/205	3·5%
25 mm	BS 4486: 1980	25	491	3·9	835	1030	505	170/205	3·5%
32 mm	BS 4486: 1980	32	804	6·66	835	1030	828	170/205	3·5%
40 mm	BS 4486: 1980	40	1257	10	835	1030	1300	170/205	3·5%
50 mm	BS 4486: 1980	50	1963	16·02	835	1030	2022	170/205	3·5%
Cold-drawn wire									
7 mm	BS 5896: 1980	7	38·5	302	1300	1570	60·4	205	2·5%
	BS 5896: 1980				1390	1670	64·3	205	2·5%
5 mm	BS 5896: 1980	5	19·6	154	1390	1670	32·7	205	2·5%
	BS 5896: 1980				1470	1770	34·7	205	2·5%

1.2 Types of pre-stressing

1.2.1 Pre-tensioning & Post-tensioning

In pre-tensioning the tendons are tensioned before the concrete is placed. The tendons are temporarily anchored to abutments or stressing beds. Then the concrete member is cast between and over the wires. After the concrete has attained the required strength, the wires are cut from the bulkhead and pre-stress is transferred to the concrete member.

In post-tensioning the concrete member is cast with ducts for the wires. After concrete has attained sufficient strength, wires are threaded into the ducts, tensioned from both or one end by means of jack/jacks and at the precise level of pre-stress the wires are anchored by means of wedges to the anchorage plates at the ends.

1.2.2 Bonded & Un-bonded tendon

In post-tensioned members, the wires are either left free to slide in the ducts or the duct is filled with grout. In the former, the tendon is un-bonded and in the latter it is bonded.

Stages of loading

Initial stage

The member is under pre-stress but is not subjected to any superimposed external loads. Further subdivision of this stage is possible.

1. Before pre-stressing: Concrete is weak in carrying loads. Yielding of supports must be prevented.
2. During pre-stress:
   a. Steel: This stage is critical for the strength of tendons. Often the maximum stress to which the wires will be subjected throughout their life may occur at this stage.
   b. Concrete: As concrete has not aged at this stage, crushing of concrete at anchorages is possible, if its quality is inferior or the concrete is honeycombed. Order of pre-stressing is important to avoid overstress in the concrete.

3. At transfer of pre-stress: For pre-tensioned members, where transfer is within a short period, and for post-tensioned members where transfer may be gradual, there are no external loads on the member except its own weight.
4. De-shuttering: The removal of form-work must be done after due consideration

Thus the initial pre-stress with little loss imposes a serious condition n the concrete and often controls the design of the member.

Final stage

This is the stage when actual working loads come on the structure. The designer must consider various combinations of live loads on different parts of the structure with lateral loads such as wind and earthquake forces and strain loads produced by settlement of supports and temperature. The major loads in this stage are:

1. Sustained load: It is often desirable to limit the deflection under sustained loads sue to its own weight and dead loads.
2. Working load: The member must be designed for the working load. Check for excessive stress and deflection must be made. But this design may not guarantee sufficient strength to carry overloads.
3. Cracking load: Cracking in a pre-stress member signifies a sudden change in bond and shearing stresses. This stage is also important
4. Ultimate load: This strength denotes the maximum load the member can carry before collapse.

METHODS OF PRESTRESSING

*** Pretensioning**

In this method steel *tendons,* in the form of wires or strands, are tensioned between end-anchorages and the concrete members cast around the tendons. Once the concrete has hardened sufficiently, the end-anchorages are released and the prestress force is transferred to the concrete through the bond between the steel and concrete. The protruding ends of the tendons are then cut away to produce the finished concrete member. Pretensioned members usually have a large number of wires or strands to provide the prestress force, since the force in them is developed by bond to the surrounding concrete, and as large an area of surface contact as possible is desirable.

METHODS OF PRESTRESSING

* Pretensioning

This method is ideally suited to factory production since large anchorages are required to anchor all the tendons, and several members can be cast along the same set of tendons (Fig. 1.9). It is important to ensure that the members are free to move along the prestressing bed, otherwise undesirable tensile stresses may be set up in them when the end-anchorages are released.

It is interesting to note the use of *in situ* prestressing in cable-stayed and suspension bridge construction. In the former, the stays are tensioned in order to reduce the deflections of the bridge and also to optimize the deck cross-section. Pretensioning of suspension bridge cables has also been employed to ensure that the grout used to protect them from corrosion remains in compression, and therefore crack-free, under all load conditions. Some examples of pretensioned members, other than beams, which are commonly produced are shown in Fig. 1.10.

METHODS OF PRESTRESSING

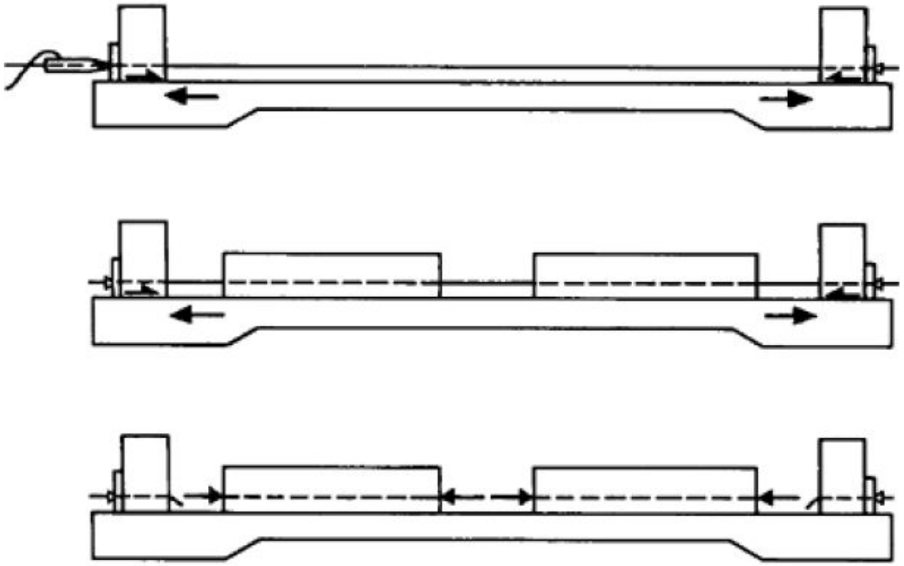


Figure 1.9 Pretensioning.

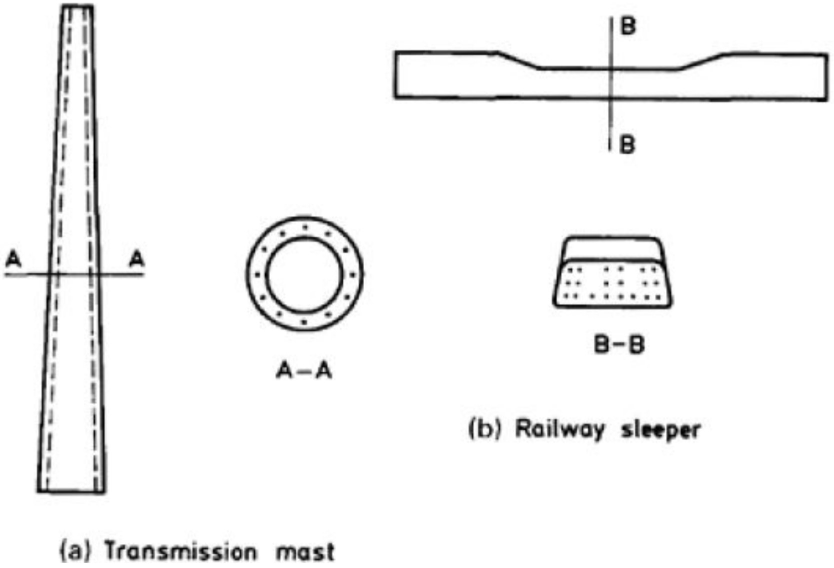
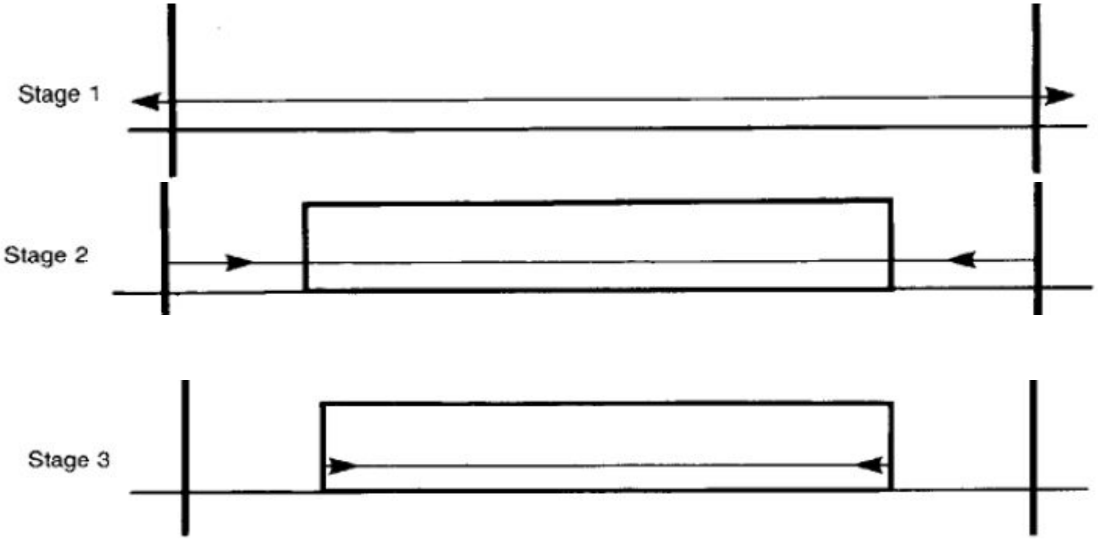


Figure 1.10 Examples of pretensioned members:

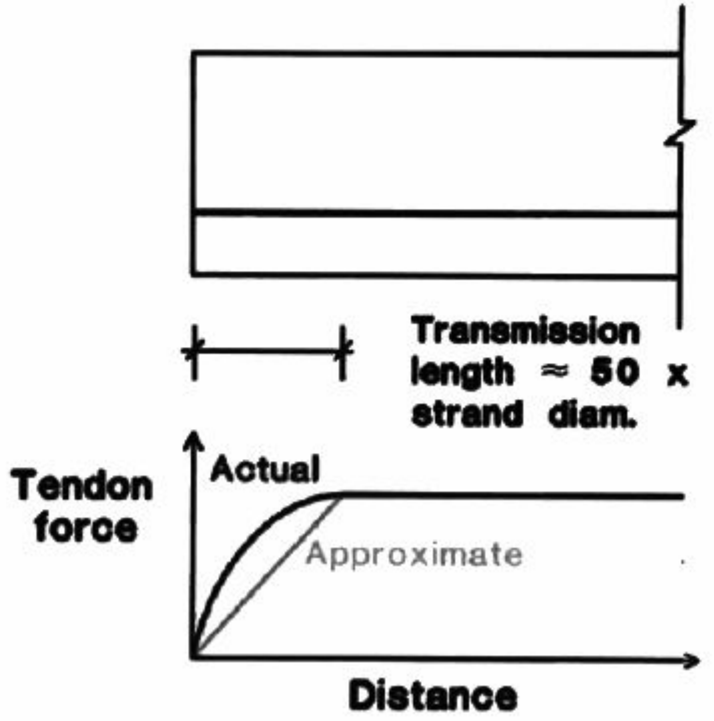
(a) transmission mast; (b) railway sleeper.

Pre-tensioning

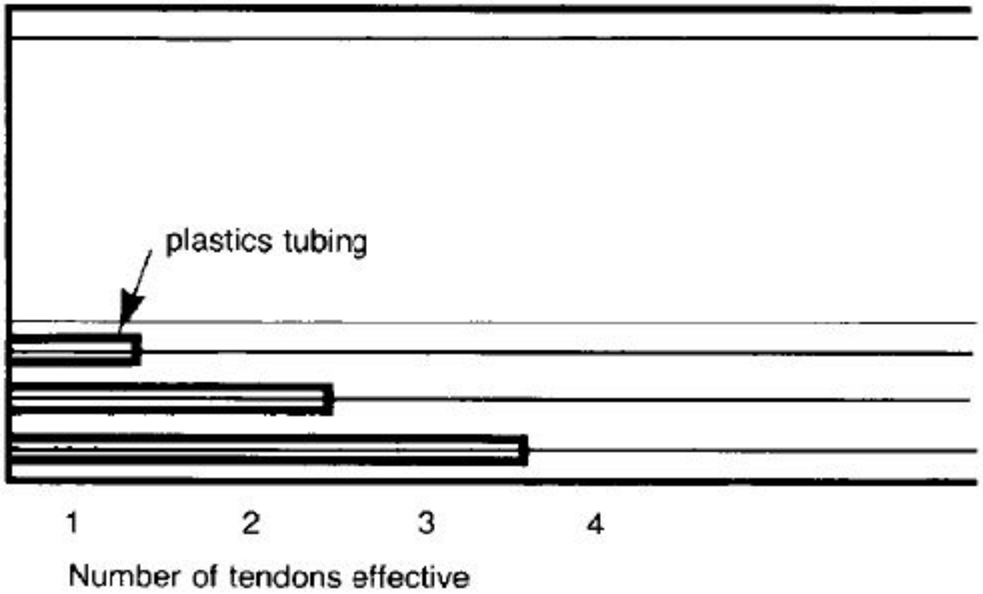
This is the most common form for precast sections. In Stage 1 the wires or strands are stressed; in Stage 2 the concrete is cast around the stressed wires/strands; and in Stage 3 the prestressed in *transferred* from the external anchorages to the concrete, once it has sufficient strength:



In pre-tensioned members, the strand is directly bonded to the concrete cast around it. Therefore, at the ends of the member, there is a transmission length where the strand force is transferred to the concrete through the bond:



At the ends of pre-tensioned members it is sometimes necessary to debond the strand from the concrete. This is to keep the stresses within allowable limits where there is little stress induced by self with or other loads:



Post-tensioning

The prestress force is applied in this case by jacking steel tendons against an already cast concrete member. Nearly all *in situ* prestressing is carried out using this method. The tendons are threaded through ducts cast into the concrete, or in some cases pass outside the concrete section. Once the tendons have been tensioned to their full force, The jacking force is transferred to the concrete through special built-in anchorages. The prestress force in post-tensioned members is usually provided by many individual wires or strands grouped into large tendons and fixed to the same anchorage. The concentrated force applied through the anchorage sets up a complex state of stress within the surrounding concrete, and reinforcement is required around the anchorage to prevent the concrete from splitting. In most post-tensioned concrete applications the space between the tendon and the duct is injected with a cement grout. This not only helps to protect the tendons, but also improves the ultimate strength capacity of the member.

Post-tensioning

One advantage of post-tensioning over pre-tensioning is that the tensioning can be carried out in stages, for all tendons in a member, or for some of them. This can be useful where the load is applied in well-defined stages. An important different between pretensioned and post-tensioned systems is that it is easy to incorporate curved tendons in the latter. The flexible ducts can be held to a curved shape while the concrete is poured around them (Fig. 1.11). The advantages of having curved tendons will become apparent later. With pretensioned members, it would be extremely difficult to arrange for a pretensioned curved tendon, although it is possible to have a sharp change of direction, as shown in Fig. 1.12. This involves providing a holding-down force at the point of deflection, and this is another reason why such members are nearly always cast in a factory, or precasting yard, where the holding-down force can be accommodated more easily.

An **anchorage zone** refers to the region of the structure in which the prestress force is transferred from the prestressing steel to the concrete and distributed more widely to the member. In post-tensioned structures the **anchorage zone** is subdivided into the local **zone** and general **zone**.

➢ In post-tensioned prestressed concrete members, the prestress forces are directly applied to the end of the members with relatively very small mechanical anchorages and large forces.

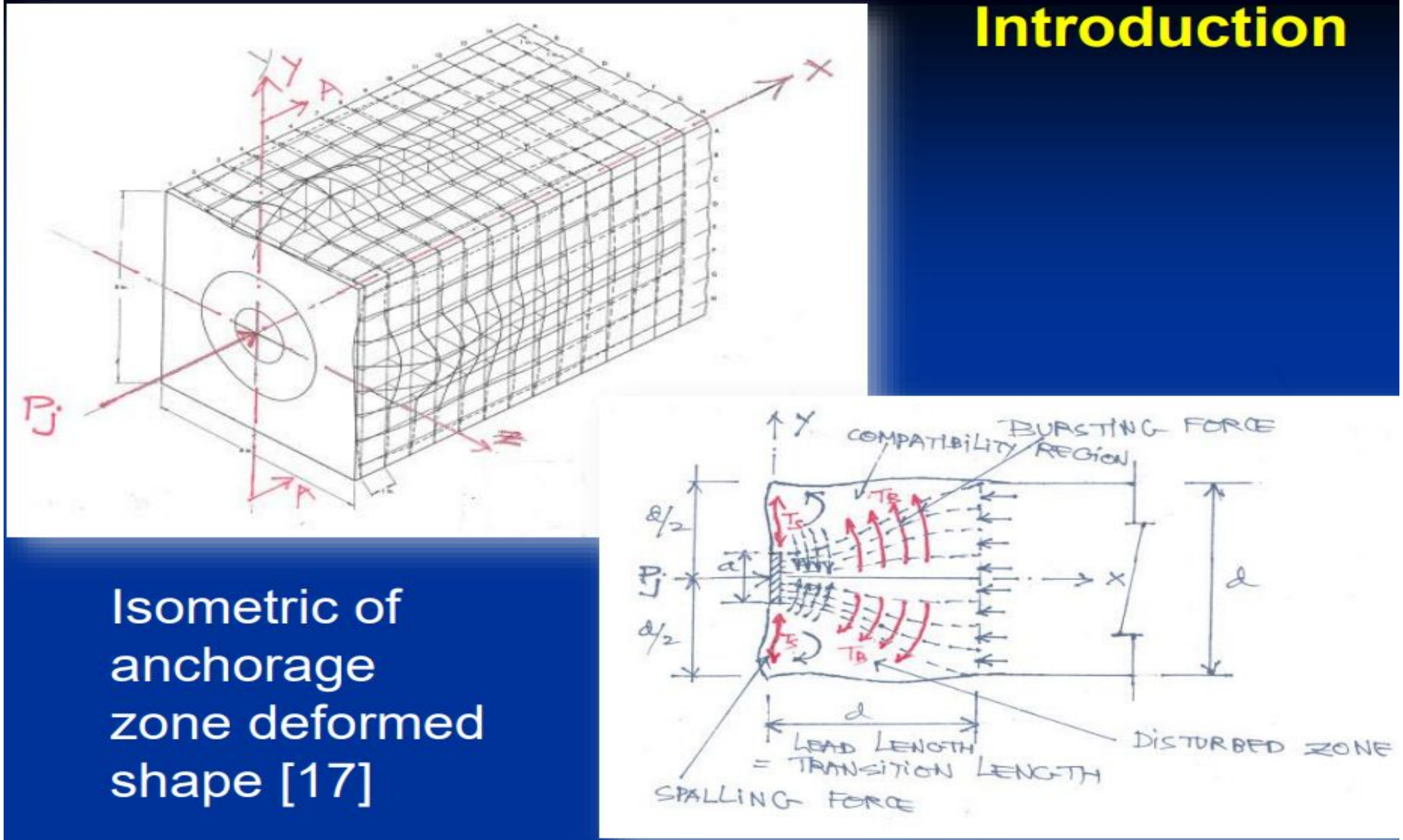
➢The PT concentrated force induces a complex 3D stress pattern near the anchorage zone.

➢For practical purposes the anchorage zone design is simplified from 3D to 2D.

➢A single tendon jacking force could vary from 100 tons to about 1000 tons.

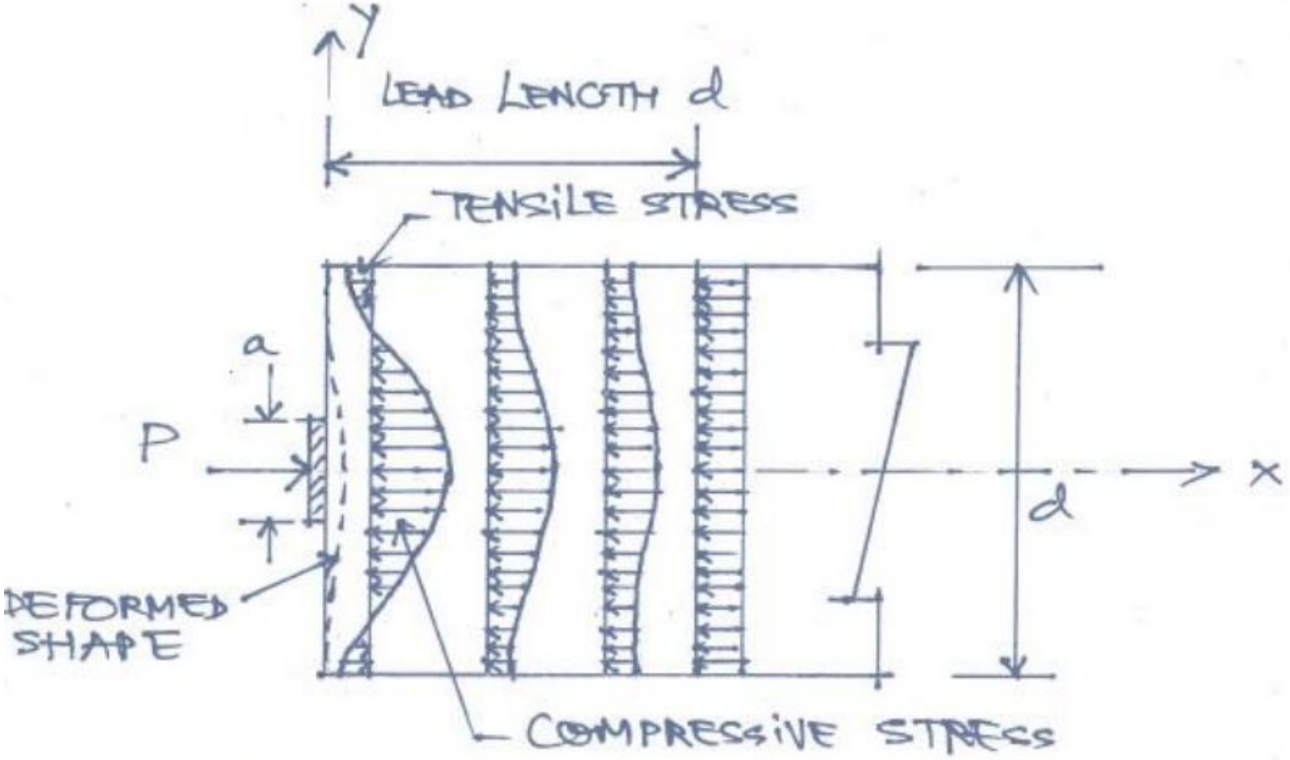
➢Single PT anchorage has been studied both theoretically and experimentally. However, in reality multiple anchorages with different configurations and cross sections exist. ➢Improperly design and detailing of anchorage zone can cause longitudinal and vertical cracks around anchorage zone.

Introduction



Isometric of anchorage zone deformed shape [17]

Section A - A

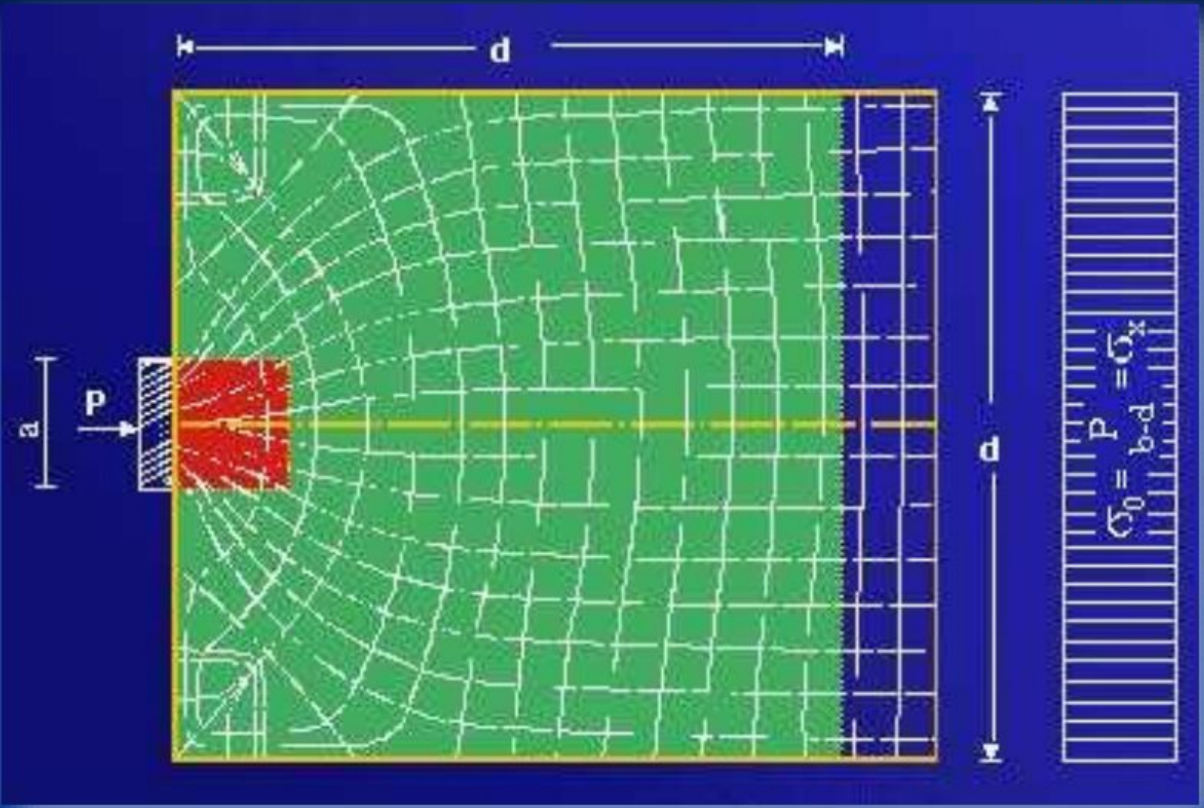


STRESS DISTRIBUTION IN THE DISTURBED ZONE

Introduction

🟥 *Local Zone*

🟩 *General Zone*

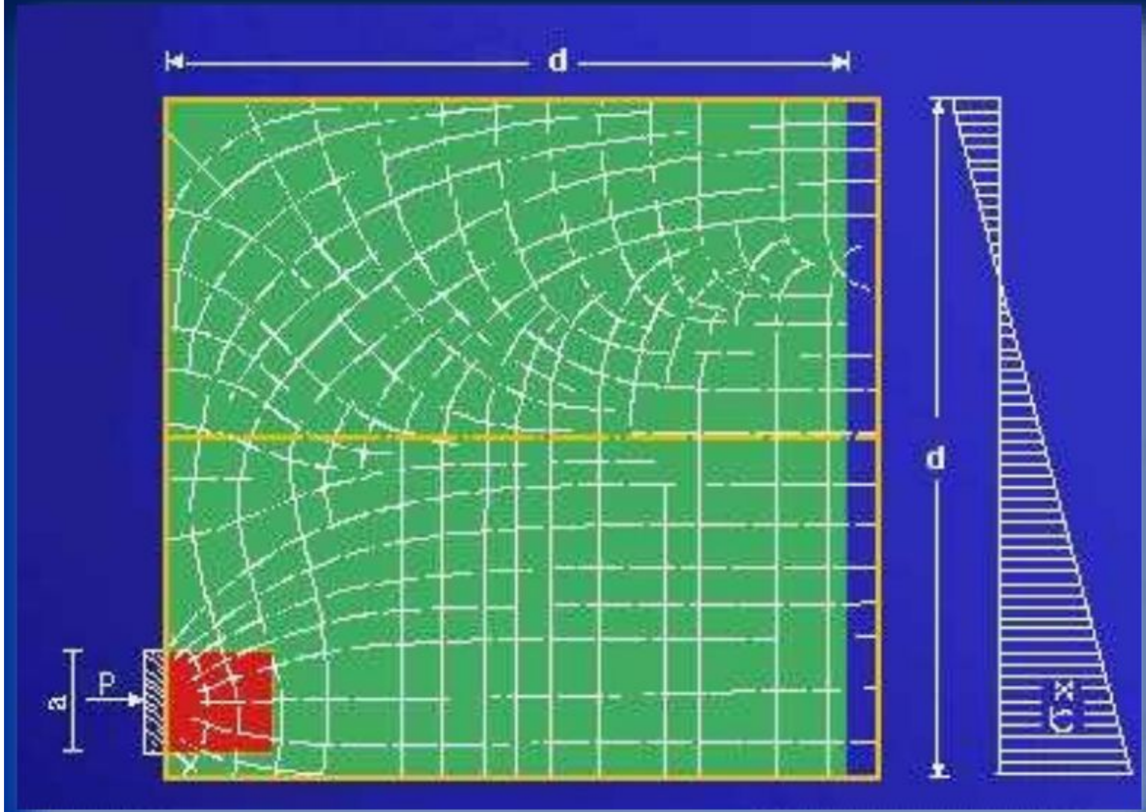


Local and General Zone Limit

Introduction

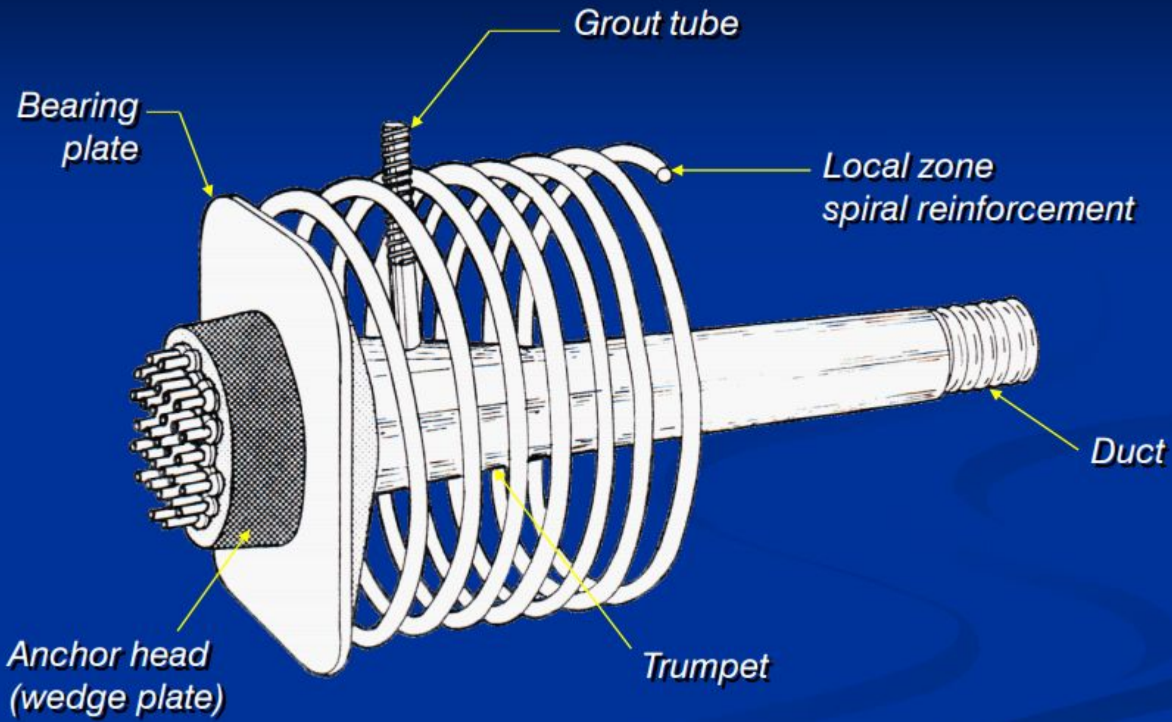
🟥 *Local Zone*

🟩 *General Zone*



Local and General Zone Limit

Introduction



Local Zone Confinement Reinforcement

Development of PT Anchorages

VSL PT anchorage for flexible filler

Development of PT Anchorages

Figure 1.11 Post-tensioning.

Figure 1.12 Deflected pretensioning tendons.

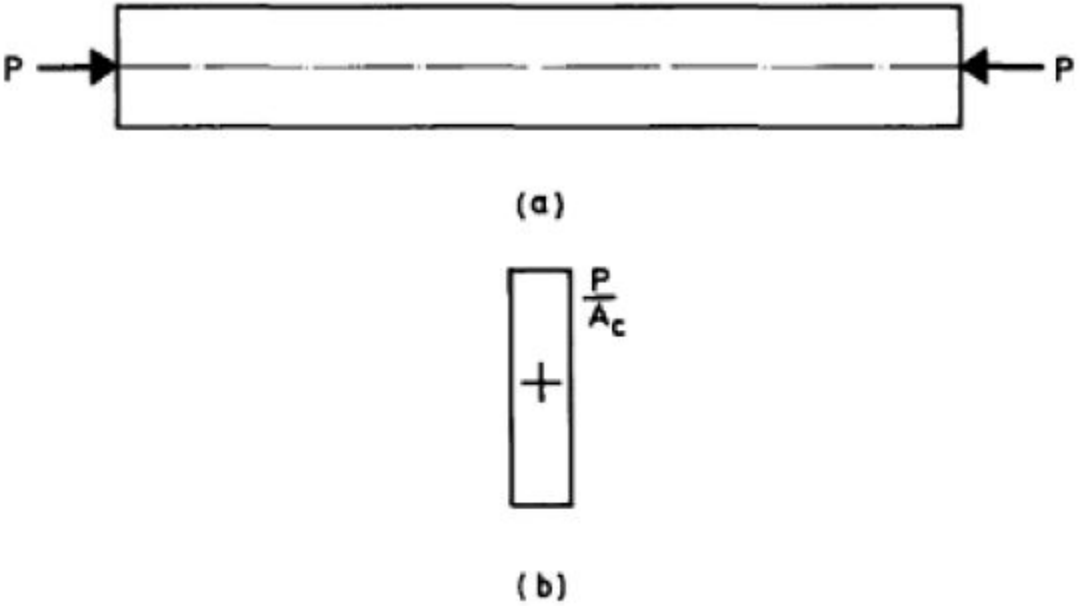
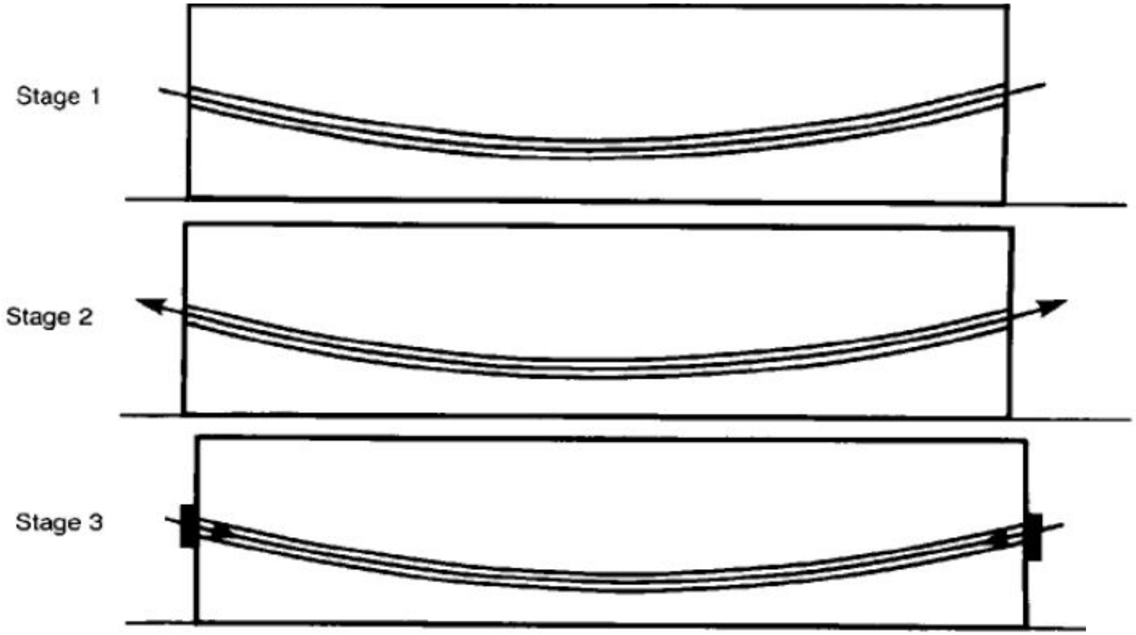


Figure 1.13 Axially loaded member.

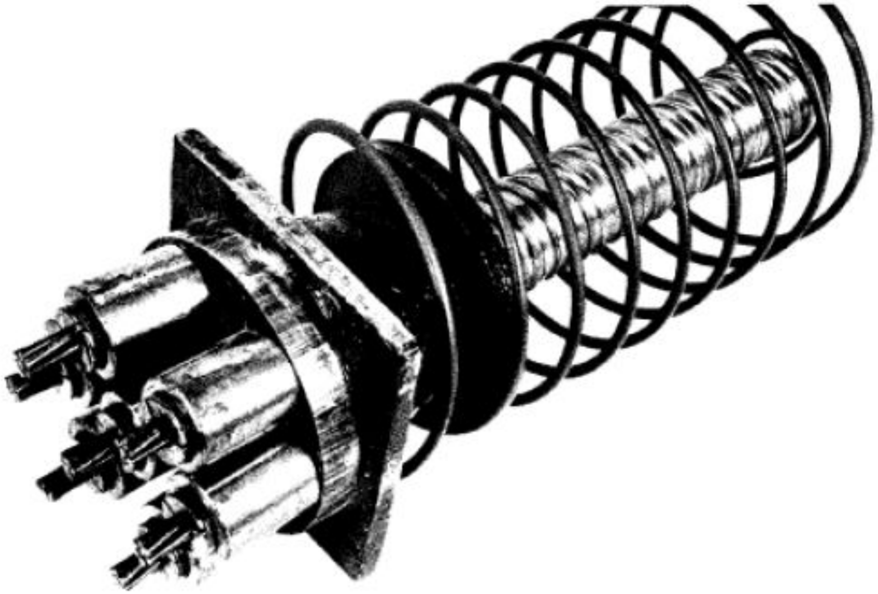
. There are other methods available for prestressing concrete (Ramaswamy, 1976) but the ones described above are by far the most common.

Post-tensioned

In this method, the concrete has already set but has ducts cast into it. The strands or tendons are fed through the ducts (Stage 1) then tensioned (Stage 2) and then anchored to the concrete (Stage 3):



The anchorages to post-tensioned members must distribute a large load to the concrete, and must resist bursting forces as a result. A lot of ordinary reinforcement is often necessary. A typical tendon anchorage is:



3.2 – Prestressing Tendons

Strand Types

Seven-Wire Strands

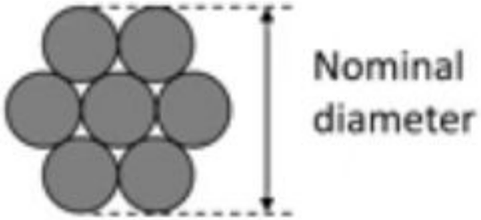
Seven-wire strands are most common type of reinforcement for prestressing

3.2 – Prestressing Tendons

Strand Types

Seven-Wire Strands

- Seven-wire strand is most widely used type of prestressing reinforcement

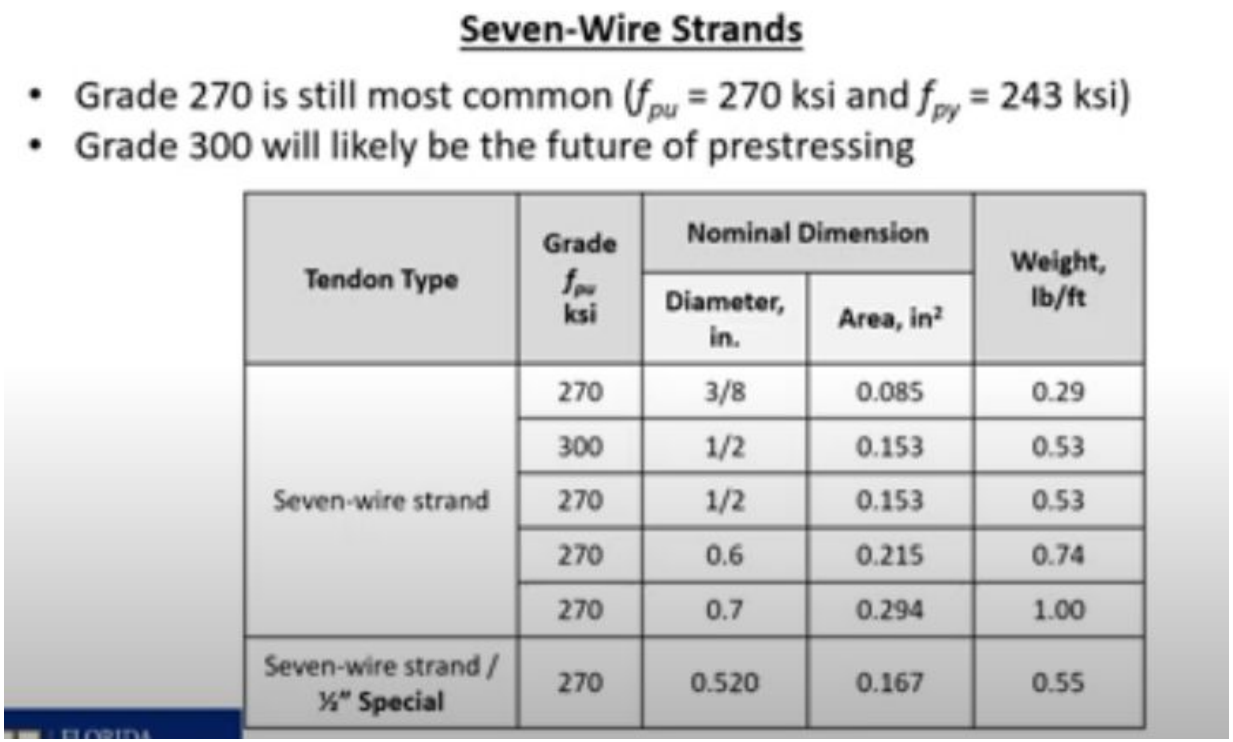


Why do we use seven-wire strands?

- Standard nominal diameters used worldwide and given in inches
- Most popular sizes are: 3/8", 1/2", 1/2" special and 0.6"
- Research is currently being conducted on 0.7" strands, so these strands may be the future

3.2 – Prestressing Tendons

Strand Types



Seven-Wire Strands

- Grade 270 is still most common (f_{pu} = 270 ksi and f_{py} = 243 ksi)
- Grade 300 will likely be the future of prestressing

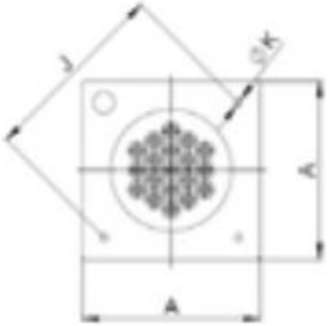
Tendon Type	Grade f_{pu} ksi	Nominal Dimension		Weight, lb/ft
		Diameter, in.	Area, in^2	
Seven-wire strand	270	3/8	0.085	0.29
	300	1/2	0.153	0.53
	270	1/2	0.153	0.53
	270	0.6	0.215	0.74
	270	0.7	0.294	1.00
Seven-wire strand / ½" Special	270	0.520	0.167	0.55

3.2 – Prestressing Tendons

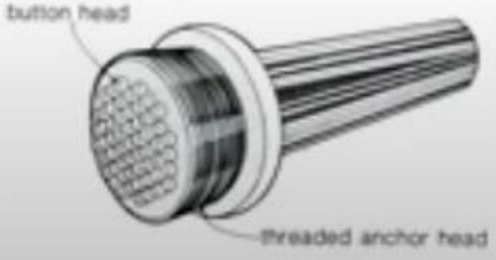
Strand Types

Other Types

- Multi-strand tendons



- Multi-wire tendons

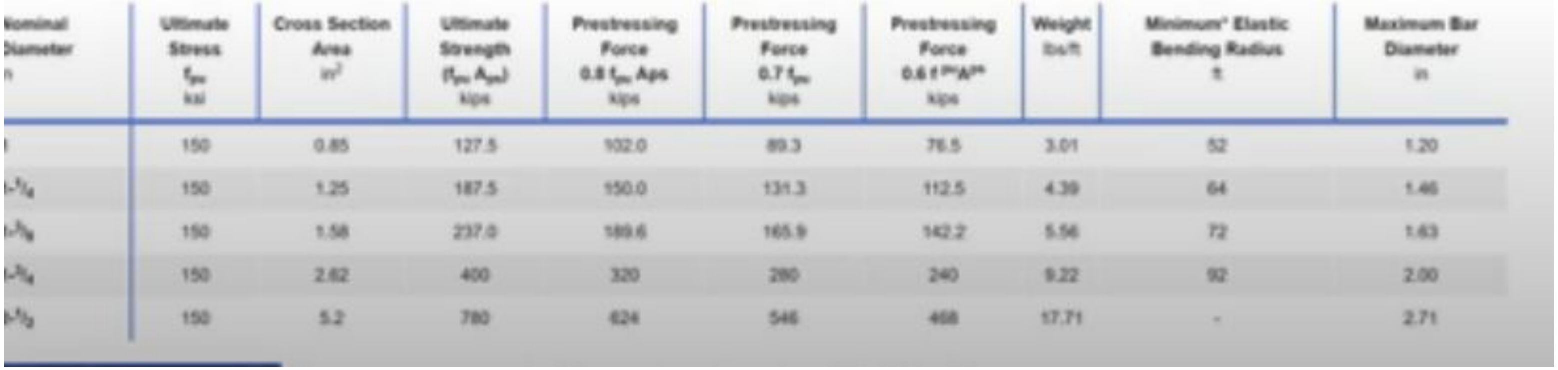


- Individual wires

3.2 – Prestressing Tendons

Strand Types

Deformed Prestressing Bars

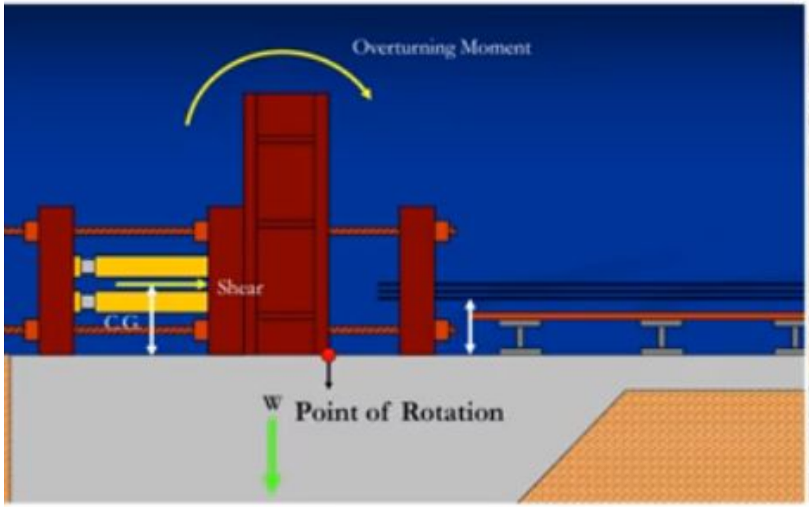


ominal Diameter n	Ultimate Stress f_{pu} ksi	Cross Section Area in^2	Ultimate Strength (f_{pu} A_{ps}) kips	Prestressing Force 0.8 f_{pu} Aps kips	Prestressing Force 0.7 f_{pu} kips	Prestressing Force $0.6 f^{pu} A^{ps}$ kips	Weight lbs/ft	Minimum* Elastic Bending Radius ft	Maximum Bar Diameter in
1	150	0.85	127.5	102.0	89.3	76.5	3.01	52	1.20
-1/4	150	1.25	187.5	150.0	131.3	112.5	4.39	64	1.46
-3/8	150	1.58	237.0	189.6	165.9	142.2	5.56	72	1.63
-3/4	150	2.62	400	320	280	240	9.22	92	2.00
-1/2	150	5.2	780	624	546	468	17.71	-	2.71

3.3 – Pretensioning Operations

Components of Prestressing Bed

End Blocks



End block shown can hold over 4 million pounds of prestressing force

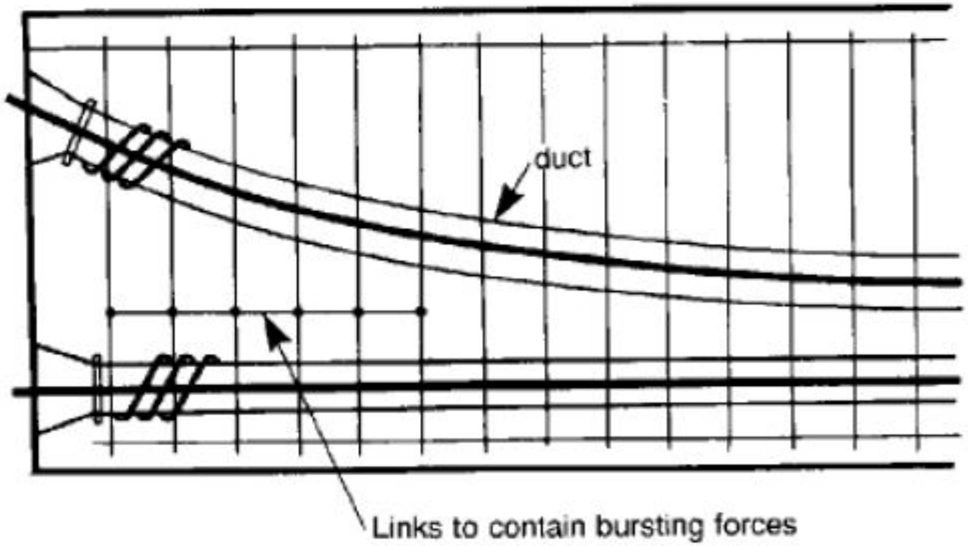
3.3 – Pretensioning Operations

Single-Strand versus Multi-Strand Stressing

Single Strand **Multi Strand**

Single-strand stressing is the most common
Multi-strand stressing also called “gang stressing”

And the end of a post-tensioned member has reinforcement such as:



Losses

From the time the prestress is applied, the prestress force gradually reduces over time to an equilibrium level. The sources of these losses depend on the method by which prestressing is applied.

In both methods:

- The member shortens due to the force and this relieves some of the prestress;
- The concrete shrinks as it further cures;
- The steel 'relaxes', that is, the steel stress reduces over time;
- The concrete creeps, that is, continues to strain over time.

In post-tensioning, there are also losses due to the anchorage (which can 'draw in' an amount) and to the friction between the tendons and the duct and also initial imperfections in the duct setting out.

For now, losses will just be considered as a percentage of the initial prestress.

1.6 Uses of Prestressed Concrete

There are a huge number of uses:

- Railway Sleepers;
- Communications poles;
- Pre-tensioned precast "hollowcore" slabs;
- Pre-tensioned Precast Double T units - for very long spans (e.g., 16 m span for car parks);
- Pre-tensioned precast inverted T beam for short-span bridges;
- Pre-tensioned precast PSC piles;
- Pre-tensioned precast portal frame units;
- Post-tensioned ribbed slab;
- In-situ balanced cantilever construction - post-tensioned PSC;
- This is "glued segmental" construction;
- Precast segments are joined by post-tensioning;
- PSC tank - precast segments post-tensioned together on site. Tendons around circumference of tank;
- Barges;
- And many more.

2. Stresses in Prestressed Members

2.1 Background

The codes of practice limit the allowable stresses in prestressed concrete. Most of the work of PSC design involves ensuring that the stresses in the concrete are within the permissible limits.

Since we deal with allowable stresses, only service loading is used, i.e. the SLS case. For the SLS case, at any section in a member, there are two checks required:

At Transfer

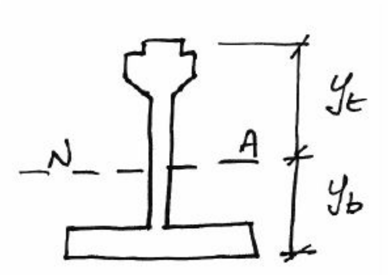
This is when the concrete first feels the prestress. The concrete is less strong but the situation is temporary and the stresses are only due to prestress and self weight.

At Service

The stresses induced by the SLS loading, in addition to the prestress and self weight, must be checked. At service stage, the concrete has its full strength but losses will have occurred and so the prestress force is reduced.

The ultimate capacity at ULS of the PSC section (as for RC) must also be checked. If there is insufficient capacity, you can add non-prestressed reinforcement. This often does not govern.

For a typical prestressed section:



We have:

Z_t	Section modulus, top fibre $= I/y_t$;
Z_b	Section modulus, bottom fibre $= -I/y_b$ (taken to be negative);
f_{tt}	Allowable tensile stress at transfer;
f_{tc}	Allowable compressive stress at transfer;
f_{st}	Allowable tensile stress in service;
f_{sc}	Allowable compressive stress ain service;
M_t	The applied moment at transfer;
M_s	The applied moment in service;
α	The ratio of prestress after losses (service) to prestress before losses, (transfer).

FORCES EXERTED BY TENDONS

From Fig. 1.24 it can be seen that, by deflecting a tendon from the straight position, a downwards force is required to maintain the tendon in the deflected position, and this force is transmitted to the concrete as an upwards force. In the case of a continuously curved tendon, there must be a distributed force applied to the concrete to maintain the tendon in position (Fig. 1.28). In order to determine the value of this force, consider a small, but finite, section of the tendon (Fig. 1.29(a)). If the frictional forces between the tendon and the surrounding concrete are ignored, the force in the tendon at either end of the element Δs is equal to T. If w is the uniformly distributed load on the tendon required to maintain it in position, then, from the triangle of forces Fig. 1.29(b):

$$w\Delta s=2T\sin(\Delta\theta/2).$$

For small changes of angle, $\sin(\Delta\theta/2)=\Delta\theta/2$. If the element is made smaller and smaller, in the limit the force at a point on the tendon is given by

$$w=T\mathrm{d}\theta/\mathrm{d}s.$$

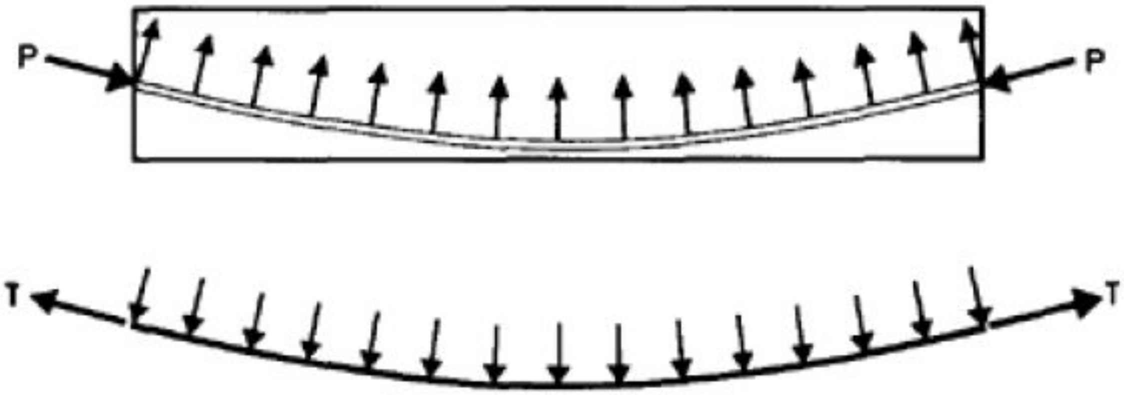


Figure 1.28 Free bodies of concrete and curved tendon.

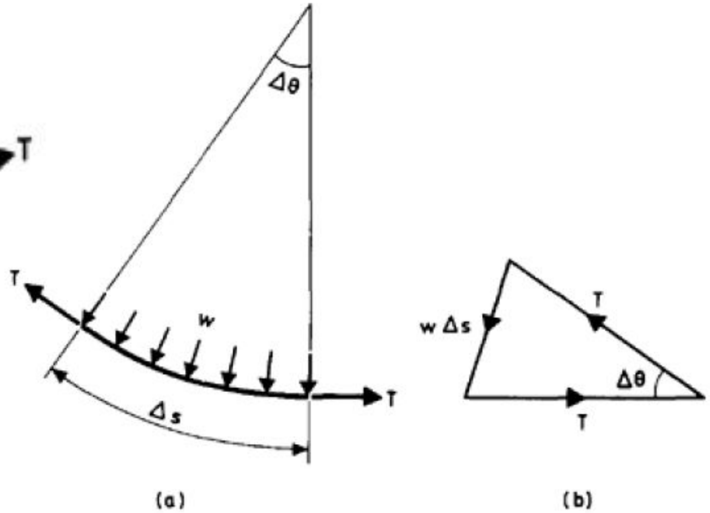


Figure 1.29 Small length of tendon.

Now, $d\theta/ds=1/r_{ps}$, where r_{ps} is the radius of curvature, so that

$$w=T/r_{ps}.$$

Although this force is theoretically directed towards the centre of curvature at any given point, in practice most tendon profiles are reasonably flat and it can be assumed that the force at any point is vertical.

The vertical force produced by a sharp change of profile, such as that found in pretensioned beams, is shown in Fig. 1.30. In this case

$$W=T(\sin\theta+\sin\varphi).$$

Example 1.2 ■■

A simply supported beam of length L has a parabolic tendon profile with maximum eccentricity e, as shown in Fig. 1.31. Determine the upwards force on the beam exerted by the tendon and draw the shear force and bending moment diagrams due to the prestress force.

If the parabolic curve is given a set of x and y coordinates with the origin at the left-hand end, the equation of the tendon profile is

$$y=4ex\ (L-x)/L^2.$$

For a reasonably flat curve, $1/r_{ps}$ may be approximated by d^2y/dx^2.

Therefore

$$1/r_{ps}=-8e/L^2$$
$$\therefore\ w=P/r_{ps}=-8Pe/L^2,$$

(1.1)

(1.1)

where *w* is an upwards force.

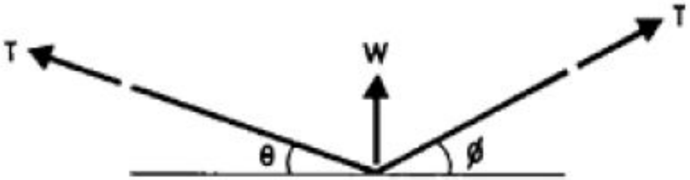


Figure 1.30 Sharp change of tendon profile.

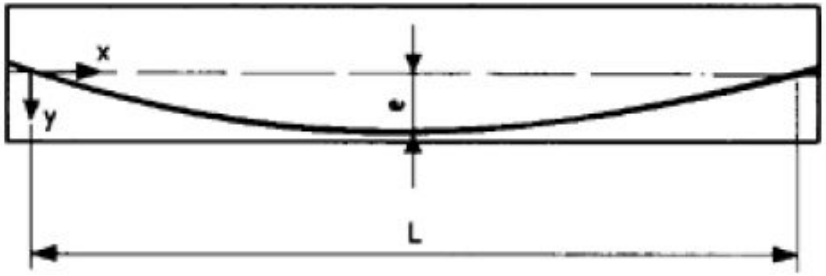


Figure 1.31

The maximum bending moment in the beam is given by

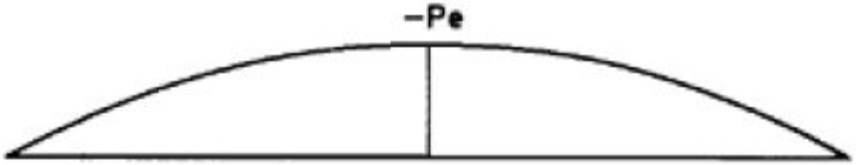
$$M_{\max}=\mathrm{w}L^2/8=(-8Pe/L^2)(L^2/8)$$
$$=-Pe.$$

■■

The prestress moment and shear force diagrams are shown in Fig. 1.32(a) and (b) respectively. Note that the prestress moment diagram is a scaled version of the shape of the tendon profile. The moments are negative because the prestress force is below the centroid at midspan, causing a hogging moment in the beam.

The observation that the prestress moment diagram is of the same shape as the tendon profile is true of all statically determinate members. It is particularly useful in drawing the prestress moment diagram, and for determining the deflections (see Chapter 6) for the member shown in Fig. 1.33(a), which has a varying section but a straight tendon. There can be no vertical loads in this case, since the tendon is straight, but the prestress moment diagram can be drawn simply by considering the distance between the tendon location and the centroid of the member at any section, Fig. 1.33(b).

The fact that a deflected tendon must exert a force on the surrounding concrete is the basis of the load balancing method which has useful application in the design of indeterminate structures, and in particular for designing prestressed concrete flat slabs (Chapter 12). However, it is not applicable for members with straight tendons, and account must be taken of any moments due to eccentricity at the ends of the member.



(a)

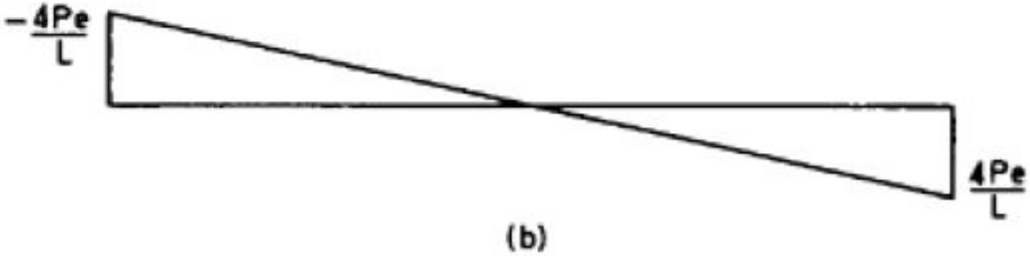
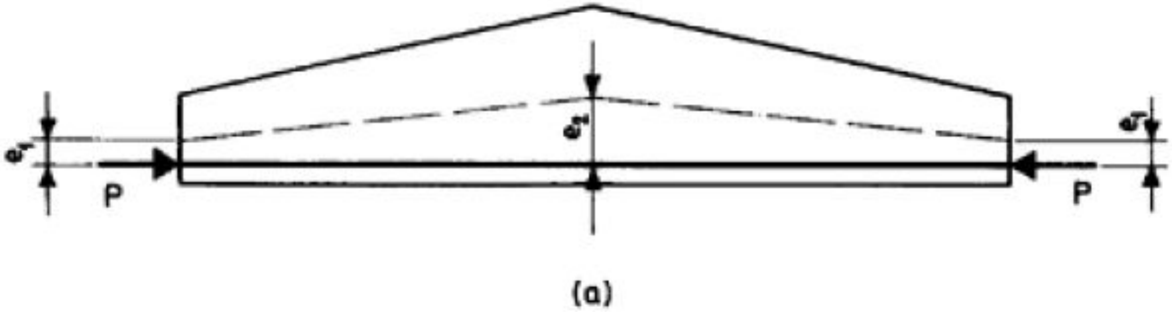


Figure 1.32 (a) Prestress moment and (b) shear force diagrams.



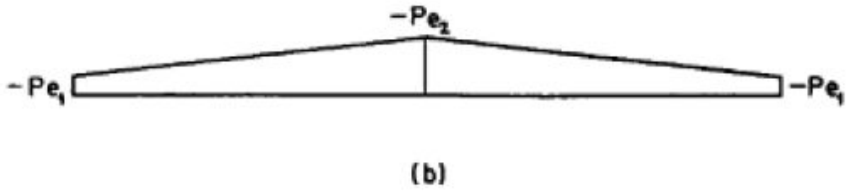


Figure 1.33 Member with varying section.

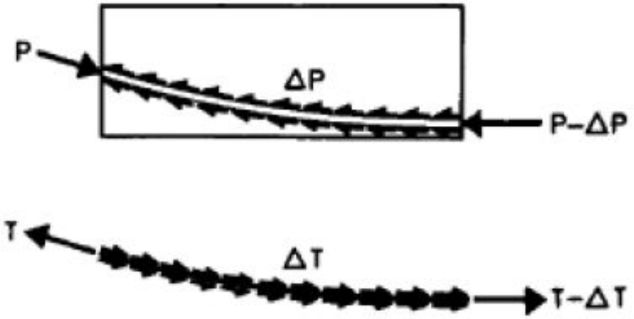


Figure 1.34 Loss of prestress due to friction.

1.8 LOSS OF PRESTRESS FORCE

In all the prestressed concrete members considered so far, it has been assumed that the force in the tendon is constant. However, during tensioning of a post-tensioned member, there is friction between the tendon and the sides of the duct. This is caused by changes in curvature of the tendon profile along its length. However, even in a straight tendon there is friction present since the tendon does not lie exactly along the centreline of the duct, and there is contact at points along its length.

The effect of friction on the behaviour of post-tensioned members is that, at any section away from the tensioning end, the force in the tendon is less than that applied to the tendon through the jack. This is shown by considering once again the free bodies of the steel tendon and concrete in a portion of a member (Fig. 1.34).

Friction is only one of the causes of loss of prestress force, and applies to post-tensioned members only. Another cause of loss which applies to both pretensioned and post-tensioned members is initial elastic shortening of the concrete which also shortens the steel tendon, reducing the prestress force. Long-term changes in length of the concrete member due to creep and shrinkage also cause reduction of prestress force. All these effects will be considered in more detail in Chapter 4.

1.10 SAFETY

Since high stresses exist in prestressed concrete members at both maximum and minimum load conditions, there must be careful control of the quality of the materials used. In reinforced concrete or steelwork structures these high stresses occur only under maximum load conditions, and are rarely reached. In prestressed concrete structures they are present at all stages of loading. In one sense it can be said that a prestressed concrete structure has been pre-tested, in that the presence of low-standard concrete or steel will generally be detected before the structure enters service.

Since a small change in tendon eccentricity can have a large effect on the stresses induced in a prestressed concrete member, care must be taken during construction that the correct profile for the prestressing steel is maintained during casting of the concrete.

Another important feature of the construction of prestressed concrete structures is the very large jacking forces that are required. Adequate provision must be made to protect site personnel against sudden failure of a steel tendon during tensioning, a not-uncommon occurrence, large amount of strain energy suddenly released is potentially lethal.

An aspect of prestressed concrete structures that is beginning to concern engineers is how to demolish them. As the early structures reach the end of their useful lives, the problem arises of how to break up prestressed concrete members which have such large amounts of energy stored in them. In some cases it is possible to reduce the force in the tendons to allow safe demolition. The problem, however, will assume greater significance as more prestressed concrete structures require demolition.

1.3 Losses

Elastic Shortening (ES) – Cl 18.5.2.4

Shortening in steel that occurs as soon as F_i is transferred to the concrete member and the member as a whole shortens.

F_i = Pre-stress just before transfer

F = Final stress after losses

F_o = Immediately after transfer – very difficult to estimate

Note: The value of F_o may not be known, but it is not necessary, as the losses from F_i to F_o is only a small percentage of F_i. Total accuracy is relative anyway, as Ec – the young's modulus of concrete – cannot be determined accurately.

ES = $E_s\delta$

where δ is the shortening in steel that occurs as soon as F_i is transferred to the concrete member and the member as a whole shortens. Thus δ is the shortening in the member due to F_i at the level of steel.

$$\delta = \frac{f_c}{E_C}$$

$$= \frac{F_O}{A_C E_C}$$

Since f_c is the stress in concrete at level of steel and is $\frac{F_O}{A_C}$

$$ES = \frac{F_O}{A_C E_C} E_S$$

Taking $\frac{E_S}{E_C} = n$

$$ES = \frac{F_O}{A_C E_C} E_S$$

$$= n\frac{F_O}{A_C}$$

As F_o cannot be estimated, F_i can be used to calculate ES.

$$\delta \text{ at level of steel} = \frac{F_i}{A_C E_C + A_S E_S}$$

$$ES = E_s \delta$$

$$= E_s \frac{F_i}{A_C E_C + A_S E_S}$$

$$= \frac{nF_i}{A_C + nA_S}$$

Taking $A_t = A_C E_C + A_S E_S$

$$ES = \frac{nF_i}{A_T}$$

$\therefore$ whichever way the ES is calculated

ES = n (concrete stress at level of steel)

If external loads are acting on the member, then concrete, then concrete stress due to all loads at level of steel must be determined.

$$f_c = -\frac{F_O}{A_G} - \frac{F_O e^2}{I} + \frac{M_G e}{I}$$

Note: A_G, the gross-area, instead of the transformed sectional area, leads to simpler calculations and fairly accurate results.

$F_o \approx 0.9 F_i$ for pre-tensioned member

$$f_c = -\frac{F_O}{A_G}$$

$$ES = nf_c$$

Creep (CR) Cl 18.5.2.1

Among the many factors affecting creep are volume to surface ratio, age of concrete at time of pre-stress, relative humidity, type of concrete (lightweight / normal). Creep is assumed to occur in the member after permanent loads are imposed after pre-stress. Creep occurs over a long period of time under sustained load. Part of initial compressive strain induced in concrete immediately after transfer is reduced by the tensile strain produced by superimposed permanent loads.

Therefore for bonded members, loss due to creep

$$CR = \theta n\left(f_{cir} - f_{cds}\right)f_c$$

θ = Creep coefficient − Cl 4.5.3 & Cl 5.2.5.1

$$n = \frac{E_S}{E_C}$$

f_{cir} = concrete stress at level of steel immediately after transfer.

f_{cds} = stress in concrete at steel level due to superimposed dead loads applied to the member after transfer of pre-stress

Shrinkage of concrete (SH) Cl 18.5.2.2

Factors like volume to surface ratio, relative humidity, time from end of moist curing to application of pre-stress, affect shrinkage in concrete. Shrinkage is time-dependant and about 80% of the final loss due to shrinkage occurs in the first year and 100% after several years.

Shrinkage strain

$$\varepsilon_{sh} = 0.0003 \text{ for pretensioned member}$$

$$= \frac{0.0002}{\log_{10}(t+2)} \text{ for posttensioned member and}$$

Cl 5.2.4.1

$$\text{may be increased by } 50\% \text{ in dry condition}$$

$$\text{but not more than } 0.0003$$

Relaxation of steel (RE) Cl 18.5.2.3

When elongation is sustained over pre-stressing cable for a long time, the pre-stress will decrease gradually. The RE – loss due to relaxation depends on type of steel, time, as well as the ratio of $\frac{f_i}{f_p}$ where f_i is the initial pre-stress and f_p is the characteristic strength of steel.

RELAXATION LOSSES FOR PRESTRESSING STEEL AT 1 000 H AT 27°C INITIAL STRESS RELAXATION	
INITIAL STRESS	RELAXATION LOSS N/mm2
0.5 *fp*	0
0.6 *fp*	35
0.7 *fp*	70
0.8 *fp*	90

Anchorage slip (ANC) Cl 18.5.2.5

In post-tensioning, when the jack is released, the full pre-stress is transferred to the anchorage and they tend to deform, allowing the tendon to slacken. Friction wedges will slip a little before they grip the wire firmly. So, in post-tensioning the wedges are positively engaged before the jack is released. In pre-tensioning also, the anchorage slip is compensated for during stressing operation.

The loss is caused by a fixed shortening of the anchorages, so the percentage loss is higher in shorter wires than in long ones.

If a tendon is stressed to 1035 MPa, with $E_s = 2\times10^5$ MPa and the anchorage slips by 2.5 mm,

$$\varepsilon_{Total} = \frac{1035}{2\times10^5} = 0.005175$$

In a cable of 3m length, elongation $\delta_l = 0.005175\times 3000 = 15.53 \text{ mm}$, ie $\%\delta_l = \dfrac{2.5}{15.53}\times100 = 16\%$

But in a cable of 30 m length, elongation $\delta_l = 0.005175\times 30000 = 155.30 \text{ mm}$, ie

$\%\delta_l = \dfrac{2.5}{155.30}\times100 = 1.6\%$ only

Frictional loss Cl 18.5.2.6

Frictional loss comprise of two parts: (1) The length effect and (2) The curvature effect.

The length effect or the wobble effect of the duct is the friction that will exist between straight tendon and the surrounding material. This loss is dependant on the length and stress in the tendon, the coefficient of friction between the contact materials, the workmanship and the method used in aligning and obtaining the duct.

The curvature effect is the loss due to intended curvature of the tendon. This again depends on the coefficient of friction between the materials and the pressure exerted by the tendon on the curvature.

For un-bonded tendon, lubrication, in the form of grease and plastic tube wrapping can be used to advantage.

For bonded tendon lubricant in the form of water soluble oils are used during stressing operation and flushed off with after before grouting.

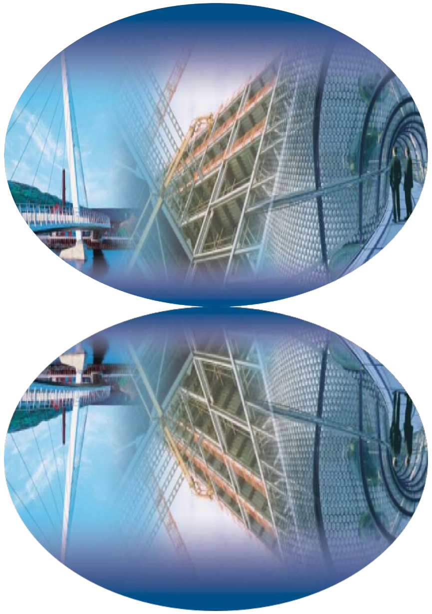
Jacking from both ends of the beam will also reduce loss due to friction.

For straight or moderately curved structures, with curved or straight cables, the value of pre-stressing force P_x at a distance x meters from tensioning end and acting in the direction of the tangent to the curve of the cable, shall be calculated as below:

$$P_x = P_o e^{-(\mu\alpha + kx)}.$$

Where P_o = pre-stressing force in the pre-stressed steel at the tensioning end acting in the direction of the tangent to the curve of the cable, α = cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration, μ = coefficient of friction in curve; unless otherwise proved by tests, μ may be taken as: 0.55 for steel moving on smooth concrete, 0.30 for steel moving on steel fixed to duct, and 0.25 for steel moving on lead, k = coefficient for wobble or wave effect varying from 15 × 10−4 to 50 × 10−4 per meter. The expansion of the equation for P_x for small values of $(\mu\alpha + kx)$ may be $P_x = P_o (1 - \mu\alpha - kx)$.

Closing Thank you for your attention! Any questions?

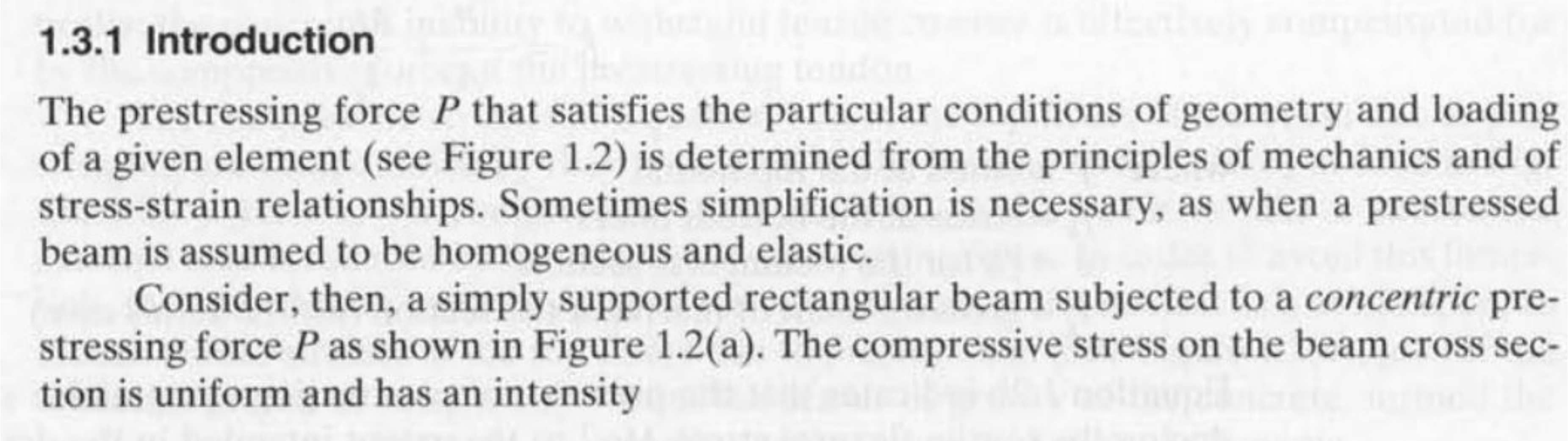


UNIVERSITY OF ANBAR

Lecturer: 2

Mohammed Mohammed

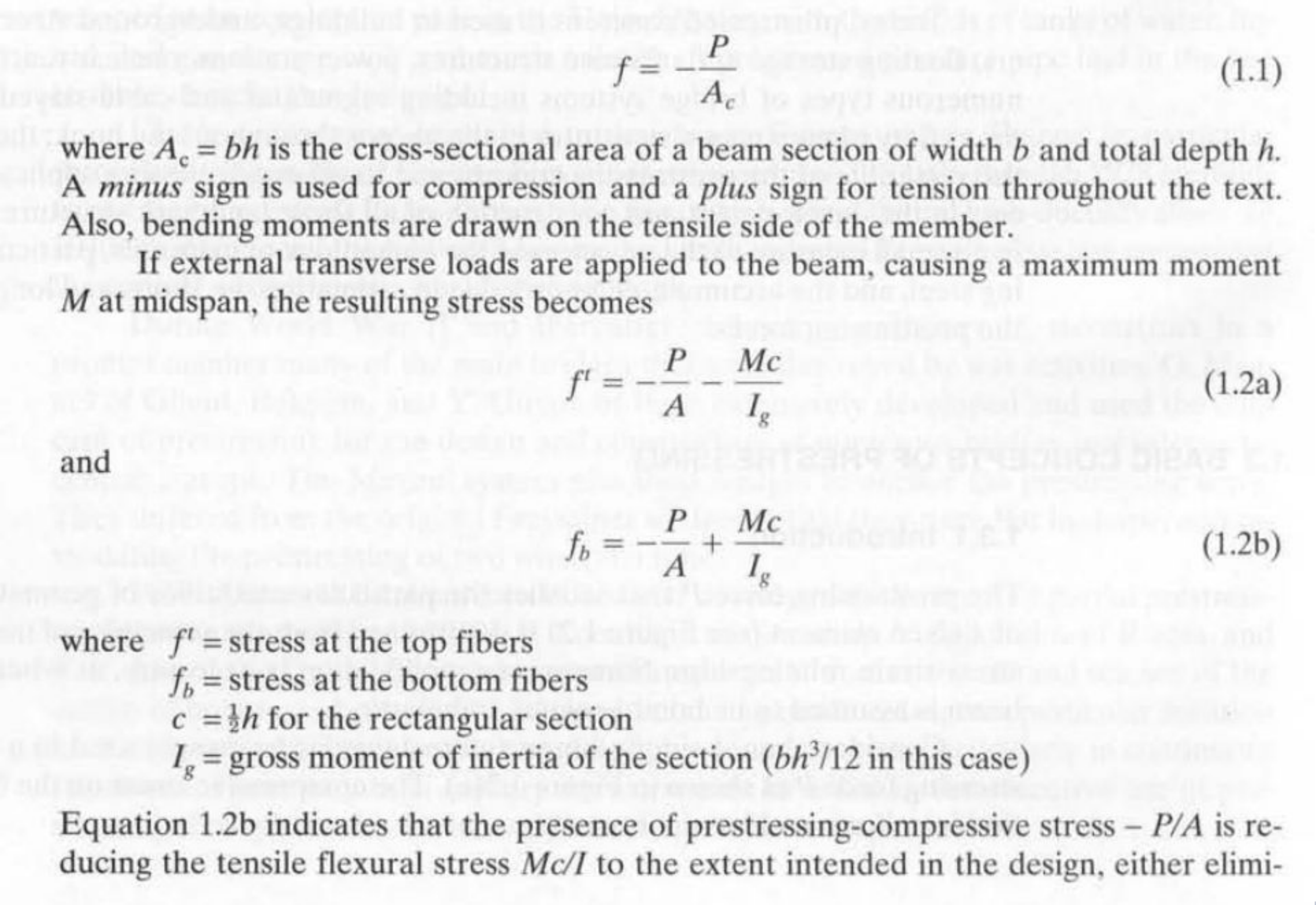
Flexural Analysis



1.3.1 Introduction

The prestressing force P that satisfies the particular conditions of geometry and loading of a given element (see Figure 1.2) is determined from the principles of mechanics and of stress-strain relationships. Sometimes simplification is necessary, as when a prestressed beam is assumed to be homogeneous and elastic.

Consider, then, a simply supported rectangular beam subjected to a *concentric* prestressing force P as shown in Figure 1.2(a). The compressive stress on the beam cross section is uniform and has an intensity



$$f = -\frac{P}{A_c} \tag{1.1}$$

where $A_c = bh$ is the cross-sectional area of a beam section of width b and total depth h. A *minus* sign is used for compression and a *plus* sign for tension throughout the text. Also, bending moments are drawn on the tensile side of the member.

If external transverse loads are applied to the beam, causing a maximum moment M at midspan, the resulting stress becomes

$$f^t = -\frac{P}{A} - \frac{Mc}{I_g} \tag{1.2a}$$

and

$$f_b = -\frac{P}{A} + \frac{Mc}{I_g} \tag{1.2b}$$

where f^t = stress at the top fibers
f_b = stress at the bottom fibers
$c = \frac{1}{2}h$ for the rectangular section
I_g = gross moment of inertia of the section ($bh^3/12$ in this case)

Equation 1.2b indicates that the presence of prestressing-compressive stress $-P/A$ is reducing the tensile flexural stress Mc/I to the extent intended in the design, either elimi-

Basic Concepts of Prestressing

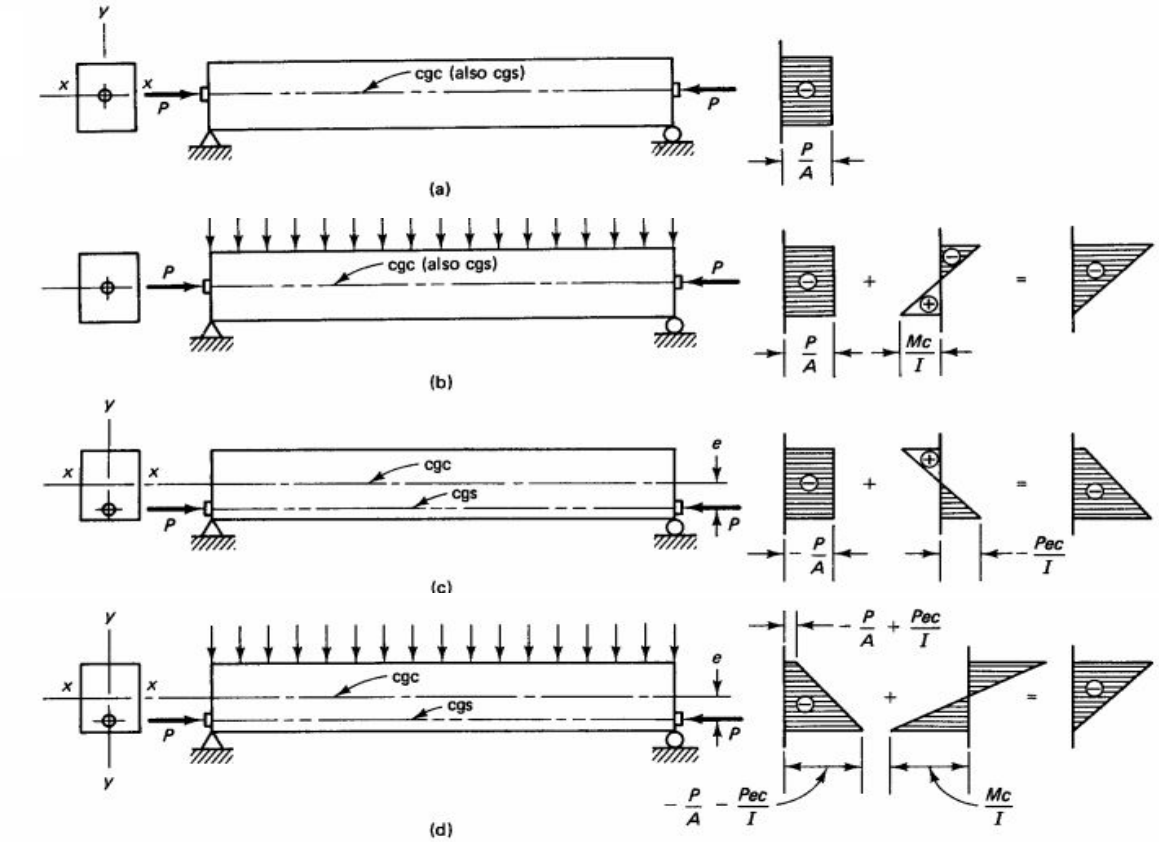
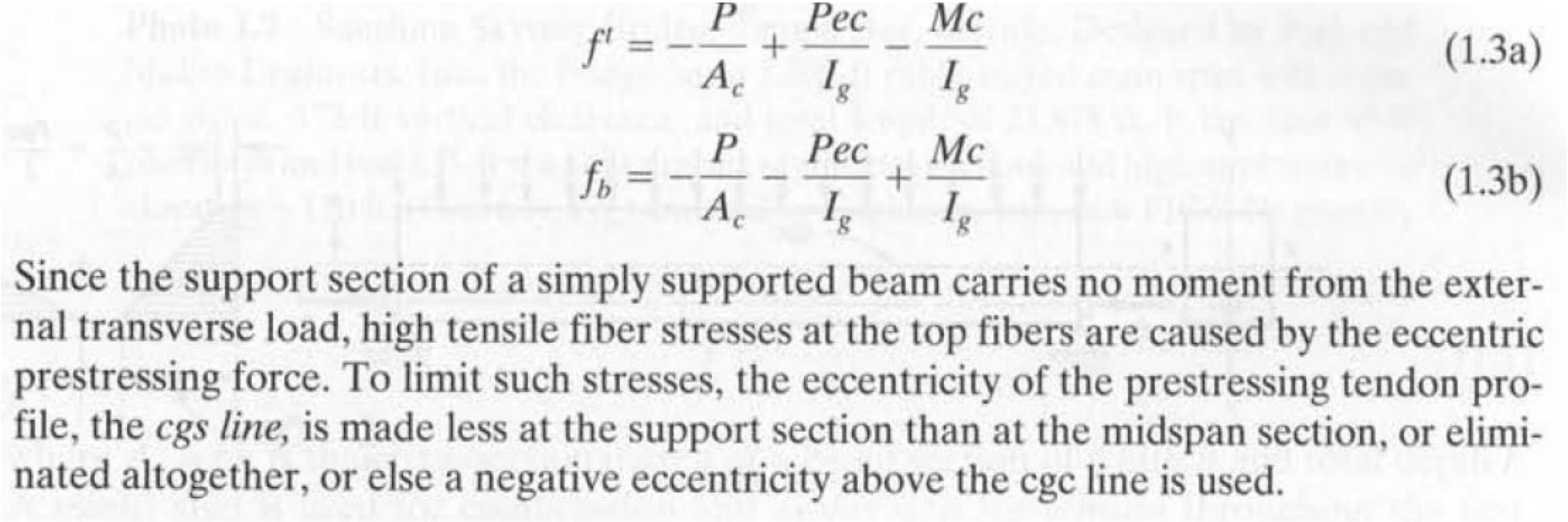


Figure 1.2 Concrete fiber stress distribution in a rectangular beam with straight tendon. (a) Concentric tendon, prestress only. (b) Concentric tendon, self-weight added. (c) Eccentric tendon, prestress only. (d) Eccentric tendon, self-weight added.

nating tension totally (even inducing compression), or permitting a level of tensile stress within allowable code limits. The section is then considered uncracked and behaves elastically: the concrete's inability to withstand tensile stresses is effectively compensated for by the compressive force of the prestressing tendon.

The compressive stresses in Equation 1.2a at the top fibers of the beam due to prestressing are compounded by the application of the loading stress $-Mc/I$, as seen in Figure 1.2(b). Hence, the compressive stress capacity of the beam to take a substantial external load is reduced by the *concentric* prestressing force. In order to avoid this limitation, the prestressing tendon is placed *eccentrically* below the neutral axis at midspan, to induce tensile stresses at the top fibers due to prestressing. [See Figure 1.2(c), (d).] If the tendon is placed at eccentricity e from the center of gravity of the concrete, termed the *cgc line,* it creates a moment Pe, and the ensuing stresses at midspan become



$$f^t = -\frac{P}{A_c} + \frac{Pec}{I_g} - \frac{Mc}{I_g} \tag{1.3a}$$

$$f_b = -\frac{P}{A_c} - \frac{Pec}{I_g} + \frac{Mc}{I_g} \tag{1.3b}$$

Since the support section of a simply supported beam carries no moment from the external transverse load, high tensile fiber stresses at the top fibers are caused by the eccentric prestressing force. To limit such stresses, the eccentricity of the prestressing tendon profile, the *cgs line,* is made less at the support section than at the midspan section, or eliminated altogether, or else a negative eccentricity above the cgc line is used.

1.3.2 Basic Concept Method

In the basic concept method of designing prestressed concrete elements, the concrete fiber stresses are *directly* computed from the external forces applied to the concrete by longitudinal prestressing and the external transverse load. Equations 1.3a and b can be modified and simplified for use in calculating stresses at the initial prestressing stage and at service load levels. If P_i is the initial prestressing force before stress losses, and P_e is the effective prestressing force after losses, then

$$\gamma = \frac{P_e}{P_i} \tag{1.3c}$$

can be defined as the residual prestress factor. Substituting r^2 for I_g/A_c in Equations 1.3, where r is the radius of gyration of the gross section, the expressions for stress can be rewritten as follows:

(a) Prestressing Force Only

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) \tag{1.4a}$$

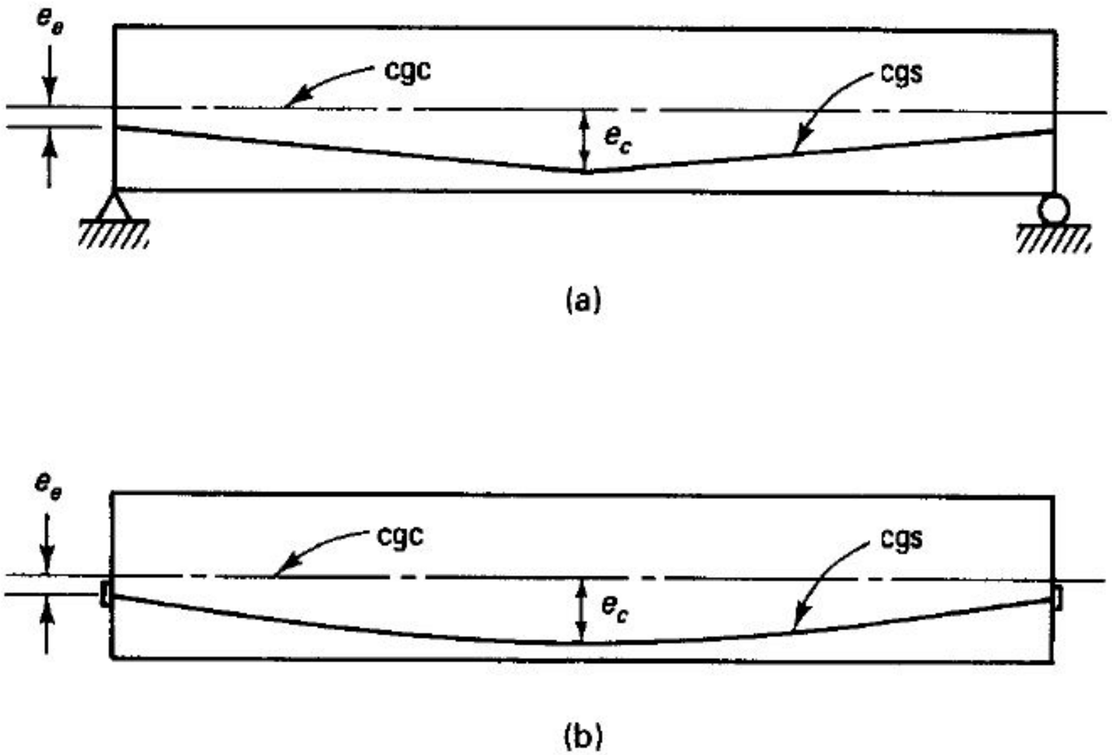


Figure 1.3 Prestressing tendon profile. (a) Harped tendon. (b) Draped tendon.

where S^t and S_b are the moduli of the sections for the top and bottom fibers, respectively.

The change in eccentricity from the midspan to the support section is obtained by raising the prestressing tendon either abruptly from the midspan to the support, a process called harping, or gradually in a parabolic form, a process called draping. Figure 1.3(a) shows a harped profile usually used for pretensioned beams and for concentrated transverse loads. Figure 1.3(b) shows a draped tendon usually used in post-tensioning.

Subsequent to erection and installation of the floor or deck, live loads act on the structure, causing a superimposed moment M_s. The full intensity of such loads normally occurs after the building is completed and some time-dependent losses in prestress have already taken place. Hence, the prestressing force used in the stress equations would have to be the effective prestressing force P_e. If the total moment due to gravity loads is M_T, then

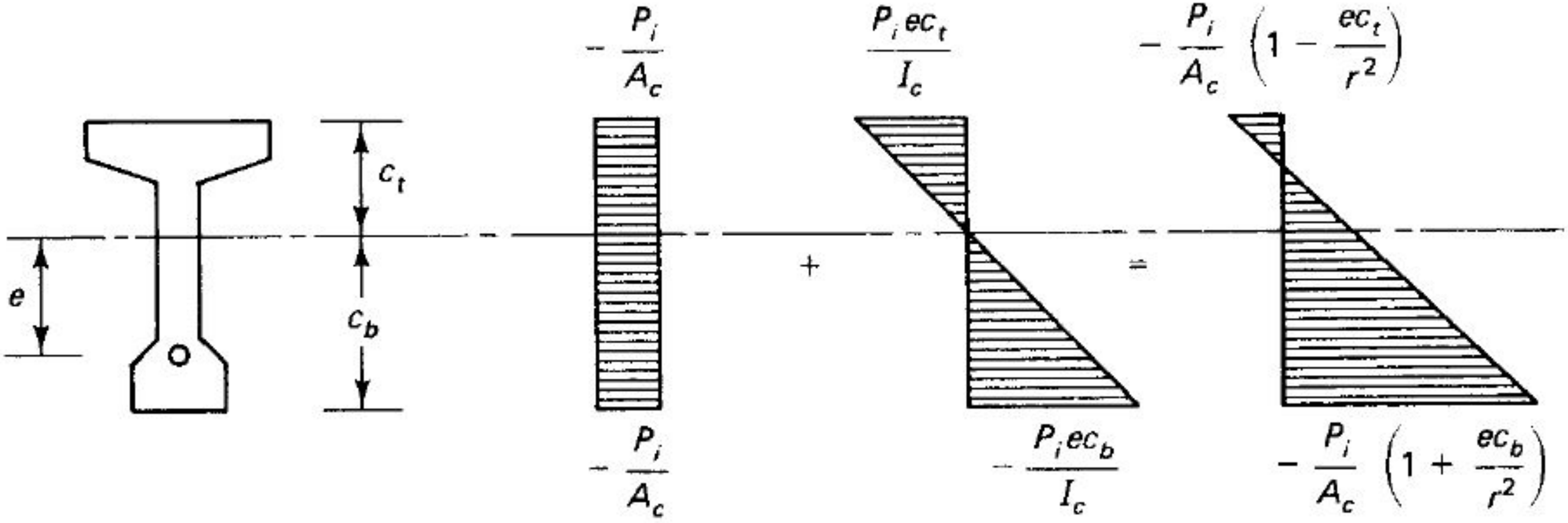
$$M_T = M_D + M_{SD} + M_L \tag{1.6}$$

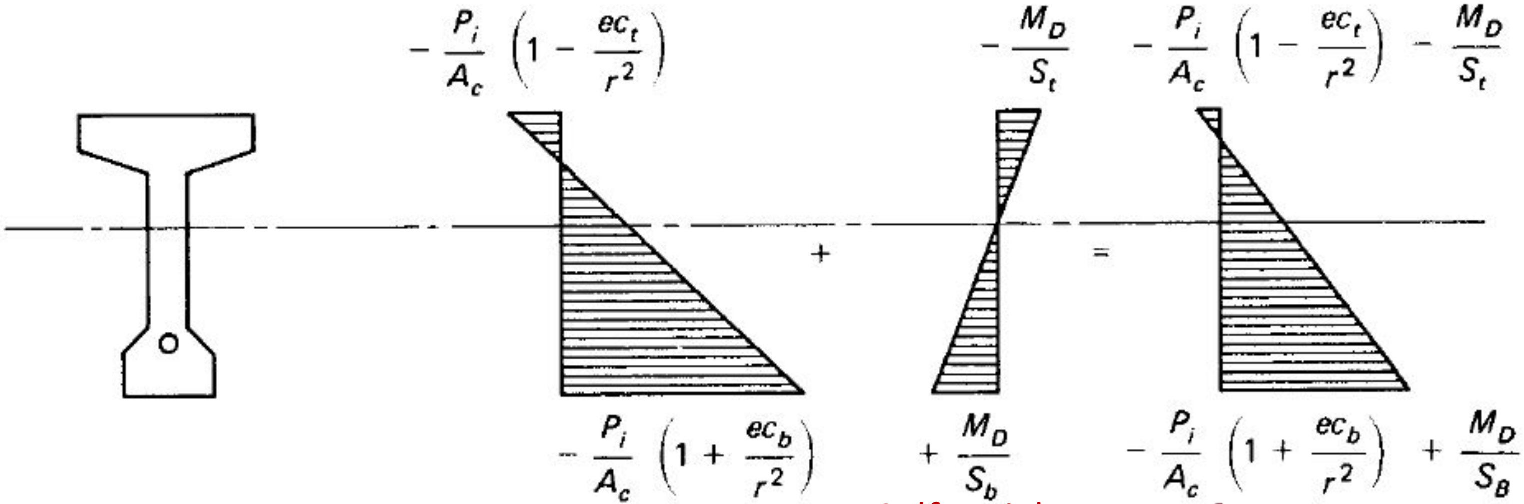
where M_D = moment due to self-weight

M_{SD} = moment due to superimposed dead load, such as flooring

M_L = moment due to live load, including impact and seismic loads if any

Equations 1.5 then become


$-\frac{P_i}{A_c}$
$\frac{P_i e c_t}{I_c}$
$-\frac{P_i}{A_c}\left(1-\frac{ec_t}{r^2}\right)$
c_t
e
c_b
$+$
$=$
$-\frac{P_i}{A_c}$
$-\frac{P_i e c_b}{I_c}$
$-\frac{P_i}{A_c}\left(1+\frac{ec_b}{r^2}\right)$



Self-weight f_{bottom} Service load

$P_i = initial\ prestressing\ force(N)$;

Ac: area of cross section (mm2);

I: moment of inertia of the cross section (mm4);

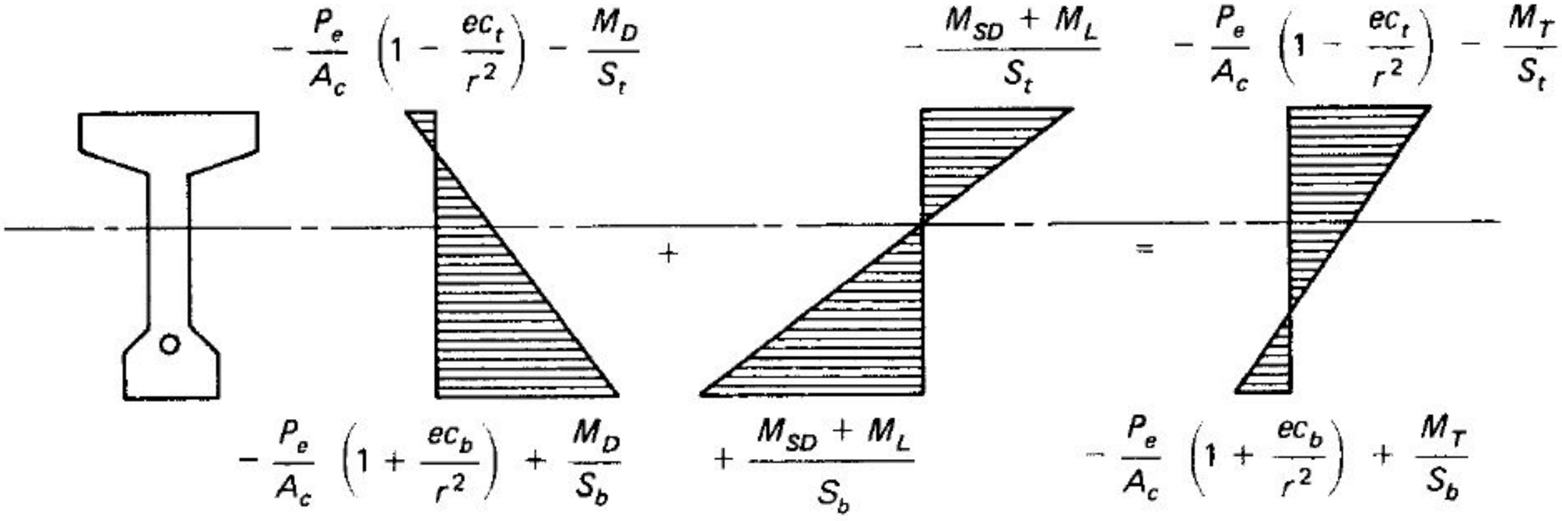
e: eccentricity of prestressing steel below centroid (mm);

C_t, C_b: top and bottom fiber distance from centroid (mm);

r: radius of gyration =($\sqrt{\frac{I}{A}}$) (mm);

P_e: $effective\ prestressing\ force\ (after\ all\ losses)(N)$;

f: stress (MPa)



(c)

Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

Allowable stresses of concrete

f_{ci} = maximum allowable compressive stress in concrete immediately after transfer and prior to losses

= $0.60\, f'_{ci}$

f_{ti} = maximum allowable tensile stress in concrete immediately after transfer and prior to losses

= $3\sqrt{f'_{ci}}$ (the value can be increased to $6\sqrt{f'_{ci}}$ at the supports for simply supported members)

f_c = maximum allowable compressive stress in concrete after losses at service-load level

= $0.45\, f'_c$ or $0.60\, f'_c$ when allowed by the code

f_t = maximum allowable tensile stress in concrete after losses at service load level

= $6\sqrt{f'_c}$ (the value can be increased in one-way systems to $12\sqrt{f'_c}$ if long-term deflection requirements are met)

Summary of equations of stress

Stress at Transfer

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_{ci}$$

Effective Stresses after Losses

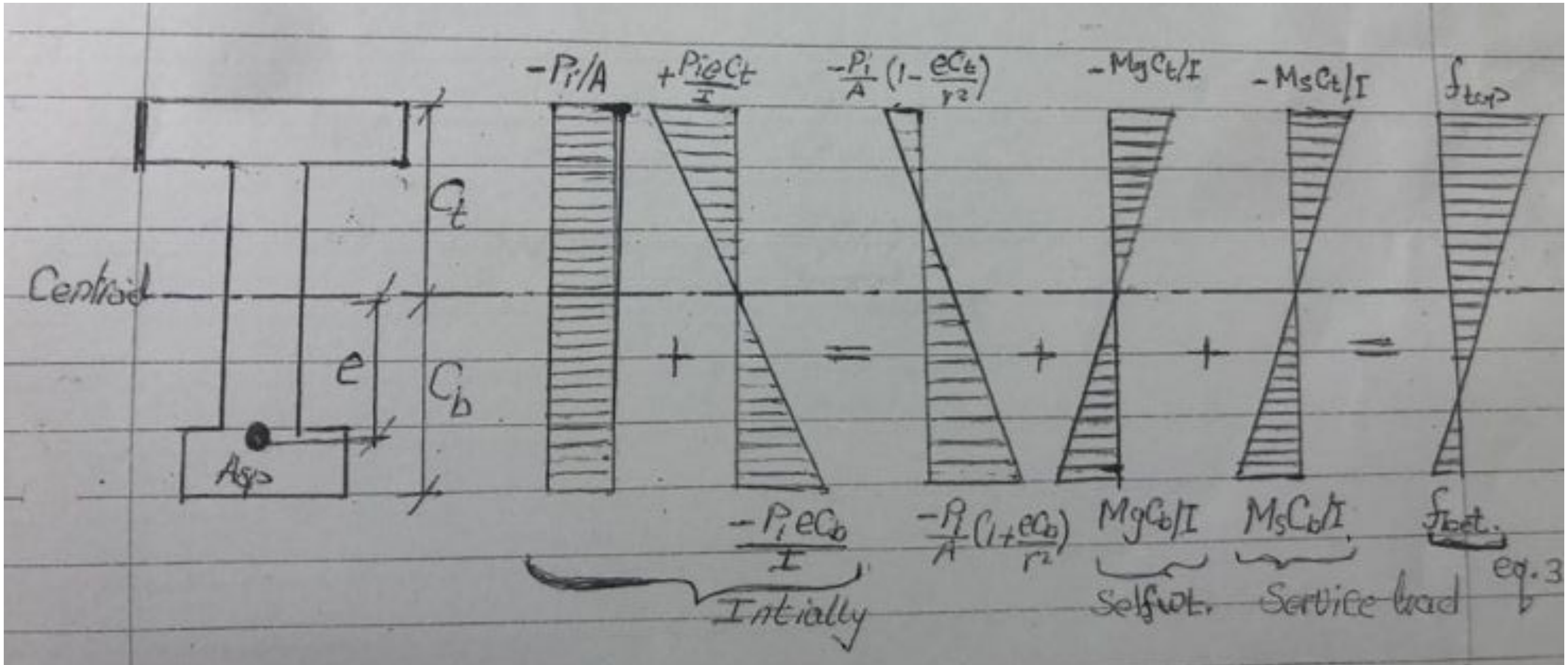
$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_t$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_c$$

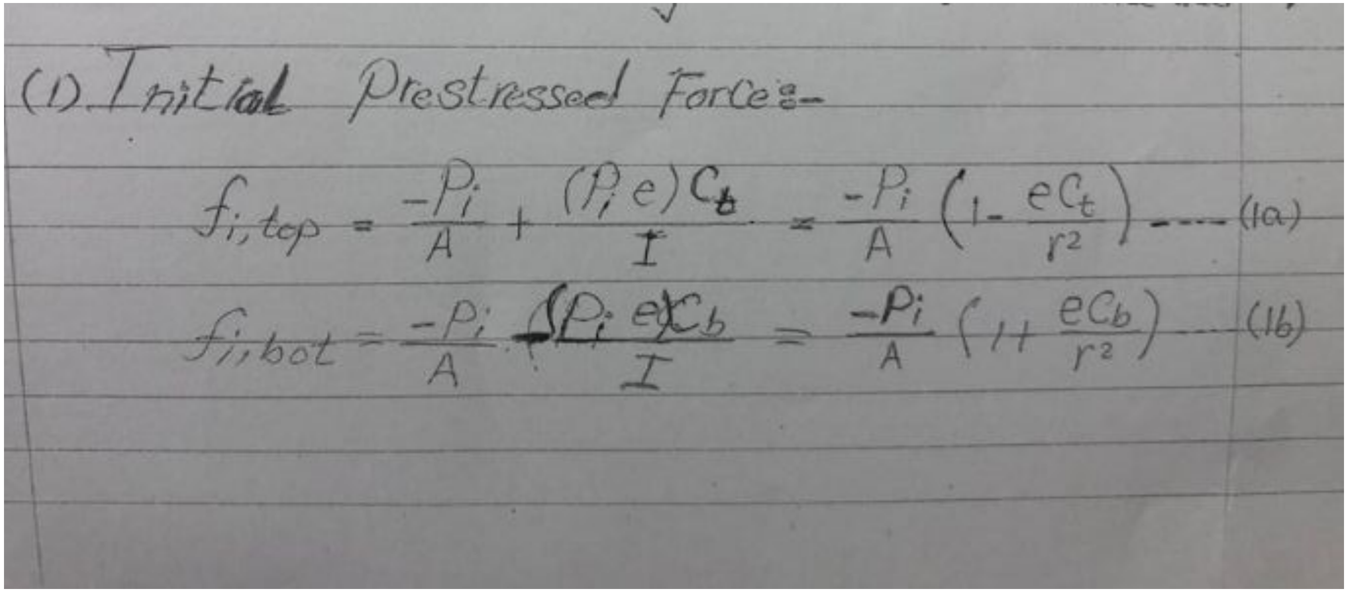
Service-load Final Stresses

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \leq f_c$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \leq f_t$$



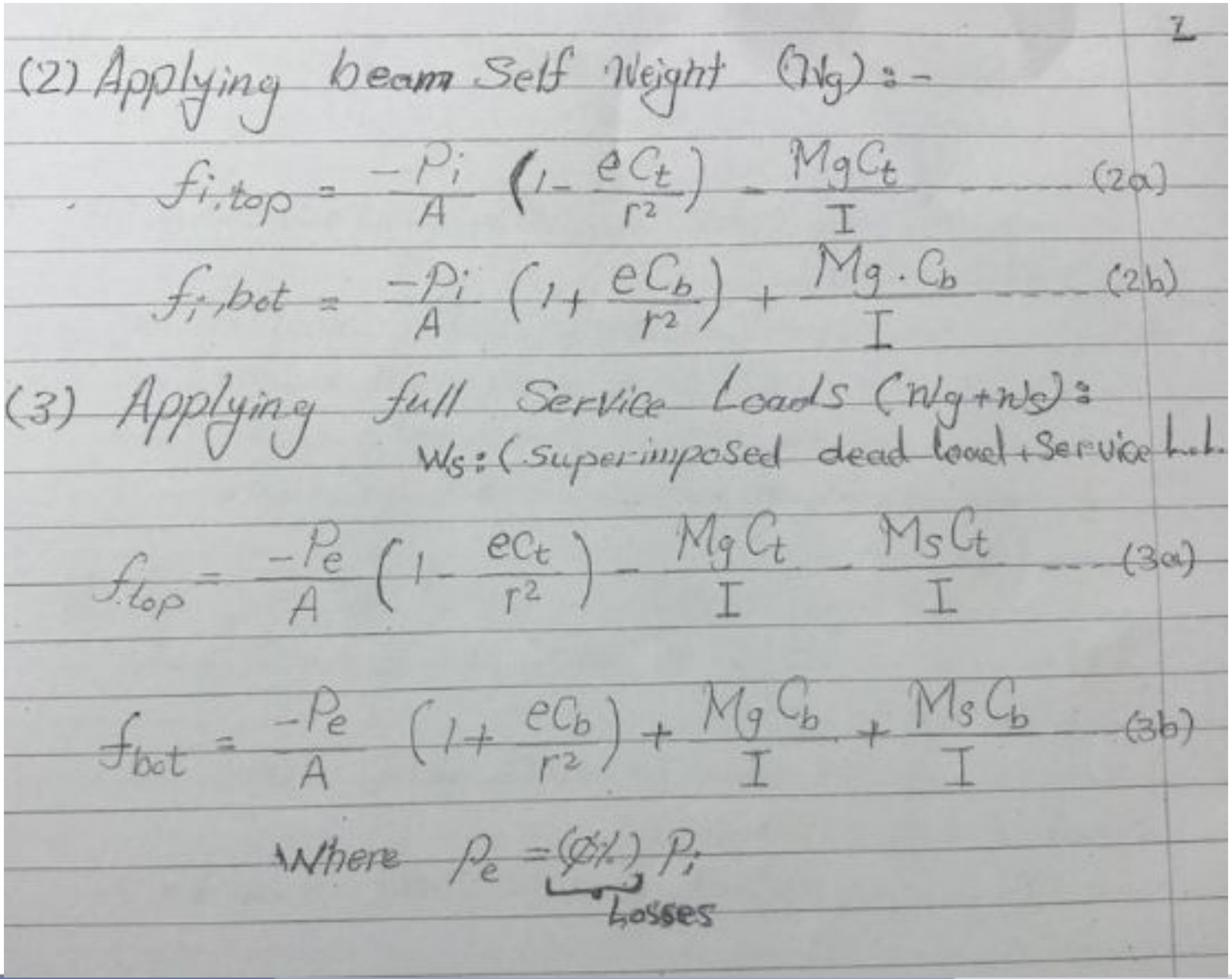
(1) Initial Prestressed Forces:-



(1) Initial Prestressed Force:-

$$f_{i,top} = \frac{-P_i}{A} + \frac{(P_i e)C_t}{I} = \frac{-P_i}{A}\left(1 - \frac{eC_t}{r^2}\right) \text{-----} (1a)$$

$$f_{i,bot} = \frac{-P_i}{A} - \frac{(P_i e)C_b}{I} = \frac{-P_i}{A}\left(1 + \frac{eC_b}{r^2}\right) \text{-----} (1b)$$



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(2) Applying beam Self Weight (Wg) :-

$$f_{i,top} = \frac{-P_i}{A}\left(1 - \frac{eC_t}{r^2}\right) - \frac{M_g C_t}{I} \quad \text{------ (2a)}$$

$$f_{i,bot} = \frac{-P_i}{A}\left(1 + \frac{eC_b}{r^2}\right) + \frac{M_g \cdot C_b}{I} \quad \text{------ (2b)}$$

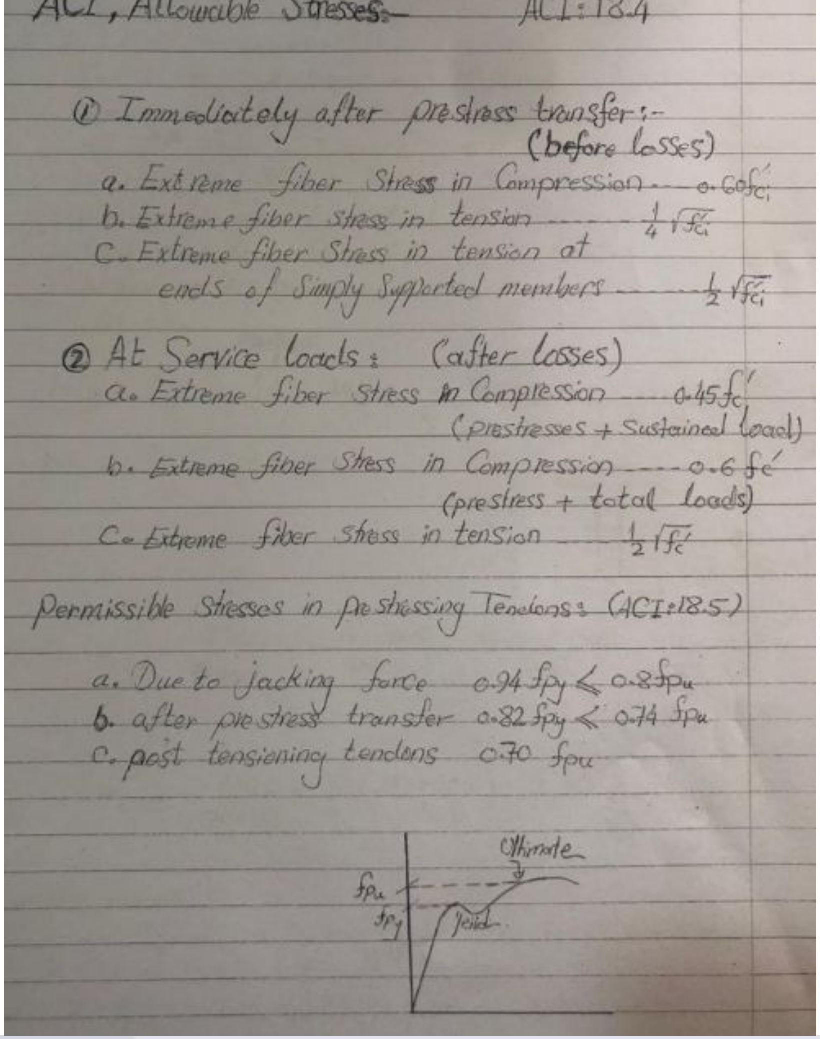
(3) Applying full Service Loads (Wg+Ws):

Ws: (superimposed dead load + Service L.L.

$$f_{top} = \frac{-P_e}{A}\left(1 - \frac{eC_t}{r^2}\right) - \frac{M_g C_t}{I} - \frac{M_s C_t}{I} \quad \text{---- (3a)}$$

$$f_{bot} = \frac{-P_e}{A}\left(1 + \frac{eC_b}{r^2}\right) + \frac{M_g C_b}{I} + \frac{M_s C_b}{I} \quad \text{---- (3b)}$$

Where $P_e = \underbrace{(\phi\%)}_{\text{Losses}} P_i$



ACI, Allowable Stresses:- ACI:18.4

① Immediately after prestress transfer:- (before losses)

a. Extreme fiber Stress in Compression --- $0.60f'_{ci}$

b. Extreme fiber stress in tension ------ $\frac{1}{4}\sqrt{f'_{ci}}$

c. Extreme fiber Stress in tension at ends of Simply Supported members ------- $\frac{1}{2}\sqrt{f'_{ci}}$

② At Service loads: (after losses)

a. Extreme fiber Stress in Compression ---- $0.45f'_c$ (prestresses + Sustained load)

b. Extreme fiber Stress in Compression ---- $0.6f'_c$ (prestress + total loads)

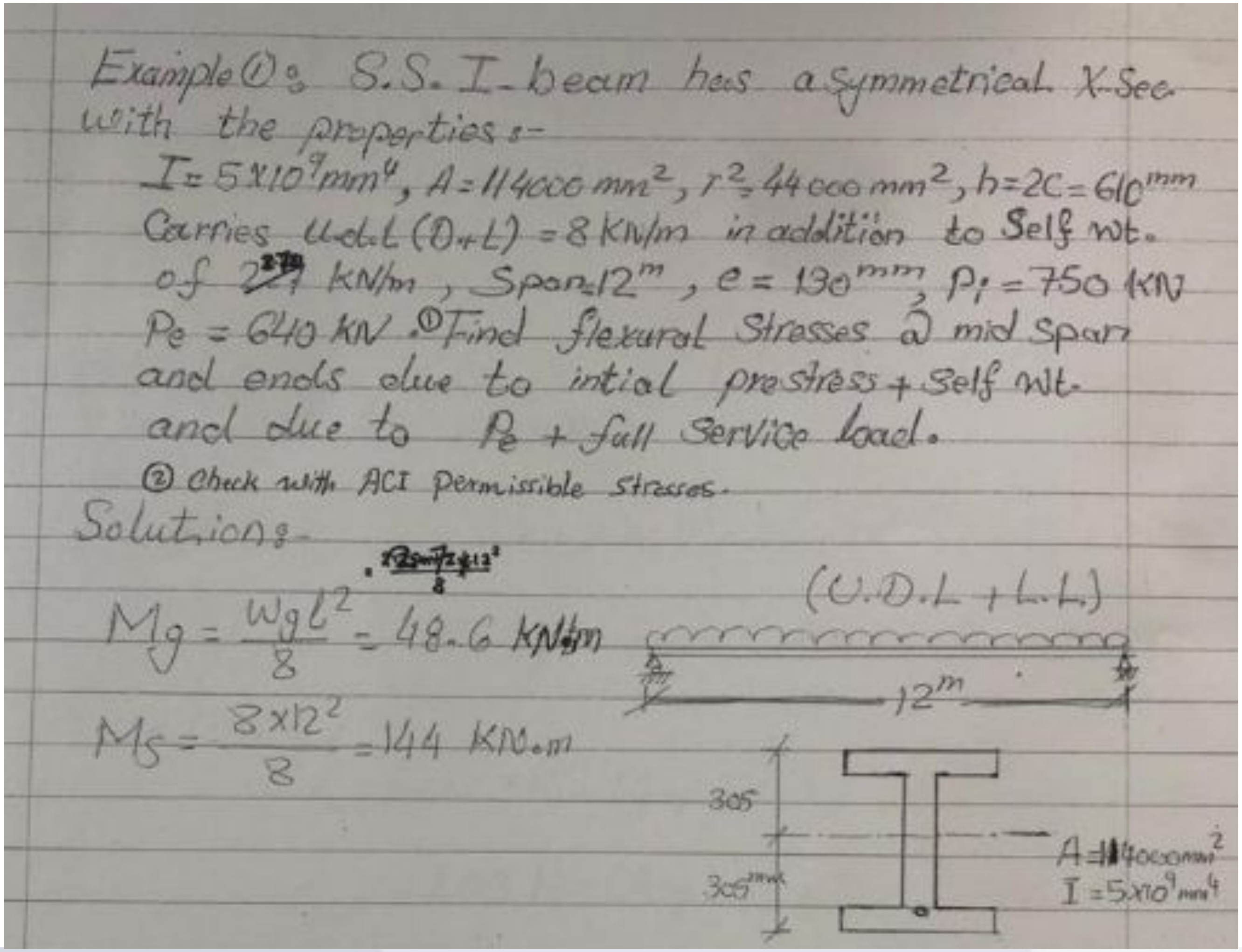
c. Extreme fiber stress in tension ----- $\frac{1}{2}\sqrt{f'_c}$

Permissible Stresses in prestressing Tendons: (ACI:18.5)

a. Due to jacking force $0.94f_{py} \leqslant 0.8f_{pu}$

b. after prestress transfer $0.82f_{py} \leqslant 0.74f_{pu}$

c. post tensioning tendons $0.70f_{pu}$



Example ①: S.S. I-beam has a symmetrical X-Sec with the properties:-

$I = 5\times10^9 mm^4$, $A = 114000 mm^2$, $r^2 = 44000 mm^2$, $h = 2C = 610^{mm}$

Carries u.d.L (D+L) = 8 kN/m in addition to Self wt. of 2.7 kN/m, Span = 12^m, $e = 130^{mm}$, $P_i = 750$ kN

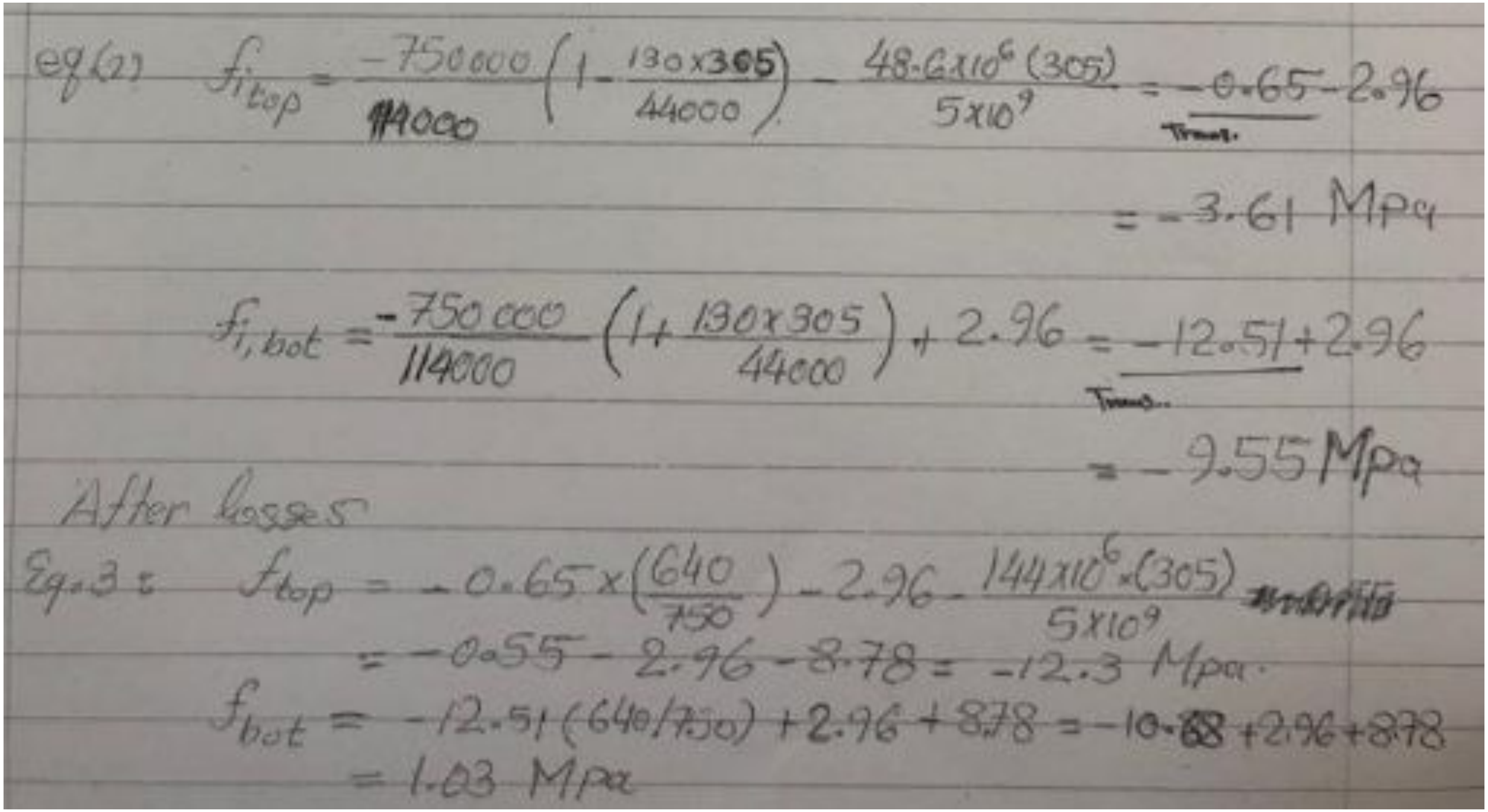
$P_e = 640$ kN. ① Find flexural Stresses @ mid span and ends due to intial prestress + Self wt. and due to P_e + full Service load.

② Check with ACI Permissible stresses.

Solution:-

$$M_g = \frac{w_g l^2}{8} = \frac{2.7 \times 12^2}{8} = 48.6 \text{ kN.m}$$

$$M_S = \frac{8\times12^2}{8} = 144 \text{ kN.m}$$



eq(2)
$$f_{i,top} = \frac{-750000}{114000}\left(1 - \frac{130 \times 305}{44000}\right) - \frac{48.6 \times 10^6 (305)}{5 \times 10^9} = \underbrace{-0.65}_{\text{Trans.}} - 2.96$$

$$= -3.61 \text{ Mpa}$$

$$f_{i,bot} = \frac{-750000}{114000}\left(1 + \frac{130 \times 305}{44000}\right) + 2.96 = \underbrace{-12.51}_{\text{Trans.}} + 2.96$$

$$= -9.55 \text{ Mpa}$$

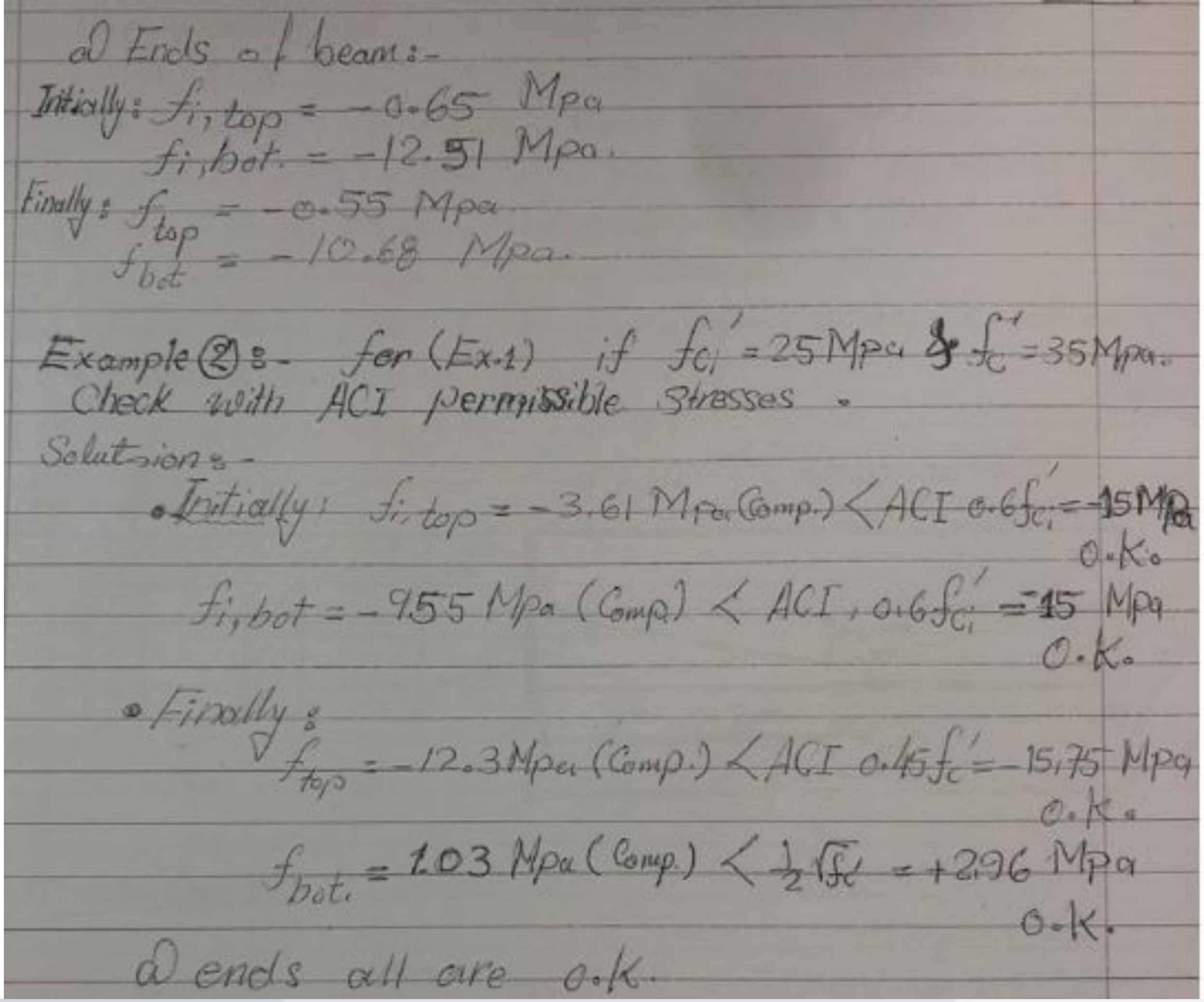
After losses

Eq. 3 :
$$f_{top} = -0.65 \times \left(\frac{640}{750}\right) - 2.96 - \frac{144 \times 10^6 \times (305)}{5 \times 10^9}$$

$$= -0.55 - 2.96 - 8.78 = -12.3 \text{ Mpa.}$$

$$f_{bot} = -12.51 (640/750) + 2.96 + 878 = -10.68 + 2.96 + 878$$

$$= 1.03 \text{ Mpa}$$



@ Ends of beam:-

Initially: $f_{i,top} = -0.65$ Mpa

$f_{i,bot.} = -12.51$ Mpa.

Finally: $f_{top} = -0.55$ Mpa

$f_{bot} = -10.68$ Mpa.

Example ② :- for (Ex-1) if $f'_{ci} = 25$ Mpa & $f'_c = 35$ Mpa.
Check with ACI permissible stresses.

Solution:-

- Initially: $f_{i,top} = -3.61$ Mpa (Comp.) $<$ ACI $0.6 f'_{ci} = -15$ Mpa o.k.

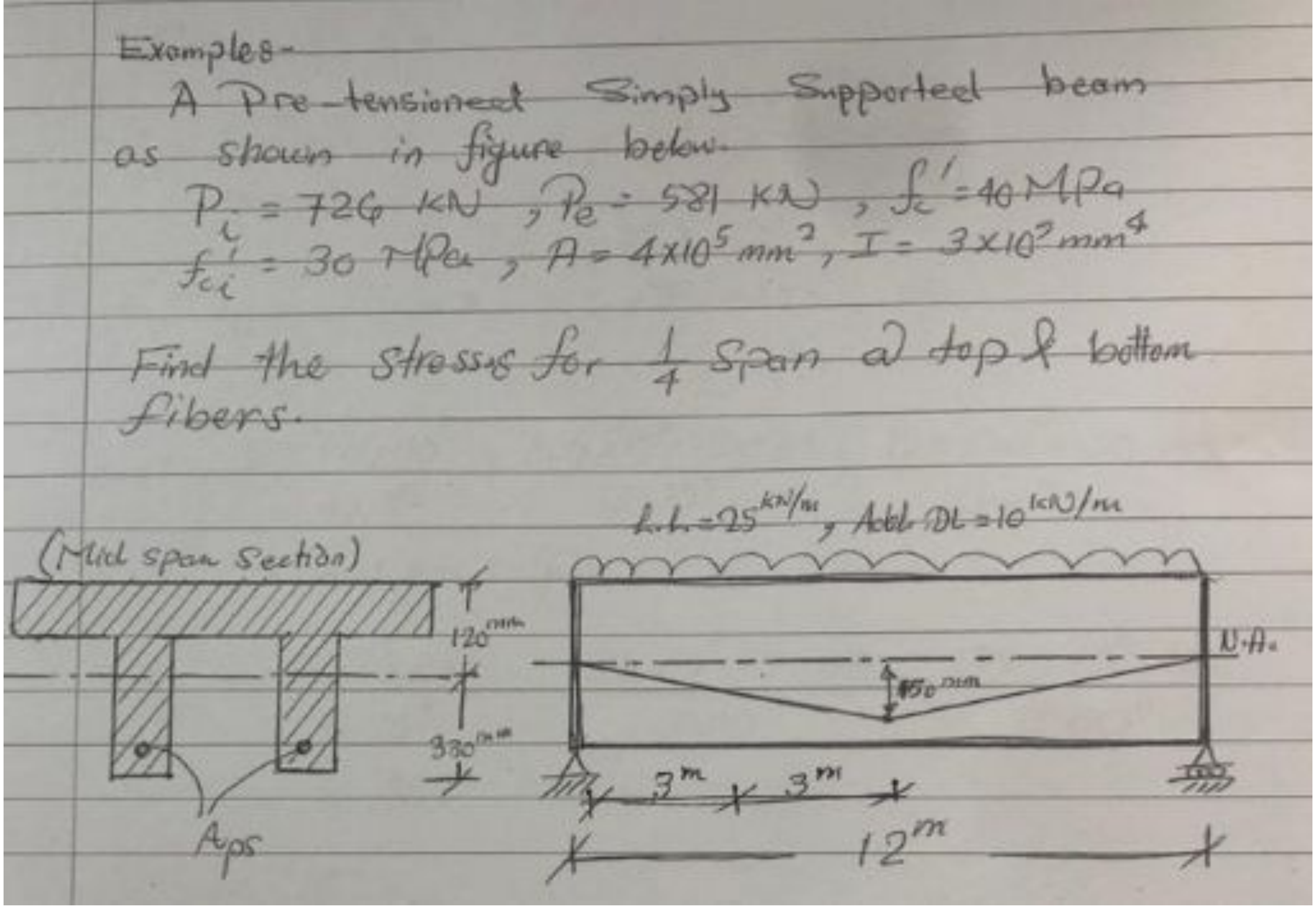
  $f_{i,bot} = -9.55$ Mpa (Comp) $<$ ACI, $0.6 f'_{ci} = -15$ Mpa o.k.

- Finally:

  $f_{top} = -12.3$ Mpa (Comp.) $<$ ACI $0.45 f'_c = -15.75$ Mpa o.k.

  $f_{bot.} = 1.03$ Mpa (Comp.) $< \frac{1}{2}\sqrt{f'_c} = +2.96$ Mpa o.k.

@ ends all are o.k.



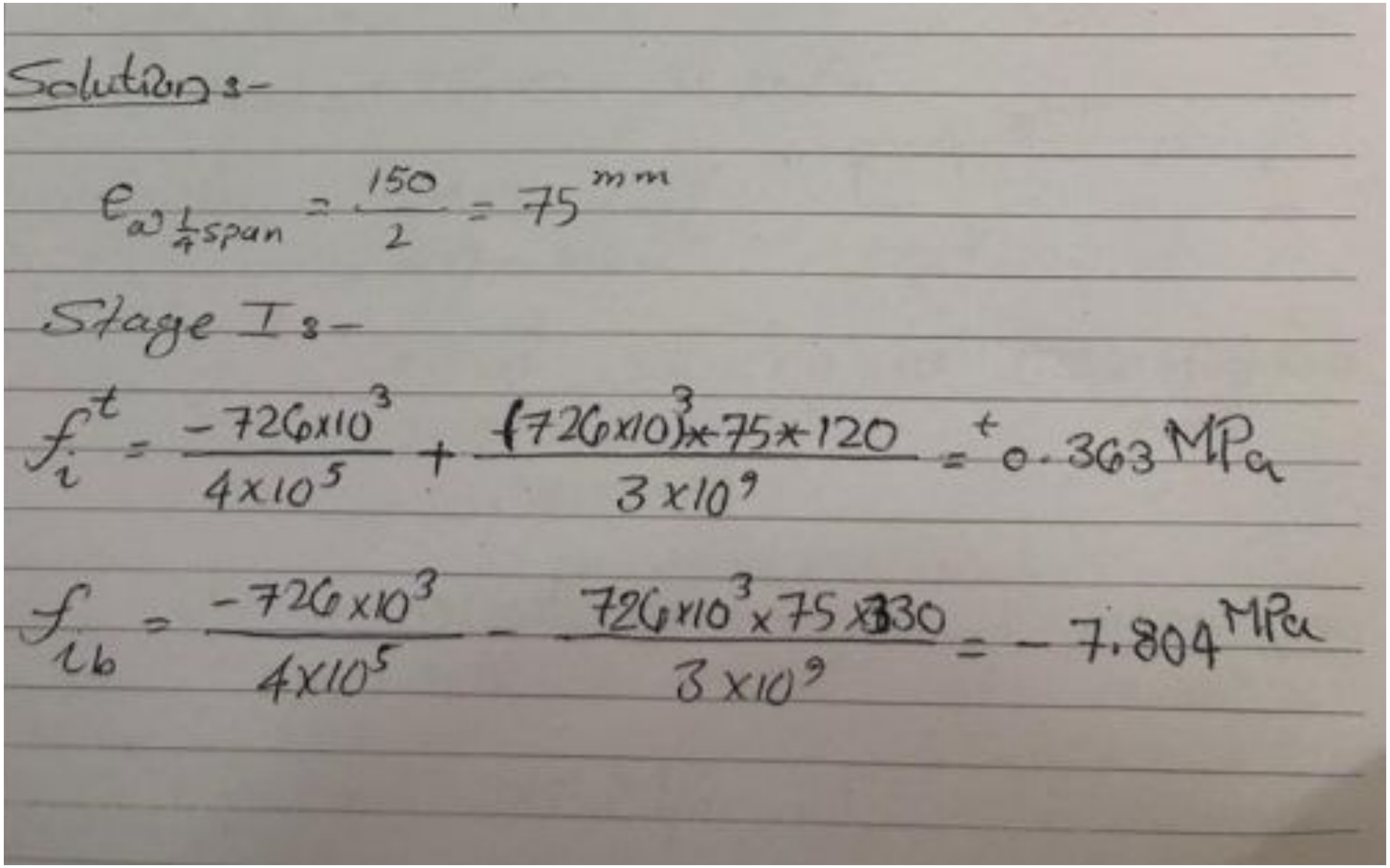
Examples-

A Pre-tensioned Simply Supported beam as shown in figure below.

$P_i = 726$ kN, $P_e = 581$ kN, $f_c' = 40$ MPa

$f_{ci}' = 30$ MPa, $A = 4 \times 10^5$ mm^2, $I = 3 \times 10^2$ mm^4

Find the stresses for $\frac{1}{4}$ Span @ top & bottom fibers.



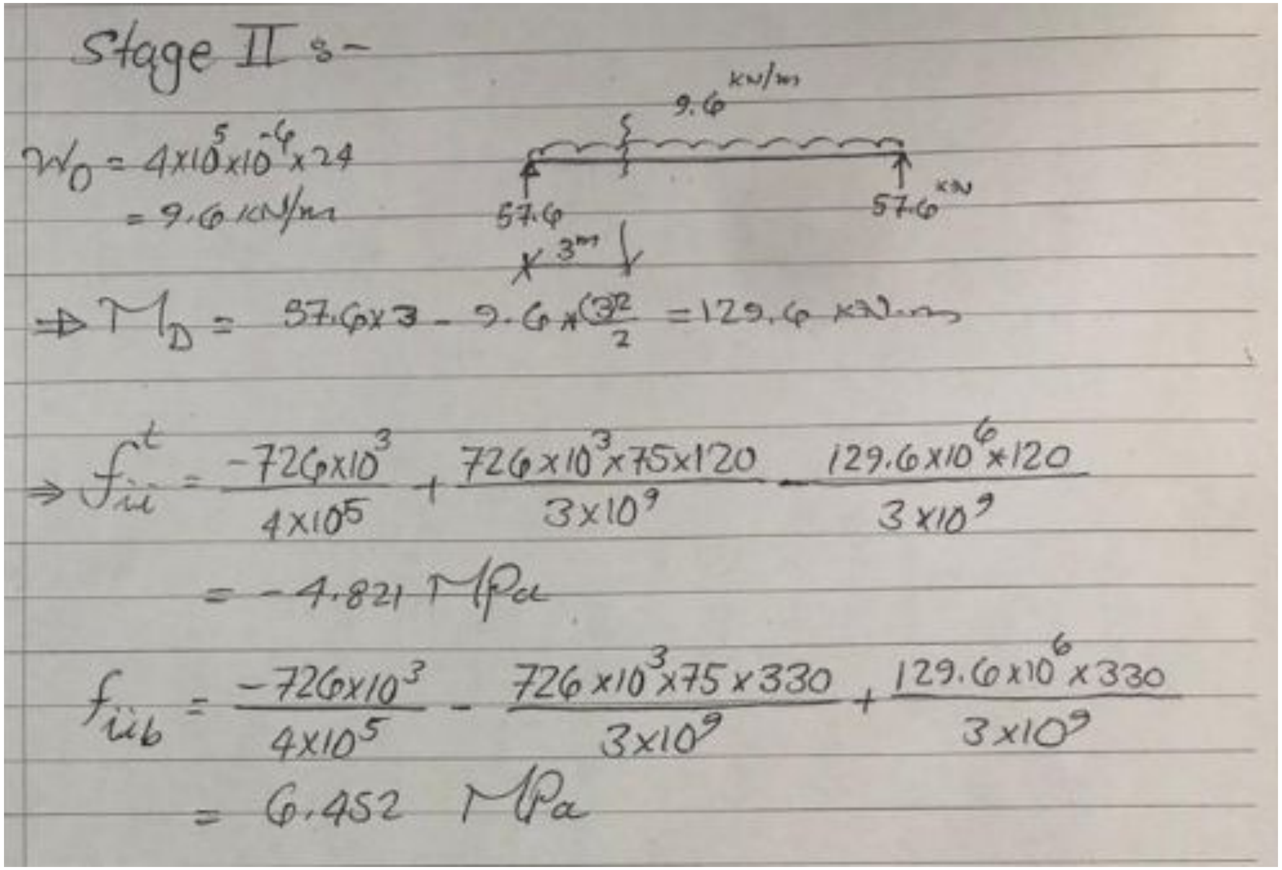
Solutions:-

$$e_{@\frac{1}{4}span} = \frac{150}{2} = 75^{mm}$$

Stage I:-

$$f_i^t = \frac{-726\times10^3}{4\times10^5} + \frac{(726\times10^3)*75*120}{3\times10^9} = {}^{+}0.363\ MPa$$

$$f_{ib} = \frac{-726\times10^3}{4\times10^5} - \frac{726\times10^3\times75\times330}{3\times10^9} = -7.804\ MPa$$



Stage II :-

$w_0 = 4\times10^5\times10^{-6}\times24$

$= 9.6$ kN/m

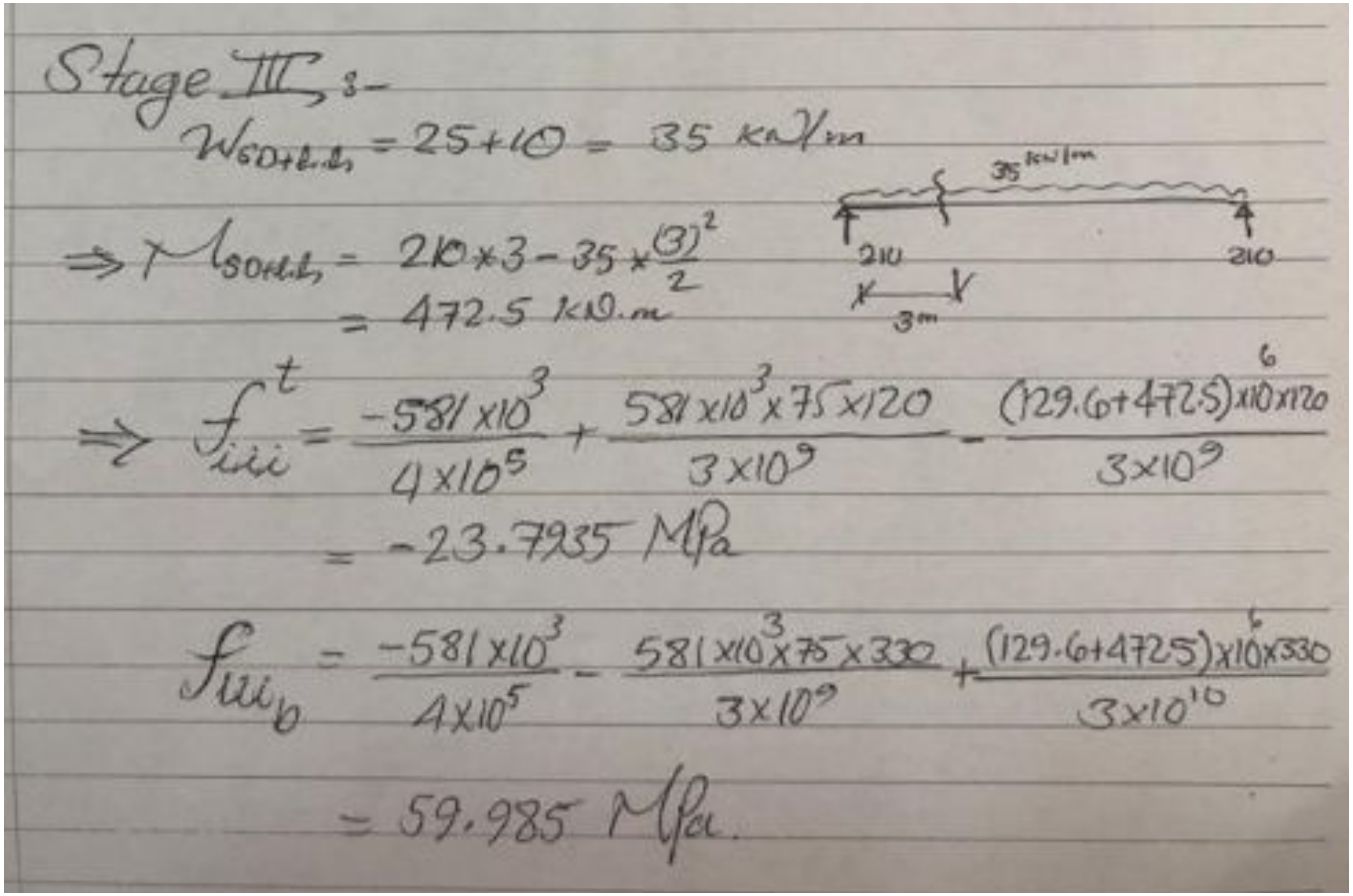
$\Rightarrow M_D = 57.6\times3 - 9.6\times\frac{(3)^2}{2} = 129.6$ kN.m

$$\Rightarrow f_{ii}^t = \frac{-726\times10^3}{4\times10^5} + \frac{726\times10^3\times75\times120}{3\times10^9} - \frac{129.6\times10^6\times120}{3\times10^9}$$

$= -4.821$ MPa

$$f_{iib} = \frac{-726\times10^3}{4\times10^5} - \frac{726\times10^3\times75\times330}{3\times10^9} + \frac{129.6\times10^6\times330}{3\times10^9}$$

$= 6.452$ MPa



Stage III :-

$W_{SD+L.L} = 25+10 = 35\ kN/m$

$\Rightarrow M_{SD+L.L} = 210 \times 3 - 35 \times \frac{(3)^2}{2}$

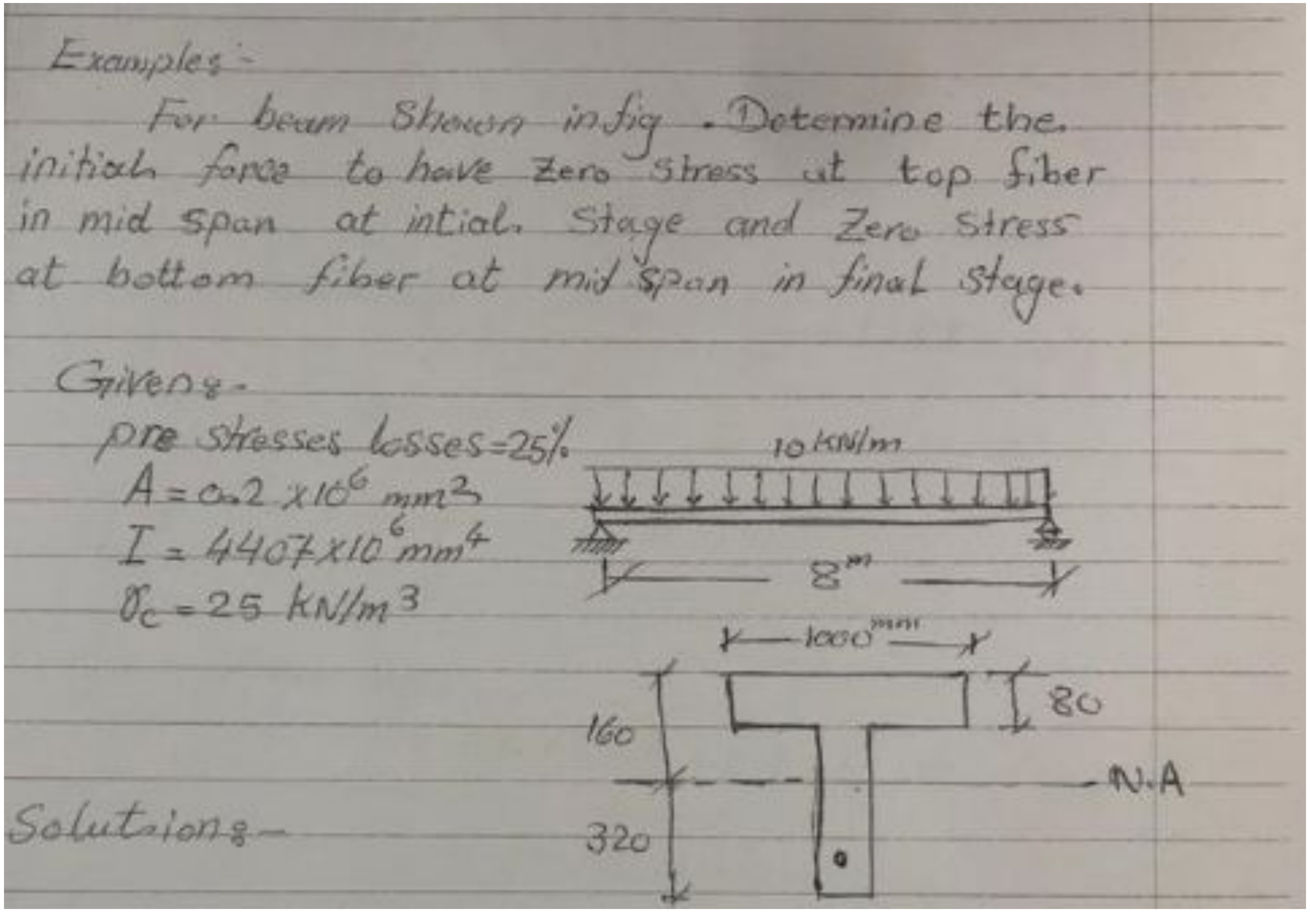
$= 472.5\ kN.m$

$$\Rightarrow f_{iii}^{t} = \frac{-581 \times 10^3}{4 \times 10^5} + \frac{581 \times 10^3 \times 75 \times 120}{3 \times 10^9} - \frac{(129.6+472.5) \times 10^6 \times 120}{3 \times 10^9}$$

$$= -23.7935\ MPa$$

$$f_{iii_b} = \frac{-581 \times 10^3}{4 \times 10^5} - \frac{581 \times 10^3 \times 75 \times 330}{3 \times 10^9} + \frac{(129.6+4725) \times 10^6 \times 330}{3 \times 10^{10}}$$

$$= 59.985\ MPa.$$



Examples:-

For beam Shown in fig. Determine the initial force to have Zero Stress at top fiber in mid span at intial. Stage and Zero Stress at bottom fiber at mid span in final stage.

Given:-

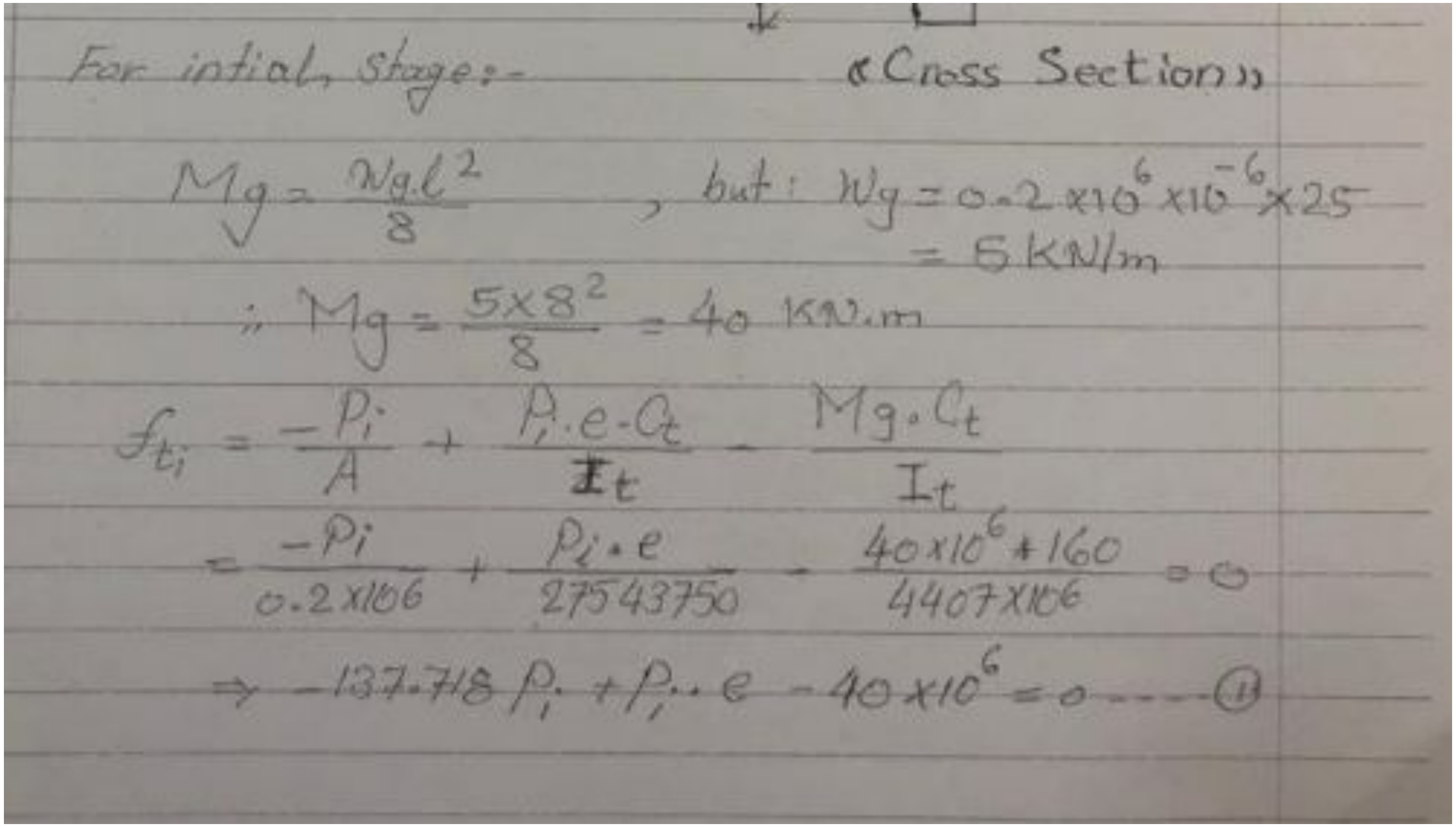
pre stresses losses=25%

$A = 0.2 \times 10^6\ mm^2$

$I = 4407 \times 10^6\ mm^4$

$\gamma_c = 25\ kN/m^3$

Solution:-



For intial, stage:- «Cross Section»

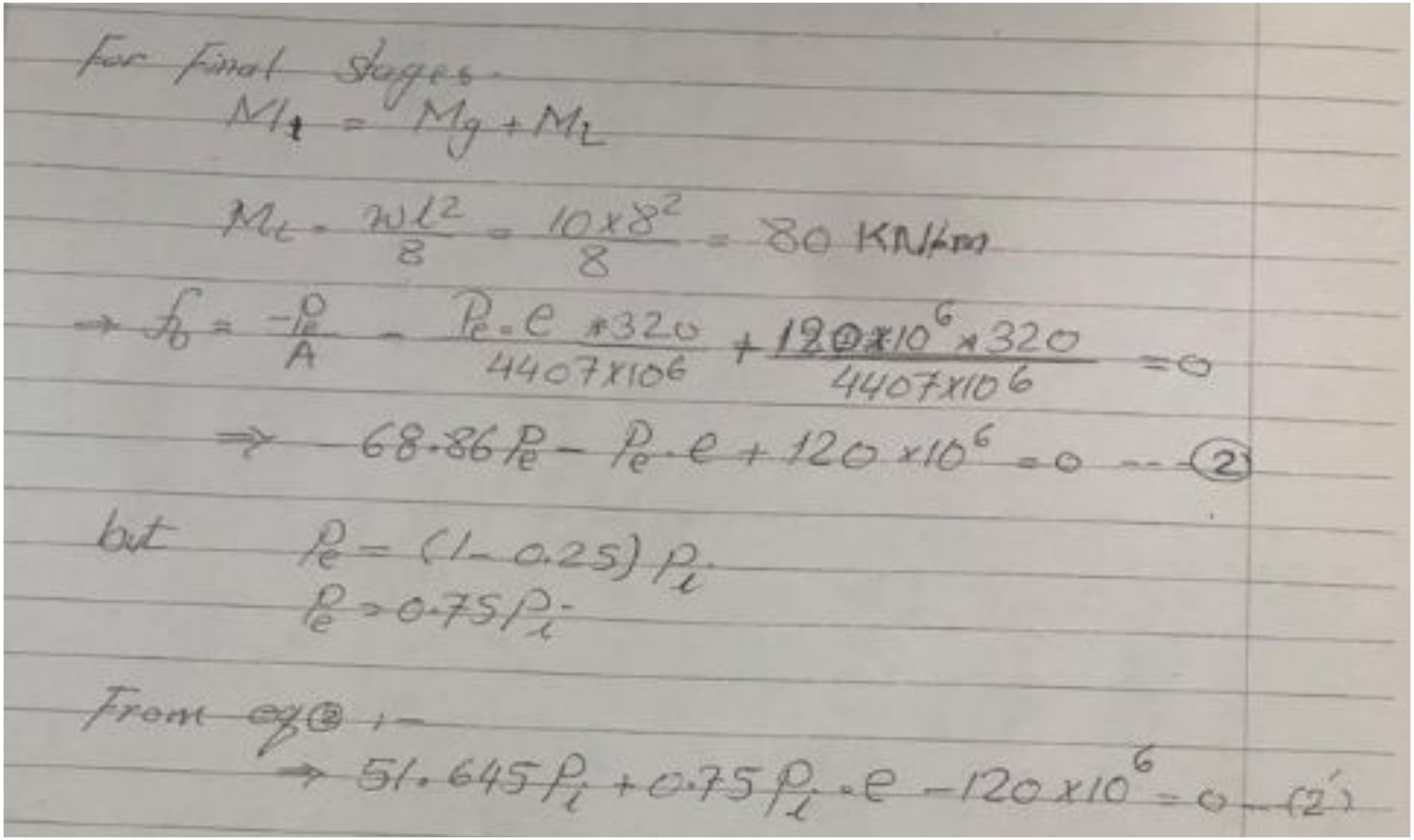
$$M_g = \frac{W_g . l^2}{8} \quad , \text{but}: W_g = 0.2 \times 10^6 \times 10^{-6} \times 25$$
$$= 5 \text{ KN/m}$$

$$\therefore M_g = \frac{5 \times 8^2}{8} = 40 \text{ KN.m}$$

$$f_{t_i} = \frac{-P_i}{A} + \frac{P_i . e . C_t}{I_t} - \frac{M_g . C_t}{I_t}$$

$$= \frac{-P_i}{0.2 \times 106} + \frac{P_i . e}{27543750} - \frac{40 \times 10^6 * 160}{4407 \times 106} = 0$$

$$\Rightarrow -137.718 P_1 + P_1 . e - 40 \times 10^6 = 0 \text{ ---- } ①$$



For final stages.

$$M_t = M_g + M_L$$

$$M_L = \frac{wl^2}{8} = \frac{10\times8^2}{8} = 80 \text{ KN/m}$$

$$\rightarrow f_b = \frac{-P_e}{A} - \frac{P_e \cdot e \times 320}{4407\times10^6} + \frac{120\times10^6\times320}{4407\times10^6} = 0$$

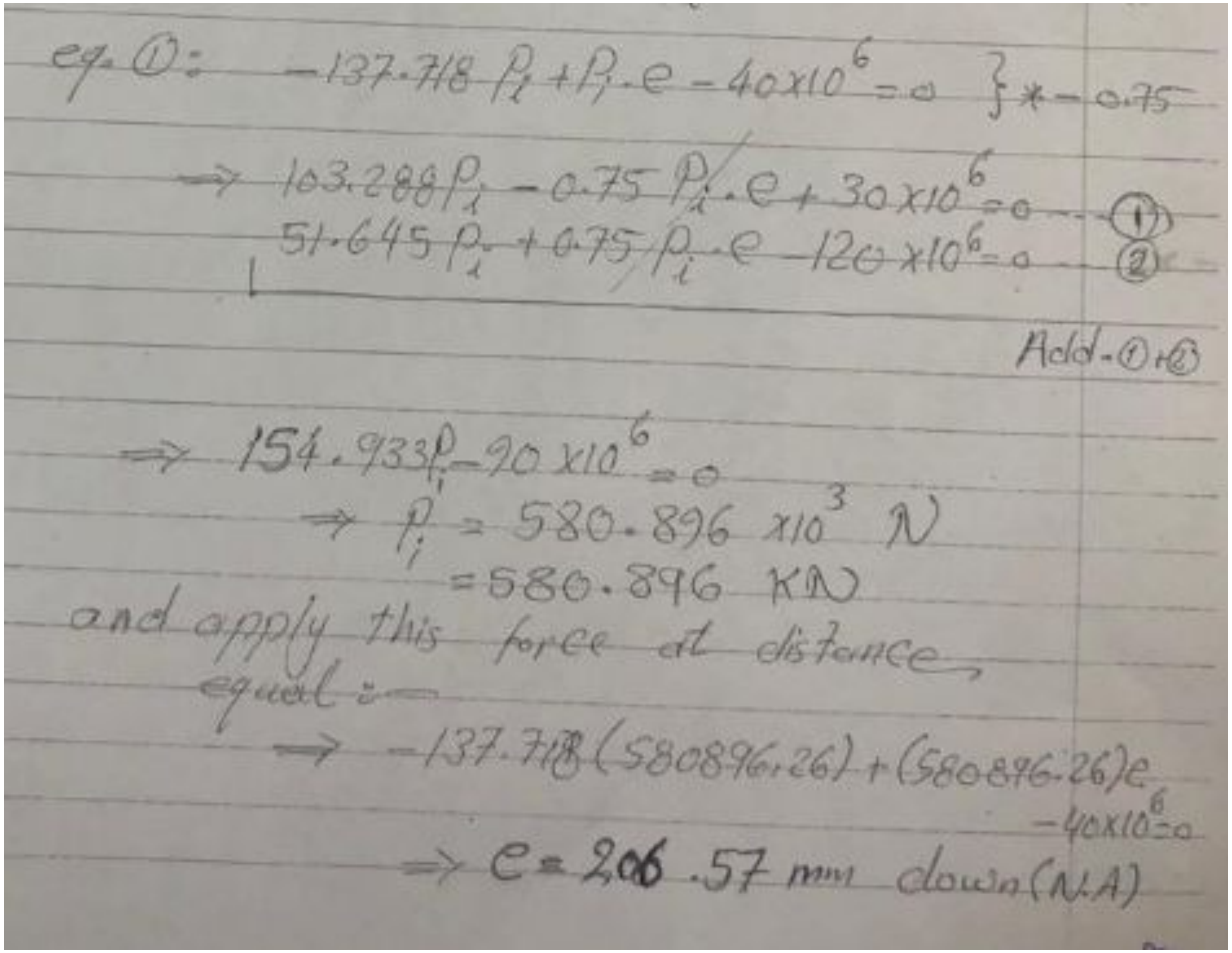
$$\Rightarrow -68.86P_e - P_e \cdot e + 120\times10^6 = 0 \quad \text{---} \quad ②$$

but $P_e = (1-0.25)P_i$

$P_e = 0.75P_i$

From eq ② :-

$$\rightarrow 51.645P_i + 0.75P_i \cdot e - 120\times10^6 = 0 \quad \text{--} (2')$$



eq. ①: $-137.718 P_i + P_i . e - 40\times10^6 = 0$ } $* -0.75$

$\Rightarrow 103.288 P_i - 0.75 P_i . e + 30\times10^6 = 0$ ...... ①

$51.645 P_i + 0.75 P_i . e - 120\times10^6 = 0$ ...... ②

Add ① + ②

$\Rightarrow 154.933 P_i - 90\times10^6 = 0$

$\Rightarrow P_i = 580.896 \times10^3$ N

$= 580.896$ KN

and apply this force at distance, equal :-

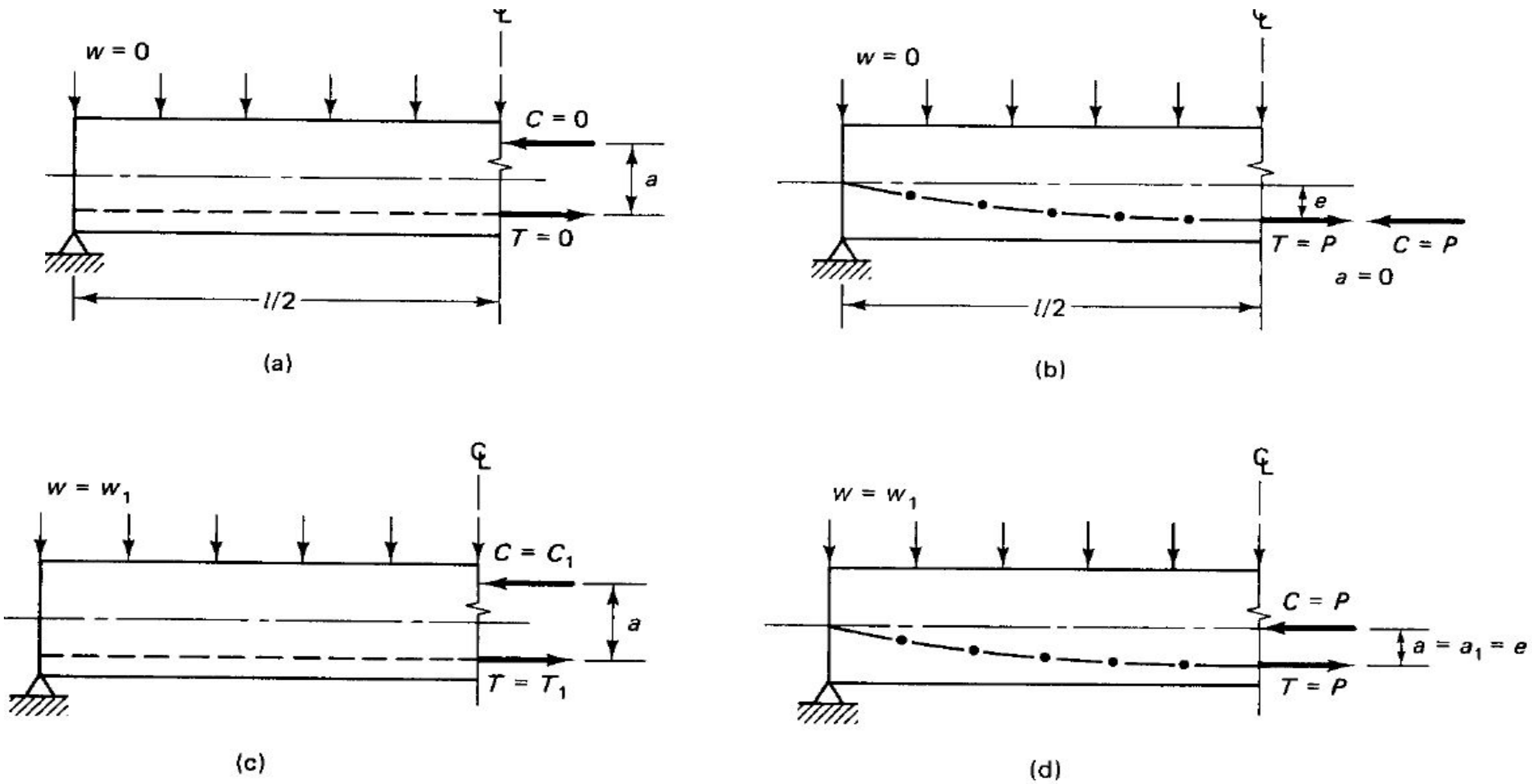
$\Rightarrow -137.718(580896.26) + (580896.26)e - 40\times10^6 = 0$

$\Rightarrow e = 206.57$ mm down (N.A)

ternal compressive force, with a constant tensile force T in the tendon throughout the span. In this manner, the effects of external gravity loads are disregarded. Equilibrium equations $\Sigma H = 0$ and $\Sigma M = 0$ are applied to maintain equilibrium in the section.

Figure 1.5 shows the relative line of action of the compressive force C and the tensile force T in a reinforced concrete beam as compared to that in a prestressed concrete beam. It is plain that in a reinforced concrete beam, T can have a finite value only when transverse and other external loads act. The moment arm a remains basically constant throughout the elastic loading history of the reinforced concrete beam while it changes from a value $a = 0$ at prestressing to a maximum at full superimposed load.

Taking a free-body diagram of a segment of a beam as in Figure 1.6, it is evident that the C-line, or center-of-pressure line, is at a varying distance a from the T-line. The moment is given by


w = 0
℄
C = 0
a
T = 0
l/2
(a)
w = 0
℄
e
T = P
C = P
a = 0
l/2
(b)
w = w₁
℄
C = C₁
a
T = T₁
(c)
w = w₁
℄
C = P
a = a₁ = e
T = P
(d)

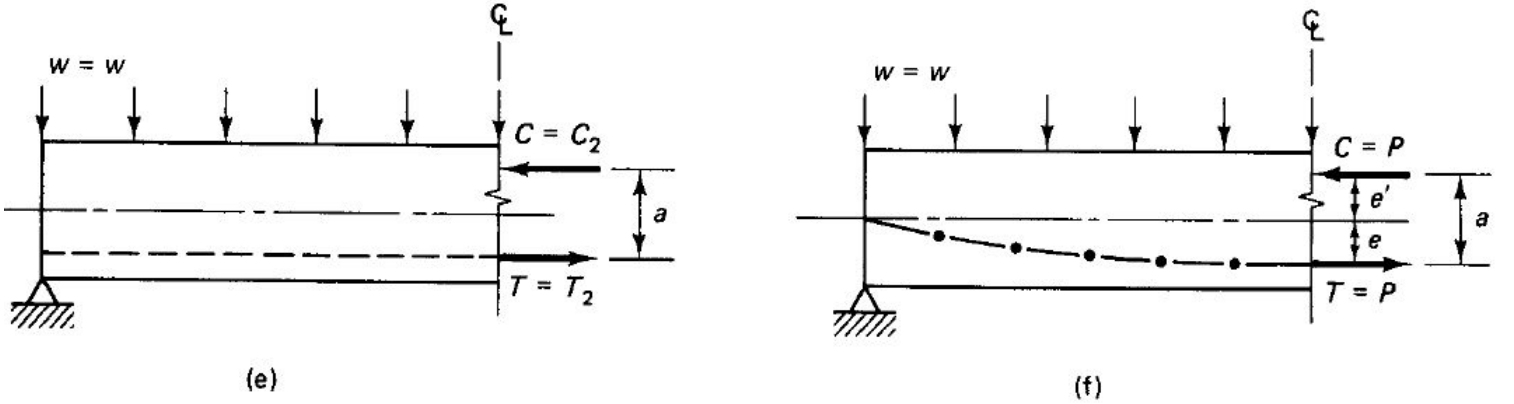


Figure 1.5 Comparative free-body diagrams of a reinforced concrete (R.C.) beam and a prestressed concrete (P.C.) beam. (a) R.C. beam with no load. (b) P.C. beam with no load. (c) R.C. beam with load w_1. (d) P.C. beam with load w_1. (e) R.C. beam with typical load w. (f) P.C. beam with typical load w.

$$M = Ca = Ta \tag{1.8}$$

and the eccentricity e is known or predetermined, so that in Figure 1.6,

$$e' = a - e \tag{1.9a}$$

Since $C = T$, $a = M/T$, giving

$$e' = \frac{M}{T} - e \tag{1.9b}$$

From the figure,

$$f^t = -\frac{C}{A_c} - \frac{Ce'c_t}{I_c} \tag{1.10a}$$

$$f_b = -\frac{C}{A_c} + \frac{Ce'c_b}{I_c} \tag{1.10b}$$

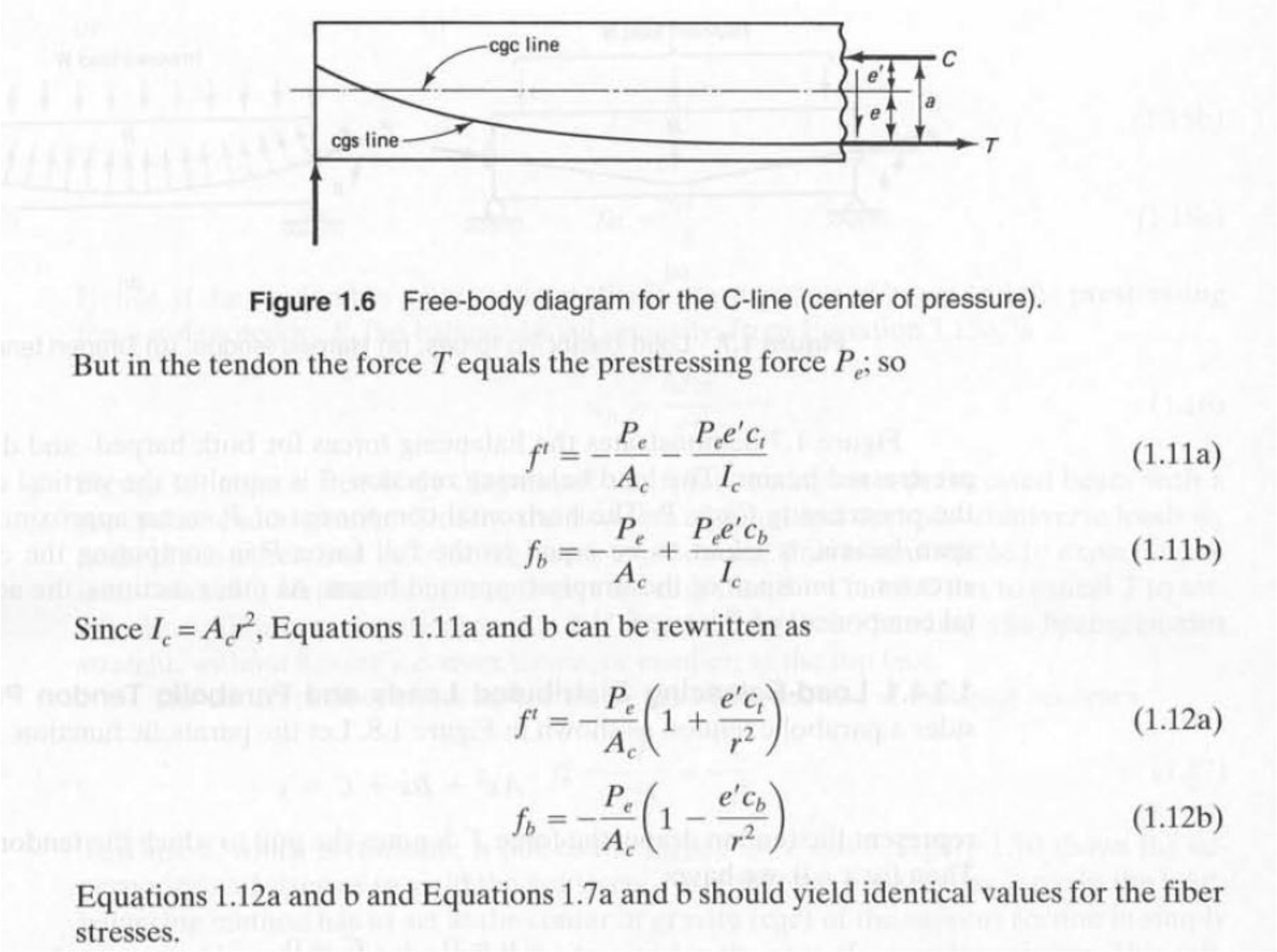


Figure 1.6 Free-body diagram for the C-line (center of pressure).

But in the tendon the force T equals the prestressing force P_e; so

$$f^t = -\frac{P_e}{A_c} - \frac{P_e e' c_t}{I_c} \qquad (1.11a)$$

$$f_b = -\frac{P_e}{A_c} + \frac{P_e e' c_b}{I_c} \qquad (1.11b)$$

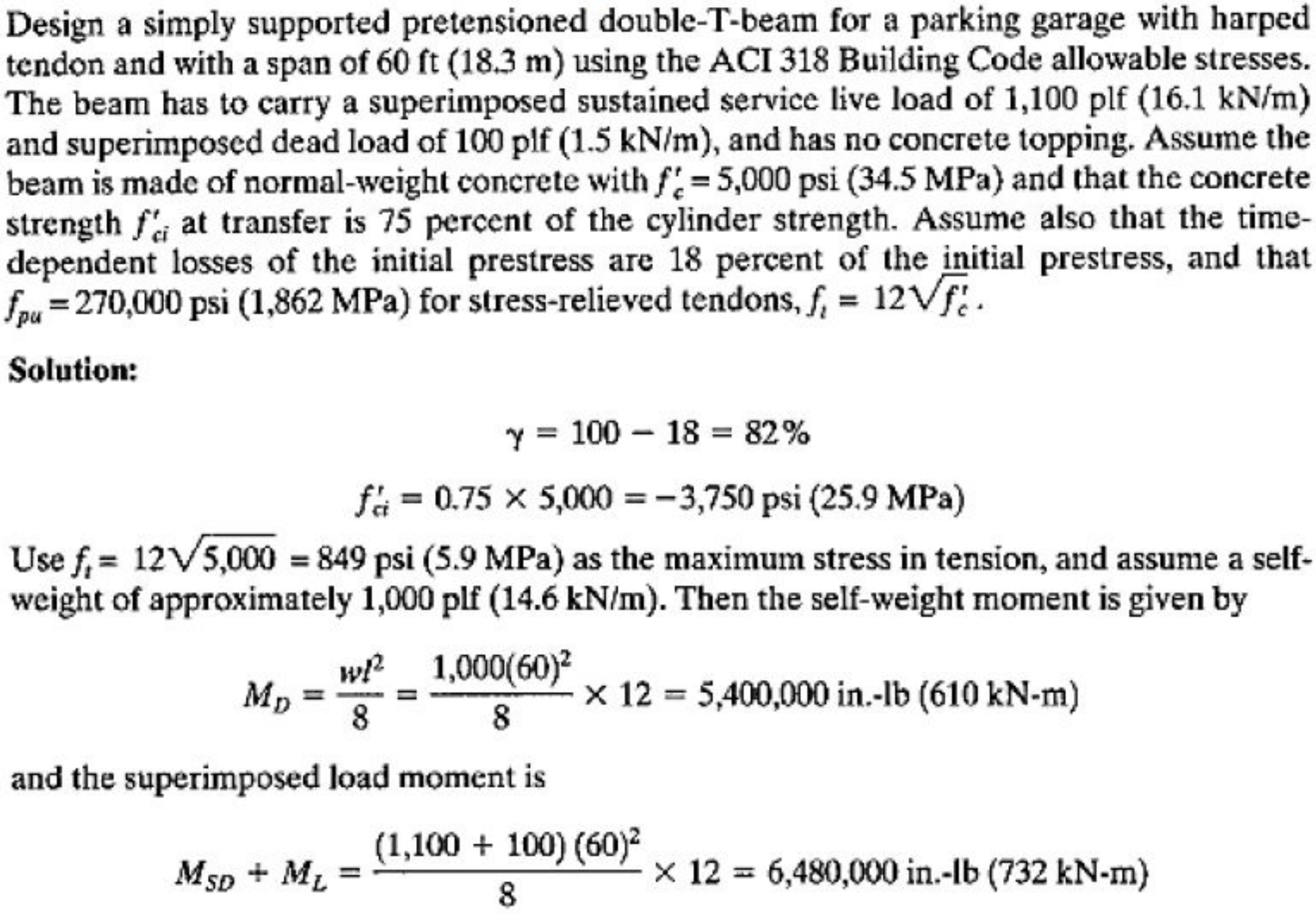
Since $I_c = A_c r^2$, Equations 1.11a and b can be rewritten as

$$f^t = -\frac{P_e}{A_c}\left(1 + \frac{e' c_t}{r^2}\right) \qquad (1.12a)$$

$$f_b = -\frac{P_e}{A_c}\left(1 - \frac{e' c_b}{r^2}\right) \qquad (1.12b)$$

Equations 1.12a and b and Equations 1.7a and b should yield identical values for the fiber stresses.

Service load design example (1)
Variable tendon eccentricity



Design a simply supported pretensioned double-T-beam for a parking garage with harped tendon and with a span of 60 ft (18.3 m) using the ACI 318 Building Code allowable stresses. The beam has to carry a superimposed sustained service live load of 1,100 plf (16.1 kN/m) and superimposed dead load of 100 plf (1.5 kN/m), and has no concrete topping. Assume the beam is made of normal-weight concrete with $f'_c = 5{,}000$ psi (34.5 MPa) and that the concrete strength f'_{ci} at transfer is 75 percent of the cylinder strength. Assume also that the time-dependent losses of the initial prestress are 18 percent of the initial prestress, and that $f_{pu} = 270{,}000$ psi (1,862 MPa) for stress-relieved tendons, $f_t = 12\sqrt{f'_c}$.

Solution:

$$\gamma = 100 - 18 = 82\%$$

$$f'_{ci} = 0.75 \times 5{,}000 = -3{,}750 \text{ psi (25.9 MPa)}$$

Use $f_t = 12\sqrt{5{,}000} = 849$ psi (5.9 MPa) as the maximum stress in tension, and assume a self-weight of approximately 1,000 plf (14.6 kN/m). Then the self-weight moment is given by

$$M_D = \frac{wl^2}{8} = \frac{1{,}000(60)^2}{8} \times 12 = 5{,}400{,}000 \text{ in.-lb (610 kN-m)}$$

and the superimposed load moment is

$$M_{SD} + M_L = \frac{(1{,}100 + 100)\,(60)^2}{8} \times 12 = 6{,}480{,}000 \text{ in.-lb (732 kN-m)}$$

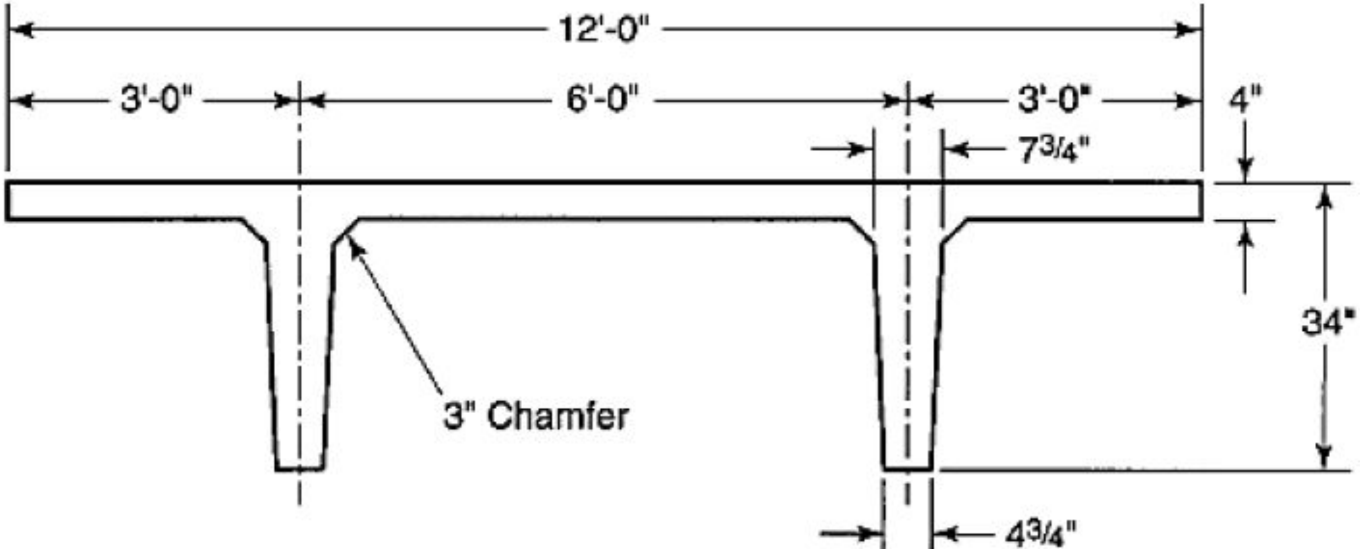
$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$\geq \frac{(1-0.82)5{,}400{,}000 + 6{,}480{,}000}{0.82 \times 184 - (-2{,}250)} = 3{,}104 \text{ in}^3 \text{ } (50{,}860 \text{ cm}^3)$$

$$S_b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

$$\geq \frac{(1-0.82)\ 5{,}400{,}000 + 6{,}480{,}000}{849 - 0.82(-2{,}250)} = 2{,}766 \text{ in.}^3 \text{ } (45{,}330 \text{ cm})^3$$

'rom the PCI design handbook, select a nontopped normal weight concrete double-T 12 DT 4 168-D1, since it has the bottom-section modulus value S_b closest to the required value.



The section properties of the concrete are as follows:

$$A_c = 978 \text{ in.}^2 \qquad c_t = 8.23 \text{ in.}$$

$$I_c = 86{,}072 \text{ in.}^4 \qquad c_b = 25.77 \text{ in.}$$

$$r^2 = \frac{I_c}{A_c} = 88.0 \text{ in.}^2 \qquad e_c = 22.02 \text{ in.}$$

$$S^t = 10{,}458 \text{ in.}^3 \qquad e_e = 12.77 \text{ in.}$$

$$S_b = 3{,}340 \text{ in.}^3 \qquad W_D = 1{,}019 \text{ plf}$$

$$\frac{V}{S} = 2.39 \text{ in.}$$

Design of Strands and Check of Stresses. The assumed self-weight is close to the actual

$$M_D = \frac{1{,}019}{1{,}000} \times 5{,}400{,}000 = 5{,}502{,}600 \text{ in.-lb}$$

$$f_{pi} = 0.70 \times 270{,}000 = 189{,}000 \text{ psi}$$

$$f_{pe} = 0.82 f_{pi} = 0.82 \times 189{,}000 = 154{,}980 \text{ psi}$$

(a) *Analysis of Stresses at Transfer.* From Equation 4.1a,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti} = 184 \text{ psi}$$

Then

$$184 = -\frac{P_i}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{5{,}502{,}600}{10{,}458}$$

$$P_i = (184 + 526.16)\,\frac{978}{1.06} = 655{,}223 \text{ lb.}$$

Required number of tendons $= \dfrac{655{,}223}{189{,}000 \times 0.153} = 22.66$ $\frac{1}{2}$-in. dia. tendons.

Try sixteen $\frac{1}{2}$-in. dia. strands for the standard section:

$$A_{ps} = 16 \times 0.153 = 2.448 \text{ in}^2 \text{ } (15.3 \text{ cm}^2)$$

$$P_i = 2.448 \times 189{,}000 = 462{,}672 \text{ lb } (2{,}058 \text{ kN})$$

$$P_e = 2.448 \times 154{,}980 = 379{,}391 \text{ lb } (1{,}688 \text{ kN})$$

(b) *Analysis of Stresses at Service Load at Midspan*

$$P_e = 379{,}391 \text{ lb}$$

$$M_{SD} = \frac{100(60)^2 12}{8} = 540{,}000 \text{ in.-lb (61 kN-m)}$$

$$M_L = \frac{1{,}100(60)^2 12}{8} = 5{,}940{,}000 \text{ in.-lb (788 kN-m)}$$

Total moment $M_T = M_D + M_{SD} + M_L = 5{,}502{,}600 + 6{,}480{,}000$
$= 11{,}982{,}600$ in.-lb (1,354 kN-m)

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{379{,}391}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{11{,}982{,}600}{10{,}458}$$

$$= +411 - 1146 = -735 \text{ psi} < f_c = -2{,}250 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{379{,}391}{978}\left(1 + \frac{22.02 \times 25.77}{88.0}\right) + \frac{11{,}982{,}600}{3{,}340}$$

$$= -2{,}889 + 3{,}587 = +698 \text{ psi (T)} < f_t = +849 \text{ psi, O.K.}$$

(c) *Analysis of Stresses at Support Section*

$$e_e = 12.77 \text{ in. } (324 \text{ mm})$$

$$f_{ti} = 6\sqrt{f'_{ci}} = 6\sqrt{3{,}750} \cong 367 \text{ psi}$$

$$f_t = 12\sqrt{f'_c} = 12\sqrt{5{,}000} = 849 \text{ psi}$$

(i) *At Transfer*

$$f^t = -\frac{462{,}672}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +92 \text{ psi } (T)$$

$$f_b = -\frac{462{,}672}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -2{,}240 \text{ psi } (C)$$

$$< f_{ci} = -2{,}250 \text{ psi, O.K.}$$

If $f_b > f_{ci}$, the support eccentricity has to be changed.

(ii) *At Service Load*

$$f^t = -\frac{379{,}391}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +75 \text{ psi } (T) \quad < f_t = 849 \text{ psi, O.K.}$$

$$f_b = -\frac{379{,}391}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -1{,}840 \text{ psi } (C)$$

$$< f_c = -2{,}250 \text{ psi, O.K.}$$

Adopt the section for service-load conditions using sixteen $\frac{1}{2}$-in. (1.7 mm) strands with midspan eccentricity $e_c = 22.02$ in. (560 mm) and end eccentricity $e_e = 12.77$ in. (324 mm).

Service load design example (2)

Variable tendon eccentricity with no height limitation

Design an I-section for a beam having a 65-ft (19.8 m) span to satisfy the following section modulus values: Use the same allowable stresses and superimposed loads as in Example 4.1.

$$\text{Required } S^t = 3{,}570 \text{ in}^3 \text{ } (58{,}535 \text{ cm}^3)$$

$$\text{Required } S_b = 3{,}780 \text{ in}^3 \text{ } (61{,}940 \text{ cm}^3)$$

Solution

Since the section moduli at the top and bottom fibers are almost equal, a symmetrical section is adequate.

$I_c = 70{,}688$ in.^{4}

$r^2 = 187.5$ in.^{2}

$A_c = 377$ in.^{2}

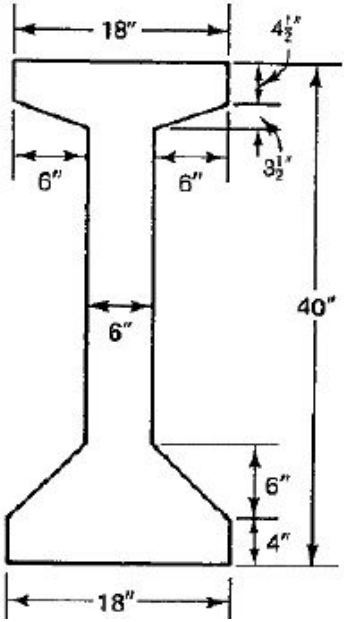
$c_t = 21.16$ in.

$S^t = 3340$ in.^{3}

$c_b = 18.84$ in.

$S_b = 3750$ in.^{3}

$w_D = 393$ plf



Analysis of Stresses at Transfer.

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= +184 - \frac{21.16}{40}(+184 + 2{,}250) \cong -1{,}104 \text{ psi } (C) \text{ (7.6 MPa)}$$

$$P_i = A_c \bar{f}_{ci} = 377 \times 1{,}104 = 416{,}208 \text{ lb (1,851 kN)}$$

$$M_D = \frac{393(65)^2}{8} \times 12 = 2{,}490{,}638 \text{ in.-lb (281 kN-m)}$$

the eccentricity required at the section of maximum moment at midspan is

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

$$= (184 + 1{,}104)\frac{3{,}340}{416{,}208} + \frac{2{,}490{,}638}{416{,}208}$$

$$= 10.34 + 5.98 = 16.32 \text{ in. (415 mm)}$$

Since c_b = 18.84 in., and assuming a cover of 3.75 in., try e_c = 18.84 – 3.75 ≅ 15.0 in. (381 mm).

$$\text{Required area of strands } A_p = \frac{P_i}{f_{pi}} = \frac{416{,}208}{189{,}000} = 2.2 \text{ in}^2 \text{ (14.2 cm}^2\text{)}$$

$$\text{Number of strands} = \frac{2.2}{0.153} = 14.38$$

Try thirteen $\frac{1}{2}$-in. strands, $A_p = 1.99$ in.^{2} (12.8 cm^2), and an actual $P_i = 189{,}000 \times 1.99 =$ 376,110 lb (1,673 kN), and check the concrete extreme fiber stresses.

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{376{,}110}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{2{,}490{,}638}{3{,}340}$$

$$= +691.2 - 745.7 = -55 \text{ psi } (C), \text{ no tension at transfer, O.K.}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b}$$

$$= -\frac{376{,}110}{377}\left(1 + \frac{15 \times 18.84}{187.5}\right) + \frac{2{,}490{,}638}{3{,}750}$$

$$= -2{,}501.3 + 664.2 = -1{,}837 \text{ psi } (C) < f_{ci} = 2{,}250 \text{ psi, O.K.}$$

Analysis of Stresses at Service Load.

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$P_e = 13 \times 0.153 \times 154{,}980 = 308{,}255 \text{ lb } (1{,}371 \text{ kN})$$

$$M_{SD} + M_L = \frac{(100 + 1100)(65)^2}{8} \times 12 = 7{,}605{,}000 \text{ m} - \text{lb}$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 2{,}490{,}638 + 7{,}605{,}000$$

$$= 10{,}095{,}638 \text{ in.-lb } (1{,}141 \text{ kN-m})$$

$$f^t = -\frac{308{,}225}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{10{,}095{,}638}{3{,}340}$$

$$= +566.5 - 3{,}022.6 = -2{,}456 \text{ psi } (C) \; > f_c = -2{,}250 \text{ psi}$$

Hence, either enlarge the depth of the section or use higher strength concrete. Using $f'_c = 6{,}000$ psi,

$$f_c = 0.45 \times 6{,}000 = -2{,}700 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} = -\frac{308{,}255}{377}\left(1 + \frac{15.0 \times 18.84}{187.5}\right) + \frac{10{,}095{,}638}{3{,}750}$$

$$= -2{,}050 + 2{,}692.2 = 642 \text{ psi } (T), \text{ O.K.}$$

Check Support Section Stresses

$$\text{Allowable} \quad f'_{ci} = 0.75 \times 6{,}000 = 4{,}500 \text{ psi}$$

$$f_{ci} = 0.60 \times 4{,}500 = 2{,}700 \text{ psi}$$

$$f_{ti} = 3\sqrt{f'_{ci}} = 201 \text{ psi for midspan}$$

$$f_{ti} = 6\sqrt{f'_{ci}} = 402 \text{ psi for support}$$

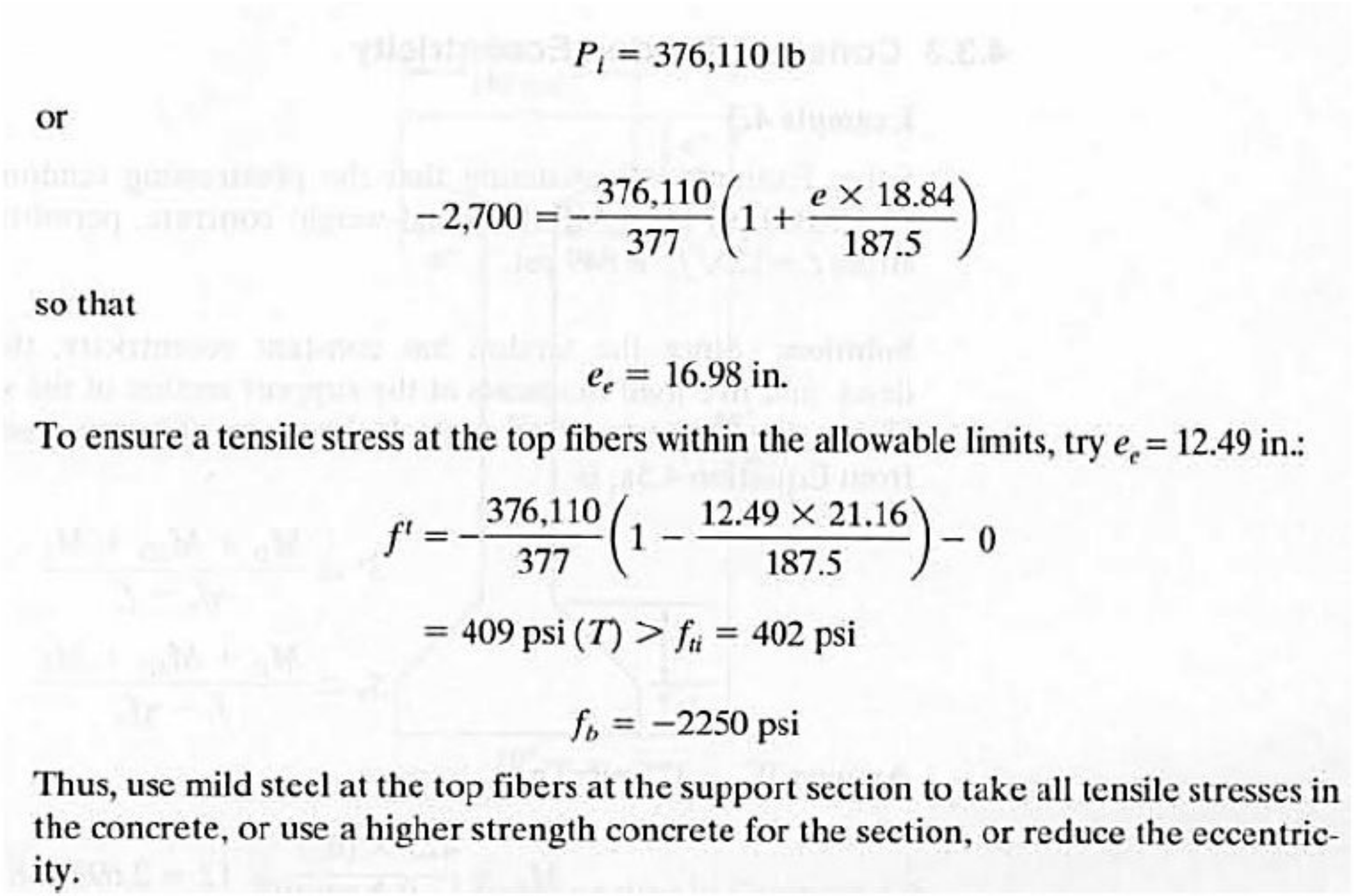
$$f_c = 0.45 f'_c = 2{,}700 \text{ psi}$$

$$f_{t1} = 6\sqrt{f'_c} = 465 \text{ psi}$$

$$f_{t2} = 12\sqrt{f'_c} = 930 \text{ psi}$$

(a) *At Transfer*. Support section compressive fiber stress.

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + 0$$



$$P_i = 376{,}110 \text{ lb}$$

or

$$-2{,}700 = -\frac{376{,}110}{377}\left(1 + \frac{e \times 18.84}{187.5}\right)$$

so that

$$e_e = 16.98 \text{ in.}$$

To ensure a tensile stress at the top fibers within the allowable limits, try $e_e = 12.49$ in.:

$$f^t = -\frac{376{,}110}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0$$

$$= 409 \text{ psi } (T) > f_{ti} = 402 \text{ psi}$$

$$f_b = -2250 \text{ psi}$$

Thus, use mild steel at the top fibers at the support section to take all tensile stresses in the concrete, or use a higher strength concrete for the section, or reduce the eccentricity.

(b) *At Service Load*

$$f^t = -\frac{308{,}255}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0 = 335 \text{ psi } (T) \ < 930 \text{ psi, O.K.}$$

$$f_b = -\frac{308{,}255}{377}\left(1 + \frac{12.49 \times 18.84}{187.5}\right) + 0 = -1{,}844 \text{ psi } (C) < -2{,}700 \text{ psi, O.K.}$$

Hence, adopt the 40-in. (102-cm)-deep I-section prestressed beam of f'_c equal to 6,000 psi (41.4 MPa) normal-weight concrete with thirteen $\frac{1}{2}$-in. tendons having midspan eccentricity $e_c = 15.0$ in. (381 mm) and end section eccentricity $e_e = 12.5$ in. (318 m).

An alternative to this solution is to continue using $f'_c = 5{,}000$ psi, but change the number of strands and eccentricities.

Service load design example (3)
Constant tendon eccentricity

Solve Example 4.2 assuming that the prestressing tendon has constant eccentricity. Use $f'_c = 5{,}000$ psi (34.5 MPa) normal-weight concrete, permitting a maximum concrete tensile stress $f_t = 12\sqrt{f'_c} = 849$ psi.

Solution:

The required section modulus

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

Assume $W_D = 425$ plf. Then

$$M_D = \frac{425 \times (65)^2}{8} \times 12 = 2{,}693{,}438 \text{ in.-lb } (304 \text{ kN-m})$$

$$M_{SD} + M_L = 7{,}605{,}000 \text{ in.-lb } (859 \text{ kN-m})$$

Thus, the total moment $M_T = 10{,}298{,}438$ in.-lb (1,164 kN-m), and we also have

Allowable $f_{ci} = -2{,}250$ psi

$f'_{ci} = -3{,}750$ psi

$f_{ti} = 6\sqrt{f'_{ci}}$ for support section $= 367$ psi

$f_c = -2{,}250$ psi (15.5 MPa)

$f_t = +849$ psi

$\gamma = 0.82$

$$\text{Required } S^t = \frac{10{,}298{,}438}{0.82 \times 184 + 2{,}250} = 4{,}289 \text{ in}^3 \; (72{,}210 \text{ cm}^3)$$

$$\text{Required } S_b = \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} = \frac{10{,}298{,}438}{849 + 0.82 \times 2{,}250}$$

$$= 3{,}823 \text{ in}^3 \; (62{,}713 \text{ cm}^3)$$

First Trial.

$I_c = 92{,}700 \text{ in}^4$

$r^2 = 228.9 \text{ in}^2$

$A_c = 405 \text{ in}^2$

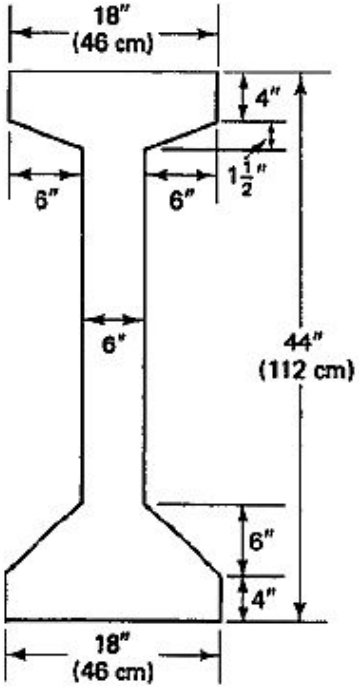
$c_t = 23.03$ in.

$S^t = 4{,}030 \text{ in}^3$

$c_b = 20.97$ in

$S_b = 4{,}420 \text{ in}^3$

$W_D = 422$ plf



the required eccentricity at the critical section at the support is

$$e_e = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i}$$

where

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= 367 - \frac{23.03}{44}(367 + 2{,}250) = -1{,}002 \text{ psi } (6.9 \text{ MPa})$$

and

$$P_i = A_c\bar{f}_{ci} = 405 \times 1{,}002 = 405{,}810 \text{ lb } (1{,}805 \text{ kN})$$

Hence,

$$e = (367 + 1{,}002)\frac{4{,}030}{405{,}810} = 13.60 \text{ in. } (346 \text{ mm})$$

The required prestressed steel area is

$$A_p = \frac{P_i}{f_{pi}} = \frac{405{,}810}{189{,}000} = 2.15 \text{ in}^2 \ (14.4 \text{ cm}^2)$$

So we try $\frac{1}{2}$in. strands tendon. The required number of strands is 2.15/0.153 = 14.05. Accordingly, use fourteen $\frac{1}{2}$in. (12.7 mm) tendons. As a result,

$$P_i = 14 \times 0.153 \times 189{,}000 = 404{,}838 \text{ lb } (1{,}801 \text{ kN})$$

(a) *Analysis of Stresses at Transfer at End Section.*

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} = -\frac{404{,}838}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0$$

$$= +368.2 \text{ psi } (T) \cong f_{ti} = 367, \text{ O.K.}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} = -\frac{404{,}838}{405}\left(1 + \frac{13.6 \times 20.97}{228.9}\right) + 0$$

$$= -2{,}245 \text{ psi } (C) \cong f_{ci} = -2{,}250, \text{ O.K.}$$

(b) *Analysis of Final Service-Load Stresses at Support*

$$P_e = 14 \times 0.153 \times 154{,}980 = 331{,}967 \text{ lb } (1{,}477 \text{ kN})$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 0$$

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{331{,}967}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0 = 302 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{331{,}967}{405}\left(1 + \frac{13.60 \times 20.97}{228.9}\right) + 0$$

$$= -1{,}841 \text{ psi } (12.2 \text{ MPa}) \ (C) < f_c = -2{,}250 \text{ psi, O.K.}$$

(c) *Analysis of Final Service-Load Stresses at Midspan.* From before, the total moment $M_T = M_D + M_{SD} + M_L = 10{,}298{,}438$ in.-lb. Revised $w_D = 422$ plf = assumed $w_D = 425$ plf; hence, $M_T \simeq 10{,}298{,}438$ in.-lb is sufficiently accurate. So the extreme concrete fiber stress due to M_T is

$$f_1^t = \frac{M_T}{S^t} = -\frac{10{,}298{,}438}{4{,}030} = -2{,}555 \text{ psi } (C)\ (17.6 \text{ MPa})$$

$$f_{1b} = \frac{M_T}{S_b} = \frac{10{,}298{,}438}{4{,}420} = +2{,}330 \text{ psi } (T)\ (16.1 \text{ MPa})$$

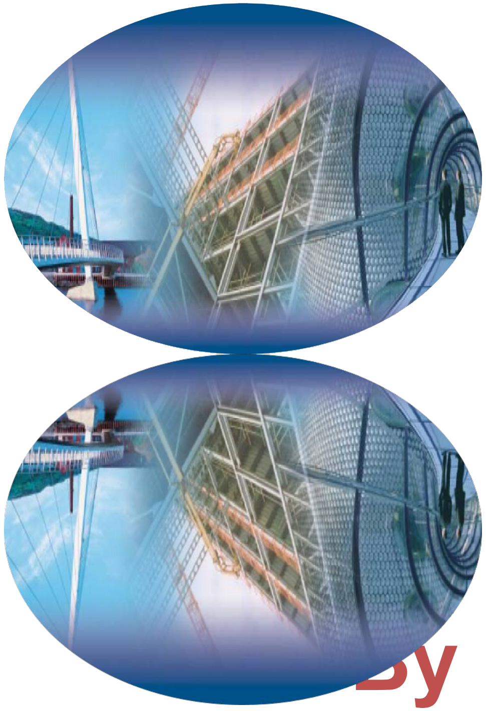
Hence, the final midspan fiber stresses are

$$f^t = +302 - 2{,}555 = -2{,}253 \text{ psi } (C) \cong f_c = -2{,}250 \text{ psi, accept}$$

$$f_b = -1{,}841 + 2{,}330 = +489 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

Consequently, accept the trial section with a constant eccentricity $e = 13.60$ in. (345 mm) for the fourteen $\frac{1}{2}''$ (12.7 mm dia.) tendons.

Thanks for Listening



UNIVERSITY OF ANBAR

Lecturer 4

By : Mohammed Mohammed

Shear Design

Analysis for Shear

Introduction:

- The analysis of reinforced concrete and prestressed concrete members for shear is more difficult compared to the analyses for axial load or flexure.

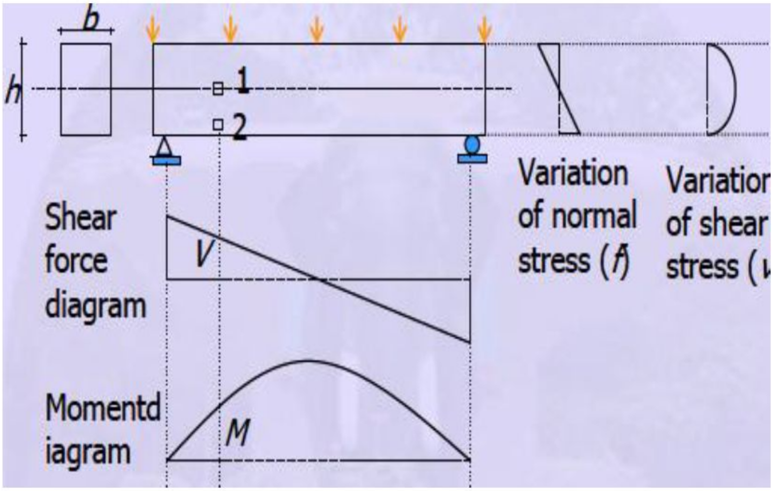
Concrete has more shear resistance, where as shear resistance of R.C.C is less.

The analysis for axial load and flexure are based on the following principles of mechanics.

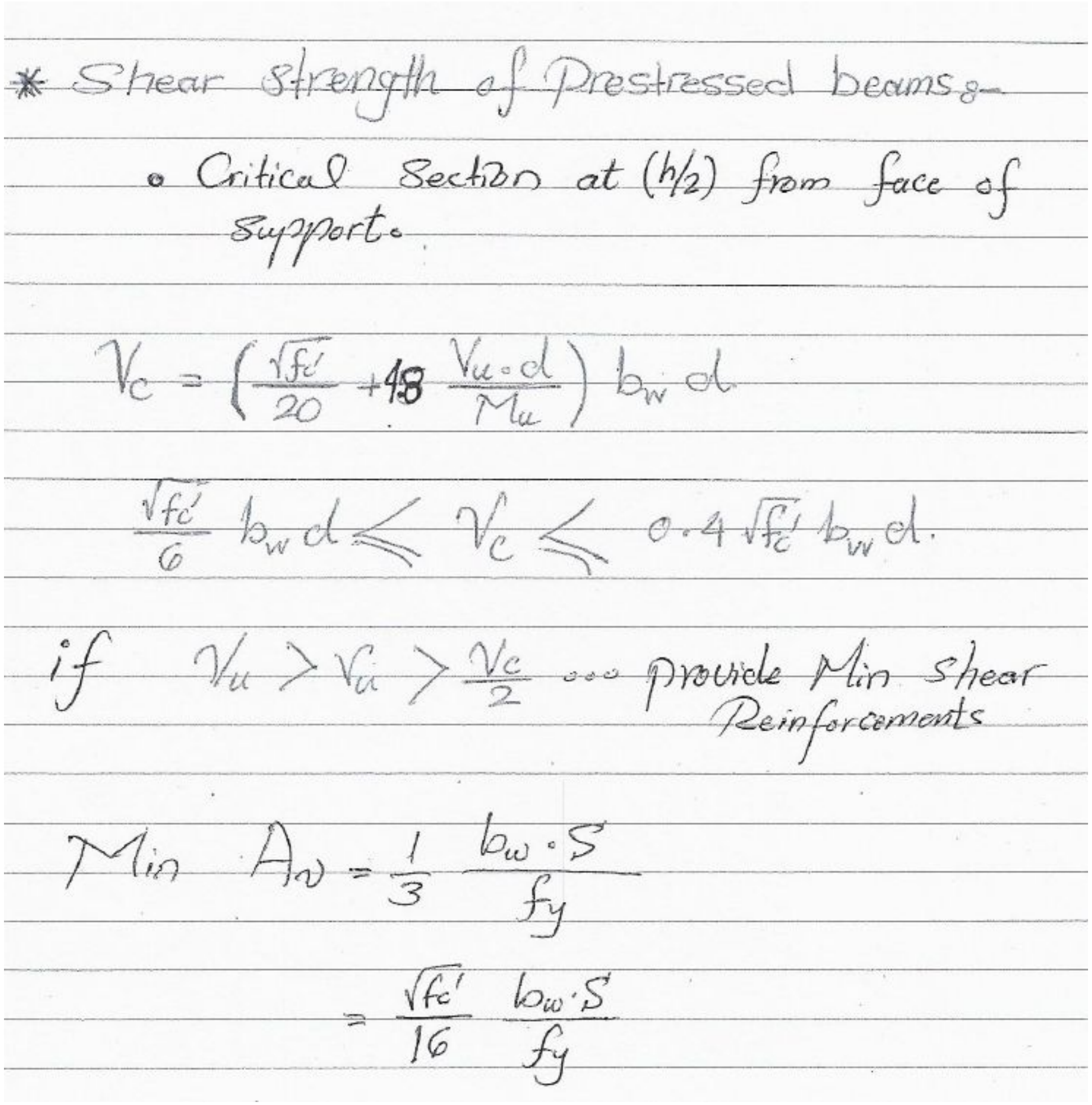
- 1) **Equilibrium** of internal and external forces
- 2) **Compatibility** of strains in concrete and steel
- 3) **Constitutive relationships** of materials.

Stresses in an Uncracked Beam

Stresses in an Uncracked Beam



Variations of forces and stresses in a simply supported beam



* Shear strength of Prestressed beams:-

- Critical Section at $(h/2)$ from face of support.

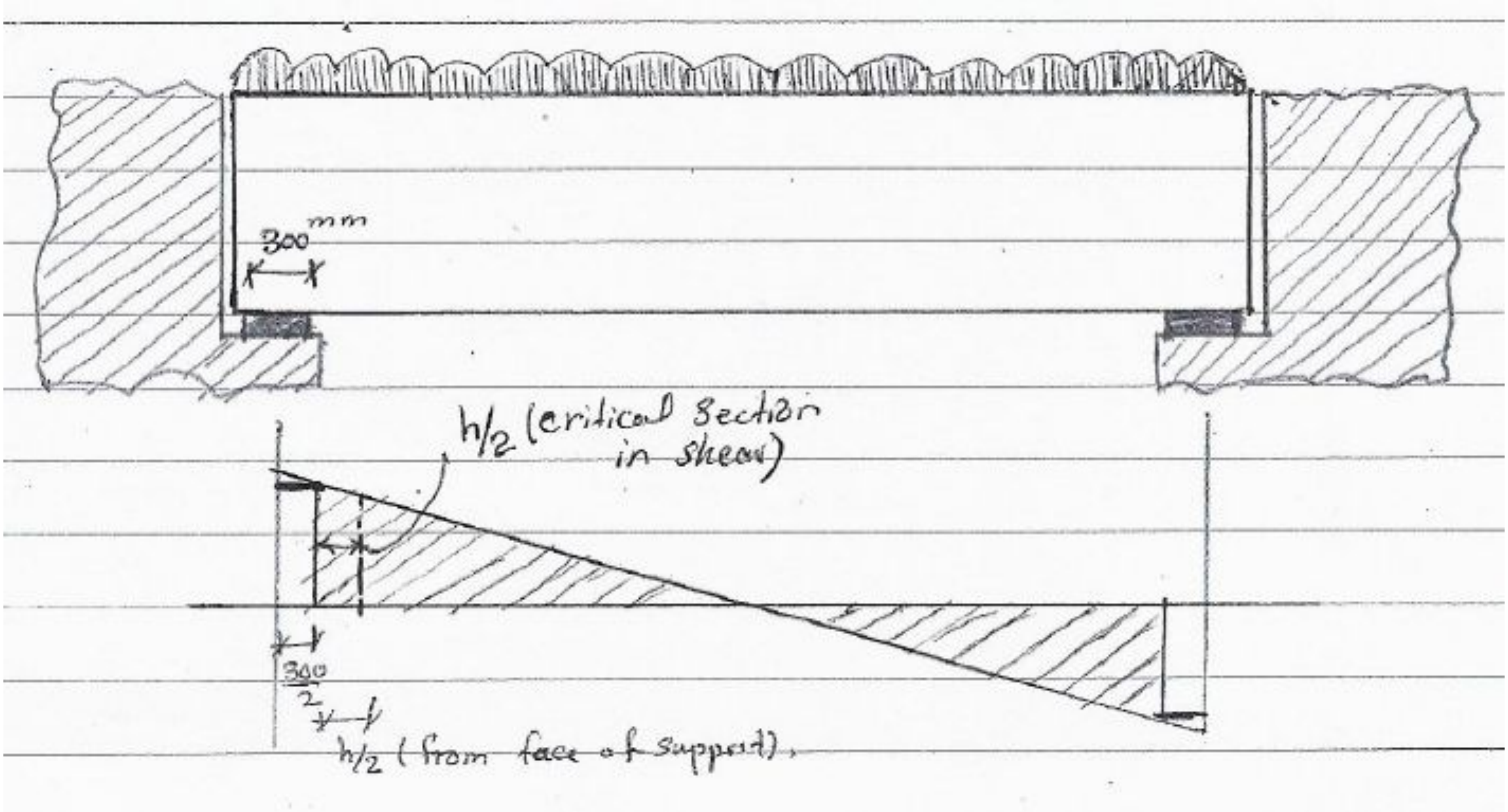
$$V_c = \left(\frac{\sqrt{f_c'}}{20} + 4.8\,\frac{V_u \cdot d}{M_u}\right) b_w\, d$$

$$\frac{\sqrt{f_c'}}{6}\, b_w\, d \leqslant V_c \leqslant 0.4\sqrt{f_c'}\, b_w\, d.$$

if $V_u > V_u > \frac{V_c}{2}$ ... provide Min shear Reinforcements

$$\text{Min } A_v = \frac{1}{3}\,\frac{b_w \cdot S}{f_y}$$

$$= \frac{\sqrt{f_c'}}{16}\,\frac{b_w \cdot S}{f_y}$$


300 mm
h/2 (critical section in shear)
300/2
h/2 (from face of support).

$S_{max} \leqslant 0.75h$ or 600 mm.

$$V_c = \left[\lambda \frac{\sqrt{f_c'}}{20} + \overbrace{4.8 \frac{V_u d}{M_u}}^{\text{Shouldbe} \leqslant 1.0} \right] b_w \cdot d_p < 0.4\sqrt{f_c'}\, b_w \cdot d_p$$

Where:-

$\lambda = 1.0$ ------ Normal Concrete.

$\lambda = 0.85$ ---- Sand light weight. Concr.

$\lambda = 0.75$ ---- All light weight Conc.

$(min)\ \frac{\sqrt{f_c'}}{6} b_w \cdot d_p$ & $\frac{1}{3} b_w d_p$ — whichever greater.

$(max)\ 0.4\sqrt{f_c'}\, b_w \cdot d_p$

Example

Design the Shear reinforcement for the Prestressed beam as shown in figure below.

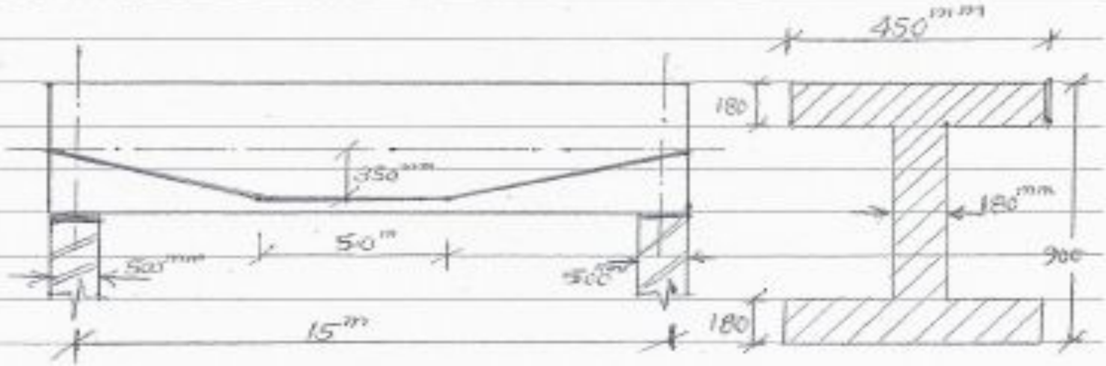
$f_c' = 35$ MPa

$f_y = 400$ MPa (stirrups)

$w_u = 55.86$ kN/m

$f_{pu} = 1860$ MPa

$A_{ps} = 1200$ mm^2



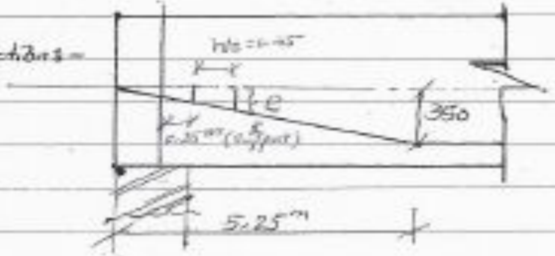
Solutions-

$$(V_u)_{h/2} = (14.5 - 0.45) * 55.86 = 379.85^{kN}$$

$$V_n = V_u/\phi = \frac{379.85}{0.75} = 506.5^{kN}$$

$$(M_u)_{h/2} = \frac{55.86*15}{2} * 0.7 - \left[\frac{55.86*0.7^2}{2}\right] = 279.6^{kN.m}$$

To find value of (e) at (h/2) sections-



$$\frac{350^{mm}}{5.25^{m}} = \frac{e}{(0.45+0.25+0.25)}$$

$$\Rightarrow e = 63.3^{mm}$$

$\therefore d_p = h/_2 + e = 450 + 63.3 = 513.3^{mm}$

$$\Rightarrow \frac{V_u \cdot d_p}{M_u} = \frac{379.85 * 513.3 * 10^{-3}}{279.6} = 0.697 < 1$$
(OK.)

$$V_c = \left[\lambda \sqrt{f_c'}/20 + \frac{V_u \cdot d_p}{M_u} \right] b_w \cdot d < 0.4\sqrt{f_c'}\, b_w \cdot d$$

$\lambda = 1.0$ —— Normal Concrete.

$$\Rightarrow V_c = \left[\frac{\sqrt{35}}{20} + 0.697 \right] 180 * 513 * 10^{-3} = 399.1^{kN}$$

or $V_c = 0.4\sqrt{35} \times 180 * 513 \times 10^{-3} = 218.5^{kN}$ —— Governe

$\therefore V_c = 218.5^{kN}$

$\phi V_c = 163.875^{kN}$

$\because V_u = 379.85^{kN} > \phi V_c$

$\therefore$ Need Shear Reinforcements.

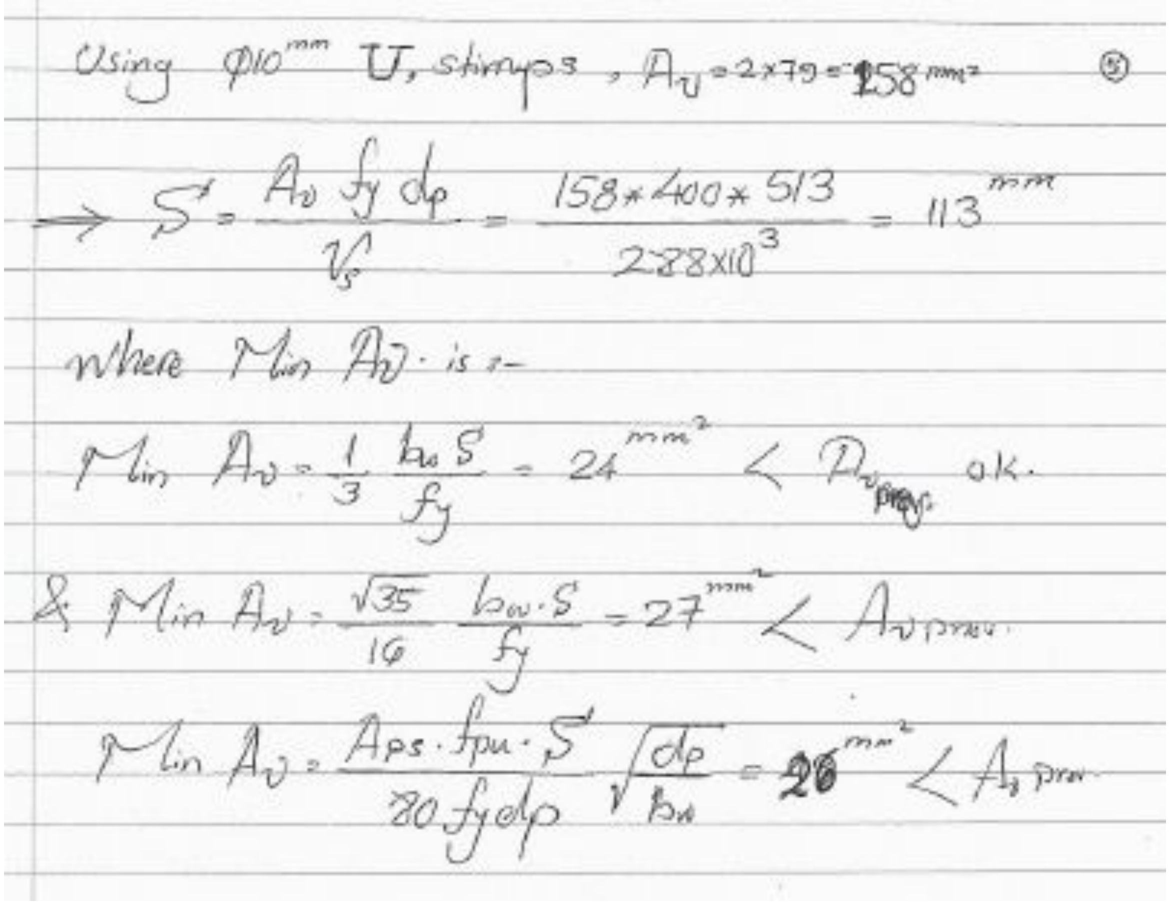
$\phi V_s = V_u - \phi V_c$

$\Rightarrow \phi V_s = 379.85 - 163.875 = 215.98^{kN}$

$\Rightarrow V_s = 288^{kN}$

$\text{Min } V_s = \frac{1}{3} b_w d_p / = \frac{1}{3} \times 180 \times 513 \times 10^{-3} = 30.78^{kN} < V_{s\,req}$ ok.

$= \sqrt{f_c'}/16\, b_w\, d_p = \frac{\sqrt{35}}{16} \times 180 \times 513 \times 10^{-3} = 34.13^{kN} < V_{s\,req}$



Using $\phi 10^{mm}$ U, stirrups , $A_v = 2\times79 = 158^{mm^2}$ ⑤

$$\rightarrow S = \frac{A_v\, f_y\, d_p}{V_s} = \frac{158*400*513}{228\times10^3} = 113^{mm}$$

where Min A_v · is :-

$$Min\ A_v = \frac{1}{3}\,\frac{b_w S}{f_y} = 24^{mm^2} < A_{v_{prov.}} \text{ o.k.}$$

$$\&\ Min\ A_v = \frac{\sqrt{35}}{16}\,\frac{b_w \cdot S}{f_y} = 27^{mm^2} < A_{v\,prov.}$$

$$Min\ A_v = \frac{A_{ps}\cdot f_{pu}\cdot S}{80 f_y d_p}\sqrt{\frac{d_p}{b_w}} = 26^{mm^2} < A_{v\,prov}$$

* Shear in Prestressed Sections:- ACI (11.3)

The Code provides two methods for estimating the Shear strength that the Concrete of Prestress Section Can resist.

The approximate Method, which can be used only when effective Prestress force is equal to at least 40% of the tensile strength of flexural reinforcement (f_{pu})

$$f_{pe} \geqslant 0.4 f_{pu} \quad ---①$$

$$\boxed{V_c = \left(0.05\lambda\sqrt{f_c'} + \frac{4.8 V_u d_p}{M_u}\right) b_w d_p}$$

V_c need ~~to~~ to limit:-

- $\underset{0.17\lambda\sqrt{f_c'}\,b_w d_p}{\frac{1}{6}\lambda\sqrt{f_c'}\,b_w d_p} \leqslant V_c \leqslant \underset{0.42\lambda\sqrt{f_c'}\,b_w d_p}{\frac{5}{12}\lambda\sqrt{f_c'}\,b_w d_p}$ 5
- $\frac{V_u d_p}{M_u} \leqslant 1.0$
- V_u is Calculated at the Same Section for which (M_u)s Code

& Detailed analysis where V_c is lesser of V_{ci} & V_{cw}

$$V_{ci} = 0.05\lambda\sqrt{f_c'}\, b_w d_p + V_d + \frac{V_i M_{cr}}{M_{max}} \leqslant 0.14\lambda\sqrt{f_c'}\, b_w d$$

$$\Rightarrow \underline{\underline{d_p \leqslant 0.8h}}$$

$$M_{cr} = \left(\frac{I_c}{y_t}\right)\left(0.5\lambda\sqrt{f_c'} + f_{pe} - f_d\right)$$

d: stress due to unfactored dead load

e: Compressive stress in Concrete (after all losses)

$\underline{\underline{M_{max}\ \&\ V_i}}$ Shall be Computed from the load Combination Causing maximum factored moment to ccurs at the Section.

$$V_{ci} \leqslant 0.14\lambda\sqrt{f_c'}\, b_w d_p$$

&

$$V_{cw} = \left(0.29\lambda\sqrt{f_c'} + 0.3 f_{pc}\right) b_w d_p + V_p$$

$$\Rightarrow \underline{d_p \leqslant 0.8h.}$$

Note

- The determination of nominal strength value $V_n = \frac{V_u}{\phi}$ at distance $\left(\frac{h}{2}\right)$ from the face of Support, $\phi = 0.75$.

3. If $V_u \leq \frac{\phi V_c}{2}$ --- no web steel is needed.

If $\frac{\phi V_c}{2} < V_u \leq \phi V_c$ --- provide minimum reinforcement

If $V_u > \phi V_c$ and

$$\phi V_s = V_u - \phi V_c \leq \frac{2}{3}\sqrt{f_c'}\, b_w dp \text{ : then,}$$

Design web steel reinforcement

Notes-

If, $\phi V_s = V_u - \phi V_c > \frac{2}{3}\sqrt{f_c'}\, b_w dp$

OR, $V_u > \phi\left(V_c + \frac{2}{3}\sqrt{f_c'}\, b_w d\right)$

--- Enlarge the section.

4. Calculate the required Min. web reinforcement,

The spacing is $S \leq 0.75h$ or 600^{mm}
→ which ever is smaller.

Min $A_{v,min} = 0.062\sqrt{f_c'}\,\frac{b_w S}{f_y}$ ACI. 11.13

OR

$A_{v\,min} = \frac{0.35\, b_w S}{f_y}$ whichever is Larger

$$\left.\begin{array}{l} A_{v,min} = 0.062\sqrt{f_c'}\,\frac{b_w S}{f_y} \\ A_{v\,min} = \frac{0.35\, b_w S}{f_y} \end{array}\right\} \text{whichever is Larger}$$

If $f_{pe} \geq 0.4 f_{pe}$

تضاف اليها هذه المعادلة ع (معادلة 11.13)

$$\left.\begin{array}{l} A_{v\,min} = \frac{A_{ps} \cdot f_{pu} \cdot S}{80 \cdot f_y \cdot d_p}\sqrt{\frac{d_p}{b_w}} \\ A_{v\,min} = 0.062\sqrt{f_c'}\,\frac{b_w S}{f_y} \\ A_{v\,min} = \frac{0.35\, b_w S}{f_y} \end{array}\right\} \text{whichever is larger.}$$

5. Calculate the required web reinforcement size and spacing:-

ⓐ If $\phi V_S = V_u - \phi V_c \leq 0.33 \lambda \sqrt{f_c'} \, b_w d_p$

$$\Rightarrow \quad S_1 = \frac{A_v f_y d_p}{(\phi V_u) - V_c} = \frac{A_v \cdot \phi f_y d_p}{V_u - \phi V_c} \leq 0.75h \leq 600^{mm}$$

Also $S > S_{min}$

If $\quad 0.33\sqrt{f_c'} b_w d < \phi V_S = V_u - \phi V_c \leq 0.67 \sqrt{f_c'} \, b_w d_p$

$$\Rightarrow \quad S = \frac{S_1}{2} = \frac{A_v f_y d_p}{2[(\phi V_u) - V_c]} = \frac{0.5 A_v \cdot \phi f_y \cdot d_p}{V_u - \phi V_c} \leq 0.35h \leq 300$$

6. Draw Shear Reinforcement band.

Example:- ①

Design the bonded beam to be Saffe against Shear failure. (Check at $\left(\frac{d_p}{2}\right)$ Pbaune) from support.

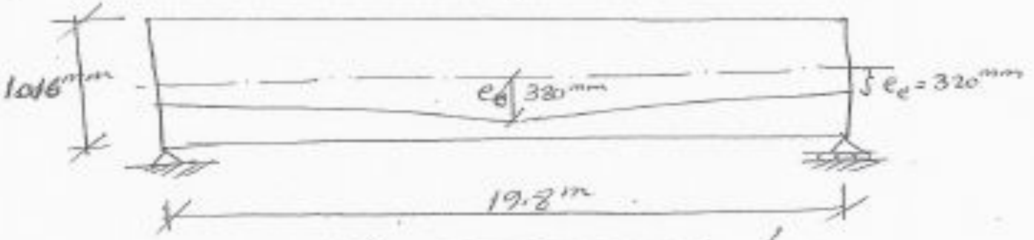
Given. $f_{pu} = 1862$, $f_y = 414$, $f_{pe} = 1069$, $f'_c = 34.5$ MPa Normal Conc

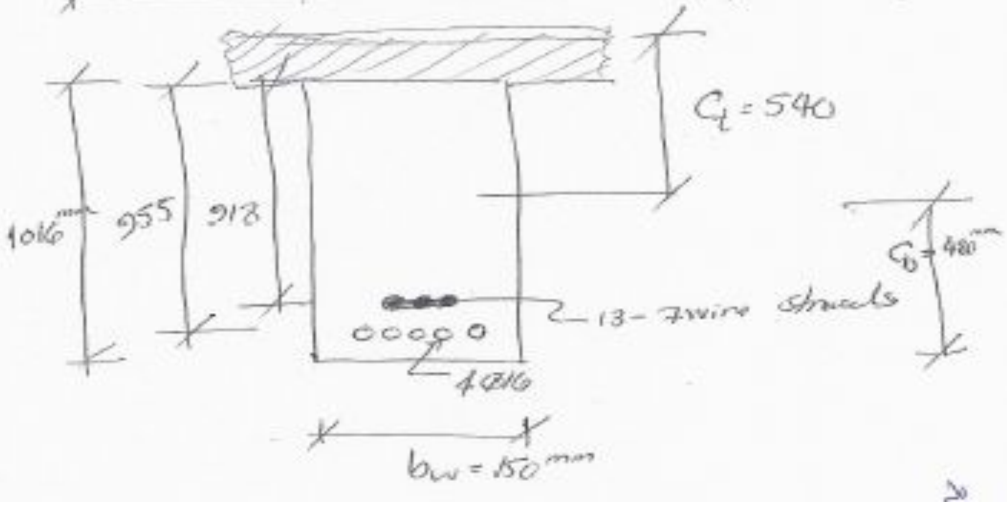
$A_{ps} = 1280\ mm^2$, $A_s = 1140\ mm^2$, Span - 19.8^m , $W_L = 16.1\ kN/m$

$W_{SD} = 1.46$, $W_D = 5.7\ kN/m$, $h = 1016$, $d_p = 918\ mm$

$I_c = 18.09 \times 10^6\ mm^4$, $A_c = 243200\ mm^2$, $r^2 = 121000\ mm^2$

$P_e = 1371\ kN$





$$w_u = 1.2\,DL + 1.6\,LL$$

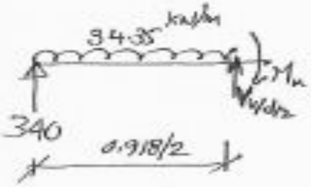
$$= 1.2\,(1.46 + 5.7) + (1.6 * 16.1) = 34.35^{kN/m}$$

Factored shear force at face of Support:-

$$V_u = \frac{w_u \times L}{2} = \frac{34.35 * 19.8}{2} = 340^{kN}$$

Plane at $\left(\frac{d_p}{2}\right)$ from face of Support:-

$$V_{u(@d_p/2)} = 340 * \left(\frac{\frac{19.8}{2} - 0.918}{(19.8/2)}\right) = \begin{matrix} 324.23\ kN \\ \text{~~[illegible]~~} \end{matrix}$$



$$f_{pe} = \frac{1371 \times 10^3}{1280} = 1070^{MPa}.$$

$$0.4 f_{pu} = 0.4 * 1862 = 744.8^{MP}$$

$$\because\ f_{pe} > 0.4 f_{pu} \Longrightarrow \text{Eq. 11-9 in (ACI-318, 11-3)}$$

$$d_p = 918 > 0.8h = 813 \ldots ok.$$

$$\phi V_c = \phi\left(0.05\sqrt{f_c'} + \frac{4.8 V_u\, d_p}{M_u}\right) b_w\, d_p \geq 0.17\sqrt{f_c'}\, b_w\, d_p$$

$$\leq 0.42\sqrt{f_c'}\, b_w d_p$$

$$M_{u(\text{at } d/2 \text{ from face})} = \underset{\downarrow_{340}}{\text{reaction}} * \frac{0.918}{2} - \frac{34.35 * \left(\frac{0.918}{2}\right)^2}{2}$$

$$= 152.44^{kN.m}$$

$$\Rightarrow \frac{V_u d_p}{M_u} = \frac{306.3 * 0.918}{152.44} = 1.844 > 1.0$$

$$\Rightarrow \text{So use } \frac{V_u d_p}{M_u} = 1.0$$

$$\phi V_c = 0.75 * \left(0.05\sqrt{345} + 4.8 * 1\right) * 150 * 918 * 10^{-3}$$

$$= 526 \text{ kN}. \quad >> \phi V_{c\,max}.$$

$$\text{but } Min(\phi V_c) = \phi 0.17\sqrt{f_c'}\, b_w d_p = 103 \text{ kN}.$$

$$Max(\phi V_c) = \phi 0.42\sqrt{f_c'}\, b_w d_p = 254 \text{ kN}$$

$$\Rightarrow \phi V_c = 254 \text{ kN}.$$

$$V_{u_{d/2}} = 306.3 > \phi V_c = 254 \text{ kN}.$$

hence web steel reinforcements:.

$$\phi V_s = 306.3 - 254 = 52.3 \text{ kN}$$

$$\phi \frac{2}{3}\sqrt{f_c'}\, b_w d_p = 404 \text{ kN} > \phi V_s \quad \text{ok}.$$

So the Section depth (Dimensions) is adequate.

2. Minimum web steel Reinforcemt.

$$A_{v_{min}} = \frac{A_{ps} \times f_{pu} \times S}{80 f_y d_p} \sqrt{\frac{d_p}{b_w}}$$

$$= \frac{1280 * 1862 * S}{80 * 414 * 918} \sqrt{\frac{918}{150}}$$

$$\frac{A_{v_{min}}}{S} = 0.194$$

. Required web steel Reinforcemt:-

$$S = \frac{A_v f_y d_p}{\frac{V_u}{\phi} - V_c} \leq 0.75h \leq 600^{mm}$$

or (Use $\phi 10$-U stirrups

$$S = \frac{A_v f_y d_p}{V_s} = \frac{2 * 79 * 414 \times 918}{\frac{52.3 \times 10^3}{0.75}}$$

$$= 861 \text{ mm} > 0.75h = 762^{mm}$$

$$> 600^{mm}$$

$$\Rightarrow S = 600^{mm}$$

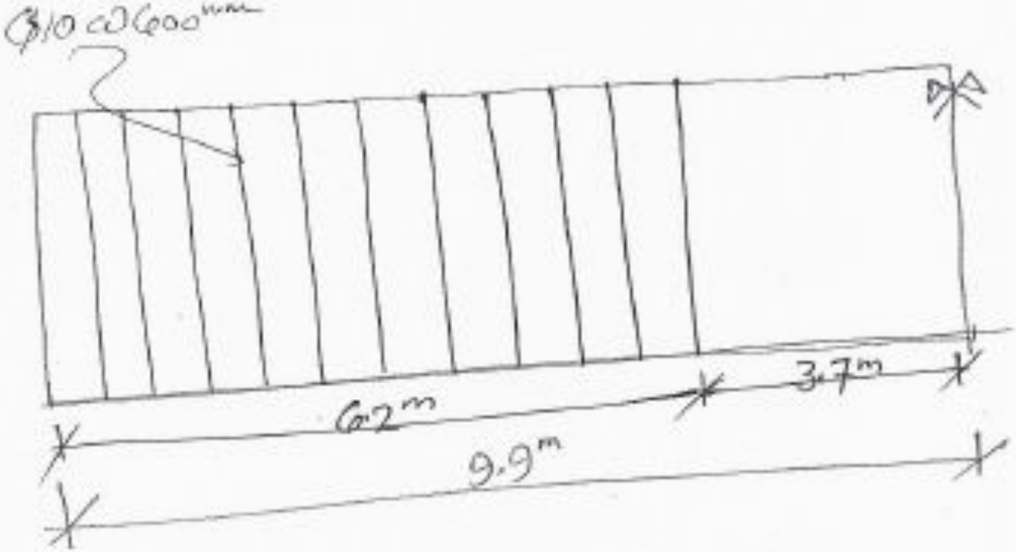
∴ Use $\phi 10$ @ 600^{mm} - U Stirrups.

Plane at which No Web Steel is Needed

$$\frac{\phi V_c}{2} = \frac{254}{2} = 127^{kN} = 340 * \left(\frac{\frac{19.8}{2} - x}{(19.8/2)}\right)$$

$$\Rightarrow 0.373 * 9.9 = 9.9 - x$$

$$\Rightarrow x = 6.2^{m}$$



Example②

Re-Solve Eq.① Using detailed Procedures:

$f_c' = 41.4\,MPa$

$w_D = 5.7\ kN/m$

Solution:-

Plane at d/2 from face of Support.

$$V_{u_{d/2}} = 324.23^{kN}$$

① Flexural-shear Cracking V_{ci}

$$V_{ci} = 0.05\lambda\sqrt{f_c'}\ b_w d_p + V_d + \frac{V_i \cdot M_{cr}}{M_{max}} \leq 0.14\lambda\sqrt{f_c'}\ b_w d$$

& $d_p \leq 0.8h$

$$M_{cr} = \left(\frac{I_c}{y_t}\right)\left(0.5\lambda\sqrt{f_c'} + f_{ce} - f_d\right)$$

y_t :distance from Centroid to the extreme tension fibers.

$I_c =$ ~~[illegible]~~ $= 2.94 \times 10^{10}\ mm^4$

$C_b = 480^{mm}$

$P_e = 1374\ kN$

$S_b = 61\,500\,051\ mm^3$

$r^2 = 121000\ mm^2$, $A = 243200^{mm^2}$

↓ Concrete stress at exterme bottom fibers due to prestress only (f_{ce})

$$f_{ce} = \frac{-P_e}{A} - \frac{P_e . e c_b}{I}$$

$$= \frac{-P_e}{A}\left(1 + \frac{e\,C_b}{r^2}\right)$$

(e) eccentricity @ $d/2 = 459^{mm}$ is

$$e_{d/2} = 320 + (380-320) * \frac{459}{\frac{19800}{2}} = 323^{mm}$$

$$\therefore f_{ce} = \frac{-1371 \times 10^3}{243200} - \left(\frac{1371 \times 10^3 * 323 * 480}{2.94 \times 10^{10}}\right) = -12.86^{MPa}_{(comp)}.$$

$(w_D = 5.7^{kN/m})$

$$M_{d/2} = \frac{w_d x(l-x)}{2} = \frac{5.7 * 0.459(19.8 - 0.459)}{2}$$

$$= 25.4 \text{ kN.m}$$

• Stress due to the Unfactored dead load at externe fibers.

$$f_d = \frac{M_{d/2} \times C_b}{I_c} = \frac{25.4 \times 10^6 * 480}{2.94 \times 10^{10}} = 0.414^{MPa}$$

Also M_{cr} :-

$$M_{cr} = \left[\left(\frac{2.94 \times 10^{10}}{480}\right) * \left(0.5 * \sqrt{41.4} + (12.86) - 0.414\right)\right] * 10^{-6}$$

$$= 959.4 \text{ kN.m}$$

$$V_d = W_D\left(\frac{l}{2} - x\right)$$

$$= 5.7 * (9.9 - 0.459) = 54^{kN}$$

$W_{SD} = 1.46^{kN/m}$

$W_{LL} = 16.1^{kN/m}$

$W_u = 1.2 \times 1.46 + 1.6 * 16.1 = 27.5 \text{ kN/m}$

The factored shear force at the section due to externally applied loads occuring simultaneously with M_{max} is :-

$$V_i = W_u * \left(\frac{l}{2} - x\right) = 27.5\left(\frac{19.8}{2} - 0.459\right) = 259^{kN}$$

$$M_{max} = \frac{W_u\, x(l - x)}{2} = \frac{27.5(0.459) * (19.8 - 0.459)}{2}$$

$$= 122 \text{ kN.m}.$$

$$\therefore V_{ci} = (0.05 \times 1 * \sqrt{41.4} * 150 * 918) * 10^{-3} + 54 + \left(\overset{10^{-3}*}{\frac{259\times10^{3}\times 959.4\times10^{6}}{122\times10^{+6}}}\right)$$

$$= 44.3 + 54 + 2036$$
$$= 2135^{kN}$$

should be more than $0.14\sqrt{f_c'}\, b_w d_p = 124^{kN} < V_{ci}$

ok.

$$V_{cw} = \left(0.29 \lambda \sqrt{f_c'} + 0.3 f_{pc}\right) b_w \, d_p + V_p$$

$\bar{f}_{pc}$ = Compressive stress in Concrete at the (cgc)

$$= \frac{P_e}{A_c} = \frac{1371 \times 10^3}{243200} = 5.6^{MPa}.$$

V_p : Vertical Component of effective prestresse @ Section

$$= P_e \tan\theta \quad (\text{more acuratly } P_e \sin\theta)$$

where θ: is the angle between the inclined tendon and horizontal.

$$V_p = 1371 * \frac{(380-320)}{\frac{19.8}{2} \times 10^3} = 8.3^{kN}$$

Hence:

$$V_{cw} = \left(0.29 * \sqrt{41.4} + (0.3 \times 5.6)\right) \times 150 \times 918 \times 10^{-3} + 8.3$$

$$= 497 \text{ kN}$$

Controls (lesser then).

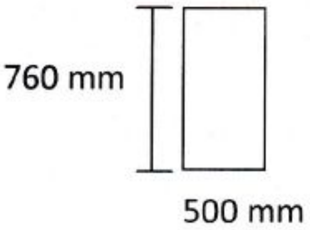
$$\therefore V_c = V_{cw} = 497^{kN}$$

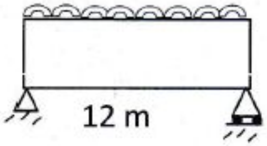
Example1:

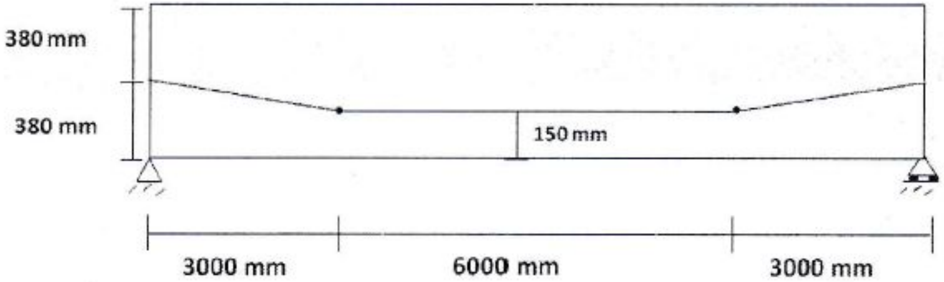
Given : eff. Prest. (F_i)=1450 kN
Superimposed, D.L. = 7 kN/m
Superimposed, L.L. = 15 kN/m
Selfweight, W_G = 9.1 kN/m
f_y = 400 N/mm^2
f_c'=35 N/mm^2
A_{ps}= 2036 mm^2
f_{pu} = 1720 N/mm^2

Required.

Design for shear of distance (1.2m) from support.







Solution:

380-150=230mm

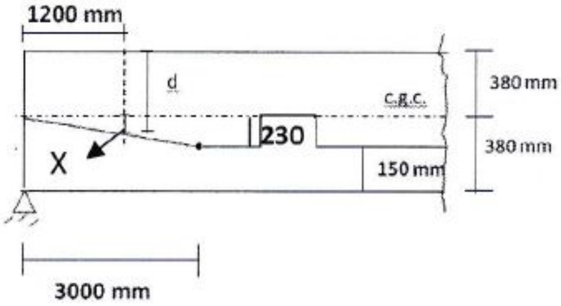
$$\frac{230}{3000} = \frac{X}{1200}$$

$$\therefore X = 92\ mm$$

$$\therefore d = 380 + 92 = 472\ mm$$

0.8 h=0.8*760 =608 mm (control).

$$\therefore d = 608\ mm$$



Concrete shear resistance (V_c)

Flexure shear

$$V_{ci} = \left[\frac{\sqrt{f'_c}}{20}\ b_w.d + V_d + \frac{V_i\,M_{cr}}{M_{max}}\right]$$

Vd : Shear due to selfweight (non factored)

$$\therefore V_d = 9.1\left(\frac{12}{2} - 1.2\right) = 43.7\ kN.$$

V_i =shear due to supper imposed loads (factored)

$$\therefore w = 1.2 * 7 + 1.6 * 15 = 32.4\ kN/m$$

$$\therefore\ V_i = (32.4) * \left(\frac{12}{2} - 1.2\right) = 155.52\ kN$$

$$M_{cr} = \left(\frac{I}{C_t}\right)\left[0.5\sqrt{f'_c} + f_{pe} - f_d\right]$$

$$I = \frac{500 * (760)^3}{12} = 1.829 * 10^{10}\ mm^4$$

$$C_t = 380\ mm$$

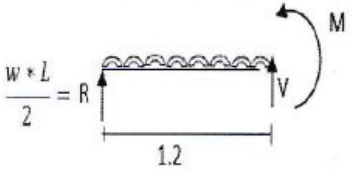
e= 472-380 = 92 mm =X

$$f_{pe} = \frac{F_i}{A} + \frac{F_i * e * C_t}{I}$$

$$= \frac{1450 * 10^3}{500 * 760} + \frac{1450 * 10^3 * 92 * 380}{1.829 * 10^{10}}$$

$$f_{pe} = 3.82 + 2.77 = 6.59\ N/mm^2$$

$$V = \frac{w * L}{2} - w * 1.2$$



$$V = w\left(\frac{L}{2} - 1.2\right)$$

$$M = \frac{w * L}{2}(1.2) - w * \frac{(1.2)^2}{2}$$

$$M = w * \frac{1.2}{2}\left(L - \frac{1.2}{2}\right)$$

M_G =moment due to selfweight (non-factored)

$$M_G = \frac{9.1 * 1.2}{2} * (12 - 1.2) = 58.97\ kN.m$$

$$f_d = \frac{M_G * \quad c_t}{I} = \frac{58.97 * 10^6 * 380}{1.829 * 10^{10}} = 1.23\ N/mm^2$$

$$M_{cr} = \frac{1.829 * 10^{10}}{380} * (\,0.5\sqrt{35}\ + 6.59 - 1.23) *\ 10^{-6}$$

M_{cr}= 400.4 kN.m

M_{max}= moment due to supper imposed loads, no selfweight (factored)

$$M_{max} = (32.4) * \frac{1.2}{2}\ (12 - 1.2)$$

M_{max}=209.95 kN.m

$$\therefore\ V_{ci} = [\frac{\sqrt{35}}{20} * 500 * 608 * 10^{-3} +\ 43.7 + \frac{155.52 * 400.4}{209.95}]$$

V_{ci}=430.2 kN

$$V_{ci}\ \geq \frac{\sqrt{f_c'}}{7}\ b_w * d = \frac{\sqrt{35}}{7} * 500 * 608 * 10^{-3} = 256\ kN$$

$$\therefore V_{ci} = 430.2\ kN$$

$$\therefore V_{ci} = \left[\frac{\sqrt{f'_c}}{20}\, b_w * d + \frac{V_u * \ M_{ct}}{M_u}\right] \geq \ \frac{\sqrt{f'_c}}{7}\, b_w * d$$

$$M_{ct} = \left(\frac{I}{C_t}\right)\left[0.5\sqrt{f'_c}\ + f_{pe}\right]$$

$$M_{ct} = \frac{1.829 * 10^{10}}{380}\left[0.5\sqrt{35}\ + 6.59\right] * \ 10^{-6}$$

M_{ct}= 459.56 kN.m

V_u : shear due to supper imposed load and selfweight (factored)

$$\therefore w = 1.2 * (7 + 9.1) + 1.6 * 15 = 43.32\ kN/m$$

$$\therefore V_u = (43.32) * \left(\frac{12}{2} - 1.2\ \right) = 207.94\ kN$$

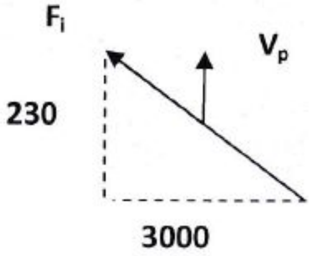
M_u : moment due to supper imposed loads and selfweight (factored)

$$M_u = 43.32 * \ \frac{1.2}{2}(12 - 1.2\ ) = 280.71\ kN.m$$

$$V_{ci} = \left[\frac{\sqrt{35}}{20} * 500 * 608 * 10^{-3} + \ \frac{207.94 * 459.56}{280.71}\right] = 430.35\ kN$$

$$\therefore V_{ci} = 430.35\ kN \quad > \frac{\sqrt{f_c}}{7}\ b_w * d \qquad o.k.$$

(b) Web-Shear Strength.



$$V_{cw} = 0.3\left[\sqrt{f'_c} + f_{pc}\right] b_w * d + V_p$$

$$f_{pc} = \frac{F_i}{A} = \frac{1450 * 10^3}{500 * 700} = 3.82\ N/mm^2$$

$$V_p = 1450 * \frac{230}{\sqrt{(230)^2 + (3000)^2}}$$

$$V_p = 111\ kN$$

$$\therefore V_{cw} = 0.3\left[\sqrt{35} + 3.82\right] * 500 * 608 * 10^{-3} + 111$$

$$\therefore V_{cw} = 998.93\ kN$$

$$\therefore use\ V_c = 430.2\ kN \quad (the\ smallest)$$

$$\therefore V_c = 430.2\ kN$$

$$\frac{V_c}{2} = 215.1\ kN$$

$$V_u = 207.94\ kN$$

$$\frac{V_u}{\emptyset} = \frac{207.9}{0.75} = 277.25\ kN$$

$$\frac{V_c}{2} < \frac{V_u}{\emptyset} < V_c$$

use min. reinf.

use 8 mm stirrups

$$A_v = 2 * \frac{\pi}{4}\,(8)^2 = 100\,mm^2$$

$$S = \frac{3\,A_v *\ f_y}{b_w} = \ \frac{3 * 100 * 400}{500} = 240\,mm$$

$$\text{eff. pres.} > \ 40\,\%\,ult.strength\,;\ \left(\frac{1450 * 10^3}{2036 * 1720}\right) = 41.4\%$$

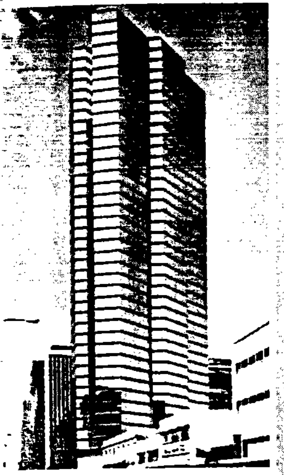
$$\therefore S = \frac{80\,A_v *\ f_y * d}{A_{ps}\ f_{pu}}\ \ \sqrt{\frac{b_w}{d}}$$

$$\therefore S = \frac{80 *\ 100 *\ 400 * 608}{2036 * 1720}\ \ \sqrt{\frac{500}{608}}$$

S= 504 mm

$$\therefore use\ \emptyset\ 8\ @240\ mm$$

Thanks for Listening



3

PARTIAL LOSS OF PRESTRESS

3.1 INTRODUCTION

It is a well-established fact that the initial prestressing force applied to the concrete element undergoes a progressive process of reduction over a period of approximately five years. Consequently, it is important to determine the level of the prestressing force at each loading stage, from the stage of transfer of the prestressing force to the concrete, to the various stages of prestressing available at service load, up to the ultimate. Essentially, the reduction in the prestressing force can be grouped into two categories:

- Immediate elastic loss during the fabrication or construction process. including elastic shortening of the concrete, anchorage losses, and frictional losses.
- Time-dependent losses such as creep, shrinkage, and those due to temperature effects and steel relaxation, all of which are determinable at the service-load limit state of stress in the prestressed concrete element.

An exact determination of the magnitude of these losses—particularly the time-dependent ones—is not feasible, since they depend on a multiplicity of interrelated factors. Empirical methods of estimating losses differ with the different codes of practice or

Executive Center, Honolulu, Hawaii. (*Courtesy*, Post-Tensioning Institute.)

Table 3.1 AASHTO Lump-Sum Losses

Type of prestressing steel	Total loss $f'_c = 4{,}000$ psi (27.6 N/mm^2)	$f'_c = 5{,}000$ psi (34.5 N/mm^2)
Pretensioning strand		45,000 psi (310 N/mm^2)
Post-tensioning[a] wire or strand	32,000 psi (221 N/mm^2)	33,000 psi (228 N/mm^2)
Bars	22,000 psi (152 N/mm^2)	23,000 psi (159 N/mm^2)

[a] Losses due to friction are excluded. Such losses should be computed according to Section 6.5 of the AASHTO specifications.

recommendations, such as those of the Prestressed Concrete Institute, the ACI-ASCE joint committee approach, the AASHTO lump-sum approach, the Comité Eurointernationale du Béton (CEB), and the FIP (Federation Internationale de la Précontrainte). The degree of rigor of these methods depends on the approach chosen and the accepted practice of record.

A very high degree of refinement of loss estimation is neither desirable nor warranted, because of the multiplicity of factors affecting the estimate. Consequently, lump-sum estimates of losses are more realistic, particularly in routine designs and under average conditions. Such lump-sum losses can be summarized in Table 3.1 of AASHTO and Table 3.2 of PTI. They include elastic shortening, relaxation in the prestressing steel, creep, and shrinkage, and they are applicable only to routine, standard conditions of loading; normal concrete, quality control, construction procedures, and environmental conditions; and the importance and magnitude of the system. Detailed analysis has to be performed if these standard conditions are not fulfilled.

A summary of the sources of the separate prestressing losses and the stages of their occurrence is given in Table 3.3, in which the subscript i denotes "initial" and the subscript j denotes the loading stage after jacking. From this table, the total loss in prestress can be calculated for pretensioned and post-tensioned members as follows:

(i) Pretensioned Members

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH} \tag{3.1a}$$

Table 3.2 Approximate Prestress Loss Values for Post-Tensioning

Post-tensioning tendon material	Prestress loss, psi Slabs	Beams and joists
Stress-relieved 270-K strand and stress-relieved 240-K wire	30,000 (207 N/mm^2)	35,000 (241 N/mm^2)
Bar	20,000 (138 N/mm^2)	25,000 (172 N/mm^2)
Low-relaxation 270-K strand	15,000 (103 N/mm^2)	20,000 (138 N/mm^2)

Note: This table of approximate prestress losses was developed to provide a common post-tensioning industry basis for determining tendon requirements on projects in which the magnitude of prestress losses is not specified by the designer. These loss values are based on use of normal-weight concrete and on average values of concrete strength, prestress level, and exposure conditions. Actual values of losses may vary significantly above or below the table values where the concrete is stressed at low strengths, where the concrete is highly prestressed, or in very dry or very wet exposure conditions. The table values do not include losses due to friction.

Source: Post-Tensioning Institute.

Table 3.3 Types of Prestress Loss

Type of prestress loss	Stage of occurrence		Tendon stress loss	
	Pretensioned members	Post-tensioned members	During time interval (t_i, t_j)	Total or during life
Elastic shortening of concrete *(ES)*	At transfer	At sequential jacking	...	Δf_{pES}
Relaxation of tendons *(R)*	Before and after transfer	After transfer	$\Delta f_{pR}(t_i, t_j)$	Δf_{pR}
Creep of concrete *(CR)*	After transfer	After transfer	$\Delta f_{pC}(t_i, t_j)$	Δf_{pCR}
Shrinkage of concrete *(SH)*	After transfer	After transfer	$\Delta f_{pS}(t_i, t_j)$	Δf_{pSH}
Friction *(F)*	...	At jacking	...	Δf_{pF}
Anchorage seating loss *(A)*	...	At transfer	...	Δf_{pA}
Total	Life	Life	$\Delta f_{pT}(t_i, t_j)$	Δf_{pT}

where $\Delta f_{pR} = \Delta f_{pR}(t_0, t_{tr}) + \Delta f_{pR}(t_{tr}, t_s)$
t_0 = time at jacking
t_{tr} = time at transfer
t_s = time at stabilized loss

Hence, computations for steel relaxation loss have to be performed for the time interval t_1 through t_2 of the respective loading stages.

As an example, the transfer stage, say, at 18 h would result in $t_{tr} = t_2 = 18$ h and $t_0 = t_1 = 0$. If the next loading stage is between transfer and 5 years (17,520 h), when losses are considered stabilized, then $t_2 = t_s = 17{,}520$ h and $t_1 = 18$ h. Then, if f_{pi} is the initial prestressing stress that the concrete element is subjected to and f_{pJ} is the jacking stress in the tendon, then

$$f_{pi} = f_{pJ} - \Delta f_{pR}(t_0, t_{tr}) - \Delta f_{pES} \tag{3.1b}$$

(ii) Post-tensioned Members

$$\Delta f_{pT} = \Delta f_{pA} + \Delta f_{pF} + \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH} \tag{3.1c}$$

where Δf_{pES} is applicable only when tendons are jacked sequentially, and not simultaneously.

In the post-tensioned case, computation of relaxation loss starts between the transfer time $t_1 = t_{tr}$ and the end of the time interval t_2 under consideration. Hence

$$f_{pi} = f_{pJ} - \Delta f_{pA} - \Delta f_{pF} \tag{3.1d}$$

3.2 ELASTIC SHORTENING OF CONCRETE *(ES)*

Concrete shortens when a prestressing force is applied. As the tendons that are bonded to the adjacent concrete simultaneously shorten, they lose part of the prestressing force that they carry.

3.2.1 Pretensioned Elements

For pretensioned (precast) elements, the compressive force imposed on the beam by the tendon results in the longitudinal shortening of the beam, as shown in Figure 3.1. The unit shortening in concrete is $\epsilon_{ES} = \Delta_{ES}/L$, so

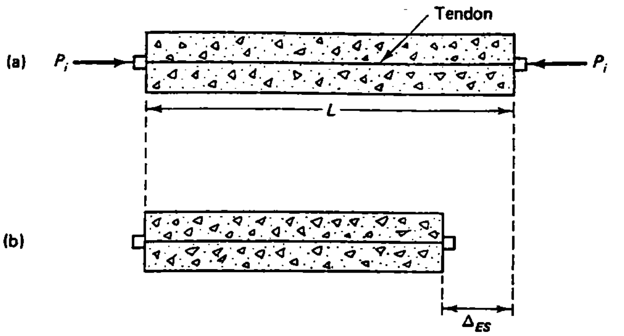


Figure 3.1 Elastic shortening. (a) Unstressed beam. (b) Longitudinally shortened beam.

$$\epsilon_{ES} = \frac{f_c}{E_c} = \frac{P_i}{A_c E_c} \tag{3.2a}$$

Since the prestressing tendon suffers the same magnitude of shortening,

$$\Delta f_{pES} = E_s \epsilon_{ES} = \frac{E_s P_i}{A_c E_c} = \frac{nP_i}{A_c} = nf_{cs} \tag{3.2b}$$

The stress in the concrete at the centroid of the steel due to the initial prestressing is

$$f_{cs} = -\frac{P_i}{A_c} \tag{3.3}$$

If the tendon in Figure 3.1 has an eccentricity *e* at the beam midspan and the self-weight moment M_D is taken into account, the stress the concrete undergoes at the midspan section at the level of the prestressing steel becomes

$$f_{cs} = -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_D e}{I_c} \tag{3.4}$$

where P_i has a lower value after transfer of prestress. The *small* reduction in the value of P_J to P_i occurs because the force in the prestressing steel immediately after transfer is less than the initial jacking prestress force P_J. However, since it is difficult to accurately determine the reduced value of P_i, and since observations indicate that the reduction is only a few percentage points, it is possible to use the initial value of P_i before transfer in Equations 3.2 through 3.4, or reduce it by about 10 percent for refinement if desired.

3.2.1.1 Elastic shortening loss in pretensioned beams

Example 3.1

A pretensioned prestressed beam has a span of 50 ft (15.2 m), as shown in Figure 3.2. For this beam,

$$f'_c = 6{,}000 \text{ psi } (41.4 \text{ MPa})$$

$$f_{pu} = 270{,}000 \text{ psi } (1{,}862 \text{ MPa})$$

$$f'_{ci} = 4{,}500 \text{ psi } (31 \text{ MPa})$$

$$A_{ps} = 10 - \frac{1}{2}\text{-in dia. seven-wire-strand tendon}$$

$$= 10 \times 0.153 = 1.53 \text{ in.}^2$$

$$E_{ps} = 27 \times 10^6 \text{ psi } (1.862 \text{ MPa})$$

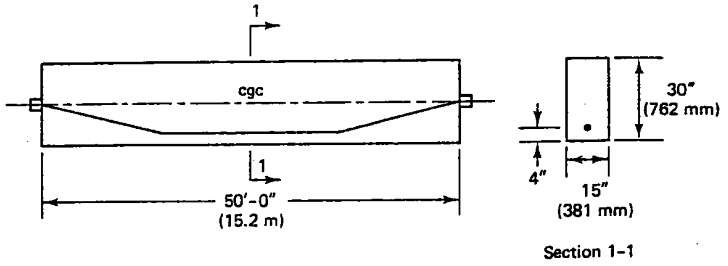


Figure 3.2 Beam in Example 3.1.

Calculate the concrete fiber stresses at transfer at the centroid of the tendon for the midspan section of the beam, and the magnitude of loss in prestress due to the effect of elastic shortening of the concrete. Assume that prior to transfer, the jacking force on the tendon was 75% f_{pu}.

Solution:

$$A_c = 15 \times 30 = 450 \text{ in.}^2$$

$$I_c = \frac{15(30)^3}{12} = 33{,}750 \text{ in.}^4$$

$$r^2 = \frac{I_c}{A_c} = 75 \text{ in.}^2$$

$$A_{ps} = 10 \times 0.153 = 1.53 \text{ in.}^2$$

$$e_c = \frac{30}{2} - 4 = 11 \text{ in.}$$

$$P_i = 0.75 f_{pu} A_{ps} = 0.75 \times 270{,}000 \times 1.53 = 309{,}825 \text{ lb}$$

$$M_D = \frac{wl^2}{8} = \frac{15 \times 30}{144} \times 150 \frac{(50)^2}{8} \times 12 = 1{,}757{,}813 \text{ in.-lb}$$

From Equation 3.4, the concrete fiber stress at the steel centroid of the beam at the moment of transfer, assuming that $P_i \cong P_J$, is

$$\begin{aligned} f_{cs} &= -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_D e}{I_c} \\ &= -\frac{309{,}825}{450}\left(1 + \frac{11^2}{75}\right) + \frac{1{,}757{,}813 \times 11}{33{,}750} \\ &= -1{,}799.3 + 572.9 = -1{,}226.4 \text{ psi (8.50 MPa)} \end{aligned}$$

We also have

$$\text{Initial } E_{ci} = 57{,}000\sqrt{f'_{ci}} = 57{,}000\sqrt{4{,}500} = 3.824 \times 10^6 \text{ psi}$$

$$\text{Initial modular ratio } n = \frac{E_s}{E_{ci}} = \frac{27 \times 10^6}{3.824 \times 10^6} = 7.06$$

$$\text{28 days' strength } E_c = 57{,}000\sqrt{6{,}000} = 4.415 \times 10^6 \text{ psi}$$

$$\text{28 days' modular ratio } n = \frac{27 \times 10^6}{4.415 \times 10^6} = 6.12$$

From Equation 3.2b, the loss of prestress due to elastic shortening is

$$\Delta f_{pES} = nf_{cs} = 7.06 \times 1.226.4 = 8{,}659.2 \text{ psi (59.7 MPa)}$$

If a reduced P_i is used with assumed 10 percent reduction,

$$\Delta f_{pES} = 0.90 \times 8{,}659.2 = 7{,}793.3 \text{ psi (53.7 MPa)}.$$

The difference of 865.9 psi in steel stress is insignificant compared to the total loss in prestress due to all factors of about 45,000 to 55,000 psi.

3.2.2 Post-tensioned Elements

In post-tensioned beams, the elastic shortening loss varies from zero if all tendons are jacked simultaneously to half the value calculated in the pretensioned case if several sequential jacking steps are used, such as jacking two tendons at a time. If n is the number of tendons or pairs of tendons sequentially tensioned, then

$$\Delta f_{pES} = \frac{1}{n}\sum_{j=1}^{n} (\Delta f_{pES})_j \tag{3.5}$$

where j denotes the number of jacking operations. Note that the tendon that was tensioned last does not suffer any losses due to elastic shortening, while the tendon that was tensioned first suffers the maximum amount of loss.

3.2.2.1 Elastic shortening loss in post-tensioned beam

Example 3.2

Solve Example 3.1 if the beam is post-tensioned and the prestressing operation is such that

(a) Two tendons are jacked at a time.
(b) One tendon is jacked at a time.
(c) All tendons are simultaneously tensioned.

Solution:

(a) From Example 3.1, $\Delta f_{pE} = 8{,}659.2$ psi. Clearly, the last tendon suffers no loss of prestress due to elastic shortening. So only the first four pairs have losses, with the first pair suffering the maximum loss of 8,659.2 psi. From Equation 3.5, the loss due to elastic shortening in the post-tensioned beam is

$$\Delta f_{pES} = \frac{4/4 + 3/4 + 2/4 + 1/4}{5}(8{,}659.2)$$

$$= \frac{10}{20} \times (8{,}659.2) = 4{,}330 \text{ psi (29.9 MPa)}$$

(b)

$$\Delta f_{pES} = \frac{9/9 + 8/9 + \cdots + 1/9}{10}(8{,}659.2)$$

$$= \frac{45}{90} \times (8{,}659.2) = 4{,}330 \text{ psi (29.9 MPa)}$$

In both cases the loss in prestressing in the post-tensioned beam is half that of the pretensioned beam.

(c) $\Delta f_{pES} = 0$

3.3 STEEL STRESS RELAXATION *(R)*

Stress-relieved tendons suffer loss in the prestressing force due to constant elongation with time, as discussed in Chapter 2. The magnitude of the decrease in the prestress depends not only on the duration of the sustained prestressing force, but also on the ratio

f_{pi}/f_{py} of the initial prestress to the yield strength of the reinforcement. Such a loss in stress is termed *stress relaxation.* The ACI 318-02 Code limits the tensile stress in the prestressing tendons to the following:

(a) For stresses due to the tendon jacking force, $f_{pJ} = 0.94 f_{py}$, but not greater than the lesser of $0.80 f_{pu}$ and the maximum value recommended by the manufacturer of the tendons and anchorages.

(b) Immediately after prestress transfer, $f_{pi} = 0.82 f_{py}$, but not greater than $0.74 f_{pu}$.

(c) In post-tensioned tendons, at the anchorages and couplers immediately after force transfer $= 0.70 f_{pu}$.

The range of values of f_{py} is given by the following:

Prestressing bars: $f_{py} = 0.80 f_{pu}$
Stress-relieved tendons: $f_{py} = 0.85 f_{pu}$
Low-relaxation tendons: $f_{py} = 0.90 f_{pu}$

If f_{pR} is the remaining prestressing stress in the steel after relaxation, the following expression defines f_{pR} for stress relieved steel:

$$\frac{f_{pR}}{f_{pi}} = 1 - \left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \tag{3.6}$$

In this expression, log t in hours is to the base 10, f_{pi}/f_{py} exceeds 0.55, and $t = t_2 - t_1$. Also, for low-relaxation steel, the denominator of the log term in the equation is divided by 45 instead of 10. A plot of Equation 3.6 is given in Figure 3.3.

An approximation of the term ($\log t_2 - \log t_1$) can be made in Equation 3.6 so that $\log t = \log(t_2 - t_1)$ without significant loss in accuracy. In that case, the stress-relaxation loss becomes

$$\Delta f_{pR} = f'_{pi}\frac{\log t}{10}\left(\frac{f'_{pi}}{f_{py}} - 0.55\right) \tag{3.7}$$

where f'_{pi} is the initial stress in steel to which the concrete element is subjected.

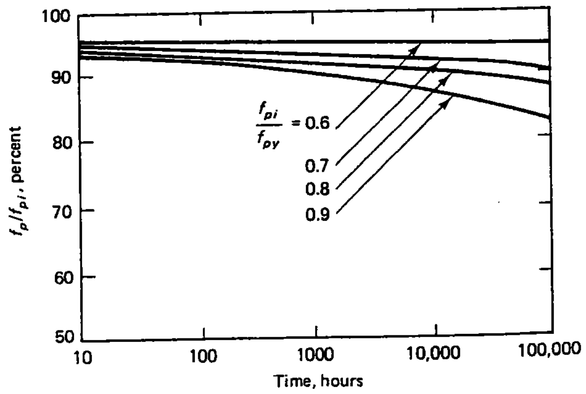


Figure 3.3 Stress-relaxation relationship in stress-relieved strands. (*Courtesy,* Post-Tensioning Institute.)

If a step-by-step loss analysis is necessary, the loss increment at any particular stage can be defined as

$$\Delta f_{pR} = f'_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f'_{pi}}{f_{py}} - 0.55\right) \tag{3.8}$$

where t_1 is the time at the beginning of the interval and t_2 is the time at the end of the interval from jacking to the time when the loss is being considered.

For low relaxation steel, the divider is 45 instead of 10 in Equation 3.8, as shown in Equation 2.19.

3.3.1 Relaxation Loss Computation

Example 3.3

Find the relaxation loss in prestress at the end of 5 years in Example 3.1, assuming that relaxation loss from jacking to transfer, from elastic shortening, and from long-term loss due to creep and shrinkage over this period is 20 percent of the initial prestress. Assume also that the yield strength $f_{py} = 230{,}000$ psi (1,571 MPa).

Solution: From Equation 3.1b for this stage

$$f_{pi} = f_{pJ} - \Delta f_{pR}(t_0, t_{tr})$$
$$= 0.75 \times 270{,}000 = 202{,}500 \text{ psi (1,396 MPa)}$$

The reduced stress for calculating relaxation loss is

$$f'_{pi} = (1 - 0.20) \times 202{,}500 = 162{,}000 \text{ psi (1,170 MPa)}$$

The duration of the stress-relaxation process is

$$5 \times 365 \times 24 \cong 44{,}000 \text{ hours}$$

From Equation 3.7,

$$\Delta f_{pR} = f'_{pi}\frac{\log t}{10}\left(\frac{f'_{pi}}{f_{py}} - 0.55\right)$$
$$= 162.000\frac{\log 44{,}000}{10}\left(\frac{162{,}000}{230{,}000} - 0.55\right)$$
$$= 162.000 \times 0.4643 \times 0.1543 = 11{,}606 \text{ psi (80.0 MPa)}$$

3.3.2 ACI-ASCE Method of Accounting for Relaxation Loss

The ACI-ASCE method uses the separate contributions of elastic shortening, creep, and shrinkage in the evaluation of the steel stress-relaxation loss by means of the equation

$$\Delta f_{pR} = [K_{re} - J\Delta(f_{pES} + f_{pCR} + f_{pSH})] \times C$$

The values of K_{re}, J, and C are given in Tables 3.4 and 3.5.

3.4 CREEP LOSS *(CR)*

Experimental work over the past half century indicates that flow in materials occurs with time when load or stress exists. This lateral flow or deformation due to the longitudinal stress is termed *creep*. A more detailed discussion is given in Ref. 3.9. It must be emphasized that creep stresses and stress losses result *only* from *sustained* loads during the loading history of the structural element.

Table 3.4 Values of C

f_{pi}/f_{pu}	Stress-relieved strand or wire	Stress-relieved bar or low-relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Source: Post-Tensioning Institute.

The deformation or strain resulting from this time-dependent behavior is a function of the magnitude of the applied load, its duration, the properties of the concrete including its mixture proportions, curing conditions, the age of the element at first loading, and environmental conditions. Since the stress-strain relationship due to creep is essentially linear, it is feasible to relate the creep strain ϵ_{CR} to the elastic strain ϵ_{EL} such that a creep coefficient C_u can be defined as

Table 3.5 Values of K_{RE} and J

Type of tendon[a]	K_{RE}	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5,000	0.040
250 Grade low-relaxation wire	4,630	0.037
240 or 235 Grade low-relaxation wire	4,400	0.035
145 or 160 Grade stress-relieved bar	6,000	0.05

[a] In accordance with ASTM A416-74, ASTM A421-76, or ASTM A722-75.

Source: Prestressed Concrete Institute.

$$C_u = \frac{\epsilon_{CR}}{\epsilon_{EL}} \tag{3.9a}$$

Then the creep coefficient at any time t in days can be defined as

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}} C_u \tag{3.9b}$$

As discussed in Chapter 2, the value of C_u ranges between 2 and 4, with an average of 2.35 for ultimate creep. The loss in prestressed members due to creep can be defined for bonded members as

$$\Delta f_{pCR} = C_t \frac{E_{ps}}{E_c} f_{cs} \tag{3.10}$$

where f_{cs} is the stress in the concrete at the level of the centroid of the prestressing tendon. In general, this loss is a function of the stress in the concrete at the section being analyzed. In post-tensioned, nonbonded members, the loss can be considered essentially uniform along the whole span. Hence, an average value of the concrete stress f_{cs} between the anchorage points can be used for calculating the creep in post-tensioned members.

The ACI-ASCE Committee expression for evaluating creep loss has essentially the same format as Equation 3.10, viz.,

$$\Delta f_{pCR} = K_{CR} \frac{E_{ps}}{E_c} (\bar{f}_{cs} - \bar{f}_{csd}) \tag{3.11a}$$

or

$$\Delta f_{pCR} = nK_{CR} (\bar{f}_{cs} - \bar{f}_{csd}) \tag{3.11b}$$

where K_{CR} = 2.0 for pretensioned members
= 1.60 for post-tensioned members (both for normal concrete)
$\bar{f}_{cs}$ = stress in concrete at level of steel cgs immediately after transfer
$\bar{f}_{csd}$ = stress in concrete at level of steel cgs due to all superimposed dead loads applied after prestressing is accomplished
n = modular ratio

Note that K_{CR} should be reduced by 20 percent for lightweight concrete.

3.4.1 Computation of Creep Loss

Example 3.4

Compute the loss in prestress due to creep in Example 3.1 given that the total superimposed load, excluding the beam's own weight after transfer, is 375 plf (5.5 kN/m).

Solution: At full concrete strength,

$$E_c = 57{,}000\sqrt{6{,}000} = 4.415 \times 10^6 \text{ psi } (30.4 \times 10^3 \text{ MPa})$$

$$n = \frac{E_s}{E_c} = \frac{27.0 \times 10^6}{4.415 \times 10^6} = 6.12$$

$$M_{SD} = \frac{375(50)^2}{8} \times 12 = 1.406{,}250 \text{ in.-lb } (158.9 \text{ kN-m})$$

$$\bar{f}_{csd} = \frac{M_{SD}e}{I_c} = \frac{1{,}406{,}250 \times 11}{33{,}750} = 458.3 \text{ psi } (3.2 \text{ MPa})$$

From Example 3.1,

$$\bar{f}_{cs} = 1{,}226.4 \text{ psi } (8.5 \text{ MPa})$$

Also, for normal concrete use, $K_{CR} = 2.0$ (pretensioned beam); so from Equation 3.11a,

$$\begin{aligned} \Delta f_{pCR} &= nK_{CR}(\bar{f}_{cs} - \bar{f}_{csd}) \\ &= 6.12 \times 2.0(1{,}226.4 - 458.3) \\ &= 9{,}401.5 \text{ psi } (64.8 \text{ MPa}) \end{aligned}$$

3.5 SHRINKAGE LOSS *(SH)*

As with concrete creep, the magnitude of the shrinkage of concrete is affected by several factors. They include mixture proportions, type of aggregate, type of cement, curing time, time between the end of external curing and the application of prestressing, size of the member, and the environmental conditions. Size and shape of the member also affect shrinkage. Approximately 80 percent of shrinkage takes place in the first year of life of the structure. The average value of ultimate shrinkage strain in both moist-cured and steam-cured concrete is given as 780×10^{-6} in./in. in ACI 209 R-92 Report. This average value is affected by the length of initial moist curing, ambient relative humidity, volume-surface ratio, temperature, and concrete composition. To take such effects into account, the average value of shrinkage strain should be multiplied by a correction factor γ_{SH} as follows

$$\epsilon_{SH} = 780 \times 10^{-6}\gamma_{SH} \tag{3.12}$$

Components of γ_{SH} are factors for various environmental conditions and tabulated in Ref. 3.12, Sec. 2.

The Prestressed Concrete Institute stipulates for standard conditions an average value for nominal ultimate shrinkage strain $(\epsilon_{SH})_u = 820 \times 10^{-6}$ in./in. (mm/mm), (Ref. 3.4). If ϵ_{SH} is the shrinkage strain after adjusting for relative humidity at volume-to-surface ratio *V/S*, the loss in prestressing in pretensioned member is

$$\Delta f_{pSH} = \epsilon_{SH} \times E_{ps} \tag{3.13}$$

For post-tensioned members, the loss in prestressing due to shrinkage is somewhat less since some shrinkage has already taken place before post-tensioning. If the relative humidity is taken as a percent value and the *V/S* ratio effect is considered, the PCI general expression for loss in prestressing due to shrinkage becomes

$$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} E_{ps}\left(1 - 0.06\frac{V}{S}\right)(100 - RH) \tag{3.14}$$

where RH = relative humidity

Table 3.6 Values of K_{SH} for Post-Tensioned Members

Time from end of moist curing to application of prestress, days	**1**	**3**	**5**	**7**	**10**	**20**	**30**	**60**
K_{sh}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

Source: Prestressed Concrete Institute.

Photo 3.1 101/280/680 interchange connectors, South San Jose, California.

where $K_{SH} = 1.0$ for pretensioned members. Table 3.6 gives the values of K_{SH} for post-tensioned members.

Adjustment of shrinkage losses for standard conditions as a function of time t in days after 7 days for moist curing and 3 days for steam curing can be obtained from the following expressions

(a) Moist curing, after 7 days

$$(\epsilon_{SH})_t = \frac{t}{35 + t}(\epsilon_{SH})_u \tag{3.15a}$$

where $(\epsilon_{SH})_u$ is the ultimate shrinkage strain, t = time in days after shrinkage is considered.

(b) Steam curing, after 1 to 3 days

$$(\epsilon_{SH})_t = \frac{t}{55 + t}(\epsilon_{SH})_u \tag{3.15b}$$

It should be noted that separating creep from shrinkage calculations as presented in this chapter is an accepted engineering practice. Also, significant variations occur in the creep and shrinkage values due to variations in the properties of the constituent materials from the various sources, even if the products are plant-produced such as pretensioned beams. Hence it is recommended that information from actual tests be obtained especially on manufactured products, large span-to-depth ratio cases and/or if loading is unusually heavy.

3.5.1 Computation of Shrinkage Loss

Example 3.5

Compute the loss in prestress due to shrinkage in Examples 3.1 and 3.2 at 7 days after moist curing using both the ultimate K_{SH} method of Equation 3.14 and the time-dependent method of Equation 3.15. Assume that the relative humidity RH is 70 percent and the volume-to-surface ratio is 2.0.

Solution A

K_{SH} method

(a) Pretensioned beam, $K_{SH} = 1.0$:
From Equation 3.14,

$$\Delta f_{pSH} = 8.2 \times 10^{-6} \times 1.0 \times 27 \times 10^6(1 - 0.06 \times 2.0)(100 - 70)$$
$$= 5{,}845.0 \text{ psi (40.3 MPa)}$$

(b) Post-tensioned beam, from Table 3.6, $K_{SH} = 0.77$:

$$\Delta f_{pSH} = 0.77 \times 5{,}845 = 4{,}500.7 \text{ psi (31.0 MPa)}$$

Solution B

Time-dependent method

From Equation 3.15a,

$$\epsilon_{SH,t} = \frac{t}{35 + t}\epsilon_{SH_u} = \frac{7}{35 + 7} \times 780 \times 10^{-6} = 130 \times 10^{-6} \text{ in/in}$$

$$\Delta f_{pSH} = \epsilon_{SH,t} E_s = 130 \times 10^{-6} \times 27 \times 10^6 = 3{,}510 \text{ psi (24.0 MPa)}$$

3.6 LOSSES DUE TO FRICTION *(F)*

Loss of prestressing occurs in post-tensioning members due to friction between the tendons and the surrounding concrete ducts. The magnitude of this loss is a function of the tendon form or alignment, called the *curvature effect,* and the local deviations in the alignment, called the *wobble effect.* The values of the loss coefficients are often refined while preparations are made for shop drawings by varying the types of tendons and the duct alignment. Whereas the curvature effect is predetermined, the wobble effect is the result of accidental or unavoidable misalignment, since ducts or sheaths cannot be perfectly placed.

It should be noted that the maximum frictional stress loss would be at the far end of the beam if jacking is from one end. Hence frictional loss varies linearly along the beam span and can be interpolated for a particular location if such refinement in the computations is warranted.

3.6.1 Curvature Effect

As the tendon is pulled with a force F_1 at the jacking end, it will encounter friction with the surrounding duct or sheath such that the stress in the tendon will vary from the jacking plane to a distance L along the span as shown in Figure 3.4. If an infinitesimal length of the tendon is isolated in a free-body diagram as shown in Figure 3.5, then, assuming that μ denotes the coefficient of friction between the tendon and the duct due to the curvature effect, we have

$$dF_1 = -\mu F_1 \, d\alpha$$

or

$$\frac{dF_1}{F_1} = -\mu d\alpha \tag{3.16a}$$

Integrating both sides of this equation yields

$$\log_e F_1 = -\mu\alpha \tag{3.16b}$$

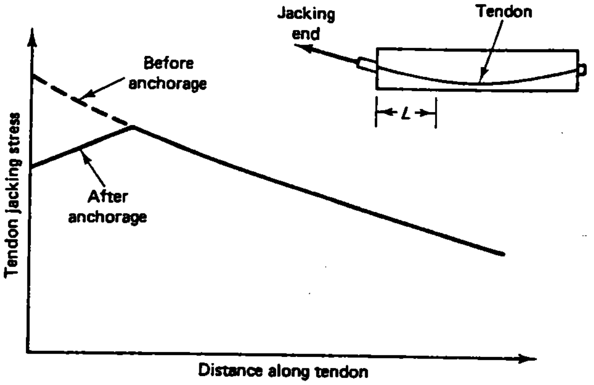


Figure 3.4 Frictional force stress distribution in tendon.

If $\alpha = L/R$, then

$$F_2 = F_1 e^{-\mu\alpha} = F_1 e^{-\mu(L/R)} \tag{3.17}$$

3.6.2 Wobble Effect

Suppose that K is the coefficient of friction between the tendon and the surrounding concrete due to wobble effect or length effect. Friction loss is caused by imperfection in alignment along the length of the tendon, regardless of whether it has a straight or draped alignment. Then by the same principles described in developing Equation 3.16,

$$\log_e F_1 = -KL \tag{3.18}$$

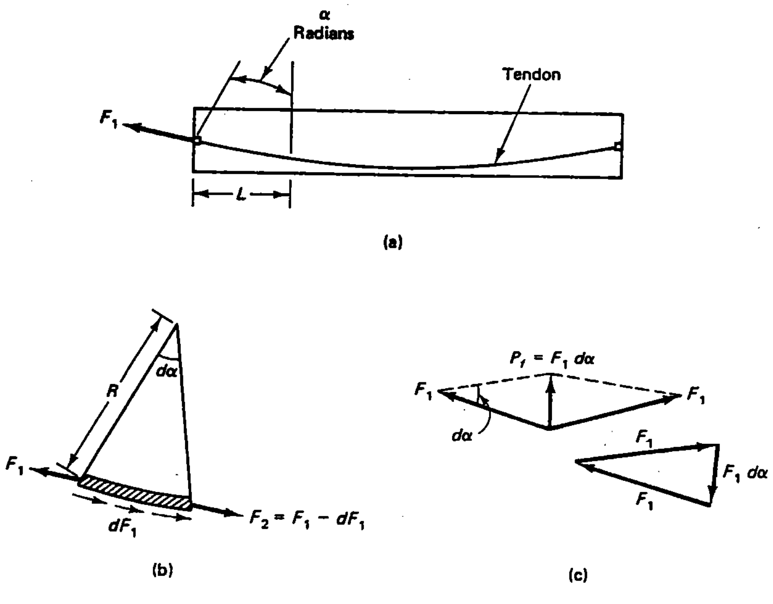


Figure 3.5 Curvature friction loss. (a) Tendon alignment. (b) Forces on infinitesimal length where F_1 is at the jacking end. (c) Polygon of forces assuming $F_1 = F_2$ over the infinitesimal length in (b).

or

$$F_2 = F_1 e^{-KL} \tag{3.19}$$

Superimposing the wobble effect on the curvature effect gives

$$F_2 = F_1 e^{-\mu\alpha - KL}$$

or, in terms of stresses,

$$f_2 = f_1 e^{-\mu\alpha - KL} \tag{3.20}$$

The frictional loss of stress Δf_{pF} is then given by

$$\Delta f_{pF} = f_1 - f_2 = f_1(1 - e^{-\mu\alpha - KL}) \tag{3.21}$$

Assuming that the prestress force between the start of the curved portion and its end is small ($\cong$ 15 percent), it is sufficiently accurate to use the initial tension for the entire curve in Equation 3.21. Equation 3.21 can thus be simplified to yield

$$\Delta f_{pF} = -f_1(\mu\alpha + KL) \tag{3.22}$$

where L is in feet.

Since the ratio of the depth of beam to its span is small, it is sufficiently accurate to use the projected length of the tendon for calculating α. Assuming the curvature of the tendon to be based on that of a circular arc, the central angle α along the curved segment in Figure 3.6 is twice the slope at either end of the segment. Hence,

$$\tan\frac{\alpha}{2} = \frac{m}{x/2} = \frac{2m}{x}$$

If

$$y \cong \frac{1}{2}m \quad \text{and} \quad \alpha/2 = 4y/x$$

then

$$\alpha = 8y/x \text{ radian} \tag{3.23}$$

Table 3.7 gives the design values of the curvature friction coefficient μ and the wobble or length friction coefficient K adopted from the ACI 318 Commentary.

3.6.3 Computation of Friction Loss

Example 3.6

Assume that the alignment characteristics of the tendons in the post-tensioned beam of Example 3.2 are as shown in Figure 3.7. If the tendon is made of 7-wire uncoated strands in flexible metal sheathing, compute the frictional loss of stress in the prestressing wires due to the curvature and wobble effects.

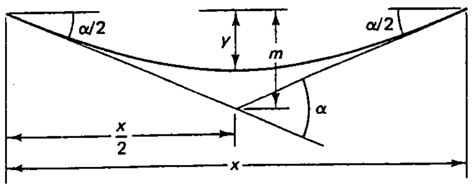


Figure 3.6 Approximate evaluation of the tendon's central angle.

Table 3.7 Wobble and Curvature Friction Coefficients

Type of tendon	Wobble coefficient, K per foot	Curvature coefficient, μ
Tendons in flexible metal sheathing		
Wire tendons	0.0010–0.0015	0.15–0.25
7-wire strand	0.0005–0.0020	0.15–0.25
High-strength bars	0.0001–0.0006	0.08–0.30
Tendons in rigid metal duct		
7-wire strand	0.0002	0.15–0.25
Mastic-coated tendons		
Wire tendons and 7-wire strand	0.0010–0.0020	0.05–0.15
Pregreased tendons		
Wire tendons and 7-wire strand	0.0003–0.0020	0.05–0.15

Source: Prestressed Concrete Institute.

Solution:

$$P_i = 309{,}825 \text{ lb}$$

$$f_{pi} = \frac{309{,}825}{1.53} = 202{,}500 \text{ psi}$$

From Equation 3.23,

$$\alpha = \frac{8y}{x} = \frac{8 \times 11}{50 \times 12} = 0.1467 \text{ radian}$$

From Table 3.7, use $K = 0.0020$ and $\mu = 0.20$. From Equation 3.22, the prestress loss due to friction is

$$\begin{aligned}\Delta f_{pF} &= f_{pi}(\mu\alpha + KL)\\ &= 202.500(0.20 \times 0.1467 + 0.0020 \times 50)\\ &= 202{,}500 \times 0.1293 = 26{,}191 \text{ psi } (180.6 \text{ MPa})\end{aligned}$$

This loss due to friction is 12.93 percent of the initial prestress.

3.7 ANCHORAGE-SEATING LOSSES *(A)*

Anchorage-seating losses occur in post-tensioned members due to the seating of wedges in the anchors when the jacking force is transferred to the anchorage. They can also occur in the prestressing casting beds of pretensioned members due to the adjustment

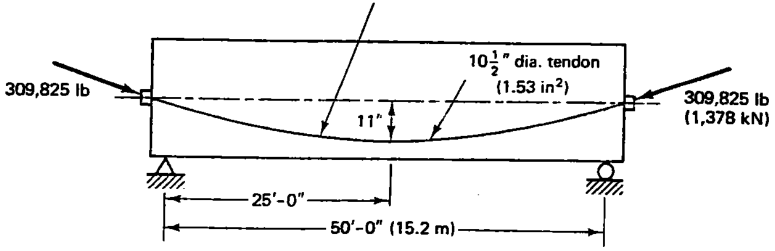


Figure 3.7 Prestressing tendon alignment.

expected when the prestressing force is transferred to these beds. A remedy for this loss can be easily effected during the stressing operations by overstressing. Generally, the magnitude of anchorage-seating loss ranges between $\frac{1}{4}$ in. and $\frac{3}{8}$ in. (6.35 mm and 9.53 mm) for the two-piece wedges. The magnitude of the overstressing that is necessary depends on the anchorage system used since each system has its particular adjustment needs, and the manufacturer is expected to supply the data on the slip expected due to anchorage adjustment. If Δ_A is the magnitude of the slip, L is the tendon length, and E_{ps} is the modulus of the prestressing wires, then the prestress loss due to anchorage slip becomes

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_{ps} \tag{3.24}$$

3.7.1 Computation of Anchorage-Seating Loss

Example 3.7

Compute the anchorage-seating loss in the post-tensioned beam of Example 3.2 if the estimated slip is $\frac{1}{4}$ in. (6.35 m).

Solution:

$$E_{ps} = 27 \times 10^6 \text{ psi}$$

$$\Delta_A = 0.25 \text{ in.}$$

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_{ps} = \frac{0.25}{50 \times 12} \times 27 \times 10^6 = 11{,}250 \text{ psi (77.6 MPa)}$$

Note that the percentage of loss due to anchorage slip becomes very high in short-beam elements and thus becomes of major significance in short-span beams. In such cases, it becomes difficult to post-tension such beams with high accuracy.

Photo 3.2 Terracentre, Denver, Colorado. (*Courtesy,* Post-Tensioning Institute.)

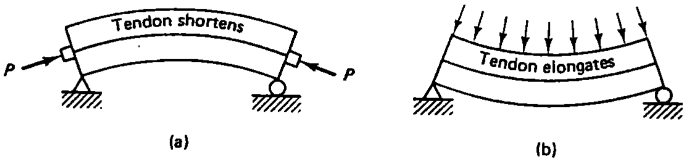


Figure 3.8 Change in beam longitudinal shape. (a) Due to prestressing. (b) Due to external load.

3.8 CHANGE OF PRESTRESS DUE TO BENDING OF A MEMBER (Δf_{pB})

As the beam bends due to prestress or external load, it becomes convex or concave depending on the nature of the load, as shown in Figure 3.8. If the unit compressive strain in the concrete along the level of the tendon is ϵ_c, then the corresponding change in prestress in the steel is

$$\Delta f_{pB} = \epsilon_c E_{ps}$$

where E_{ps} is the modulus of the steel. Note that any loss due to bending need not be taken into consideration if the prestressing stress level is measured after the beam has already bent, as is usually the case.

Figure 3.9 presents a flowchart for step-by-step evaluation of time-dependent prestress losses without deflection.

3.9 STEP-BY-STEP COMPUTATION OF ALL TIME-DEPENDENT LOSSES IN A PRETENSIONED BEAM

Example 3.8

A simply supported pretensioned 70-ft-span lightweight steam-cured double T-beam as shown in Figure 3.10 is prestressed by twelve ½-in. diameter (twelve 12.7 mm dia) 270-K grade stress-relieved strands. The strands are harped, and the eccentricity at midspan is 18.73 in. (476 mm) and at the end 12.98 in. (330 mm). Compute the prestress loss at the critical section in the beam of 0.40 span due to dead load and superimposed dead load at

(a) stage I at transfer
(b) stage II after concrete topping is placed
(c) two years after concrete topping is placed

Suppose the topping is 2 in. (51 mm) normal-weight concrete cast at 30 days. Suppose also that prestress transfer occurred 18 h after tensioning the strands. Given

$$f'_c = 5{,}000 \text{ psi, lightweight (34.5 MPa)}$$
$$f'_{ci} = 3{,}500 \text{ psi (24.1 MPa)}$$

and the following noncomposite section properties.

$$A_c = 615 \text{ in.}^2 \text{ (3,968 cm}^2\text{)}$$
$$I_c = 59{,}720 \text{ in.}^4 \text{ (}2.49 \times 10^6 \text{ cm}^4\text{)}$$
$$c_b = 21.98 \text{ in. (55.8 cm)}$$
$$c^t = 10.02 \text{ in. (25.5 cm)}$$
$$S_b = 2{,}717 \text{ in.}^3 \text{ (44,520 cm}^3\text{)}$$
$$S^t = 5{,}960 \text{ in.}^3 \text{ (97,670 cm}^3\text{)}$$

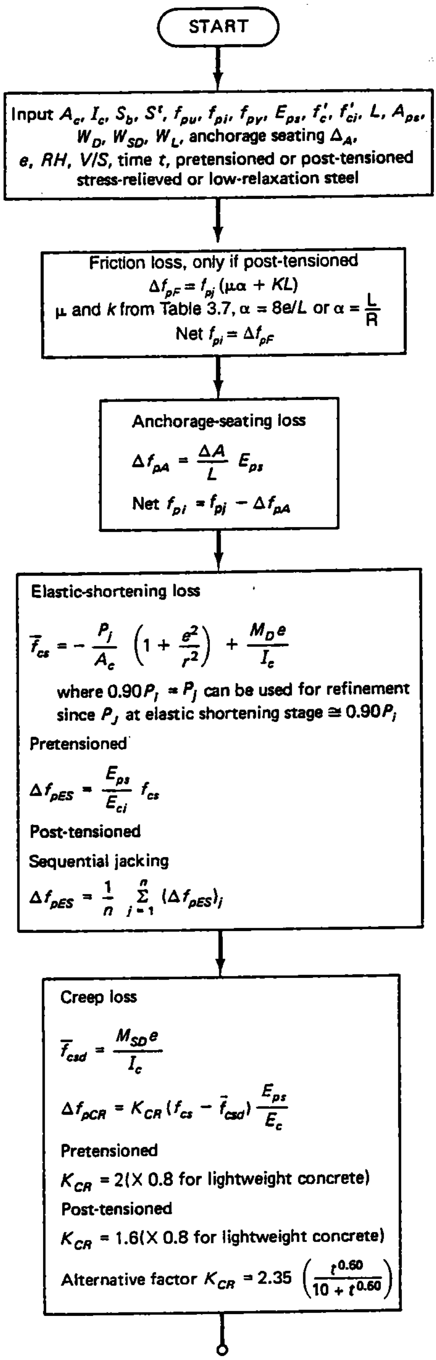
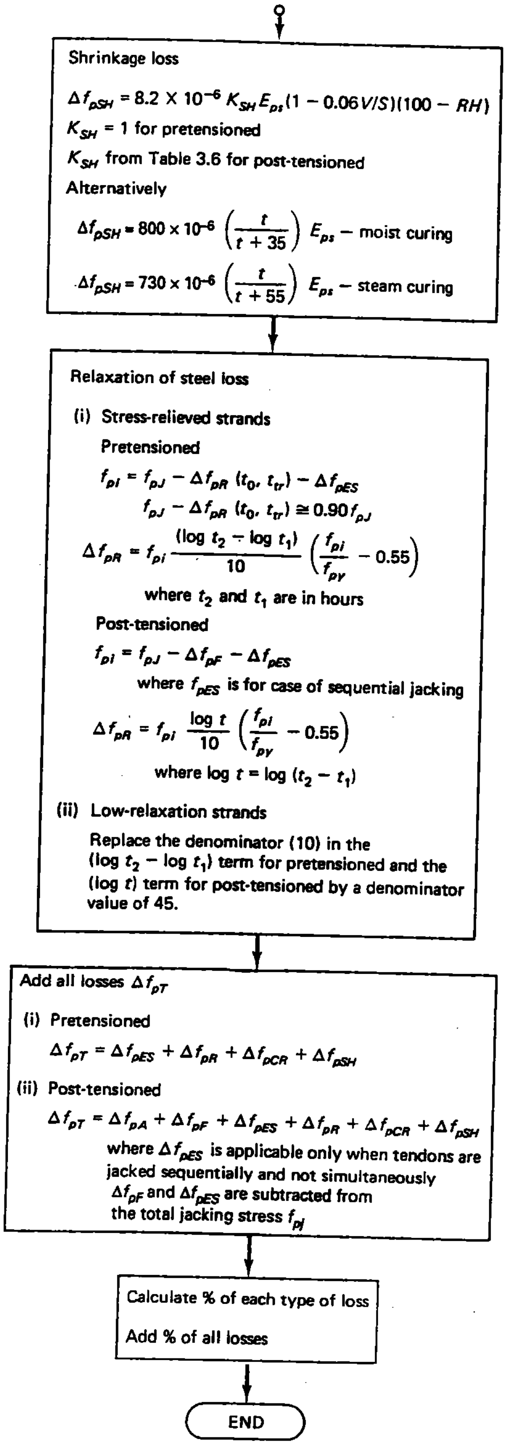


Figure 3.9 Flowchart for step-by-step evaluation of prestress losses.



Shrinkage loss

$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} E_{ps}(1 - 0.06 V/S)(100 - RH)$

$K_{SH} = 1$ for pretensioned

K_{SH} from Table 3.6 for post-tensioned

Alternatively

$\Delta f_{pSH} = 800 \times 10^{-6} \left(\frac{t}{t+35}\right) E_{ps}$ – moist curing

$\Delta f_{pSH} = 730 \times 10^{-6} \left(\frac{t}{t+55}\right) E_{ps}$ – steam curing

Relaxation of steel loss

(i) Stress-relieved strands

Pretensioned

$f_{pi} = f_{pJ} - \Delta f_{pR}(t_0, t_{tr}) - \Delta f_{pES}$

$f_{pJ} - \Delta f_{pR}(t_0, t_{tr}) \cong 0.90 f_{pJ}$

$$\Delta f_{pR} = f_{pi} \frac{(\log t_2 - \log t_1)}{10} \left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

where t_2 and t_1 are in hours

Post-tensioned

$f_{pi} = f_{pJ} - \Delta f_{pF} - \Delta f_{pES}$

where f_{pES} is for case of sequential jacking

$$\Delta f_{pR} = f_{pi} \frac{\log t}{10} \left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

where $\log t = \log (t_2 - t_1)$

(ii) Low-relaxation strands

Replace the denominator (10) in the ($\log t_2 - \log t_1$) term for pretensioned and the ($\log t$) term for post-tensioned by a denominator value of 45.

Add all losses Δf_{pT}

(i) Pretensioned

$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH}$

(ii) Post-tensioned

$\Delta f_{pT} = \Delta f_{pA} + \Delta f_{pF} + \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH}$

where Δf_{pES} is applicable only when tendons are jacked sequentially and not simultaneously

Δf_{pF} and Δf_{pES} are subtracted from the total jacking stress f_{pJ}

Calculate % of each type of loss

Add % of all losses

Figure 3.9 *Continued*

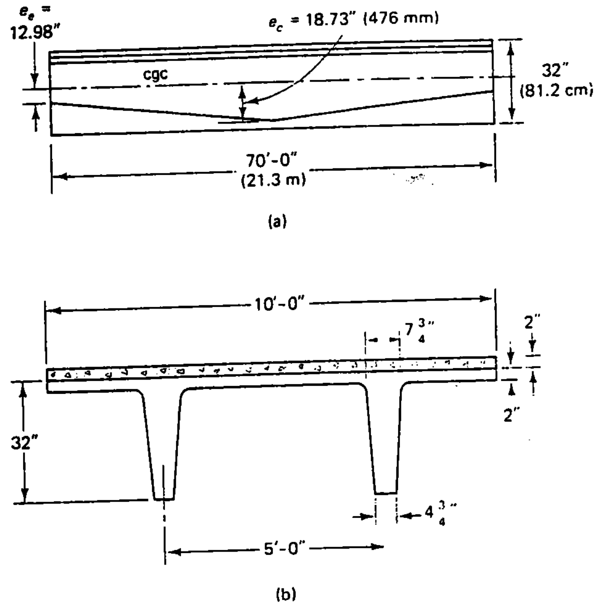


Figure 3.10 Double-T pretensioned beam. (a) Elevation. (b) Pretensioned section.

$$W_D \text{ (no topping)} = 491 \text{ plf } (7.2 \text{ kN/m})$$

$$W_{SD}(\text{2-in. topping}) = 250 \text{ plf } (3.65 \text{ kN/m})$$

$$W_L = 40 \text{ psf } (1{,}915 \text{ Pa})\text{—Transient}$$

$$f_{pu} = 270{,}000 \text{ psi } (1{,}862 \text{ MPa})$$

$$f_{py} = 0.85 f_{pu} \approx 230{,}000 \text{ psi } (1{,}589 \text{ MPa})$$

$$f_{pi} = 0.70 f_{pu} = 0.82 f_{py} = 0.82 \times 0.85 f_{pu} \cong 0.70 f_{pu}$$

$$= 189{,}000 \text{ psi } (1{,}303 \text{ MPa})$$

$$E_{ps} = 28 \times 10^6 \text{ psi } (193.1 \times 10^3 \text{ MPa})$$

Solution:

$$f'_{ci} = 3{,}500 \text{ psi}$$

$$E_{ci} = 115^{1.5}(33\sqrt{3{,}500}) = 2.41 \times 10^6 \text{ psi}$$

$$E_c = 115^{1.5}(33\sqrt{5{,}000}) = 2.88 \times 10^6 \text{ psi}$$

Stage 1: Stress Transfer

(a) *Elastic shortening.* Given critical section distance from support = 0.40 × 70 = 28 ft, e at critical section = 12.98 + 0.8(18.73 − 12.98) = 17.58 in. Dead-load moment M_D at 0.40 of the span is

$$M_D = W_D\frac{x}{2}(L - x) = 491\left(\frac{28}{2}\right)(70 - 28)$$

$$= 288{,}708 \text{ ft-lb} = 3{,}464{,}496 \text{ in.-lb (391 kN-m)}$$

$$f_{pi} = 0.70f_{pu} = 0.70 \times 270{,}000 = 189.000 \text{ psi}$$

Assume elastic-shortening loss and steel-relaxation loss ≅ 18,000 psi; then net-steel stress $f_{pi} = 189{,}000 - 18.000 = 171{,}000$ psi, and

$$P_i = A_{ps}f_{pi} = 12 \times 0.153 \times 171.000 = 313{,}956 \text{ lb}$$

$$r^2 = \frac{I_c}{A_c} = \frac{59{,}720}{615} = 97.11 \text{ in}^2$$

$$\bar{f}_{cs} = -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_De}{I_c}$$

$$= -\frac{313{,}956}{615}\left(1 + \frac{(17.58)^2}{97.11}\right) + \frac{3{,}464{,}496 \times 17.58}{59{,}720}$$

$$= -2{,}135.2 + 1{,}019.9 = -1.115.3 \text{ psi (7.7 MPa)}$$

$$n = \frac{E_{ps}}{E_{ci}} = \frac{28 \times 10^6}{2.41 \times 10^6} = 11.62$$

$$\Delta f_{pES} = n\bar{f}_{CS} = 11.62 \times 1{,}115.3 = 12{,}958 \text{ psi (85.4 MPa)}$$

If $f_{pi} = 189{,}000$ psi is used, then the net $f_{pi} = 189{,}000 - 12{,}958 = 176{,}042$ psi, and we have

$$\bar{f}_{cs} = -2{,}135.20 \times \frac{176{,}042}{171{,}000} + 1{,}019.90 = -1178.3 \text{ psi}$$

$$\Delta f_{pES} = nf_{CS} = 11.62 \times 1178.3 = 13{,}690 \text{ psi (94 MPa)}$$

vs. 12,985 psi in the refined solution, a small difference of ~6 percent. Thus, an assumption of 10-percent loss at the beginning in estimating $P_i \cong 0.9P_J$ would have been adequate.

(b) *Steel-Stress Relaxation.* Calculate the steel relaxation at transfer.

$$f_{py} = 230{,}000 \text{ psi}$$

$$f_{pi} = 189{,}000 \text{ psi (or net } f_{pi} = 171{,}000 \text{ psi could be used)}$$

$$t = 18 \text{ hours}$$

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

$$= 189{,}000\left(\frac{\log 18}{10}\right)\left(\frac{189{,}000}{230{,}000} - 0.55\right)$$

$$= 6{,}446.9 \text{ psi} \cong 6{,}447 \text{ psi}$$

$$\Delta f_{pES} + \Delta f_{pR} = 12{,}958 + 6{,}447 = 19{,}405 \cong 18{,}000 \text{ psi, assumed OK.}$$

(c) *Creep Loss*

$$\Delta f_{pCR} = 0$$

(d) *Shrinkage Loss*

$$\Delta f_{pSH} = 0$$

The stage-I total losses are

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH}$$

$$= 12{,}958 + 6{,}447 + 0 + 0 = 19{,}405 \text{ psi (134 MPa)}$$

The strand stress f_{pi} at the end of stage I = 189,000 – 19,405 = 169,595 psi (1,169 MPa), giving P_i = 311,376 psi.

Stage II: Transfer to Placement of Topping after 30 Days

(a) *Creep Loss*

$$E_c = 2.88 \times 10^6 \text{ psi}$$

$$E_{ps} = 28 \times 10^6 \text{ psi}$$

$$n = \frac{E_{ps}}{E_c} = \frac{28 \times 10^6}{2.88 \times 10^6} = 9.72$$

$$\bar{f}_{cs} = 1{,}115.3 \text{ psi}$$

Intensity of 2-in. normal-weight concrete topping:

$$W_{SD} = \frac{2}{12} \times 10 \times 150 = 250 \text{ plf}$$

The moment due to the 2-in. topping is

$$M_{SD} = W_{SD}\left(\frac{x}{2}\right)(L - x) = 250\left(\frac{28}{2}\right)(70 - 28) \times 12 = 1{,}764{,}000 \text{ in.-lb (199 kN-m)}$$

$$\bar{f}_{csd} = \frac{M_{SD}e}{I_c} = \frac{1{,}764{,}000 \times 17.58}{59{,}720} = 519.3 \text{ psi}$$

Although 30 days' duration is short for long-term effects. sufficient approximation can be justified in stage II using the creep factor K_{CR} of Equation 3.11 to account for stage III as well (see stage-III creep calculations).

For lightweight concrete, use $K_{CR} = 2.0 \times 80\% = 1.6$. Then, from Equation 3.10, the prestress loss due to long-term creep is

$$\Delta f_{pCR} = nK_{CR}(\bar{f}_{cs} - \bar{f}_{csd}) = 9.72 \times 1.6(1{,}115.3 - 519.3) = 9{,}269 \text{ psi (63.3 MPa)}$$

(b) *Shrinkage Loss.* Assume relative humidity $RH = 70\%$. Then, from Equation 3.14, the prestress loss due to long-term shrinkage is

$$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} E_{ps}\left(1 - 0.06\frac{V}{S}\right)(100 - RH)$$

$$\frac{V}{S} = \frac{615}{364} = 1.69 \text{ from geometry}$$

$K_{SH} = 1.0$ for pretensioned members; hence,

$$\Delta f_{pSH} = 8.2 \times 10^{-6} \times 1.0 \times 28 \times 10^6(1 - 0.06 \times 1.69)(100 - 70)$$
$$= 6{,}190 \text{ psi (42.7 MPa)}$$

(c) *Steel Relaxation Loss at 30 Days*

$$t_1 = 18 \text{ hours}$$

$$t_2 = 30 \text{ days} = 30 \times 24 = 720 \text{ hours}$$

$$f_{ps} = 169{,}595 \text{ psi from stage I}$$

$$\Delta f_{pR} = 169{,}595\left(\frac{\log 720 - \log 18}{10}\right)\left(\frac{169{,}595}{230{,}000} - 0.55\right) = 5{,}091 \text{ psi (35.1 MPa)}$$

Stage-II total loss is

$$\Delta f_{pT} = \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_{pR} = 9{,}269 + 6{,}190 + 5{,}091 = 20{,}550 \text{ psi (142 MPa)}$$

The increase in stress in the strands due to the addition of topping is

$$f_{SD} = n\bar{f}_{csd} = 9.72 \times 519.3 = 5.048 \text{ psi (34.8 MPa)}$$

Hence, the strand stress at the end of stage II is

$$f_{pe} = f_{ps} - \Delta f_{pT} + f_{SD} = 169{,}595 - 20{,}550 + 5.048 = 154{,}093 \text{ psi (1,062 MPa)}$$

Stage III: At End of Two Years

The values for long-term creep and long-term shrinkage evaluated for stage II are assumed not to have increased significantly, since the long-term values of K_{CR} for creep and K_{SH} for shrinkage were used in stage-II computations. Accordingly,

$$f_{pe} = 154{,}093 \text{ psi (1,066 MPa)}$$

$$t_1 = 30 \text{ days} = 720 \text{ hours}$$

$$t_2 = 2 \text{ years} \times 365 \times 24 = 17.520 \text{ hours}$$

The steel relaxation stress loss is

$$\Delta f_{pR} = 154{,}093\left(\frac{\log 17.520 - \log 720}{10}\right)\left(\frac{154{,}093}{230{,}000} - 0.55\right) = 2{,}563 \text{ psi (17.7 MPa)}$$

So the strand stress f_{pe} at the end of stage III ≅ 154,093 − 2,563 = 151,530 psi (1,033 MPa).

Summary of Stresses

Stress level at various stages	Steel stress, psi	Percent
After tensioning (0.70 f_{pu})	189,000	100.0
Elastic shortening loss	−12,958	−6.9
Creep loss	−9,269	−4.9
Shrinkage loss	−6,190	−3.3
Relaxation loss (6,447 + 5,091 + 2,563)	−14,101	−7.5
Increase due to topping	5,048	2.7
Final net stress f_{pe}	151,530 psi (1,045 MPa)	80.1

Percentages of total losses = 100 − 80.1 = 19.9%, say, 20% for this pretensioned beam.

3.10 STEP-BY-STEP COMPUTATION OF ALL TIME-DEPENDENT LOSSES IN A POST-TENSIONED BEAM

Example 3.9

Solve Example 3.8 assuming that the beam is post-tensioned. Assume also that the anchorage seating loss is ¼ in. and that all strands are simultaneously tensioned in a flexible duct. Also assume that the total jacking force prior to the friction and anchorage seating losses resulted in f_{pi} = 189,000 psi ($f_{pJ} = f_{pi}$ of Equation 3.1d in this case).

Solution:

(a) *Anchorage seating loss*

$$\Delta_A = \frac{1''}{4} = 0.25'' \qquad L = 70 \text{ ft}$$

From Equation 3.24, the anchorage slip stress loss is

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_{Ps} = \frac{0.25}{70 \times 12} \times 28 \times 10^6 \cong 8333 \text{ psi (40.2 MPa)}$$

(b) *Elastic shortening.* Since all jacks are simultaneously post-tensioned, the elastic shortening will precipitate during jacking. As a result, no elastic shortening stress loss takes place in the tendons. Hence, $\Delta f_{pES} = 0$.

(c) *Frictional loss.* Assume that the parabolic tendon approximates the shape of an arc of a circle. Then, from Equation 3.23,

$$\alpha = \frac{8y}{x} = \frac{8(18.73 - 12.98)}{70 \times 12} = 0.0548 \text{ radian}$$

From Table 3.7, use $K = 0.001$ and $\mu = 0.25$. Then, from Example 3.8,

$$f_{pi} = 189{,}000 \text{ psi (1,303 MPa)}$$

From Equation 3.22, the stress loss in prestress due to friction is

$$\begin{aligned}\Delta f_{pF} &= f_{pi}(\mu\alpha + KL)\\ &= 189{,}000(0.25 \times 0.0548 + 0.001 \times 70)\\ &= 15{,}819 \text{ psi (109 MPa)}\end{aligned}$$

The stress remaining in the prestressing steel after all initial instantaneous losses is

$$f_{pi} = 189{,}000 - 8{,}333 - 0 - 15{,}819 = 164{,}848 \text{ psi (1,136 MPa)}$$

Hence, the net prestressing force is

$$P_i = 164{,}848 \times 12 \times 0.153 = 296{,}726 \text{ lb}$$

compared to $P_i = 311{,}376$ lb in the pretensioned case of Example 3.8.

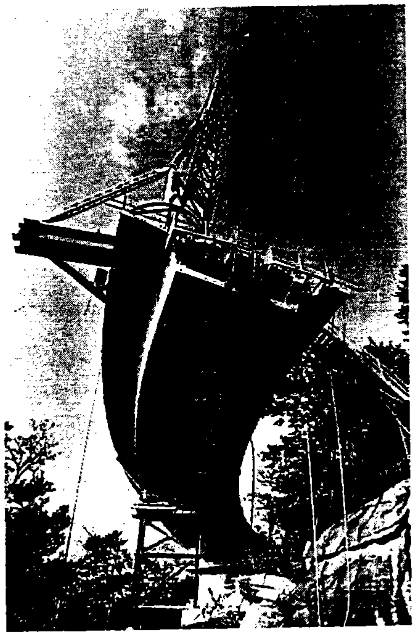


Photo 3.3 Linn Cove Viaduct, Grandfather Mountain, North Carolina. A 90° cantilever and a 10 percent superelevation in one direction to a full 10 percent in the opposite direction within 180 ft. Designed by Figg and Muller Engineers, Inc., Tallahassee, Florida. (*Courtesy,* Figg and Muller Engineers, Inc.)

Stage I: Stress at Transfer

(a) *Anchorage Seating Loss*

$$\text{Loss} = 8{,}333 \text{ psi}$$
$$\text{Net stress} = 164.848 \text{ psi}$$

(b) *Relaxation Loss*

$$\Delta f_{pR} = 164{,}848\left(\frac{\log 18}{10}\right)\left(\frac{164{,}848}{230{,}000} - 0.55\right)$$
$$\cong 3{,}450 \text{ psi } (23.8 \text{ MPa})$$

(c) *Creep Loss*

$$\Delta f_{pCR} = 0$$

(d) *Shrinkage Loss*

$$\Delta f_{pSH} = 0$$

So the tendon stress f_{pi} at the end of stage I is

$$164{,}848 - 3{,}450 = 161{,}398 \text{ psi } (1{,}113 \text{ MPa})$$

Stage II: Transfer to Placement of Topping after 30 Days

(a) *Creep Loss*

$$P_i = 161{,}398 \times 12 \times 0.153 = 296{,}327 \text{ lb}$$
$$\bar{f}_{cs} = -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_D e}{I_c}$$
$$= -\frac{296{,}327}{615}\left(1 + \frac{(17.58)^2}{97.11}\right) + \frac{3{,}464{,}496 \times 17.58}{59{,}720}$$
$$= -2{,}016.20 + 1{,}020.00 = -996.2 \text{ psi } (6.94 \text{ MPa})$$

Hence, the creep loss:
For lightweight concrete, K_{CR} is reduced by 20%, hence $= 1.6 \times 0.80 = 1.28$.

$$\Delta f_{pCR} = nK_{CR}(\bar{f}_{cs} - \bar{f}_{csd})$$
$$= 9.72 \times 1.28(996.2 - 519.3) \cong 5{,}933 \text{ psi } (41 \text{ MPa})$$

(b) *Shrinkage Loss.* From Example 3.8, for $K_{SH} = 0.58$ at 30 days, Table 3.6,

$$\Delta f_{pSH} = 6{,}190 \times 0.58 = 3{,}590 \text{ psi } (24.8 \text{ MPa})$$

(c) *Steel Relaxation Loss at 30 Days*

$$f_{ps} = 161{,}398 \text{ psi}$$

The relaxation loss in stress becomes

$$\Delta f_{pR} = 161{,}398\left(\frac{\log 720 - \log 18}{10}\right)\left(\frac{161{,}398}{230{,}000} - 0.55\right)$$
$$\cong 3{,}923 \text{ psi } (27.0 \text{ MPa})$$

Stage II: Total Losses

$$\Delta f_{pT} = \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_{pR}$$
$$= 5{,}933 + 3{,}590 + 3{,}923 = 13{,}446 \text{ psi } (93 \text{ MPa})$$

From Example 3.8, the increase in stress in the strands due to the addition of topping, is $f_{SD} = 5{,}048$ psi (34.8 MPa); hence, the strand stress at the end of stage II is

$$f_{pe} = f_{ps} - \Delta f_{pT} + \Delta f_{SD} = 161{,}398 - 13{,}446 + 5{,}048 = 153{,}000 \text{ psi (1,055 MPa)}$$

Stage III: At End of 2 Years

$$f_{pe} = 153{,}000 \text{ psi}$$
$$t_1 = 720 \text{ hours}$$
$$t_2 = 17{,}520 \text{ hours}$$

The steel relaxation stress loss is

$$\Delta f_{pR} = 153{,}000\left(\frac{\log 17{,}520 - \log 720}{10}\right)\left(\frac{153{,}000}{230{,}000} - 0.55\right)$$
$$\cong 2{,}444 \text{ psi (16.9 MPa)}$$

Using the same assumptions for stage III creep and shrinkage as in Example 3.8, the strand stress f_{pe} at the end of stage III is approximately

$$153{,}000 - 2{,}444 = 150{,}536 \text{ psi (1,038 MPa)}$$

Summary of Stresses

Stress level at various stages	Steel stress psi	Percent
After tensioning ($0.70f_{pu}$)	189,000	100.0
Elastic shortening loss	0	0.0
Anchorage loss*	–8,333	–4.4
Frictional loss*	–15,819	–8.4
Creep loss	–5,933	–3.1
Shrinkage loss	–3,590	–1.9
Relaxation loss (3,450 + 3,923 + 2,444)	–9,817	–5.2
Increase due to topping	+5,048	+2.7
Final net stress f_{pe}	150,556	79.7
Percentage of total losses = 100 – 79.7 = 20.3% beam.		

*Frictional and anchorage seating losses are included in this table since the total jacking stress is given as 189,000 psi; otherwise the tendons would have to be jacked an additional stress of such a magnitude as to neutralize the frictional and anchorage seating losses.

3.11 LUMP-SUM COMPUTATION OF TIME-DEPENDENT LOSSES IN PRESTRESS

Example 3.10

Solve Examples 3.8 and 3.9 by the approximate lump-sum method, and compare the results.

Solution for Example 3.8. From Table 3.1, the total loss $\Delta f_{pT} = 45{,}000$ psi (228 MPa). So the net final strand stress by this method is

$$f_{pe} = 189{,}000 - 45{,}000 = 144{,}000 \text{ psi (993 MPa)}$$
$$\text{Step-by-step } f_{pe} \text{ value} = 151{,}530$$
$$\text{Percent difference} = \frac{151{,}530 - 144{,}000}{151{,}530} = 5.0\%$$

Solution for Example 3.9. From Table 3.2, the total loss $\Delta_{pT} = 35{,}000$ psi (241 MPa). So the net final strand stress by the lump-sum method is

$$f_{pe} = 189{,}000 - 35{,}000 = 154{,}000 \text{ psi } (1{,}062 \text{ MPa})$$

$$\text{Step-by-step } f_{pe} \text{ value} = 150{,}556$$

$$\text{Percent difference} = \frac{154{,}000 - 149{,}232}{154{,}000} = 2.3\%$$

In both cases, the difference between the step-by-step "exact" method and the approximate lump-sum method is quite small, indicating that in normal, standard cases both methods are equally reliable.

3.12 SI PRESTRESS LOSS EXPRESSIONS

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f'_{pi}}{f_{py}} - 0.55\right) \tag{3.8}$$

for stress-relieved tendons where t is in hours. The denominator 10 becomes 45 for low-relaxation tendons.

$$\Delta f_{pCR} = nK_{CR}(\bar{f}_{cs} - \bar{f}_{csd}) \tag{3.11b}$$

where for normal concrete

$$\begin{aligned} K_{CR} &= 2.0 \text{ for pretensioned} \\ &= 1.6 \text{ for post-tensioned} \end{aligned}$$

reduced by 20% for lightweight concrete.

$$n = \text{modular ratio} = \frac{E_{ps}}{E_c}$$

$$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} E_{ps}\left(1 - 0.06\frac{V}{S}\right)(100 - RH) \tag{3.14}$$

$$\begin{aligned} K_{SH} &= 1.0, \text{ pretensioned} \\ &= \text{range of } 0.92 \text{ (1 day) to } 0.45 \text{ (60 days)} \end{aligned}$$

Equation 3.15a, moist curing for 7 days

$$\epsilon_{SH,t} = \left[\frac{t}{t + 35}\right]\epsilon_{SH,u}$$

where $\epsilon_{SH,t} = 800 \times 10^{-6}$ mm/mm

Equation 3.15b, steam curing 1 to 3 days max

$$\epsilon_{SH,t} = \left[\frac{t}{t + 55}\right]\epsilon_{SH,u}$$

where $\epsilon_{SH,t} = 730 \times 10^{-6}$ mm/mm

$$\Delta f_{pF} = -f_1(\mu\alpha + 3.28KL) \tag{3.22}$$

where L, meter.

$$\Delta f_{pA} = \left(\frac{\Delta_A}{L}\right)E_{ps} \tag{3.24}$$

$$E_c = w^{1.5}0.043\sqrt{f'_c} \quad w\text{ (lightweight)} \simeq 1830\text{ Kg/m}^3.$$
$$E_{ci} = w^{1.5}0.043\sqrt{f'_{ci}}$$
$$MPa = 10^6\text{ N/m}^2 = \text{N/mm}^2$$
$$\text{(psi) } 0.006895 = \text{MPa}$$
$$\text{(lb/ft) } 14.593 = \text{N/m}$$
$$\text{(in.-lb)} = 0.113 = \text{N-m}$$

3.12.1 SI Prestress Loss Example

Example 3.11

Solve Example 3.9 using SI units for losses in prestress, considering self-weight and superimposed dead load only.

Data

$f'_c = 34.5$ MPa

$f'_{ci} = 24.1$ MPa

$A_c = 3{,}968\text{ cm}^2$ $\quad S^t = 97{,}670\text{ cm}^3$

$I_c = 2.49 \times 10^6\text{ cm}^4$ $\quad S_b = 44{,}520\text{ cm}^3$

$r^2 = I_c/A_c = 626$

$c_b = 55.8$ cm $\quad c^t = 25.5$ cm

$e_c = 47.6$ cm $\quad e_e = 33.0$ cm

$f_{pu} = 1{,}860$ MPa

$f_{py} = 0.85f_{pu} = 1{,}580$ MPa

$f_{pi} = 0.82f_{py} = (0.82 \times 0.85)f_{pu} = 0.7f_{pu} = 1{,}300$ MPa

$E_{ps} = 193{,}000$ MPa

Span $l = 21.3$ m

A_{ps} = twelve tendons, 12.7-mm diameter (99 mm^2)

$= 12 \times 99 = 1{,}188\text{ mm}^2$

$M_D = 391$ kN-m $\quad M_{SD} = 199$ kN-m

$\Delta_A = 0.64$ cm

$V/S = 1.69$ $\quad RH = 70\%$

Solution:

(a) *Anchorage seating loss*

$$\Delta A = 0.64\text{ cm} \qquad l = 21.3\text{ m}$$

$$\Delta f_{pA} = \frac{\Delta_A}{l}(E_{ps}) = \frac{0.64}{21.3 \times 100} \times 193{,}000 = 58.0\text{ MPa}$$

(b) *Elastic Shortening*

Since all jacks are simultaneously tensioned, the elastic shortening will simultaneously precipitate during jacking. As a result, no elastic shortening loss takes place in the tendons.

Hence

$$\Delta f_{pES} = 0.$$

(c) *Frictional Loss*

Assume that the parabolic tendon approximates the shape of an arc of a circle. Then, from Equation 3.23,

$$\alpha = \frac{8y}{x} = \frac{8(e_c - e_e)}{l} = \frac{8(47.6 - 33.0)}{21.3 \times 100}$$
$$= 0.055 \text{ radians}$$

From Table 3.7, $K = 0.001$, $\mu = 0.25$, $f_{pi} = 1{,}300$ MPa
From Equation 3.22,

$$\begin{aligned}\Delta f_{pF} &= f_{pi}(\mu\alpha + 3.28KL)\\ &= 1{,}300(0.25 \times 0.055 + 0.001 \times 3.28 \times 21.3)\\ &= 109 \text{ MPa}\end{aligned}$$

The stress remaining in the prestressing steel after all instantaneous stresses

$$f_{pi} = 1{,}300 - (58 + 0 + 109) = 1{,}133 \text{ MPa}$$

Hence, the net prestressing force is

$$P_i = f_{pi}A_{ps} = 1{,}133 \times 1{,}188 = 1.35 \times 10^6 \text{ N}$$

Stage I: Stress at Transfer

(a) *Anchorage Seating Loss* $\Delta_{fA} = 58$ MPa

(b) *Relaxation Loss*
From Equation 3.8,

$$\begin{aligned}\Delta f_{pR} &= f_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f'_{pi}}{f_{py}} - 0.55\right)\\ &= 1{,}133\left(\frac{\log 18 - \log 0}{10}\right)\left(\frac{1{,}133}{1{,}580} - 0.55\right)\\ &= 24.4 \text{ MPa}\end{aligned}$$

(c) *Creep Loss* $\Delta f_{cR} = 0$

(d) *Shrinkage Loss* $\Delta f_{SH} = 0$
Tendon stress at the end of stage I

$$f_{ps} = 1{,}133 - 24.4 \cong 1{,}108 \text{ MPa}$$

Stage II: Transfer to Placement of Topping After 30 Days

(a) *Creep Loss*

$P_i = 1{,}108 \times 1{,}188 = 1.32 \times 10^6$ N

$$\bar{f}_{cs} = -\frac{P_i}{A_c}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_D e_b}{I_c} \quad \text{at tendons centroid}$$

e at 0.4 of span = 17.58 in. = 44.7 cm

$$\bar{f}_{cs} = -\frac{1.32 \times 10^6}{3{,}968 \times 10^2}\left[1 + \frac{(44.6)^2}{626}\right] + \frac{3.91 \times 10^7 \text{ N-cm} \times 44.6 \text{ N/mm}^2}{2.49 \times 10^6} \times \frac{1}{100}$$
$$= -13.90 + 7.00 \text{ N/mm}^2 = 6.90 \text{ MPa at cgs}$$

w (lightweight) = 1,800 Kg/m^3

$$E_c\,(\text{lightweight}) = w^{1.5}\,0.043\sqrt{34.5}$$

$$= 1{,}830^{1.5} \times 0.043\sqrt{34.5} = 19{,}770\ \text{MPa}$$

$$n = \frac{E_{ps}}{E_c} = \frac{193{,}000}{19{,}770} = 9.76$$

$\bar{f}_{csd}$ = stress in concrete at cgs due to all superimposed dead loads after prestressing is accomplished.

$$\bar{f}_{csd} = \frac{M_{SD}e}{I_c} = \frac{1.99 \times 10^7\ \text{N-cm} \times 44.7}{2.49 \times 10^6} \times \frac{1}{100}\ \text{N/mm}^2$$

$$= 3.57\ \text{MPa}$$

$$K_{CR} = 1.6 \text{ for post-tensioned beam}$$

From Equation 3.11b,

$$\Delta f_{pCR} = nK_{CR}(\bar{f}_{cs} - \bar{f}_{csd})$$

$$= 9.76 \times 1.6(6.90 - 3.57) = 52.0\ \text{MPa}$$

(b) *Shrinkage Loss at 30 Days*
From Equation 3.14,

$$\Delta f_{pSH} = 8.2 \times 10^{-6}\,K_{SH}\,E_{ps}\left(1 = 0.06\frac{V}{S}\right)(100 - RH)$$

K_{SH} at 30 days = 0.58 (Table 3.6)

$$\Delta f_{pSH} = 8.2 \times 10^{-6} \times 0.58 \times 193{,}000(1 - 0.06 \times 1.69)(100 - 70)$$

$$= 24.7\ \text{MPa}$$

(c) *Relaxation Loss at 30 Days (720 Hrs)*

$$f_{ps} = 1{,}108\ \text{MPa}$$

$$\Delta f_{pR} = 1{,}108\left(\frac{\log 720 - \log 18}{10}\right)\left(\frac{1{,}108}{1{,}580} - 0.55\right)$$

$$= 110.8(2.85 - 1.25)0.151 = 26.8\ \text{MPa}$$

Stage II: Total Losses

$$\Delta f_{pT} = \Delta f_{pCR} + \Delta f_{pSH} + \Delta f_R$$

$$= 52.0 + 24.7 + 26.8 = 104\ \text{MPa}$$

Increase of tensile stress at bottom cgs fibers due to addition of topping is from before,

$$\Delta f_{SD} = nf_{CSD} = 9.76 \times 3.57 = 34.8\ \text{MPa}$$

$$f_{pe} = f_{ps} - \Delta f_{pT} + \Delta f_{SD}$$

$$= 1{,}108 - 103.5 + 34.5 = 1{,}039\ \text{MPa}$$

Stage III: At End of Two Years

f_{pe} = 1,039 MPa

t_1 = 720 hrs. t_2 = 17,520 hrs.

$$\Delta f_{pR} = 1{,}039\left(\frac{\log 17{,}520 - \log 720}{10}\right)\left(\frac{1{,}039}{1{,}580} - 0.55\right)$$

$$= 103.9(4.244 - 2.857)0.108 = 15.6\ \text{MPa}$$

On the assumption that Δf_{pCR} and Δf_{pSH} were stable in this case, the stress in the tendons at end of stage III can approximately be $f_{ps} = 1{,}039 - 15.6 \cong 1.020$ MPa.

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3.9 Nawy, E. G. *Reinforced Concrete—A Fundamental Approach*. 5th ed. Prentice-Hall, Upper Saddle River, NJ: 2003, 866 pp.

3.10 Cohn, M. Z. *Partial Prestressing, From Theory to Practice*. NATO-ASI Applied Science Series, vols 1 and 2. Dordrecht, The Netherlands: Martinus Nijhoff, in Cooperation with NATO Scientific Affairs Division, 1986.

3.11 ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-02)* and *Commentary (ACI 318 R-02)*, American Concrete Institute, Farmington Hills, MI, 2002, pp. 446.

3.12 ACI Committee 435, "Control of Deflection in Concrete Structures," ACI Committee Report R435-95, E. G. Nawy, Chairman, American Concrete Institute, Farmington Hills, MI, 1995, pp. 77.

PROBLEMS

3.1 A simply supported pretensioned beam has a span of 75 ft (22.9 m) and the cross section shown in Figure P3.1. It is subjected to a uniform gravitational live-load intensity $W_L = 1{,}200$ plf (17.5 kN/m) in addition to its self-weight and is prestressed with 20 stress-relieved $\frac{1}{2}$in. dia (12.7 mm dia) 7-wire strands. Compute the total prestress losses by the step-by-step method, and compare them with the values obtained by the lump-sum method. Take the following values as given:

$f'_c = 6{,}000$ psi (41.4 MPa), normal-weight concrete
$f'_{ci} = 4{,}500$ psi (31 MPa)
$f_{pu} = 270.000$ psi (1,862 MPa)
$f_{pi} = 0.70 f_{pu}$
Relaxation time $t = 5$ years
$e_c = 19$ in. (483 mm)
Relative humidity $RH = 75\%$
$V/S = 3.0$ in. (7.62 cm)

Assume SD load = 30% LL.

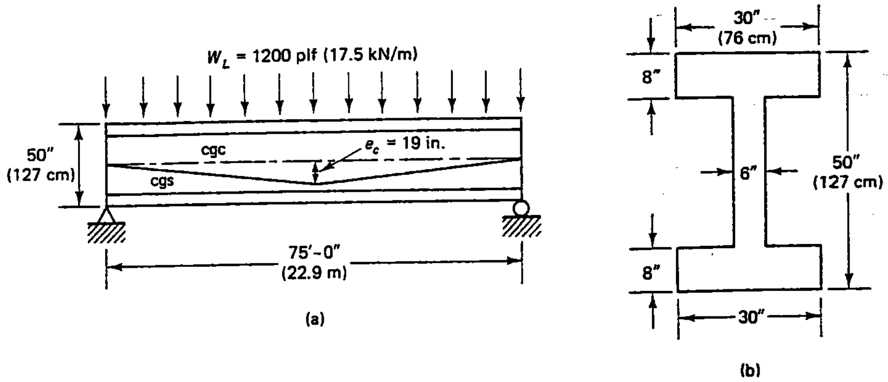


Figure P3.1 (a) Elevation. (b) Section.

3.2 Compute, by the detailed step-by-step method, the total losses in prestress of the 10-ft (3.28-m)-wide flange double T-beam in Example 1.1 which has a span of 64 ft (19.5 m) for a steel relaxation period of 7 years. Use $RH = 70\%$ and $V/S = 3.5$ in. (8.9 cm), and solve for both pretensioned and post-tensioned prestressing conditions. Assume SD load = 30% LL. In the post-tensioned case, assume that the total jacking stress prior to the friction and anchorage seating losses is 189,000 psi.

3.3 Compute, by the detailed step-by-step method, the total losses of prestress in the AASHTO 36-in. (91.4 cm)-deep beam used in Problem 1.1 and which has a span of 34 ft (10.4 m) for both the pretensioned and the post-tensioned case. Use all the data of Problem 1.1 in your solution, and assume that the relative humidity $RH = 70\%$ and the volume-to-surface ratio $V/S = 3.2$. Determine the steel relaxation losses at the end of the first year after erection and at the end of 4 years.

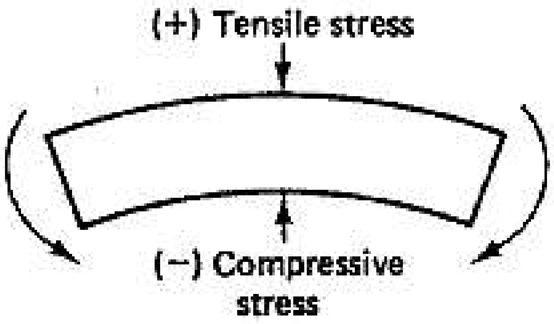
3.4 Compute, by the detailed step-by-step method, the total prestress losses of the simply supported double T-beam of Example 3.9 if it was post-tensioned using flexible ducts for the tendon. Assume that the tendon profile is essentially parabolic. Assume also that all strands are tensioned simultaneously and that the anchorage slip $\Delta_A = \frac{3}{8}$ in. (9.5 mm). All the data are identical to those of Example 3.8; the critical section is determined to be at a distance 0.4 times the span from the face of the support.

General design procedure

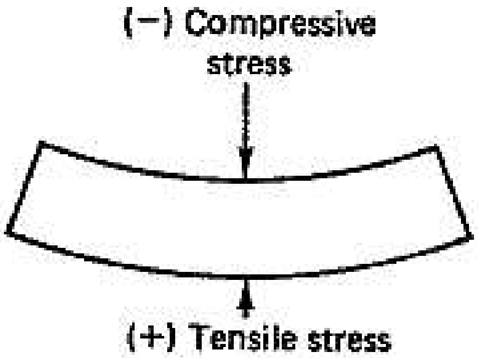
- Design process starts with the choice of a preliminary geometry.
- By trial and adjustment, it converge to the final section with geometrical details of the concrete section and the alignment of the prestressing strands.
- The section has to satisfy the flexural (bending) requirements of concrete stress and steel stress limitations.
- Other factors such as shear and torsion capacity, deflection and cracking are analyzed and satisfied.
- Additional checks are required at the load transfer and limit state at service load, as well as the limit state at failure, with the failure load indicating the reverse strength for overload conditions.
- All these checks are needed to ensure that at service load cracking is negligible and the long term effects on deflection are well controlled.
- A good understanding of the principles of analysis and the alternative presented thereby significantly simplifies the task of designing the section.

Sign convention

- A negative sign (-) is used to denote compressive stress and a positive sign (+) is used to denote the tensile stress in the concrete section.
- A convex or hogging shape indicates negative bending moment (a); a concave or sagging shape indicates positive bending moment (b).



(a)



(b)

Loading stages of prestressed concrete

Unlike the case of reinforced concrete members, the external dead load and partial live load are applied to the prestressed concrete member at varying concrete strengths at various loading stages. These loading stages can be summarized as follows:

- Initial prestress force P_i is applied; then, at transfer, the force is transmitted from the prestressing strands to the concrete.
- The full self-weight W_D acts on the member together with the initial prestressing force, provided that the member is simply supported, i.e., there is no intermediate support.
- The full superimposed dead load W_{SD} including topping for composite action, is applied to the member.
- Most short-term losses in the prestressing force occur, leading to a reduced prestressing force P_{eo}.
- The member is subjected to the full service load, with long-term losses due to creep, shrinkage, and strand relaxation taking place and leading to a net prestressing force P_e.
- Overloading of the member occurs under certain conditions up to the limit state at failure.

Stress distribution across the depth of the critical section

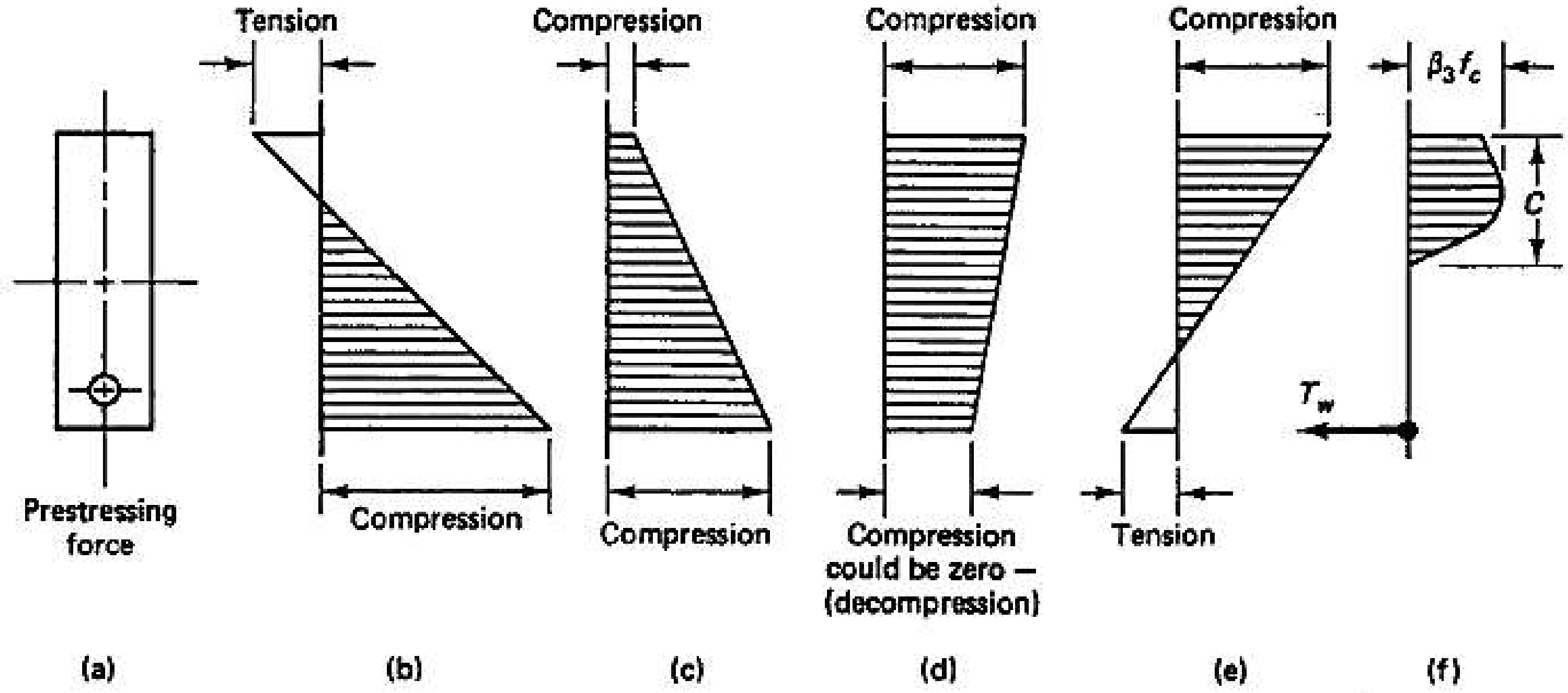


Figure 4.2 Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

Allowable stresses of concrete

f_{ci} = maximum allowable compressive stress in concrete immediately after transfer and prior to losses

$= 0.60\,f'_{ci}$

f_{ti} = maximum allowable tensile stress in concrete immediately after transfer and prior to losses

$= 3\sqrt{f'_{ci}}$ (the value can be increased to $6\sqrt{f'_{ci}}$ at the supports for simply supported members)

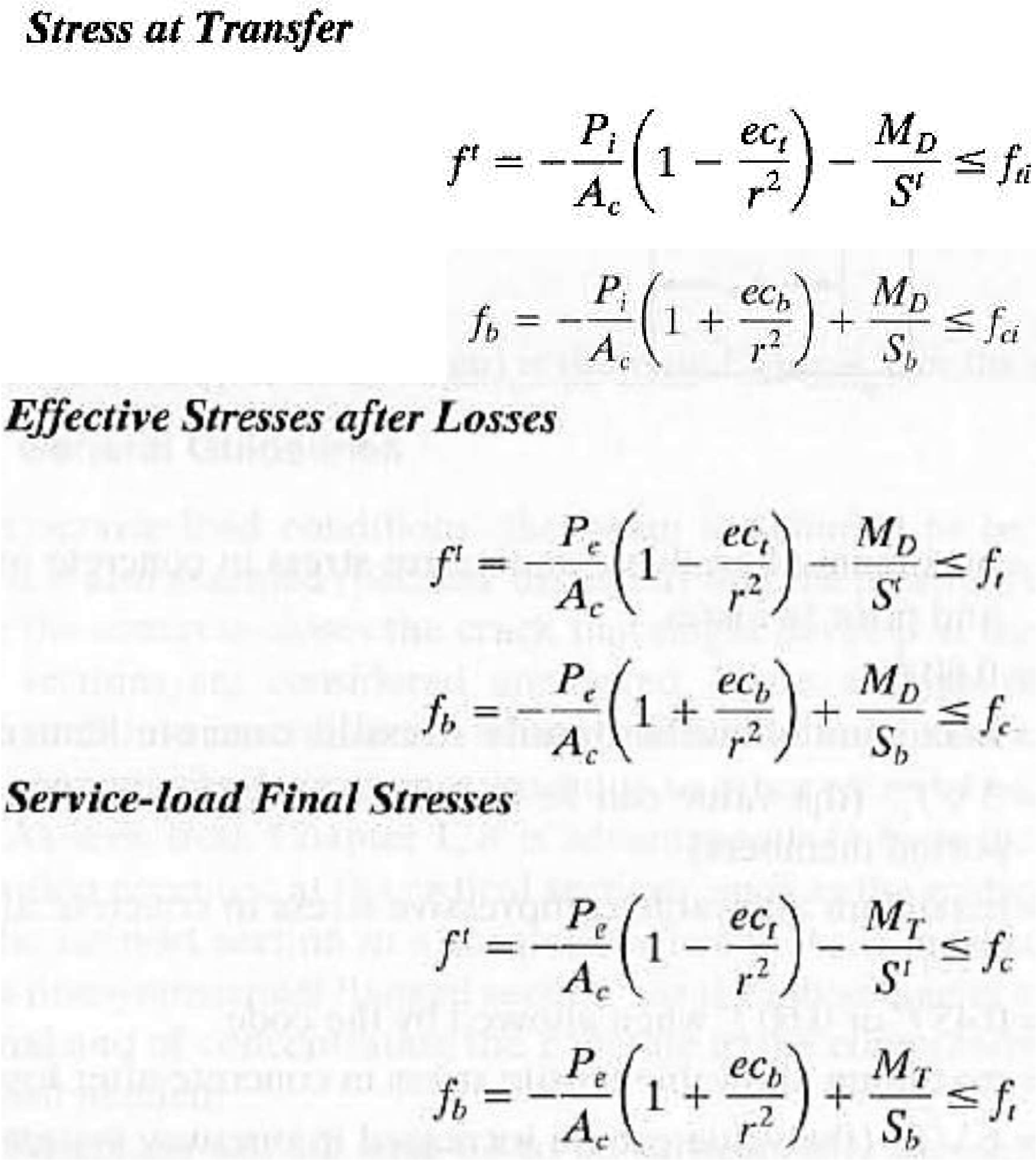
f_c = maximum allowable compressive stress in concrete after losses at service-load level

$= 0.45\,f'_c$ or $0.60\,f'_c$ when allowed by the code

f_t = maximum allowable tensile stress in concrete after losses at service load level

$= 6\sqrt{f'_c}$ (the value can be increased in one-way systems to $12\sqrt{f'_c}$ if long-term deflection requirements are met)

Summary of equations of stress



Stress at Transfer

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_{ci}$$

Effective Stresses after Losses

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_t$$

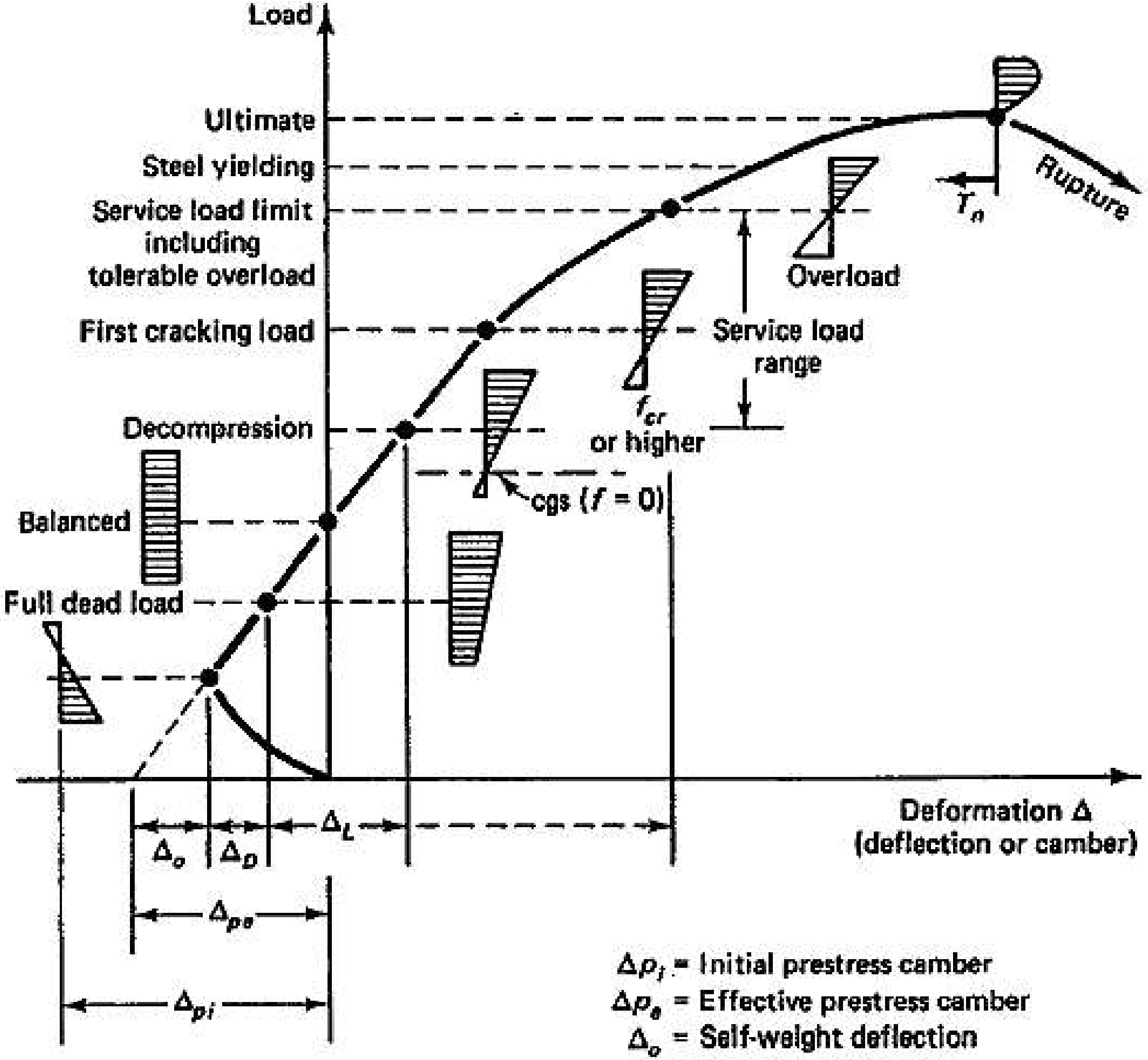
$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \leq f_c$$

Service-load Final Stresses

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \leq f_c$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \leq f_t$$

Load-deformation curve of typical prestressed beam



Δp_i = Initial prestress camber
Δp_e = Effective prestress camber
Δ_o = Self-weight deflection
Δ_D = Dead load deflection
Δ_L = Live load deflection

Decompression stage

- The decompression stage denotes the increase in steel strain due to the increase in load from the stage when the effective prestress (Pe) acts alone to the stage when additional load causes the compressive stress in the concrete at the cgs level reduce to zero.
- The change in concrete stress due to decompression is:

$$f_{\text{decomp}} = \frac{P_e}{A_c}\left(1 + \frac{e^2}{r^2}\right)$$

Selection of geometrical properties

1. *Minimum Section Modulus*:

- To design or choose the section, a determination of the required minimum section modulus has to be made first.
- Determination of the minimum section modulus is dependent on the prestressing steel profile where different equation can be used.

1.1 Beams with variable tendon eccentricity

The section should have section moduli values:

$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S_b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

The required eccentricity of prestressing tendons at the critical section (Mid span) is:

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

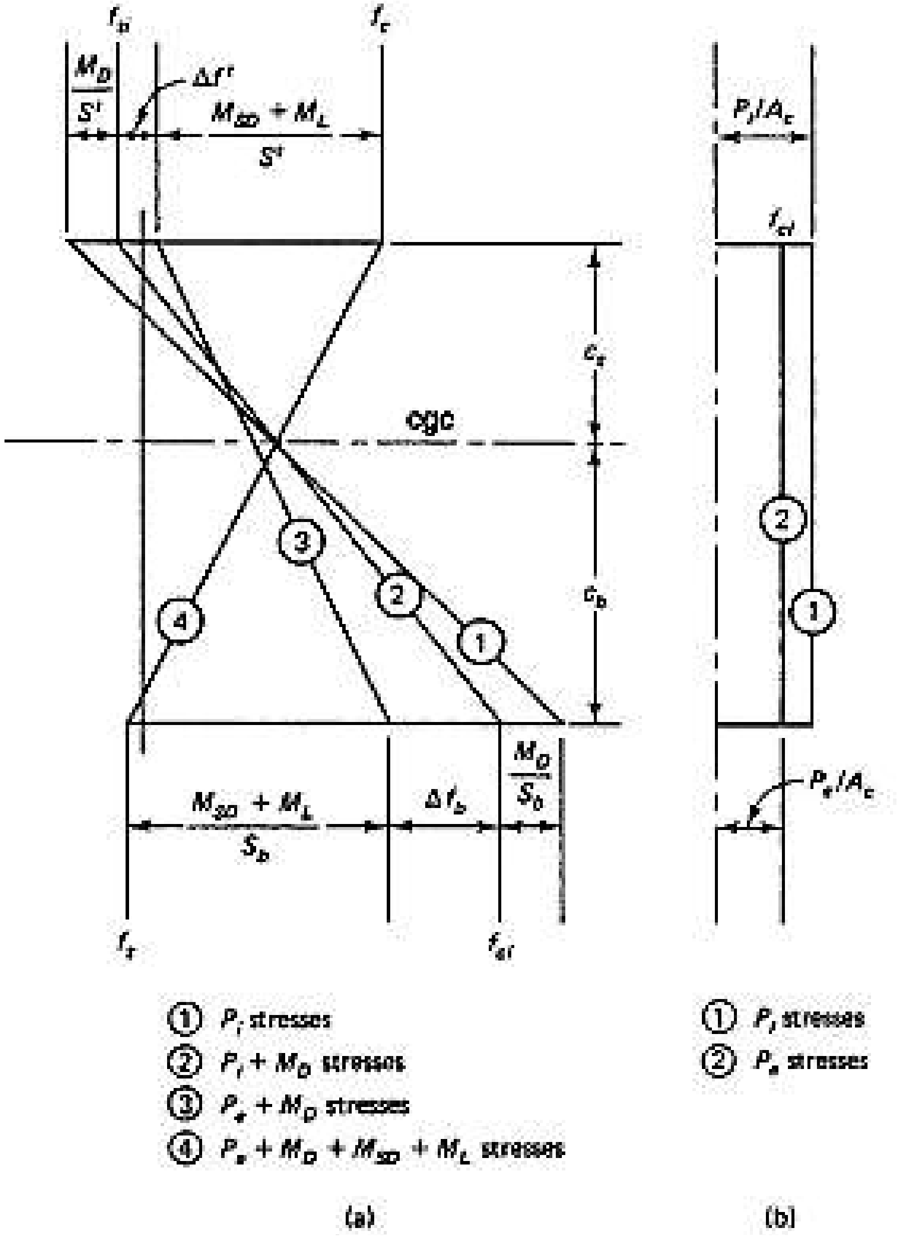
$$P_i = \bar{f}_{ci} A_c$$

Where $\bar{f}_{ci}$ is the concrete stress at transfer at the level of the centroid cgc of the concrete section and given as:

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

Maximum fiber stresses of Beams with variable tendon eccentricity

(a) Critical section. (b) Support section of simply supported beam.



1.2 Beams with constant tendon eccentricity

The section should have section moduli values:

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

The required eccentricity of prestressing tendons at the critical section is:

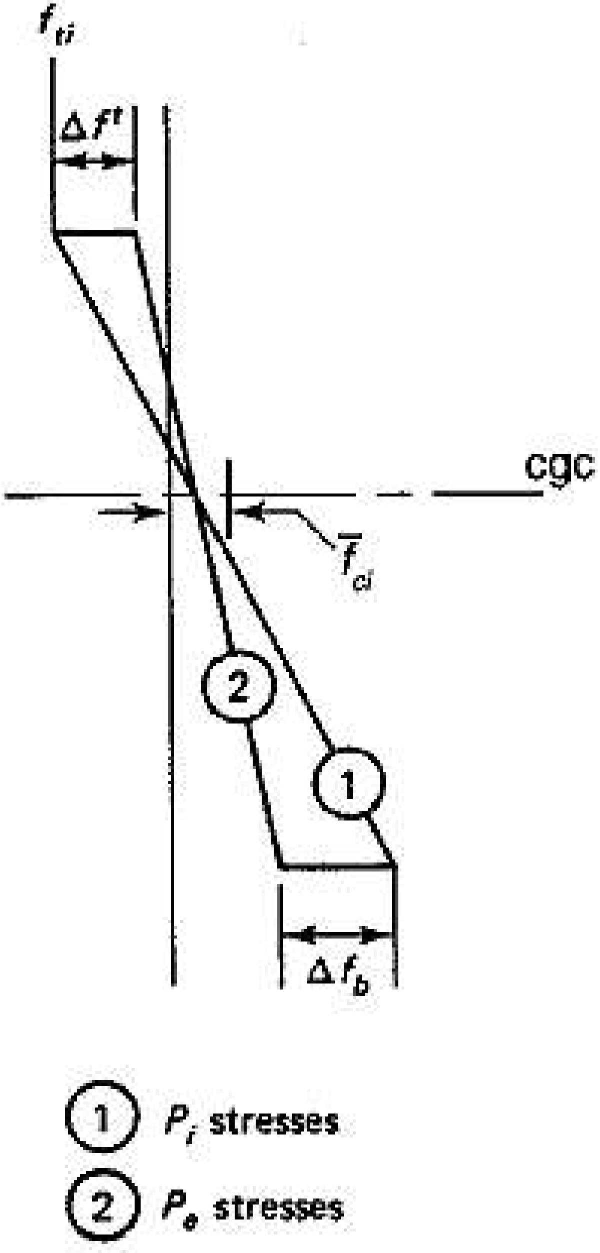
$$e_e = (f_{ti} - \bar{f}_{ci}) \frac{S^t}{P_i}$$

$$P_i = \bar{f}_{ci} A_c$$

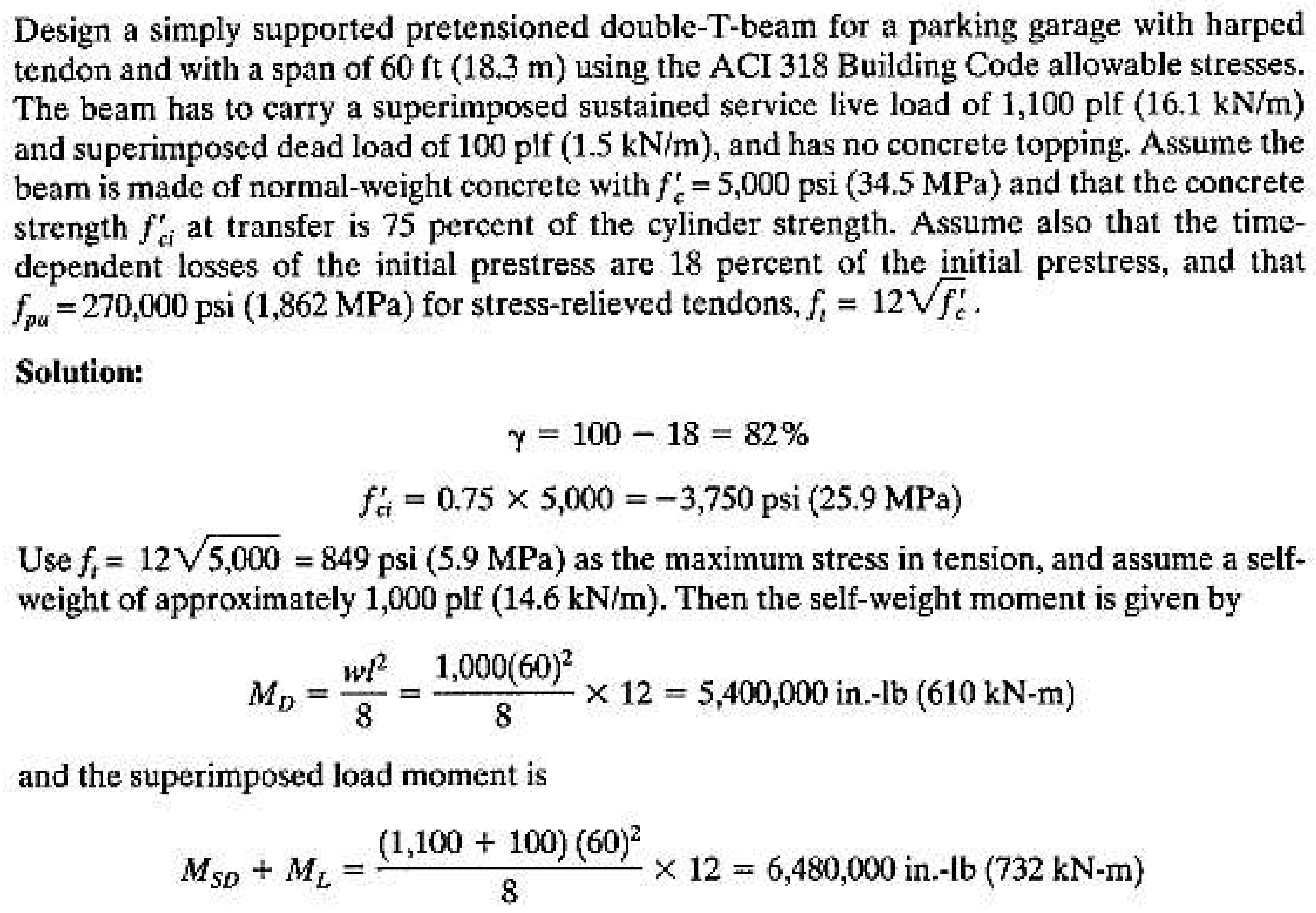
Where $\bar{f}_{ci}$ is the concrete stress at transfer at the level of the centroid cgc of the concrete section and given as:

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

Maximum fiber stresses of beam with straight tendons



Service load design example (1)
Variable tendon eccentricity



Design a simply supported pretensioned double-T-beam for a parking garage with harped tendon and with a span of 60 ft (18.3 m) using the ACI 318 Building Code allowable stresses. The beam has to carry a superimposed sustained service live load of 1,100 plf (16.1 kN/m) and superimposed dead load of 100 plf (1.5 kN/m), and has no concrete topping. Assume the beam is made of normal-weight concrete with $f'_c = 5{,}000$ psi (34.5 MPa) and that the concrete strength f'_{ci} at transfer is 75 percent of the cylinder strength. Assume also that the time-dependent losses of the initial prestress are 18 percent of the initial prestress, and that $f_{pu} = 270{,}000$ psi (1,862 MPa) for stress-relieved tendons, $f_t = 12\sqrt{f'_c}$.

Solution:

$$\gamma = 100 - 18 = 82\%$$

$$f'_{ci} = 0.75 \times 5{,}000 = -3{,}750 \text{ psi } (25.9 \text{ MPa})$$

Use $f_t = 12\sqrt{5{,}000} = 849$ psi (5.9 MPa) as the maximum stress in tension, and assume a self-weight of approximately 1,000 plf (14.6 kN/m). Then the self-weight moment is given by

$$M_D = \frac{wl^2}{8} = \frac{1{,}000(60)^2}{8} \times 12 = 5{,}400{,}000 \text{ in.-lb } (610 \text{ kN-m})$$

and the superimposed load moment is

$$M_{SD} + M_L = \frac{(1{,}100 + 100)\,(60)^2}{8} \times 12 = 6{,}480{,}000 \text{ in.-lb } (732 \text{ kN-m})$$

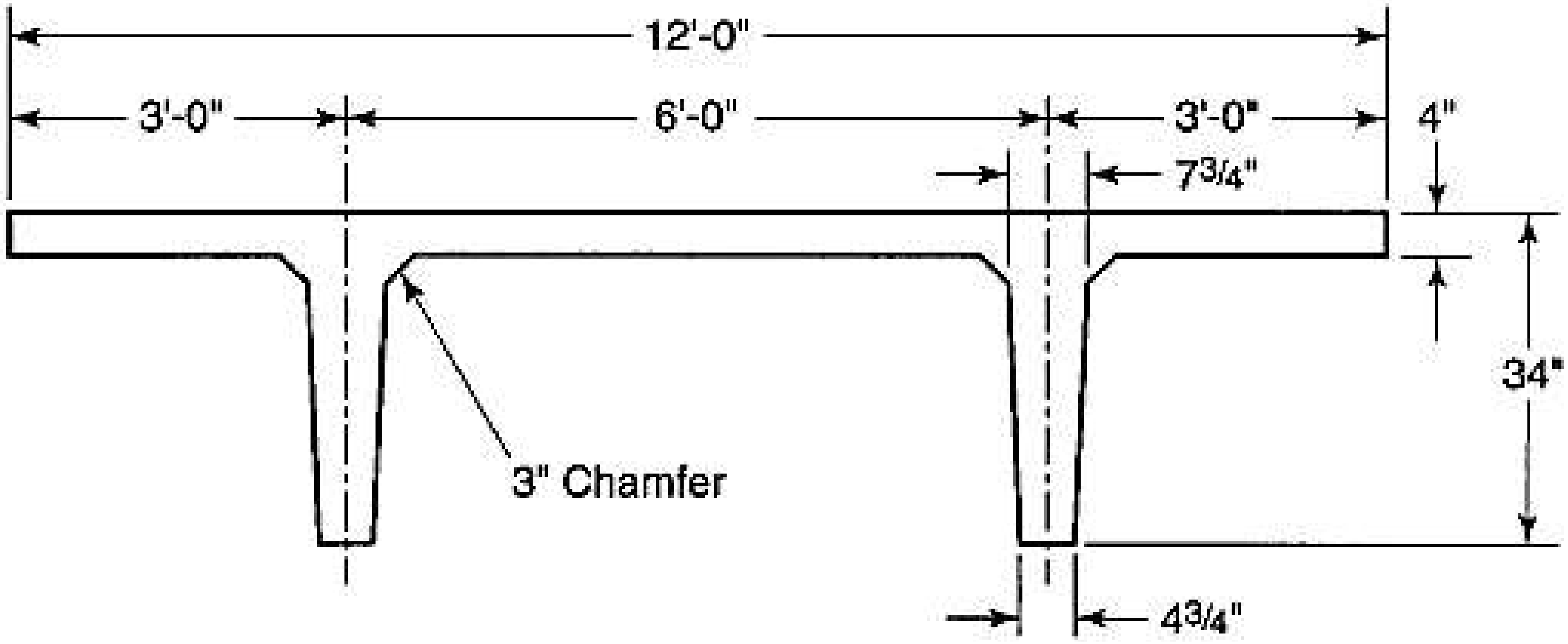
$$S^t \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$\geq \frac{(1-0.82)5{,}400{,}000 + 6{,}480{,}000}{0.82 \times 184 - (-2{,}250)} = 3{,}104 \text{ in}^3 \ (50{,}860 \text{ cm}^3)$$

$$S_b \geq \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

$$\geq \frac{(1-0.82)\ 5{,}400{,}000 + 6{,}480{,}000}{849 - 0.82(-2{,}250)} = 2{,}766 \text{ in.}^3 \ (45{,}330 \text{ cm})^3$$

From the PCI design handbook, select a nontopped normal weight concrete double-T 12 DT 34 168-D1, since it has the bottom-section modulus value S_b closest to the required value.



The section properties of the concrete are as follows:

$$A_c = 978 \text{ in.}^2$$

$$I_c = 86{,}072 \text{ in.}^4$$

$$r^2 = \frac{I_c}{A_c} = 88.0 \text{ in.}^2$$

$$S^t = 10{,}458 \text{ in.}^3$$

$$S_b = 3{,}340 \text{ in.}^3$$

$$c_t = 8.23 \text{ in.}$$

$$c_b = 25.77 \text{ in.}$$

$$e_c = 22.02 \text{ in.}$$

$$e_e = 12.77 \text{ in.}$$

$$W_D = 1{,}019 \text{ plf}$$

$$\frac{V}{S} = 2.39 \text{ in.}$$

Design of Strands and Check of Stresses. The assumed self-weight is close to the actual

$$M_D = \frac{1{,}019}{1{,}000} \times 5{,}400{,}000 = 5{,}502{,}600 \text{ in.-lb}$$

$$f_{pi} = 0.70 \times 270{,}000 = 189{,}000 \text{ psi}$$

$$f_{pe} = 0.82 f_{pi} = 0.82 \times 189{,}000 = 154{,}980 \text{ psi}$$

(a) *Analysis of Stresses at Transfer.* From Equation 4.1a,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \leq f_{ti} = 184 \text{ psi}$$

Then

$$184 = -\frac{P_i}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{5{,}502{,}600}{10{,}458}$$

$$P_i = (184 + 526.16)\,\frac{978}{1.06} = 655{,}223 \text{ lb.}$$

$$\text{Required number of tendons} = \frac{655{,}223}{189{,}000 \times 0.153} = 22.66\ \tfrac{1}{2}\text{-in. dia. tendons.}$$

Try sixteen ½-in. dia. strands for the standard section:

$$A_{ps} = 16 \times 0.153 = 2.448 \text{ in}^2\ (15.3 \text{ cm}^2)$$

$$P_i = 2.448 \times 189{,}000 = 462{,}672 \text{ lb } (2{,}058 \text{ kN})$$

$$P_e = 2.448 \times 154{,}980 = 379{,}391 \text{ lb } (1{,}688 \text{ kN})$$

(b) *Analysis of Stresses at Service Load at Midspan*

$$P_e = 379{,}391 \text{ lb}$$

$$M_{SD} = \frac{100(60)^2 12}{8} = 540{,}000 \text{ in.-lb } (61 \text{ kN-m})$$

$$M_L = \frac{1{,}100(60)^2 12}{8} = 5{,}940{,}000 \text{ in.-lb } (788 \text{ kN-m})$$

Total moment $M_T = M_D + M_{SD} + M_L = 5{,}502{,}600 + 6{,}480{,}000$
$= 11{,}982{,}600$ in.-lb (1,354 kN-m)

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{379{,}391}{978}\left(1 - \frac{22.02 \times 8.23}{88.0}\right) - \frac{11{,}982{,}600}{10{,}458}$$

$$= +411 - 1146 = -735 \text{ psi} < f_c = -2{,}250 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{379{,}391}{978}\left(1 + \frac{22.02 \times 25.77}{88.0}\right) + \frac{11{,}982{,}600}{3{,}340}$$

$$= -2{,}889 + 3{,}587 = +698 \text{ psi (T)} < f_t = +849 \text{ psi, O.K.}$$

(c) *Analysis of Stresses at Support Section*

$$e_e = 12.77 \text{ in. } (324 \text{ mm})$$

$$f_{ti} = 6\sqrt{f'_{ci}} = 6\sqrt{3{,}750} \cong 367 \text{ psi}$$

$$f_t = 12\sqrt{f'_c} = 12\sqrt{5{,}000} = 849 \text{ psi}$$

(i) *At Transfer*

$$f^t = -\frac{462{,}672}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +92 \text{ psi } (T)$$

$$f_b = -\frac{462{,}672}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -2{,}240 \text{ psi } (C)$$

$$< f_{ci} = -2{,}250 \text{ psi, O.K.}$$

If $f_b > f_{ci}$, the support eccentricity has to be changed.

(ii) *At Service Load*

$$f^t = -\frac{379{,}391}{978}\left(1 - \frac{12.77 \times 8.23}{88.0}\right) - 0 = +75 \text{ psi } (T) \quad < f_t = 849 \text{ psi, O.K.}$$

$$f_b = -\frac{379{,}391}{978}\left(1 + \frac{12.77 \times 25.77}{88.0}\right) + 0 = -1{,}840 \text{ psi } (C)$$

$$< f_c = -2{,}250 \text{ psi, O.K.}$$

Adopt the section for service-load conditions using sixteen $\frac{1}{2}$-in. (1.7 mm) strands with midspan eccentricity $e_c = 22.02$ in. (560 mm) and end eccentricity $e_e = 12.77$ in. (324 mm).

Service load design example (2)

Variable tendon eccentricity with no height limitation

Design an I-section for a beam having a 65-ft (19.8 m) span to satisfy the following section modulus values: Use the same allowable stresses and superimposed loads as in Example 4.1.

$$\text{Required } S^t = 3{,}570 \text{ in}^3 \ (58{,}535 \text{ cm}^3)$$

$$\text{Required } S_b = 3{,}780 \text{ in}^3 \ (61{,}940 \text{ cm}^3)$$

Solution

Since the section moduli at the top and bottom fibers are almost equal, a symmetrical section is adequate.

$I_c = 70{,}688 \text{ in.}^4$

$r^2 = 187.5 \text{ in.}^2$

$A_c = 377 \text{ in.}^2$

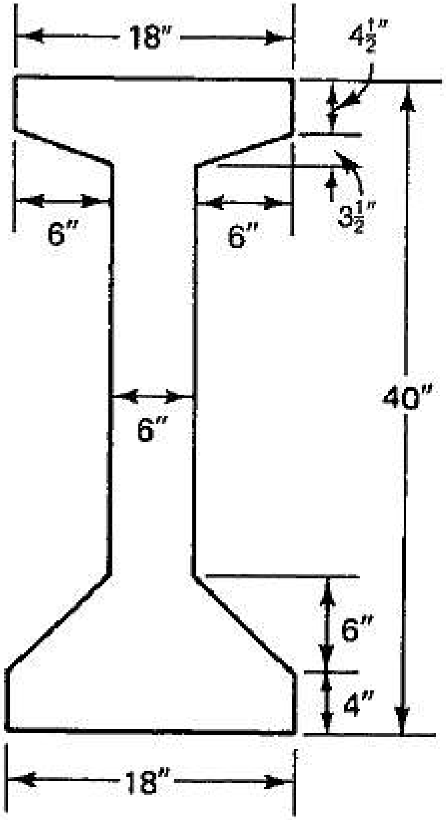
$c_t = 21.16 \text{ in.}$

$S^t = 3340 \text{ in.}^3$

$c_b = 18.84 \text{ in.}$

$S_b = 3750 \text{ in.}^3$

$w_D = 393 \text{ plf}$



Analysis of Stresses at Transfer.

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= +184 - \frac{21.16}{40}(+184 + 2{,}250) \cong -1{,}104 \text{ psi } (C)\ (7.6 \text{ MPa})$$

$$P_i = A_c \bar{f}_{ci} = 377 \times 1{,}104 = 416{,}208 \text{ lb } (1{,}851 \text{ kN})$$

$$M_D = \frac{393(65)^2}{8} \times 12 = 2{,}490{,}638 \text{ in.-lb } (281 \text{ kN-m})$$

the eccentricity required at the section of maximum moment at midspan is

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

$$= (184 + 1{,}104)\frac{3{,}340}{416{,}208} + \frac{2{,}490{,}638}{416{,}208}$$

$$= 10.34 + 5.98 = 16.32 \text{ in. } (415 \text{ mm})$$

Since $c_b = 18.84$ in., and assuming a cover of 3.75 in., try $e_c = 18.84 - 3.75 \cong 15.0$ in. (381 mm).

$$\text{Required area of strands } A_p = \frac{P_i}{f_{pi}} = \frac{416{,}208}{189{,}000} = 2.2 \text{ in}^2\ (14.2 \text{ cm}^2)$$

$$\text{Number of strands} = \frac{2.2}{0.153} = 14.38$$

Try thirteen $\frac{1}{2}$-in. strands, $A_p = 1.99$ in.^{2} (12.8 cm^2), and an actual $P_i = 189{,}000 \times 1.99 =$ 376,110 lb (1,673 kN), and check the concrete extreme fiber stresses.

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{376{,}110}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{2{,}490{,}638}{3{,}340}$$

$$= +691.2 - 745.7 = -55 \text{ psi } (C), \text{ no tension at transfer, O.K.}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b}$$

$$= -\frac{376{,}110}{377}\left(1 + \frac{15 \times 18.84}{187.5}\right) + \frac{2{,}490{,}638}{3{,}750}$$

$$= -2{,}501.3 + 664.2 = -1{,}837 \text{ psi } (C) < f_{ci} = 2{,}250 \text{ psi, O.K.}$$

Analysis of Stresses at Service Load.

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$P_e = 13 \times 0.153 \times 154{,}980 = 308{,}255 \text{ lb } (1{,}371 \text{ kN})$$

$$M_{SD} + M_L = \frac{(100 + 1100)(65)^2}{8} \times 12 = 7{,}605{,}000 \text{ m} - \text{lb}$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 2{,}490{,}638 + 7{,}605{,}000$$

$$= 10{,}095{,}638 \text{ in.-lb } (1{,}141 \text{ kN-m})$$

$$f^t = -\frac{308{,}225}{377}\left(1 - \frac{15.0 \times 21.16}{187.5}\right) - \frac{10{,}095{,}638}{3{,}340}$$

$$= +566.5 - 3{,}022.6 = -2{,}456 \text{ psi } (C) \;\; > f_c = -2{,}250 \text{ psi}$$

Hence, either enlarge the depth of the section or use higher strength concrete. Using $f'_c = 6{,}000$ psi,

$$f_c = 0.45 \times 6{,}000 = -2{,}700 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} = -\frac{308{,}255}{377}\left(1 + \frac{15.0 \times 18.84}{187.5}\right) + \frac{10{,}095{,}638}{3{,}750}$$

$$= \; - 2{,}050 + 2{,}692.2 = 642 \text{ psi } (T), \text{ O.K.}$$

Check Support Section Stresses

$$\text{Allowable} \quad f'_{ci} = 0.75 \times 6{,}000 = 4{,}500 \text{ psi}$$

$$f_{ci} = 0.60 \times 4{,}500 = 2{,}700 \text{ psi}$$

$$f_{ti} = 3\sqrt{f'_{ci}} = 201 \text{ psi for midspan}$$

$$f_{ti} = 6\sqrt{f'_{ci}} = 402 \text{ psi for support}$$

$$f_c = 0.45 f'_c = 2{,}700 \text{ psi}$$

$$f_{t1} = 6\sqrt{f'_c} = 465 \text{ psi}$$

$$f_{t2} = 12\sqrt{f'_c} = 930 \text{ psi}$$

(a) *At Transfer.* Support section compressive fiber stress.

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + 0$$

$$P_i = 376{,}110 \text{ lb}$$

or

$$-2{,}700 = -\frac{376{,}110}{377}\left(1 + \frac{e \times 18.84}{187.5}\right)$$

so that

$$e_e = 16.98 \text{ in.}$$

To ensure a tensile stress at the top fibers within the allowable limits, try $e_e = 12.49$ in.:

$$f^t = -\frac{376{,}110}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0$$

$$= 409 \text{ psi } (T) > f_{ti} = 402 \text{ psi}$$

$$f_b = -2250 \text{ psi}$$

Thus, use mild steel at the top fibers at the support section to take all tensile stresses in the concrete, or use a higher strength concrete for the section, or reduce the eccentricity.

(b) *At Service Load*

$$f^t = -\frac{308{,}255}{377}\left(1 - \frac{12.49 \times 21.16}{187.5}\right) - 0 = 335 \text{ psi } (T) \; < 930 \text{ psi, O.K.}$$

$$f_b = -\frac{308{,}255}{377}\left(1 + \frac{12.49 \times 18.84}{187.5}\right) + 0 = -1{,}844 \text{ psi } (C) < -2{,}700 \text{ psi, O.K.}$$

Hence, adopt the 40-in. (102-cm)-deep I-section prestressed beam of f'_c equal to 6,000 psi (41.4 MPa) normal-weight concrete with thirteen $\frac{1}{2}$-in. tendons having midspan eccentricity $e_c = 15.0$ in. (381 mm) and end section eccentricity $e_e = 12.5$ in. (318 m).

An alternative to this solution is to continue using $f'_c = 5{,}000$ psi, but change the number of strands and eccentricities.

Service load design example (3)
Constant tendon eccentricity

Solve Example 4.2 assuming that the prestressing tendon has constant eccentricity. Use $f'_c = 5{,}000$ psi (34.5 MPa) normal-weight concrete, permitting a maximum concrete tensile stress $f_t = 12\sqrt{f'_c} = 849$ psi.

Solution:

The required section modulus

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

Assume $W_D = 425$ plf. Then

$$M_D = \frac{425 \times (65)^2}{8} \times 12 = 2{,}693{,}438 \text{ in.-lb (304 kN-m)}$$

$$M_{SD} + M_L = 7{,}605{,}000 \text{ in.-lb (859 kN-m)}$$

Thus, the total moment $M_T = 10{,}298{,}438$ in.-lb (1,164 kN-m), and we also have

$$\text{Allowable } f_{ci} = -2{,}250 \text{ psi}$$

$$f'_{ci} = -3{,}750 \text{ psi}$$

$$f_{ti} = 6\sqrt{f'_{ci}} \text{ for support section } = 367 \text{ psi}$$

$$f_c = -2{,}250 \text{ psi (15.5 MPa)}$$

$$f_t = +849 \text{ psi}$$

$$\gamma = 0.82$$

$$\text{Required } S^t = \frac{10{,}298{,}438}{0.82 \times 184 + 2{,}250} = 4{,}289 \text{ in}^3 \text{ (72,210 cm}^3\text{)}$$

$$\text{Required } S_b = \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} = \frac{10{,}298{,}438}{849 + 0.82 \times 2{,}250}$$

$$= 3{,}823 \text{ in}^3 \text{ (62,713 cm}^3\text{)}$$

First Trial.

$I_c = 92{,}700$ in^4

$r^2 = 228.9$ in^2

$A_c = 405$ in^2

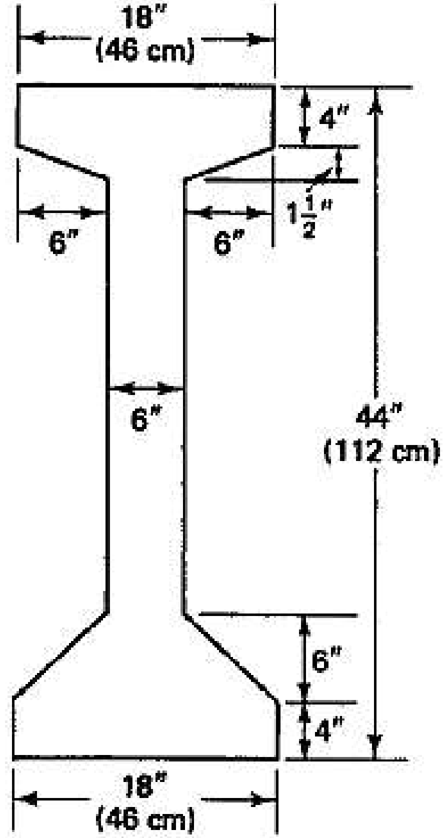
$c_t = 23.03$ in.

$S^t = 4{,}030$ in^3

$c_b = 20.97$ in

$S_b = 4{,}420$ in^3

$W_D = 422$ plf



the required eccentricity at the critical section at the support is

$$e_e = (f_{ti} - \bar{f}_{ci}) \frac{S^t}{P_i}$$

where

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$= 367 - \frac{23.03}{44}(367 + 2{,}250) = -1{,}002 \text{ psi } (6.9 \text{ MPa})$$

and

$$P_i = A_c \bar{f}_{ci} = 405 \times 1{,}002 = 405{,}810 \text{ lb } (1{,}805 \text{ kN})$$

Hence,

$$e = (367 + 1{,}002) \frac{4{,}030}{405{,}810} = 13.60 \text{ in. } (346 \text{ mm})$$

The required prestressed steel area is

$$A_p = \frac{P_i}{f_{pi}} = \frac{405{,}810}{189{,}000} = 2.15 \text{ in}^2 \ (14.4 \text{ cm}^2)$$

So we try $\frac{1}{2}$in. strands tendon. The required number of strands is 2.15/0.153 = 14.05. Accordingly, use fourteen $\frac{1}{2}$in. (12.7 mm) tendons. As a result,

$$P_i = 14 \times 0.153 \times 189{,}000 = 404{,}838 \text{ lb } (1{,}801 \text{ kN})$$

(a) *Analysis of Stresses at Transfer at End Section.*

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} = -\frac{404{,}838}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0$$

$$= +368.2 \text{ psi } (T) \cong f_{ti} = 367, \text{ O.K.}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} = -\frac{404{,}838}{405}\left(1 + \frac{13.6 \times 20.97}{228.9}\right) + 0$$

$$= -2{,}245 \text{ psi } (C) \cong f_{ci} = -2{,}250, \text{ O.K.}$$

(b) *Analysis of Final Service-Load Stresses at Support*

$$P_e = 14 \times 0.153 \times 154{,}980 = 331{,}967 \text{ lb } (1{,}477 \text{ kN})$$

$$\text{Total moment } M_T = M_D + M_{SD} + M_L = 0$$

From Equation 4.3a,

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{331{,}967}{405}\left(1 - \frac{13.60 \times 23.03}{228.9}\right) - 0 = 302 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{331{,}967}{405}\left(1 + \frac{13.60 \times 20.97}{228.9}\right) + 0$$

$$= -1{,}841 \text{ psi } (12.2 \text{ MPa}) (C) < f_c = -2{,}250 \text{ psi, O.K.}$$

(c) *Analysis of Final Service-Load Stresses at Midspan.* From before, the total moment $M_T = M_D + M_{SD} + M_L = 10{,}298{,}438$ in.-lb. Revised $w_D = 422$ plf = assumed $w_D = 425$ plf; hence, $M_T = 10{,}298{,}438$ in.-lb is sufficiently accurate. So the extreme concrete fiber stress due to M_T is

$$f_1^t = \frac{M_T}{S^t} = -\frac{10{,}298{,}438}{4{,}030} = -2{,}555 \text{ psi } (C)\ (17.6 \text{ MPa})$$

$$f_{1b} = \frac{M_T}{S_b} = \frac{10{,}298{,}438}{4{,}420} = +2{,}330 \text{ psi } (T)\ (16.1 \text{ MPa})$$

Hence, the final midspan fiber stresses are

$$f^t = +302 - 2{,}555 = -2{,}253 \text{ psi } (C) \cong f_c = -2{,}250 \text{ psi, accept}$$

$$f_b = -1{,}841 + 2{,}330 = +489 \text{ psi } (T) < f_t = 849 \text{ psi, O.K.}$$

Consequently, accept the trial section with a constant eccentricity $e = 13.60$ in. (345 mm) for the fourteen ½″ (12.7 mm dia.) tendons.

Envelops for tendon placement

- The tensile stress in the extreme concrete fiber under service load conditions cannot exceed the stresses allowable by codes.
- It is important to establish the limiting zone in the concrete section; an envelope within which the prestressing force can be applied without causing tension in the extreme concrete fibers.
- It can be determined by assuming the allowable stress equals to zero,

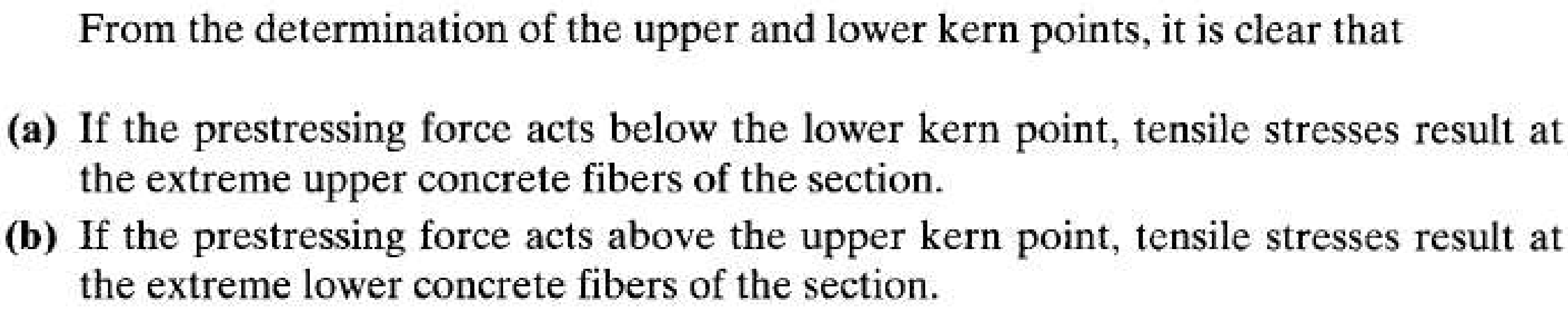
$$f_t = 0 = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right)$$

Giving $e = \frac{r^2}{c_t}$; Hence, the *lower kern point* is: $k_b = \frac{r^2}{c_t}$

Similarly, if $f_b = 0$,

$e = \frac{r^2}{c_\mathrm{b}}$; Hence, the *upper kern point* is: $k_t = \frac{r^2}{c_b}$

Envelops for tendon placement

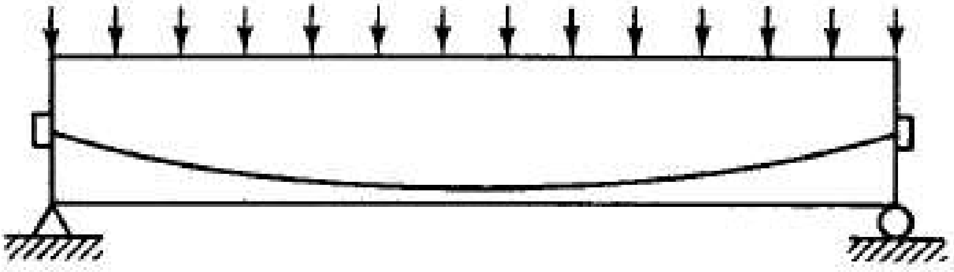


From the determination of the upper and lower kern points, it is clear that

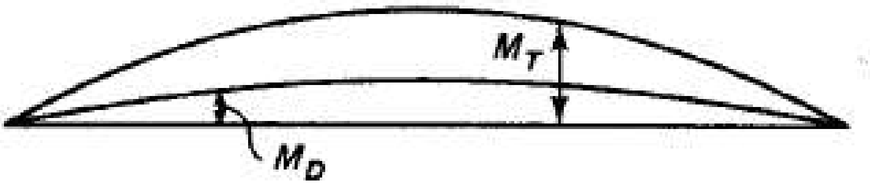
(a) If the prestressing force acts below the lower kern point, tensile stresses result at the extreme upper concrete fibers of the section.

(b) If the prestressing force acts above the upper kern point, tensile stresses result at the extreme lower concrete fibers of the section.

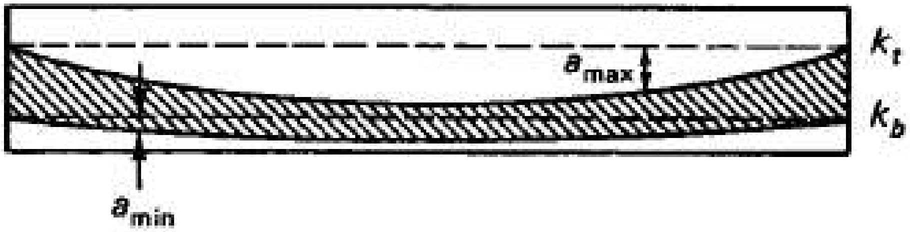
Limiting eccentricity envelopes



(a) One tendon location in beam.



(b) Bending moment diagram.



(c) Limiting cgs envelope.

Cgs envelope determination.

Limiting eccentricity envelopes

Lower cgs Envelope. The minimum arm of the tendon couple is

$$a_{\min} = \frac{M_D}{P_i}$$

This defines the maximum distance below the *bottom* kern where the cgs line is to be located so that the C-line does not fall *below* the *bottom* kern line, thereby preventing tensile stresses at the *top* extreme fibers. Hence, the limiting bottom eccentricity is

$$e_b = (a_{\min} + k_b)$$

Upper cgs Envelope. The maximum arm of the tendon couple is

$$a_{\max} = \frac{M_T}{P_e}$$

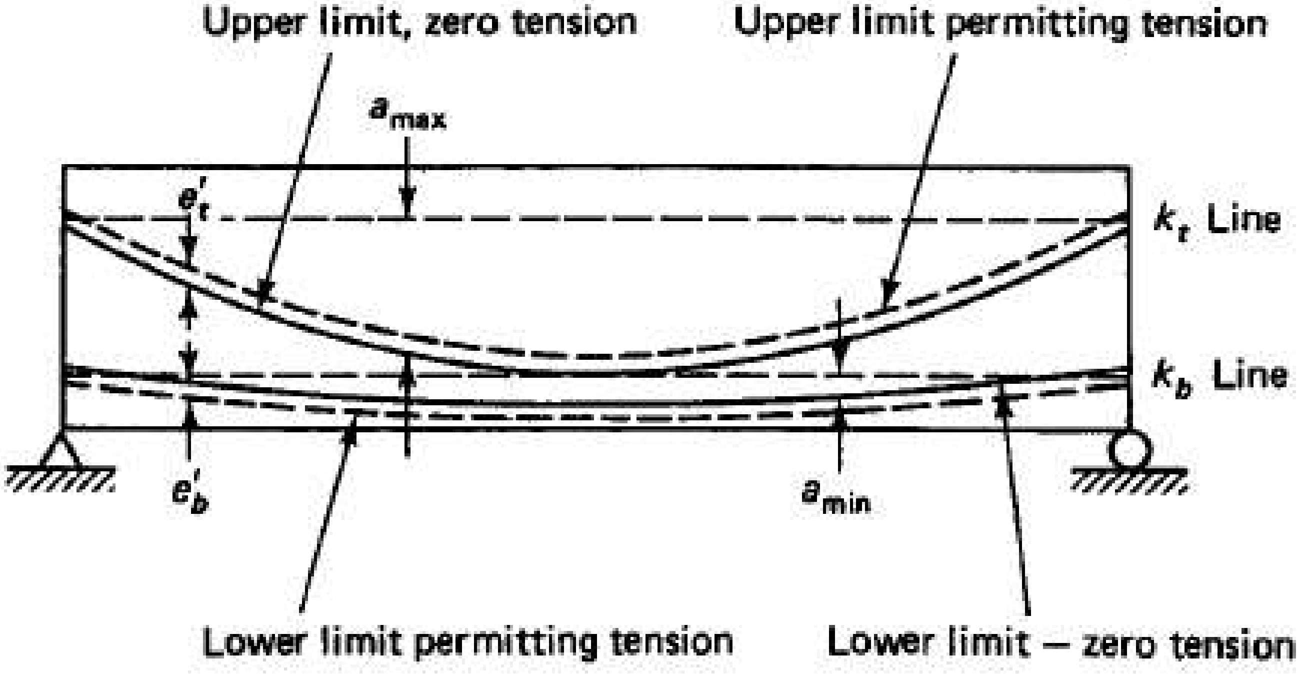
This defines the minimum distance below the *top* kern where the cgs line is to be located so that the C-line does not fall *above* the *top* kern, thereby preventing tensile stresses at the *bottom* extreme fibers. Hence, the limiting top eccentricity is

$$e_t = (a_{\max} - k_t)$$

If an additional eccentricity e'_b, e'_t is superimposed on the cgs-line envelope that results in limited tensile stress at both the top and bottom extreme concrete fibers, the additional top stress $f^{(t)}$ and bottom stress $f_{(b)}$ would be

$$f^{(t)} = \frac{P_i e'_b c_t}{I_c} \qquad f_{(b)} = \frac{P_e e'_t c_b}{I_c}$$

$$e'_b = \frac{f^{(t)} A_c k_b}{P_i} \qquad e'_t = \frac{f_{(b)} A_c k_t}{P_e}$$



Envelope permitting tension in concrete extreme fibers.

Prestressing tendon envelope, Example

Suppose that the beam in example 4.2 is a post-tensioned bonded beam and that the prestressing tendon is draped in a parabolic shape. Determine the limiting envelope for tendon location such that the limiting concrete fiber stresses are at no time exceeded. Consider the midspan, quarter-span, and beam ends as the controlling sections. Assume that the magnitude of prestress losses is the same as in Example 4.2 but that $P_i = 549{,}423$ lb, $P_e = 450{,}526$ lb, $f'_c = 6000$ psi, $e_c = 13$ in. and $e_e = 6$ in.

The design moments of the I-beam with the section properties needed here:

$P_i = 549{,}423$ lb (2,431 kN)

$P_e = 450{,}526$ lb (2,004 kN)

$M_D = 2{,}490{,}638$ in.-lb (281 kN-m)

$M_{SD} + M_L = 7{,}605{,}000$ in.-lb (859 kN-m)

$M_T = M_D + M_{SD} + M_L = 10{,}095{,}638$ in.-lb (1,141 kN-m)

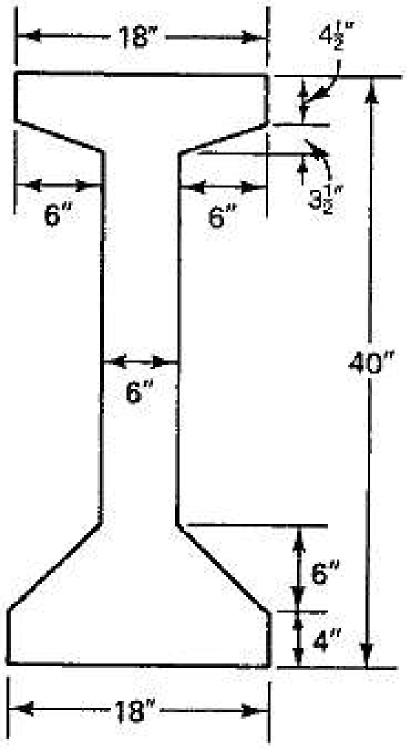
$A_c = 377$ in^2 (2,536 cm^2)

$f'_c = 6{,}000$ psi

$r^2 = 187.5$ in^2 (1,210 cm^2)

$c_t = 21.16$ in. (537 mm)

$c_b = 18.84$ in. (479 mm)



Since bending moments in this example are due to a uniformly distributed load, the shape of the bending moment diagram is parabolic, with the moment value being zero at the simply supported ends. Hence, quarter-span moments are

$$M_D = 0.75 \times 2{,}490{,}638 = 1{,}867{,}979 \text{ in.-lb } (211 \text{ kN-m})$$

$$M_T = 0.75 \times 10{,}095{,}638 = 7{,}571{,}729 \text{ in.-lb } (856 \text{ kN-m})$$

the kern point limits are

$$k_t = \frac{r^2}{c_b} = \frac{187.5}{18.84} = 9.95 \text{ in. } (253 \text{ mm})$$

$$k_b = \frac{r^2}{c_t} = \frac{187.5}{21.16} = 8.86 \text{ in. } (225 \text{ mm})$$

Lower Envelope the maximum distance that the cgs line is to be placed below the *bottom* kern to prevent tensile stress at the top fibers is determined as follows:

(i) *Midspan*

$$a_{\min} = \frac{M_D}{P_i} = \frac{2{,}490{,}638}{549{,}423} = 4.53 \text{ in. } (115 \text{ mm})$$

giving

$e_1 = k_b + a_{\min} = 8.86 + 4.53 = 13.39$ in. (340 mm) vs $e_c = 15$ in. used in Ex. 4.2 allowing tension at top at transfer

(ii) *Quarter span*

$$a_{\min} = \frac{1,867,979}{549,423} = 3.40 \text{ in. } (86 \text{ mm})$$

giving

$$e_2 = 8.86 + 3.40 = 12.26 \text{ in. } (311 \text{ mm})$$

(iii) *Support*

$$a_{\min} = 0$$

giving

$$e_3 = 8.86 + 0 = 8.86 \text{ in. } (225 \text{ mm})$$

Upper Envelope the maximum distance that the cgs line is to be placed below the *top* kern to prevent tensile stress at the bottom extreme fibers is determined as follows:

(i) *Midspan*

$$a_{\max} = \frac{M_T}{P_e} = \frac{10,095,638}{450,526} = 22.41 \text{ in. } (569 \text{ mm})$$

$$e_1 = a_{\max} - k_t = 22.41 - 9.95 = 12.46 \text{ in. } (316 \text{ mm})$$

Clear minimum cover = 3.0 in.

Note that e_1 cannot exceed c_b otherwise tendon is outside the section.

(ii) *Quarter span*

$$a_{\max} = \frac{7{,}571{,}729}{450{,}526} = 16.80 \text{ in. } (427 \text{ mm})$$

$$e_2 = 16.80 - 9.95 = 6.85 \text{ in. } (174 \text{ mm})$$

(iii) *Support*

$$a_{\max} = 0$$

$$e_3 = 0 - 9.95 = -9.95 \text{ in. } (-253 \text{ mm}) \quad (9.95 \text{ in. above cgc line})$$

Now, assume for practical purposes that the maximum fiber tensile stresses under working-load conditions for the purpose of constructing the cgs envelopes does not exceed $f_t = 6\sqrt{f'_c} = 465$ psi for both top and bottom fibers both at midspan and the support, since $f'_c = 6{,}000$ psi from Example 4.2. From Equation 4.9a, this additional eccentricity to add to the *lower* cgs envelope in order to allow limited tension at the *top* fibers is

$$e'_b = \frac{f^{(t)} A_c\, k_b}{P_i} = \frac{465 \times 377 \times 8.86}{549{,}423} = 2.83 \text{ in. } (72 \text{ mm})$$

Similarly, from Equation 4.9b, the additional eccentricity to add to the *upper* cgs envelope in order to allow limited tension at the *bottom* fibers is

$$e'_t = \frac{f_{(b)} A_c\, k_t}{P_e} = \frac{465 \times 377 \times 9.95}{450{,}526} = 3.87 \text{ in. } (98 \text{ mm})$$

We thus have the following summary of cgs envelope eccentricities:

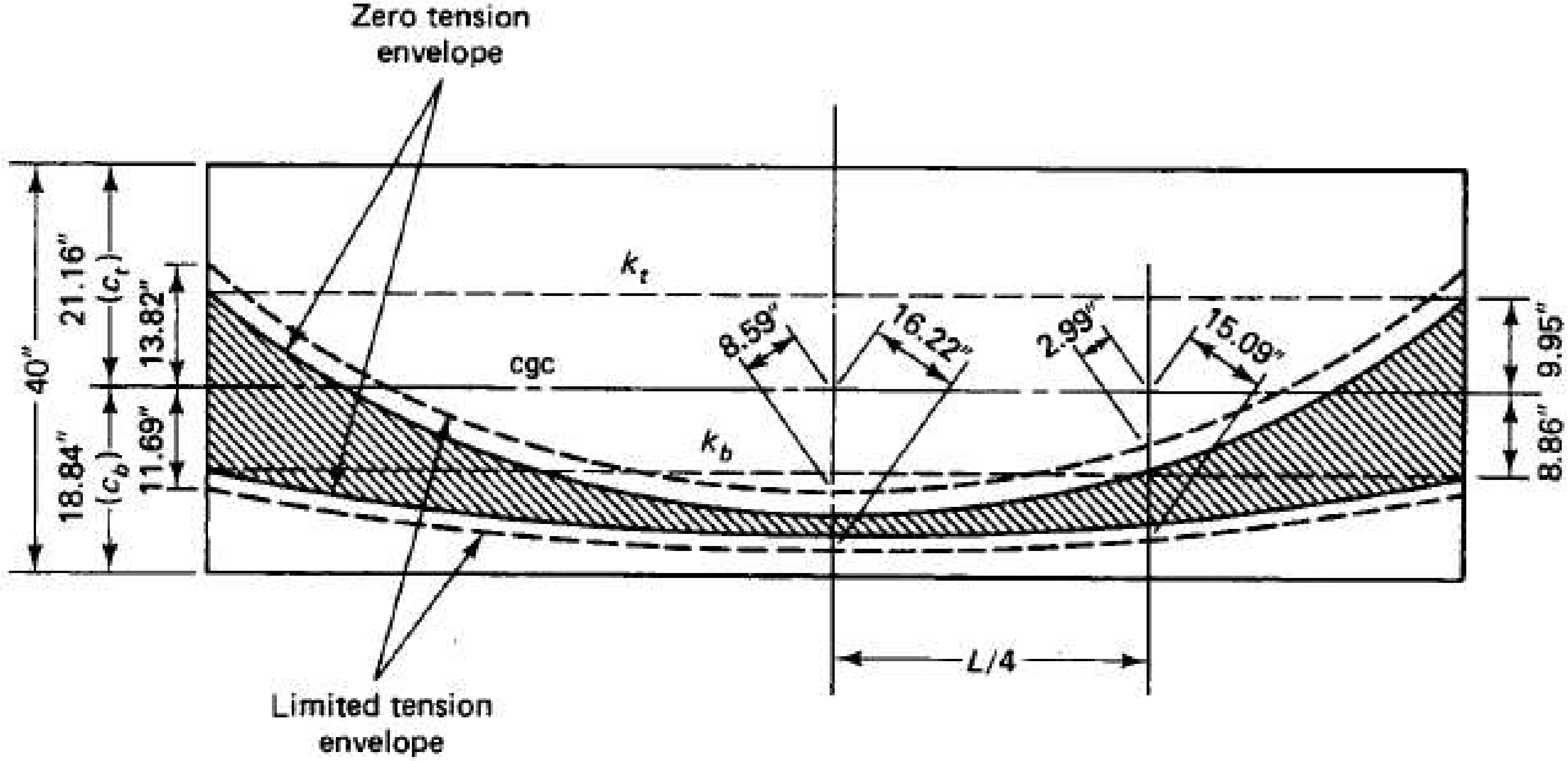
	Zero tension, in.	Increment	Allowable tension, in.
Midspan			
Lower envelope	13.39	+2.83	16.22
Upper envelope	12.46	−3.87	8.59
Quarter span			
Lower envelope	12.26	+2.83	15.09
Upper envelope	6.86	−3.87	2.99
Support			
Lower envelope	8.86	+2.83	11.69
Upper envelope	−9.95	−3.87	−13.82

Actual midspan eccentricity $e_c = 13$ in. < 16.22 in.

Hence, tendon is inside envelope at midspan.

Actual support eccentricity $e_e = 6$ in. < 11.69 in.

Hence, tendon is also inside envelope at support.


Zero tension envelope
40"
21.16"
(c_t)
13.82"
18.84"
(c_b)
11.69"
k_t
cgc
k_b
8.59"
16.22"
2.99"
15.09"
9.95"
8.86"
L/4
Limited tension envelope

(i) precast slab;
(ii) precast slab and topping;
(iii) precast slab and topping, with slab propped at midspan;
(iv) propping removed and an imposed load of 3 kN/m^2 applied.

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11

Indeterminate structures

11.1 INTRODUCTION

All of the prestressed concrete members so far considered have been statically determinate. This reflects the major use of prestressed concrete in building structures, since the most common type of prestressed concrete construction is in the form of simply supported beams. However, there are important applications of prestressed concrete in statically indeterminate structures. Many of the features of the analysis and design of these structures are similar to those used for statically determinate structures, as outlined in previous chapters. There are two important differences, however: the introduction of secondary moments and the behaviour at the ultimate limit state. These will be discussed in the following sections.

The most important application of prestressed concrete indeterminate structures is in the field of multi-span bridges. This is a specialized area of design and construction and is well beyond the scope of this book, but many excellent reference books on the subject may be found in the Bibliography.

In the field of building structures, continuous prestressed concrete beams are sometimes employed, but a more widespread use is in prestressed concrete flat slabs. The design of these will be discussed in detail in Chapter 12.

11.2 SECONDARY MOMENTS

It was shown in Chapter 1 that for a statically determinate prestressed concrete member the line of pressure in the concrete is coincident with the resultant force due to the prestressing tendons, provided that there is no applied axial load on the member. For statically indeterminate prestressed concrete structures, this is not necessarily the case. The prestress moment in a statically determinate member at any section is

Pe, which is known as the *primary* prestress moment. In statically indeterminate structures, *secondary* or ‘parasitic’ prestress moments may be introduced into the structure due to prestressing. Support reactions and shear forces will also be present in this case, even though there is no applied load on the structure. The presence of these secondary moments involves extra work in the analysis and design of statically indeterminate prestressed concrete structures, although in nearly all other respects the design and analysis procedures outlined in the preceding chapters are applicable.

In order to understand how these secondary moments arise, consider a two-span continuous beam, as shown in Fig. 11.1(a), which has a constant prestressing force *P* acting at an eccentricity *e*.

If the central support were unable to restrain vertical upward movement of the beam, the deflected shape of the beam due to the prestressing force would be as shown in Fig. 11.1(b). The beam is now effectively statically determinate and the prestress moment at any section would be the primary moment *Pe* (Fig. 11.2(a)). However, in practice the beam would be restrained at the central support, and in order to maintain compatibility of displacements at this position, a downward reaction *R* must be applied at the support. The distribution of secondary moments induced in the beam by this reaction is shown in Fig. 11.2(b), whilst Fig. 11.2(c) shows the total distribution of the moments along the beam. Note that the secondary moment diagram varies linearly between supports, since it is produced only by the reactions at the supports induced by prestressing.

The resulting prestress moment at any section shown in Fig. 11.2(c) may be written as *Py,* where *y* is some displacement. This may then be considered as the eccentricity of the resultant line of pressure in the concrete, and the locus of *y* along the beam may be considered as an effective tendon profile. The effective profile is obtained by raising or

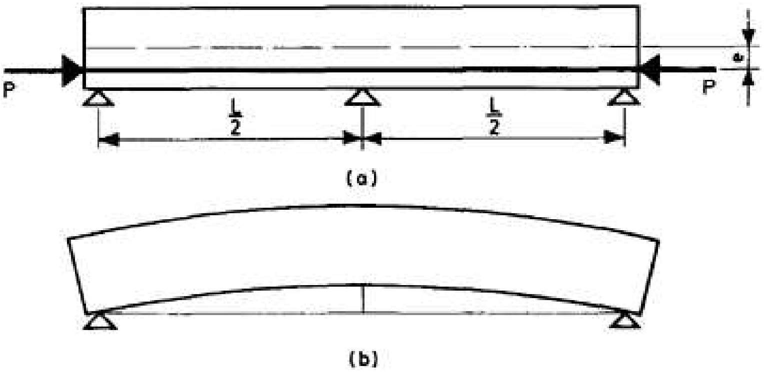

Figure 11.1 Continuous prestressed concrete beam.

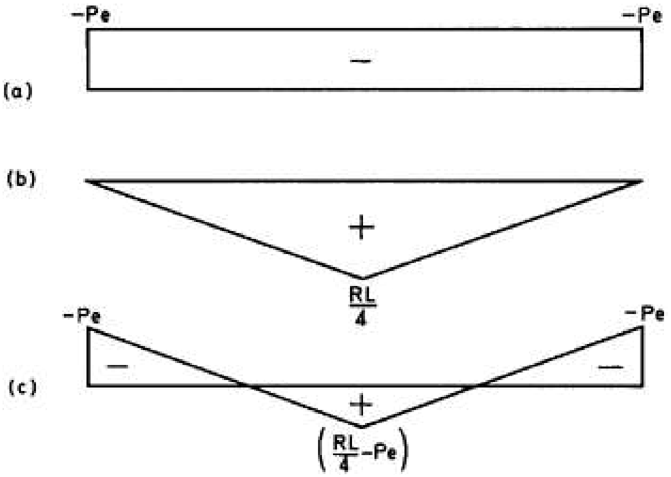

Figure 11.2 Secondary moments.

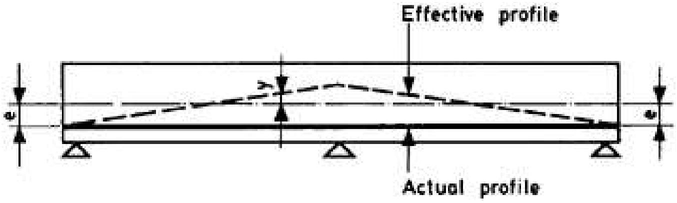

Figure 11.3 Effective tendon profile.

lowering the actual profile at interior supports, while keeping the basic shape of the profile constant, as shown in Fig. 11.3.

All the expressions given in Chapter 9 may be used for the design of statically indeterminate structures if, once the secondary moments have been determined, the actual eccentricity *e* is replaced by the effective eccentricity *y*. However, determination of the cable profile is generally then an iterative procedure.

Example 11.1 ■■

A two-span continuous beam ABC has spans of 10 m and a prestress force of 1500 kN acting at a constant eccentricity of 300 mm. Determine the distribution of prestress moments along the beam and the support reactions induced by prestressing.

On the assumption that there is no vertical restraint at the central support, the beam is subjected to a pair of end-moments equal to *Pe*, that is 1500×0.3=450 kNm. The midspan upward deflection of a beam subjected to a pair of end-moments *M* is given by

$$\delta_M = ML2/8EI,$$

where *EI* is the constant flexural stiffness of the beam and *L* is the span.

Thus, for this example,

$$\delta_M = 450\times20^2/8EI = 22\ 500/EI.$$

For a downward force *R* at the central support, the downward deflection at this point is given by

$$\delta_R = RL3/48EI$$
$$= R\times20^3/48EI = 166.7\ R/EI.$$

For compatibility of displacements at the central support, these two deflections must be numerically equal. Thus:

$$22500/EI = 166.7R/EI$$
$$\therefore R = 135\ \text{kN}.$$

The end-support reactions are thus each 67.5 kN, upward.

The primary, secondary and total distributions of prestress moments along the beam are shown in Fig. 11.4(a)–(c), respectively, whilst the effective tendon profile is shown in Fig. 11.5.

An alternative method of analysis for the secondary moments is to consider the primary moment as a distributed applied moment on the structure and then to analyse it using any of the common methods of structural analysis. For the beam in this example, the method of moment distribution will be used. The fixed-end moments for each span may be

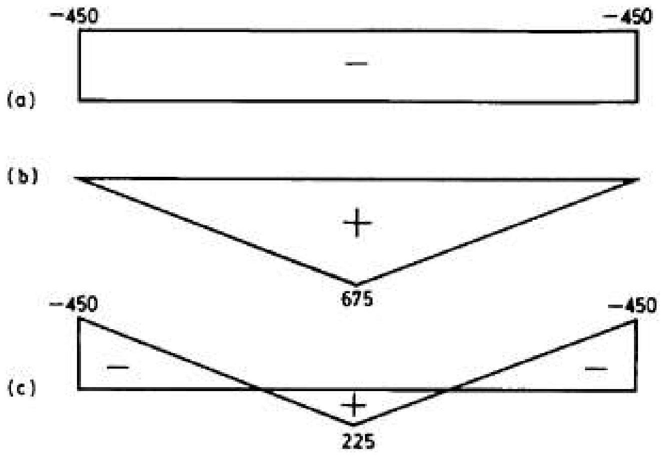

Figure 11.4 Prestress moments for beam in Example 11.1 (kNm).

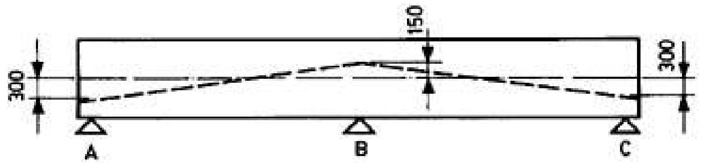

Figure 11.5 Effective tendon profile for beam in Example 11.1.

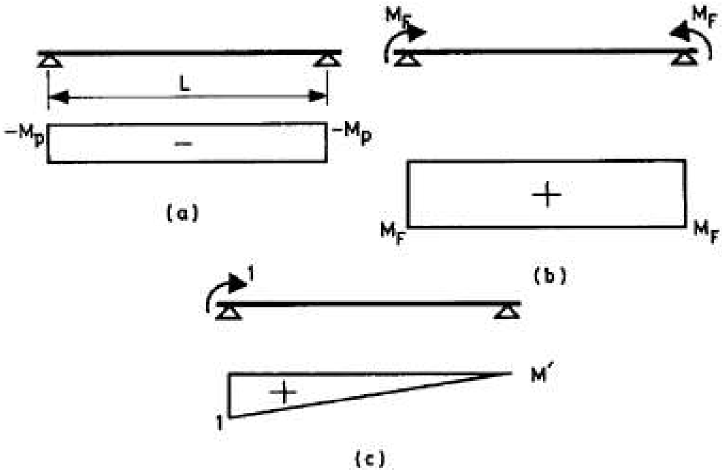

Figure 11.6 Primary moment as distributed applied moment.

found by considering the span as simply supported and with distributed moment, M_P, and end-moments, M_F, applied as shown in Fig. 11.6(a) and (b), respectively. From symmetry, the fixed-end moments M_F at each end of the span must be equal. The rotations at each end of the span due to the combination of M_P and M_F are zero for a fixed-end condition. This rotation may be found conveniently using the virtual work method. The same simply supported span is shown in Fig. 11.6(c), with a unit moment applied to the left-hand end. The rotation at this end due to the moments M_P and M_F is then given by

$$1 \times \theta_A = \int_0^L [M'(M_p + M_F)/EI]\mathrm{d}x.$$

From Simpson’s rule:

$$\theta_A = (L/6EI)[(-M_P+M_F)(1)+4(-M_P+M_F)(1/2)]$$
$$=(-M_p+M_F)L/2EI.$$

Since this rotation must be zero, M_p=M_F. In this example, M_p= 450 kNm and so the fixed-end moments are 450 kNm. The moment distribution is shown in Fig. 11.7, showing that the secondary moment

	A		B			C
	AB		BA	BC		CB
D.F.			0.5	0.5		
F.E.M.	450		−450	450		−450
Balanace	−450					450
Carry-over		↘	−225	225	↙	
Total	0		−675	675		0

Figure 11.7 Moment distribution for beam in Example 11.1 (kNm).

at support B is 675 kNm, as found previously. The resulting total distribution of prestress moments throughout the structure is shown in Fig. 11.4(c).

■■

For less simple tendon profiles, the primary moment diagram shown in Fig. 11.6(a) is found by plotting the ordinates *Pe* along the span. The two fixed-end moments at either end of the span will, in general, be unequal, and the condition of zero end-slope must be applied to each end to enable solution of the unknowns.

The straight tendon profile shown in Fig. 11.1 was used only to illustrate how secondary moments arise. In practice the profile in continuous members is determined according to the same underlying principle that is used for simply supported members, namely that the prestressing tendons are so positioned as to counteract any tension induced by the applied load. In continuous members, hogging support bending moments produce tension at the top surface, and so the eccentricity of the tendons is usually above the centroid at the supports. A typical tendon profile is shown in Fig. 11.8(a) and an enlarged detail of the tendon geometry near the support is shown in Fig. 11.8(b). The inflexion point for the profile is commonly taken as one-tenth of the span.

A useful method of determining the *total* prestress moments in an indeterminate structure is to analyse the structure under the equivalent loading applied to the concrete by the prestressing tendons. For a smoothly draped, or a sharply deflected, tendon a vertical force is exerted on the concrete member and the total distribution of prestress moments may be determined by any of the usual methods of structural analysis. The equivalent loading for the tendon profile in Fig. 11.8(a) is shown in Fig. 11.9.

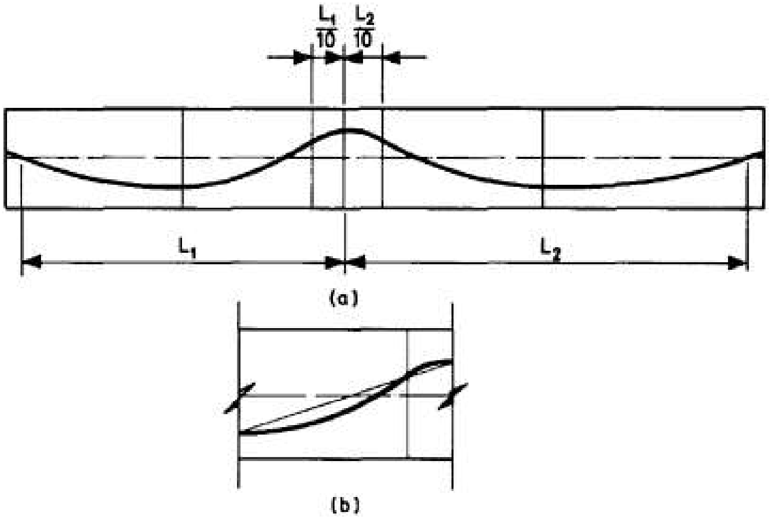

Figure 11.8 Practical tendon geometry: (a) typical profile; (b) enlarged detail near the support.

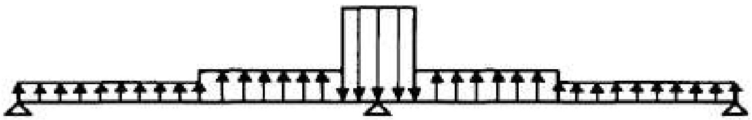

Figure 11.9 Equivalent loading from tendons.

For the straight tendon in Example 1.1, there is no vertical force exerted on the concrete, but there are end-moments as shown in Fig. 11.10. The moment distribution for the beam subjected to these end-moments is shown in Fig. 11.11 and the resulting distribution of total prestress moments is identical to that shown in Fig. 11.4(c). The secondary moments may be found by deducting the primary from the total prestressing moments, a procedure useful in analysis at the ultimate limit state, described in section 11.4.

Example 11.2 ■■

Determine the distribution of total prestress moments due to a prestress force of 1000 kN for the beam shown in Fig. 11.12. Also determine the support reactions induced by prestressing.

The equivalent uniform vertical load exerted on the concrete is given by

$$w=P/r_{ps}.$$

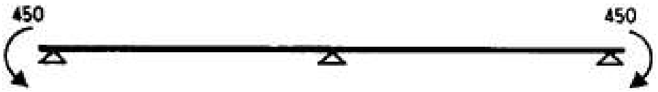

Figure 11.10 Applied moments for straight tendon profile (kNm).

	A		*B*	*C*
	AB	*BA*	*BC*	*CB*
D.F.		0.5	0.5	
F.E.M.	−450	−225	225	450
Total	−450	−225	225	450

Figure 11.11 Moment distribution for beam subjected to end-moments (kNm).

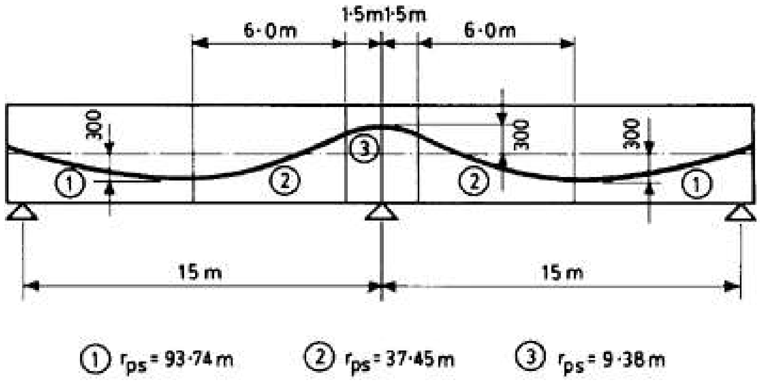

Figure 11.12

Thus:

w_1=1000/93.74=10.67 kN/m
w_2=1000/37.45=26.70 kN/m
w_3=1000/9.38=106.61 kN/m.

The beam can now be analysed under the loading shown in Fig. 11.13.

The resulting distributions of total and secondary prestress moments are shown in Fig. 11.14(a) and (b), respectively, the latter also showing the support reactions, and the effective tendon profile in Fig. 11.15. Note that, once again, the effective tendon profile has been obtained by raising or lowering the actual profile at the supports, keeping the shape constant between the supports.

■■

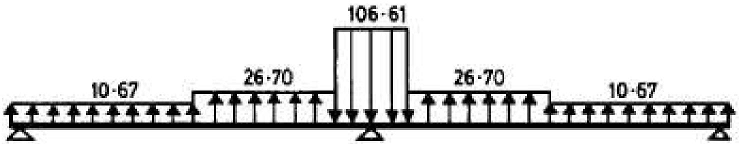

Figure 11.13 Equivalent loading for beam in Example 11.2 (kN/m).

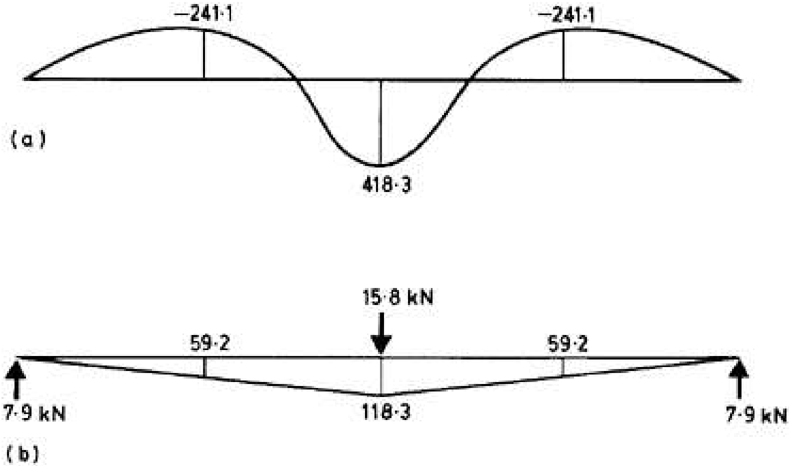

Figure 11.14 (a) Total and (b) secondary prestress moments for beam in Example 11.2 (kNm).

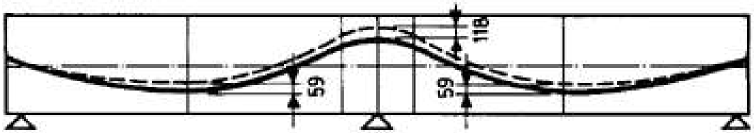

Figure 11.15 Effective tendon profile for beam in Example 11.2.

An alternative method of determining a tendon profile for the given eccentricities is shown in Concrete Society (1994). However, this will not always give maximum eccentricity at the desired points in the spans.

The profile shown in Example 11.2 gives rise to equivalent uniform loads. If there is a sharp change in curvature, then the equivalent force on the concrete member is concentrated, as described in Chapter 1.

In many long continuous prestressed concrete structures, such as bridge decks or floor slabs, some of the tendons are stopped off within a span. They may end there or be connected to other tendons some time later. At these points the effect of a concentrated vertical load and moment must be taken into account when applying equivalent loads to the structure in order to determine the total prestress moment distribution.

Once the secondary moments have been determined, the total stresses at the serviceability limit state may be found by adding the total prestress moment to the applied load bending moment.

In assessing the elastic distribution of bending moments throughout the structure, the following load cases should be considered for beams in buildings without cantilevers and with predominantly uniform loads:

(i) alternate spans loaded with dead load plus imposed load;
(ii) any two adjacent spans loaded with dead load plus imposed load and all other spans loaded with 1.0×dead load.

In the above, the imposed load is either the frequent or rare load combination, depending on whether crack widths or concrete stresses are to be determined, respectively.

For framed structures, the axial shortening in the beams caused by prestressing must also be considered in determining the secondary moments (see Problem 11.5).

11.3 LINEAR TRANSFORMATION AND CONCORDANCY

It was shown in the previous section that the line of pressure can be obtained from the actual tendon profile by raising or lowering the profile at an interior support while keeping the same basic shape in the spans either side of the support. This is an example of a *linear transformation* of a profile, since the amount by which the profile is raised or lowered at any point is directly proportional to the distance of that point from the support that is adjusted. Linear transformations of successive spans can be superimposed to produce a combined transformation.

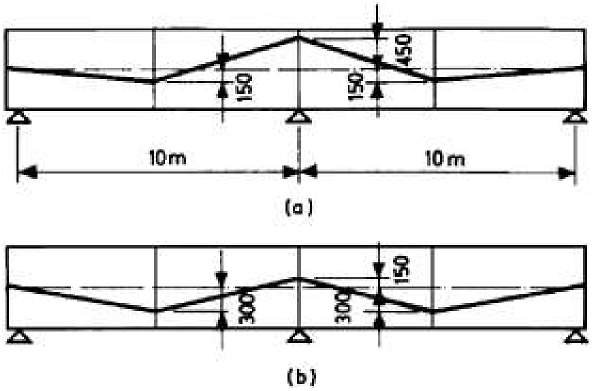

Figure 11.16 Linear transformation.

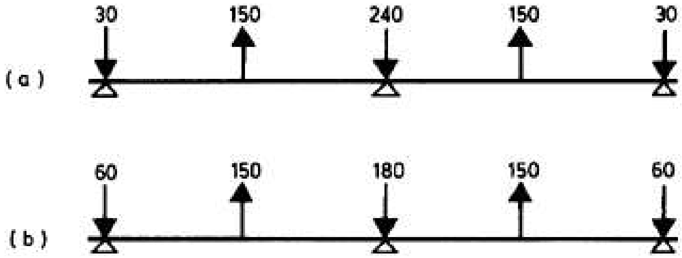

Figure 11.17 Equivalent loading for linear transformation (kN).

Consider the beams shown in Fig. 11.16(a) and (b). The profile in Fig. 11.16(b) is a linear transformation of that in Fig. 11.16(a). The equivalent loads on the concrete in the two cases are shown in Fig. 11.17(a) and (b). The equivalent loads within the spans in each case are the same, although the different inclinations of the tendons at the supports give rise to different vertical forces there. Since the loads within the spans are the same the distributions of total prestress moments must be the same. The lines of pressure in each case must thus be equal. However, the distributions of primary and secondary prestress moments, and the support reactions induced by prestressing, will be different in each case as shown in Fig. 11.18(a) and (b).

In Fig. 11.16 the tendon profile is shown with a sharp change of curvature at the central support, for simplicity. In practice the profile would be more like that shown in Fig. 11.8. For this profile, a linear transformation would slightly alter the inflexion point between the two curved regions and similarly affect the equivalent loads within the span. A linear transformation of such a profile, in theory, would thus cause a change in the total distribution of moments due to the prestress force, but in practice this change is very small and is usually ignored.

A general rule can now be stated, that if a tendon profile undergoes a linear transformation, the line of pressure in the concrete remains constant. This property is particularly useful in modifying tendon profiles when the limits to the cable zone, determined from Inequalities 9.6(a)–(e), do not permit a practical tendon profile. A profile can be chosen to lie within the theoretical cable zone, and a linear transformation performed to bring the actual profile into a more practical location.

If the eccentricity of the straight tendon profile shown in Fig. 11.1 (a) is gradually reduced, the free upward movement of the member at the central support position also becomes smaller. The magnitude of the reaction required to maintain the beam in contact with the support is also lessened, and this implies that the secondary moments reduce. In the limiting case, the eccentricity becomes zero, and the beam is

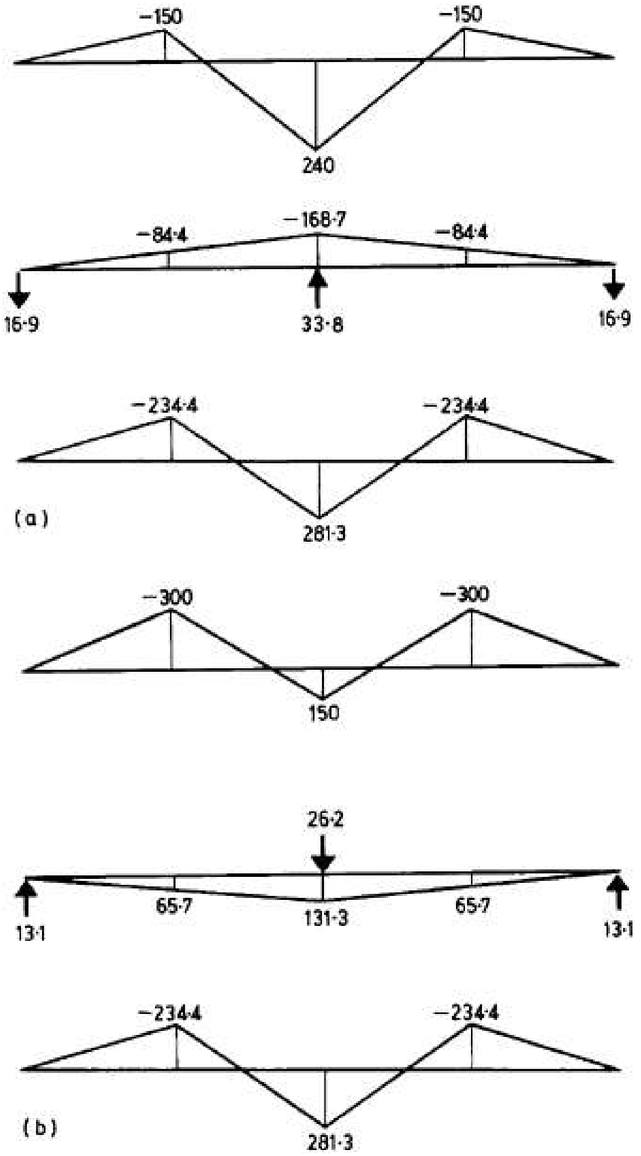

Figure 11.18 Primary, secondary and total prestress moments for beams (a) and (b) in Fig. 11.16 (kNm).

concentrically prestressed. The central support reaction and the secondary moments are then also zero. The total prestress moment in the beam at every section would be equal to the primary moment *Pe,* and the line of pressure would be everywhere coincident with the tendon profile.

Any tendon profile in a prestressed concrete member that has this property is known as a *concordant* profile. All profiles in statically determinate members are thus concordant, but in statically indeterminate members most profiles are non-concordant. For any given member, there can be many different basic profiles that are concordant.

In the design of a statically indeterminate prestressed concrete member, it is not necessary to ensure that the chosen profile is concordant, although this simplifies the calculations. In practice it is found that concordant profiles are not the most economical, but in the design process it is quite useful to start with a concordant profile and then to modify it as necessary.

Consider now the continuous beam shown in Fig. 11.19(a) with a uniform load on each span. The distribution of bending moments is shown in Fig. 11.19(b). If tendons are fixed in the beam according to a profile determined from Fig. 11.19(b), then the equivalent load on the beam from the tendons must be of the form shown in Fig. 11.19(a), since any elastic bending moment distribution within a given structure can only correspond to one distribution of applied loads. The distribution of total prestress moments within the member must therefore be similar to that shown in Fig. 11.19(b). Since this distribution of moments is consistent with zero vertical deflection at the central support, this tendon profile must be concordant.

A general rule thus emerges for determining concordant profiles, that the bending moment diagram for any given loading on a structure yields a concordant profile.

Example 11.3 ■■

Determine a concordant profile for the beam shown in Fig. 11.20, using a prestress force of 500 kN.

A uniform load of 12 kN/m will be used to find a concordant profile. The bending moment diagram for this loading is shown in Fig. 11.21(a).

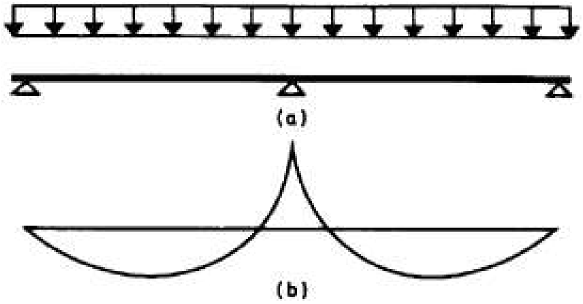

Figure 11.19 Continuous beam: (a) uniform load on each span; (b) bending moment distribution resulting from UDL.

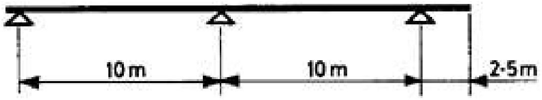

Figure 11.20

The concordant tendon profile is obtained by dividing the ordinates of the bending moment diagram in Fig. 11.21(a) by the prestress force. The resultant profile is shown in Fig. 11.21(b).

■■

This profile is just one such concordant profile, since any linear transformation of it will not alter the position of the pressure line in the concrete and thus the profile will remain concordant. The design could then proceed exactly as for a statically determinate structure, since, provided the chosen tendon profile is a linear transformation of that shown in Fig. 11.21(b), the secondary moments will be zero.

The above method of finding a concordant profile is strictly valid only if the prestress force is constant along the member. In practice the prestress force varies, and the concordant profile should be obtained by dividing the ordinate of the bending moment diagram in Fig. 11.21(a) at any section by the effective prestress force at that section.

11.4 ULTIMATE LOAD BEHAVIOUR

The analysis of prestressed concrete members at the ultimate limit state outlined in Chapter 5 is sufficient for statically determinate structures, since, for these structures, once the ultimate moment of resistance has been reached at any section, a mechanism is formed and the structure cannot support any more load.

The situation is different, however, for statically indeterminate structures. As the applied load is increased, the ultimate moment of resistance will be reached at some point in the structure, but in this case a mechanism will not form. Provided that the member at this point will allow rotation to take place at the *plastic hinge* which has formed, additional load can be carried by the structure, which effectively redistributes the load to less heavily loaded regions until sufficient plastic hinges have formed to produce a mechanism. This description of the plastic analysis of prestressed concrete structures is equally applicable to steel, timber or reinforced concrete structures, and the general background and details of the theory may be found in Coates, Coutie and Kong (1980). The full methods of plastic analysis may be used for prestressed concrete structures, but there are important

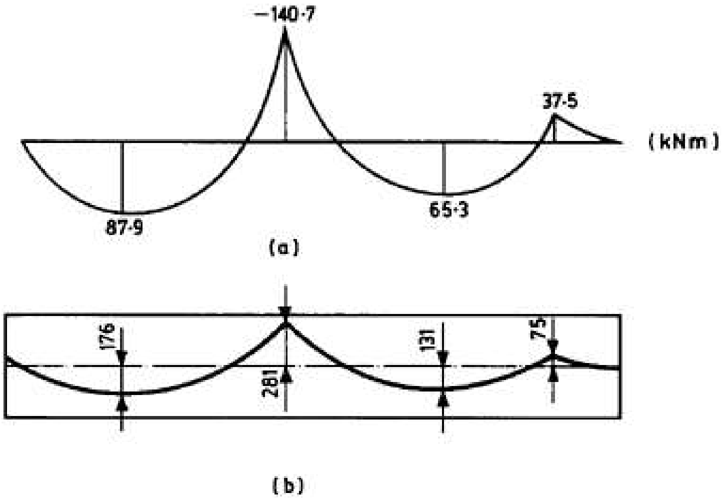

Figure 11.21 Concordant profile for beam in Example 11.3.

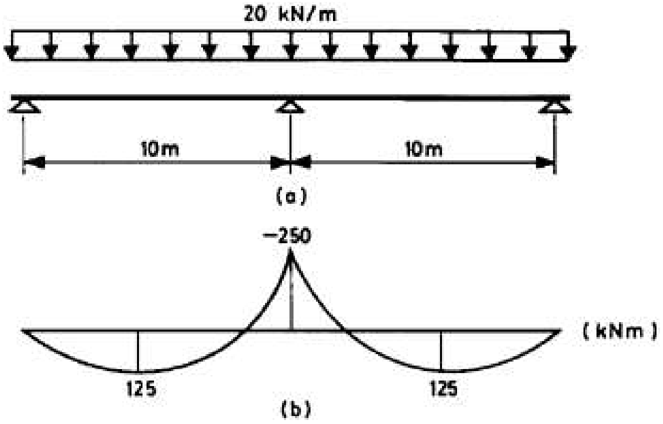

Figure 11.22

limitations imposed in EC2 on the amount of rotation that takes place at a given section once a plastic hinge has formed there.

Consider the two-span continuous beam shown in Fig. 11.22(a), which is subjected to a total ultimate uniform load of 20 kN/m. An elastic analysis of the structure would give the bending moment distribution shown in Fig. 11.22(b). If the ultimate moment of resistance of the beam at the central support is 175 kNm, then the bending moment distribution of Fig. 11.22(b) can never be achieved. At a load of 14 kN/m, the bending moment diagram would be as shown in Fig. 11.23(a). As the uniform load is increased from 14 kN/m to 20 kN/m, the

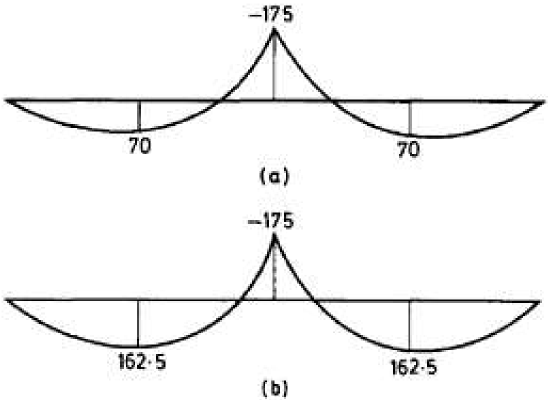

Figure 11.23 Moment redistribution (kNm).

bending moment at the central support is assumed to remain constant at 175 kNm, since a plastic hinge has formed there. In order to maintain equilibrium, the bending moment diagram becomes that shown in Fig. 11.23(b). Comparison of this with Fig. 11.22(b) shows that the elastic bending moment of 250 kNm at the support has been *redistributed* by 30% to give the value of 175 kNm in Fig. 11.23(b). However, the ultimate moment of resistance to be provided at the midspan sections has now increased to 162.5 kNm.

Moment redistribution may also be applied to the midspan sections. In this case it is the moment of resistance at the supports which must be increased to maintain equilibrium.

The 30% redistribution shown in the above example is the maximum permitted in EC2 for post-tensioned tendons. However, in practice the actual amount of redistribution permitted for a given section may be less than this figure. This is because for a statically indeterminate structure to resist an increase in load once the first plastic hinge has formed, rotation must take place at that hinge. The ability of a prestressed concrete member to undergo the required rotation once the ultimate moment of resistance has been reached is dependent on the position of the neutral axis within the section. Typical plots of moment-curvature for a given rectangular prestressed concrete section with varying amounts of prestressing steel are shown in Fig. 11.24. Each curve also corresponds to a different location of the neutral axis at the ultimate moment of resistance. For positions of the neutral axis high in the section, the amount of rotation that can take place after the plastic hinge has formed is much greater than for positions of the neutral axis lower in the section.

Strictly according to plastic theory, the ultimate strength of a prestressed concrete structure is independent of any secondary moments induced by prestressing, in a similar way that it is also independent of such effects as settlement of supports; it depends solely on the mechanisms which can form. However, it is stated in EC2 that secondary moments and shear forces should be included in ultimate strength calculations, with a value of γ_p of 1.0. The inclusion of secondary moments generally increases span bending moments and decreases those at supports. For framed structures, such as prestressed flat slabs, described in Chapter 12, secondary moments will have an effect on the design moments for the columns.

The amount of redistribution allowed is linked directly in EC2 to the position of the neutral axis. The permitted values for prestressing tendons of *δ,* the ratio of the moment at a section after redistribution to that before redistribution, are shown in Table 11.1.

In assessing the elastic distribution of bending moments throughout the structure due to ultimate loads, the load combinations described in Section 11.2 should be used, in combination with the partial factors of safety given in Chapter 3.

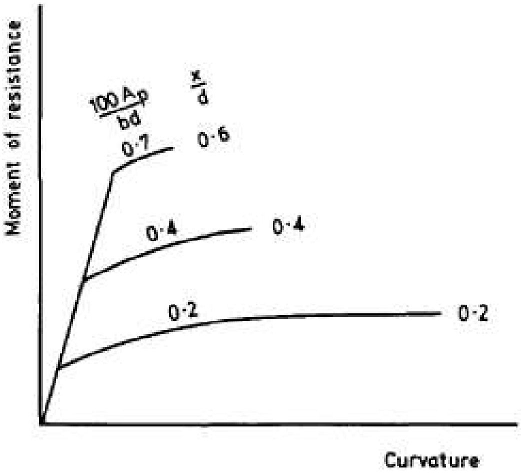

Figure 11.24 Moment-curvature relationships.

Table 11.1 Limits to redistribution

Concrete grade	*Minimum value of δ*		*Maximum value of x/d*
	Post-tensioning	*Pretensioning*	
≤C35/45	0.70	0.85	0.8 δ–0.35≤0.45
>C35/45	0.70	0.85	0.8 δ–0.45≤0.35

Example 11.4 ■■

Determine the maximum ultimate uniform load that can be supported by the beam in Example 11.2, if the beam dimensions are 750 mm deep by 400 mm wide and the area of prestressing steel is 1030 mm^2. Assume that the concrete is grade C40/50 and f_{pk}=1850 N/mm2.

At the central support:

$$A\text{p}_{\text{fp}}\text{k}/(bdf_{ck})=(1030\times1850)/(400\times675\times40)$$
$$=0.176.$$

From the design chart shown in Fig. 5.16,

$$M_u=0.121\times400\times675^2\times40\times10^{-6}$$
$$=882.1 \text{ kNm}$$

Also from Fig. 5.16, x/d is 0.3. Thus from Table 11.1, δ=0.94 (>0.70).

For a uniform load, w, over the full length of the beam the maximum elastic support bending moment is $wL^2/8$, or 28.13 w. From Fig. 11.14 the secondary moment at the support is 18.3 kNm (with γ_p=1.0).

Thus:

$$0.94\,(-28.13\,w+118.3)=-882.1$$
$$\therefore\ w_{ult}=37.6 \text{ kN/m}$$

With no redistribution, w_{ult}=31.4 kN/m.

The secondary moment at midspan is 59.2 kNm. For a support bending moment of 882.1 kNm the corresponding midspan moment is

$$=37.6\times15^2/16+59.2$$
$$=588.0 \text{ kNm.}$$

Since this is less than the ultimate moment of resistance of the midspan section no additional untensioned reinforcement is required.

■■

PROBLEMS

11.1 For the beam in Example 11.1 determine the midspan deflection under the action of prestress force only.

11.2 For the beam shown in Fig. 11.25, determine the support reactions induced by prestressing:

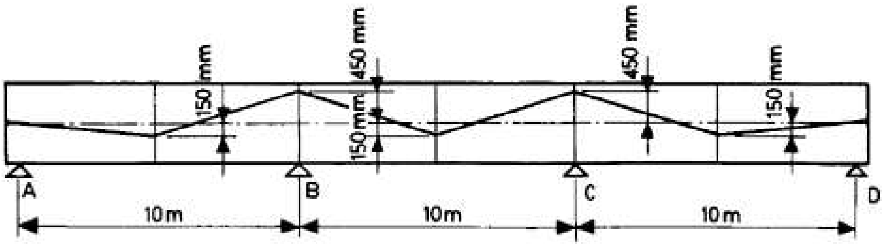

Figure 11.25

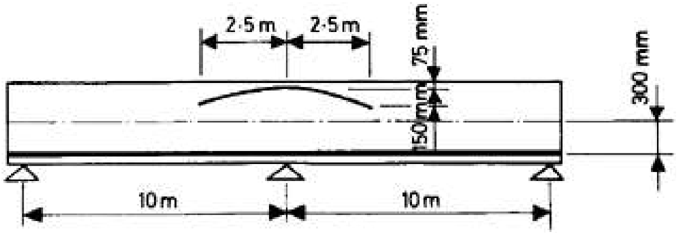

Figure 11.26

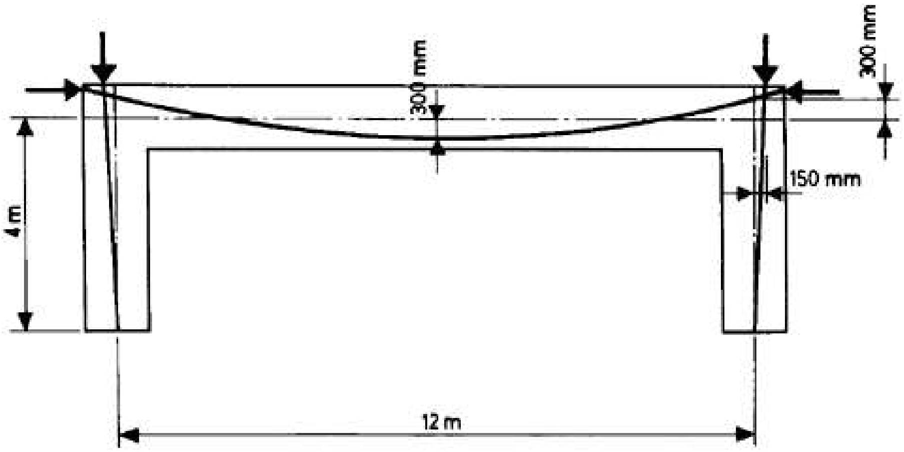

Figure 11.27

(i) with the profile as shown;

(ii) with the profile at support B lowered by 300 mm, the basic shapes of the profiles in the adjacent spans remaining unaltered.

11.3 For the beam in Example 11.1, show that, if the tendon is placed along the original line of pressure, the new profile is concordant.

11.4 The beam in Example 11.1 is to have a circular cap cable over the central support, as shown in Fig. 11.26. The radius of the cable is 20.91 m and the prestress force is 900 kN. If the dimensions of the beam are 750 mm deep by 400 mm wide, determine the stresses in the top of the beam over the central support under the effect of prestress, self weight and an imposed load of 55 kN/m:

(i) with no cap cable;
(ii) with cap cable.

For case (i), determine the maximum and minimum compressive stresses in the beam at midspan.

11.5 The pinned-base portal frame shown in Fig. 11.27 has a constant cross-section of 900×300 mm. It has a parabolic tendon profile in the beam with constant prestress force of 2000 kN, and straight tendons in the columns with prestress force of 500 kN. Determine the primary and secondary prestress moment diagrams and find the secondary support reactions. Assume E_{cm}=35×103 N/mm^2.

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Coates, R.C., Coutie, M.G. and Kong, F.K. (1980) *Structural Analysis,* Nelson, Walton-on-Thames.

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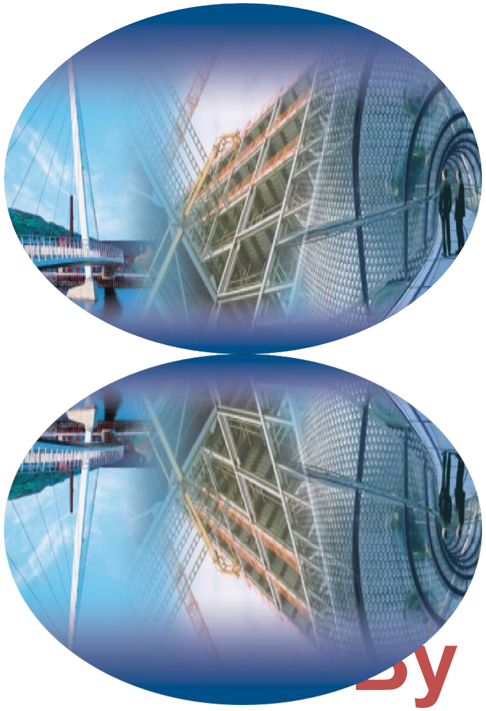


UNIVERSITY OF ANBAR

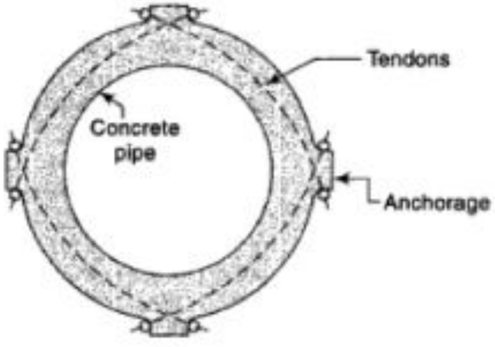
Lecturer 12

By : Mohammed Mohammed

Prestressed pipes & tanks

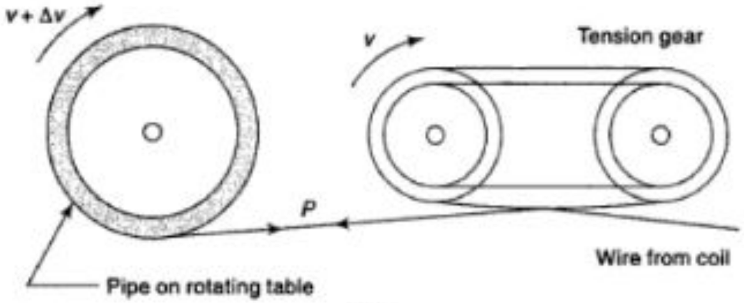
- Liquid retaining structures, such as circular pipes , tanks ad pressure vessels are admirably suited for circular prestressing.
- The circumferential hoop compression induced in concrete by prestressing counterbalances the hoop tension developed due to the internal fluid pressure.
- A reinforced concrete pressure pipe requires a large amount of reinforcement to ensure low-tensile stresses resulting in a crack free structure.
- Advantages in using circular prestressing are
  - Eliminates cracks
  - Economical use of materials
  - Safeguards against shrinkage cracks

Pipes



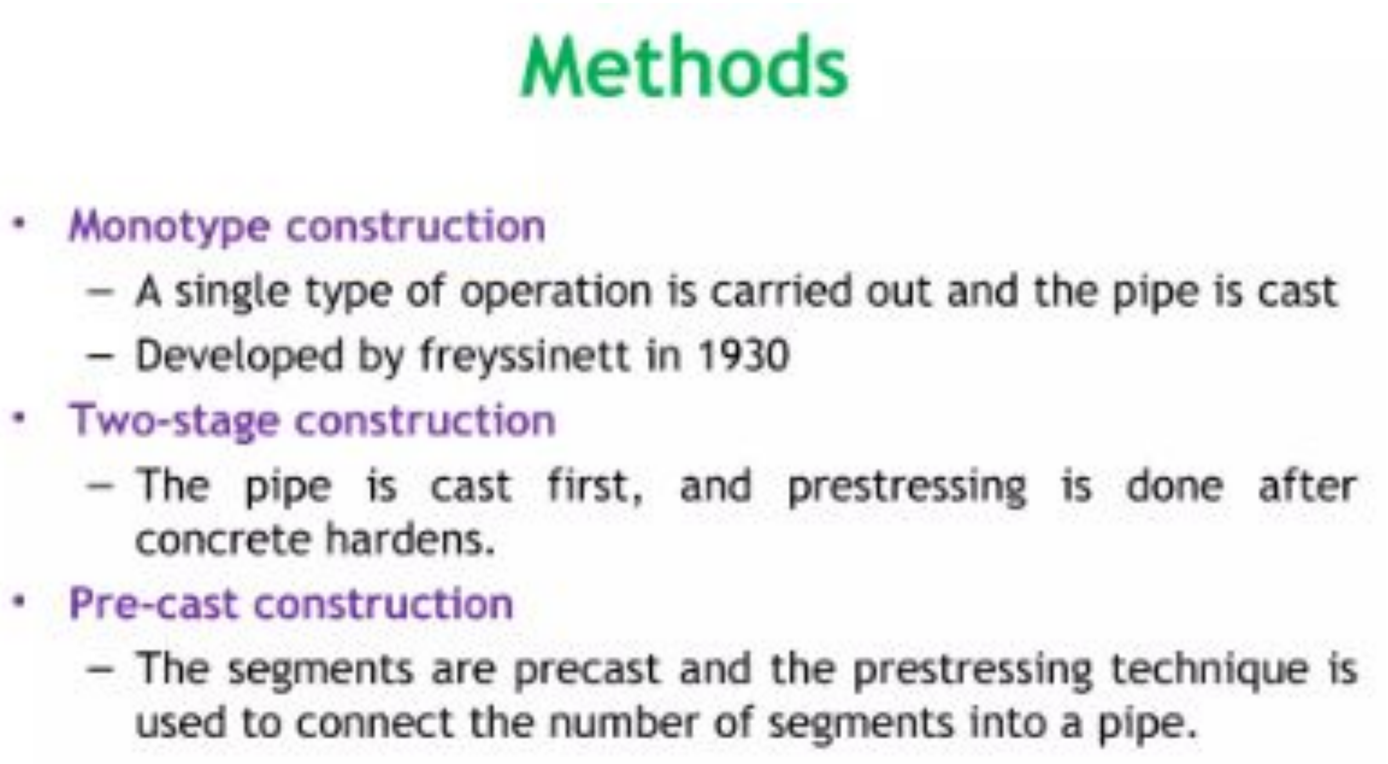
(a)

Overlapping of tendons within the ducts to minimize frictional loss



(b)

Wrap the high tensile wires under tension around precast cylindrical members.



Methods

- **Monotype construction**
  - A single type of operation is carried out and the pipe is cast
  - Developed by freyssinett in 1930
- **Two-stage construction**
  - The pipe is cast first, and prestressing is done after concrete hardens.
- **Pre-cast construction**
  - The segments are precast and the prestressing technique is used to connect the number of segments into a pipe.

Criteria of design

According to Indian standard Is 784:2001, the design of prestressed concrete pipe should cover the following five stages:

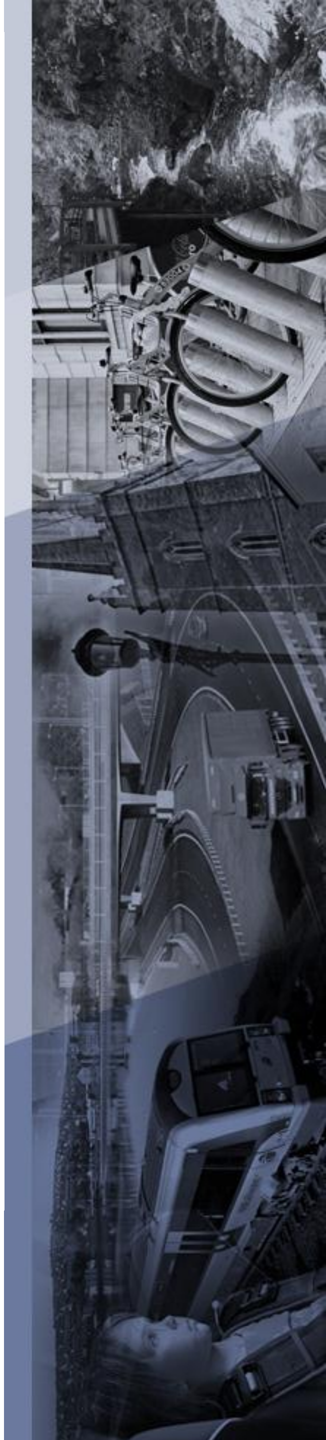
- **Circumferential prestressing**, winding with or without longitudinal prestressing.
- **Handling stresses** with or without longitudinal prestressing.
- Condition in which a pipe is supported by saddles at extreme points with full water load but zero **hydrostatic pressure**.
- Full working pressure conforming to the **limit state of serviceability.**
- The first crack stage corresponding to the **limit state of local damage**.
- Examine the stage of bursting or failure of pipes corresponding to the **limit state of collapse**, mainly to ensure a desirable load factor against collapse.

Design of Non-cylinder pipe

- The tensioning of the prestressing steel induces a circumferential compression, f_c in the pipe and should not exceed the permissible compressive stress at transfer.
- The working pressure, p should not be less than f_{min} . Thus the permissible range of stress is ($\eta f_{c -} f_{min}$).
- The circumferential stress is given by the following equation:

Where

- D=inside diameter
- T= hoop tension=pD/2
- f_{ct} = allowable stress in concrete
- η = loss ratio
- f_{min} = permissible stress in concrete under working pressure=0 as per IS 784
- t= thickness of wall in mm
- f_c = compressive stress in concrete in N/mm^2



$$\frac{pD}{2t} < \left(\eta f_{ct} - f_{\min}\right)$$

↓

$$t > \frac{\left(pD/2\right)}{\left(\eta f_{ct} - f_{\min}\right)}$$

↓

$$t > \frac{T}{\left(\eta f_{ct} - f_{\min}\right)}$$

↓

$$\eta f_{ct} - f_{\min} = \frac{T}{t}$$

↓

$$f_{ct} = \frac{T}{\eta t} + \frac{f_{\min}}{\eta}$$

Where T=N_d = pD/2

The prestressing force is P per metre length

Where, t= thickness of wall in mm

f_c = compressive stress in concrete in N/mm²

$$P = 2000 t f_c$$

Referring to Figure

Where,

N=number of turns

d= diameter of wire

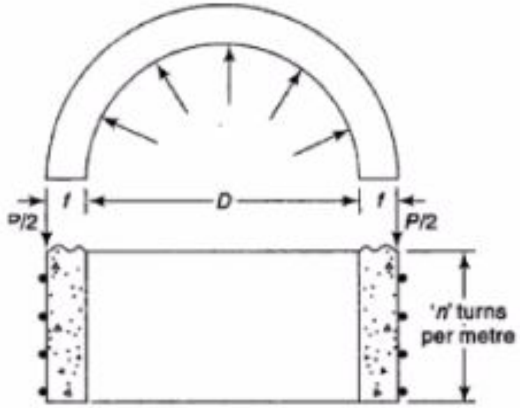
A_s = Area of steel

f_s = Stress in steel

$$P = A_s f_s \rightarrow \quad P = 2\left(\frac{\pi d^2}{4}\right) n f_s$$

Water pressure after winding

$$P_w = \frac{2t}{D}\left(f_c - f_{\min}\right)$$



Using force equilibrium condition

$$2000 t f_c = 2\left(\frac{\pi d^2}{4}\right) n f_s$$

$$n = \frac{4000 t f_c}{\pi d^2 f_s}$$

Loss of prestress due to elastic shortening

- There will be contraction in the pipe due to the application of circumferential tension in the wire wounds. Also when the adjacent length is wound, there will be further contraction of the diameter of the pipe.
- The loss due to elastic shortening is calculated as follows.

$$f_{se} = \frac{f_s}{1+\alpha_e \rho}$$

Where,
f_s = initial stress in steel
f_{se} = Effective stress after winding
$\acute{\alpha}_e$ = modular ratio =E_s/E_c
ρ= reinforcement ratio= f_c/f_s

Guidelines
Percentage of reinforcement= 0.5 to 1 %
Modular ratio =5 to 6
Loss due to elastic shortening =3 to 6 %

Problem-1

Design a non - cylinder prestressed concrete pipe of 600 mm internal diameter to withstand a working hydrostatic pressure of 1.05 N/mm^2, using a 2.5 mm high - tensile wire stressed to 1000 N/mm^2 at transfer. Permissible maximum and minimum stresses in concrete at transfer and service loads are 14 and 0.7 N/mm^2. The loss ratio is 0.8. calculate also the test pressure required to produce a tensile stress of 0.7 N/mm^2 in concrete when applied immediately after tensioning and also the winding stress in steel if E_s = 28 kN/mm^2 and E_C = 35 kN/mm^2.

$$t > \frac{N_d}{\eta f_{ct} - f_{\min.w}} > \frac{1.05(600/2)}{0.8 \times 14 - 0.7} > 30 \text{ mm}$$

For a 30 mm thick concrete pipe, the actual compressive stress in concrete $f_c = 14$ N/mm^2.

The number of turns of the 2.5 mm wire stressed to 1000 N/mm^2 per metre length of the pipe is given by,

$$n = \frac{4000 t f_c}{\pi d^2 f_s} = \frac{4000 \times 30 \times 14}{\pi \times 2.5^2 \times 1000} = 86 \text{ turns/m}$$

$$\text{Pitch of circumferential wire winding} = \frac{1000}{86} = 11.6 \text{ mm}$$

If W_w = test pressure required immediately after winding, ($\eta = 1$)

$$f_c = \frac{W_w D}{2\eta t} + \frac{f_{\min.w}}{\eta}$$

$$W_w = \frac{2t}{D}(f_c - f_{\min,w})$$

$$= \frac{2 \times 30}{600}[14 - (-0.7)] = 1.47 \text{ N/mm}^2$$

If f_{si} = winding stress in steel,

$$f_{si} = (1 + \alpha_e \rho) f_{se}$$

$$\alpha_e = 6 \text{ and } \rho = \frac{f_c}{f_s} = \frac{14}{1000} = 0.014$$

$$f_{si} = (1 + 6 \times 0.014)1000$$

$$= 1084 \text{ N/mm}^2$$

Problem-2

A non - cylinder prestressed concrete pipe of internal diameter 1000 mm and thickness of concrete shell 75 mm is required to convey water at a working pressure of 1.5 N/mm^2. The length of each pipe is 6 m. the maximum direct compressive stresses in concrete are 15 and 2 N/mm^2. The loss ratio is 0.8. i. Design the circumferential wire winding using 5 mm diameter wires stressed to 1000 N/mm^2. ii. Design the longitudinal prestressing using 7 mm wires tensioned to 1000 N/mm^2. The maximum permissible tensile stress under the critical transient loading (wire wrapping at spigot end) should not exceed 0.8 root fci , where fci is the cube strength of concrete at transfer = 40 N/mm^2. iii. Check for safety against longitudinal stresses that develop, considering the pipe as a hollow circular beam as per IS: 784 provisions.

$D = 1000\text{ mm}$

$W_w = 1.5\text{ N/mm}^2$

$t = 75\text{ mm}$

$L = 6\text{ m}$

$f_{ct} = 15\text{ N/mm}^2$

$f_{min,w} = 2\text{ N/mm}^2$

$f_s = 1000\text{ N/mm}^2$

(a) Circumferential wire winding

Compressive stress in concrete,

$$f_c = \frac{N_d}{\eta t} + \frac{f_{\min.w}}{\eta} = \frac{1.5(1000/2)}{0.8 \times 75} + \frac{2}{0.8} = 15 \text{ N/mm}^2$$

Number of turns,

$$n = \frac{4000 r f_c}{\pi d^2 f_s} = \frac{4000 \times 75 \times 15}{\pi \times 5^2 \times 1000} = 57 \text{ turns/m}$$

$$\text{Pitch of winding} = \frac{1000}{57} = 17.5 \text{ mm}$$

(b) Longitudinal prestressing

Critical transient stress at spigot end $= 0.6 \times$ hoop stress $= 0.6 \times 15$
$= 9$ N/mm^2

Maximum permissible tensile stress $= 0.8\sqrt{f_{ci}} = 0.8\sqrt{40} = 5$ N/mm^2

Hence the tensile stress of $9 - 5 = 4$ N/mm^2 should be counterbalanced by longitudinal prestressing. Cross-sectional area of the pipe

$$= (\pi \times 1.075 \times 0.075) \text{ m}^2$$

If P is the longitudinal prestressing force required, then

$$P = \frac{\pi \times 1.075 \times 0.075 \times 10^6 \times 4}{10^3} = 1013 \text{ kN}$$

Using 7 mm wires stressed to 1000 N/mm^2,

Force in each wire = 38.5 kN

$$\therefore \quad \text{Number of wires} = \frac{1013}{38.5} = 27$$

(c) Check for flexural stresses as per IS: 784

Considering the pipe as a beam of hollow cicular section over a span of 6 m.

Three times self-weight $= 3\pi \times 1.075 \times 0.075 \times 24 = 18.30$ kN/m

Weight of water $= (\pi \times 1^2 \times 10)/4 = 7.90$ kN/m

Total u.d.l on pipe $= 26.20$ kN/m

Maximum bending moment $= \dfrac{26.2 \times 6^2}{8} = 118$ kN m

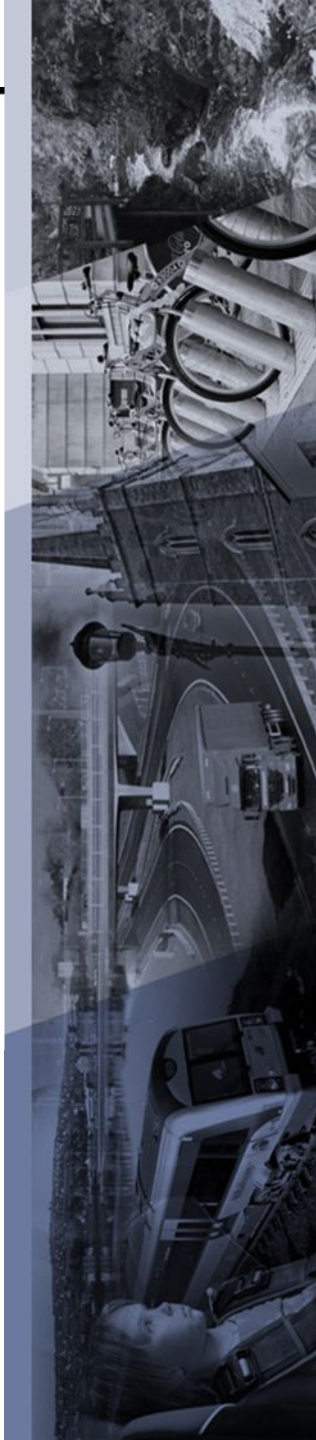
Second moment of area, $I = \dfrac{\pi(1.15^4 - 1^4)}{64} = 0.0365\ \text{m}^4$

Flexural tensile stress $= \dfrac{118 \times 10^6 \times 575}{0.0365 \times 10^{12}} = 1.88\ \text{N/mm}^2$ (tension)

Longitudinal prestress $= 4\ \text{N/mm}^2$

$\therefore$ Resultant stress in concrete $= 4 - 1.88 = 2.12\ \text{N/mm}^2$ (compression)

The resultant stress being compressive, the pipe is safe against cracking.



Longitudinal stress in prestressed pipes

- The winding of pipe with wires and tensioning causes the stresses.
- In additional to the bending moment and shear stresses, the **longitudinal moments develop due to the reduction in diameter from the unwound to wound length of pipe**.
- This wire winding in the circumferential direction causes **longitudinal tensile stresses**.
- The suggested transient stress is 0.6 times the hoop stress.
- The design longitudinal stress given by curtis and cowan is

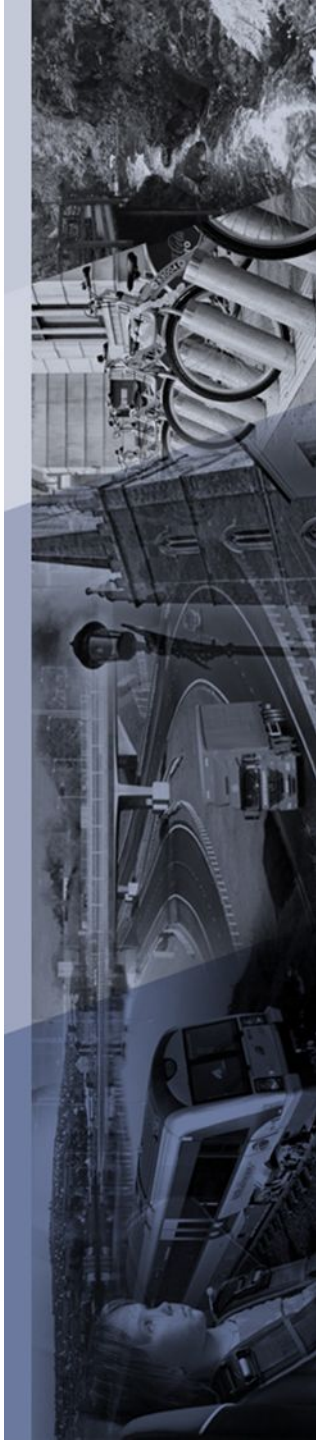
Where

$$P_i = 0.275T_i + tf_{\min}$$

P_i = Longitudinal prestressing force per unit of circumference

T_i = Tangential prestressing force per unit length

$f_{\min}$ = permissible stress in concrete



Creep Separation

- A prestressed pipe is given outer mortar coat.
- The mortar as such is not prestressed.
- It tends to separate as the creep reduces the diameter.
- Let f_b be the radial stress tending to separate from the rest of the pipe.
- This stress can be estimated by considering the equilibrium of portion of a prestressed concrete as follows:

$$f_b = \frac{2(\gamma f_t - \varepsilon_1)}{D\left(\frac{1}{tE_c} + \frac{1}{\frac{t'}{E'_C}}\right)}$$

Where

f_b =radial stress

γ= creep strain/unit of strain

f_t = circumferential stress at transfer

D= Diameter of pipe

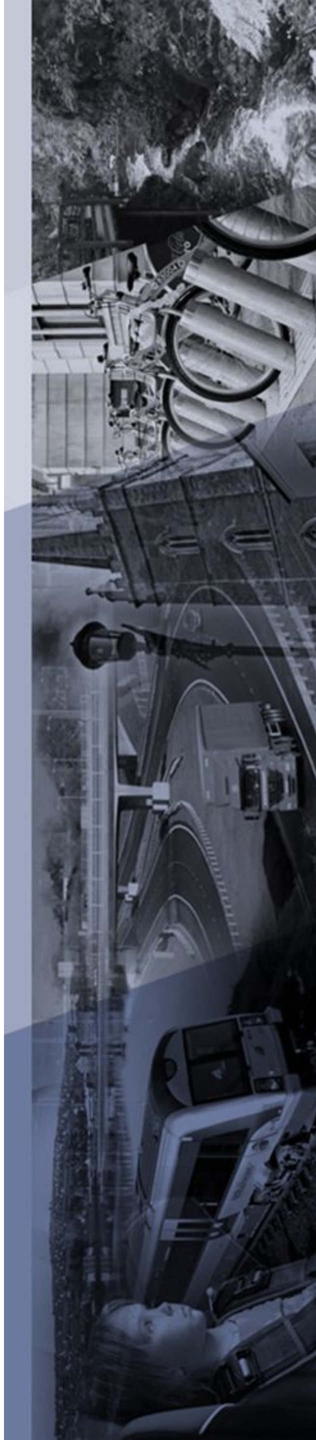
t= thickness of pipe

t'= mortar thickness

ξ_1 = differential shrinkage

E_c= Modulus of elasticity of concrete

E'_c= Modulus of elasticity of mortar



Design of cylinder pipe

- The design principles, in general , follow the design of non-cylinder pipe, and the thickness of concrete is found out by using equivalent area of concrete of light gauge steel cylinder.
- The thickness of concrete wall can be known by

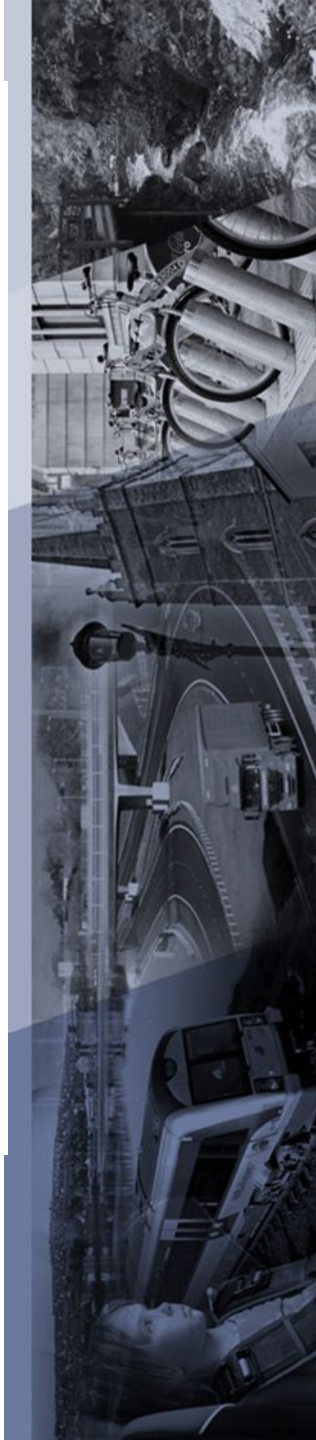
$$t = \frac{T}{\eta f_{ct} - f_{\min}} - \alpha_e t_s$$

- The prestress required in concrete at transfer is given as follows

$$f_c = \frac{T}{\eta(t + \alpha_e t_s)} + \frac{f_{\min}}{\eta}$$

- The number of turns of wire per meter length of pipe is as follows

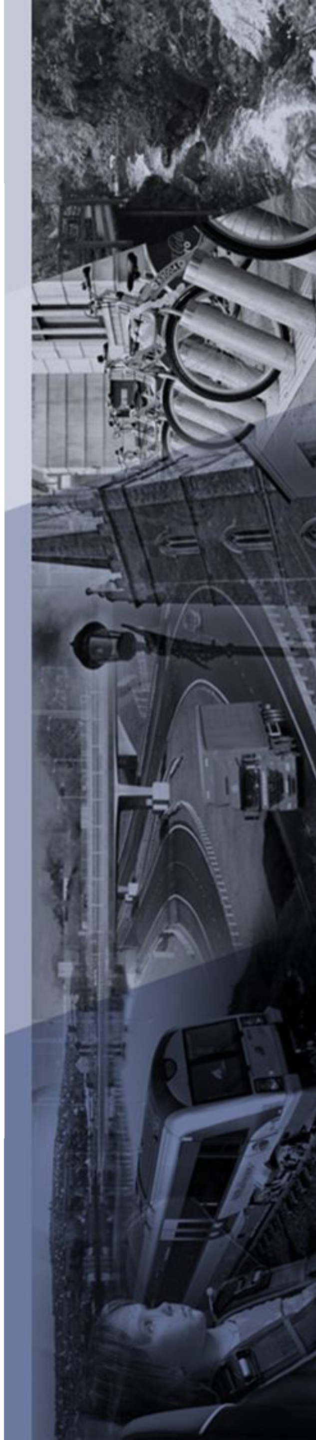
$$n = \frac{4000(t + \alpha_e t_s) f_s}{\pi d^2 f_s}$$



- In cylinder pipe, the failure occurs due to the yielding of the steel cylinder and followed by excessive elongation or fracture of hard drawn wires. The bursting fluid pressure is estimated as follows:

$$P_u = \frac{0.00157 d^2 n f_{pu} + 2 t_s f_y}{D}$$

Where
D = diameter of the pipe
ts = thickness of the cylinder
f_{ct} *= permissible compressive stress in concrete*
$f_{min,w}$ *= allowable tensile stress*
$a_e = (E_S/E_C)$*= Modular ratio*
P_u = Bursting pressure in N/mm^2
d= diameter of wire winding in mm.
f_{pu},f_{y-} ultimate and yield stress of prestressing steel



A prestressed cylinder pipe is to be designed using a steel cylinder of 1000 mm internal diameter and thickness 1.6 mm. The cucumferential wire winding consists of a 4 mm high tensile wire, initially tensioned to a stress of 1000 N/mm^2. Ultimate tensile strength of the wire = 1600 N/mm^2. Yield stress of the steel cylinder = 280 N/mm^2. The maximum permissible compressive stress in concrete at transfer is 14 N/mm^2 and no tensile stresses are permitted under working pressure of 0.8 N/mm^2. Determine the thickness of the concrete lining required, the number of turns of circumferential wire winding and the factor of safety against bursting. Assume modular ratio as 6.

$$t > \frac{N_d}{\eta f_{ct} - f_{min.w}} - \alpha_e t_s.$$

$$> \frac{0.8(1000/2)}{0.8 \times 14 - 0} - 6 \times 1.6 > 25.9 \text{ mm}$$

Using 26 mm thick concrete lining,

$$f_c = 14 \text{ N/mm}^2$$

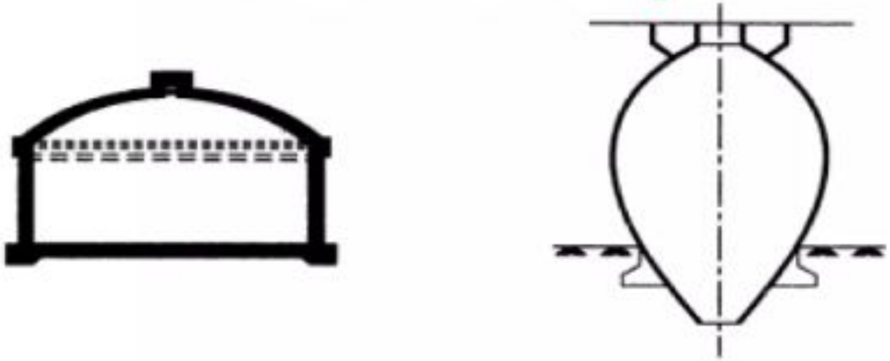
$$n = \frac{4000(26 + 6 \times 1.6)14}{\pi \times 4^2 \times 1000} = 40 \text{ turns/metre}$$

$$\text{Bursting pressure, } p_u = \frac{(0.00157 \times 4^2 \times 40 \times 1600) + (2 \times 1.6 \times 280)}{1000}$$

$$= 2.516 \text{ N/mm}^2$$

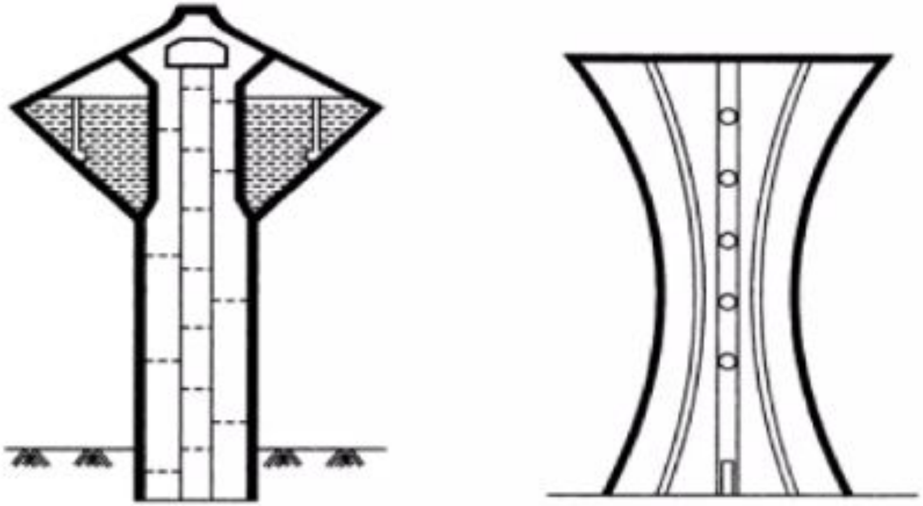
$$\text{Factor of safety against bursting} = \frac{\text{bursting pressure}}{\text{working pressure}} = \frac{2.516}{0.08} = 3.14$$

Tanks-Shapes



Circular cylindrical tank
(a)

Conical tank
(b)

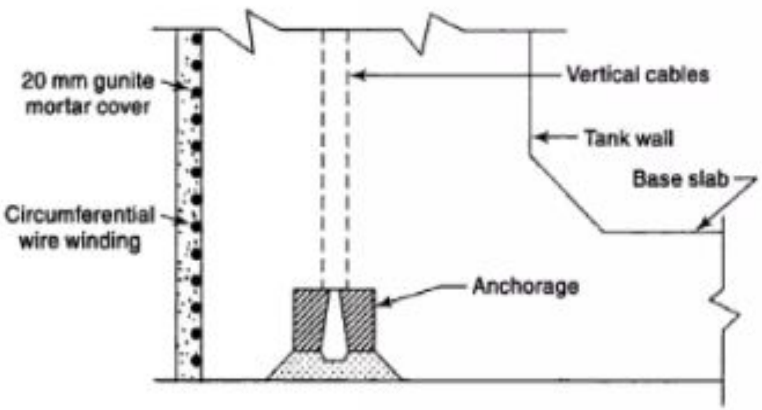


Water tower with conical tank
(c)

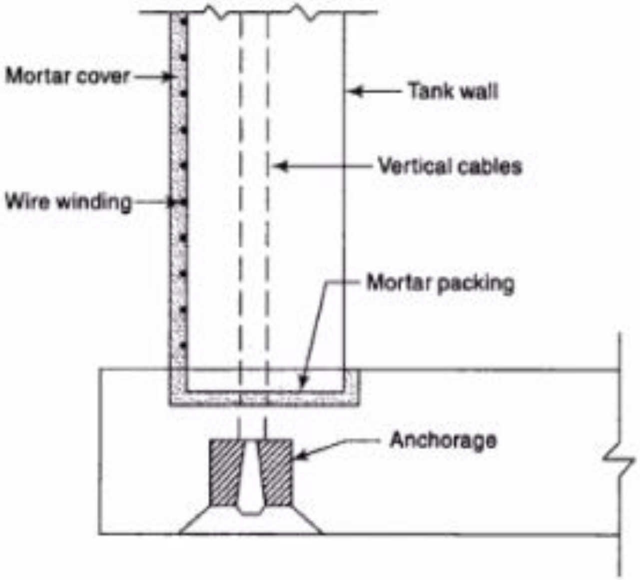
Water tower of doubly curved shell
(d)

Shapes of Prestressed Concrete Tanks

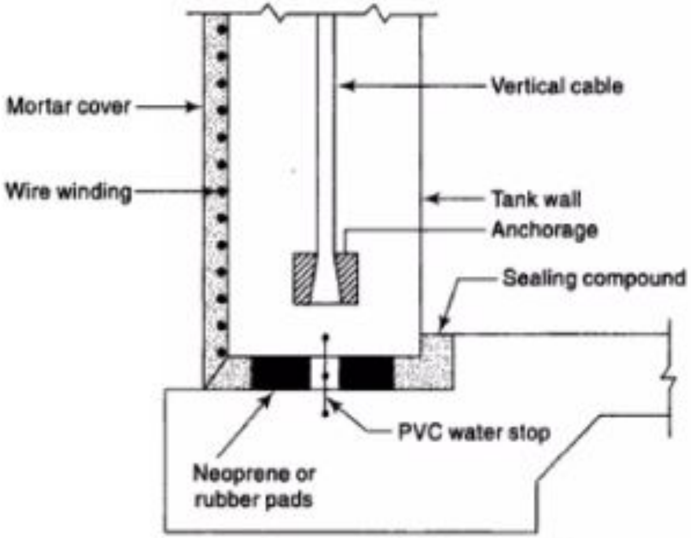
Tank with different base



Tank Wall with Fixed Base

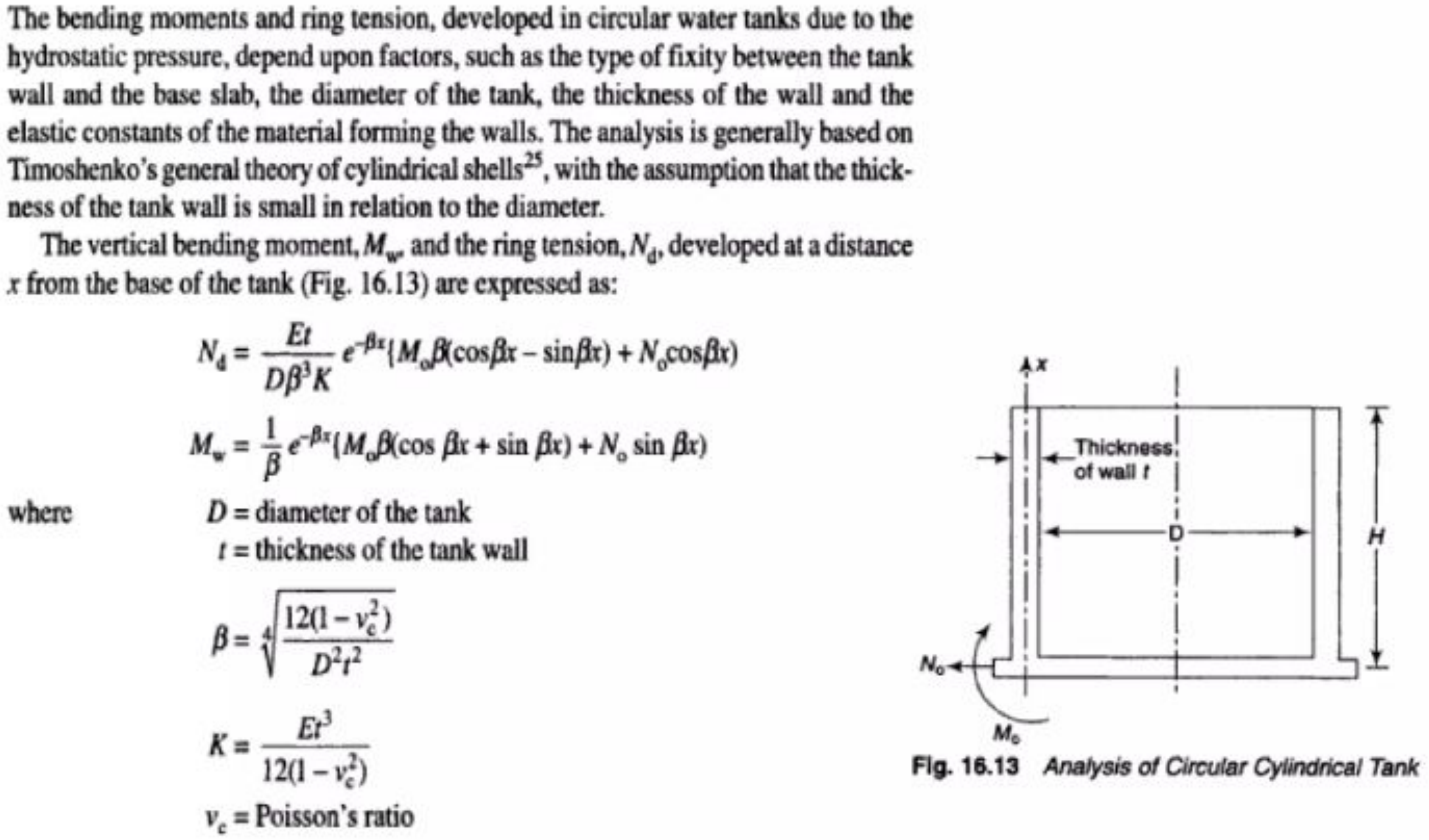


Tank Wall with Hinged Base



Tank Wall with Sliding Base

Analysis of prestressed concrete tanks (Krishnaraju pg no:490-498)



The bending moments and ring tension, developed in circular water tanks due to the hydrostatic pressure, depend upon factors, such as the type of fixity between the tank wall and the base slab, the diameter of the tank, the thickness of the wall and the elastic constants of the material forming the walls. The analysis is generally based on Timoshenko's general theory of cylindrical shells[25], with the assumption that the thickness of the tank wall is small in relation to the diameter.

The vertical bending moment, M_w, and the ring tension, N_d, developed at a distance x from the base of the tank (Fig. 16.13) are expressed as:

$$N_d = \frac{Et}{D\beta^3 K} e^{-\beta x}\{M_o\beta(\cos\beta x - \sin\beta x) + N_o\cos\beta x)$$

$$M_w = \frac{1}{\beta} e^{-\beta x}\{M_o\beta(\cos\beta x + \sin\beta x) + N_o\sin\beta x)$$

where

D = diameter of the tank

t = thickness of the tank wall

$$\beta = \sqrt[4]{\frac{12(1-v_c^2)}{D^2t^2}}$$

$$K = \frac{Et^3}{12(1-v_c^2)}$$

v_c = Poisson's ratio

Fig. 16.13 *Analysis of Circular Cylindrical Tank*

M_0 and N_0 are the moment and shear acting at the base of the tank, with their values depending upon the pressure distribution and the conditions of fixity at the base. A diagrammatic representation of the variation of bending moments and ring tension in the walls of tanks for different types of bases is shown in Fig. 16.14. Maximum bending moments develop in the case of tanks with a fixed base while the ring tension is maximum for the free-base condition. In the case of tanks with walls resting on rubber or neoprene pads, a comparatively smaller magnitude of bending moments is generated due to the radial frictional force developed at the base junction. According to Crom[26], the base shear can be estimated for a maximum coefficient of friction μ of 0.5, which is not likely to be exceeded.

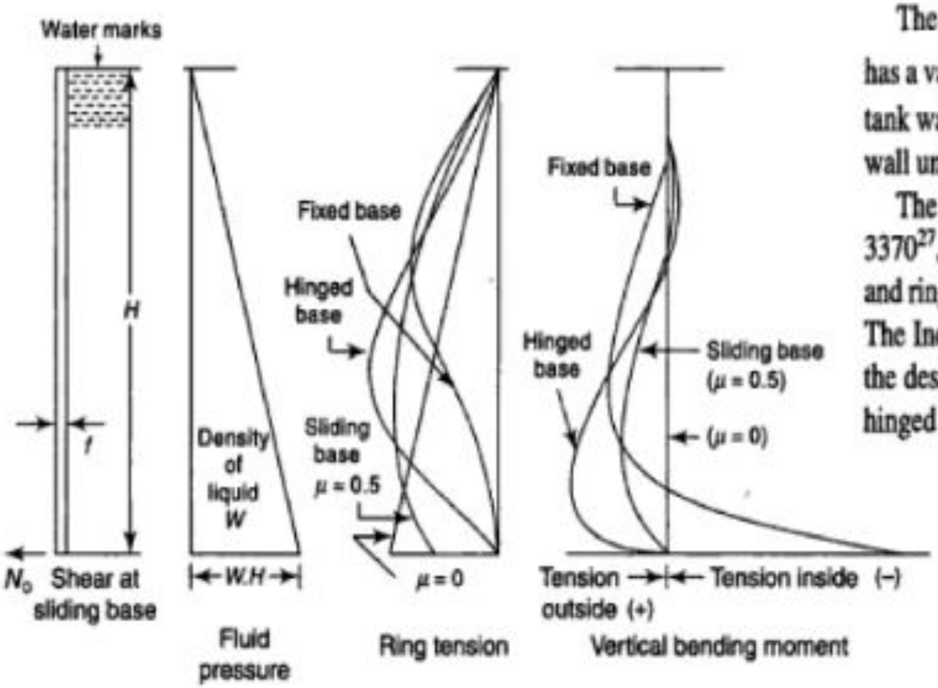
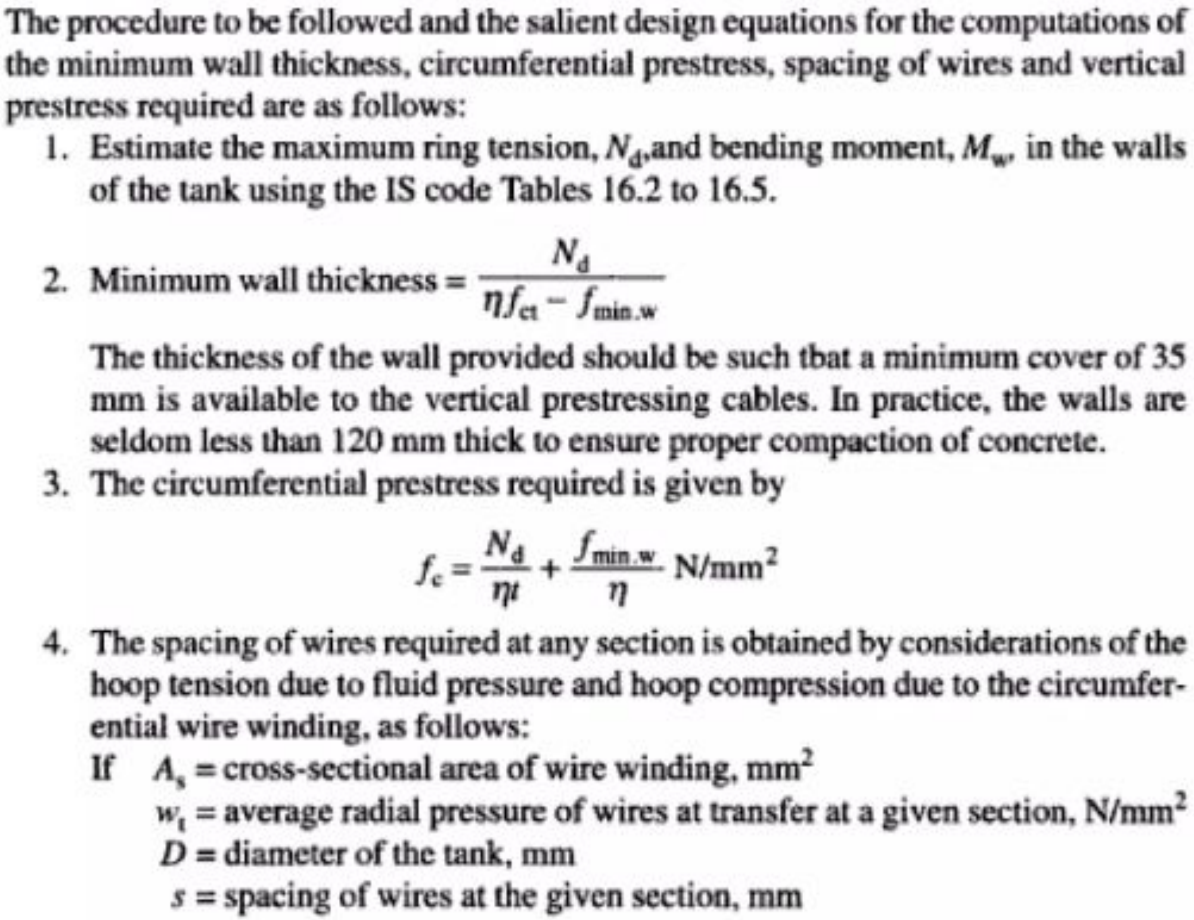


Fig. 16.14 *Ring Tension and Bending Moments in Cylindrical Tank Walls*

The maximum bending moment developed in the tank wall due to a base shear N_0 has a value of $0.247\, N_0 \sqrt{Rt}$, where R is the internal radius and t the thickness of the tank wall. The maximum moment occurs at approximately one-fifth the height of the wall under pressure, measured from the base.

The coefficients compiled in Tables 16.2 to 16.5, which are recommended in IS : 3370[27], are immensely useful in the design office for computing bending moments and ring tension developed in circular tanks with different types of base connections. The Indian standard code also contains tables of moment and shear coefficients for the design of rectangular tanks with walls fixed or hinged at the bottom and free or hinged at the top.

Design procedure of circular tanks



The procedure to be followed and the salient design equations for the computations of the minimum wall thickness, circumferential prestress, spacing of wires and vertical prestress required are as follows:

1. Estimate the maximum ring tension, N_d,and bending moment, M_w, in the walls of the tank using the IS code Tables 16.2 to 16.5.
2. Minimum wall thickness = $\dfrac{N_d}{\eta f_{ct} - f_{\min.w}}$

   The thickness of the wall provided should be such that a minimum cover of 35 mm is available to the vertical prestressing cables. In practice, the walls are seldom less than 120 mm thick to ensure proper compaction of concrete.
3. The circumferential prestress required is given by

$$f_c = \frac{N_d}{\eta t} + \frac{f_{\min.w}}{\eta}\ \text{N/mm}^2$$

4. The spacing of wires required at any section is obtained by considerations of the hoop tension due to fluid pressure and hoop compression due to the circumferential wire winding, as follows:

   If A_s = cross-sectional area of wire winding, mm^2
   w_t = average radial pressure of wires at transfer at a given section, N/mm^2
   D = diameter of the tank, mm
   s = spacing of wires at the given section, mm

f_s = stress in wires at transfer, N/mm^2
t = thickness of the tank wall, mm
f_c = compressive stress in concrete, N/mm^2

$\therefore$ Hoop compression due to prestressing = $\dfrac{w_t D}{2}$

Equating $\dfrac{w_t D}{2} = \dfrac{f_s A_s}{s}$

$$\therefore \qquad w_t = \frac{2 f_s A_s}{sD} \qquad (16.8)$$

If N_d = hoop tension due to hydrostatic working pressure, w_w
N_t = hoop compression due to radial pressure of wires, w_t

$$\text{then} \qquad N_t = N_d\left(\frac{w_t}{w_w}\right) \qquad (16.9)$$

$$\text{also} \qquad N_t = t f_c$$

From Eqs 16.9 and 16.10, the spacing of the wire winding

$$s = \frac{2N_d}{w_w} \times \frac{f_s A_s}{f_c D t} \text{ mm} \qquad (16.10)$$

5. The vertical prestress required to resist the bending moments in the wall due to the circumferential wire winding and hydrostatic pressure as a consequence of end restraint is computed as follows:

If M_t = vertical moment due to the prestress at transfer,

and Mw = vertical moment due to hydrostatic pressure

then $M_t = M_w\left(\frac{w_t}{w_w}\right)$

The compressive prestress required in concrete is expressed as

$$f_c = \frac{f_{\min.w}}{\eta} + \frac{M_w}{\eta Z}$$

where Z is the section modulus of a unit length of wall about an axis in the tangential direction and passing through the centroid.

When the tank is empty, the prestress required

$$f_c = \frac{f_{\min.w}}{\eta} + \frac{M_t}{Z} \tag{16.11}$$

The vertical prestressing force required is given by,

$$P = f_c A_c$$

where A_c is the cross-sectional area of concrete per unit length along the circumference.

According to the Indian standard code, the vertical prestressing force is to be designed for 30 per cent of the hoop compression.

Problem-4

A cylindrical prestressed concrete water tank of internal diameter 30 m is required to store water over a depth of 7.5 m. The permissible compressive stress in concrete at transfer is 13 N/mm^2 and the minimum compressive stress under working pressure is 1 N/mm^2, the loss ratio is 0.75, Wires of 5 mm dia with an initial stress of 1000 N/mm^2 are available for circumferential winding and freyssinet cables made up of 12 wires of 8 mm dia stressed to 1200 N/mm^2 are to be used for vertical prestressing. Design the tank walls assuming the base as fixed. The cube strength of concrete is 40 N/mm^2. For the thickness of wall is 150 mm.

For the required depth of storage of 7.5 m and diameter 30 m, an average wall thickness of 150 mm is tentatively assumed based on Table 16.1.

$D = 30$ m, $H = 7.5$ m and $t = 150$ mm, $\eta = 0.75$

$$\frac{H^2}{Dt} = \frac{7.5^2}{30 \times 0.15} = 12.5$$

$$w_w = wH = (10 \times 7.5)\ \text{kN/m}^2 = 0.075\ \text{N/mm}^2$$

maximum ring tension $N_d = (0.64 \times 10 \times 7.5 \times 15) = 720$ kN/m = 720 N/mm

maximum moments $M_w = (0.01 \times 10 \times 7.5^3) = 42.5$ kN m/m = 42500 N mm/mm

Minimum wall thickness

$$t = \frac{N_d}{\eta f_{ct} - f_{min.w}} = \frac{720}{(0.75 \times 13) - (1)} = 82.3\ \text{mm}$$

Net thickness available (allowing for vertical cables of diameter 30 mm) is = (150 – 30) = 120 mm

Required circumferential prestress is,

$$f_c = \frac{N_d}{\eta t} + \frac{f_{min.w}}{\eta}$$

$$f_c = \frac{720}{0.75 \times 120} + \frac{1}{0.75} = 9.4\ \text{N/mm}^2$$

Spacings of circumferential wire winding at base is,

$$s = \frac{2N_d}{w_w}\frac{f_s A_s}{f_c Dt} = \frac{2 \times 720}{0.075} \times \frac{1000 \times 20}{9.4 \times 30 \times 10^3 \times 120} = 11.4\ \text{mm}$$

Number of wires/metre = 87

Ring tension N_d at 0.1 H(0.75 m) from top is

$$N_d = (0.097 \times 10 \times 7.5 \times 15) = 109 \text{ kN/m} = 109 \text{ N/mm}$$

$$f_c = \frac{109}{0.75 \times 120} + \frac{1}{0.75} = 2.5 \text{ N/mm}^2$$

$$s = \frac{2 \times 109}{0.075} \times \frac{1000 \times 20}{2.5 \times 30 \times 10^3 \times 120} = 64 \text{ mm}$$

Number of wires/metre at the top of tank = 16

Maximum radial pressure due to prestress is,

$$w_t = \frac{2 f_s A_s}{sD} = \frac{2 \times 1000 \times 20}{11.4 \times 30 \times 10^3} = 0.117 \text{ N/mm}^2$$

Maximum vertical moment due to prestress is,

$$M_t = M_w \left(\frac{w_t}{w_w}\right) = 42500\left(\frac{0.117}{0.075}\right) = 67{,}000 \text{ N mm/mm}$$

$$= 67 \times 10^6 \text{ Nmm/m}$$

Considering one metre length of tank along the circumference, the section modulus is,

$$Z = \frac{1000 \times 150^2}{6} = 375 \times 10 \text{ mm}^3$$

Vertical prestress required is,

$$f_c = \frac{f_{\min.w}}{\eta} + \frac{M_t}{Z} = \frac{1}{0.75} + \frac{67 \times 10^6}{375 \times 10^4} = 19.2 \text{ N/mm}^2$$

Since this stress exceeds the permissible value of $f_{ct} = 13$ N/mm², the thickness of the tank wall at base is increased to 200 mm. Thus,

$$Z = \frac{1000 \times 200^2}{6} = 666 \times 10^4 \text{ mm}^3$$

$$f_c = \frac{1}{0.75} + \frac{67 \times 10^6}{666 \times 10^4} = 12 \text{ N/mm}^2$$

$$\text{Vertical prestressing force} = f_c A = \frac{(12 \times 1000 \times 200)}{(1000)} = 2400 \text{ kN}$$

Using 8 mm diameter (12 nos.) Freyssinet cables

$$\text{Force/cable} = \frac{(50 \times 12 \times 1200)}{(1000)} = 720 \text{ kN}$$

$$\therefore \quad \text{Spacings of vertical cables} = \frac{1000 \times 720}{2400} = 300 \text{ mm}$$

The approximate vertical prestress required to counteract winding stresses as per IS code is

$$= 0.3 f_c = (0.3 \times 9.4) = 2.82 \text{ N/mm}^2$$

$$\text{Vertical prestressing force required} = \frac{(2.82 \times 1000 \times 200)}{(1000)} = 564 \text{ kN}$$

$$\text{Ultimate tensile force in wires at base of tank} = \frac{(87 \times 20 \times 1500)}{(1000)} = 2610 \text{ kN}$$

$$\text{Load factor against collapse} = \frac{(2610)}{(720)} = 3.6$$

$$\text{Direct tensile strength of concrete} = 0.267\sqrt{40} = 1.7 \text{ N/mm}^2$$

$$\text{Cracking load} = (1000 \times 200)\frac{(0.75 \times 9.4 + 1.7)}{(1000)} = 1760 \text{ kN}$$

$$\therefore \quad \text{Factor of safety against cracking} = \frac{(1760)}{(720)} = 2.45$$

Prestressed concrete poles

- PC poles ate widely used for overhead power transmission, lighting poles and telecommunication lines.
- These poles have virtually replaced the traditional poles made of wood, steel and reinforced concrete.
- The prestressed concrete poles are lighter, durable, and more economical.
- The poles may be pretensioned or post tensioned and may be designed in accordance with IS 1678 and IS 1343.
- The poles should be designed for the following load conditions
  - Wind load on the conductors and the pole
  - Torsion due to snapping of a conductor
  - Bending due to snapping of all conductors on either side of the pole
  - Handling and erection stresses and
  - Snow loads

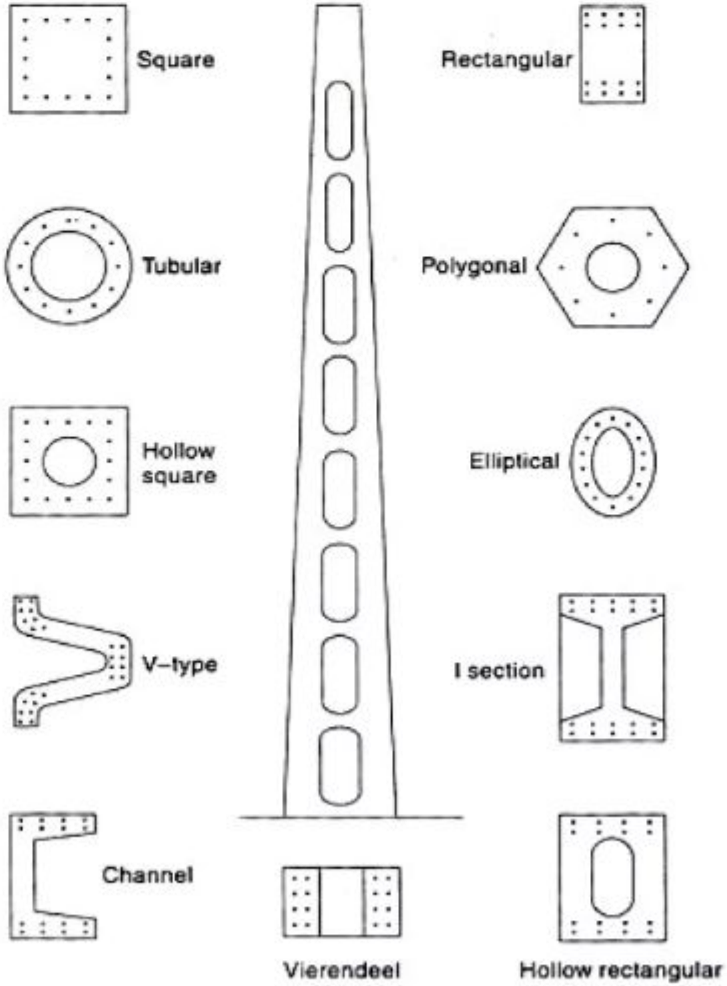


Fig. *Cross-sections of Prestressed Concrete Poles*

Codal provisions

- IS 1678-1998 gives minimum length, minimum design loads and detailing requirements. It defines four stages of load acting on a PSC pole:
  - Working load- maximum load in the transverse direction including the wind pressure, ever likely to occur, on the pole. This load is assumed to act a point 600 mm below the top of the pole.
  - Transverse load at first crack- **at least equal to the working load for design purpose**
  - Average permanent load- it is the fraction of the working load which may be considered of long duration over a period of one year. It is taken equal to 40% of the load at the first crack.
  - Ultimate transverse load- it is maximum transverse load acting at 600 mm below the top at which failure occurs.

- The load factor on the transverse strength for PSC pole is taken between 2 and 2.5.
- The code further specifies that in the case of poles used of power transmission lines, the strength of the poles in the direction of the line should not be less than 25% of the strength required in the transverse direction.

General Considerations

- A PSC pole is essentially a vertical cantilever.
- The bending moment increases from zero at the top to the maximum at the base.
- consequently, the maximum moment of resistance and the maximum cross-sectional area is required at the base.
- Generally rectangular and square cross sections are used in PSC poles.
- The width of the pole is kept constant while the depth is tapered from top to bottom.
- Since the pole is subjected to reversible wind pressure, the prestress has to be uniform over the whole section.
- The eccentricity ‘e’ is taken as zero. Thus a PSC Pole is an axially prestressed member.
- It may be designed as a fully prestressed member or a partially prestressed member as per IS 1343.

References

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Thanks for listening-

All the best