University of Anbar
College of Engineering
Civil Engineering Department
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Strength of Materials I, Semester 1, 2022 **Dr. Dhafer K. Jadaan**

Syllabus

1. Equilibrium

2. Stresses

3. Strains

4. Mechanical Properties

5. Axial Load

6. Torsion

7. Shear and Bending diagrams

Introduction

International System of Units

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

Example:

$$
\begin{aligned}
(50\ \text{kN})(60\ \text{nm}) &= [50(10^3)\ \text{N}][60(10^{-9})\ \text{m}] \\
&= 3000(10^{-6})\ \text{N}\cdot\text{m} = 3(10^{-3})\ \text{N}\cdot\text{m} = 3\ \text{mN}\cdot\text{m}
\end{aligned}
$$

Dimensional Homogeneity

$$s = vt + \frac{1}{2}at\char`^2$$

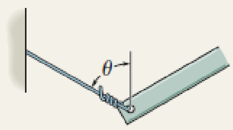
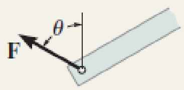
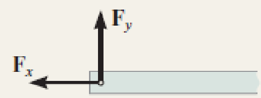
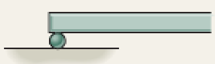
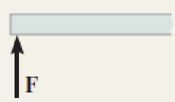
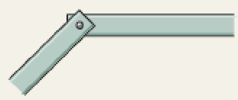
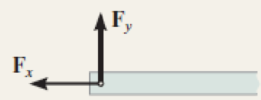
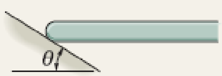
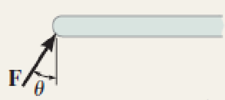
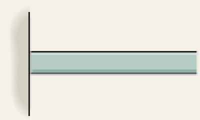
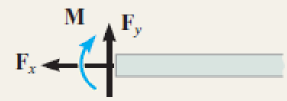
$$m = \frac{m}{\cancel{s}}\cancel{s} + \frac{1}{2}\frac{m}{\cancel{s}^2}\cancel{s}^2$$

Chapter One

Equilibrium

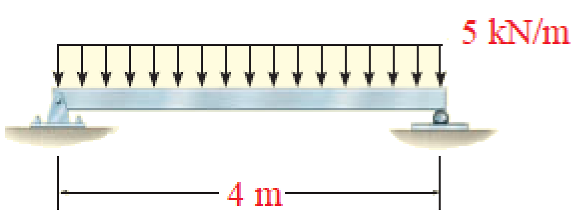
Support Reactions

TABLE 1–1

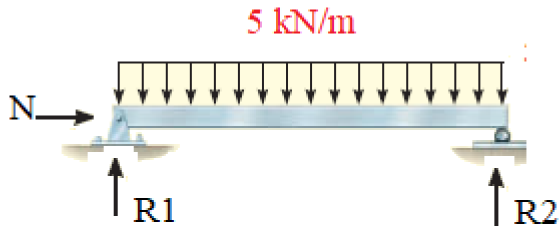
Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Examples

Determine the reactions of the simply supported beam.



Solution

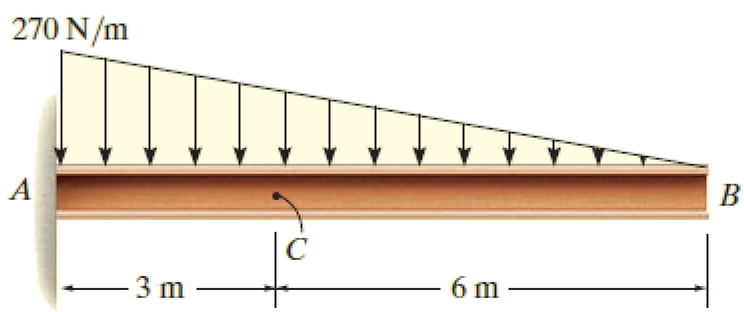


$$\Sigma F_x = 0 \rightarrow N = 0$$

$$\Sigma M_{R2} = 0 \rightarrow R_1 * 4 - 5 * 4 * 4/2 = 0 \rightarrow R_1 = 10\ kN$$

$$\Sigma F_y = 0 \rightarrow R_1 + R_2 - 5 * 4 = 0 \rightarrow R_2 = 10\ kN$$

Determine the resultant internal loadings acting on the cross section at *C* of the cantilevered beam shown in Fig. 1–4*a*.

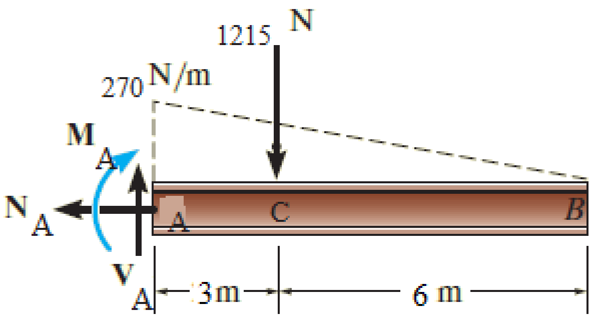


(a)

Fig. 1–4

Solution

Free-body diagram. From table 1-4, fixed support has three reactions



(b)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_A = 0$$

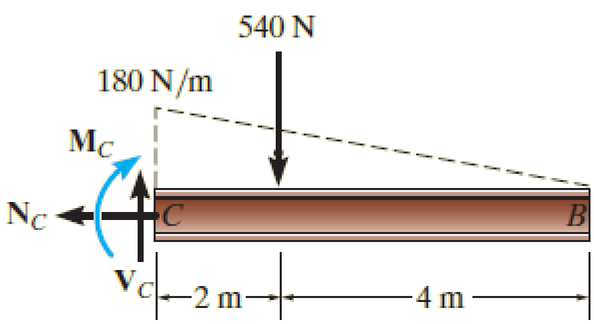
$$N_A = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad V_A - 1215\,\text{N} = 0$$

$$V_A = 1215\,\text{N}$$

$$\circlearrowright + \Sigma M_A = 0; \qquad -M_A - 1215\,\text{N}(3\,\text{m}) = 0$$

$$M_A = -3645\,\text{N}\cdot\text{m}$$



(b)

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_C = 0$$

$$N_C = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C - 540\,\text{N} = 0$$

$$V_C = 540\,\text{N}$$

$$\circlearrowright + \Sigma M_C = 0; \qquad -M_C - 540\,\text{N}(2\,\text{m}) = 0$$

$$M_C = -1080\,\text{N}\cdot\text{m}$$

Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. 5–12*a*. Neglect the weight of the beam.

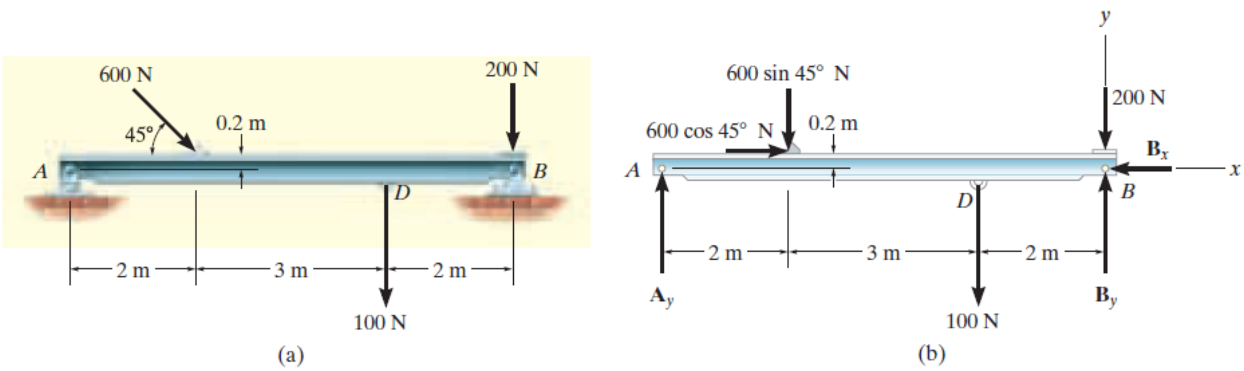
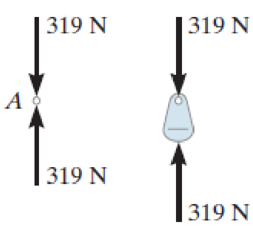


Fig. 5–12



(c)

SOLUTION

Free-Body Diagram. The supports are *removed*, and the free-body diagram of the beam is shown in Fig. 5–12*b*. (See Example 5.1.) For simplicity, the 600-N force is represented by its x and y components as shown in Fig. 5–12*b*.

Equations of Equilibrium. Summing forces in the x direction yields

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 600 \cos 45^\circ \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100 \text{ N}(2 \text{ m}) + (600 \sin 45^\circ \text{ N})(5 \text{ m})$$

$$- (600 \cos 45^\circ \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N} \qquad \textit{Ans.}$$

Summing forces in the y direction, using this result, gives

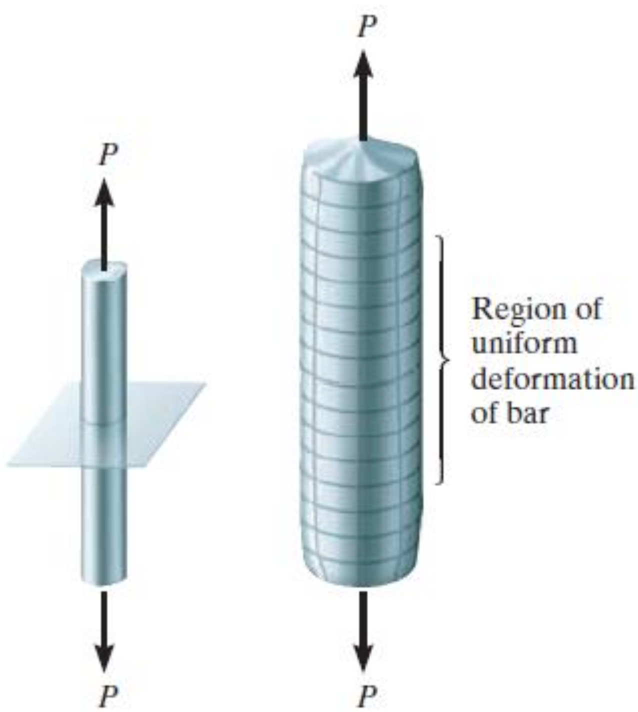
$$+\uparrow \Sigma F_y = 0; \qquad 319 \text{ N} - 600 \sin 45^\circ \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N} \qquad \textit{Ans.}$$

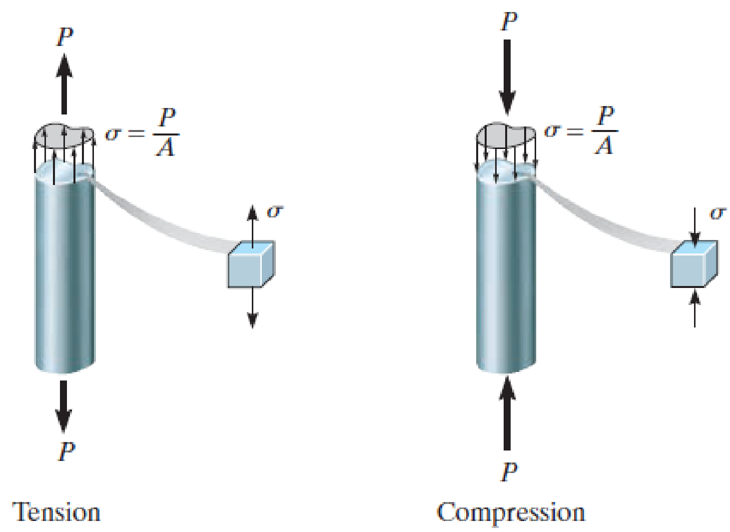
Average Normal Stress in an Axially Loaded Bar

This bar is **prismatic** since all cross sections are the same

Homogeneous material has the same physical and mechanical properties throughout its volume, and ***isotropic material*** has these same properties in all directions.



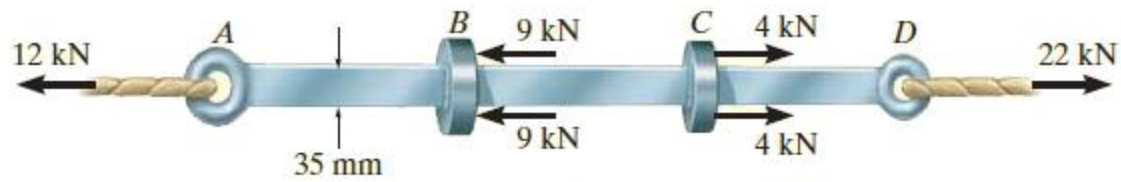
(a) (b)



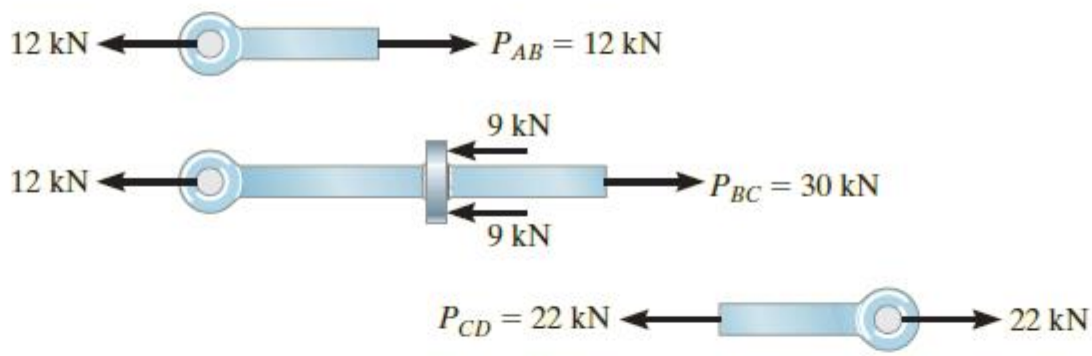
$$\sigma = \frac{P}{A}$$

Examples

The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.

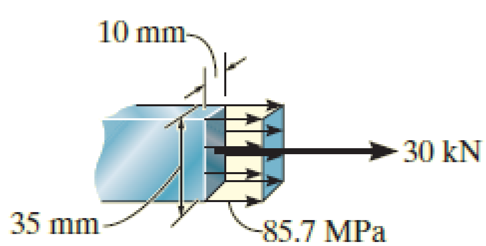


(a)



(b)

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$



The casting shown in Fig. 1–18*a* is made of steel having a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$. Determine the average compressive stress acting at points *A* and *B*.

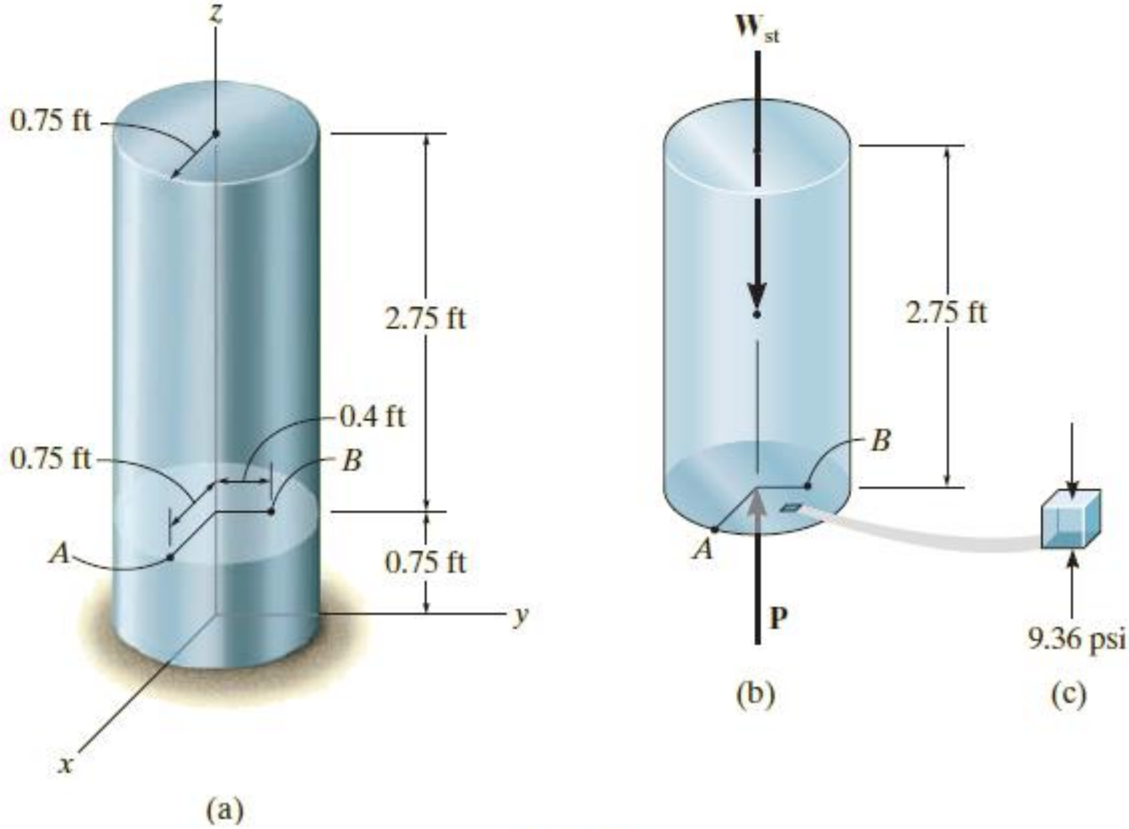


Fig. 1–18

SOLUTION

Internal Loading. A free-body diagram of the top segment of the casting where the section passes through points *A* and *B* is shown in Fig. 1–18*b*. The weight of this segment is determined from $W_{st} = \gamma_{st} V_{st}$. Thus the internal axial force *P* at the section is

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (490 \text{ lb/ft}^3)(2.75 \text{ ft})[\pi(0.75 \text{ ft})^2] = 0$$

$$P = 2381 \text{ lb}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75 \text{ ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381 \text{ lb}}{\pi(0.75 \text{ ft})^2} = 1347.5 \text{ lb/ft}^2$$

$$\sigma = 1347.5 \text{ lb/ft}^2 (1 \text{ ft}^2/144 \text{ in}^2) = 9.36 \text{ psi} \qquad \textit{Ans.}$$

Member AC shown in Fig. 1–19a is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.

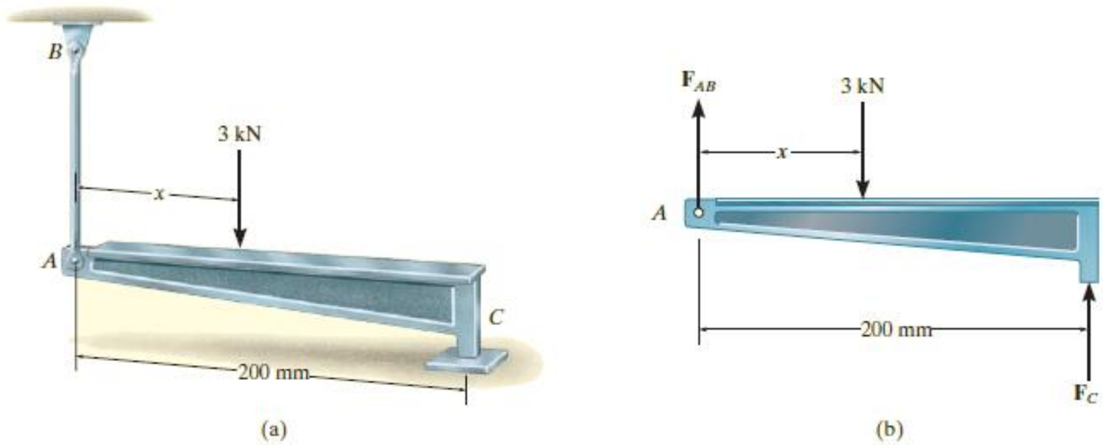


Fig. 1–19

SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000\text{ N} = 0 \qquad (1)$$
$$\circlearrowleft + \Sigma M_A = 0; \qquad -3000\text{ N}(x) + F_C(200\text{ mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the bar AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400\text{ mm}^2} = \frac{F_C}{650\text{ mm}^2}$$
$$F_C = 1.625F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143\text{ N}$$
$$F_C = 1857\text{ N}$$

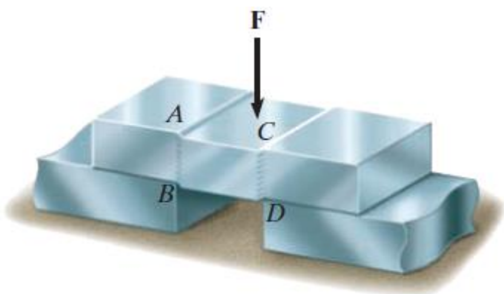
The position of the applied load is determined from Eq. 2,

$$x = 124\text{ mm} \qquad \textit{Ans.}$$

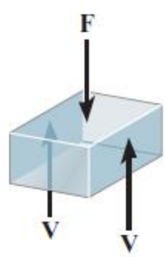
NOTE: $0 < x < 200$ mm, as required.

Average Shear Stress

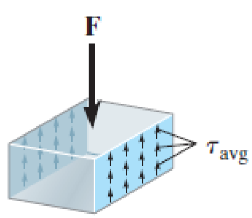
Shear stress has been defined in Section 1.3 as the stress component that acts *in the plane* of the sectioned area.



(a)



(b)



(c)

Fig. 1–20

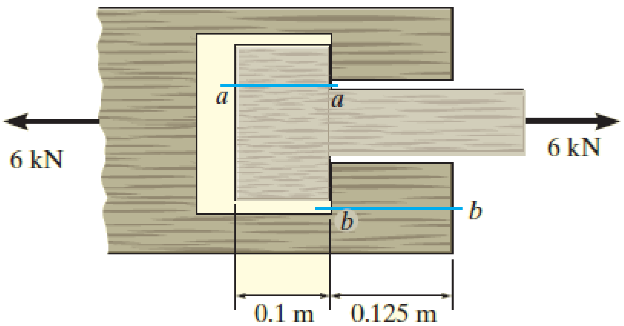
Fig. 1–20*b*, indicates that the shear force $V = F/2$ must be applied at each section to hold the segment in equilibrium. The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

$$\tau_{avg} = \frac{V}{A} \tag{1–7}$$

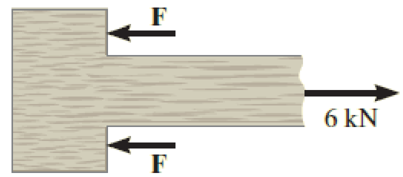
The loading case discussed here is an example of ***simple or direct shear***, since the shear is caused by the *direct action* of the applied load **F**.

Examples

If the wood joint in Fig. 1–23*a* has a width of 150 mm, determine the average shear stress developed along shear planes *a–a* and *b–b*. For each plane, represent the state of stress on an element of the material.



(a)



(b)

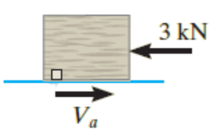
Fig. 1–23

SOLUTION

Internal Loadings. Referring to the free-body diagram of the member, Fig. 1–23*b*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 6\text{ kN} - F - F = 0 \qquad F = 3\text{ kN}$$

Now consider the equilibrium of segments cut across shear planes *a–a* and *b–b*, shown in Figs. 1–23*c* and 1–23*d*.

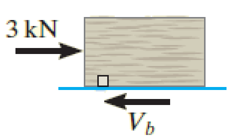


(c)

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad V_a - 3\text{ kN} = 0 \qquad V_a = 3\text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3\text{ kN} - V_b = 0 \qquad V_b = 3\text{ kN}$$

Average Shear Stress.

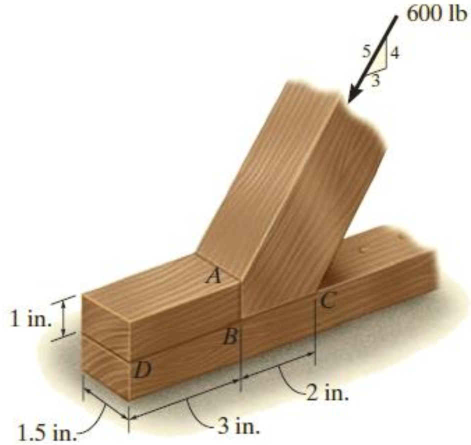


(d)

$$(\tau_a)_{\text{avg}} = \frac{V_a}{A_a} = \frac{3(10^3)\text{ N}}{(0.1\text{ m})(0.15\text{ m})} = 200\text{ kPa} \qquad \textit{Ans.}$$

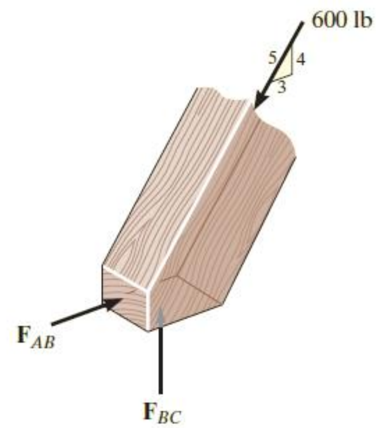
$$(\tau_b)_{\text{avg}} = \frac{V_b}{A_b} = \frac{3(10^3)\text{ N}}{(0.125\text{ m})(0.15\text{ m})} = 160\text{ kPa} \qquad \textit{Ans.}$$

The inclined member in Fig. 1–24*a* is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by *AB* and *BC*, and the average shear stress along the horizontal plane defined by *DB*.

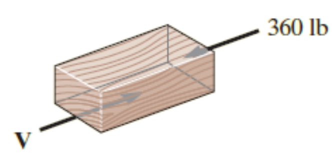


(a)

Fig. 1–24



(b)



(c)

SOLUTION

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–24*b*. The compressive forces acting on the areas of contact are

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 360\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 480\text{ lb}$$

Also, from the free-body diagram of the top segment *ABD* of the bottom member, Fig. 1–24*c*, the shear force acting on the sectioned horizontal plane *DB* is

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V = 360\text{ lb}$$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

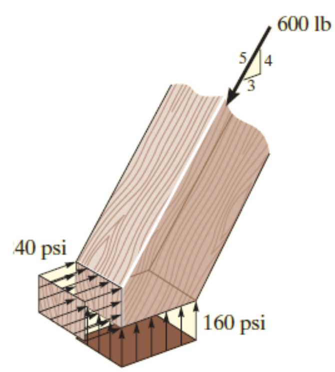
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360\text{ lb}}{(1\text{ in.})(1.5\text{ in.})} = 240\text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480\text{ lb}}{(2\text{ in.})(1.5\text{ in.})} = 160\text{ psi} \qquad \textit{Ans.}$$

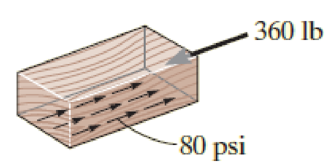
These stress distributions are shown in Fig. 1–24*d*.

The average shear stress acting on the horizontal plane defined by *DB* is

$$\tau_{\text{avg}} = \frac{360\text{ lb}}{(3\text{ in.})(1.5\text{ in.})} = 80\text{ psi} \qquad \textit{Ans.}$$

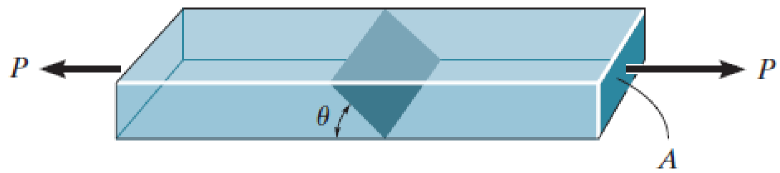


(d)



(e)

Show that the average normal and shear stresses on the shade section in the figure below are given as

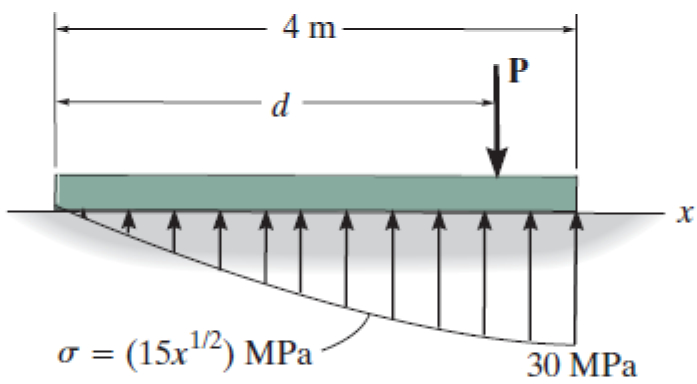


$$V = P\cos\theta,\ N = P\sin\theta,$$

$$\sigma = \frac{P}{A}\sin^2\theta,\ \tau_{avg} = \frac{P}{2A}\sin 2\theta$$

The exercise below is on the normal stress (DIY)

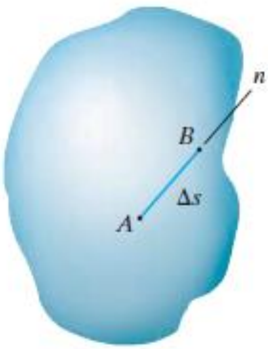
The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance *d* to where it is applied.



Chapter 3

Strains

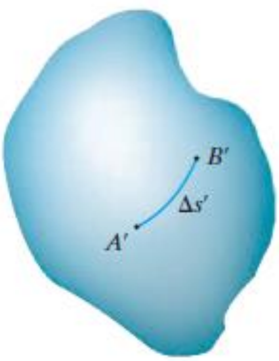
Normal Strain. If we define the normal strain as the change in length of a line per unit length, then we will not have to specify the *actual length* of any particular line segment.



Undeformed body
(a)

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$

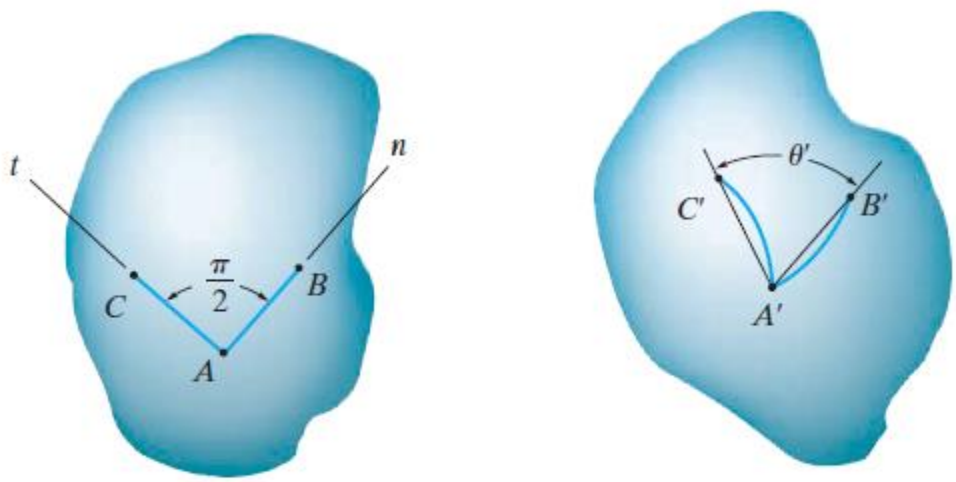


Deformed body
(b)

Units: In SI units: m/m or cm/cm or mm/mm or $\frac{\mu m}{m} = 10^{-6} m/m$ and in Inch-pound in/in.

for experimental work, strain is expressed as a percent, e.g., 0.001 m/m = 0.1%. As an example, a normal strain of $480(10^{-6})$ can be reported as $480(10^{-6})$ in./in., 480 μm/m, or 0.0480%. Also, one can state this answer as simply 480 μ (480 "micros").

Shear Strain. Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between these two line segments is referred to as ***shear strain***.



Undeformed body
(a)

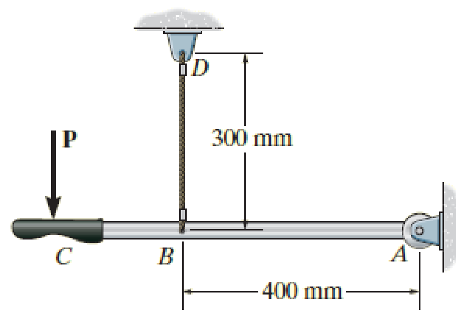
Deformed body
(b)

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

Examples:

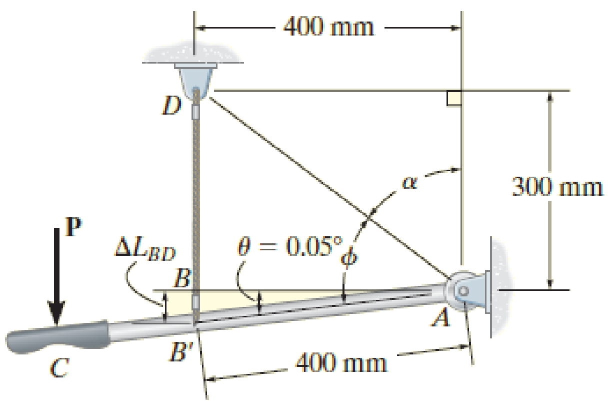
When force **P** is applied to the rigid lever arm ABC in Fig. 2–5*a*, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain developed in wire BD.

SOLUTION I



Elongation in rod L_{BD}

$$\Delta L_{BD} = \sin(0.05) * 400mm = 0.3491\ mm$$



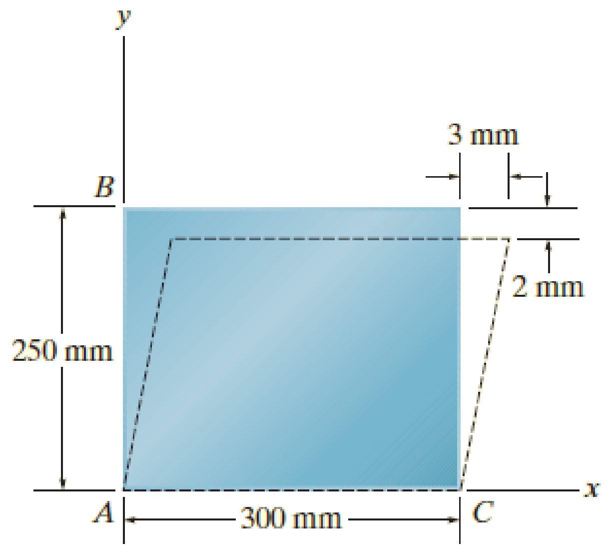
(b)

Fig. 2–5

Strain in rod L_{BD} is

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.3491\ \text{mm}}{300\ \text{mm}} = 0.00116\ \text{mm/mm} \qquad \textit{Ans.}$$

Due to a loading, the plate is deformed into the dashed shape shown in Fig. 2–6*a*. Determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate at A relative to the x and y axes.

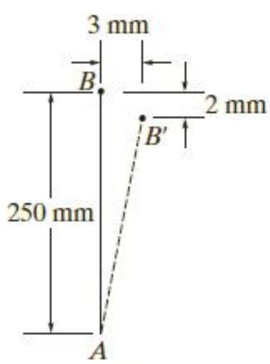


(a)

Fig. 2–6

SOLUTION

Part (a). Line AB, coincident with the y axis, becomes line AB' after deformation, as shown in Fig. 2–6*b*. The length of AB' is



(b)

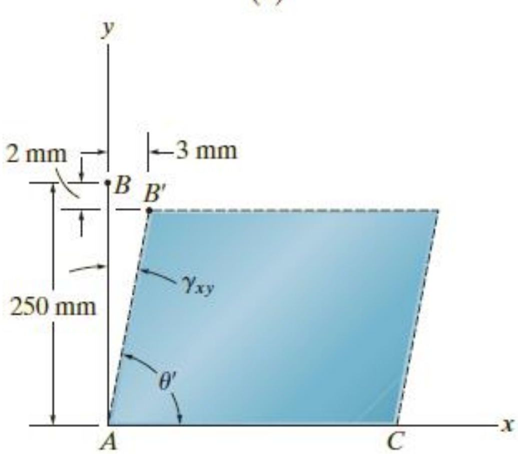
$$AB' = \sqrt{(250\text{ mm} - 2\text{ mm})^2 + (3\text{ mm})^2} = 248.018\text{ mm}$$

The average normal strain for AB is therefore

$$(\epsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018\text{ mm} - 250\text{ mm}}{250\text{ mm}}$$

$$= -7.93(10^{-3})\text{ mm/mm} \qquad \textit{Ans.}$$

The negative sign indicates the strain causes a contraction of AB.



Part (b). As noted in Fig. 2–6*c*, the once 90° angle BAC between the sides of the plate at A changes to θ' due to the displacement of B to B'. Since $\gamma_{xy} = \pi/2 - \theta'$, then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3\text{ mm}}{250\text{ mm} - 2\text{ mm}}\right) = 0.0121\text{ rad} \qquad \textit{Ans.}$$

Chapter 4

Mechanical properties

The Stress–Strain Diagram

Conventional Stress–Strain Diagram. We can determine the ***nominal*** or ***engineering stress*** by dividing the applied load P by the specimen's *original* cross-sectional area A_0.

$$\sigma = \frac{P}{A_0}$$

Likewise, the ***nominal*** or ***engineering strain*** is found directly from the strain gauge reading, or by dividing the change in the specimen's gauge length, δ, by the specimen's original gauge length L_0.

$$\epsilon = \frac{\delta}{L_0}$$

Elastic behavior

If the load is removed, the specimen will return back to its original shape.

proportional limit

The stress-strain relationship up to this point is linear elastic.

Elastic limit

After this point the stress-strain curve is not linear but still elastic.

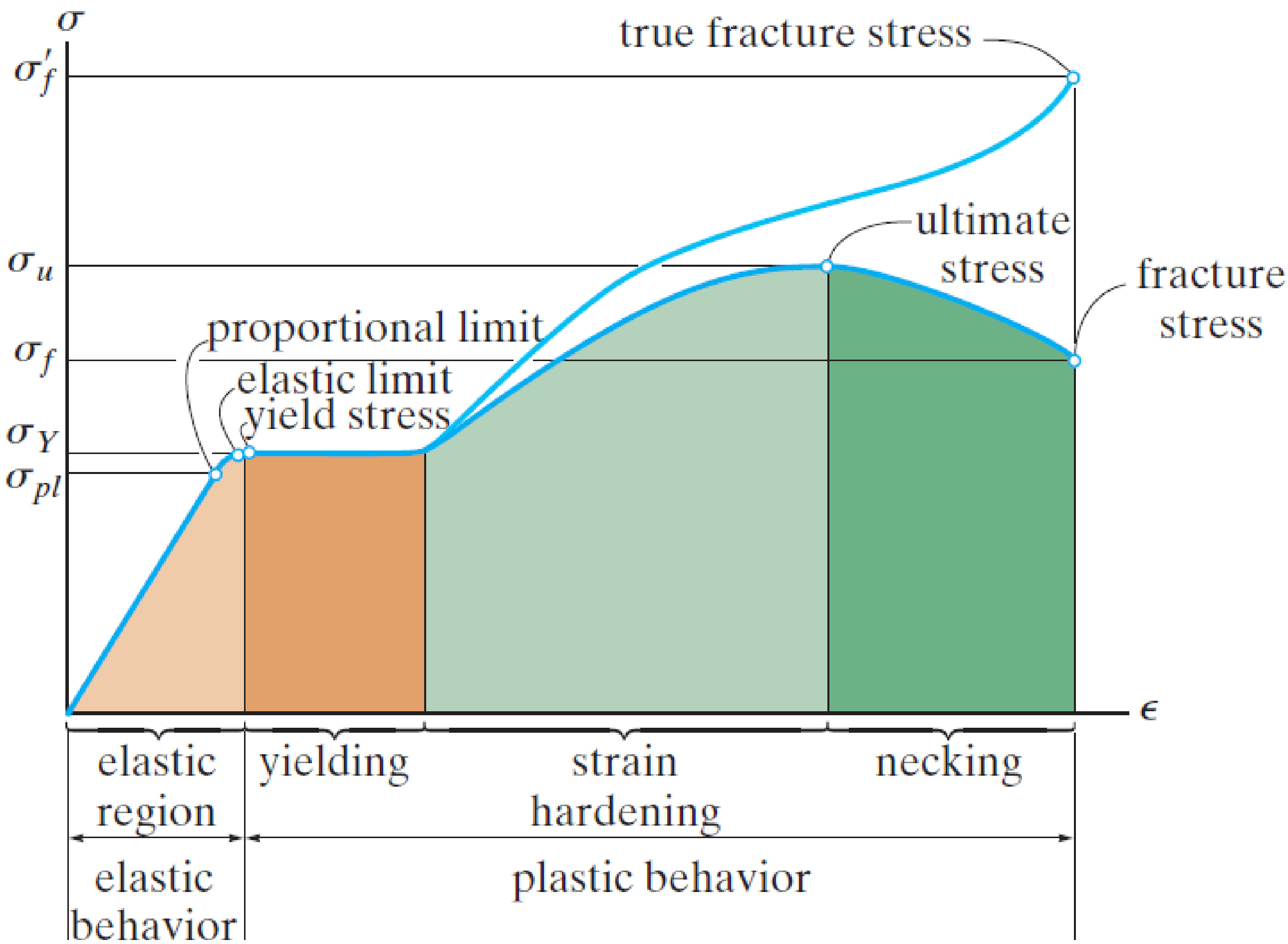
Yielding

A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently* and becomes ***plastic***.

Strain hardening

Increase in the stress after yielding until it reaches maximum stress σ_u.

Necking



When the

Figure: Stress strain curve for steel.

Necking

A "neck" tends to form due to the elongation in the specimen and finally the specimen fails at the fracture stress σ_f.

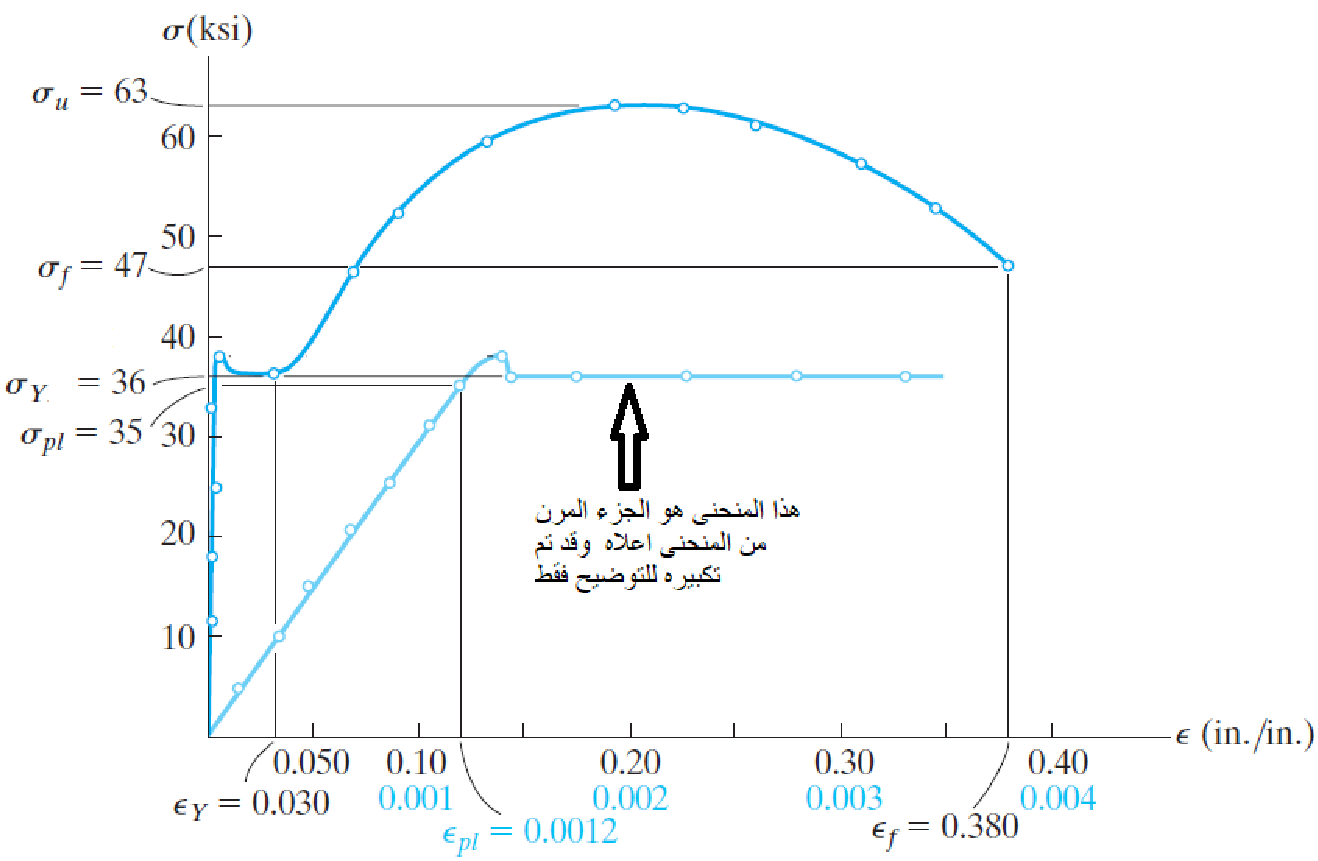
Necking

Failure of a ductile material

True Stress–Strain Diagram.

In reality, due to the elongation of the specimen, the cross-sectional area decreases and the length increases. As a result, the true stress-strain curve is obtained by using the real area A then $\sigma = \frac{P}{A}$ and the true strain is obtained bu using the true length L then $\epsilon = \frac{\delta L}{L}$.

Ductile Materials. Any material that can be subjected to large strains before it fractures is called a ***ductile material*** *(an example is steel).*



Example of stress-strain curve for steel.

From the above figure we see:

Proportional limit stress is $\sigma_{pl} = 35\ ksi$.

Yield stress is $\sigma_Y = 36\ ksi$

Ultimate stress is $\sigma_Y = 63\ ksi$

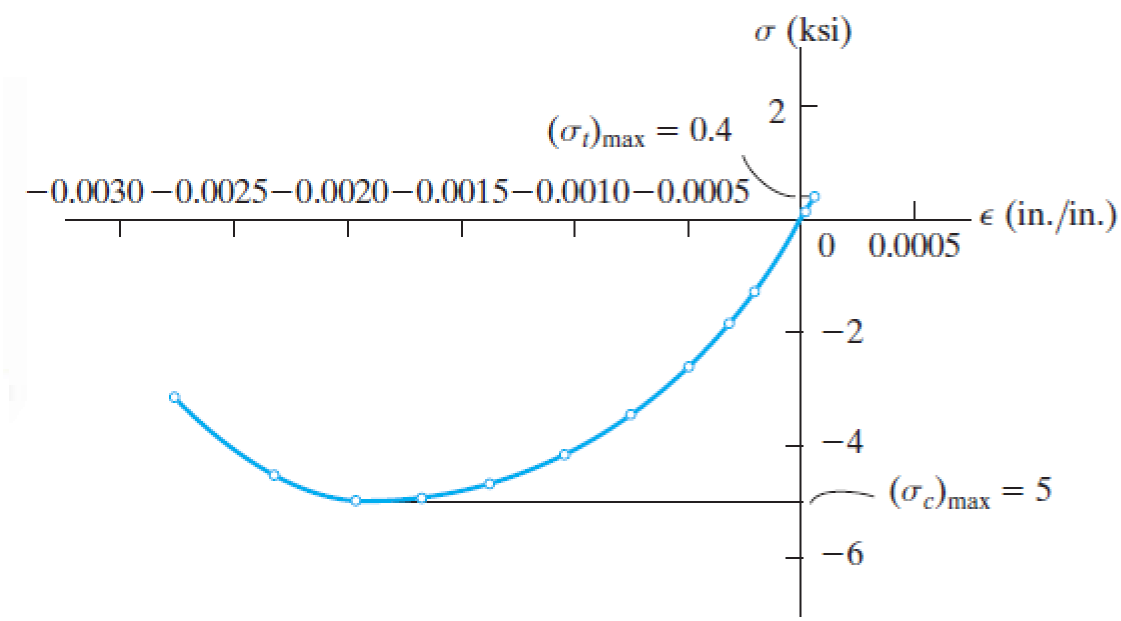
Failure stress is $\sigma_f = 47\ ksi$

Proportional limit strain $\epsilon_{pl} = 0.0012$

Yield strain $\epsilon_Y = 0.03$

Failure strain $\epsilon_f = 0.38$

Brittle Materials. *Materials that exhibit little or no yielding before failure are referred to as* ***brittle materials*** *(an example is concrete).*



Hook's law (Hook 1676)

$$\sigma = E\epsilon$$

From the above stress-strain curve of the steel

$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{35\ \text{ksi}}{0.0012\ \text{in./in.}} = 29(10^3)\ \text{ksi}$$

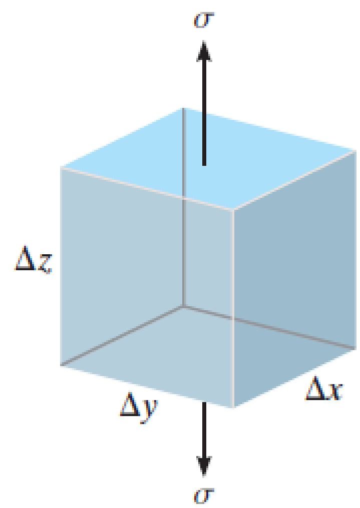
And E *is Young's modulus of elasticity (Young 1807)*

Strain Energy density

Energy is work= Force*distance and strain energy is work due to deformations.

For the specimen in tension in this figure, assume the elongation is δL and the cross-sectional area is A, then

Strain energy density is the strain energy divided by the volume



$$Strain\ energy\ density = \frac{\frac{1}{2}F * \delta L}{\Delta V}$$

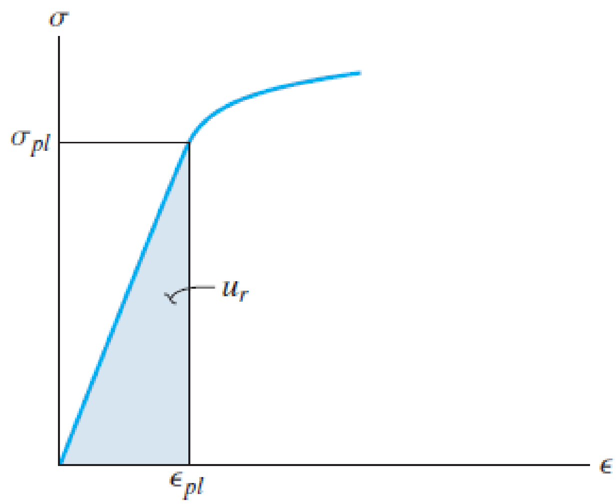
$$= \frac{1}{2\Delta V}(\sigma * A) * (\epsilon * \Delta z) = \frac{1}{2\Delta V}(\sigma\epsilon) * (A * \Delta z) = \frac{1}{2\Delta V}(\sigma\epsilon) * (\Delta V)$$

$$Strain\ energy\ density = \frac{1}{2}(\sigma\epsilon) = \frac{1}{2}(\sigma * \sigma/E) = \frac{1}{2E}\sigma^2$$

Modulus of Resilience. In particular, when the stress σ reaches the proportional limit, the strain-energy density is referred to as the ***modulus of resilience***, i.e.,

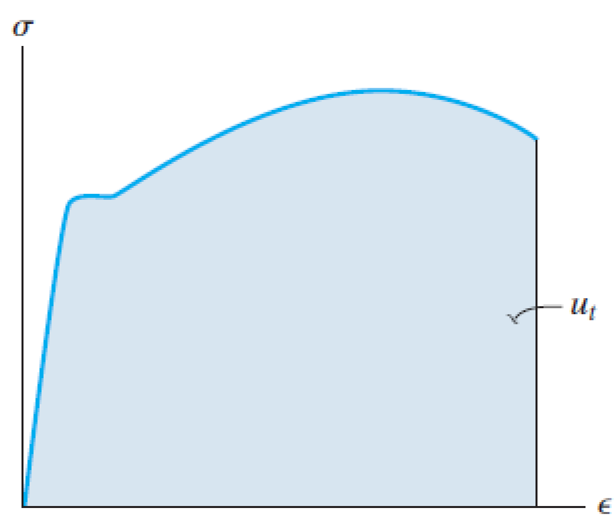
$$u_r = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}$$

We notice that strain energy density and modulus of resilience are the area under the stress-strain curve.



Modulus of resilience u_r

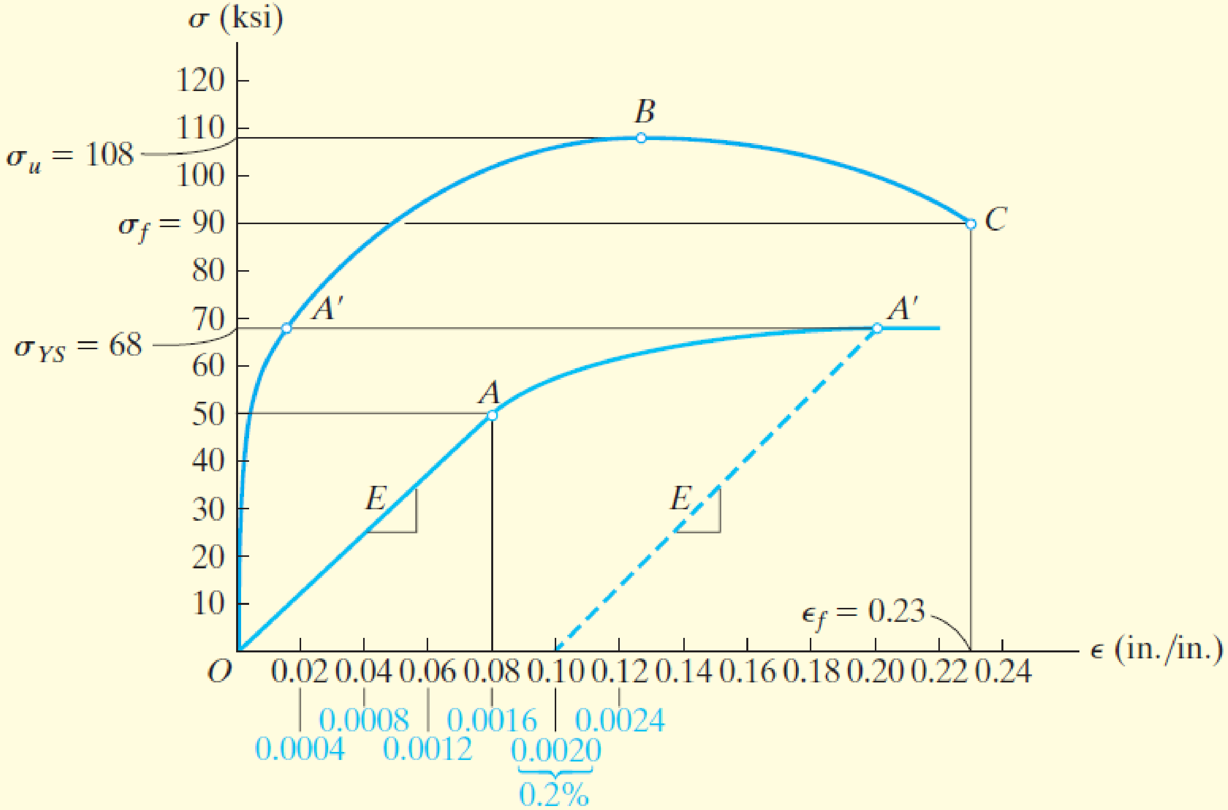
Modulus of Toughness. Another important property of a material is the ***modulus of toughness***, u_t.
entire area under the stress–strain diagram



Modulus of toughness u_t

Examples

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.



Solution

$$E = \frac{50 \text{ ksi}}{0.0016 \text{ in./in.}} = 31.2(10^3) \text{ ksi}$$

Yield Strength. For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 in./in. and graphically extend a (dashed) line parallel to OA until it intersects the $\sigma-\epsilon$ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 68 \text{ ksi} \qquad \textit{Ans.}$$

Ultimate Stress. This is defined by the peak of the σ–ϵ graph, point B in Fig. 3–18.

$$\sigma_u = 108 \text{ ksi} \qquad \textit{Ans.}$$

Fracture Stress. When the specimen is strained to its maximum of $\epsilon_f = 0.23$ in./in., it fractures at point C. Thus,

$$\sigma_f = 90 \text{ ksi} \qquad \textit{Ans.}$$

Example

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.

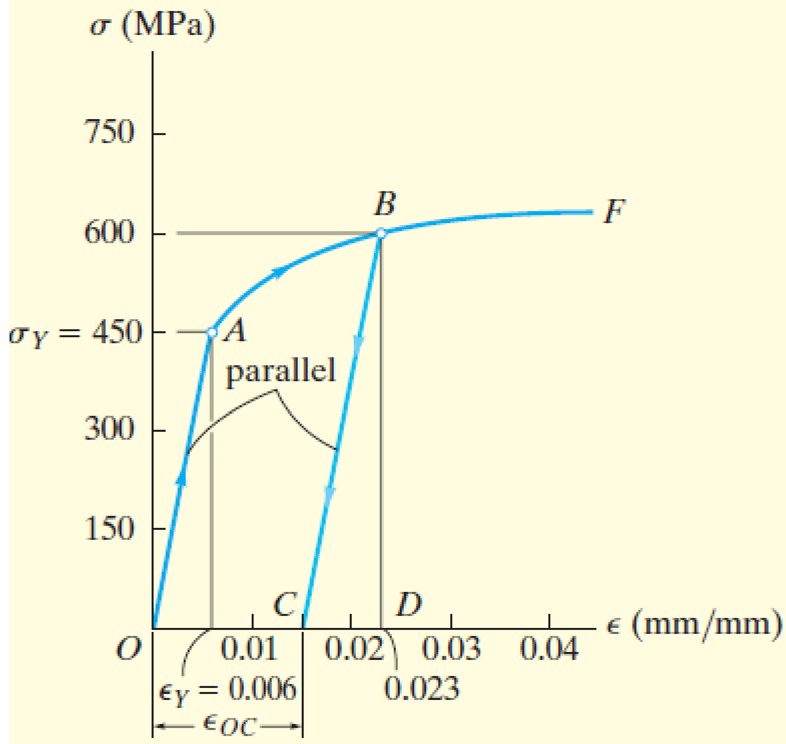


Fig. 3–19

SOLUTION

Permanent Strain. When the specimen is subjected to the load, it strain-hardens until point B is reached on the $\sigma-\epsilon$ diagram. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA.

$$E = \frac{450 \text{ MPa}}{0.006 \text{ mm/mm}} = 75.0 \text{ GPa}$$

From triangle CBD, we require

$$E = \frac{BD}{CD}; \qquad 75.0(10^9) \text{ Pa} = \frac{600(10^6) \text{ Pa}}{CD}$$

$$CD = 0.008 \text{ mm/mm}$$

This strain represents the amount of *recovered elastic strain*. The permanent strain, ϵ_{OC}, is thus

$$\epsilon_{OC} = 0.023 \text{ mm/mm} - 0.008 \text{ mm/mm} = 0.0150 \text{ mm/mm}$$

Modulus of Resilience. Area under the curve OA

$$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(450 \text{ MPa})(0.006 \text{ mm/mm}) = 1.35 \text{ MJ/m}^3$$ *Ans.*

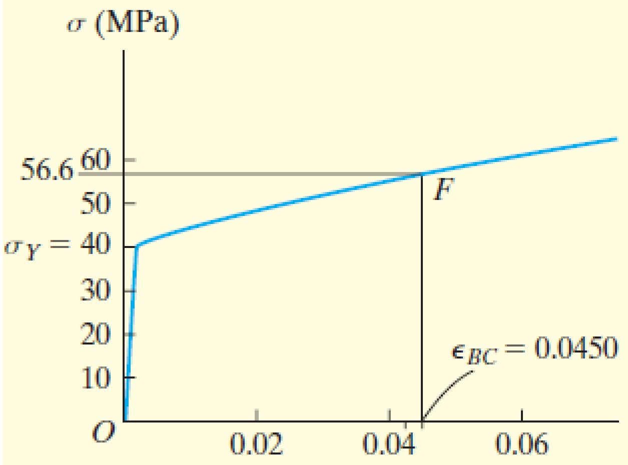
Area under the curve CB

$$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(600 \text{ MPa})(0.008 \text{ mm/mm}) = 2.40 \text{ MJ/m}^3$$ *Ans.*

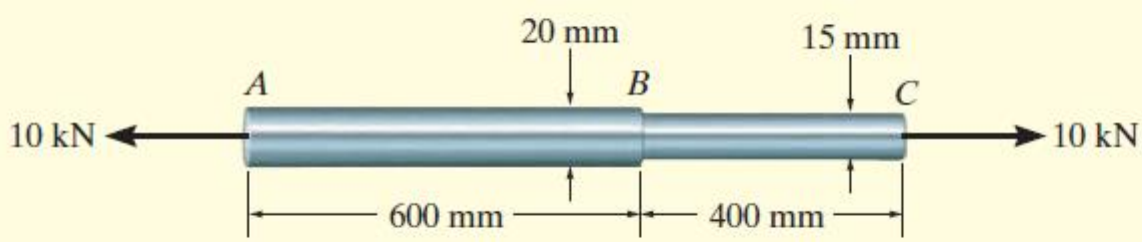
* Work in the SI system of units is measured in joules, where $1 \text{ J} = 1 \text{ N} \cdot \text{m}$.

An aluminum rod shown in Fig. 3–20*a* has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram is shown in Fig. 3–20*b*, determine the approximate elongation of the rod when the load is applied. Take $E_{al} = 70$ GPa.

Example



(b)



(a)

$$\sigma_{AB} = \frac{P}{A} = \frac{10(10^3)\ \text{N}}{\pi(0.01\ \text{m})^2} = 31.83\ \text{MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10(10^3)\ \text{N}}{\pi(0.0075\ \text{m})^2} = 56.59\ \text{MPa}$$

From the stress–strain diagram, the material in segment AB is strained *elastically* since $\sigma_{AB} < \sigma_Y = 40$ MPa. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{al}} = \frac{31.83(10^6)\text{ Pa}}{70(10^9)\text{ Pa}} = 0.0004547\text{ mm/mm}$$

The material within segment BC is strained plastically, since $\sigma_{BC} > \sigma_Y = 40$ MPa. From the graph, for $\sigma_{BC} = 56.59$ MPa, $\epsilon_{BC} \approx 0.045$ mm/mm.

The total elongation of the rod is the elongation in AB (which is elastic) and the elongation in BC (which is plastic) as follow

$$\delta = \Sigma \epsilon L = 0.0004547(600\text{ mm}) + 0.0450(400\text{ mm})$$
$$= 18.3\text{ mm} \qquad \textit{Ans.}$$

بقية هذا الفصل ياخذها الطالب من محاضرات د. شامل

Chapter 5

Axial Loading

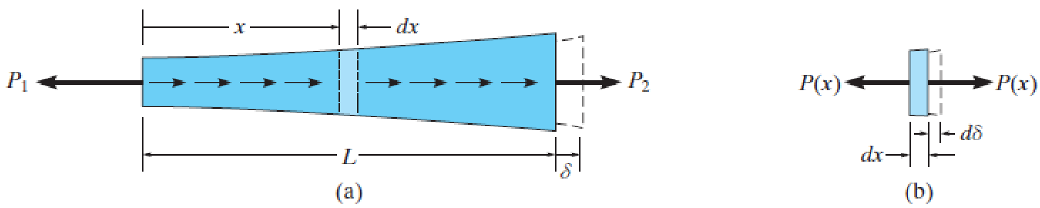
$$\sigma = \frac{P(x)}{A(x)} \qquad \text{and} \qquad \epsilon = \frac{d\delta}{dx}$$

Provided the stress does not exceed the proportional limit, we can apply Hooke's law; i.e.,

$$\sigma = E\epsilon$$

$$\frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)\,dx}{A(x)E}$$



For the entire length L of the bar, we must integrate this expression to find δ. This yields

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E}$$

IF the P, E and A are constants then

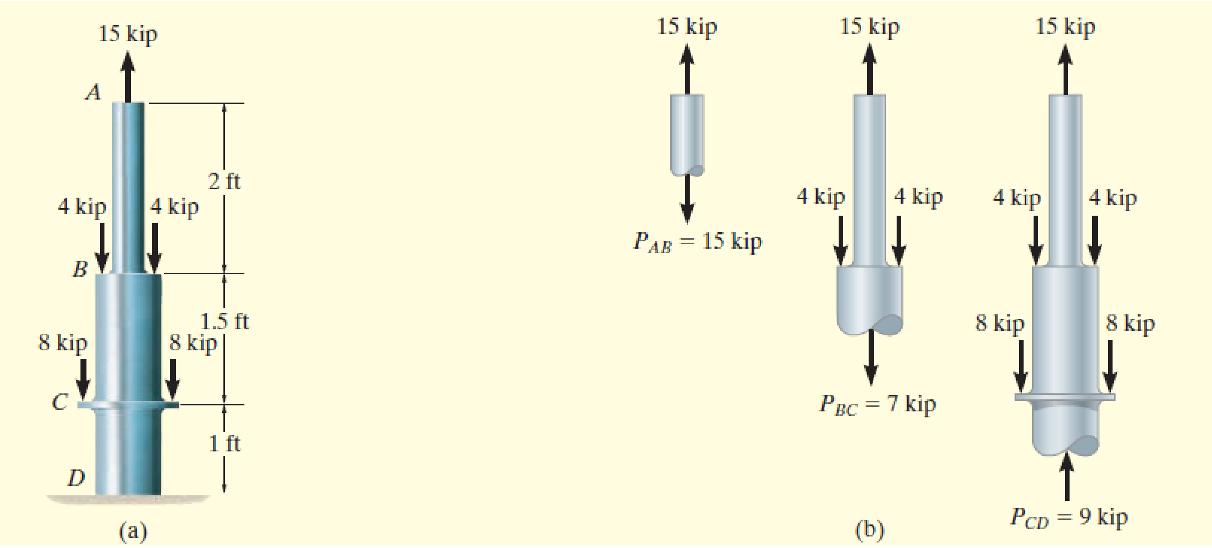
$$\delta = \frac{PL}{AE}$$

If many members have same P, E and A then

$$\delta = \sum \frac{PL}{AE}$$

Examples

The A-36 steel bar shown in Fig. 4–6*a* is made from two segments having cross-sectional areas of $A_{AB} = 1\ \text{in}^2$ and $A_{BD} = 2\ \text{in}^2$. Determine the vertical displacement of end A and the displacement of B relative to C.



$$\delta_A = \sum \frac{PL}{AE} = \frac{[+15\ \text{kip}](2\ \text{ft})(12\ \text{in./ft})}{(1\ \text{in}^2)[29(10^3)\ \text{kip/in}^2]} + \frac{[+7\ \text{kip}](1.5\ \text{ft})(12\ \text{in./ft})}{(2\ \text{in}^2)[29(10^3)\ \text{kip/in}^2]} + \frac{[-9\ \text{kip}](1\ \text{ft})(12\ \text{in./ft})}{(2\ \text{in}^2)[29(10^3)\ \text{kip/in}^2]} = +0.0127\ \text{in.} \qquad \textit{Ans.}$$

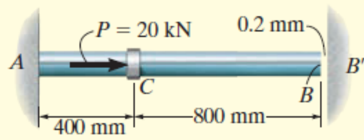
$$\delta_{B/C} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{[+7\ \text{kip}](1.5\ \text{ft})(12\ \text{in./ft})}{(2\ \text{in}^2)[29(10^3)\ \text{kip/in}^2]} = +0.00217\ \text{in.} \qquad \textit{Ans.}$$

Positive displacements mean elongation.

Statically Indeterminate Axially Loaded Member

If the equilibrium equations are not enough to solve the problem, use the compatibility equations with the equilibrium equations to solve the problem.

The steel rod shown in Fig. 4–12*a* has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B' and the rod. Determine the reactions at A and B' if the rod is subjected to an axial force of $P = 20$ kN as shown. Neglect the size of the collar at C. Take $E_{st} = 200$ GPa.



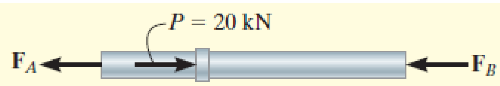
(a)

Let's check if the load is enough to cause the end B to contact the wall.

$\delta_B = \frac{20000*400}{AE} = 0.509\ mm > 0.2mm$ then B will contact the wall.

Equilibrium

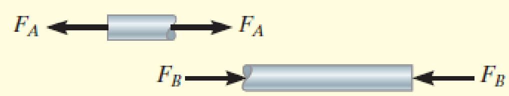
$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_A - F_B + 20(10^3)\text{ N} = 0 \qquad (1)$$



(b)

Two unknown and one equation then indeterminate.

Compatibility



(c)

$$\delta_{B/A} = 0.0002\text{ m} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE}$$

$$0.0002\text{ m} = \frac{F_A(0.4\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} - \frac{F_B(0.8\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]}$$

or

$$F_A(0.4\text{ m}) - F_B(0.8\text{ m}) = 3141.59\text{ N}\cdot\text{m} \qquad (2)$$

Solving Eqs. 1 and 2 yields

$$F_A = 16.0\text{ kN} \qquad F_B = 4.05\text{ kN} \qquad \textit{Ans.}$$

Another solution (Called flexibility or force method)

SOLUTION

Compatibility. Here we will consider the support at B' as redundant. Using the principle of superposition, Fig. 4–17*b*, we have

$$(\stackrel{+}{\rightarrow}) \qquad 0.0002 \text{ m} = \delta_P - \delta_B \qquad (1)$$

The deflections δ_P and δ_B are determined from Eq. 4–2.

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{[20(10^3)\text{ N}](0.4\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} = 0.5093(10^{-3})\text{ m}$$

$$\delta_B = \frac{F_B L_{AB}}{AE} = \frac{F_B(1.20\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} = 76.3944(10^{-9})F_B$$

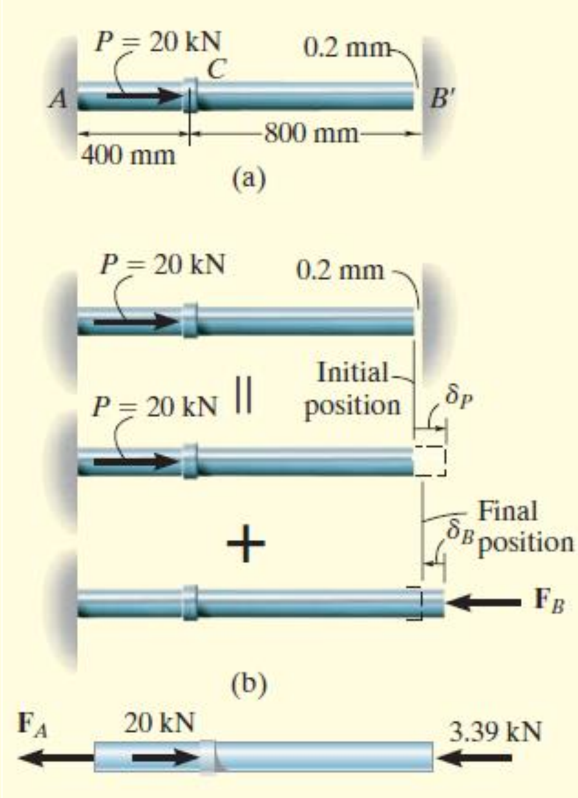
Substituting into Eq. 1, we get

$$0.0002\text{ m} = 0.5093(10^{-3})\text{ m} - 76.3944(10^{-9})F_B$$

$$F_B = 4.05(10^3)\text{ N} = 4.05\text{ kN} \qquad \textit{Ans.}$$

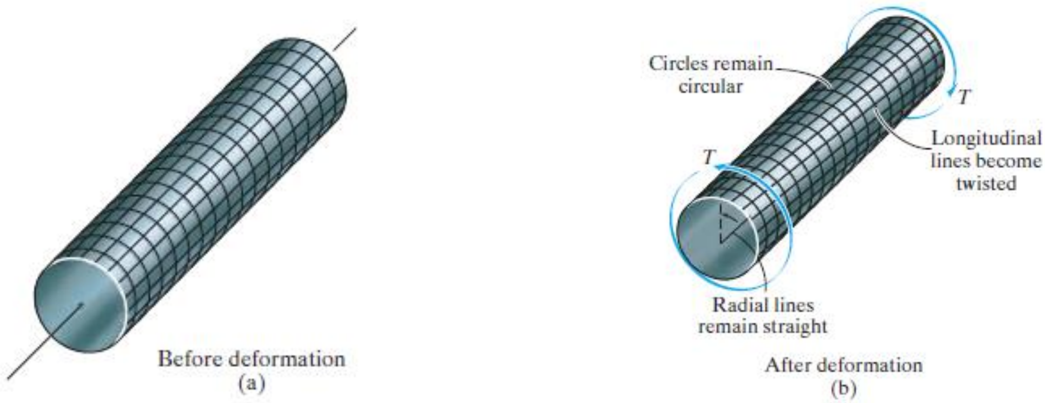
Equilibrium. From the free-body diagram, Fig. 4–17*c*,

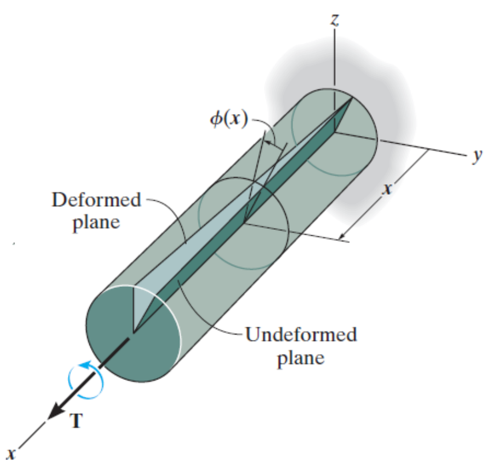
$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \quad -F_A + 20\text{ kN} - 4.05\text{ kN} = 0 \quad F_A = 16.0\text{ kN} \qquad \textit{Ans.}$$

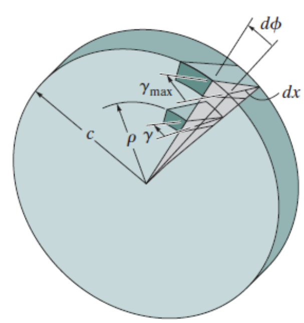


Chapter 6

Torsion







$$\gamma dx = d\phi\rho \rightarrow \gamma = \rho \frac{d\phi}{dx}$$

Since dx and $d\phi$ is the same for all elements located at points on the cross section at x, then $\frac{dx}{d\phi}$ is constant over the cross section, γ increases linearly with distance from the center of the axis of the shaft,

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

If the material is linear-elastic, then Hooke's law applies, $\tau = G\gamma$, and consequently a ***linear variation in shear strain***, as noted in the previous section, leads to a corresponding ***linear variation in shear stress*** along any radial line on the cross section.

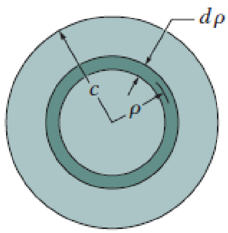
$$\tau = \left(\frac{\rho}{c}\right)\tau_{\max}$$

$$T = \int_A \rho(\tau\, dA) = \int_A \rho\left(\frac{\rho}{c}\right)\tau_{\max}\, dA$$

$$T = \frac{\tau_{\max}}{c}\int_A \rho^2\, dA$$

$$\tau_{\max} = \frac{Tc}{J}$$

Combing above equations, $$\tau = \frac{T\rho}{J}$$



Solid Shaft. If the shaft has a solid circular cross section, the polar moment of inertia J can be determined using an area element in the form of a *differential ring* or annulus having a thickness $d\rho$ and circumference $2\pi\rho$, Fig. 5–6. For this ring, $dA = 2\pi\rho\, d\rho$, and so

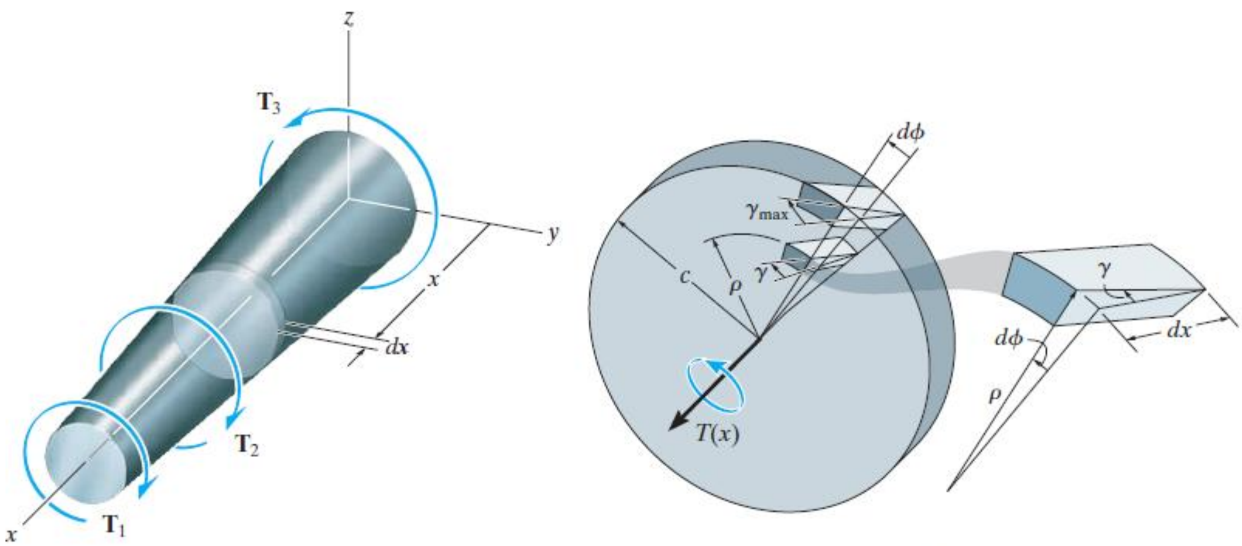
$$J = \int_A \rho^2\, dA = \int_0^c \rho^2(2\pi\rho\, d\rho) = 2\pi\int_0^c \rho^3\, d\rho = 2\pi\left(\frac{1}{4}\right)\rho^4\Big|_0^c$$

$$J = \frac{\pi}{2}c^4$$

Tabular shafts

$$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

Angle of twist



(a) (b)

$$d\phi = \gamma \frac{dx}{\rho}$$

Since Hooke's law, $\gamma = \tau/G$, applies and the shear stress can be expressed in terms of the applied torque using the torsion formula $\tau = T(x)\rho/J(x)$, then $\gamma = T(x)\rho/J(x)G$. Substituting

$$d\phi = \frac{T(x)}{J(x)G}\,dx$$

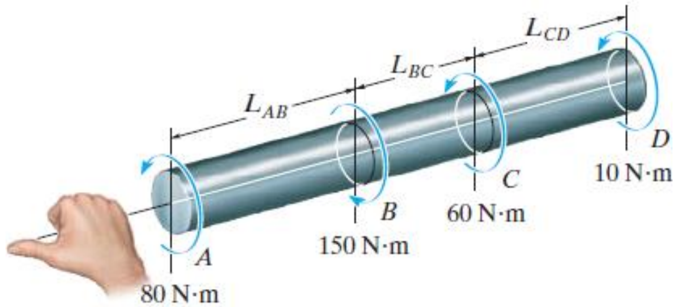
Integrating over the entire length L of the shaft, we obtain the angle of twist for the entire shaft, namely,

$$\phi = \int_0^L \frac{T(x)\,dx}{J(x)G}$$

Multiple torques

$$\phi = \sum \frac{TL}{JG}$$

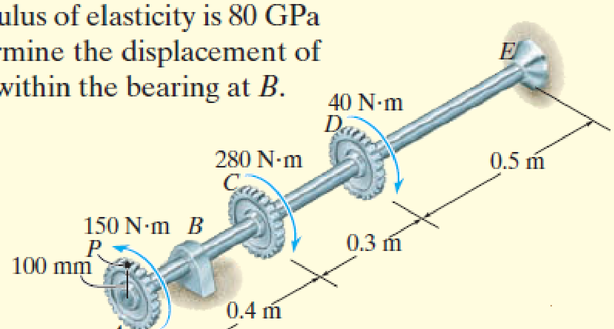
Example



$$\phi_{A/D} = \frac{(+80\ \text{N}\cdot\text{m})\ L_{AB}}{JG} + \frac{(-70\ \text{N}\cdot\text{m})\ L_{BC}}{JG} + \frac{(-10\ \text{N}\cdot\text{m})\ L_{CD}}{JG}$$

Example

The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 5–19*a*. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth *P* on gear *A*. The shaft turns freely within the bearing at *B*.



(a)

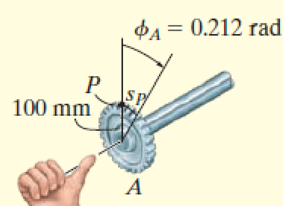
SOLUTION

$$\phi_A = \sum \frac{TL}{JG} = \frac{(+150\ \text{N}\cdot\text{m})(0.4\ \text{m})}{3.771(10^{-9})\ \text{m}^4\ [80(10^9)\ \text{N/m}^2]} + \frac{(-130\ \text{N}\cdot\text{m})(0.3\ \text{m})}{3.771(10^{-9})\ \text{m}^4\ [80(10^9)\ \text{N/m}^2)]} + \frac{(-170\ \text{N}\cdot\text{m})(0.5\ \text{m})}{3.771(10^{-9})\ \text{m}^4\ [80(10^9)\ \text{N/m}^2)]} = -0.2121\ \text{rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed *toward* the end *E* of the shaft, and therefore gear *A* will rotate as shown in Fig. 5–19*d*.

The displacement of tooth *P* on gear *A* is

$$s_P = \phi_A r = (0.2121\ \text{rad})(100\ \text{mm}) = 21.2\ \text{mm}$$ *Ans.*

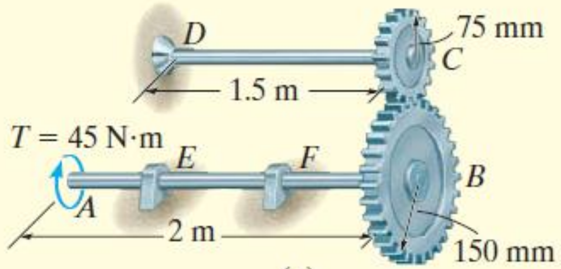


(d)

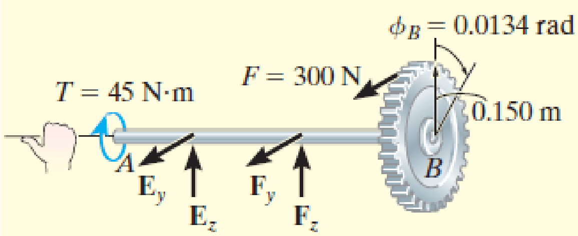
Example

The two solid steel shafts shown in Fig. 5–20*a* are coupled together using the meshed gears. Determine the angle of twist of end *A* of shaft *AB* when the torque $T = 45\ \text{N}\cdot\text{m}$ is applied. Take $G = 80$ GPa. Shaft *AB* is free to rotate within bearings *E* and *F*, whereas shaft *DC* is fixed at *D*. Each shaft has a diameter of 20 mm.

d



(a)



(b)

SOLUTION

Internal Torque. Free-body diagrams for each shaft are shown in Fig. 5–20*b* and 5–20*c*. Summing moments along the *x* axis of shaft *AB* yields the tangential reaction between the gears of $F = 45\ \text{N}\cdot\text{m}/0.15\ \text{m} = 300$ N. Summing moments about the *x* axis of shaft *DC*, this force then creates a torque of $(T_D)_x = 300\ \text{N}\ (0.075\ \text{m}) = 22.5\ \text{N}\cdot\text{m}$ on shaft *DC*.

Angle of Twist. To solve the problem, we will first calculate the rotation of gear *C* due to the torque of 22.5 N · m in shaft *DC*, Fig. 5–20*c*. This angle of twist is

$$\phi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5\ \text{N}\cdot\text{m})(1.5\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0269\ \text{rad}$$

Since the gears at the end of the shaft are in mesh, the rotation ϕ_C of gear *C* causes gear *B* to rotate ϕ_B, Fig. 5–20*b*, where

$$\phi_B(0.15\ \text{m}) = (0.0269\ \text{rad})(0.075\ \text{m})$$

We will now determine the angle of twist of end *A* with respect to end *B* of shaft *AB* caused by the 45 N · m torque, Fig. 5–20*b*. We have

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45\ \text{N}\cdot\text{m})(2\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0716\ \text{rad}$$

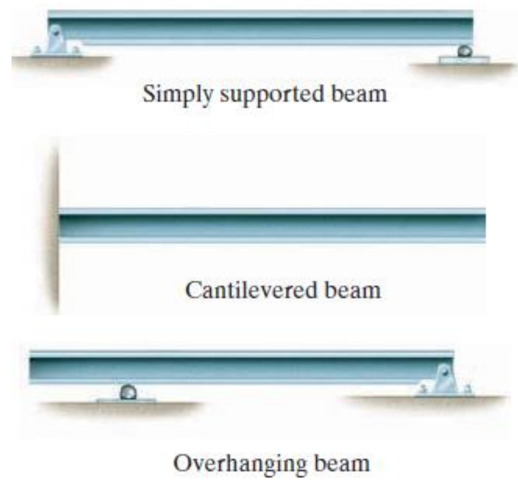
The rotation of end *A* is therefore determined by adding ϕ_B and $\phi_{A/B}$, since both angles are in the *same direction*, Fig. 5–20*b*. We have

$$\phi_A = \phi_B + \phi_{A/B} = 0.0134\ \text{rad} + 0.0716\ \text{rad} = +0.0850\ \text{rad} \qquad \textit{Ans.}$$

Chapter 7

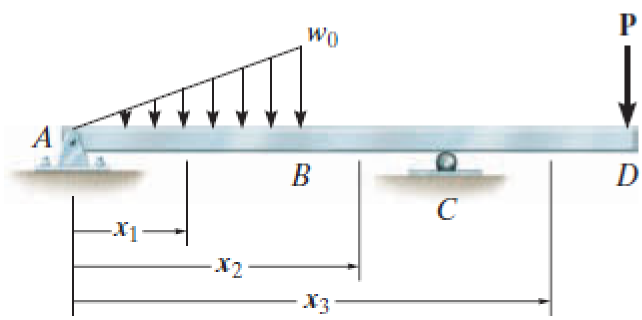
Shear and Bending diagrams

Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called **beams** and classified as how they are supported as shown below.



To design beams, we need to know the shear and bending moment diagrams and find the maximum values.

Because shear and moment are discontinuous functions, we divide their diagrams into regions as shown below.



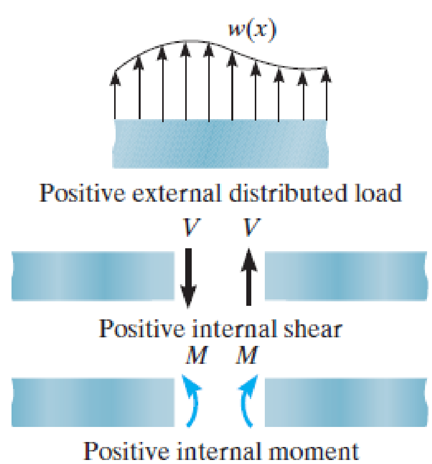
Consider the origin to the left. Then x1 will be used for region AB, x2 for region BC and x3 for region CD.

Beams sign convention

It is arbitrary but the one often used in engineering practice is as shown below.

OR,

When starting from left side of the beam, upward forces cause positive shear and bending moment and downward forces cause negative shear and bending moment.



Beam sign convention

EXAMPLE 6.1

Draw the shear and moment diagrams for the beam shown in Fig. 6–4*a*.

SOLUTION

Support Reactions. The support reactions are shown in Fig. 6–4*c*.

Shear and Moment Functions. A free-body diagram of the left segment of the beam is shown in Fig. 6–4*b*. The distributed loading on this segment, wx, is represented by its resultant force only *after* the segment is isolated as a free-body diagram. This force acts through the centroid of the area comprising the distributed loading, a distance of $x/2$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \tag{2}$$

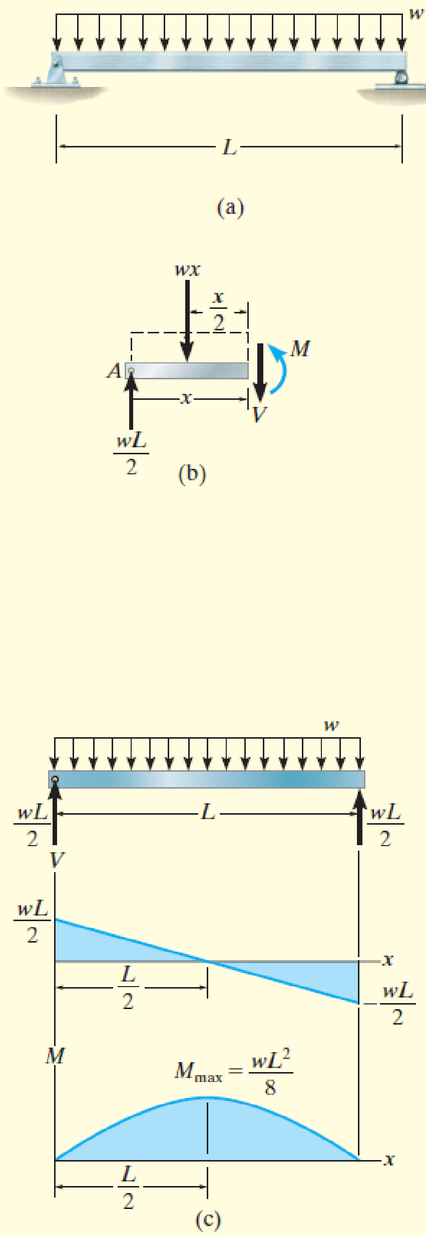


Fig. 6–4

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–4*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$

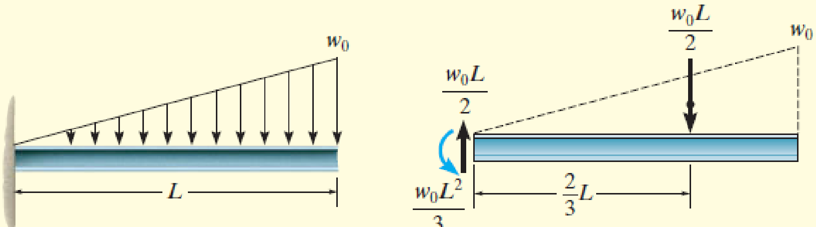
NOTE: From the moment diagram, this value of x represents the point on the beam where the *maximum moment* occurs, since by Eq. 6–2 (see Sec. 6.2) the *slope* $V = dM/dx = 0$. From Eq. 2, we have

$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

EXAMPLE 6.2

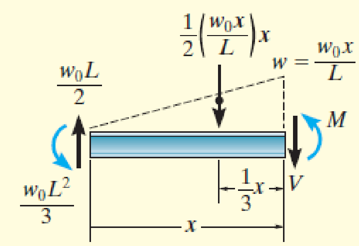
Draw the shear and moment diagrams for the beam shown in Fig. 6–5a.



(a) (b)

SOLUTION

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–5b.



(c)

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5c. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0L}{2} - \frac{1}{2}\left(\frac{w_0x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad \frac{w_0L^2}{3} - \frac{w_0L}{2}(x) + \frac{1}{2}\left(\frac{w_0x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

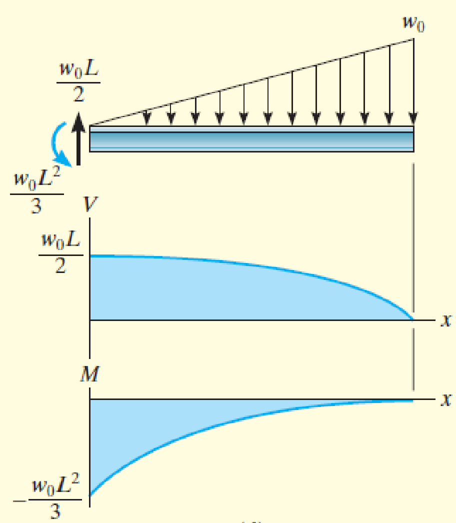
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2x - x^3) \tag{2}$$

These results can be checked by applying Eqs. 6–1 and 6–2 of Sec. 6.2, that is,

$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–5d.

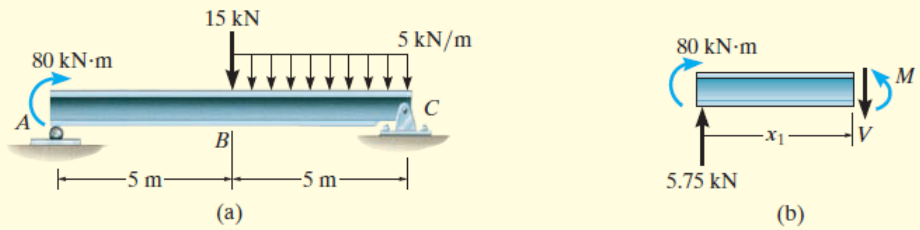


(d)

Fig. 6–5

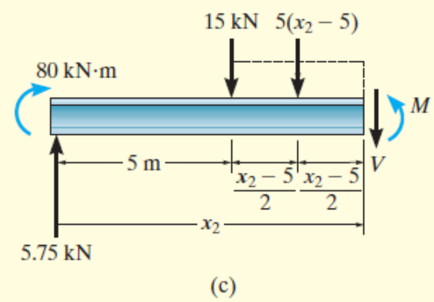
EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



SOLUTION

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–7*d*.



Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.

$0 \le x_1 < 5$ m, Fig. 6–7*b*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m} \qquad (2)$$

$5 \text{ m} < x_2 \le 10$ m, Fig. 6–7*c*:

$$+\uparrow \Sigma F_y = 0; \quad 5.75 \text{ kN} - 15 \text{ kN} - 5 \text{ kN/m}(x_2 - 5 \text{ m}) - V = 0$$

$$V = (15.75 - 5x_2) \text{ kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_2 + 15 \text{ kN}(x_2 - 5 \text{ m})$$

$$+ 5 \text{ kN/m}(x_2 - 5 \text{ m})\left(\frac{x_2 - 5 \text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–7*d*.

Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–7*d*.

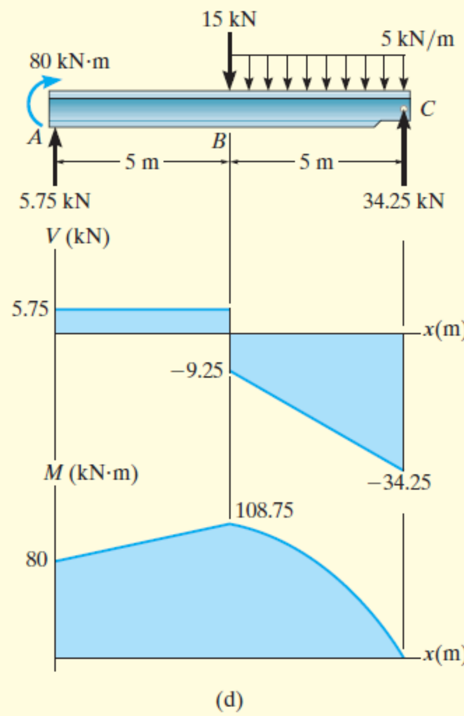
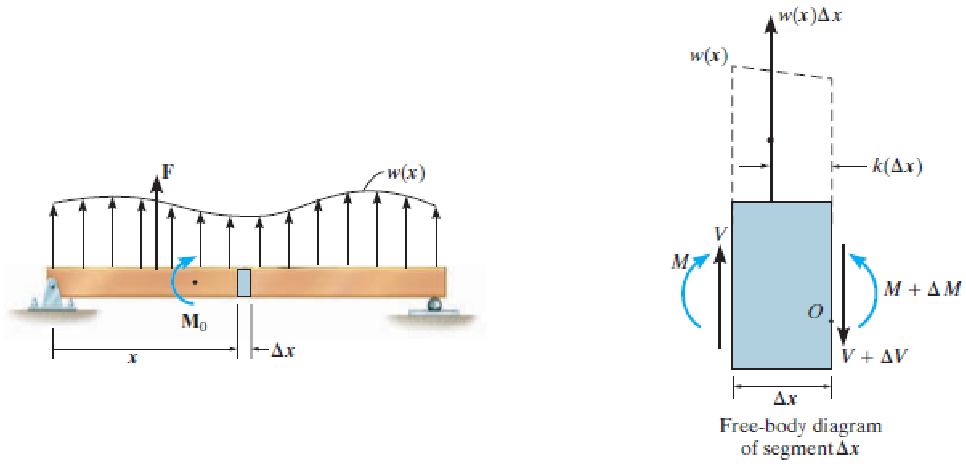


Fig. 6–7

Graphical Method for Constructing Shear and Moment Diagrams



(a) (b)

$$+\uparrow \Sigma F_y = 0; \qquad V + w(x)\,\Delta x - (V + \Delta V) = 0$$
$$\Delta V = w(x)\,\Delta x$$

$$\circlearrowleft + \Sigma M_O = 0; \quad -V\,\Delta x - M - w(x)\,\Delta x[k(\Delta x)] + (M + \Delta M) = 0$$
$$\Delta M = V\,\Delta x + w(x)\,k(\Delta x)^2$$

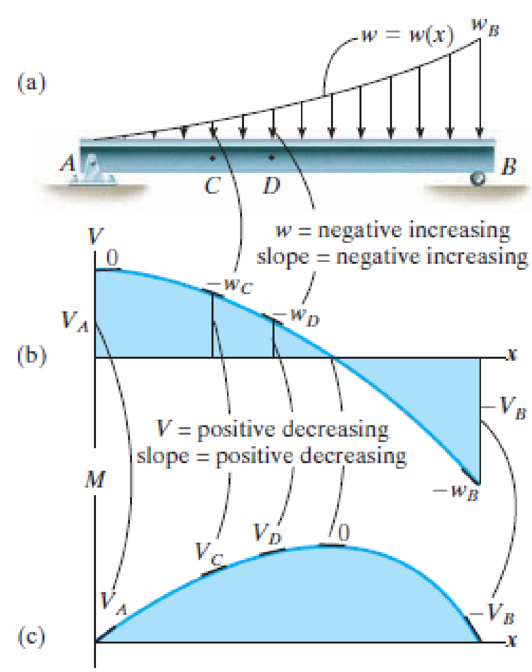
Dividing by Δx and taking the limit as $\Delta x \rightarrow 0$, the above two equations become

$$\frac{dV}{dx} = w(x) \tag{6–1}$$

slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V \tag{6–2}$$

slope of moment diagram at each point = shear at each point



Integrating equations 6-1 and 6-2,

$$\Delta V = \int w(x)\,dx \qquad (6\text{–}3)$$

change in shear = area under distributed loading

$$\Delta M = \int V(x)\,dx \qquad (6\text{–}4)$$

change in moment = area under shear diagram