Ministry of Higher Education
and Scientific Research
University of Anbar

College of Engineering
Mechanical Engineering Department
2nd Year – ME2303

THERMODYNAMICS

Instructor: ***Asst. Prof. Dr. SAAD M. JALIL***

➢ Textbooks:

1. *T.D. ESTOP – A. MCCNKEY* "Applied Thermodynamics".
2. *RAYNER JOEL* "Basic Engineering Thermodynamics".

➢ Other useful books:

1. *SONNTAG, BORGNAKKE and VAN WYLEN"* Fundamental of Thermodynamics".
2. *YUNUS A. CENGEL and MICHAEL A. BOLES"* Thermodynamics an Engineering Approach".
3. *MERLE C. POTTER and CRAIG W. SOMERTON* "Engineering thermodynamics".

➢ Course Contents:

Week	Subject	Week	Subject
1	***Concepts, Definitions, And Basic Principles***	17	***The Second Law of Thermodynamics***
2	Dimensions and Units	18	Heat Engine and Its Types
3	Thermodynamic Systems	19	Refrigerators & Heat Pumps
4	Properties and State of Substance	20	***Entropy***
5	Processes and Cycles	21	Entropy of Single Phase (Ideal Gas)
6	Forms of Energy & Reversibility	22	Entropy of Tow Phase (Vapor)
7	The Zeroth & First Laws of Thermodynamics	23	***Heat Engine Cycles***
8	The Steady & Non-Flow Energy Equations	24	Carnot & Otto Cycles
9	***Ideal Gas (Single Phase System) and Its Laws***	25	Brayton & Diesel Cycles
10	Reversible & Irreversible Processes	26	Dual Cycle and Mean Effective Pressure
11	Specific Heats	27	***Steam Cycles (Carnot & Rankine Cycles)***
12	***Steam and Tow Phase System (Vapor)***	28	Superheated & Reheated Rankine Cycles
13	Important Terms for Steam	29	***Refrigeration System***
14	Reversible & Irreversible Processes	30	Carnot Cycle
15	Measurement of Quality	31	Refrigeration Cycles
16	***First Semester Exam (Fall Semester)***	32	***Second Semester Exam (Spring Semester)***

THERMODYNAMICS

1

CHAPTER ONE

Introduction and Basic Concepts

.Thermodynamics:-

can be defined as the science of energy. The name thermodynamics stems from the Greek words therme (Heat) and dynamics (Power).

Applied thermodynamics: is the science of relationship between heat, work, and the properties of the System. It is concerned with the means necessary to convert heat energy from available sources such as chemical fuels or nuclear piles into mechanical work.

.Working Substance:-

the working substances are, in general fluids which are capable of deformation in that they can readily be expanded and compressed. Common examples of working substance used in thermodynamic System air and Steam.

.Pure Substance:- Is a single substance or mixture of substance which has the same consistent composition throughout. Such as water, Nitrogen, Helium and Carbon dioxide

In other words, it is a homogenous substance and its molecular structure dose not vary. For example, Steam or water or mixture of both can be considered as a pure substance. This is because it has the same molecular or chemical structure through its mass.

1. Dimensions and Units:

Any phisical quantity can be characterized by «dimensions». The arbitrary magnitudes assigned to the dimensions are called «Units».

Some basic dimensions such as mass (m), Length (L), time (t), and temperature (T) are selected as [primary] or fundamental dimensions. While others such as velocity (V), energy (E) and volume (V) are expressed in terms of the primary dimensions and are called [secondary] dimensions, or derived dimensions.

Two sets of Units are still in common use today:

1. The English System
2. System International (SI) Units

Dimension	Unit
Length	meter (m)
mass	kilogram (kg)
Time	second (s)
Temperature	Kelvin (K)
Electric current	Ampere (A)
Amount of Light	Candela (C)
Amount of matter	mole (mol)

Quantity	SI Units	from English to SI Units
Area	m^2	1 ft = 0.3048 m
Density	kg/m^3	1 in = 0.0254 m
Energy	J	1 lbm = 0.4536 kg
Energy per unit mass	J/kg	1 Slug = 14.594 kg
Force	N	1 lbf = 4.448 N
Heat flux	W/m^2	
Heat transfer rate	W	1 Btu = 1055.06 J 1 cal = 4.1868 J 1 Btu/lbm = 2326 J/kg
mass flow rate	kg/s	
Power	W (J/s)	1 hp = 745.7 W
pressure & Stress	N/m^2	1 lbf/ft^2 = 47.88 N/m^2
Specific heat	J/kg·K	1 psi = 1 lbf/in^2 = 6894.8 N/m^2
Surface tension	N/m	
Velocity	m/s	1 standard atmosphere = 1.0133×10^5 N/m^2
Viscosity (Kin.)	m^2/s	
Volume	m^3	1 gal = 0.003785 m^3
Volume flow rate	m^3/s	
Specific Volume	m^3/kg	

-. Heat Engine:

is the name given to a system which by operating in a cyclic manner produces net work from a supply of heat.

-. Heat, work, and the System:

Heat is a form of energy which is transferred from one body to another body at a lower temperature by virtue of the temperature difference between the bodies.

Note that the heat and work are a transient quantities, not properties.

In mechanics, work is defined as the product of the force (F) and the distance moved (x) in the direction of the force ⇒ $W = F * x$

If a system exists in which a force at the boundary of the system is moved through a distance, then work is done by or on the System.

$$\boxed{\text{Work} = \text{Force} * \text{distance}} \quad * $$

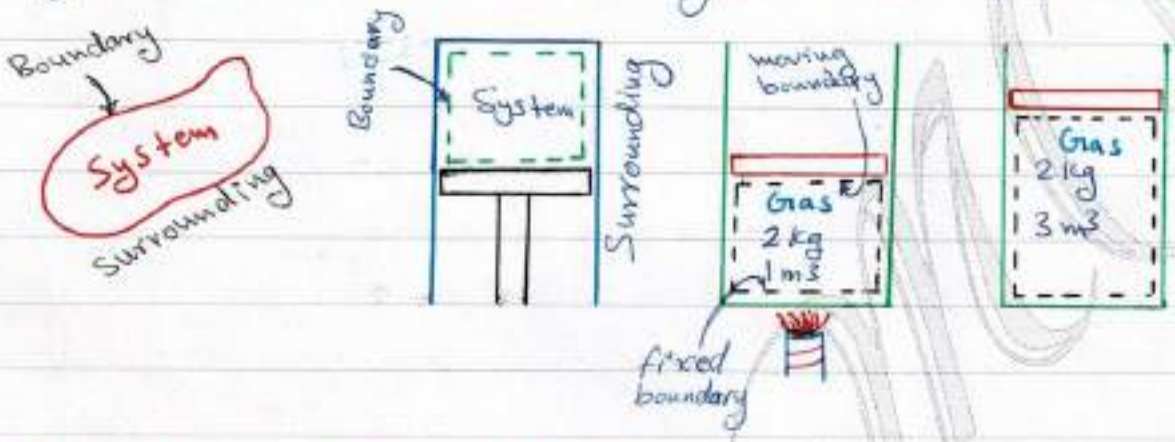
A system my be defined as a collection of matter within prescribed and identifiable boundaries.

or

a quantity of matter or a region in space chosen for study.

The mass or region outside the System is called the **Surrounding**.

The real or imaginary surface that separates the system from its surrounding is called the **boundary**.

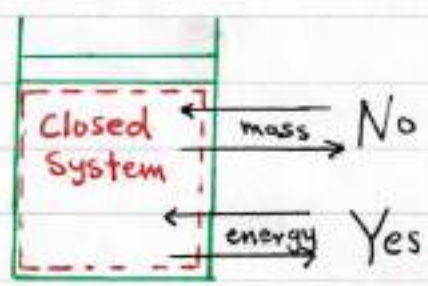


- Closed and Open System:

System may be considered to be closed or open depending on whether a fixed mass or a fixed volume in space is chosen for study.

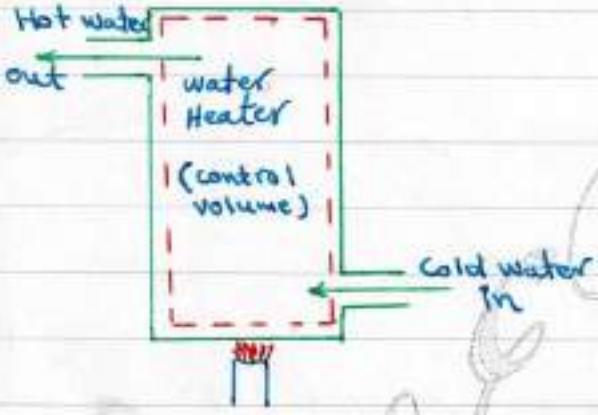
A closed **System** (also known as a **control mass**) consists of a fixed amount of mass, and no mass can cross its boundary. That is, no mass can enter or leave a closed System.

But energy in the form of heat or work, can cross the boundary, and the volume of a closed System dose not have to be fixed.

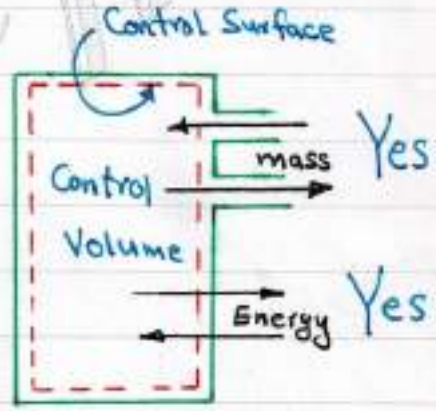


If, as a special case, even energy is not allowed to cross the boundary, that System is called an **isolated System**.

An Open System, or a control volume, as it is often called, is a properly selected region in space. It usually encloses a device that involves mass flow, such as a compressor, turbine, or nozzle. Both mass and energy can cross the boundary of a control volume which is called **a control surface**.



[An open system (a control volume) with one inlet and one exit]



[Both mass and energy can cross the boundaries of a control volume]

- Properties of a System :

Any characteristic of a system is called a property. Some familiar examples are pressure **P**, temperature **T**, volume **V**, and mass **m**.

The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.

Not all properties are independent, some are defined in terms of other ones, such as density (ρ), Specific volume (v), Specific gravity (ρ_s).

Properties are considered to be either **intensive** or **extensive**.

Intensive properties are those that are independent of the size of a system, such as temperature, pressure, and density.

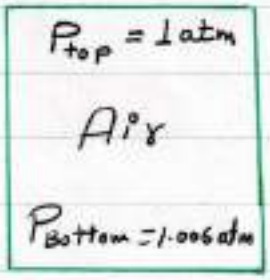
Extensive properties are those whose values depend on the size -or extent- of the System, such as mass, volume, and total energy (E).

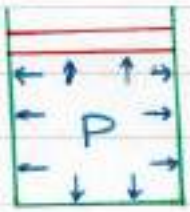
m	→	$\frac{1}{2}$m | $\frac{1}{2}$m	Extensiv Properties	ρ, kg/m³
V		$\frac{1}{2}$V | $\frac{1}{2}$V		v, m³/kg
T		T | T	Intensive properties	e, kJ/kg
P		P | P		U, kJ/kg
ρ		ρ | ρ		

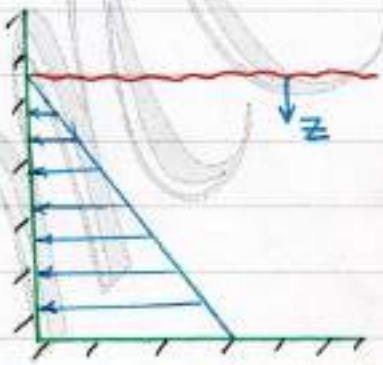
Intensive properties are independent of the size of System

Pressure:

is the force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is stress.







In a container filled with a gas the variation of pressure with height is negligible

The pressure of a fluid at rest increases with depth (as a result of added weight)

Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **pascal** (Pa).

$$\boxed{P = F/A}$$

$1\,N = 1\,kg \cdot m/s^2$

$1\,Pa = 1\,N/m^2$

$1\,MPa = 10^6\,Pa$

$1\,kPa = 10^3\,Pa$

Two other common pressure units are **bar** and **standard atmosphere**:

$$1 \text{ bar} = 10^5 \text{ pa} = 0.1 \text{ Mpa} = 100 \text{ kpa}$$

$$1 \text{ atm} = 101325 \text{ pa} = 101.325 \text{ kpa} = 1.01325 \text{ bars}$$

In the English System, the pressure unit is [pound-force per square inch] lb_f / in^2 or psi

$$1 \text{ atm} = 14.696 \text{ psi} = 760 \text{ mm Hg}$$

the actual pressure at a given position is called **the absolute pressure**, and it is measured relative to absolute vacuum, i.e., absolute zero pressure.

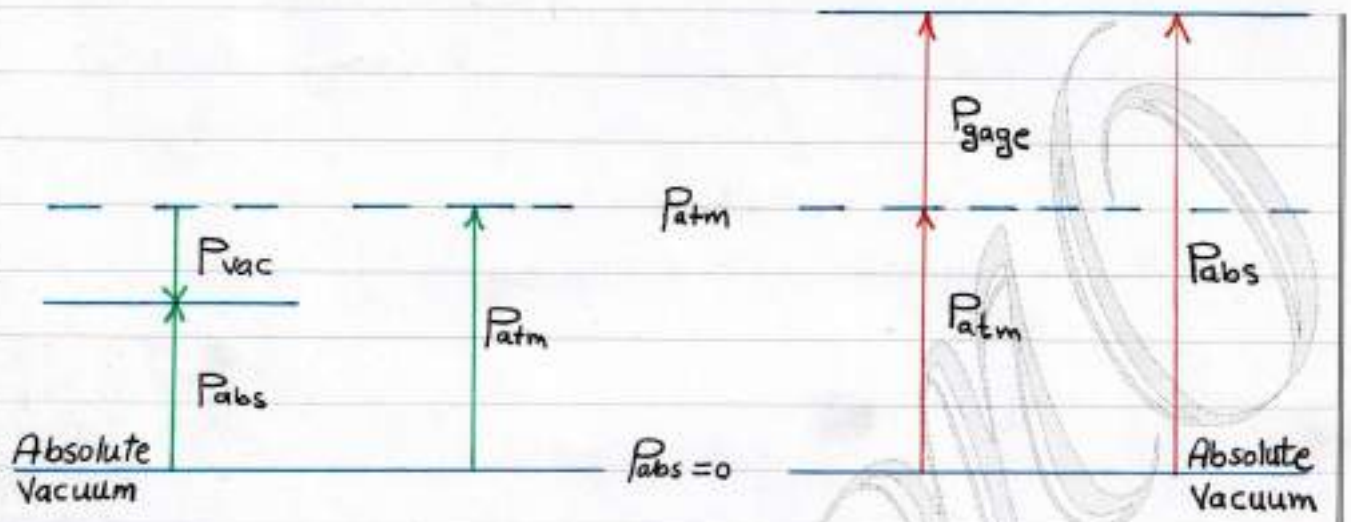
the difference between the absolute pressure and the local atmospheric pressure is called the **gage pressure**.

Pressures below atmospheric pressure are called **Vacuum pressure**, and are measured by vacuum gage.

Absolute, gage, and vacuum pressure are all positive quantities and are related to each other by:

$$\boxed{P_{gage} = P_{abs} - P_{atm}}$$ (For pressure above P_{atm})

$$\boxed{P_{vac.} = P_{atm} - P_{abs}}$$ (" " below ")



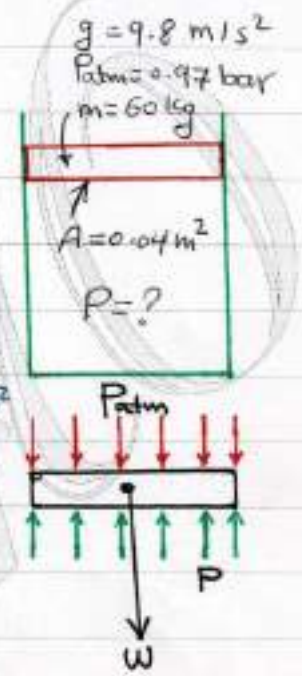
Ex: A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

Sol:

$$P_{abs} = P_{atm} - P_{vac}$$
$$= 14.5 - 5.8$$
$$= 8.7 \text{ psi}$$

Ex: The piston of a piston-cylinder device containing a gas has a mass of 60 kg and a cross-sectional area of 0.04 m², as shown below. The local atmospheric pressure is 0.97 bar and the gravitational acceleration is 9.8 m/s². a- Determine the pressure inside the cylinder b- If some heat is transferred to the gas and its volume doubles, do you expect the pressure inside the cylinder to change.

Sol:- a. Equilibrium $\Rightarrow \Sigma F = 0$



$$P \cdot A = P_{atm} \cdot A + W \quad , \quad W = mg$$

$$P = P_{atm} + \frac{mg}{A}$$

$$= 0.97 \text{ bar} + \frac{60 \text{ kg} \cdot 9.8 \frac{m}{s^2}}{0.04 m^2} \cdot \frac{1 N}{1 kg \cdot \frac{m}{s^2}} \cdot \frac{1 \text{ bar}}{10^5 N/m^2}$$

$$= 1.117 \text{ bars}$$

- Temperature:-

Although we are familiar with temperature as a measure of "hotness" or "coldness" it is not easy to give an exact definition for it.

Based on our physiological sensations, we express the level of temperature qualitatively with words like frezzing cold, cold, warm, hot, and red-hot.

The variation of an easily measurable property of a substance with temperature can be used to provide a temperature-measuring instrument. For example, the length of a column of mercury will vary with temperature due to the expansion and contraction of the mercury or another Liquid, gas, and Solid

- Temperature Scales:

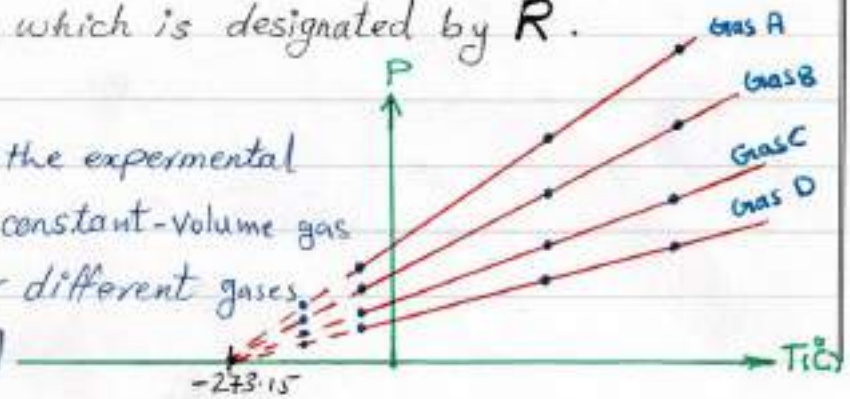
The temperature scales used in the SI and in the English System today are the **Celsius Scale** (formerly called the **Centigrade scale**) and the **Fahrenheit Scale**.

On the Celsius scale, the ice and Steam point, are assigned the values of **0** and **100°C**, respectively, the corresponding values on the Fahrenheit scale are **32** and **212°F**

In thermodynamics, it is very desirable (مهم) to have a temperature scale that is independent of the properties of any substance or substances. The thermodynamic temperature scale in the SI is the **Kelvin scale**. The temperature unit on this scale is the Kelvin, which is designated by **K** (not °K). The lowest temperature on the Kelvin scale is **0 K**.

The thermodynamic temperature scale in the English system is the **Rankine scale**, the temperature unit on this scale is the rankine, which is designated by **R**.

[P versus T plots at the expermental data obtained from a constant-volume gas thermometer using four different gases at different pressures]



$T(K) = T(^\circ C) + 273.15$ *

$T(R) = T(^\circ F) + 459.67$ *

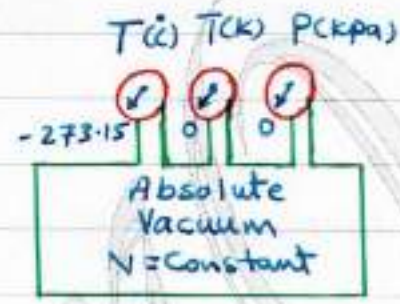
$T(R) = 1.8\, T(K)$ *

$T(^\circ F) = 1.8\, T(^\circ C) + 32$ *

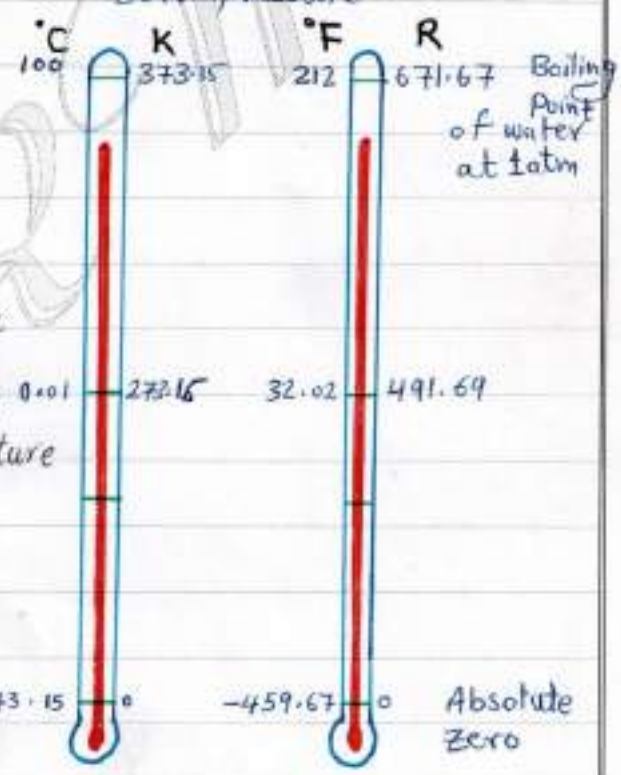
$\Delta T(R) = 1.8\, \Delta T(K)$

$\Delta T(K) = \Delta T(^\circ C)$ *

$\Delta T(R) = \Delta T(^\circ F)$ *



A constant-Volume gas thermometer would read -273.15 at absolute zero pressure



Comparision of temperature scales

Ex: During a heating process, the temperature of a system rises by 10°C. Express this rise in temperature in K, °F, and R.

Sol:

$\Delta T(K) = \Delta T(^\circ C) = 10\,^\circ C$

$\Delta T(R) = 1.8\, \Delta T(K)$

$= 1.8 * 10 = 18\ R$

$\Delta T(^\circ F) = \Delta T(R) = 18\,^\circ F$

قال رسول الله (صلى الله عليه وسلم): «من دخل على غني او حاكم وتضعضع دينه لغناه او لحكمه ذهب ثلثا دينه»

Volume :

volume is a property, being that property which is associated with cubic measure.

The unit of volume is the cubic meter (m^3). Sometimes the liter L may be used.

1 Liter = 1 Cubic decimeter ($1\ dm^3 = [10^{-1}\ m]^3$)

If the volume of a substance increases then the substance is said to have been **Expanded**.

If the volume of a substance decreases then the substance is said to have been **Compressed**.

Specific volume is given the symbol (v)

$$\boxed{v = \frac{V}{m}} \quad m^3/kg$$

$v \equiv$ It is the reciprocal of density and is defined as the volume per unit mass.

Density defined as mass per unit volume.

$$\boxed{\rho = \frac{m}{V}} \quad kg/m^3$$

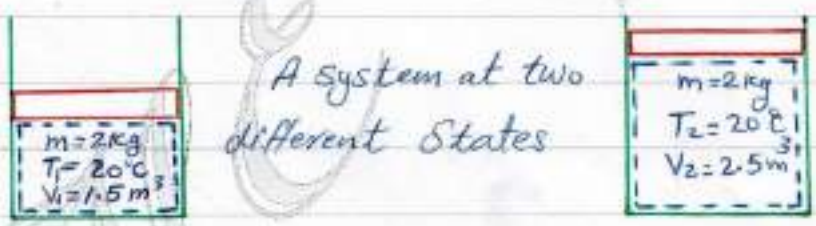
Specific gravity or relative density :-

is defined as the ratio of the density of a substance to the density of some standard substance at specified temperature, [(usually water at 4°C) for which $\rho_{H_2O} = 1000\ kg/m^3$]

$$\rho_s = \frac{\rho}{\rho_{H_2O}}$$

- State and Equilibrium :-

Consider a system that is not undergoing change. At this point, all properties can be measured or calculated throughout the entire system, which gives as a set of properties that completely describe the condition, or the state of the system.



A system at two different states

a- State 1 b- State 2

thermodynamics deals with equilibrium states. The word equilibrium implies a state of balance. In an equilibrium state, there are no unbalanced potentials (or driving forces) within the system.

Types of Equilibrium:

* **Thermal equilibrium:** If the temperature is the same throughout the entire System, then the System is in thermal equilibrium.

a. Befor b. Aftey

A closed System reaching thermal equilibrium

* **Mechanical Equilibrium:** is related to pressure, and a System is in mechanical equilibrium if there is no change in pressure at any point of the System with time.

* **Phase Equilibrium:** If a system involves two phases, it is phase equilibrium when the mass of each phase reaches an equilibrium level and Stays there.

* **Chemical Equilibrium:** A System is in chemical equilibrium if its chemical composition dose not change with time (no chemical reactions occur)

A System will not be in equilibrium unless all the relevant equilibrium criteria are satisfied.

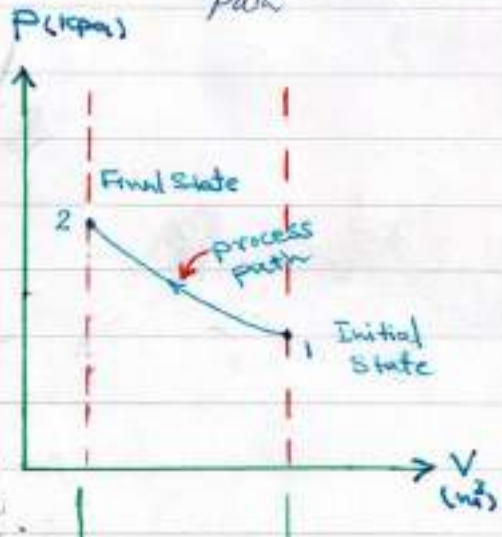
- Processes and Cycles :-

Any change that a system undergoes from one equilibirum state to another is called a **Process**, and the series of states through which a system passes during a process is called the **Path** of the process.

A process between states 1 and 2 and the process path

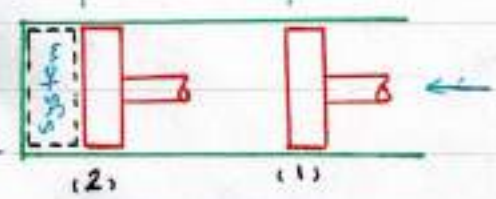
Process diagrams that plotted by employing thermodynamic properties as coordinates are very useful in visualizing the processes.

Some common properties that are used as coordinates are temperature **T**, Pressure **P**, and volume **V** (or Specific volume **v**).



The prefix **iso-** is often used to designate a process for which a particular property remains constant.

An **isothermal** process, for example, is a process during which the temperature T remains constant.



The PV diagram of a compression process

An **isobaric** process is a process during which the pressure P remains constant.

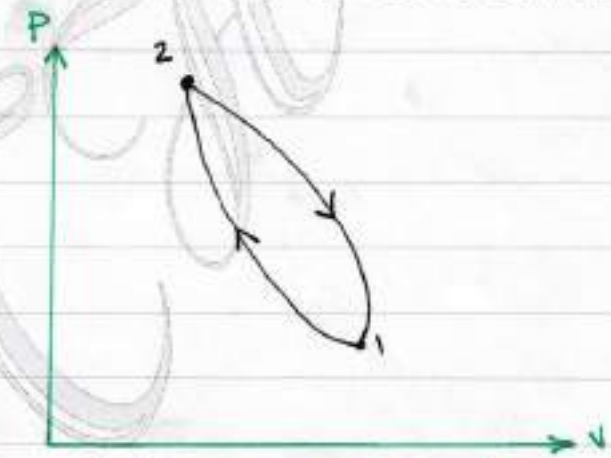
An **isochoric** (or **isometric**) process is a process during which the specific volume v remains constant.

A system is said to have undergone a cycle if it returns to its initial state at the end of the process. That is, for a cycle the initial and final states are identical.

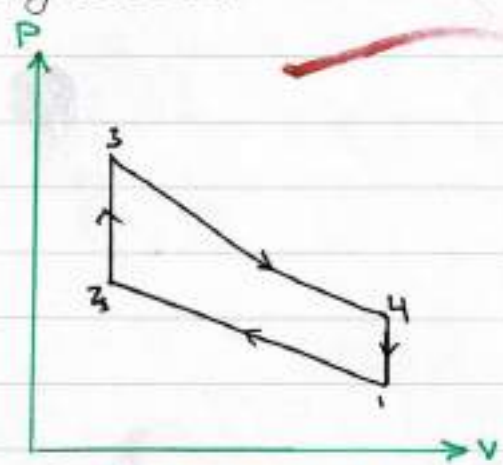
* Types of thermal Cycle :

1- Thermodynamic cycles: In this cycle no change in the chemical structure and the properties of fluid, for example the steam in power plant.

2. Mechanical Cycles: In this cycle the properties of fluid or substance will be change, for example the Internal-Combustion Engine (I.C.E).



a- A two-process cycle



b- A Four-process cycle

- Energy :

Energy is defined as that capacity abody or substance possesses which can result in the performance of work. From the Law of conservation of energy the energy cannot be created or destroyed.

- Forms of Energy :

Energy can exist in numerous forms such as thermal, mechanical, kinetic, Potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the Total Energy "E" of a System. The total energy of a System on unit mass basis denoted by "e" and is defined as:

$$\boxed{e = \frac{E}{m}} \quad kJ/kg$$

In thermodynamics analysis, it is often helpful to consider the various forms of energy that make up the total energy of a system in two groups:

1. Microscopic 2. Macroscopic

the microscopic forms of energy are those related to the molecular structure of a System and the degree

of the molecular activity, and they are independent of outside reference frams.

The sum of all the microscopic forms of energy is called the "Internal Energy" of a System and is denoted by "U"

$$U = m \cdot u \quad kJ$$

$m \equiv$ mass kg

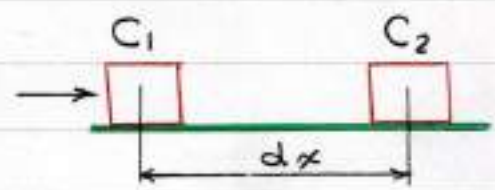
$u \equiv$ Specific internal energy kJ/kg

The macroscopic energy of a System is related to motion and the influence of some external effect such as gravity, magnetism, electricity, and Surface tension.

the energy that a system possesses as a result of its motion relative to some reference fram is called "Kinetic Energy" KE

$$dE = F \cdot dx$$

$$\int dE = \int F \cdot dx$$



$$F = m \cdot a = m \cdot \frac{dc}{dt}$$

$c =$ velocity

$t =$ time

$$\int dE = \int m \cdot \frac{dc}{dt} \cdot dx$$

$$\int dE = m \int \frac{dx}{dt} \cdot dc \quad , \quad \frac{dx}{dt} = c$$

$$= m \int c \, dc$$

$$\int_0^E dE = m \int_{c_1}^{c_2} c \, dc$$

$$= m \left[\frac{c^2}{2} \right]_{c_1}^{c_2}$$

$$\boxed{K.E = \frac{1}{2} m (c_2^2 - c_1^2)} \quad , \quad c_1 = 0 \quad , \quad E \equiv K.E$$

$$\boxed{K.E = \frac{1}{2} m c_2^2} \quad kJ$$

or, on a unit mass basis (Specific Kinetic energy)

$$\boxed{k.e = \frac{c^2}{2}} \quad kJ/kg$$

- Single phase Point (Triple Point) of water:

The State in which the solid, liquid and vapor exist together.

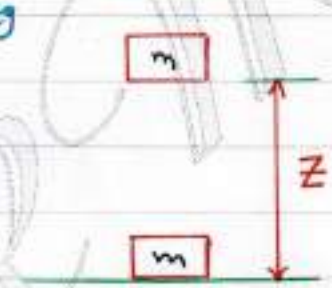
The energy that a System possesses as a result of its elevation in a gravitational field is called Potential Energy "PE" and is expressed as:

$$PE = W \cdot Z$$

$$= mgZ \quad \text{Jole (N.m)}$$

or, on a unit mass basis, (Specific potential energy)

$$Pe = gZ \quad \text{J/kg}$$



$g \equiv$ gravitational acceleration.
$Z \equiv$ elevation.

Any volume of fluid entering or leaving a System displace an equal volume, the energy produced due to this flow is called Flow Energy "FE".

$$F.E = P.V$$

Heat Energy: It is one form of energy that produced only when Temperature difference between the System and Surrounding. There are three types of Heat transfer:

1. Conduction 2. Convection 3. Radiation

Heat energy is given the symbol "Q", to indicat a rate of heat transfer a dot is placed over the Symbol Q, thus

$$\dot{Q} = \text{heat transfer / unit time}$$

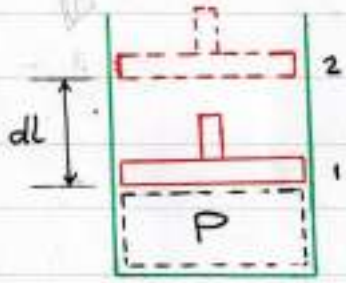
- Work Energy :-

It is a form of energy defined as the multiplication of the force that effected on the mass by the distance that the mass will be moved due to this effect.

$$dW = F \cdot dL \quad , F = P \cdot A$$
$$= P \cdot A \cdot dL \quad , dV = A \cdot dL$$
$$= P \cdot dV$$

$$\int_1^2 dW = \int_{V_1}^{V_2} P \cdot dV$$

$$\boxed{W_{1-2} = \int_{V_1}^{V_2} P \cdot dV} \quad kJ$$

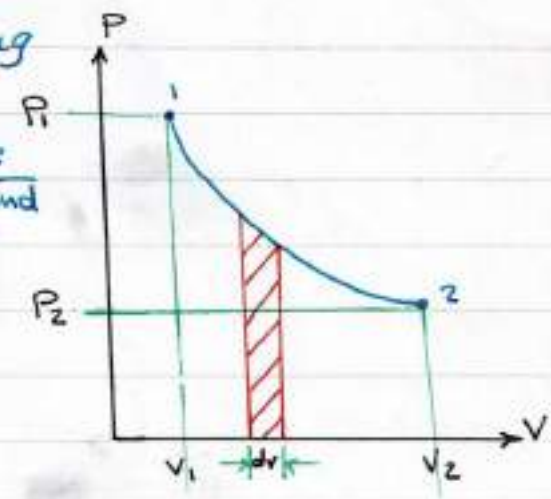


$$w_{1-2} = \int_{v_1}^{v_2} P \cdot dv \quad kJ/kg$$

Power :- is the rate of doing work.

$$\text{Power} = \frac{\text{work done}}{\text{Time taken}} \quad , \frac{\text{Joule}}{\text{Second}}$$

$$P = \frac{W}{t} \qquad J/s \equiv \text{Watt}$$



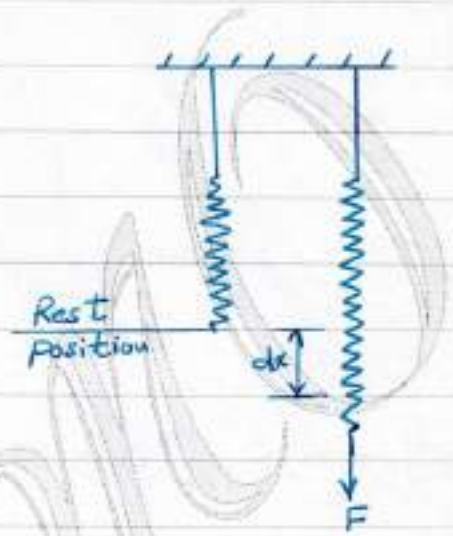
$$\text{Power} = w \cdot \dot{m}$$
$$\equiv \frac{kJ}{kg} \cdot \frac{kg}{s} \equiv \frac{kJ}{s} = kW$$

w = Area under the curve

$\dot{m} \equiv$ flow rate $= \rho C A$

ρ : density (kg/m^3) , C : velocity (m/s) , A : Area (m^2)

- Spring Work :



$$dW = F \cdot dx \quad \text{.... (1)}$$

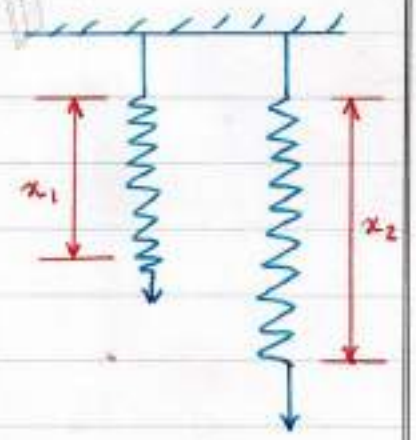
$$F = K \cdot X \quad \text{.... (2)}$$

$K \equiv$ Spring Constant (kN/m)

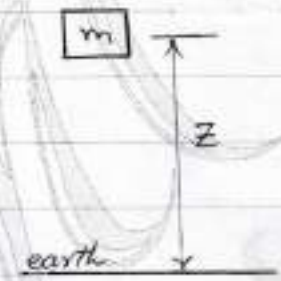
Sub eqn (2) in (1)

$$\int dW = \int_{x_1}^{x_2} K \cdot x \cdot dx$$

$$\boxed{W_{spring} = \frac{1}{2} K (x_2^2 - x_1^2)}$$



Ex: An insulated 2kg box falls from a ballon 3.5Km above the earth. what's the change in potential energy of the box after it has hit the earth Surface.



Sol:

$$P.E = mgZ$$
$$= 2 * 9.81 * 3500 * 10^{-3}$$
$$= 68.67 \text{ KJ}$$

Ex: A force, F, is proportional to (x^2) and has the value of 133N when $x=2$. Determine the work done by it moves in object from $x=1$ to $x=4$ where (x) in meter.

Sol:

$$W = \int_{x_1}^{x_2} F \cdot dx$$

$$= \int_1^4 k x^2 dx$$

$$= k \int_1^4 x^2 dx$$

$$= k \left[\frac{x^3}{3}\right]_1^4 = \frac{133}{4*3}\left[4^3 - 1^3\right]$$

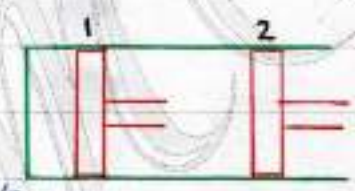
$$= 69825 \text{ N.m}$$
$$= 69.825 \text{ KJ}$$

$$F \propto x^2$$
$$F = k \cdot x^2$$
$$133 = k(2)^2$$
$$\therefore k = \frac{133}{4} \text{ N/m}^2$$

Ex:- Let the pressure in the cylinder in the figure given by the equation $P = C/V$ as a function of Volume.

a. find the work done, if the initial pressure is 400 kpa, the initial volume $0.02\,m^3$ and the final volume is $0.08\,m^3$

b. Is the sign correct?



Sol:- a. $P = C/V \Rightarrow P_1 V_1 = P_2 V_2$

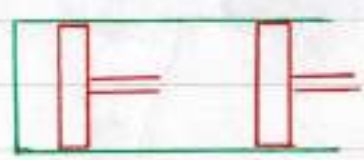
$$W = \int_{V_1}^{V_2} P \cdot dV = \int_{V_1}^{V_2} \frac{C}{V}\, dV = C \int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = P_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} = P_1 V_1 \left[\ln V\right]_{V_1}^{V_2} = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$W = 400 \times 0.02 * \ln(0.08/0.02) = 11.09 \text{ KJ}$$

b. the sign correct since it is positive and the work is done by the System.

Ex:- The pressure in the cylinder shown below varies in the following manner with volume $P = C/V^2$, if the initial volume is $0.05\,m^3$ and the final pressure is 200 kpa, find the work done by the System?



Sol:- $$W = \int_{V_1}^{V_2} P\, dV$$
$$= \int_{V_1}^{V_2} \frac{C}{V^2}\, dV$$

initial pressure = 500 kpa,

$$w = c\left[\frac{1}{V_1} - \frac{1}{V_2}\right]$$

$$c = P_1 V_1^2 = P_2 V_2^2 \Rightarrow V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/2}$$

$$w = P_1 V_1 - P_2 V_2$$
$$= P_1 V_1 - (P_2 P_1)^{1/2} \cdot V_1$$

$$= 500 * 0.05 - (500 * 200)^{1/2} * 0.05$$
$$= 9.19 \text{ kN.m} = 9.19 \text{ kJ}$$ +ve work done by the System

Ex:- A fluid in a cylinder is at a pressure of 700 kN/m². It is expanded at constant pressure from a volume of 0.28 m³ to a volume of 1.68 m³. Determine the work done?

Sol:-

$$\text{Work done} = W = P(V_2 - V_1)$$
$$= 700 * 10^3 (1.68 - 0.28)$$
$$= 7 * 10^5 * 1.4$$
$$= 0.98 \text{ MJ}$$

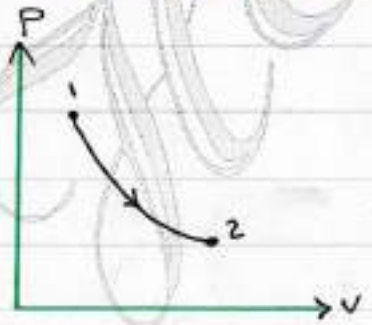
- Reversibility:

A more rigorous definition of reversibility is as follows: When a fluid undergoes a reversible process, both the fluid and its surroundings can always be restored to their original state. The criteria of reversibility are as follows:

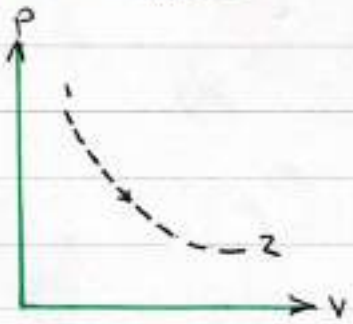
1. The process must be frictionless.
2. The difference in pressure between the fluid and its surroundings during the process must be infinitely small.
3. The difference in temperature between the fluid and its surroundings during the process must be infinitely small.

A reversible process between two states drawn as a line on any diagram of properties.

An irreversible process is usually represented by a dotted line joining the end states to indicate that the intermediate states are indeterminate.



reversible process



irreversible process

- Internal reversibility:

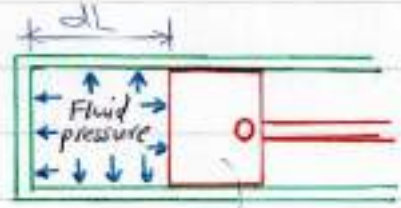
It is may be obtained, because no process in practice is truly reversible, but with conditions below:

1. the surrounding can never be restored to their original state,
2. the fluid itself is at all times in an equilibrium state and the path of the process can be exactly retraced to the initial state.

- Reversible work:

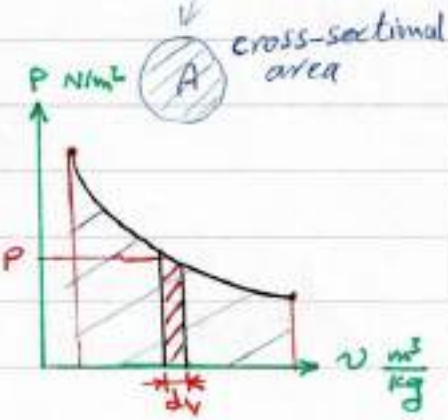
Consider an ideal frictionless fluid contained in a cylinder behind a piston, with the following assumptions:

1. the pressure and temperature of the fluid are uniform,
2. no friction between the piston and the cylinder walls.



work done by fluid $= (PA) * dL$

$= P\,dV$



Per unit mass

Work done $= P\,dv$

(where v is the specific volume)

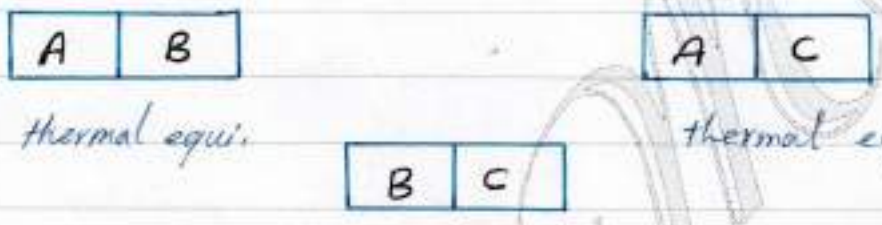
w.

work done = shaded area

$= \int_1^2 P\,dV$

- The zeroth Law (law number zero) :-

This law is concerned with thermal equilibrium. It states that "if two bodies are separately in thermal equilibrium with a third body then they must be in thermal equilibrium with each other".



If $T_A = T_B$

$T_A = T_C$

$\therefore T_B = T_C$ (thermal equilibrium)

- The first Law of thermodynamics :-

the concept of energy and the hypothesis that it can be neither created nor destroyed were developed by scientists in the early part of the nineteenth century, and became known as the Principle of the Conservation of Energy. The First Law of Thermodynamics is merely one Statement of this general principle with particular reference to heat energy and mechanical energy (work).

The First law of Thermodynamics can be stated as follows:

"When a System undergoes a thermodynamic cycle then the net heat supplied to the system from its surrounding is equal to the net work done by the system on its surroundings". In symbols.

$\dot{Q} = q \cdot \dot{m}$ $\qquad \sum dQ = \sum dW$ $\quad$ kW

$\dot{W} = w \cdot \dot{m}$ $\qquad \sum dq = \sum dw$ $\quad$ kJ/kg

where $\sum$ represents the sum for a complete cycle.

Ex: In a certain Steam plant the turbine develops 1000 kW. the heat supplied to the Steam in the boiler is 2800 kJ/kg, the heat rejected by the System to cooling water in the condenser is 2100 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 kW. Calculate the steam flow in $\frac{kg}{s}$.

Sol:

$\sum dq = 2800 - 2100 = 700$ kJ/kg

$\sum dw = 1000 - 5 = 995$ kW

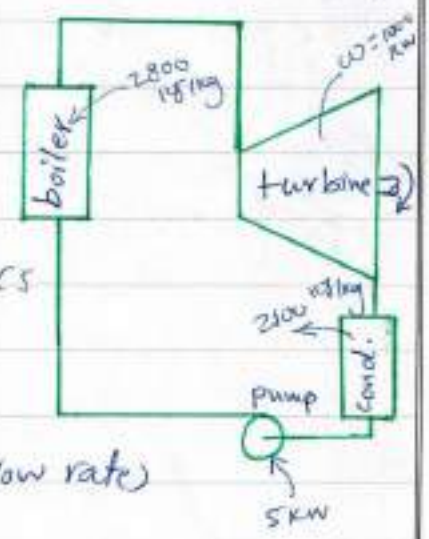
$Q = \dot{m}\, q$

from the first law of thermodynamics

$\sum Q = \sum W$

$\dot{m} \cdot 700 = 995$

$\therefore \dot{m} = 1.421$ kg/s (Steam flow rate)



- The Steady-flow energy equation :- (SFEE)

This equation is a mathematical statement of the principle of conservation of energy as applied to the flow of a fluid through a thermodynamic system.

The various forms of energy which the fluid can have are as follows:-

a. Potential Energy : $P.E = mgz$ kJ

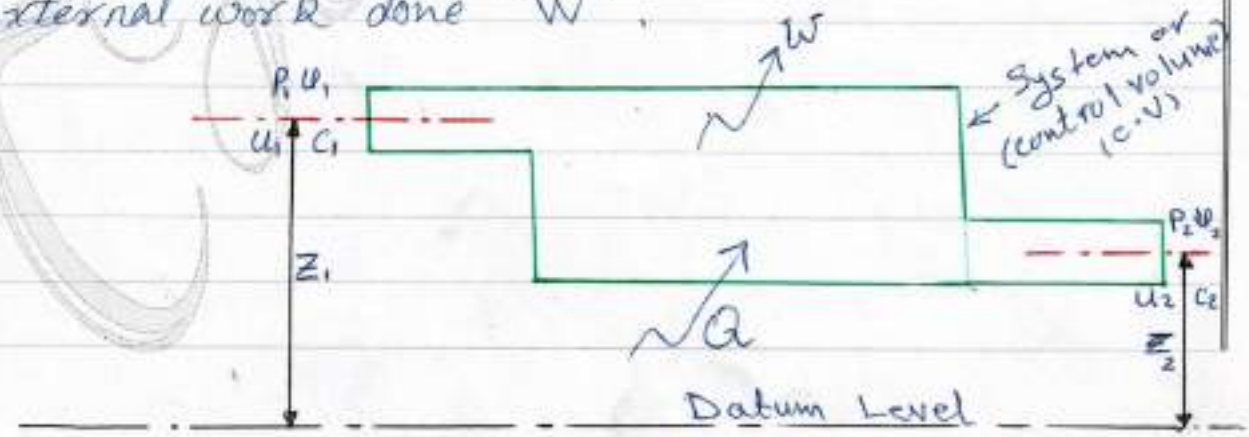
b. Kinetic Energy : $K.E. = \frac{1}{2} m C^2$ kJ

c. Internal Energy : $U = m.u$ kJ

d. Flow or displacement Energy :- $F.E = P.V$ kJ

e. Heat received or rejected "Q".

f. External work done "W".



Applying the principle of conservation of energy to the System (or C.V.) then.

Total energy entering the system = Total energy leaving the System

$$E_{input} = E_{output}$$

$$P.E_1 + I.E_1 + F.E_1 + K.E_1 + Q = P.E_2 + I.E_2 + F.E_2 + K.E_2 + W$$

for 1 kg of fluid mass :-

$$gz_1 + u_1 + P_1 v_1 + \frac{1}{2} C_1^2 + q = gz_2 + u_2 + P_2 v_2 + \frac{1}{2} C_2^2 + w$$

or

$$gz_1 + \overbrace{(u_1 + P_1 v_1)}^{h_1} + \frac{1}{2} C_1^2 + q = gz_2 + \overbrace{(u_2 + P_2 v_2)}^{h_2} + \frac{1}{2} C_2^2 + w$$

$$\boxed{gz_1 + h_1 + \frac{1}{2} C_1^2 + q = gz_2 + h_2 + \frac{1}{2} C_2^2 + w}$$

" Steady flow Energy Equation " SFEE

. h = Enthalpy

Where q & w per unit mass.

Mass flow rate :-

$$\dot{m} = \frac{CA}{v} = \rho C A \quad , \quad \dot{m}_1 = \dot{m}_2$$

*This equation is known as the continuity of mass equation. $\dot{m} = \frac{C_1 A_1}{v_1} = \frac{C_2 A_2}{v_2}$

- The non-flow energy equation : "NFEE"

In the case of a closed system, however, in which the fluid mass remains constant, no substance passing through the system boundary. the flow terms in steady flow E.E. will not apply. Thus the terms Pv and $C^2/2$ are neglected. The system is then said to be non-flow.

from the SFEE:

$$\cancel{gz_1}^{0} + u_1 + \cancel{P_1 v_1}^{0} + \cancel{\tfrac{1}{2}C_1^2}^{0} + q = \cancel{gz_2}^{0} + u_2 + \cancel{P_2 v_2}^{0} + \cancel{\tfrac{1}{2}C_2^2}^{0} + W$$

$$u_1 + q = u_2 + W$$

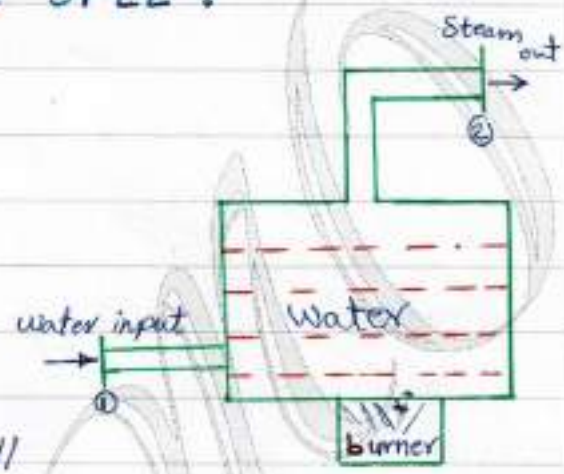
$$\boxed{q - w = u_2 - u_1}$$ "NFEE" per 1 kg

When, $\Delta z, \Delta C^2 \simeq 0 \Rightarrow$

$$\boxed{q - w = h_2 - h_1}$$ "SFEE" per 1 kg

– Some Applications of the SFEE :

1. Steam Boilers :



$$gz_1 + h_1 + \frac{C_1^2}{2} + q = gz_2 + h_2 + \frac{C_2^2}{2} + w$$

in boiler :∘ 1. $w = 0$

2. $C_2^2 - C_1^2/2$ very small

3. $\Delta Z \simeq 0$

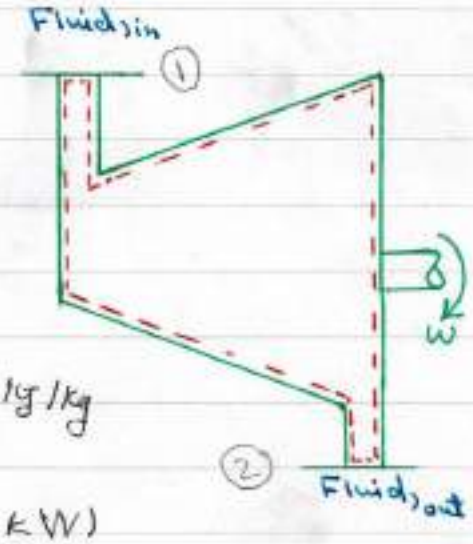
$$\therefore q = h_2 - h_1 \quad kJ/kg$$

$$\dot{Q} = \dot{m}_s (h_2 - h_1) \quad kJ/s \ (kW)$$

$\dot{m}_s$ = Steam flow rate kg/s

$\dot{m}_s = \rho C A$

2. Turbine :



$$gz_1 + h_1 + \frac{C_1^2}{2} + q = gz_2 + h_2 + \frac{C_2^2}{2} + w$$

in turbine :∘ 1. $Q = 0$

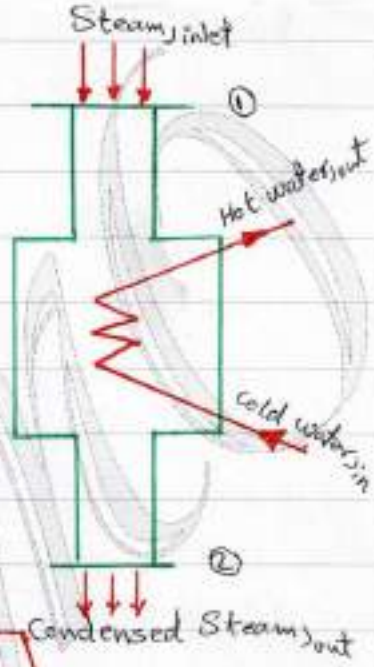
2. $C_2^2 - C_1^2/2 \simeq 0$

3. $\Delta Z \simeq 0$

$$\therefore h_1 = h_2 + w \Rightarrow w = h_1 - h_2 \quad kJ/kg$$

$$\dot{W} = \dot{m}_s (h_1 - h_2) \quad kJ/s \ (kW)$$

- Heat Exchanger :



in H.E. :

1. $w = 0$
2. $C_2^2 - C_1^2/2 \simeq 0$ (neglect)
3. $\Delta Z \simeq 0$

from SFEE :

$\therefore \; q = h_2 - h_1$ (rejected heat)

$\dot{Q} = \dot{m}_s (h_2 - h_1)$

$$\boxed{Q = \dot{m}_s (h_2 - h_1) = \dot{m}_w C_{pw} (T_{w_{out}} - T_{w_{in}})}$$

$\dot{m}_s$ = Steam flow rate , $\dot{m}_w$ = Cooling water flow rate

C_{pw} = Specific Heat of water at constant pressure

$T_{w_{out}}$ = outlet water Temperature.

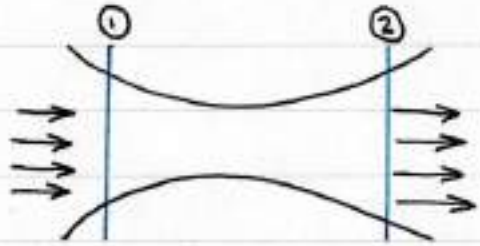
$T_{w_{in}}$ = inlet " " .

- Nozzle :

in Nozzle : 1. $Q = 0$ 2. $w = 0$ 3. $\Delta Z = 0$

from SFEE :

$$\frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2$$



when $C_1 \simeq 0$

$$\frac{C_2^2}{2} = h_1 - h_2$$

$$\therefore \; \boxed{C_2 = \sqrt{2(h_1 - h_2)}} \; m/s$$

- Specific Heats:

is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

$$\boxed{Q = m.C.\Delta T} \quad kJ$$

C = Specific heat capacity kJ/kg.K
m = mass of substance.
Q = heat transferred to produce temperature change.

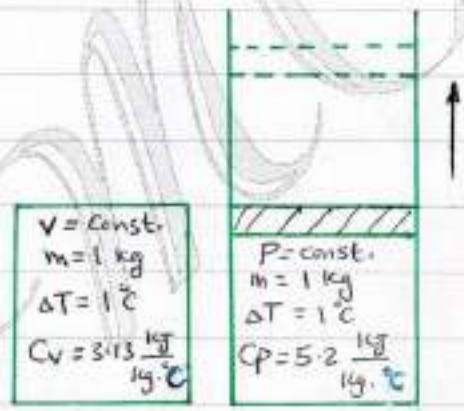
In general, the energy required to raise the temperature will depend on how the process is executed. In thermodynamics, we are interested in two kinds of specific heats:

1. Specific heat at constant volume "C_v".
2. " " " " Pressure "C_p".

physically, the specific heat at constant volume C_v can be viewed as "the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

The energy required to do the same as the pressure is maintained constant is the specific heat at constant pressure C_p.

This is illustrated in fig. below. The specific heat at constant pressure C_p is always greater than C_v because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system



(C_v & C_p values of helium)

$\because \; Q = m C \Delta T$

for unit mass

$$q = C \, \Delta T$$

$$dq = C \, dT$$

$$\therefore \; \boxed{C = \frac{dq}{dT}}$$

1. at constant volume:

from NFEE:

$$q - w = \Delta u \quad , \text{ at } V = c \Rightarrow w = 0$$

$$q = \Delta u \Rightarrow dq = du$$

$$C_v = \frac{dq}{dT} = \frac{du}{dT} \Rightarrow du = C_v dT \Rightarrow \Delta u = C_v \Delta T$$

$\therefore$ at constant volume

$$\boxed{Q = \Delta U = m C_v \Delta T}$$

2. at Constant Pressure:

$$W = P_2V_2 - P_1V_1$$

$$q = W + \Delta U$$

$$q = (P_2V_2 - P_1V_1) + (U_2 - U_1)$$

$$q = (P_2V_2 + U_2) - (P_1V_1 + U_1)$$

$$q = h_2 - h_1$$

$$q = \Delta h$$

$$dq = dh$$

$$C_p = \frac{dq}{dT} = \frac{dh}{dT}$$

$$dh = C_p \cdot dT$$

$$\Delta h = C_p \cdot \Delta T$$

$$\boxed{Q = \Delta H = m C_p \Delta T}$$

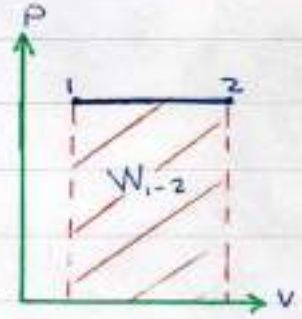
Notes: 1. Heat recived by the System q is +ve
" rejected " " " q is -ve

2. Work done by the System W is +ve
" " on " " W " -ve

Ex:- A certain fluid at 10 bar is contained in a Cylinder behind a piston, the initial volume being $0.05 m^3$. Calculate the work done by the fluid when it expands reversibly:-

a. at constant pressure to a final volume of $0.2 m^3$.
b. According to a linear law to a final volume of $0.2 m^3$ and a final pressure of 2 bar.
c. According to a law $pV = C$ to a final volume of $0.1 m^3$.
d. According to a law $pV^2 = C$ to a final volume of $0.06 m^3$.
e. According to a law $p = (A/V^2) - (B/V)$ to a final volume of $0.1 m^3$ and final pressure of 1 bar. A & B are constant.

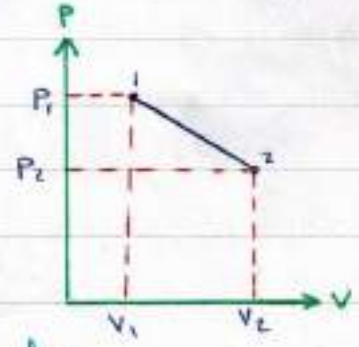
Sol:-



a. $W = \int_1^2 p\,dV \quad , p = C$

$= p_1 \int_1^2 dV = p[V]_1^2 = p(V_2 - V_1)$

$= 10 * 10^2 (0.2 - 0.05) = 150$ kJ



b. linear law $= \frac{P - P_1}{V - V_1} = \frac{P_2 - P_1}{V_2 - V_1}$

$$\frac{P - 10 * 10^2}{V - 0.05} = \frac{2 * 10^2 - 10 * 10^2}{0.2 - 0.05}$$

$$0.15P + 800V - 190 = 0$$

$$P = \frac{190 - 800V}{0.15}$$

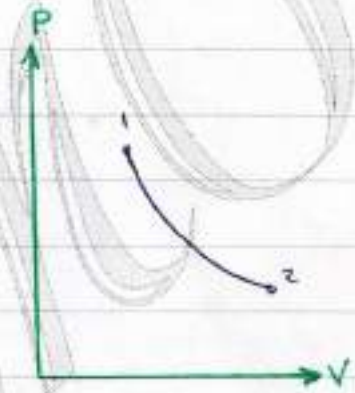
800

$$w = \int p\,dV = \int_1^2 \frac{190 - 800V}{0.15}\,dV = \left[\frac{190V - 400V^2}{0.15}\right]_{0.05}^{0.2}$$

$\therefore w = 90$ kJ

c. $pV = C \Rightarrow 10 \times 10^2 \times 0.05 = C \quad \therefore C = 50$

$\therefore p = 50/V$



$$w = \int_1^2 p\,dV = \int_1^2 50\,\frac{dV}{V} = 50 \ln V \Big]_1^2$$

$$= 50\left[\ln V_2 - \ln V_1\right]$$

$$= 50\left\{\ln(0.1) - \ln(0.05)\right\}$$

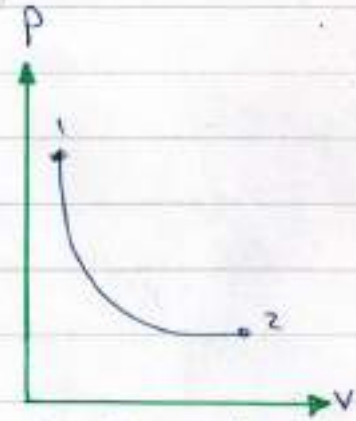
$$= 34.65 \text{ kJ}$$

d. $pV^3 = C$

$10 \times 10^2 (0.05)^3 = C \Rightarrow C = 0.125 \Rightarrow p = \frac{0.125}{V^3}$

$$w = \int_1^2 p\,dV = \int_1^2 0.125\,\frac{dV}{V^3} = \left[0.125 \times \frac{-1}{2V^2}\right]_1^2$$

$w = 7.638$ kJ



e. $p = \frac{A}{V^2} - \frac{B}{V} \Rightarrow 10 \times 10^2 = \frac{A}{0.05^2} - \frac{B}{0.05}, \quad 1 \times 10^2 = \frac{A}{0.1^2} - \frac{B}{0.1}$

$\therefore A = 4 \qquad B = 30$

$\therefore p = \frac{4}{V^2} - \frac{30}{V}$

$$w = \int_1^2 p\,dV = \int_1^2 \left(\frac{4}{V^2} - \frac{30}{V}\right) dV = \left[-\frac{4}{V} - 30 \ln V\right]_1^2$$

$\therefore w = 19.2$ kJ

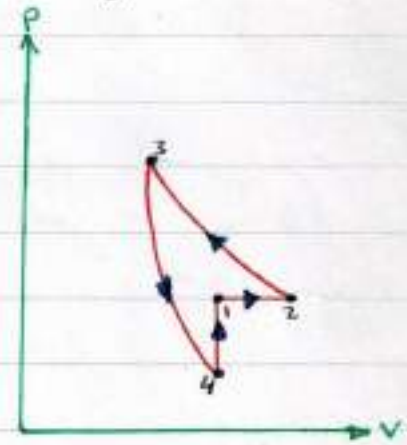
Ex: A fluid is heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of $0.1\ m^3/kg$. It is then compressed reversibly according to a law $pv=c$ to a pressure 4.2 bar, then allowed to expand reversibly according to a law $pv^{1.3}=c$ and finally heated at constant volume back to initial condition. The work done in the constant pressure process is 515 N.m and the mass of fluid present is 0.2 kg. Calculate the net work done on or by the fluid in the cycle and sketch the cycle on the pv diagrame.

Sol: $P_1 = 1.05$ bar $\quad v_1 = ?$

$P_2 = P_1 = 1.05$ bar $\quad v_2 = 0.1\ m^3/kg$

$P_3 = 4.2$ bar $\quad v_2 = ?$

$P_4 = ?$ $\quad v_4 = v_1$



- process $1 \to 2$, $W_{1-2} = 515$ N.m

$$w = \int_1^2 P\,dv = P_1 \int_1^2 dv = P(v_2 - v_1)$$

$$W = m \cdot w = m[P(v_2 - v_1)]$$

$$515 = 0.2 \times 1.05 \times 10^5 (0.1 - v_1)$$

$$v_1 = 0.075\ m^3/kg$$

- process $2 \to 3$ $\quad pv = c \Rightarrow p_2 v_2 = c$

$$1.05 \times 10^5 \times 0.1 = c \Rightarrow c = 10500$$

$$p = \frac{c}{v}$$

$$w_{2-3} = \int_2^3 P\,dv = C_2 \int_2^3 \frac{dv}{v} = 10500\,[\ln v_3 - \ln v_2]$$

$$\because P_2 v_2 = P_3 v_3 \Rightarrow v_3 = \frac{P_2}{P_3} \cdot v_2 = \frac{1.05}{4.2} * 0.1 = 0.025 \ \frac{m^3}{kg}$$

$$\therefore w = 10500\,[\ln 0.025 - \ln 0.1] = -14556 \ \frac{N.m}{kg}$$

$$W_{2-3} = m \cdot w = 0.2 * -14556 = -2911.2 \ N.m = -2911.2 \text{ Joule}$$

- process 3-4 , $pv^{1.3} = c$

$$v_4 = v_1 = 0.075 \ m^3/kg$$

$$P_3 v_3^{1.3} = P_4 v_4^{1.3} = C$$

$$\therefore P = \frac{C}{v^{1.3}}$$

$$w_{3-4} = \int_3^4 P\,dv = \int_3^4 C \frac{dv}{v^{1.3}} = P_3 v_3^{1.3} \int_3^4 \frac{dv}{v^{1.3}}$$

$$w = P_3 v_3^{1.3} \left[-\frac{1}{0.3 v^{0.3}} \right]_3^4 = 4.2 * 10^5 (0.025)^{1.3} \left[-\frac{1}{0.3(0.075)^{0.3}} + \frac{1}{0.3(0.025)^{0.3}} \right]$$

$$= 9827.4 \ J/kg$$

$$W_{3-4} = m \cdot w_{3-4} = 0.2 * 9827.4 = 1965.5 \ J$$

$$w_{4-1} = \int_1^2 P\,dv \quad \text{since } v = c \Rightarrow dv = 0 \quad \therefore w = 0$$

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$= 515 + (-2911.2) + 1965.5 + 0$$

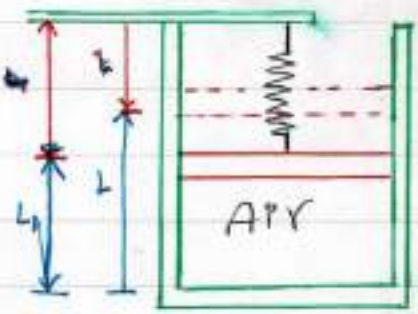
$$= -430.7 \text{ Joule}$$

$$= -0.4307 \ kJ \quad \text{(work done on the System)}$$

Ex:- consider the System shown in figure. the initial volume inside the cylinder is (0.1 m³). At this stats the pressure inside is (100) kpa, which just balance the atmospher pressure plus the piston weight, the spring is touching but exerts no force on the piston at this State. The gas now heated until the volume is doubled. The final pressure of the gas is (300 kpa), and during the process the Spring force is proportional to the displacement of the piston from the initial position. Calculate the work done by the System, what persentage of work is done against the Spring.

Sol:- $P_1 = 100$ kpa $\quad V_1 = 0.1\ m^3$

$P_2 = 300$ kpa $\quad V_2 = 0.2$



force $\propto \Delta L$, $\Delta L = L_2 - L_1 = L - L_1$

$F_s \propto (L - L_1) \Rightarrow \frac{F_s}{A} \propto \frac{L-L_1}{A} \cdot \frac{A}{A}$

$P_s \propto \frac{(L-L_1)A}{A^2} \Rightarrow P_s \propto \frac{(V-V_1)}{A^2}$

$P_s = \frac{c}{A^2}(V-V_1) = a(V-V_1)$, at $P = 300$, $V = 0.2$

$P_t = P_o + P_s = 100 + a(V-V_1)$

$300 = 100 + a(0.2 - 0.1) \Rightarrow a = 2000$

$\therefore \; P_t = 100 + 2000(V - 0.1) = 100 - 200 + 2000V = 2000V - 100$

$$W = \int_1^2 P\,dV = \int_1^2 (2000V - 100)\,dV$$

$$W_{1-2} = \left[\frac{2000V^2}{2} - 100V\right]_1^2$$

$$W_{1-2} = \left(\frac{2000(0.2)^2}{2} - 100(0.2)\right) - \left(\frac{2000 \times (0.1)^2}{2} - 100 \times 0.1\right)$$
$$= 20 \text{ kJ}$$

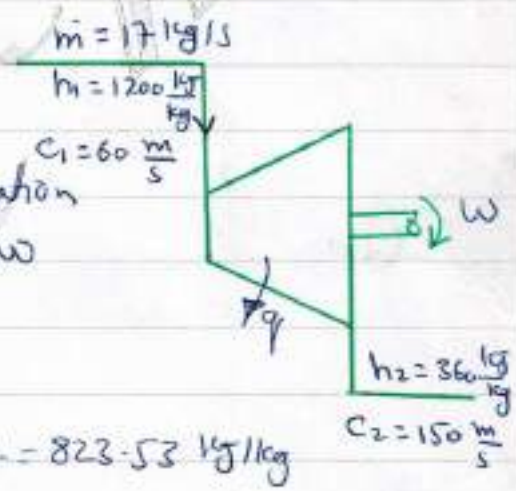
If there is no Spring the piston will rise at constant pressure

$$W = \int_1^2 P \cdot dV = P \int dV$$

$$= P(V_2 - V_1)$$
$$= 100(0.2 - 0.1)$$
$$= 10 \text{ kJ}$$

$$\% = \frac{10}{20} = 50\%$$

Ex:- In a turbine of gas turbine unit, the gases flow through the turbine at 17 kg/s and the power developed by the turbine is 14000 kW. The enthalpies of the gases at inlet and outlet are 1200 kJ/kg and 360 kJ/kg respectively and the velocities of the gas at inlet and outlet are 60 m/s and 150 m/s respectively. Find the rate at which heat is rejected from the turbine. Find also the inlet pipe cross-sectional area when the inlet specific volume is 0.5 m³/kg.



Sol:-

from Steady flow Energy Equation

$$gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w$$

$$z_1 \simeq z_2$$

power = 14000 kW

$$P = \dot{m} \cdot w \Rightarrow w = 14000/17 = 823.53 \text{ kJ/kg}$$

$$h_1 + \frac{1}{2}C_1^2 + q = h_2 + \frac{1}{2}C_2^2 + w$$

$$\therefore q = (h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) * 10^{-3} + w$$

$$= (360 - 1200) + \frac{1}{2}(150^2 - 60^2) * 10^{-3} + 823.53$$

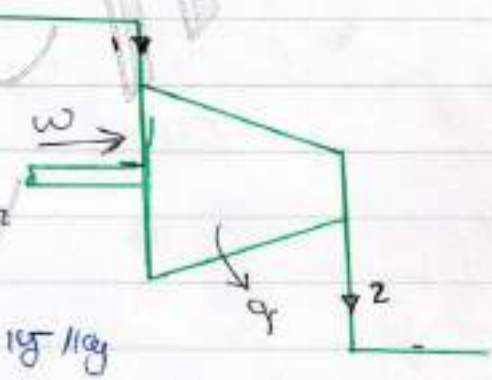
$$= -7.05 \text{ kJ/kg}$$

$$= 7.05 \text{ rejected heat}$$

$$Q = \dot{m} \cdot q = 17 * (-7.05) = -119.85 \text{ kW}$$

$$\dot{m} = \frac{C_1 A_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} \cdot v_1}{C_1} = \frac{17 * 0.5}{60} = 0.142 \text{ m}^2$$

Ex:- Air flows at a rate of 0.4 kg/s through an air compresser entering at 6 m/s, 1 bar and 0.85 m^3/kg and leaving at 4.5 m/s, 6.9 bar and 0.16 m^3/kg. The internal energy of the air leaving is greater than that of entering air by 88 kJ/kg, cooling water in the jacket surrounding the cylinder absorbs heat from the air at the rate of 59 kJ/s. Find the power required to drive the compressor and the inlet and outlet pipe cross-sectional area.



Sol:- from Steady flow EE

$$gz_1 + PV_1 + U_1 + \frac{1}{2}C_1^2 + q = gz_2 + P_2V_2 + U_2 + \frac{1}{2}C_2^2 + w$$

$$Q = \dot{m}\cdot q \Rightarrow q = 59/0.4 = 147.5 \text{ kJ/kg}$$

$$P_1V_1 + U_1 + \frac{1}{2}C_1^2 + q = P_2V_2 + U_2 + \frac{1}{2}C_2^2 + w \qquad (U_2 - U_1 = 88 \text{ kJ/kg})$$

$$w = (P_1V_1 - P_2V_2) + (U_1 - U_2) + \frac{1}{2}(C_1^2 - C_2^2)\times 10^{-3} + q \qquad (Z_1 \simeq Z_2)$$

$$w = (1\times10^2 \times 0.85 - 6.9\times10^2 \times 0.16) + (-88) + \frac{1}{2}(6^2 - 4.5^2)\times10^{-3} + (-147.5)$$

$$= -260.9 \text{ kJ/kg}$$

$$W = \dot{m}\cdot w = 0.4 \times (-260.9) = -104.35 \text{ kW}$$

$$A_1 = \frac{\dot{m}\cdot V_1}{C_1} = 0.057 \text{ m}^2$$

$$A_2 = \frac{\dot{m}\cdot V_2}{C_2} = 0.0148 \text{ m}^2$$

(Sheet No. 1)

Q1:-

1 kg of fluid is compressed reversibly according to a $pv = 0.25$, where (p) in bar and v in m^3/kg. The final volume is one fourth the initial volume. Calculate the work done on the fluid and sketch the process on p-v diagram.

Ans. [34660 N.m]

Q2:-

$0.05\ m^3$ of gas at 6.9 bar expands reversibly in a Cylinder behind a piston according to a law $pV^{1.2} = C$ until the volume is $0.08\ m^3$. Calculate the work done by the gas and sketch the process on p-v diagram.

Ans. [15300 N.m]

Q3:-

One kilograme of fluid expand reversibly according to a linear law from 4.2 bar to 1.4 bar. The initial and final volume are $0.004\ m^3$ and $0.02\ m^3$ respectively. The fluid is then cooled reversibly at constant pressure, and finally compressed reversibly according to a law $pv = c$ back to initial condition of 4.2 bar and $0.004\ m^3$. Calculate the work done for each process, the net work done and sketch the cycle on p-v diagram. Ans. [4480 J, -1120 J, -1845 J, 1515 J]

Q4: 0.09 m^3 of fluid at 0.7 bar is compressed reversibly to a pressure of 3.5 bar according to a law of $pv^n = c$. The fluid is then heated reversibly at constant volume until the pressure is 4 bar. the specifice volume is 0.5 m^3/kg. A reversible expansion according to a law $pv^2 = c$ restores the fluid to its initial state. Calculate the mass of the fluid present, the value of (n) and the net work done on or by the system. Sketch the cycle on p-v diagram.

Ans. [0.0753 kg, 1.85, 676 N.m]

Q5:

Air at 200 kpa, 30 °C is contained in a cylinder/piston with initial volume of 0.1 m^3. The inside pressure balances ambient pressure of 100 kpa plus an external imposed force that proportinal to $V^{0.5}$. Now heat is transfer to the system to final pressure of 225 kpa. Find the work done for this process.

Ans. [8.65 kJ]

Q6: In an air compresser, the compression is takes place at constant internal energy, and 50 kJ of heat is rejected to the cooling water for every one kilograme of air. find the work required for the compression.

Ans. [50 kJ/kg]

Q7: In a compression strok of gas engine the work done on the gas by the piston is 70 kJ/kg and heat rejected to the cooling water is 42 kJ/kg. Find the change of internal energy. Stating whether if is gain or lost.

Ans. [28 $\frac{kJ}{kg}$]

Q8:

A mass of gas with an internal energy of 1500 kJ is contained in a cylinder which has perfect thermal insulated. The gas is allowed to expand behind a piston until its internal energy is 1400 kJ. Calculate the work done by the gas. If the expansion follows a law $pv^2=c$ and the internal energy is changed. and initial pressure and volume are 28 bar and 0.06 m^3 respectively. Calculate the final pressure and volume.

Ans. [100 kJ, 4.59 bar, 0.48 m^3]

Q9: A Steam turbine recives a steam flow at 1.35 $\frac{kg}{s}$ and delivers 5000 kW. The heat loss from the casing is negligible. Find a. the change of enthalpy across this turbine when the velocities at entrance and exit and the difference in elevation at entrance and exit are negligible. b. the change of enthalpy across the turbine when the velocity at entrance and exit are 60 $\frac{m}{s}$ and 360 $\frac{m}{s}$, and the inlet pipe is 3m above the exhaust pipe.

Ans. [-3704 kJ/kg , -3766 kJ/kg]

Q10: A steady flow of steam enters a condenser with an enthalpy of 2300 kJ/kg and velocity of 350 m/s. The condensate leaves the condenser with an enthalpy of 160 kJ/kg and velocity of 70 m/s. Find the heat transfer to the cooling water per kg of steam.

Ans. [-2199 kJ/kg]

Q11: A turbine recives steam at 13.8 bar, 0.143 m^3/kg, internal energy 2590 kJ/kg, and 30 m/s. At leaving the condenser is 0.35 bar, 4.37 m^3/kg, internal energy 2360 kJ/kg and 90 m/s. Heat is lost to surrounding at 0.25 kJ/s. If the rate of steam flow is 0.38 kg/s. What is the power developed by the turbine.

Ans. [102.8 kW]

Q12: A nozzle is a device for increasing the velocity of fluid. At inlet to a nozzle the enthalpy is 3026 kJ/kg and the velocity is 60 m/s. At exit from the nozzle the enthalpy is 2790 kJ/kg. The nozzle is horizontal and there is negligible heat loss from the turbine.

a. find the velocity at the nozzle exit. [688 m/s]

b. If the inlet area is 0.1 m^2 and the specific volume at the inlet is 0.19 m^3/kg. find the mass flow rate. [0.36 kg/s]

c. If $v = 0.5\ m^3/kg$ find the exit area of the nozzle. [0.0229 m^2]

Q13: In a non-flow process there is a heat loss of 1055 kJ and an internal energy increase of 210 kJ. How much work is done and is the process is expansion or compression.

Ans. [-1265 kJ, Comp.]

Q14: Air and fuel enter furnace used for home heating. The air has an enthalpy of 320 kJ/kg and the fuel an enthalpy of 43027 kJ/kg. The gases leaving the furnace has an enthalpy of 616 kJ/kg. There are 17 $\frac{kg_a}{kg_f}$ water circulate through the wall furnace receiving heat. The house required 17.02 kW of heat, what is the fuel consu ption per day.

Ans. [41 kg/day]

THERMODYNAMICS

2

CHAPTER TWO

IDEAL GAS

(IDEAL GAS)

(Perfect gas)

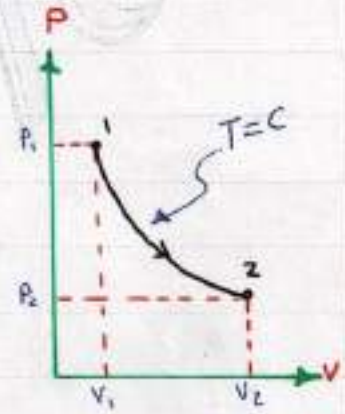
- Boyle's Law:

It is defined as: when a change of state of any gas in which the mass and the temperature remain constant, the volume varies inversely as the pressure.

$$P \propto \frac{1}{V} \Rightarrow P = C \cdot \frac{1}{V}$$

$$\therefore \boxed{PV = C} \quad \text{Boyle's Law}$$

$$\therefore P_1 V_1 = P_2 V_2$$



Ex:

A gas whose original pressure and volume were 300 kN/m² and 0.14 m³ is expanded until its new pressure is 60 kN/m² while its temperature remains constant. What is its new volume?

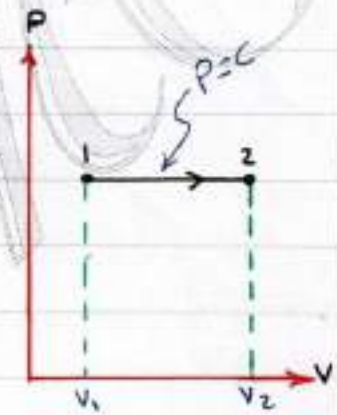
Sol: $P_1 V_1 = P_2 V_2$ or $V_2 = V_1 \frac{P_1}{P_2}$

$$V_2 = 0.14 * \frac{300}{60}$$

$$= 0.7 \, m^3$$

Charles's Law :

It is defined as: When the change of state of any gas in which the mass and pressure remain constant, the volume varies in proportion with the absolute temperature.



$$V \propto T \Rightarrow V = CT$$

$$\therefore \boxed{\frac{V}{T} = C} \quad \text{Charles's Law}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Ex: A quantity of gas whose original volume and temperature are $0.2\,m^3$ and $303^\circ C$, respectively, is cooled at constant pressure untile its volume becomes $0.1\,m^3$. What will be the final temperature of the gas?

Sol:

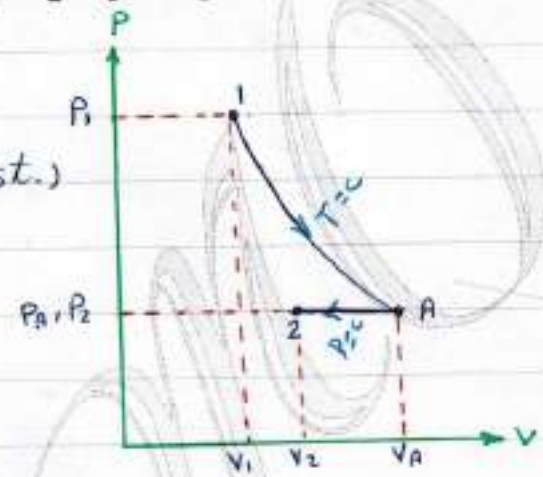
$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow T_2 = T_1 \cdot \frac{V_2}{V_1}$$

$$T_1 = 303 + 273 = 576\ K$$

$$T_2 = 576 * \frac{0.1}{0.2} = 288\ K$$

$$\therefore t_2 = 288 - 273 = 15^\circ C$$

(The characteristic equation of a perfect gas)



the process $1 \rightarrow A$

$PV = C$ (Boyle) (T = Const.)

$P_1 V_1 = P_A V_A$, $T_1 = T_A$

$$V_A = \frac{P_1}{P_A} \cdot V_1 \quad \text{------} \quad (1)$$

$V = cT$ (charles) (P = const.)

$$\frac{V_A}{T_A} = \frac{V_2}{T_2} = \frac{V}{T} = C$$

$$V_A = \frac{T_A}{T_2} \cdot V_2 \quad \text{------} \quad (2)$$

from Boyle's Law ($T_A = T_1$) Sub in eq.(2)

$$V_A = \frac{T_1}{T_2} \cdot V_2 \quad \text{----} \quad (3)$$

from eq's (1) & (3):

$$V_A = \frac{P_1}{P_A} \cdot V_1 = \frac{T_1}{T_2} \cdot V_2$$, from charles Law $P_A = P_2$

$$\frac{P_1}{P_2} \cdot V_1 = \frac{T_1}{T_2} \cdot V_2 = C$$

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = R}$$, R = gas constant

$P_1 V_1 = R T_1$ for 1 kg

$$\boxed{P_1 V_1 = m R T_1}$$ for m kg

$\because m = n \cdot M$

Where : m = mass kg

n = number of moles mol

M = molecular weight

$PV = mRT$ (In general)

$PV = nMRT$

$MR = \frac{PV}{nT}$ (P & T are constant)

$MR = C \cdot \frac{V}{n}$, $(\frac{V}{n} = C)$ (Avogadro's theorm)

$\therefore$ $\boxed{MR = R_o}$, R_o = Universal gas Constant

Avogadro's theorm: volume of one mole of any gas is same for all gases at any temperature and pressure.

O_2 32
N_2 28
CO_2 44

$\therefore \quad R_o = \frac{PV}{nT}$

$\therefore$ Volume of 1 mole at 0°c and $P = P_{atm} = 1.0132$ bar equal to $22.7\ m^3$.

$\therefore R_o = \frac{1.0132 * 10^5 * 22.7}{1 * (0+273)} = 8.314 \ kJ/kg.K$

Ex: for O_2 : $R = R_o/M = 8.314/32 = 0.259$

Ex: A volume of $3.6\ m^3$ of O_2 initially at 220°C and pressure 400 kpa, is compressed reversibly at constant temperature to a final volume of $0.06\ m^3$. Calculate the mass, the final pressure, the increase in internal energy and the work done.

Sol:

$PV = mRT$, for O_2, $R = R_o/M = 0.26 \ kJ/kg.°K$

$m = 400 * 10^3 \times 3.6 / 0.26 * 10^3 \times (220+273) \equiv \frac{N/m^2 * m^3}{J/kg.K * K}$

$= 11.23 \ kg$

$P_1V_1 = P_2V_2 \Rightarrow P_2 = (3.6/0.06) * 400 = 24 \ Mpa$

$Q = W + \Delta U$, $\Delta U = 0$ (since $T = C$)

$W = \int_1^2 P dV = C \int_1^2 \frac{dV}{V} = P_1V_1 \int \frac{dV}{V} = P_1V_1 \ln \frac{V_2}{V_1}$

$W = 400 * 10^3 * 3.6 * \ln \frac{0.06}{3.6} = Q = -5.896 \ MJ$

- Joule's Law :

It is defined as:

the internal energy of a gas is a function of temperature only and is independent of changes in pressure and volume.

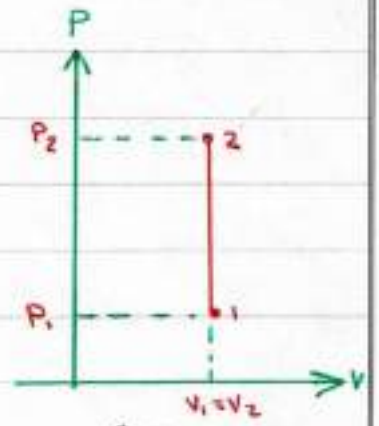
$\therefore$ If $T = C \Rightarrow \Delta U = 0$

- The specific heat Capacities of a gas :

a. the constant volume heating of a gas:

let a mass of gas m be heated at constant volume such that its temperature rises from T_1 to T_2 and its pressure rises from P_1 to P_2. Then

Heat received by the gas $= m\, C_v\, (T_2 - T_1)$



Area = 0

from NFEE , $Q = \Delta U + W$

$\therefore V = C \Rightarrow W = 0$

$Q = \Delta U$,

or $m C_v (T_2 - T_1) = U_2 - U_1$

from the perfect gas equation (Characteristic equation)

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \Rightarrow P_2 = P_1 \frac{T_2}{T_1}$$

Ex: 2 kg of gas, occupying 0.7 m^3, had an original temperature of 15 °C. It was then heated at constant volume until its temperature became 135 °C. How much heat was transferred to the gas and what was its final pressure?

Take, $C_v = 0.72$ kJ/kg K and $R = 0.29$ kJ/kg K

Sol: Heat transferred at (V=C) $= m C_v (T_2 - T_1)$

$$= 2 \times 0.72 \times (135 - 15)$$
$$= 2 \times 0.72 \times 120$$
$$= 172.8 \text{ kJ}$$

Now $P_1 V_1 = m R T_1$

$$\therefore \quad P_1 = \frac{m R T_1}{V_1} = \frac{2 \times 0.29 \times 288}{0.7} = \frac{167.04}{0.7} = 238.6 \text{ kPa}$$

Since the volume remains constant, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \therefore \quad P_2 = P_1 \frac{T_2}{T_1}$$

$$= 238.6 * \frac{408}{288}$$

$$= 338.1 \text{ kN/m}^2$$

<u>or</u> $P_2 V_2 = m R T_2$

$$P_2 = m R T_2 / V_2 = 2 * 0.29 * (135 + 273) / 0.7$$
$$= 338.1 \text{ kPa}$$

b. the constant pressure heating of a gas:

Let a mass of gas (m) be heated at constant pressure such that its temperature rises from T_1 to T_2 and its volume increases from V_1 to V_2. Then

Heat received by the gas $= m C_p (T_2 - T_1)$

$$Q = \Delta U + W$$

$$m C_p (T_2 - T_1) = (U_2 - U_1) + P(V_2 - V_1)$$
$$= (U_2 + PV_2) - (U_1 + PV_1)$$
$$= H_2 - H_1$$

or

$$U_2 - U_1 = m C_p (T_2 - T_1) - P(V_2 - V_1)$$

When $PV = mRT$ then

$$U_2 - U_1 = m C_p (T_2 - T_1) - mR(T_2 - T_1)$$
$$= m(T_2 - T_1)(C_p - R)$$

from the characteristic equation of a perfect gas:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad , P_1 = P_2$$

$$V_2 = V_1 \cdot \frac{T_2}{T_1}$$

Note: $H = m C_p T$, $h = C_p T$, $\Delta h = C_p \Delta T$
$U = m C_v T$, $u = C_v T$, $\Delta u = C_v \Delta T$

Ex:- A gas whose pressure, volume and temperature are 275 kN/m², 0.09 m³ and 185 °C, respectively, has its state changed at constant pressure until its temperature becomes 15 °C. How much heat is transferred from the gas and how much work is done on the gas during the process?

Take: $R = 0.29$ kJ/kg.K , $C_p = 1.005$ kJ/kg.K.

Sol:-

$$P_1 V_1 = m R T_1$$

$$\therefore m = P_1 V_1 / R T_1 = 275 \times 10^3 \times 0.09 / 0.29 \times 10^3 \times 458$$

$$= 0.186 \text{ kg}$$

$$\text{Heat transferred} = m C_p (T_2 - T_1)$$

$$= 0.186 \times 1.005 \times (288 - 458)$$

$$= -31.78 \text{ kJ}$$ (the heat has been extracted from the gas)

$\because$ P = constant (given)

$$\therefore \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = V_1 * \frac{T_2}{T_1} = 0.09 \times \frac{288}{458}$$

$$= 0.0566 \text{ m}^3$$

$$\text{Work done} = P(V_2 - V_1)$$

$$= 275 * (0.0566 - 0.09)$$

$$= -9.19 \text{ kJ}$$

. Relation Between Specific Heat (Cp&Cv):-

from the non-flow energy equation NFEE

$$Q = W + \Delta U \quad , \Delta U = m C_v (T_2 - T_1)$$

$$W = \int_1^2 P \cdot dV = P_2 V_2 - P_1 V_1$$

Ideal Gas law $PV = mRT$

$$W = mR(T_2 - T_1)$$

$$\therefore Q = mR(T_2 - T_1) + m C_v (T_2 - T_1)$$

$$= m(R + C_v)(T_2 - T_1)$$

$$\because Q = \Delta H = m C_p (T_2 - T_1)$$

$$\therefore m C_p (T_2 - T_1) = m(R + C_v)(T_2 - T_1)$$

$$\therefore C_p = R + C_v \Rightarrow \boxed{C_p - C_v = R}$$

$$\boxed{\gamma = \frac{C_p}{C_v}}$$, γ = the ratio of the specific heat.

$$\frac{C_p}{C_v} - \frac{C_v}{C_v} = \frac{R}{C_v}$$

$$\gamma - 1 = R / C_v \Rightarrow \boxed{C_v = \frac{R}{\gamma - 1}}$$

$$C_p = \gamma * C_v \Rightarrow \boxed{C_p = \frac{\gamma R}{\gamma - 1}}$$

Ex:- A perfect gas have specific heat as $C_p = 0.846 \frac{kJ}{kg \cdot K}$ $C_v = 0.657$ kJ/kg.K. find the gas constant and molecular weight of gas.

Sol:- $R = C_p - C_v = 0.846 - 0.657 = 0.189$ kJ/kg.K
$= 189$ J/kg.K

$M = R_o / R = 8314 / 189 = 44$

Ex:- A perfect gas has molecular weight of 26 and avalue of $\gamma = 1.26$. Calculate the heat rejected per 1kg when:-

a. the gas is contained in arigid vessel at 3bar and 315°C and cooled until the pressure fails to 1.5 bar.

b. the gas enters pipe at 280°C and flow steady in the end of the pipe where T = 250°C. Neglect any change in velocity of the gas.

Sol:- a. $Q = W + \Delta U$, arigid vessel $\Rightarrow V = C \Rightarrow dV = 0$

$W = \int P \cdot dV = 0$

$\therefore Q = \Delta U = m C_v (T_2 - T_1)$. for $V = C$ only

$M = 26$, $\gamma = 1.26$

$R = R_o / M = 8.314 / 26 = 0.3198$ kJ/kg.K

$C_v = \frac{R}{\gamma - 1} = \frac{0.3198}{1.26 - 1} = 1.229$ kJ/kg.K

$Q = m C_v (T_2 - T_1) = 1 * 1.229 * (T_2 - T_1)$

since $v = c \Rightarrow v_1 = v_2$

from the characteristic equation

$$\frac{P_1 \cancel{v_1}}{T_1} = \frac{P_2 \cancel{v_2}}{T_2} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore T_2 = \frac{P_2}{P_1} T_1 = \frac{1.5}{3} (315 + 273) = 294 \text{ K}$$

$Q = 1 * 1.229 * (294 - 588)$

$= -361$ kJ (rejected)

$= 361$ kJ rejected

b. Steady flow Energy Equation SFEE

$$gz_1 + P_1 v_1 + u_1 + \frac{1}{2} C_1^2 + q = gz_2 + P_2 v_2 + u_2 + \frac{1}{2} C_2^2 + w$$

$\Delta z \simeq 0$, $\Delta C \simeq 0$, $w \simeq 0$

$\therefore h_1 + q = h_2 \Rightarrow q = h_2 - h_1$

$Q = m(h_2 - h_1) = m C_p (T_2 - T_1)$

$C_p = \gamma \cdot C_v = 1.26 * 1.229 = 1.548$ kJ/kg.K

$\therefore Q = 1 * 1.548 * (250 - 280)$

$= -46.44$ kJ (rejected)

$= 46.44$ kJ rejected

Ex:- Five kilogram of oxygen are heated from 250 to 400K at constant pressure. Calculate the change of enthalpy, change in internal energy, heat transferred and work done. $\gamma = 1.4$

Sol:-

a non-flow energy equation

$$Q = W + \Delta U$$

$$W = \int P\,dV = P(V_2 - V_1) \quad , \quad P = C \text{ (given)}$$

$$Q = (P_2V_2 - P_1V_1) + (U_2 - U_1)$$

$$Q = (P_2V_2 + U_2) - (P_1V_1 + U_1)$$

$$= H_2 - H_1 = mC_p(T_2 - T_1) \quad \text{for } P = C \text{ only}$$

for O_2:

$$M = 32 \quad , \quad \gamma = 1.4$$

$$R = R_o/M = 8.314/32 = 0.2598 \text{ kJ/kg.K}$$

$$C_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.2598}{1.4 - 1} = 0.9093 \text{ kJ/kg.K}$$

$$\Delta H = 5 * 0.9093 * (400 - 250) = 692 \text{ kJ}$$

$$\Delta U = mC_v(T_2 - T_1)$$

$$C_v = C_p - R = 0.9093 - 0.2598 = 0.6495 \ \frac{\text{kJ}}{\text{kg.K}}$$

$$\Delta U = 5 * 0.6495 * (400 - 250) =$$

$$Q = W + \Delta U \quad , \quad \text{at } P = C \Rightarrow Q = \Delta H$$

$$\therefore W = Q - \Delta U$$

$$= 682 - 487 = 195 \text{ kJ} \quad \text{(work done by the System)}$$

Ex:- An Oxygen cylinder has a capacity of 300L, and contains O_2 at 3.1 Mpa and 18°C. The valve is open and some gas is used. If the pressure and temperature of the oxygen fall to 1.7 Mpa and 15°C respectively. Find the mass of oxygen used. If after the valve is closed the Oxygen remain in the Cylinder is gradualy attains its initial temperature of 18°C. Find the amount of heat transfer through the Cylinder wall. Oxygen density at 0°C and 0.1013 Mpa is 1.429 $\frac{kg}{m^3}$ and $\gamma = 1.4$.

Sol:- $P_1V_1 = mRT_1 \Rightarrow m_1 = \frac{P_1V_1}{RT_1}$

$$R(O_2) = 8.314/32 = 0.26 \text{ kJ/kg.K}$$

$$\text{initial mass} = m_1 = \frac{3.1*10^3 * 300*10^{-3}}{0.26*(18+273)} = 12.29 \text{ kg}$$

$$\text{final mass} = m_2 = \frac{1.7*10^3 \times 300*10^{-3}}{0.26*(15+273)} = 6.8 \text{ kg}$$

$$\text{mass used} = m_1 - m_2 = 12.29 - 6.8 = 5.49 \text{ kg}$$

$$Q = W + \Delta U \quad , V = C \Rightarrow W = 0$$

$$Q = mC_v(T_2 - T_1)$$

$$C_v = R/\gamma - 1 = 0.26/1.4-1 = 0.65 \text{ kJ/kg.K}$$

$$Q = 6.8 * 0.65 * (18-15) = 13.26 \text{ kJ}$$

(Heat gain)

(Sheet No. 2)

Q1:- The molecular weight of Carbon dioxide CO_2 is 44. In an experment the value of γ was found to be 1.3. Assuming that CO_2 is a perfect gas. Calculate the gas constant R and the Specific heats at constant P&V. Ans.[0.189, 0.63 and 0.819 kJ/kg.K]

Q2:- Calculate the internal energy and enthalpy of 1kg of air occupying 0.05 m^3 at 20bar. If the internal energy is increased by 120 kJ/kg as the air compressed to 50 bar. Calculate the new volume occupied by 1 kg of air. Ans.[250.1, 350.1 kJ/kg, 0.0296 m^3]

$R_{air} = 0.287$

Q3:- When a certine perfect gas is heated at constant pressure from 15°C to 95°C, the heat required is 1130 kJ/kg. While when the same gas is heated at constant volume between the same temperature limits above, the heat required is 808 kJ/kg. Calculate C_p, C_v, γ, R and M of the gas.

Ans.[14.2, 10.1 kJ/kg.K, 1.405, 4.1 kJ/kg.K, 2.028]

Q4:- In an air compressor the pressure at inlet and outlet are 1bar & 5bar respectively. The temperature of the air at inlet is 15°C and the volume at

beging of compression is three times that at the end of compression. Calculate the temperature of air at outlet and the increase in internal energy.

Ans. [207°C, 138 kJ/kg]

Q5:- The exhaust gas leaving an oil engine is passed through a heat exchanger which consist of tubes surrounding by water. The gas enters at 327°C and leaves at 193°C. The water enters the exchanger at 17°C and leaves at 56°C. If the mass flow rate of water is 2.54 kg/min. Determine the mass flow rate of gas. Take: $C_{p_g} = 1.04$ kJ/kg.K , $C_{p_w} = 4.186 \frac{kJ}{kg.°C}$

Ans. [2.98 kg/min]

Q6:- A working fluid enters a steady-flow system with velocity 30 m/s and leaves with velocity 140 m/s. The mass flow rate is 9 kg/s. The properties of the fluid are at entry, 13.8 bar, 0.122 m³/kg, internal energy 422 kJ/kg, and exit properties are 1.035 bar, 0.805 m³/kg, internal energy 208 $\frac{kJ}{kg}$. The heat transfer from the system is 4.22 kJ/kg. Determine the work transfer in kW from the system.

Ans. [2565 kW]

Reversible and Irreversible Processes for The Ideal Gas

1. Reversible non-flow Processes:

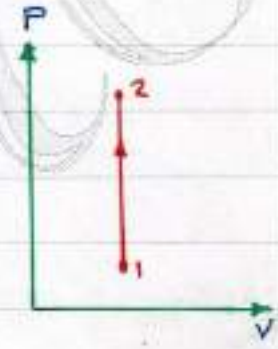
a. Constant volume Process (Isometeric)

from the non-flow E.E.

$$Q = W + \Delta U$$

$$W = \int p\, dV \quad , \quad V = c \Rightarrow dV = 0$$

$$\therefore W = 0 \Rightarrow Q = \Delta U$$



$$\boxed{Q = m C_v (T_2 - T_1)}$$

b. Constant Pressure Process (Isobaric)

from the NFEE

$$Q = W + \Delta U$$

$$W = P_2 V_2 - P_1 V_1$$

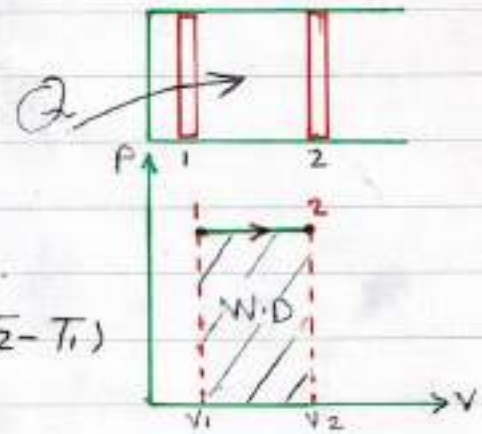
$$= m R (T_2 - T_1)$$

$$\Delta U = m C_v (T_2 - T_1)$$

$$Q = m R (T_2 - T_1) + m C_v (T_2 - T_1)$$

$$= m (R + C_v)(T_2 - T_1)$$

$$= m C_p (T_2 - T_1) = \Delta H = m (h_2 - h_1)$$



$$\boxed{Q = m C_p (T_2 - T_1)}$$

c. Constant temperature process (Isothermal)

from NFEE :

$$Q = W + \Delta U$$

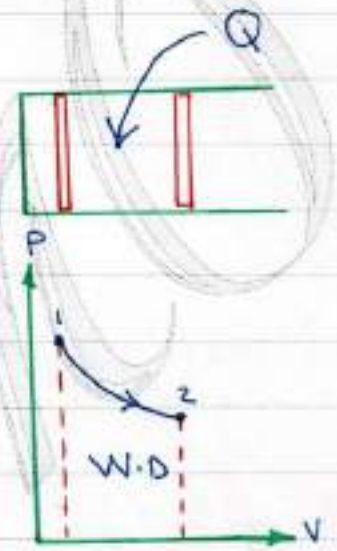
$$w = \int_1^2 p\, dv$$

we have $\frac{P_1 V_1}{T_1} = C$, since $T = C$

$\therefore P_1 V_1 = C \Rightarrow p = \frac{C}{V}$

$$w = \int_1^2 \frac{C}{V}\, dV = C \ln \frac{V_2}{V_1}$$

$$W = PV \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1}$$



$U_2 - U_1 = mC_v(T_2 - T_1)$, $T_2 = T_1 \Rightarrow \Delta U = 0$

$$\boxed{\therefore Q = PV \ln \frac{V_2}{V_1} = PV \ln \frac{P_1}{P_2} = mRT \ln \frac{P_1}{P_2}}$$

Ex: 1 kg of nitrogen (Molecular weight 28) is compressed reversibly and isothermally from 1.01 bar, 20°C to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.

Sol: $R = R_o/M = 8.314/28 = 0.297$ kJ/kg K

$$W = RT \ln \frac{P_1}{P_2} = 0.297 * 293 * \ln \frac{1.01}{4.2} = 124 \frac{kJ}{kg} \text{ (comp.)}$$

$Q = W = -124$ kJ/kg

Heat rejected $= +124$ kJ/kg

d. Adiabatic Process :

In this process the System must be in thermal insulated, to prevent the heat transfer between the System and Surrounding.

To find the Relation between P & V:
from the non-flow E.E

$$Q = W + \Delta U$$

adiabatic process $\Rightarrow Q = 0$

$$dQ = dU + dW \Rightarrow dq = du + dw$$
$$dU = m C_v dT \Rightarrow du = C_v dT$$
$$dW = p\, dV \Rightarrow dw = p\, dv$$

per unit mass

$$\therefore dq = C_v dT + p\, dv$$
$$pv = RT \Rightarrow p = RT/v$$

$$\therefore dq = C_v dT + RT \frac{dv}{v} = 0$$

$$0 = C_v dT + RT \frac{dv}{v}$$
$$0 = C_v \frac{dT}{T} + R \frac{dv}{v}$$

$$0 = \frac{dT}{T} + \frac{R}{C_v} \cdot \frac{dv}{v} \quad , \quad \frac{R}{C_v} = \gamma - 1$$

$$0 = \frac{dT}{T} + (\gamma-1)\frac{dv}{v}$$

$$\int 0 = \int_{T_1}^{T_2} \frac{dT}{T} + (\gamma-1)\int_{v_1}^{v_2} \frac{dv}{v}$$

$$\ln T + (\gamma-1)\ln V = C \quad , \quad T = \frac{Pv}{R}$$

$$\ln\left(\frac{Pv}{R}\right) + (\gamma-1)\ln v = C$$

$$\ln \frac{Pv \cdot v^{\gamma-1}}{R} = C$$

$$\frac{P v^{\gamma}}{R} = e^{C} \Rightarrow P v^{\gamma} = R e^{C} = C$$

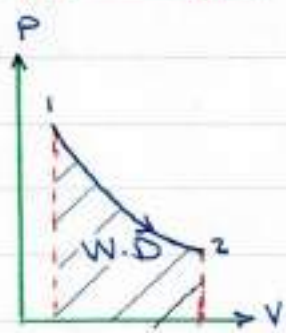
$$\therefore \boxed{PV^{\gamma} = C}$$ adiabatic process

the non-flow energy

$$Q = W + \Delta U \quad , Q = 0 \text{ (adiabatic)}$$

$$W = -\Delta U$$

$$= m C_v (T_1 - T_2) = m\left(\frac{R}{\gamma-1}\right)(T_1 - T_2)$$



$$W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$\boxed{W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}}$$ ~ Adiabatic work

The Relation Between P,V and T in Adiabatic Process:-

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = C$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \cdot \frac{T_1}{T_2} \quad \text{--- (1)}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} \quad \text{--- (2)}$$

Sub. eq (1) in eq (2)

$$\left(\frac{V_2}{V_1}\right)^{\gamma} = \left(\frac{V_2}{V_1}\right) \cdot \left(\frac{T_1}{T_2}\right)$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \text{--- (3)}$$

Sub eq (2) in eq (3) , where $\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$

$$\frac{T_1}{T_2} = \left[\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}\right]^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \boxed{\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}}$$

the relation between P, V and T for adiabatic process

Ex: Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder.

Sol:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_2 = T_1 * \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 295 * \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} = 295 * 6.67^{0.286}$$

$$= 507.5\ K$$

(Where $T_1 = 22 + 273 = 295$, $\gamma = 1.4$)

Final temperature $= 507.5 - 273 = 234.5\ °C$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} \quad \text{or} \quad \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

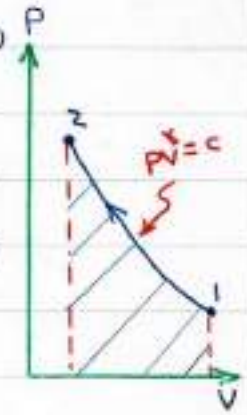
$$\therefore \frac{0.015}{V_2} = \left(\frac{6.8}{1.02}\right)^{1/1.4} \Rightarrow V_2 = 0.00388\ m^3$$

$W = U_1 - U_2$ (adiabatic process)

$= C_v(T_1 - T_2)$ (Per kg of gas)

$= 0.718(295 - 507.5) = -152.8$ kJ/kg

$\therefore$ Work input per kg $= 152.8$ kJ/kg



$$m = \frac{P_1 V_1}{R T_1} = \frac{1.02 * 10^5 * 0.015}{0.287 * 10^3 * 295} = 0.0181\ kg$$

Total work done $= 0.0181 * 152.8 = 2.76$ kJ

e. Polytropic Processes:

This general case of expansion or compression of a substance according to the law $PV^n = C$. It is called a polytropic expansion or compression or a polytrope.

It is should be noted that the value of the constant "C" will change with each change of condition. So also will the value of "n" which is called the "index" of the expansion or compression or the polytropic exponent.

$$PV^n = C$$

The non-flow energy equation

$$Q = W + \Delta U$$

$$W = \int P\,dV = C \cdot \int \frac{dV}{V^n} = PV^n \int (V^{-n})\,dV$$

$$W = PV^n \left[\frac{V^{-n+1}}{-n+1} \right]_1^2 = \frac{P_2 V_2^n \cdot V_2^{-n+1} - P_1 V_1^n \cdot V_1^{-n+1}}{-n+1}$$

$$W = \frac{P_2 V_2 - P_1 V_1}{-n+1}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$W = \frac{mR(T_1 - T_2)}{n-1}$$

$$\Delta U = m C_v (T_2 - T_1) = -m C_v (T_1 - T_2)$$

$$\Delta U = -m(R/\gamma - 1)(T_1 - T_2)$$

$$\therefore \quad Q = \frac{mR(T_1 - T_2)}{n-1} - m\frac{R}{\gamma - 1}(T_1 - T_2)$$

$$= mR(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma - 1}\right)$$

$$= mR(T_1 - T_2)\left(\frac{\gamma - 1 - n + 1}{(n-1)(\gamma - 1)}\right)$$

$$= mR(T_1 - T_2)\left(\frac{\gamma - n}{(n-1)(\gamma - 1)}\right)$$

$$= (mR(T_1 - T_2)/n-1)(\gamma - n/\gamma - 1)$$

$$\boxed{\therefore \quad Q = \left(\frac{\gamma - n}{\gamma - 1}\right) \cdot W}$$

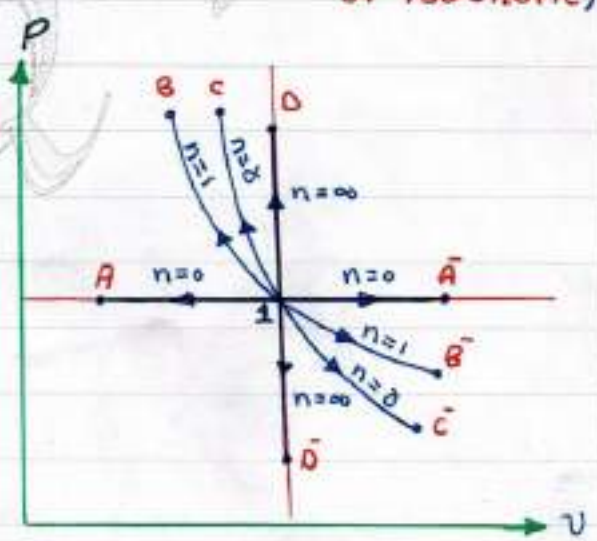
In a polytropic process the value of (n) depends on the a mount of heat added or rejected and on the work done during the process

$$PV^n = C$$

When:

1. $n=0 \rightarrow PV^0=C \rightarrow P=C$ (isobaric)
2. $n=1 \rightarrow PV^1=C \rightarrow PV=C$ (isothermal $T=C$)
3. $n=\gamma \rightarrow PV^\gamma=C \rightarrow Q=0$ (adiabatic)
4. $n=\alpha \rightarrow PV^\alpha=C \rightarrow P^{\frac{1}{\alpha}}.V=C \rightarrow V=C$ (isometeric or isochoric)

- Processes :



$1 \rightarrow A$ Cooling at $P=C$
$1 \rightarrow \bar{A}$ Heating at $P=C$

$1 \rightarrow \bar{B}$ Expansion at $T=C$
$1 \rightarrow B$ Compression at $T=C$

$1 \rightarrow C$ Adiabatic compression ($PV^\gamma=C$)
$1 \rightarrow \bar{C}$ " Expansion

$1 \rightarrow D$ Heating at $V=C$
$1 \rightarrow \bar{D}$ Cooling " "

- Work and the hyperbolic process :-

The hyperbolic process is a particular case of the polytropic process, ($PV^n = C$), being the case When ($n = 1$). Thus, the law for the hyperbolic process is,

$$\boxed{PV = C}$$

For ahyperbolic change from state 1 to state 2.

$$P_1 V_1 = P_2 V_2$$

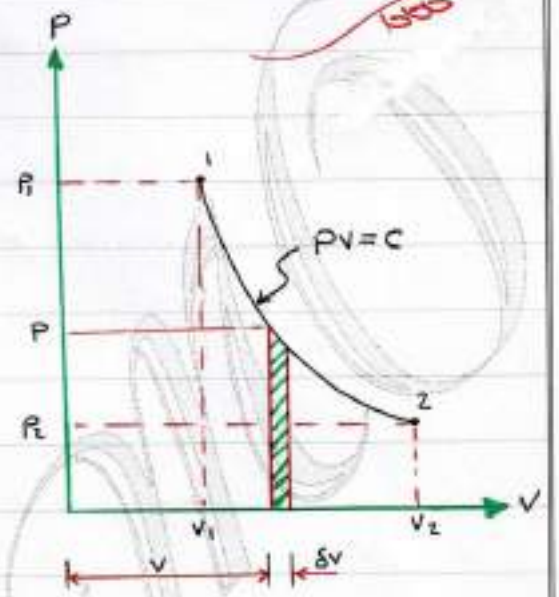
An expression for the work done during a polytropic process has already been determined,

$$\text{Work done} = \frac{P_1V_1 - P_2V_2}{n-1}$$

for hyperbolic process, $P_1V_1 = P_2V_2$ & $n = 1$

$$\therefore \text{Work done} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{0}{0}$$

$\therefore$ to compute the work done. It is necessary to determine the area under curve.



$$\text{work done} = \int_{V_1}^{V_2} P\,dV$$

$$PV = C \Rightarrow P = C/V$$

$$W = C\int_{V_1}^{V_2} \frac{dV}{V} = C\left[\ln V\right]_{V_1}^{V_2}$$

$$W = C\left[\ln V_2 - \ln V_1\right]$$

$$= C \ln \frac{V_2}{V_1} = PV \ln \frac{V_2}{V_1}$$

$$\therefore \quad \boxed{W = PV \ln \frac{V_2}{V_1}}$$

Ex:

A gas is compressed hyperbolically from a pressure and volume of 100 kN/m² and 0.056 m³, respectively. to a volume of 0.007 m³. Determine the final pressure and the work done on the gas.

Sol: since the gas is compressed hyperbolically, then

$$P_1V_1 = P_2V_2 \quad \text{or} \quad P_2 = 100 * \frac{0.056}{0.007} = 800 \text{ kPa}$$

$$\text{work done} = P_1V_1 \ln \frac{V_2}{V_1} = 100 \times 10^3 \times 0.056 \times \ln \frac{0.007}{0.056}$$

$$= -11.65 \text{ kJ}$$

work done on the gas = 11.65 kJ

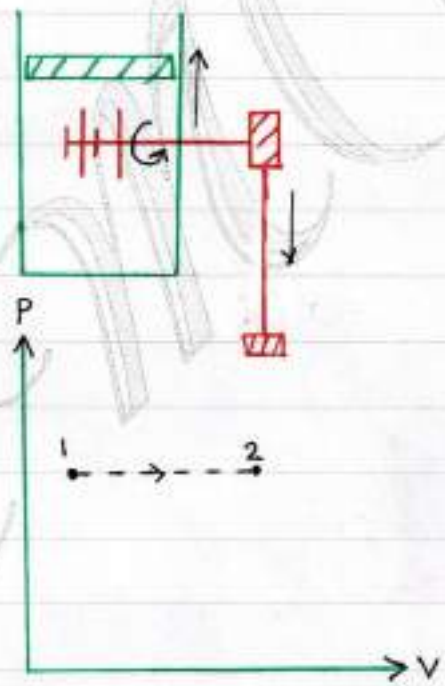
2. Irreversible Processes (Closed System)

a. Paddle work (P=C)

W_p = Paddle work

from NFEE

$$Q = \Delta U + W + W_p$$



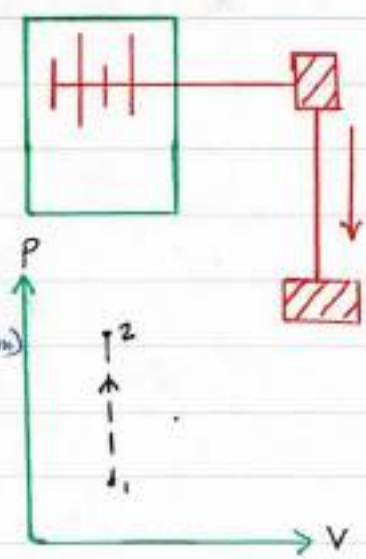
Q may be equal to zero if the process is very fast or insulated.

$$\Delta U = mC_v(T_2 - T_1)$$

$$W \equiv P(V_2 - V_1) = mR(T_2 - T_1)$$

W_p = -ve since it is done on the System.

b. Paddle work (V=C)



$$Q = W + \Delta U + W_p$$

$W = 0$ (V=C) , $Q = 0$

$$\Delta U = mC_v(T_2 - T_1)$$

$W_p \equiv$ -ve (done on the System)

3. Reversible Processes (Open System)

a. Constant Pressure Process: (such as condenser)

$$gz_1 + P_1V_1 + U_1 + \frac{1}{2}C_1^2 + q = gz_2 + P_2V_2 + U_2 + \frac{1}{2}C_2^2 + w$$

$$P_1 = P_2$$

$$w - q = (P_1V_1 - P_2V_2) + (U_1 - U_2) + \frac{1}{2}(C_1^2 - C_2^2)*10^{-3} + g(Z_1 - Z_2)*10^{-3}$$

or

$$\dot{W} - \dot{Q} = \dot{m}[w - q]$$

b. Constant temperature process:

$P = \text{Constant}$

$T = C$, $P_1V_1 = P_2V_2$, $U_1 = U_2$

$$\therefore W = q + \left[g(Z_1 - Z_2) + \frac{1}{2}(C_1^2 - C_2^2)\right]*10^{-3}$$

or $$w = \int_1^2 P\,dv = Pv \ln\frac{V_2}{V_1} = mRT \ln\frac{V_2}{V_1}$$

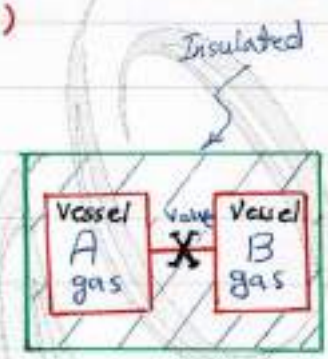
c. Polytropic Process:

from Steady flow Energy Equation SFEE.

$$W - q = (P_1V_1 - P_2V_2) + (U_1 - U_2) + \frac{1}{2}(C_1^2 - C_2^2)*10^{-3} + g(Z_1 - Z_2)*10^{-3}$$

4. Irreversible Processes (Open System)

a. Unresisted, or free, expansion :



$$Q = W + \Delta U$$

$Q = 0$ (adiabatic) , $W = 0$

$\therefore \Delta U = 0 \Rightarrow u_2 = u_1$ (Per 1 kg)

$$C_v T_2 = C_v T_1$$

$\therefore T_1 = T_2$ (Irreversible)

Ex: Air at 20 bar is initially contained in vessel "A" as shown in the fig. above, the volume of which can be assumed to be $1 m^3$. The valve X is opened and the air expands to fill vessels A and B. Assuming that the vessels are of equal volume, Calculate the final pressure of the air.

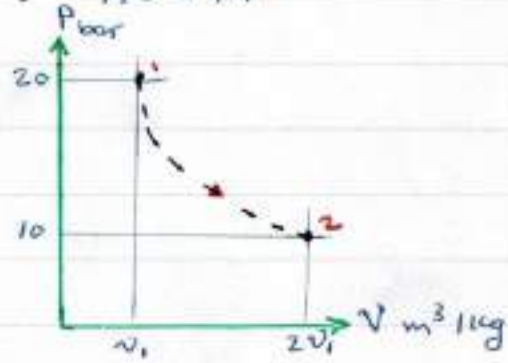
Sol:

$\because$ Free expansion $\Rightarrow T_1 = T_2$

for a perfect gas $pV = mRT$, $P_1 V_1 = P_2 V_2$

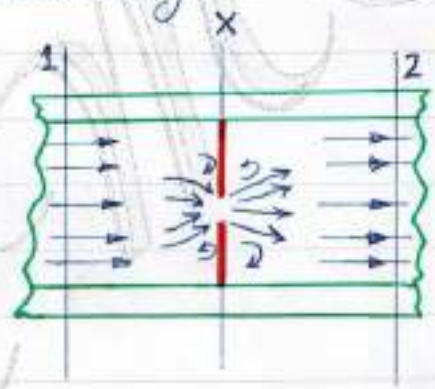
$V_2 = V_A + V_B = 1 + 1 = 2 m^3$, $V_1 = 1 m^3$

$$P_2 = P_1 \cdot \frac{V_1}{V_2} = 20 * \frac{1}{2} = 10 \text{ bar}$$



b. Throttling Process :

A flow of fluid is said to be throttled when there is som restriction to the flow. When the velocities before and after the restriction are either equal or negligibly small, and when there is a negligible heat loss to the surroundings.



from SFEE :

$$gz_1 + P_1v_1 + u_1 + \frac{1}{2}C_1^2 + q = gz_2 + P_2v_2 + u_2 + \frac{1}{2}C_2^2 + w$$

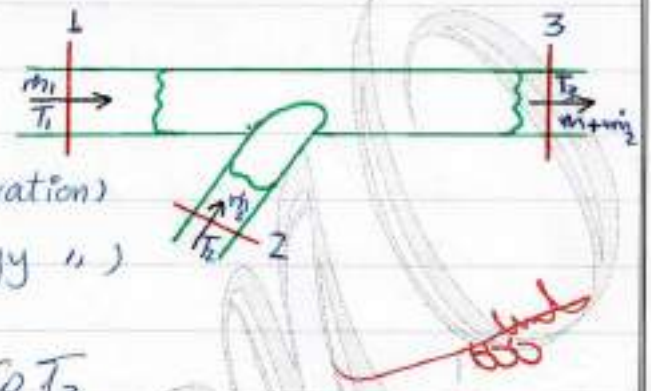
$z_1 = z_2$, $w = 0$, $Q = 0$, $C_1 \simeq C_2$

$$\therefore \quad P_1v_1 + u_1 = P_2v_2 + u_2$$

$$h_1 = h_2 \Rightarrow C_p T_1 = C_p T_2$$

$$\therefore T_1 = T_2 \quad \text{(Irreversible)}$$

c. Adiabatic mixing :



$m_1 + m_2 = m_3$ (mass conservation)

$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$ (energy ")

$$\dot{m}_1 C_p T_1 + \dot{m}_2 C_p T_2 = \dot{m}_3 C_p T_3$$

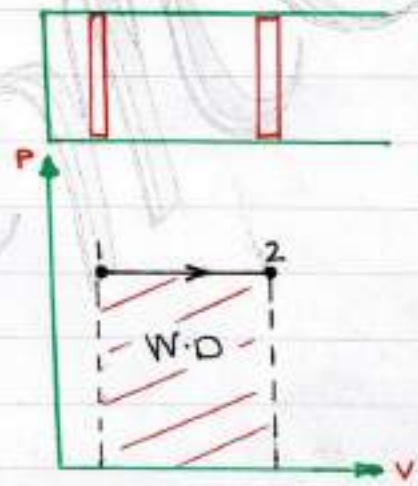
for the same gases mixing C_p = Constant

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 = \dot{m}_3 T_3$$

$$\boxed{\dot{m}_1 T_1 + \dot{m}_2 T_2 = (\dot{m}_1 + \dot{m}_2) T_3}$$

Ex: 0.05 kg of air is heated at constant pressure of 2bar until the volume occupied is $0.0658\ m^3$. Calculate the heat supplied and work done, when the initial temperature is 130°C. Take $C_p = 1.005$ kJ/kg.K, $R = 0.287$ kJ/kg.K

Sol: the non-flow energy equation



$$Q = W + \Delta U$$

$$W = \int_1^2 P \cdot dV = P(V_2 - V_1)$$

$$= mR(T_2 - T_1)$$

$$\Delta U = (U_2 - U_1) = mC_v(T_2 - T_1)$$

$$\therefore Q = mC_p(T_2 - T_1)$$

$$P_2V_2 = mRT_2 \Rightarrow T_2 = P_2V_2/mR$$

$$T_2 = 2 * 10^2 * 0.0658 / 0.05 * 0.287$$

$$= 912\ K$$

$$T_1 = 130 + 273 = 403\ K$$

$$Q = 0.05 * 1.005 * (912 - 403)$$

$$= 25.6\ kJ$$

$$W = mR(T_2 - T_1) = 0.05 * 0.287 * (912 - 403)$$

$$= 7.3\ kJ$$

OR $W = P(V_2 - V_1)$, $V_1 = mRT_1/P_1 = 0.0289\ m^3$

$$\therefore W = 2 * 10^2 (0.0658 - 0.0289) = 7.38\ kJ$$

$$\Delta U = mC_v(T_2 - T_1) = 0.05 * (1.005 - 0.287)(912 - 403)$$

$$= 18.27\ kJ$$

$$Q = W + \Delta U = 7.38 + 18.27 = 25.65\ kJ$$

Ex: A constant pressure adiabatic System contain 0.13 kg of air at 1.3 bar. The System recives paddle work. The temperature of air rises from 29 to 185°C. Find the total work. mechanical work, change in internal energy and enthalpy. Take $R = 0.287$, $\gamma = 1.4$.

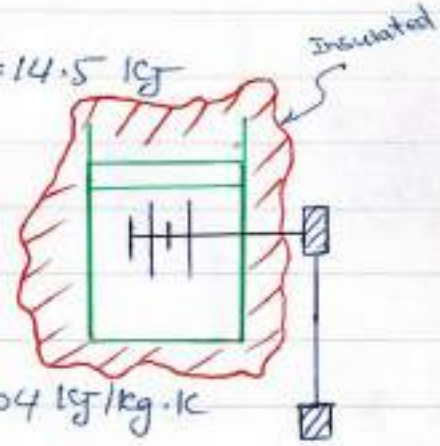
Sol: $Q = W + \Delta U + W_p$, $Q = 0$ (adiabatic)

$W = mR(T_2 - T_1) = 0.13 * 0.287 * (185 - 29) = 5.82$ kJ

$\Delta U = m C_v (T_2 - T_1)$

$C_v = \frac{R}{\gamma - 1} = \frac{0.287}{1.4 - 1} = 0.7175$

$\Delta U = 0.13 * 0.7175 * (185 - 29) = 14.5$ kJ



$Q = W + \Delta U + W_p$

$0 = 5.82 + 14.5 + W_p$

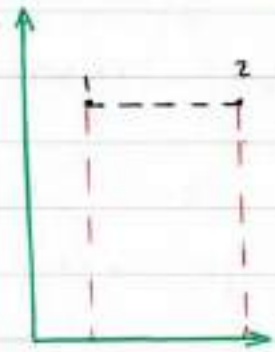
$W_p = -20.32$ kJ

$C_p = \gamma R / \gamma - 1 = \frac{1.4 * 0.287}{1.4 - 1} = 1.004$ kJ/kg.K

$\Delta H = m C_p (T_2 - T_1)$

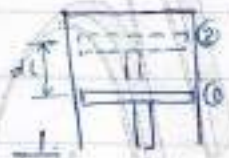
$= 0.13 * 1.004 * (185 - 29)$

$= 20.3$ kJ



Ex: Air at 1.02 bar and 22°C initialy occuping a cylinder of 0.015 m^3, is compressed reversibly and adiabaticaly by a piston to a pressure of 6.8 bar. Calculate the final volume, the final temperature and the work done on the mass of air in the Cylinder.

Sol: $Q = 0$ (adiabatic)



$\therefore \quad W = -\Delta U$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \cdot V_1 = \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} * 0.015$

$= 3.86 * 10^{-3} \ m^3$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow$

$T_2 = \left(\frac{6.8}{1.02}\right)^{\frac{0.4}{1.4}} * (22+273)$

$= 507.25 \ K$

$\Delta U = m C_v (T_2 - T_1)$

$m = P_1 V_1 / RT = 1.02 * 10^2 * 0.015 / 0.287 * 295 = 0.018 \ kg$

$\Delta U = 0.01807 * 0.717 \ (507.25 - 295) = 2.749 \ kJ$

$W = -\Delta U = -2.749 \ kJ$

(Sheet No. 3)

Q1: Oxygen (molecular weight 32) expands reversibly in a Cylinder behind a piston at constant pressure of 3 bar. The initial volume is $0.01\ m^3$ and finaly is $0.03\ m^3$, the initial temperature is $17\ ^\circ C$. Calculate the work done by the Oxygen and the Heat flow to or from the Cylinder walls during the expansion. Take $C_p = 0.917\ kJ/kg \cdot K$.

Ans. [6 kJ , 21.25 kJ]

Q2: $0.05\ m^3$ of a perfect gas at 6.3 bar undergoes a reversible isothermal process to a pressure of 1.05 bar. Calculate the heat flow to or from the gas.

Ans. [56.4 kJ]

Q3: 1 kg of air at 1 bar, $15\ ^\circ C$ is compressed reversibly adiabatically to a pressure of 4 bar. Calculate the final temperature and the work done on the air.

Ans. [$155\ ^\circ C$, 100.5 kJ/kg]

Q4: 1 kg of air at 1.02 bar, $20\ ^\circ C$ is compressed reversibly according to a Law $PV^{1.3} = constant$, to a pressure of 5.5 bar. Calculate the work done on the air and the heat flow during the compression.

Ans. [$133.5\ \frac{kJ}{kg}$, $-33.38\ \frac{kJ}{kg}$]

Q5:- Nitrogen (molecular weight 28) expands reversibly in a perfectly thermally insulated Cylinder from 3.5 bar, 200°C to a volume of $0.09 m^3$. If the initial volume occupied was $0.03 m^3$, Calculate the work done during the expansion. Assume nitrogen to be aperfect gas and take $C_v = 0.741$ kJ/kg·K Ans. [9.31 kJ]

Q6: 0.05 kg of Carbon dioxide ($M = 44$), occupying a volume of $0.03 m^3$ at 1.025 bar, is compressed reversibly until the pressure is 6.15 bar. Calculate the final temperature, the work done on the CO_2, and the heat flow to or from the Cylinder walls.

a. When the process is according to a law $PV^{1.4} = C$.

b. " " " " isothermal.

c. " " " takes place in a perfectly thermally insulated Cylinder. $\gamma = 1.3$

Ans. [270°C, 5.138 kJ, 1.713 kJ, 52.6°C, 5.51 kJ, -5.51 kJ, 219°C, 5.25 kJ, 0 kJ]

Q7: 225 kg/h of air at 40°c enter amixing chamber where it mixes with 540 kg/h of air at 15°c. Calculate the temperature of the air leaving the chamber, assuming steady flow conditions. Assume that the heat loss is negligible.

Ans. [22.4°C]

Q8: Aquantity of gas occupying $0.14 m^3$ at a pressure of $1.4 MN/m^2$ and temperature of 300°C is expanded adiabatically to $280 kN/m^2$. $C_v = 0.74 kJ/kg.K$, $C_p = 1.04 kJ/kg.K$, determine:

a. the mass of gas
b. the temperature of the gas after expansion.
c. the work done during the expansion.

Ans. [a. 1.14 kg b. 87°C c. 180 kJ]

Process	P,V,T	Work(W)	Heat(Q)	Internal Energy (Δu)
Isometric V=C	$P_1/P_2=T_1/T_2$	0	$mC_v(T_2-T_1)$	$mC_v(T_2-T_1)$
Isobaric P=C	$V_1/V_2=T_1/T_2$	$P(V_2-V_1)$	$mC_p(T_2-T_1)$	$mC_v(T_2-T_1)$
Isothermal T=C	$P_1V_1=P_2V_2$	$PV.Ln(V_2/V_1)$	$PV.Ln(V_2/V_1)$	0
Adiabatic $Pv^{\gamma}=C$	$(T_1/T_2)=(V_2/V_1)^{(\gamma-1)}$ $=(P_1/P_2)^{(\gamma-1)/\gamma}$	$(P_1V_1-P_2V_2)/\gamma-1$ OR $mR(T_1-T_2))/\gamma-1$	0	$mC_v(T_2-T_1)$
Polytropic $Pv^n=C$	$(T_1/T_2)=(V_2/V_1)^{(n-1)}$ $=(P_1/P_2)^{(n-1)/n}$	$(P_1V_1-P_2V_2)/n-1$ OR $mR(T_1-T_2))/n-1$	$mC_v(\gamma-n/1-n)(T_2-T_1)$	$mC_v(T_2-T_1)$

THERMODYNAMICS

3

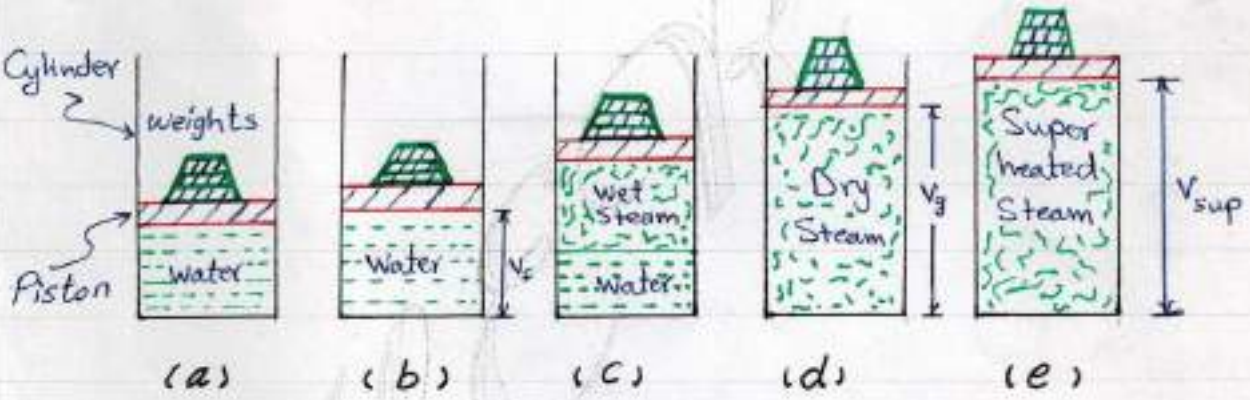
CHAPTER THREE

Steam and Two-Phase System

(Steam and Two-Phase System)

- **Steam**: is a vapour of water, and is invisible when pure and dry. It is used as the working substance in the operation of steam engines and Steam turbines. Steam does not obey laws of perfect gases, until it is perfectly dry.

- Formation of Steam at Constant Pressure from water:



Consider 1 kg of water at 0°C contained in the piston-cylinder arrangement as shown in the figure (a).

The piston and weights maintain a constant pressure in the Cylinder. If we heat the water contained in the cylinder, it will be converted into Steam as below:

1. The volume of water will increase slightly with the increase in temperature as shown in fig (b). It will cause the piston to move slightly upwards and hence work is obtained. This increase in volume of water (or work) is generally neglected for all types of calculation.

2. On further heating, temperature reaches boiling point. The boiling point of water, at normal atmospheric pressure of 1.013 bar is 100°C, but it increases with the increase in pressure. When the boiling point is reached, the temperature remains constant and the water evaporates, thus pushing the piston up against the constant pressure. Consequently, the Specific volume of Steam increase as shown in fig (c). At this stage, the steam will have some particles of water in suspension, and is termed as «Wet Steam». This process will continue till the whole water is converted into wet Steam.

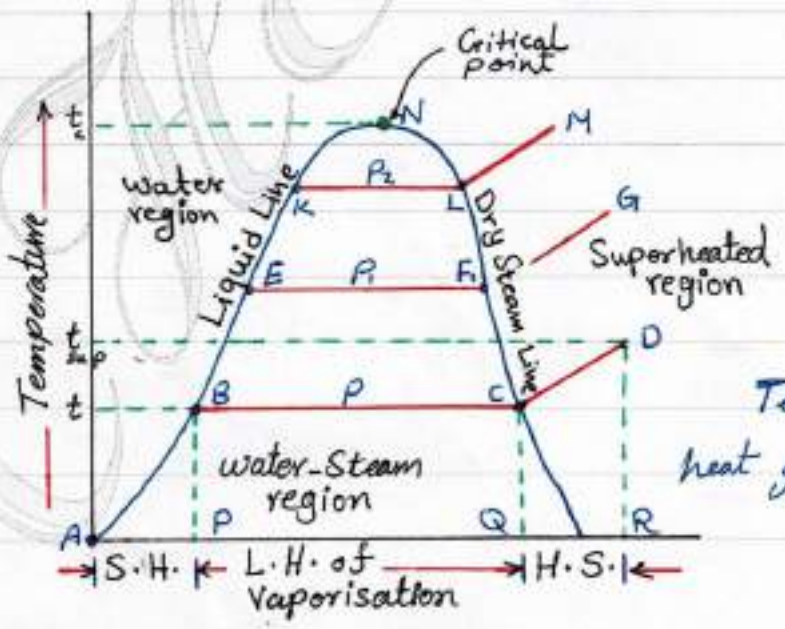
3. On further heating, the particles in suspension will be converted into Steam. The entire Steam, in such a state, is termed as **dry Steam** or Saturated Steam as shown in fig (d). Practically

the dry Steam behaves like a perfect gas.

4. On further heating, the temperature of the Steam Starts rising. The steam, in such a state is termed as Superheated Steam as Shown in fig. (e).

- Temperature vs. Total Heat Graph during Steam Formation:

The process of Steam formation may be represented on a graph, whose abscissa represents the total heat and the vertical ordinate represents the temperature. The point (A) represents the initial condition of water at 0°C and pressure p (in bars) as shown in figure below. Line ABCD shows the relation between temperature and heat at a specific pressure of p (in bars).



Temperature - total heat graph during Steam formation

During the formation of the superheated steam, from water at freezing point, the heat is absorbed in the following three stages:

1. The heating of water up to boiling temperature or saturation temperature (t) is shown by AB. The heat absorbed by the water is AP, known as Sensible heat (S.H.) or Liquid heat or total heat of water.

2. The change of state from Liquid to Steam is shown by BC. The heat absorbed during this stage is PQ, known as Latent heat (L.H.) of vaporisation.

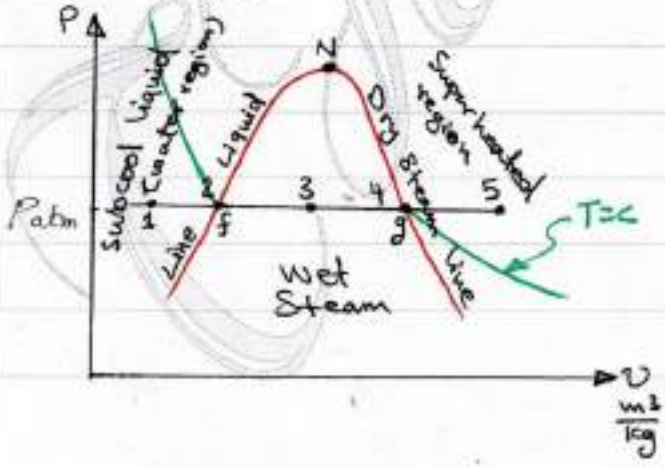
3. The Superheating process is shown by CD. The heat absorbed during this stage is QR, known as heat of Superheat (H.S.). Line AR represents the total heat of the Superheated Steam.

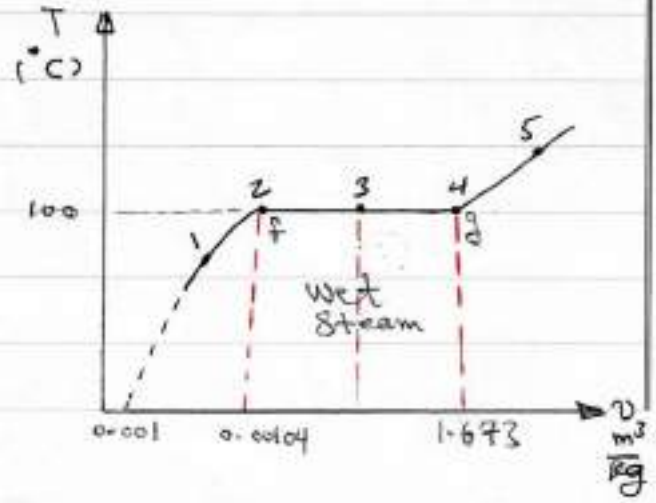
If the pressure is increased (Say P_1 bar), the boiling temperature also increas. The point (E) represents the boiling temperature or saturation temperature at pressure (P_1) and (F) is the point of dry saturated steam. Line (FG) shows the constant pressure process, in which the steam is superheated.

The line passing through the points A, B, E, K is known as «Saturated Liquid Line» which forms boundary line between water and steam. Similarly, a line passing through dry steam points L, F, C is known as «dry Saturated Steam Line» which forms boundary line between wet and superheated steam.

It may also be noted from the figure, that when the pressure and saturation temperature increases, the latent heat of vaporisation decreases. It becomes zero at point «N» where liquid and dry steam lines meet.

This point «N» is known as the critical point and at this point, the liquid and vapour phases merge. The temperature corresponding to critical point «N» is known as critical temperature and the pressure is known as critical pressure. For steam, the critical temperature is 374.15 °C and critical pressure is 221.2 bar.





Important Terms for Steam :

1. **Wet Steam :** When the steam contains moisture or particles of water in suspension, it is said to be **wet Steam**. It means that the evaporation of water is not complete, and the whole of the latent heat has not been absorbed.

2. **Dry Saturated Steam :** When the wet steam is further heated, and it does not contain any suspended particles of water, it is known as **dry saturated Steam**. The dry saturated steam has absorbed its full latent heat and behaves practically, in the same way as a perfect gas.

3. **Superheated Steam :** When the dry steam is further heated at a constant pressure, thus raising its temperature, it is said to be **superheated Steam**. Since the pressure is constant, therefore the volume of superheated Steam increases.

4. **Dryness fraction or quality of wet Steam :**
It is the ratio of the mass of actual dry Steam to the mass of same quantity of wet Steam, and generally denoted by 'x'.

Mathematically. $x = \frac{m_g}{m_g + m_f} = \frac{m_g}{m}$

m_g = Mass of actual dry steam.
m_f = Mass of water in suspension, and
m = Mass of wet Steam.

5. **Sensible heat of water:** It is the amount of heat absorbed by 1 kg of water, when heated at a constant pressure, from the freezing point (0) to the temperature of formation of steam. The Sensible heat is also known as liquid heat.

The specific heat of water at constant pressure is usually taken as 4.2 kJ/kg.K.

$$Q_{sensible} = \text{mass} * \text{Specific heat} * \text{Rise in temp.}$$
$$= m C_p (T_2 - T_1)$$
$$= m(h_2 - h_1)$$
$$= H_2 - H_1$$

h_2, h_1 ≡ Enthalpy of fluid (water), denoted by h_f

6. **Latent heat of vaporisation:** It is the amount of heat absorbed to evaporate 1 kg of water, at its boiling point or saturation temperature without change of temperature. It is denoted by h_{fg} and its value depends upon the pressure.

The heat of vaporisation of water or latent heat of Steam is 2257 kJ/kg at atmospheric pressure.

It has been experimentally found that the value of h_{fg} decreases as the pressure increases and it is **zero** at critical pressure.

If the Steam is wet with a dryness fraction x, then the heat absorbed by it during evaporation is $x h_{fg}$.

7. Enthalpy or total heat of Steam:

It is amount of heat absorbed by water from freezing point to saturation temperature **plus** the heat absorbed during evaporation.

∴ Enthalpy or total heat of Steam

= Sensible heat + Latent heat

It is denoted by **h_g** and its value for the dry Saturated Steam may be read directly from the Steam tables. The expressions for the enthalpy of wet Steam, dry Steam and superheated Steam are as follows:

i. Wet steam: the enthalpy of wet Steam is given by:

$$\boxed{h = h_f + x h_{fg}}$$

$h_f = u_f + p v_f$

ii. **Dry Steam:** We know that in case of dry Steam, $x = 1$

$$\therefore \quad h = h_f + 1 * h_{fg}$$

$$= h_f + (h_g - h_f)$$

$$\boxed{h = h_g}$$

iii. **Superheated Steam:** If we further add heat to the dry Steam, its temperature increases while pressure remaining constant. This increase in temperature shows the superheat stage of the Steam. Thus, the total heat required for the Steam to be Superheated is:

$$h_{sup} = \text{Total heat for dry Steam} + \text{Heat for Superheated Steam}$$

$$= h_f + h_{fg} + C_p(t_{sup} - t) = h_g + C_p(t_{sup} - t)$$

Where:

C_p = Specific heat at constant pressure for Superheated Steam.

t_{sup} = Temperature of the Superheated Steam.

t = Saturation temperature at the given constant pressure.

Notes:

1. The difference $(t_{sup} - t)$ is known as degree of Superheated
2. The value of C_p for Steam lies between 1.67 kJ/kg.K to 2.5 kJ/kg.K.

8. **Specific volume of Steam:** It is the volume occupied by the Steam per unit mass at a given temperature and pressure, and is expressed in m^3/kg. It is the reciprocal of density of Steam in kg/m^3.

The value of Specific volume decreases with the increase in pressure.

The expressions for the volumes occupied by Steam are as follows:

i. **Wet steam:** Consider 1kg of wet Steam of dryness fraction x. When know that this Steam will have x kg of dry Steam and $(1-x)$ kg of water. Let v_f be the volume of 1 kg of water, then

Volume of one kg of wet Steam $= x v_g + (1-x) v_f$

Since v_f is very small as compared to v_g, therefore the expression $(1-x) v_f$ may be neglected

$\therefore$ Volume of one kg of wet steam $= x \cdot v_g \quad m^3$

or Specific volume of wet Steam, $\quad v = x \cdot v_g \quad m^3/kg$

ii. Dry Steam: We know that in case of dry steam, the mass of water in suspension is zero and dryness fraction is unity. Therefore Specific volume of dry Steam $= v_g$ m^3/kg

iii. Superheated Steam: We have already discussed that when the dry saturated Steam is further heated under a constant pressure, there is an increase in volume with the rise in temperature. The superheated Steam behaves more or Less Like a perfect gas. Therefore, according to Charles' law.

$$\frac{v_{sup}}{T_{sup}} = \frac{v_g}{T} \quad \text{or} \quad \boxed{v_{sup} = \frac{v_g * T_{sup}}{T}}$$

Where:

v_{sup} = Specific volume of Superheated Steam.
v_g = Specific volume of dry Steam at the pressure of Steam formation.
$T_{sup.}$ = Absolute temperature of Superheated Steam
T = " Saturation temperature at the pressure of Steam formation.

Ex: Calculate the enthalpy of 1 kg of Steam at a pressure of 8 bar and dryness fraction of 0.8. How much heat would be required to raise 2 kg of this Steam from water at 20°c?

Sol: From Steam tables, at $P = 8$ bar

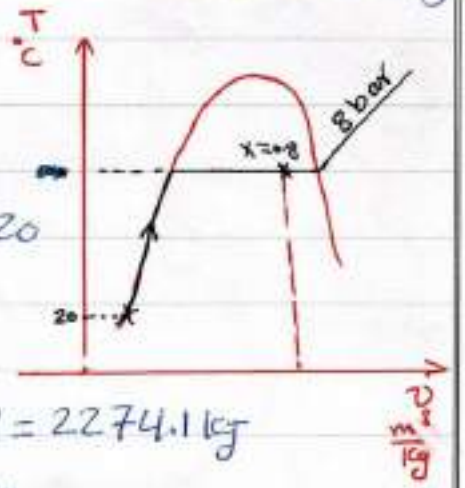
$h_f = 720.9$ kJ/kg , $h_{fg} = 2046.5$ kJ/kg

$\therefore h = h_f + x h_{fg}$

$= 720.9 + 0.8 * 2046.5 = 2358.1$ kJ

Heat required to raise 2kg of this Steam from water at 20°c.

We have calculated above the enthalpy or total heat required to raise 1kg of Steam from water at 0°C. Since the water, in this case, is already at 20°c, therefore



Heat already in water $= m C_p T$

$= 1 * 4.2 * 20$

$= 84$ kJ

Heat required per 1 kg of Steam

$= 2358.1 - 84 = 2274.1$ kJ

and Heat required for 2kg of Steam

$= 2 * 2274.1$

$= 4548.2$ kJ

Ex:- Determine the quantity of heat required to produce 1kg of steam at a pressure of 6 bar at a temperature of 25°C, under the following conditions:

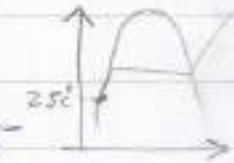
1. When the steam is wet having a dryness fraction 0.9
2. When the steam is dry saturated.
3. When it is superheated at a constant pressure at 250°C assuming the mean specific heat of superheated steam to be 2.3 kJ/kg.K.

Sol:- From Steam tables, at $p = 6$ bar

$h_f = 670.4$ kJ/kg.K , $h_{fg} = 2085$ kJ/kg.K

$t = 158.8°C$

1. When the Steam is wet:



$h = h_f + x h_{fg} = 670.4 + 0.9 * 2085$

$= 2546.9$ kJ (for 1 kg)

Since the water is at a temperature of 25°C, therefore enthalpy or Heat already in water $= m C_p T$

$= 1 * 4.2 * 25 = 105$ kJ

$\therefore$ Heat actually required $= 2546.9 - 105$

$= 2441.9$ kJ

2. When the Steam is dry saturated: $h = h_g$, $x = 1$

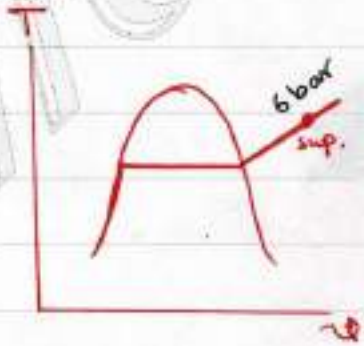
enthalpy or total heat of 1kg of dry saturated Steam, $h = h_f + 1 * h_{fg}$

$\Rightarrow h_g = 670.4 + 2085 = 2755.4$ kJ

Heat actually required = 2755.4 - 105
= 2650.4 kJ

3. When the Steam is superheated,
enthalpy or total heat of 1 kg of Superheated Steam,

$$h_{sup} = h_g + C_p(t_{sup} - t)$$
$$= 2755.4 + 2.3(250 - 158.8)$$
$$= 2965.16 \text{ kJ}$$



Heat actually required = 2965.16 - 105
= 2860.16 kJ

Ex: Determine the condition of Steam in the following cases:

1. At a pressure of 10 bar and temperature 200°C.
2. " " " " " " volume 0.175 m³/kg.

Sol:

1. From Steam tables, at $P = 10$ bar

$v_g = 0.194$ m³/kg , $h_f = 762.6$ kJ/kg.

$t_{sat} = 179.9°C \equiv t_{saturation}$

$\therefore t > t_{sat} \Rightarrow$ the Steam is Superheated

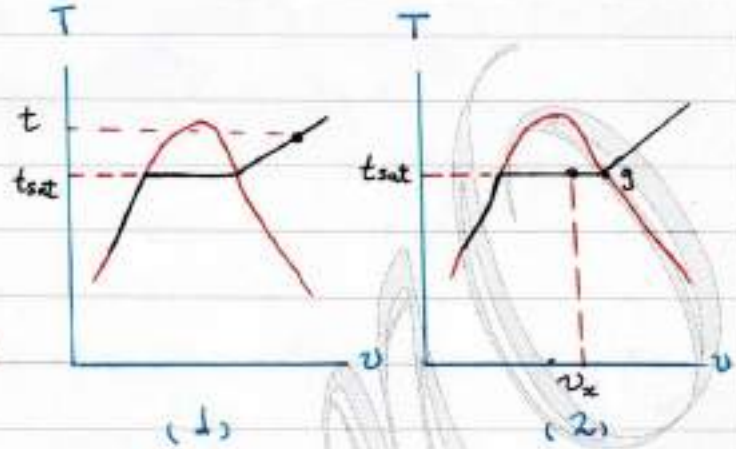
$\therefore$ the degree of Superheated $= t - t_{sat}$

$= 200 - 179.9 = 20.1°C$

2. $\because v < v_g$

$\therefore$ the Steam is wet

$$x = \frac{v}{v_g} = \frac{0.175}{0.194} = 0.902$$



Ex: Steam enters on engine at a pressure of 12 bar with a 67 °C of Superheated. It is exhausted at a pressure of 0.16 bar and 0.95 dry. Find the drop in enthalpy of the Steam. Take $C_{p_{sup}} = 2$ kJ/kg.K

Sol: from Steam tables, at 12 bar

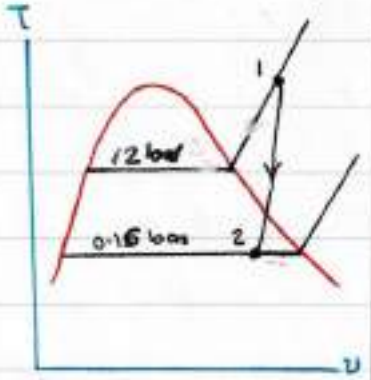
$h_f = 798$ kJ/kg. , $h_{fg} = 1986$ kJ/kg.

$$h_{sup} = h_f + h_{fg} + C_p(t_{sup} - t_{sat})$$
$$= 798 + 1986 + 2 * 67 = 2918 \text{ kJ/kg}$$

from Steam tables at 0.16 bar

$h_f = 232$ kJ/kg , $h_{fg} = 2369$ kJ/kg

$$h = h_f + x\, h_{fg} = 232 + 0.95 * 2369$$
$$= 2482.55 \text{ kJ/kg}$$



Drop in enthalpy of the steam

$$= h_{sup} - h = 2918 - 2482.55$$
$$= 435.45 \text{ kJ/kg}$$

* Interpolation :-

Ex:- When $P = 133$ bar Compute (t_{sat}, h_f, h_g, v_g, u_f)

Sol:- from Steam tables.

P [bar]	t_{sat} [°C]	v_g [m³/kg]	u_f [kJ/kg]	h_f [kJ/kg]	h_g [kJ/kg]
130	330.8	0.01278	1511	1531	2662
133	t_s ?	v_g ?	u_f ?	h_f ?	h_g ?
135	333.8	0.01211	1530	1551	2650

$$\frac{133-130}{135-130} = \frac{t_s - 330.8}{333.8-330.8} = \frac{v_g - 0.01278}{0.01211-0.01278} = \frac{u_f - 1511}{1530-1511}$$

$$= \frac{h_f - 1531}{1551-1531} = \frac{h_g - 2662}{2650-2662}$$

$$\therefore t_s = 330.8 + \frac{133-130}{135-130}(333.8-330.8) = 332.6\ °C$$

$$v_g = 0.01278 + \frac{3}{5}(0.01211-0.01278) = 0.017398\ \frac{m^3}{kg}$$

$$u_f = 1511 + \frac{3}{5}(1530-1511) = 1522.4\ kJ/kg$$

$$h_f = \quad + \frac{\quad}{\quad}(\quad - \quad) = 1543\ kJ/kg$$

$$h_g = \quad + \frac{\quad}{\quad}(\quad - \quad) = 2654.8\ \frac{kJ}{kg}$$

$$h_{fg} = \quad + \frac{\quad}{\quad}(\quad - \quad) = 1111.8\ \frac{kJ}{kg}$$

Ex: If $P = 12$ bar & $t = 255°C$, find (h) ?

Sol: at $P = 12$ bar from Steam tables

$t_{sat} = 188°C$ $\therefore t > t_{sat} \Rightarrow$ the Steam is Superheated

from Superheated Steam table:

	250 °C	(255°C)	300 °C
10 bar	2944	h_1	3052
(12)	h_1 ?	h_{sup}	h_2 ?
15 bar	2925	h_2	3039

$$h_1 = 2944 + \frac{12-10}{15-10}(2925-2944) = 2936.4 \text{ kJ/kg}$$

$$h_2 = 3052 + \frac{12-10}{15-10}(3039-3052) = 3046.8 \text{ kJ/kg}$$

$$h_{sup} = h_1 + \frac{255-250}{300-250}(h_2 - h_1)$$

$$= 2936.4 + \frac{255-250}{300-250}(3046.8 - 2936.4)$$

$$= 2947.44 \text{ kJ/kg}$$

It is called double Interpolation.

H.W: find for the same case (u, v)

Ex: Find the missing properties of water and given the phase of it:

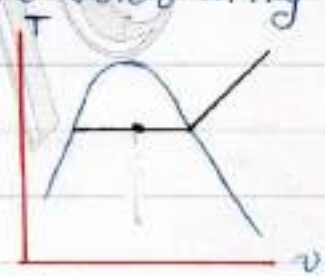
a. $T = 120^\circ C$, $v = 0.5\ m^3/kg$, u, P, X?

b. $T = 100^\circ C$, $P = 10\ Mpa$, u, X, v?

Sol: a. Saturation Condition of H_2O at $120^\circ C$ from Steam tables, $v_g = 0.8908$

$v_f = 0.00108\ m^3/kg$

Since $v < v_g \Rightarrow$ wet Steam



$v = v_f + x(v_g - v_f)$

$0.5 = 0.00108 + x(0.8908 - 0.00108)$

$\therefore x = 0.56$, $P = 198.5\ kpa = 1.985\ bar$

$u = u_f + x(u_g - u_f)$, By Interpolation

$= 503.48 + 0.56(2529.24 - 503.48)$

$u = 1639.7\ kJ/kg$

b. $T = 100^\circ C$ $P = 10\ Mpa$

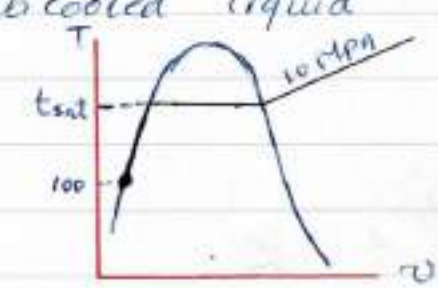
from Steam tables at $P = 10\ Mpa$

$t_{sat} = 311.06$ $\therefore t < t_{sat} \Rightarrow$ Subcooled liquid

$\therefore x = 0$

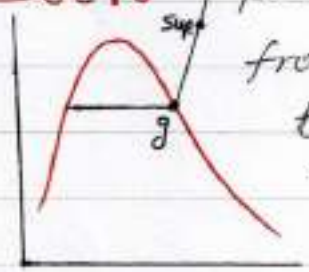
$u = u_f = 418.91\ kJ/kg$

$v = v_f = 0.00104\ m^3/kg$



Ex: Determine the volume of 1 kg of superheated Steam at a pressure of 20 bar and a temperature of 300 °C. (using charles's law)

Sol: $P = 20$ bar , $T_{sup} = 300°C = 300 + 273 = 573K$



from Steam tables, at $P = 20$ bar

$t_{sat} = 212.4°C$, $v_g = 0.09957 \ m^3/kg$

$T_{sat} = 212.4 + 273 = 485.4 \ K$

from Charles's law $\frac{v_g}{T_{sat}} = \frac{v_{sup}}{T_{sup}}$ (for 1 kg)

$\therefore v_{sup} = \frac{v_g}{T} * T_{sup} = \frac{0.09957}{485.4} * 573 = 0.1175 \ m^3$

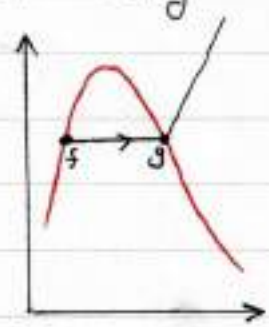
Ex: Find the external work done during evaporation per kg of Steam at a pressure of 15 bar when the Steam is (a) 90% dry (or 10% wet) (b) dry Saturated.

Sol: at $P = 15$ bar

from Steam tables, $v_g = 0.1317 \ m^3/kg$

a. $\because P = C \Rightarrow W = P(V_2 - V_1)$

$= P(V_g - V_f)$



$\because (v_f)$ very small, as compared with v_g therefore neglecting the value of v_f

$\therefore W = P * v_g$, when the steam is not completely dry, and has dryness fraction of x, the the work:

$$W = P * x v_g$$
$$= 15 * 10^2 * 0.9 * 0.1317$$
$$= 177.8 \text{ kJ/kg}$$

b. Work done when the Steam is dry Saturation

$$W = P * v_g$$
$$= 15 * 10^2 * 0.1317 = 197.5 \text{ kJ/kg}$$

Ex: A vessel having volume of 0.4 m^3 contains 2kg of a Liquid water and water vapour mixture in equilibrium of 600 kpa. Calculate the volume and mass of Liquid, the volume and mass of vapour.

Sol:

$$v = V/m = 0.4/2 = 0.2 \text{ m}^3/\text{kg}$$

at 600 kpa, from Steam tables,

$v_g = 0.3157 \text{ m}^3/\text{kg}$ since $v < v_g$

$\therefore$ wet Steam

$$v = v_f + x v_{fg}$$
$$0.2 = 0.001014 + x * 0.3146 \Rightarrow x = 0.6322$$

$$x = \frac{m_{vapour}}{m_{total}} \Rightarrow m_v = x \cdot m_t$$
$$= 0.6322 * 2$$
$$= 1.2644 \text{ kg}$$
$$m_f = m_t - m_v$$
$$= 2 - 1.2644$$
$$= 0.7356 \text{ kg} = m_{Liquid} \text{ or } m_L$$

$v_L = v_f$
$m_L = m_f$

$V = V_{Liq} + V_{vap}$

$m v = m_{Liq} \cdot v_f + m_v v_g$

$V_{Liq} = m_{Liq} * v_f$
$V_{vap} = m_v * v_g$

$$v_v = \frac{V_v}{m_v} \Rightarrow$$

$v_g = v_v$

$$V_v = m_v * v_v = 1.2644 * 0.3157$$
$$= 0.3992 \text{ m}^3$$
$$V_L = V_t - V_v = 0.4 - 0.3992 = 8.2 * 10^{-4} \text{ m}^3$$

Ex: A 200 l volume tank contain R-12 at 25°C has 10% liquid and 90% vapour by volume. Find the dryness fraction of R-12.

Sol: at 25°C, from R-12 tables.

$$v_L = 0.000763 \frac{m^3}{kg}, \quad v_g = 0.02685 \frac{m^3}{kg}$$
$$V = \frac{200}{1000} = 0.2 \text{ m}^3$$

$$V_L = 0.1 * V = 0.1 * 0.2 = 0.02 \text{ m}^3$$
$$V_g = 0.9 * V = 0.9 * 0.2 = 0.18 \text{ m}^3$$

$$v_L = \frac{V_L}{m_L} \Rightarrow m_L = \frac{V_L}{v_L} = \frac{0.02}{0.000763} = 26.2 \text{ kg}$$

$$m_g = \frac{V_g}{v_g} = \frac{0.18}{0.02685} = 6.7 \text{ kg}$$

$m_t = m_L + m_g = 26.2 + 6.7 = 32.9$ kg

$$x = \frac{m_g}{m_t} = \frac{6.7}{32.9} = 0.203$$

Ex: A vessel contains 0.1 kg Saturated Liquid and vapour water at 100 kpa, quality of 0.25. Find the total volume of vessel and the persentage of Liquid and vapour by volume.

Sol: from Steam tables, at 100 kpa

$v_f = 0.001043 \frac{m^3}{kg}$, $v_g = 1.694 \frac{m^3}{kg}$

$$v = v_f + x\, v_{fg}$$
$$= 0.001043 + 0.25(1.694 - 0.001043)$$
$$= 0.4242 \ m^3/kg$$

$v = V/m \Rightarrow V = v \cdot m = 0.4242 * 0.1$
$= 0.04242 \ m^3$

$V_L = v_L * m_L$, $V_g = v_g * m_g$

$m_g = x \cdot m_t = 0.25 * 0.1 = 0.025$ kg

$m_L = m_t - m_g = 0.1 - 0.025 = 0.075$ kg

$\therefore V_L = 0.001043 * 0.075 = 7.8225 * 10^{-5}\ m^3$

$V_g = 1.644 * 0.025 = 0.0423\ m^3$

$V_t = V_L + V_g = 7.8225 * 10^{-5} + 0.0423 = 0.0423782$

$\% V_L = \frac{V_L}{V_t} * 100 = \frac{7.8225 * 10^{-5}}{0.042378} * 100 = 0.1845\ \%$

$\% V_g = \frac{V_g}{V_t} * 100 = \frac{0.0423}{0.0423782} * 100 = 99.815\ \%$

Ex: Find the dryness fraction, the Specific volume and internal energy of Steam at 7 bar and enthalpy (2600 kJ/kg).

Sol: at 7 bar, from Steam tables,

$h_g = 2764\ kJ/kg \qquad h_f = 697\ kJ/kg$

Since $h_f < h_{steam} < h_g$

$697 < 2600 < 2764 \Rightarrow$ wet Steam

$\therefore h_x = 2600 = h_f + x h_{fg}$

$= 697 + x(2764 - 697) \Rightarrow x = 0.921$

$v_x = x v_g \quad , \quad v_g = 0.2728\ m^3/kg$

$v_x = 0.921 * 0.2728$

$= 0.2515\ m^3/kg$

$u_x = u_f + x(u_g - u_f)$,

$= 696 + 0.921(2573 - 696)$

$= 2420\ kJ/kg$

(Sheet No. 4)

Q1: Find the missing properties among (P, T, v, u, h) together with x if applicable and give the phase of Substance:

a.	H_2O	$T = 120°C$	$v = 0.5\ m^3/kg$
b.	H_2O	$P = 100\ kpa$	$v = 0.18\ m^3/kg$
c.	H_2O	$T = 340°C$	$h = 3150\ kJ/kg$
d.	N_2	$P = 750\ kpa$	$v = 0.2\ m^3/kg$
e.	R_{12}	$P = 600\ kpa$	$h = 230\ kJ/kg$
f.	R_{22}	$T = 10°C$	$u = 200\ kJ/kg$
g.	R_{134a}	$T = 40°C$	$u = 407\ kJ/kg$
h.	NH_3	$T = 20°C$	$v = 0.1\ m^3/kg$

Q2: A water Storage tank contains Liquid and vapour in equilibrium at 110°C. The distance from bottom of the tank to the Liquid level is 8 m. What is the absolute pressure at the bottom of the tank. [218 kpa]

Q3: Saturated water vapour at 60°C has it's pressure decrease to increase the volume by 10% keeping the temperature constant. To what pressure should be expands. [18.9 kpa]

Q4:- 5kg of Steam at pressure of 4.9 bar is produced from water at 20°c. Determine the amount of heat supplied if the Steam is 0.9 dry.

[12263 kJ]

Q5:-

1 kg of water at 47.8°C is heated under constant pressure of 13.7 bar until it is converted into Steam with 111 of Superheated. Determine the quantity of heat Supplied during Superheating and the total heat.

[268 kJ , 2852 kJ]

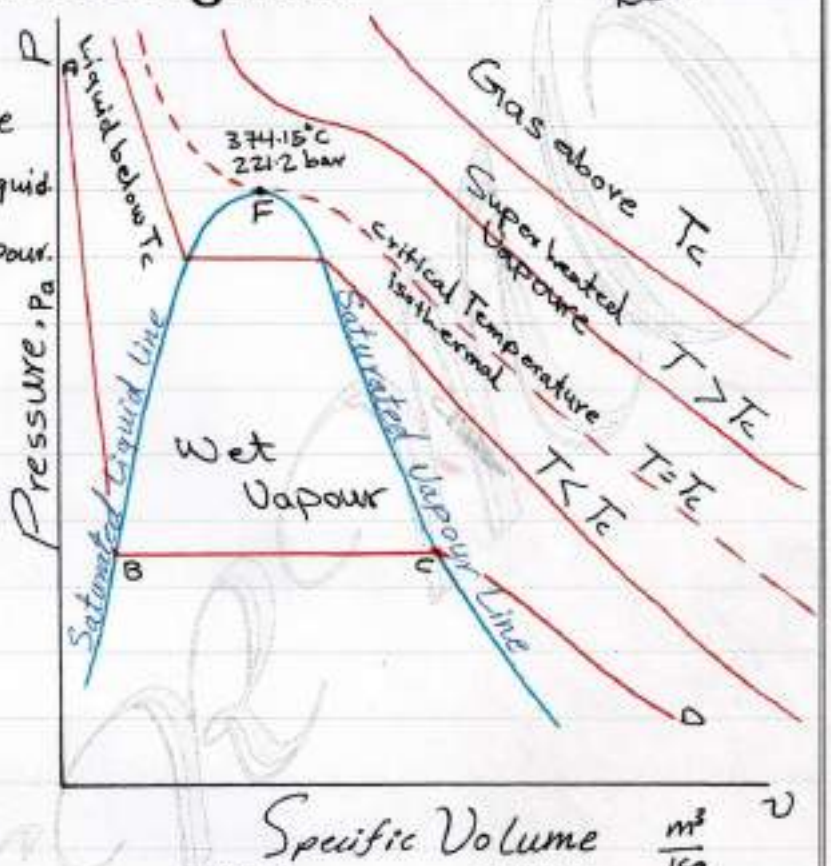
Q6:- A tank of $2m^3$ volume contains saturated ammonia at a temperature of 40°c. the tank contains 50% Liquid and 50% Vapour by Volume. Find the Specific volume, enthalpy and internal energy of the mixture.

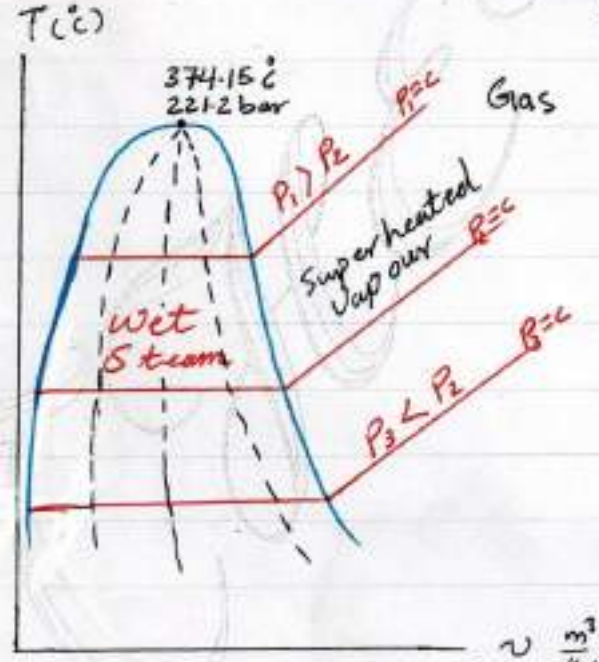
[$3.369 \times 10^{-3} \frac{m^3}{kg}$, 232908 kJ , 229816 kJ]

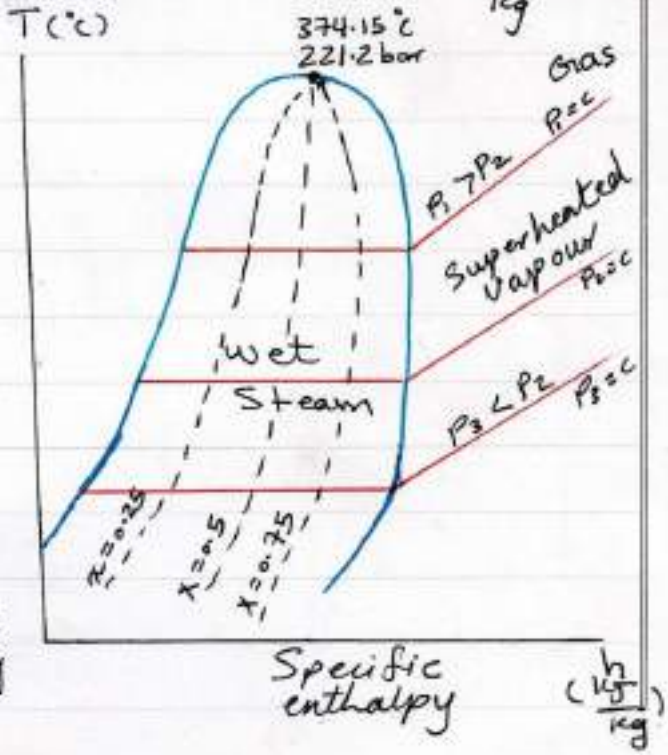
The Pressure-Volume Diagram:-

ABCD ≡ Isothermal Line
Point B ≡ Saturated liquid
" C ≡ " Vapour.
" F ≡ Critical point

Note:
Area under curve (or line) represent the magnitud of work.





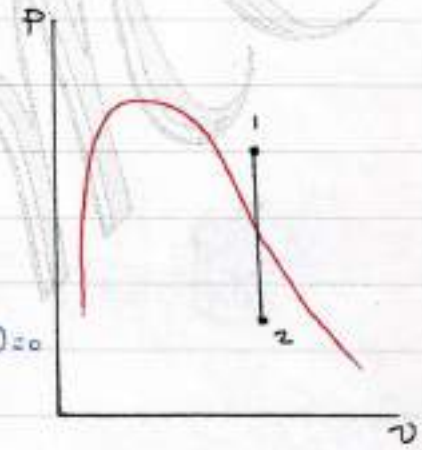


T_s ≡ Saturated Temperature at $P = P_3$

* Reversible and Irreversible Processes :

1. Closed System (Reversible)

a. Constant volume process (Isometric Process)



From non-flow Energy Equation:

$$Q = W + \Delta U$$

Per unit mass

$$q = w + \Delta u \quad , \quad v = c \Rightarrow w = 0$$

$$\therefore q = u_2 - u_1 \quad , \quad h = u + pv$$

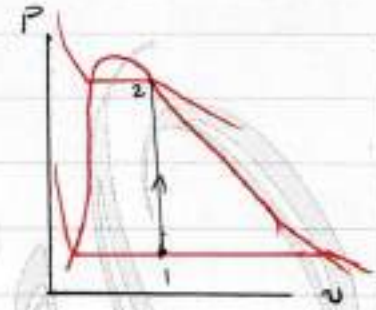
$$\therefore \boxed{q = (h_2 - h_1) - v(P_2 - P_1)}$$ heat added or rejected

Ex: A vessel having a volume of $5 m^3$ contains $0.05 m^3$ of Saturated Liquid water and $4.95 m^3$ of Saturated water vapour at 0.1 Mpa. Heat is transfered until the vessel is fitted with saturated vapour. Determine the heat transfer for this Process.

Sol: condition (1) wet Steam

at 0.1 Mpa $\quad v_f = 1.0432 * 10^{-3} \ m^3/kg$

$v_g = 1.694 \ m^3/kg$

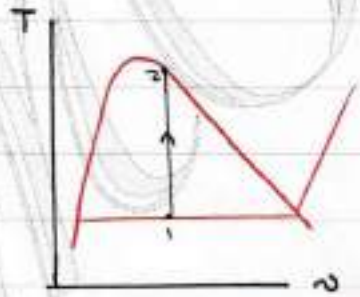


$m_l = V_l / v_f = 0.05 / 1.043 \times 10^{-3} = 47.93$ kg

$m_g = V_g / v_g = 4.95 / 1.694 = 2.92$ kg

$m_t = m_l + m_g = 47.93 + 2.92 = 50.85$ kg

$x = m_g / m_t = 2.92 / 50.85 = 0.0574$



$Q = W + \Delta U$, $V = C$ $\therefore$ $W = 0$

$= U_2 - U_1 = m(u_2 - u_1)$

$u_1 = u_f + x\, u_{fg} = 417.36 + 0.0574 * 2088.7 = 521.8 \ \frac{kJ}{kg}$

$v_1 = \frac{V}{m} = \frac{5}{50.85} = 0.09833 \ m^3/kg$, $v_1 = v_2 = 0.09833 = v_g$

from Steam tables:

By Interpolation, $u_2 = 2600.465 \ \frac{kJ}{kg}$

v_g	u_g
0.09963	2600.26
(0.09833)	u_2
0.08875	2601.98

$\therefore Q = 50.85(2600.465 - 521.8)$

$= 105700.1$ kJ

When (C_v) is given:

$$\boxed{Q = m C_v (T_2 - T_1)}$$

for Sensible Heat
(for gas)

Ex:- A closed vessel of 0.6 m^3 capacity contains dry Saturated Steam at 360 kN/m^2. The vessel is cooled until the pressure is reduced to 200 kN/m^2. Calculate:

a. the mass of Steam in the vessel,

b. the final dryness of the Steam.

c. the amount of heat transferred during the cooling process.

Sol:-

a. At 360 kN/m^2, $v_g = 0.510\ m^3/kg$

$\therefore$ mass of Steam in vessel $= \frac{0.6}{0.51} = 1.18\ kg$

b. $V = c$, $v_2 = v_g = 0.510\ m^3/kg$, $v_f \simeq 0$

$v_2 = x\, v_g$, at $P = 200\ kN/m^2$ $v_g = 0.885\ m^3/kg$

$0.510 = x * 0.885 \Rightarrow x = 0.510/0.885 = 0.576$

c. $Q = m(u_2 - u_1)$, $V = c \Rightarrow W = 0$

$u_1 = u_g$ at $P = 360\ kN/m^2$

from Steam tables, $u_g = 2549.87$ (By Interpolation)

$u_2 = u_f + x\, u_{fg}$

from Steam tables, $u_f = 504.47\ kJ/kg$, $u_{fg} = 2025.02\ kJ/kg$

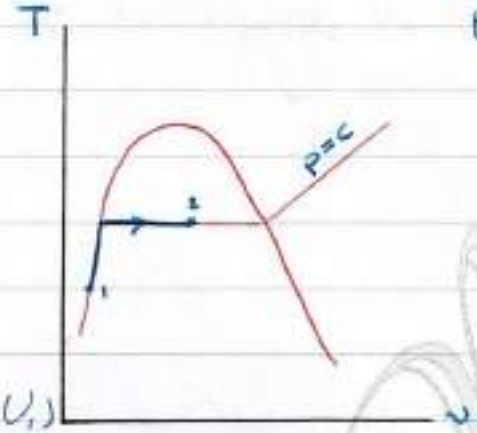
$u_2 = 504.47 + 0.576 * 2025.02$

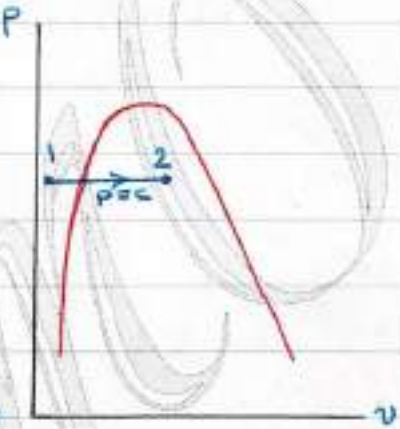
$= 1670.88\ kJ/kg$

$\therefore Q = 1.18 * (1670.88 - 2549.87)$

$= -1037.2\ kJ$

b. Constant Pressure Process (Isobaric Process)





from NFEE

$Q = W + \Delta U$

$W = \int P\,dV$

$= P(V_2 - V_1)$

$Q = (P_2V_2 - P_1V_1) + (U_2 - U_1)$

$= H_2 - H_1 \Rightarrow$

$$\boxed{Q = m(h_2 - h_1)}$$

When C_p is given (for Sensible Heat) (for gas)

$$Q = m C_p (T_2 - T_1)$$

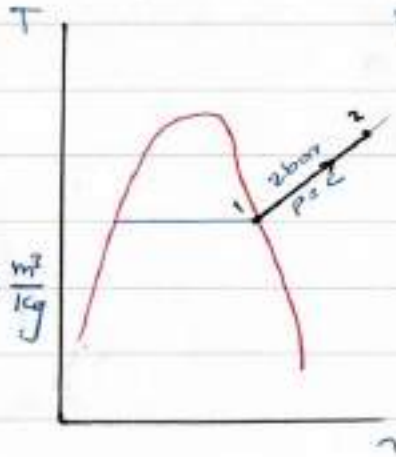
Ex: 0.05 kg of a dry Saturated Steam is heated at a constant pressure of 2 bar until the volume occupied is $0.0658\ m^3$. Calculate the heat supplied and the work done.

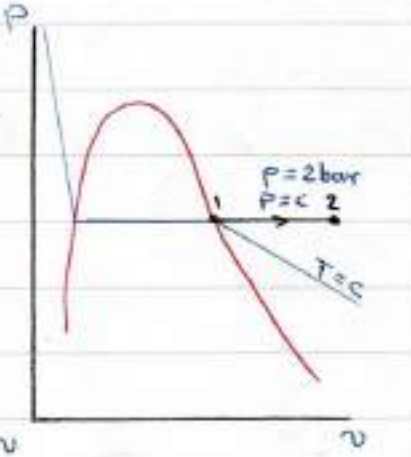
Sol:

$h_1 = h_g$ at 2 bar

$= 2707\ kJ/kg$

$v_2 = \frac{V}{m} = \frac{0.0658}{0.05} = 1.316\ \frac{m^3}{kg}$





the Steam is Superheated, from Steam tables at 2bar & $1.316 \frac{m^3}{kg}$ $\Rightarrow$ $h_2 = 3072$ kJ/kg

$Q = H_2 - H_1 = m(h_2 - h_1) = 0.05(3072 - 2707)$

$\therefore$ Heat Supplied $= 18.25$ kJ/kg

$W = p(v_2 - v_1)$, $v_1 = v_g$ at 2bar $= 0.8856$ m³/kg

$v_2 = 1.316$ m³/kg

$w = 2 * 10^5 (1.316 - 0.8856)$

$W = m \cdot w = 0.05 * 2 * 10^5 * 0.4304$

$= 4304$ J $= 4.304$ kJ

Ex: Steam at 4 MN/m² and dryness fraction 0.95 receives heat at constant pressure until its temperature becomes 350°C. Determine the heat recived by the Steam per unit mass.

Sol: At $p = 4$ Mpa and $x = 0.95$ dry , $h_1 = h_f + x h_{fg}$

$h_1 = 1087.4 + 0.95 * 1712.9 = 2714.7$ kJ/kg

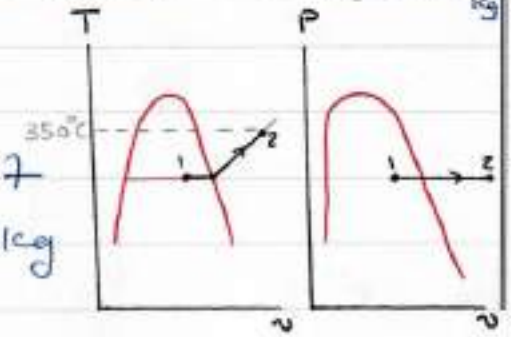
At $p = 4$ Mpa, from Steam tables $T_{sat} = 250.3$°C

$\therefore T > T_{sat} \Rightarrow$ Superheated Steam $\therefore h_2 = 3095 \frac{kJ}{kg}$

Heat received $= h_2 - h_1$

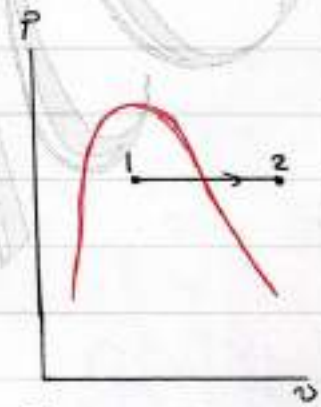
$= 3095 - 2714.7$

$= 380.3$ kJ/kg



Ex: A cylinder fitted with a piston has a volume of $0.1\ m^3$ and contains $0.5\ kg$ of Steam at $0.4\ Mpa$. Heat is transfer to the steam until the temperature is $300°C$. While the pressure remains constant. Determine the heat transfer and work done.

Sol:



$$Q = W + \Delta U = m(h_2 - h_1)$$

$v_1 = V/m = 0.1/0.5 = 0.2\ m^3/kg$

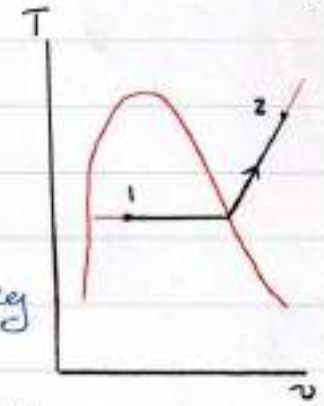
at $0.4\ Mpa$ $\quad v_g = 0.4625\ m^3/kg$

since $v < v_g$ $\quad \therefore$ wet Steam

$v = v_f + x\, v_{fg}$

$0.2 = 1.0836 * 10^{-3} + x(0.4625 - 1.0836 * 10^{-3})$

$x = 0.431$



$\therefore\ u_1 = u_f + x\, u_{fg} = 604.31 + 0.431 * 1949.3$

$= 1444.6\ kJ/kg$

$h_1 = 604.74 + 0.431 * 2133.8 = 1524.6\ kJ/kg$

point 2, $\quad P = 0.4\ Mpa$, from Steam tables

$t_s = 143.63°C$, since $T_2 > T_{sat} \Rightarrow$ Superheated

from Superheated Steam tables, $u_2 = 2804.8\ kJ/kg$

$h_2 = 3066.8\ kJ/kg$

$Q = 0.5(3066.8 - 1524.6) = 771.1\ kJ$

$W = Q - \Delta U = 771.1 - 0.5(2804.8 - 1444.6)$

$= 91\ kJ$

c. Constant Temperature process (Isothermal Process)

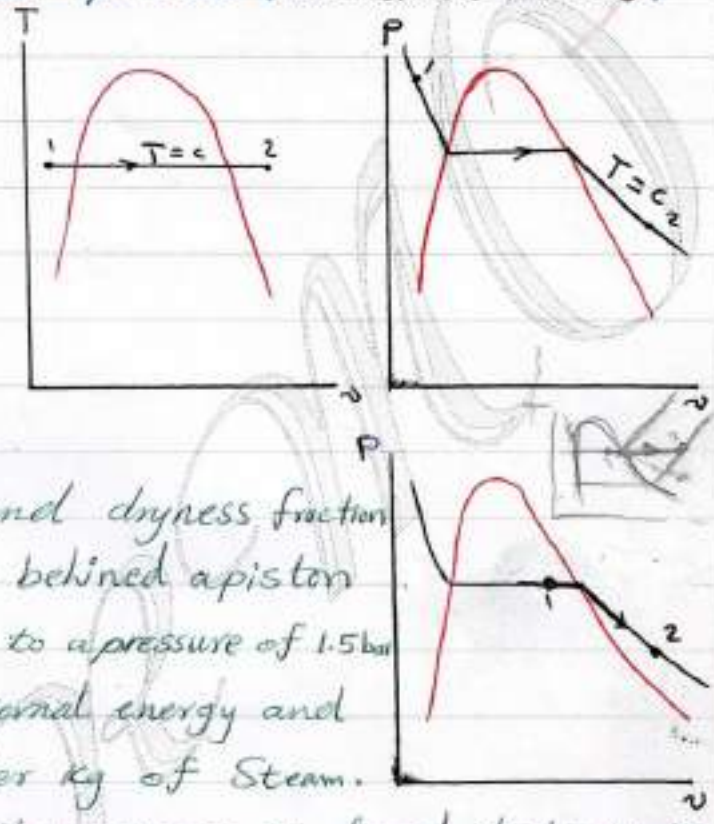
$Q = W + \Delta U$

$W \neq P V \ln \frac{V_2}{V_1}$

$\Delta U = U_2 - U_1$

$Q = H_2 - H_1$

$W = Q - \Delta U$



Ex: Steam at 7 bar and dryness fraction 0.9 expands in a Cylinder belined apiston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of Steam. The heat supplied during the process is found to be $400 \frac{kJ}{kg}$. Calculate the work done per kg of Steam.

Sol: $u_1 = u_f + x u_{fg}$ (at 7 bar), $T_{sat} = 165°C$

$= 2385.3$ kJ/kg

from Steam tables (Superheated) at 1.5 bar & 165°C

$u_2 = 2602.8$ kJ/kg , $h_2 = 2803$ kJ/kg

$\therefore \Delta u = u_2 - u_1 = 2602.8 - 2385.3 = 217.5$ kJ/kg

$h_1 = h_f + x h_{fg} = 697 + 0.9 * 2067 = 2557.3$ kJ/kg

$\therefore \Delta h = h_2 - h_1 = 2803 - 2557.3 = 245.7$ kJ/kg

$Q = \Delta U + W \Rightarrow \therefore W = q - \Delta u$

$= 400 - 217.5 = 182.5$ kJ/kg

d. The hyperbolic Process $PV = C$

$P_1 V_1 = P_2 V_2$

Work done $= W = PV \ln \frac{v_2}{v_1}$ for any mass

$W = Pv \ln \frac{v_2}{v_1}$ for unit mass

the NFEE.

$$q = \Delta u + W$$
$$= (u_2 - u_1) + Pv \ln \frac{v_2}{v_1}$$
$$= (h_2 - P_2 v_2) - (h_1 - P_1 v_1) + Pv \ln \frac{v_2}{v_1} \quad , P_1 v_1 = P_2 v_2$$
$$= (h_2 - h_1) + Pv \ln \frac{v_2}{v_1}$$

$$\therefore Q = (H_2 - H_1) + PV \ln \frac{v_2}{v_1}$$

Ex: A quantity of dry saturated steam occupies $0.2634\ m^3$ at $1.5\ MN/m^2$. Determine the final condition of the Steam if it is compressed until the volume is halved:

a. If the compression is carried out in an isothermal manner:

b. " " " follows the law $PV = C$.

Sol:

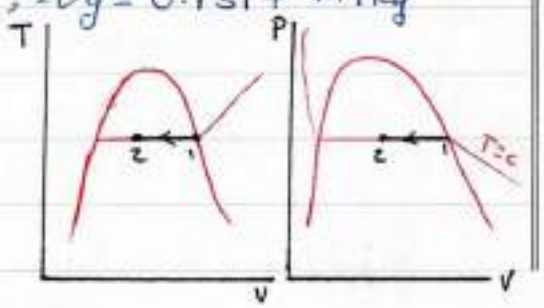
a. from Steam tables at 1.5 Mpa, $v_g = 0.1317\ m^3/kg$

$$m = \frac{V}{v} = \frac{0.2634}{0.1317} = 2\ kg$$

When the volume is halved.

$$v_2 = \frac{v_g}{2} = \frac{0.1317}{2} = 0.0659\ m^3/kg$$

$$x = \frac{0.0659}{0.1317} = 0.5$$



In this case the steam is operating in the evaporation region, since the temperature remains constant.

$h_2 = h_f + x\, h_{fg}$, from Steam tables at $P = 1.5$ Mpa

$h_f = 844.7$ kJ/kg , $h_{fg} = 1945.2$ kJ/kg

$h_2 = 844.7 + 0.5 * 1945.2 = 1817.3$ kJ/kg

$H_2 = m \cdot h_2 = 2 * 1817.3 = 3634.6$ kJ

Heat loss $= 0.5\, h_{fg} = 0.5 * 1945.2 = 972.6$ kJ/kg

for 2 kg , Heat loss $= 2 * 972.6 = 1945.2$ kJ

b. If the compression is according to the law $PV = C$, then $P_1 V_1 = P_2 V_2$

$\therefore P_2 = P_1 \cdot \frac{V_1}{V_2} = 1.5 * 2 = 3$ Mpa

Specific volume after compression, $v_2 = \frac{v_1}{2} = \frac{v_g}{2} = 0.0659$ m³/kg

from Steam tables,

at $P_2 = 3$ Mpa, $v_g = 0.0666$ m³/kg , $h_f = 1008.4$ kJ/kg

$v_2 < v_g \Rightarrow$ Wet Steam $\qquad h_{fg} = 1793.9$ kJ/kg

$x = \frac{v_2}{v_g} = \frac{0.0659}{0.0666} = 0.989$

$h_2 = h_f + x\, h_{fg} = 1008.4 + 0.989 * 1793.9 = 2782.6$ kJ/kg

$H_2 = m * h_2 = 2 * 2782.6 = 5565.2$ kJ

Ex:- 1 kg of water at 30 bar, 300°C, expand reversibly to 0.75 bar. Calculate the heat flow and the work done when the process is hyperbolic.

Sol:-

at $P = 30$ bar, $T_s = 233.9°C$, $T_1 = 300°C$

since $T_1 > T_{sat} \Rightarrow$ Superheated Steam

from Steam tables at 30 bar & 300°C

$v_1 = 0.08119\ m^3/kg$, $u_1 = 2750.1\ kJ/kg$

$P_1 v_1 = P_2 v_2 \Rightarrow v_2 = \frac{P_1}{P_2} \cdot v_1 = \frac{30}{0.75} * 0.08119$

$= 3.243\ m^3/kg$

at $P = 0.75$ bar, $v_g = 2.21711\ m^3/kg$

$\because v_2 > v_g \Rightarrow$ Superheated Steam

$\therefore$ at 0.75 bar & 3.243 from Superheated Steam tables

$h_2 = 2986.3\ kJ/kg$, $u_2 = 2742.9\ kJ/kg$

$W = P_1 V_1 \ln \frac{v_2}{v_1} = 30 * 10^2 * 0.08114 \ln \frac{3.2456}{0.08119}$

$= 897.9\ kJ/kg$

$W = m \cdot w = 1 * 897.9 = 897.9\ kJ$

$\Delta U = m(u_2 - u_1) = 1 * (2742.9 - 2750.1)$

$= -7.21\ kJ$

$Q = W + \Delta U = 897.9 - 7.21$

$= 890.68\ kJ$

e. The polytropic process $PV^n = C$

Here the steam is assumed to be expanded or compressed according to the law $PV^n = C$.

$$Q = W + \Delta U \quad , \quad W = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$q = w + \Delta u$$

$$\Delta u = u_2 - u_1 = (h_2 - P_2v_2) - (h_1 - P_1v_1)$$

$$= (h_2 - h_1) - (P_2v_2 - P_1v_1)$$

$$q = \frac{P_1v_1 - P_2v_2}{n-1} + (h_2 - h_1) - (P_2v_2 - P_1v_1)$$

$$= (h_2 - h_1) + \frac{P_1v_1 - P_2v_2}{n-1} + (P_1v_1 - P_2v_2)$$

$$= (h_2 - h_1) + (P_1v_1 - P_2v_2)(1 + n - 1/n - 1)$$

$$= (h_2 - h_1) + n \cdot \frac{P_1v_1 - P_2v_2}{n-1}$$

$$\therefore \quad \boxed{q = (h_2 - h_1) + n \cdot w} \quad *$$

$$Q = (H_2 - H_1) + n \cdot W$$

Ex: A quantity of Steam at a pressure of 2.1 Mpa and 0.9 dry occupies a volume of 0.2562 m^3. It is expanded according to the law $PV^{1.25} = C$, to a pressure of 0.7 Mpa. Determine:

a. the mass of Steam present,
b. the external work done.
c. the change of internal energy.
d. the heat exchange between the Steam and surroundings.

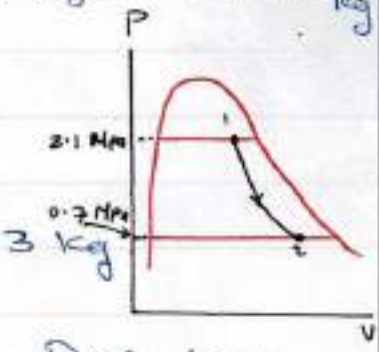
Sol:

a. $v_1 = v_f + x_1 v_{fg}$, from Steam tables at 2.1 Mpa

$v_f = 1.181 * 10^{-3}$, $v_g = 0.0949 \ \frac{m^3}{kg}$

$\therefore v_1 = 1.181 * 10^{-3} + 0.9(0.0949 - 1.181 * 10^{-3})$

$= 0.0856 \ m^3/kg$



$v = V/m \Rightarrow m = \frac{V}{v} = \frac{0.2562}{0.0856} = 3 \ kg$

b. $Q = W + \Delta U$, $W = P_1V_1 - P_2V_2 / n-1$

$P_2V_2^{1.25} = P_1V_1^{1.25} \Rightarrow V_2 = (P_1/P_2)^{1/1.25} * V_1$

$= (2.1/0.7)^{1/1.25} * 0.0856$

$= 0.206 \ m^3/kg$

$W = \frac{2.1*10^3 * 0.0856 - 0.7*10^3 * 0.206}{1.25 - 1} = 143.36 \ kJ/kg$

$W = m \cdot w = 3 * 143.36 = 430 \ kJ$

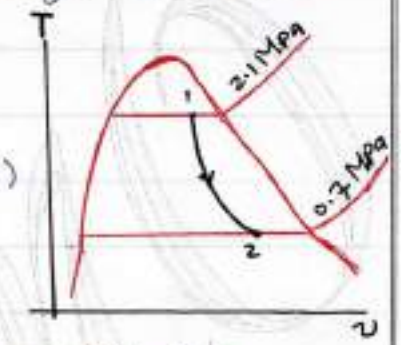
c. at $P = 0.7$ Mpa , $v_g = 0.273\ m^3/kg$, $v_2 = 0.206$

$v_g > v_2 \Rightarrow$ wet Steam

$v_2 = v_f + x\, v_{fg}$

$0.206 = 1.108 * 10^{-3} + x_2(0.273 - 1.108 * 10^{-3})$

$x_2 = 0.753$



$\therefore\ U_1 = U_f + x_1 U_{fg} = 917.5 + 0.9(2598.2 - 917.51)$

$= 2430\ kJ/kg$

$U_2 = U_f + x_2 U_{fg} = 696.3 + 0.753(2571.1 - 696.3)$

$= 2108\ kJ/kg$

$\Delta U = m(U_2 - U_1) = 3 * (2108 - 2430) = -966\ kJ$

d. $Q = W + \Delta U = 430 - 966 = -536\ kJ$ (loss to the Surrounding)

Ex: In a steam engine the steam at the beginning of the expansion process is at 7 bar, dryness fraction 0.95, and the expansion follows the law $p v^{1.1} = c$, down to a pressure of 0.34 bar. Calculate the work done per kg of steam during the expansion, and the heat flow per kg of steam to or from the cylinder walls during the expansion.

Sol:o At 7bar , $v_g = 0.2728\ m^3/kg$, $v_f \simeq 0$

$\therefore\ v_1 = x_1 v_g = 0.95 * 0.2728 = 0.259\ m^3/kg$

$P_1/P_2 = \left(\frac{v_2}{v_1}\right)^n \Rightarrow v_2 = v_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0.259 \left(\frac{7}{0.34}\right)^{\frac{1}{1.1}}$

$\therefore v_2 = 4.05\ m^3/kg$

$$W = \frac{P_1 v_1 - P_2 v_2}{n-1} = \frac{7*10^2*0.259 - 0.34*10^2*4.05}{1.1-1}$$

$\therefore W = 436\ kJ/kg$

At 0.34 bar , $v_g = 4.649\ m^3/kg$,

$v_2 < v_g \Rightarrow$ Wet Steam

$x_2 = \frac{v_2}{v_g} = \frac{4.05}{4.649} = 0.873$

$u_1 = u_f + x_1 u_{fg} = 696 + 0.95(2573 - 696) = 2476.8\ kJ/kg$

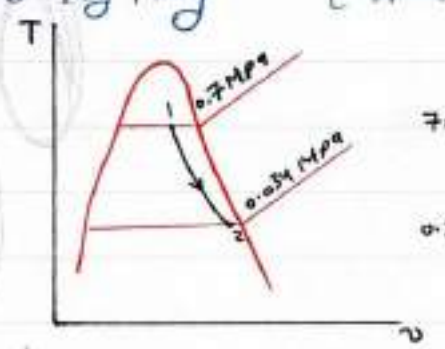
$u_2 = u_f + x_2 u_{fg} = 302 + 0.873(2472 - 302) = 2196.4\ kJ/kg$

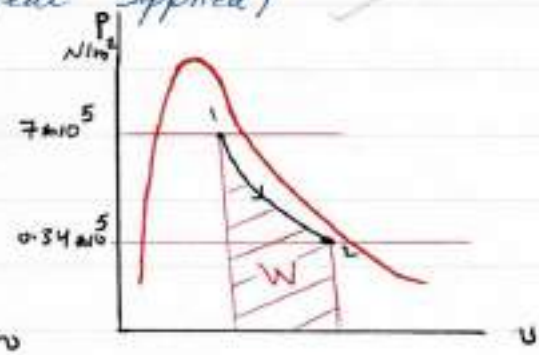
$Q = \Delta U + W$

$q = \Delta u + w$

$= (2196.4 - 2476.8) + 436$

$= 155.6\ kJ/kg$ (Heat Supplied)





f. Adiabatic Process:

An adiabatic process is one in which no heat is transferred to or from the fluid during the process.

from NFEE,

$$Q = W + \Delta U \quad , \quad Q = 0$$

$$W = -\Delta U$$

$$W = U_1 - U_2$$

Per 1 kg, $w = u_1 - u_2$

Ex: 1 kg of Steam at 100 bar and 375 °C expands reversibly in a perfectly insulated cylinder behind a piston until the pressure is 38 bar and the Steam is then dry Saturated. Calculate the work done by the System.

Sol:

$$Q = W + \Delta U \quad , \quad Q = 0 \quad \text{(perfectly insulated)}$$

$$\therefore W = U_1 - U_2$$

at 100 bar, $t_s = 311°C$, since $t > t_s \Rightarrow$ Superheated

$\therefore u_1 = 2621.78$ kJ/kg , $u_2 = u_g = 2608$ kJ/kg

$$W = m \cdot w$$

$$= m(u_2 - u_1)$$

$$= 1 * (2621 - 2608)$$

$$= 13 \text{ kJ/kg}$$

Ex: Determine the volume occupied by 2kg of Steam at 0.85 Mpa and dryness fraction of 0.95. If this Steam is expanded reversibly and adiabatically to 0.17 Mpa, the law of expansion is ($PV^{1.13} = c$). Determine the final dryness fraction and the change in internal energy during expansion.

Sol:

$v_1 = v_f + x_1(v_g - v_f)$, at 0.85 Mpa & $x_1 = 0.95$

$= 1.118 * 10^{-3} + 0.95(0.227) = 0.2167 \ m^3/kg$

$v_1 = \frac{V}{m} \Rightarrow V_1 = v * m = 0.2167 * 2 = 0.4335 \ m^3$

$P_1 v_1^{1.13} = P_2 v_2^{1.13} \Rightarrow v_2 = (0.85/0.17)^{1/1.13} * 0.2167$

$= 0.9 \ m^3/kg$

$Q = W + \Delta U$, $Q = 0$

$W = m(u_1 - u_2)$ or $u_2 - u_1 = -(P_1 v_1 - P_2 v_2)/n-1$

$u_1 = u_f + x u_{fg} = 731.1 + 0.95(2577.1 - 731.1) = 2484.8 \ kJ/kg$

at $P_2 = 0.17$ Mpa , $v_g = 1.0312 \ m^3/kg$, $v_2 = 0.9 \ m^3/kg$

$\therefore v_2 < v_g \Rightarrow$ wet Steam

$v_2 = v_f + x_2 v_{fg}$

$0.9 = 1.056 * 10^{-3} + x_2 * (1.0312 - 1.056 * 10^{-3}) \Rightarrow x_2 = 0.8717$

$u_2 = u_f + x_2 u_{fg} = 483.02 + 0.8717 * 2040.9$

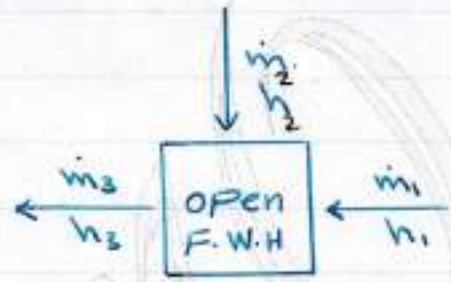
$= 2262.1 \ kJ/kg$

$W = 2 * (2484 - 2262.1)$

$= 445.26 \ kJ$

2. Irreversible Process :

1. Adiabatic mixing:



open feedwater heater

from flow Equation

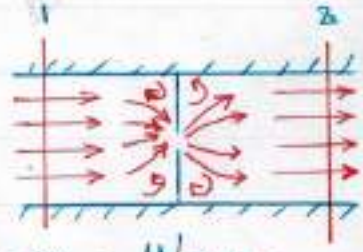
$$H_1 + H_2 = H_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\dot{m}_1 \cdot C_{p_1} \cdot T_1 + \dot{m}_2 \cdot C_{p_2} \cdot T_2 = (\dot{m}_1 + \dot{m}_2) \cdot C_{p_3} \cdot T_3$$

2. Throttling:



$$h_1 + \frac{C_1^2}{2} + q = h_2 + \frac{C_2^2}{2} + W$$

$Q = 0$, $W = 0$,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad , \; C_1^2 - C_2^2/2 \simeq 0$$

$$h_1 = h_2 \Rightarrow C_{p_1} T_1 = C_{p_2} T_2$$

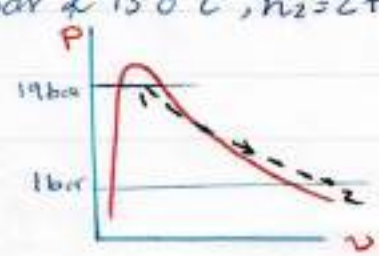
Ex: Steam at 19 bar is throttled to 1 bar and the temperature after throttling is found to be 150 °C. Calculate the initial dryness fraction of the Steam.

Sol: from Steam tables, at 1 bar & 150 °C, $h_2 = 2777$

$$h_2 = h_1 = h_f + x h_{fg} \Rightarrow$$

$$2777 = 897 + x_1 * 1901$$

$$\therefore x = 0.989$$



Ex: Steam at 1.5 Mpa and 250°c flowing at 1.5 kg/s is throttled to 700 kpa and then mixed with a steam at 700 kpa also 0.97 dryness fraction with a flow rate of 3.6 kg/s. find the condition of resulting steam.

Sol:

at 1.5 Mpa from steam tables, $t_s = 198.32°C$

Since $t_1 > t_s$ ∴ Superheated steam at point 1

from Superheated steam tables. $h_1 = 2925$ kJ/kg $= h_2$

$$\dot{m}_2 h_2 + \dot{m}_3 h_3 = \dot{m}_4 h_4$$

$h_3 = h_{f3} + x_3 h_{fg3}$, at $P = 700$ kpa , $h_f = 697$ kJ/kg
$h_{fg} = 2067$ kJ/kg

$$h_3 = 697 + 0.97 * 2067 = 2702 \text{ kJ/kg}$$

$$h_4 = \frac{\dot{m}_2 h_2 + \dot{m}_3 h_3}{\dot{m}_2 + \dot{m}_3} = \frac{1.5 * 2925 + 3.6 * 2702}{1.5 + 3.6} = 2767.6 \text{ kJ/kg}$$

at 700 kpa , $h_g = 2763.5$ kJ/kg , $T_s = 164.97°C$

Since $h_4 > h_g$ ⇒ Superheated steam

from Superheated steam tables, $T_4 = 166.64°C$

∴ Degree of Superheated $= (t - t_s)$

$= 166.64 - 164.92$

$= 1.67°C$

Ex:

Steam at 0.6 Mpa, 200°C enters an insulated nozzle with a velocity of 50 m/s. It leaves at 0.15 Mpa and velocity of 600 m/s. Determine the final temperature of Steam if it is superheated and quality if it is wet.

Sol: from SFEE

$$gz_1 + \frac{1}{2}C_1^2 + h_1 + q = gz_2 + \frac{1}{2}C_2^2 + h_2 + w$$

$z_1 \simeq z_2$, $q = 0$ (insulated) , $w = 0$ (for nozzle)

$$\therefore h_2 = h_1 + \frac{1}{2}(C_1^2 - C_2^2) * 10^{-3}$$

at 0.6 Mpa , $t_s = 158.8°C$, since $T_1 > T_s$ ∴ Superheated

from Superheated Steam tables at 0.6 Mpa & 200°C

$h_1 = 2850.1$ kJ/kg

$h_2 = 2850.1 + \frac{1}{2}(50^2 - 600^2) * 10^{-3} = 2671.4$ kJ/kg

at 0.15 Mpa $h_g = 2693.6$ kJ/kg (from Steam tables)

since $h_2 < h_g \Rightarrow$ wet Steam

$h_2 = h_{f_2} + x_2 h_{fg}$, at $P = 0.15$ Mp from Steam table

$h_f = 467.1$ kJ/kg , $h_{fg} = 2226.5 \frac{kJ}{kg}$

$\therefore 2671.4 = 467.1 + x_2 * 2226.5$

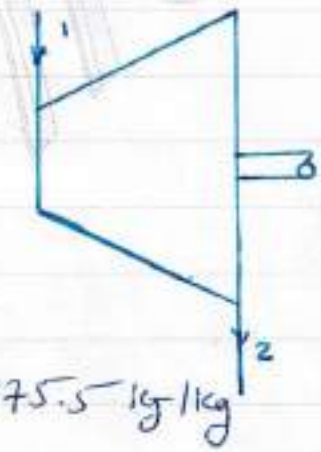
$x_2 = 0.99$

Ex: The mass flow rate of steam turbine is 1.5 kg/s and the heat transfer from the turbine is 8.5 kW. The following data are known for steam entering and leaving turbine. Find the power output of turbine.

	P (Mpa)	t (°C)	x	C (m/s)	Z (m)
input	2	350	—	50	6
output	0.1	—	100%	200	3

Solu: $gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w$

$$w = g(z_1 - z_2)\times10^{-3} + (h_1 - h_2) + \frac{1}{2}(C_1^2 - C_2^2)\times10^{-3} + \frac{Q}{\dot{m}}$$



from Superheated Steam tables at

2 Mpa & 350°C , $h_1 = 3137$ kJ/kg

at $P = 0.1$ Mpa & dry , $h_2 = h_g = 2675.5$ kJ/kg

$$w = 9.8(6-3)\times10^{-3} + (3137 - 2675.5) + \frac{1}{2}(50^2 - 200^2)\times10^{-3} + \left(\frac{-8.5}{1.5}\right)$$

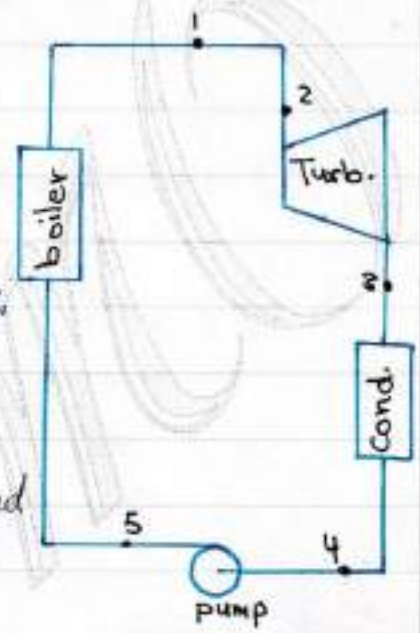
$$= 437.11 \text{ kJ/kg}$$

$$\text{Power} = \dot{W} = \dot{m}\cdot w = 1.5 \times 437.11$$

$$= 656 \text{ kW}$$

Ex: Consider the simple power plant, as shown, the following data are:

	P	t(°C)	x
Leaving boiler	2 Mpa	300	—
entering turbine	1.9 Mpa	290	—
leaving turbine	15 kpa	—	90%
leaving condenser	14 kpa	45	—



pump work = 4 kJ/kg , Find:

a. heat transfer between boiler and turbine

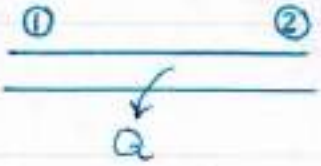
b. turbine work

c. heat transfer in condenser

d. heat transfer in boiler

Sol: a. pipe 1→2

$$gz_1 + \frac{1}{2}C_1^2 + h_1 + q = gz_2 + \frac{1}{2}C_2^2 + h_2 + w$$



$z_1 \simeq z_2$, $C_1 = C_2$, $w = 0$

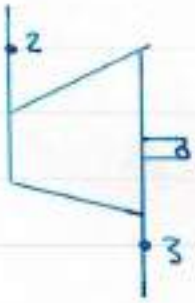
$\therefore q = h_2 - h_1$

from Superheated Steam, at 2 Mpa & 300°C , $h_1 = 3023.5$ kJ/kg

" " " , at 1.9 Mpa & 290°C , $h_2 = 3002.5$ kJ/kg

$\therefore q = 3002.5 - 3023.5 = -21$ kJ/kg

b. Turbine 2→3



$$gz_2 + \frac{1}{2}C_2^2 + h_2 + q = gz_3 + \frac{1}{2}C_3^2 + h_3 + w$$

$z_2 \simeq z_3$, $C_2 = C_3$, $q = 0$

$w = h_2 - h_3$

$h_3 = h_f + x\, h_{fg}$, at 15 kpa , $h_f = 226$ kJ/kg

$h_{fg} = 2373.1$ kJ/kg

$h_3 = 226 + 0.9 * 2373.1 = 2361.8$ kJ/kg

$\therefore w_t = 3002.5 - 2361.8 = 640.7$ kJ/kg

c. Condenser 3→4

$g z_3 + \frac{1}{2} C_3^2 + h_3 + q = g z_4 + \frac{1}{2} C_4^2 + h_4 + w$

$z_3 \simeq z_4$, $C_3 = C_4$, $w = 0$

$\therefore q = h_4 - h_3$

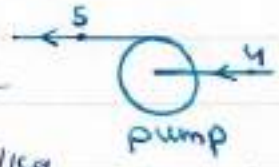
at $p = 14$ kpa $t_s = 52.5\,^\circ C$, $t_4 = 45\,^\circ C$

$\therefore t_4 < t_s \Rightarrow$ Subcooled liquid

from Steam table at $t_s = 45\,^\circ C$, $h_4 = 188.5$ kJ/kg

$q = 188.5 - 2361.8 = -2173.3$ kJ/kg

d. Pump 4→5



$\cancel{g z_4} + \cancel{\frac{1}{2} C_4^2} + h_4 + \cancel{q} = \cancel{g z_5} + \cancel{\frac{1}{2} C_5^2} + h_5 + w$

$w = h_4 - h_5 \Rightarrow -4 = 188.5 - h_5$

$\therefore h_5 = 192.5$ kJ/kg

e. Boiler 5→1

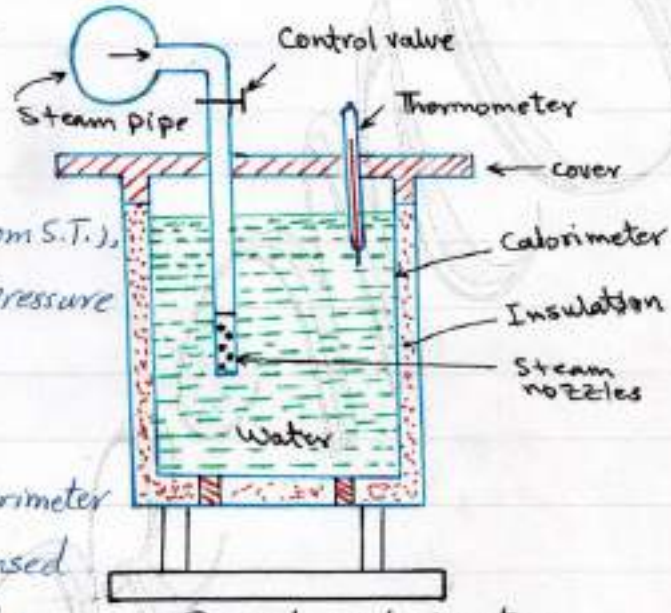
$\cancel{g z_5} + \cancel{\frac{1}{2} C_5^2} + h_5 + q = \cancel{g z_1} + \cancel{\frac{1}{2} C_1^2} + h_1 + \cancel{w}$

$\therefore q = h_1 - h_5 = 3023 - 192.5$

$= 2831$ kJ/kg

Measurement of Dryness Fraction of Steam.

1. Barrel Calorimeter



Barrel Calorimeter

p = pressure of the Steam.

t = temperature of Steam formation at pressure P (from S.T.).

h_{fg} = latent heat of steam at pressure P (from Steam tables)

m_c = Mass of the calorimeter

C_c = Specific heat of the calorimeter

m_s = Mass of the Steam Condensed

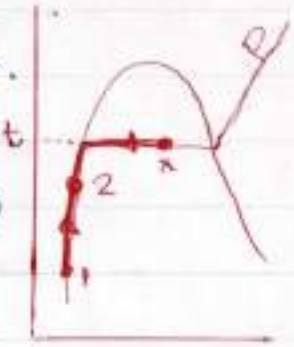
m_w = Mass of cold water in the Calorimeter

t_1 = Initial temperature of water and calorimeter.

t_2 = Final " " " " " .

C_w = Specific heat of water (4.2 kJ/kg.K)

x = Dryness fraction of Steam sample.



According to the law of conservation, the heat lost by Steam is equal to the heat gained by water and calorimeter.

$$\text{Heat lost} = m_s\left[x\,h_{fg} + C_w(t - t_2)\right]$$

$$\text{Heat gained} = (m_w C_w + m_c C_c)(t_2 - t_1)$$

$$\therefore\ m_s\left[x\,h_{fg} + C_w(t - t_2)\right] = (m_w C_w + m_c C_c)(t_2 - t_1)$$

from this equation, the dryness fraction (x) may be determined.

Ex: In a laboratory experiment on wet Steam by a barrel calorimeter, the following observations were recorded:

Mass of copper calorimeter = 1 kg = m_c
Mass of calorimeter + water = 3.8 kg = $m_c + m_w$
Mass of calorimeter + water + Steam = 4 kg = $m_c + m_w + m_s$
Initial temperature of water = 10 °C
Final temperature of water = 50 °C
Steam pressure = 5.5 bar

If the Specific heat of copper is 0.406 kJ/kg.K. Determine the dryness fraction of Steam.

Sol: $m_w = 3.8 - m_c = 3.8 - 1 = 2.8$ kg

$m_s = 4 - (m_c + m_w) = 4 - (3.8) = 0.2$ kg

from Steam tables at $P = 5.5$ bar, $t_s = 155.5$ °C

$h_{fg} = 2095.5 \frac{kJ}{kg}$

Heat lost by Steam $= m_s[x h_{fg} + C_w(t - t_2)$

$= 0.2[x * 2095.5 + 4.2(155.5 - 50)]$

$= 419.1x + 88.6$ kJ --- ①

Heat gained by water and calorimeter

$= (m_w C_w + m_c C_c)(t_2 - t_1)$

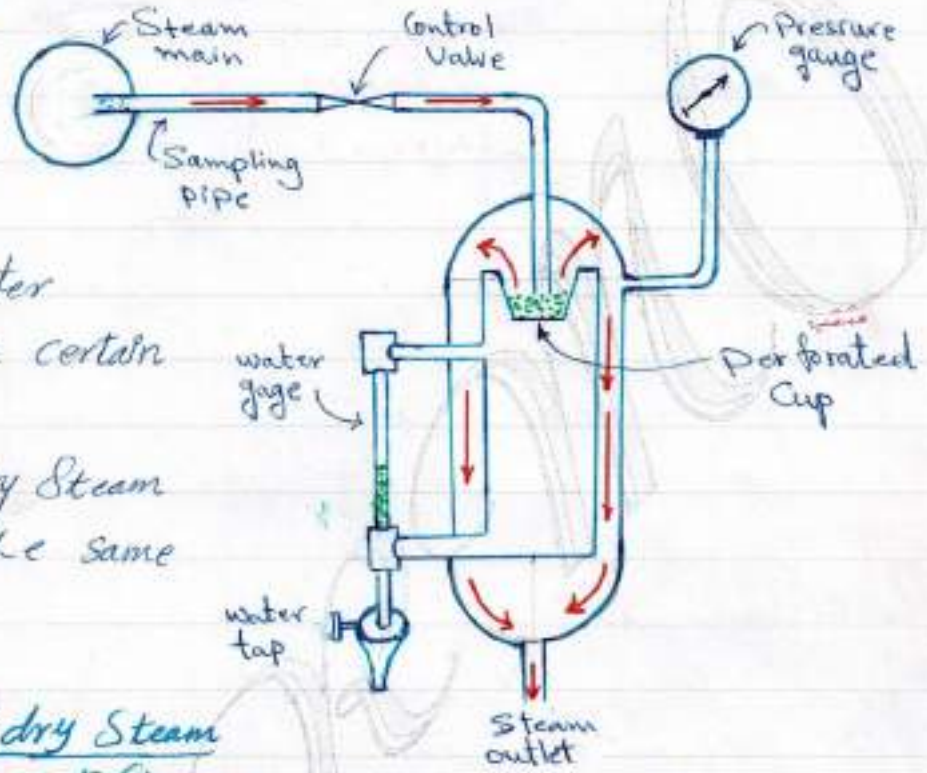
$= (2.8 \times 4.2 + 1 \times 0.406)(50 - 10) = 486.6$ kJ --- ②

Equating eqs ① and ②

$419.1x + 88.6 = 486.6$

$\therefore x = 0.95$

2. Separating Calorimeter :



Separating calorimeter

m = Mass of water collected in a certain time

M = Mass of dry Steam passing in the same time.

$$x = \frac{\text{Mass of dry Steam}}{\text{Mass of wet Steam}}$$

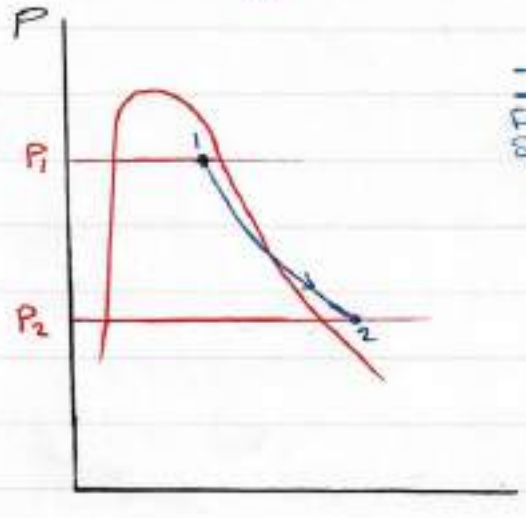
$$\boxed{x = \frac{M}{M+m}}$$

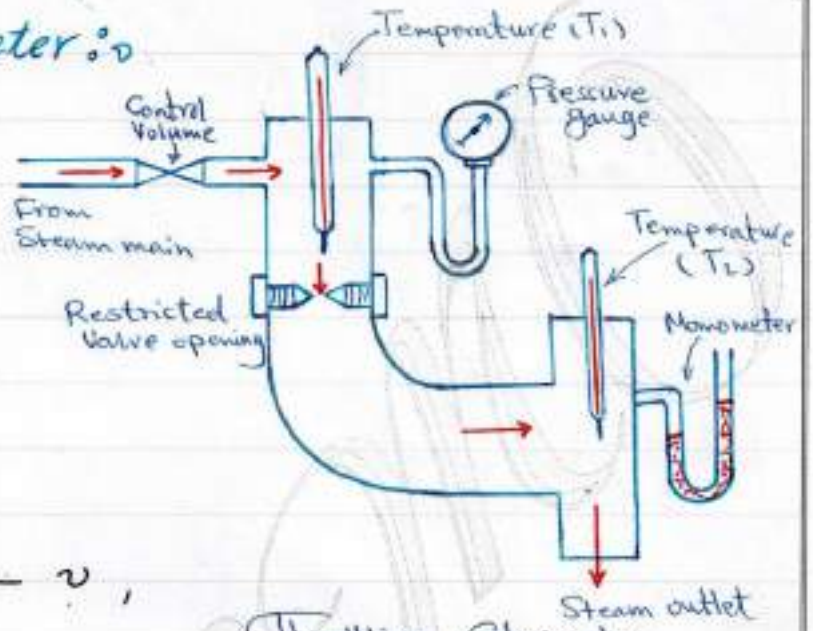
Ex: In a laboratory experiment, a sample of wet Steam is allowed to pass through a separating calorimeter. At some instant, the water collected in the chamber was 0.1 kg whereas the condensed Steam was found to be 1.25 kg. Determine the dryness fraction of Steam entering the calorimeter.

Sol: $m = 0.1$ kg , $M = 1.25$ kg

$$x = \frac{M}{M+m} = \frac{1.25}{1.25+0.1} = 0.926$$

3. Throttling Calorimeter :-





Throttling Calorimeter

the Steam in point (2) after throttling in superheated state and at a lower pressure than (P)

Total heat befor throttling = Total heat after throttling

or

$$h_1 = h_2$$

$$\underbrace{h_{f_1} + x\,h_{fg_1}}_{\text{at } P_1} = \underbrace{h_{g_2} + C_p(t_{sup} - t_2)}_{\text{at } P_2}$$

Ex:- In a throttling calorimeter, the steam is admitted at a pressure of 10 bar. If it is discharged at atmospheric pressure and 110°C after throttling, determine the dryness fraction of steam. Assume specific heat of steam as 2.2 kJ/kg.K.

Sol:- $P_1 = 10$ bar, $P_2 = 1.013$ bar, $t_{sup.} = 110°C$, $C_p = 2.2$ kJ/kg.K

from Steam tables at 10 bar, $h_{f_1} = 762.6$ kJ/kg

$h_{fg_1} = 2013.6$ kJ/kg

from Steam tables at 1.013 bar, $t_2 = 100°C$, $h_{g_2} = 2676$ kJ/kg

$$h_{f_1} + x\, h_{fg_1} = h_{g_2} + C_p(t_{sup} - t_2)$$

$$762.6 + x * 2013.6 = 2676 + 2.2(110 - 100)$$

$$x = 0.961$$

4. Combined Separating and Throttling Calorimeter :-

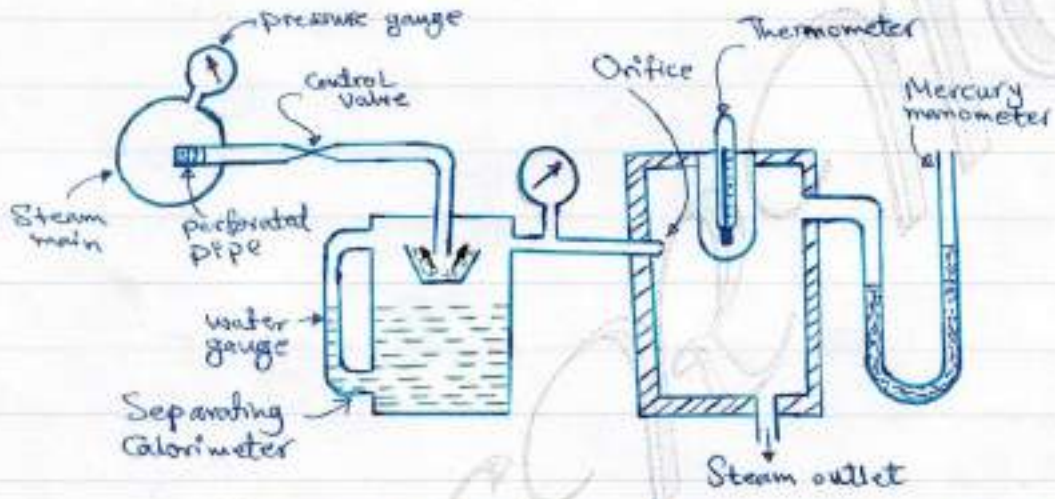
A very Successful method of measuring the dryness fraction of Steam is by a combined separating and throttling calorimeter.

In this calorimeter, the wet Steam is first collected in a perforated collecting pipe and then passed through a separating calorimeter. A part of water is removed by separating calorimeter owing to quick change of direction of flow. The resulting semi-dry steam is throttled into a throttling calorimeter. This method ensures that the Steam will be Superheated after throttling. This instrument is well insulated to prevent any loss of heat.

x_1 = Dryness fraction of Steam considering separating calorimeter.
x_2 = Dryness fraction of Steam entering the throttling " .

Now the actual dryness fraction of Steam in the Steam main,

$$x = x_1 * x_2$$



" Combined Separating and throttling calorimeter "

Note: It is not possible to obtain results with this instrument if the final condition of Steam is wet. The final condition of Steam must be just dry or Superheated

Ex: In a laboratory experiment, the following observations were recorded to find the dryness fraction of Steam by combined separating and throttling calorimeter:

Total quantity of Steam passed = 36 kg = $m_s + m_w$
Water drained from Separating = 1.8 kg = m_w
Steam pressure befor throttling = 12 bar = P_1
Temperature of Steam after throttling = 110°C = t_{sup}
Pressure after throttling = 1.013 bar = P_2
Specific heat of Steam = 2.1 kJ/kg.K
Determine the dryness fraction of Steam befor inlet to the calorimeter

Sol: $m_s \neq l$

We know that mass of dry Steam

$m_s = (m_s + m_w) - m_w = 36 - 1.8 = 34.2$ kg

$x_1 = m_s / m_s + m_w = 34.2/36 = 0.95$

from Steam tables, corresponding to $P_1 = 12$ bar

$h_{f_1} = 798.4$ kJ/kg , $h_{fg} = 1984.3$ kJ/kg

at $P_2 = 1.013$ bar

$h_{g_2} = 2676$ kJ/kg , $t_2 = 100°C$

$h_{f_1} + x_2 h_{fg_1} = h_{g_2} + C_p(t_{sup} - t_2)$

$798.4 + x_2 * 1984.3 = 2676 + 2.1(110 - 100)$

$x_2 = 0.957 \Rightarrow x = x_1 + x_2 = 0.95 + 0.957$
$= 0.909$

(Sheet No. 5)

Q1: Steam at a pressure of 28 kpa is passed into a condenser and it leaves as condensate at a temperature of 59°C. Cooling water circulates through the condenser at the rate of 45 kg/min. It enters at 15°C and leaves at 30°C. If the Steam flow rate is 1.25 kg/min, determine the dryness fraction of steam as it enters the condenser.

Q2: 1 kg of Saturated steam at 10 bar undergoes a non-flow constant volume process until the pressure becomes 3.5 bar. Determine:

a. the final condition of Steam.
b. the change in internal energy.
c. the change in Specific enthalpy
d. the heat energy transfered.

Ans. [0.37, -1273 kJ/kg, -1500 kJ/kg, -1273 kJ]

Q3: A sample of Steam at 1.4 Mpa is taken from a boiler and passed through a throttling calorimeter where after throttling to 0.11 Mpa, its temperature is observed to be 110°C. Determine the dryness fraction of the Steam leaving the boiler.

Ans. [0.95]

Q4: 0.5 kg of water at 7bar and 15°c is contained in a cylinder 0.3m diameter by friction-less piston. Heat energy is supplied until the temperature of cylinder contents becomes 204°c. the pressure of the contents remains at 7bar.

Determine: a. the heat energy supplied
b. the distance moved by the piston.
c. the work energy.
d. the change in internal energy.

Ans. [1395.8 kJ/kg, 2.14m, 105 kJ, 1290.8 kJ]

Q5: 0.075 m³ of dry saturated steam at 8bar and contained in a cylinder by friction-less piston if the steam undergose hyperbolic equation to a pressure of 4bar. Determine the work energy, the change in internal energy and the heat energy transfered.

Ans. [41.5 kJ, 0.624 kJ, 40.5 kJ]

Q6: A quantity of dry saturated steam occupies 0.2634 m³ at 1.5 Mpa. Determine the final condition of the steam if it is compressed until the volume is halved.

a. if the compression is carried out in an isothermal manner, then find the heat rejected.

b. if the compression follows the law $pv = c$.

Ans. [a. 0.5, 972.6 kJ/kg, b. 0.989]

THERMODYNAMICS

4

CHAPTER FOUR

The Second Law of Thermodynamics

The Second Law of Thermodynamics

According to the first law of thermodynamics, when a System undergoes a complet cycle then the net heat supplied is equal to the net work done.

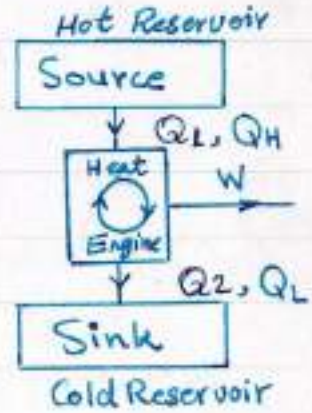
$$\oint dQ = \oint dW$$

The second law of thermodynamics indicates that, although the net heat supplied in a cycle is equal to the net work done, the gross heat supplied must be greater than the net work done, some heat must always be rejected by the System.

• The Heat Engine:-

A heat engine is a system operating in a complete cycle and developing net work from a supply of heat. The Second law implies that a source of heat supply and a sink for the rejection of heat are both necessary, since some heat must always be rejected by the System.

The heat supplied from the source is Q_1, the work done is W. and the heat rejected is Q_2.



By the first law of thermodynamics, in a complete Cycle

Net heat supplied = Net work done

$$\oint dQ = \oint dW$$

$$Q_1 - Q_2 = W$$

By the second law, the gross heat supplied must be greater than the net work done.

$$Q_1 > W$$

• Forward Heat Engine:

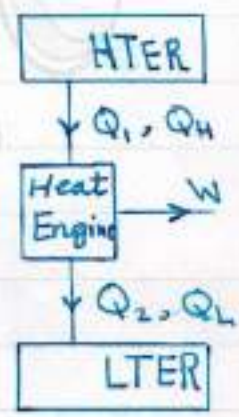
A forward heat engine has a positive work output such as Rankine or Brayton Cycle.

Appling the first law to the Cycle gives:

$Q_1 - Q_2 - W = 0$

$Q_1 > W$ (heat Supplied must be greater than W)

$Q_2 > 0$



LTER = Low Temp. Energy Reservoir

HTER = High Temp. Energy Reservoir

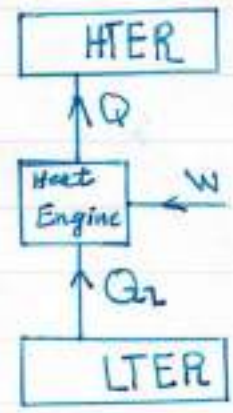
• Reverse Heat Engine:

A reverse heat engine has a positive work input such as heat pump and refrigerator.

Appling the first law to the cycle gives:

$-Q_1 + Q_2 + W = 0$

$W > 0$



• The thermal efficiency:

the thermal efficiency of a heat engine is defined as the ratio of the net work done in the cycle to the gross heat supplied in the cycle.

thermal efficiency, $\eta = \frac{W}{Q_1}$, $W = Q_1 - Q_2$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow \eta < 100\%$$

From the definition of heat, a temperature difference is necessary for heat to flow, therefore that the source of heat must be at a higher temperature than the sink.

The source can be thought of as a hot reservoir and the sink as a cold reservoir.

There are <u>two</u> statements of the Second law:

1. The Kelvin-Planck Statement.
2. The Clausius statement.

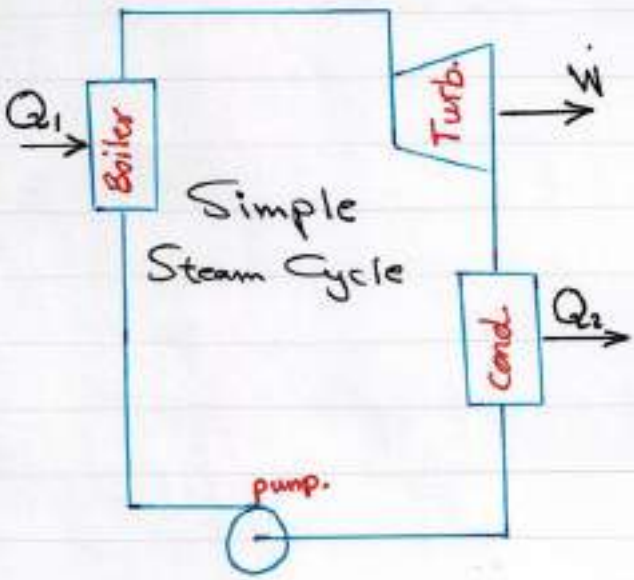
1. The Kelvin-Planck Statement:

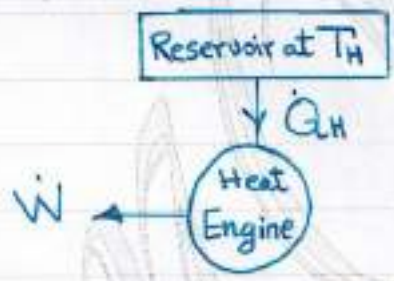
It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of work to its surroundings while exchanging heat with a single heat reservoir.

The Kelvin-Plank Statement of the 2nd law:

It is not possible to have the following system with $Q_2 = 0$

We cannot convert all of the heat from a heat reservoir at T_H to work





- heat is supplied in the Boiler
- Work is developed in the Steam turbine or engine
- heat is rejected in a condenser.

Boiler ≡ Hot Reservoire.
Condenser ≡ Cold reservoir.
Steam ≡ System.

- **Heat Reservoir:**

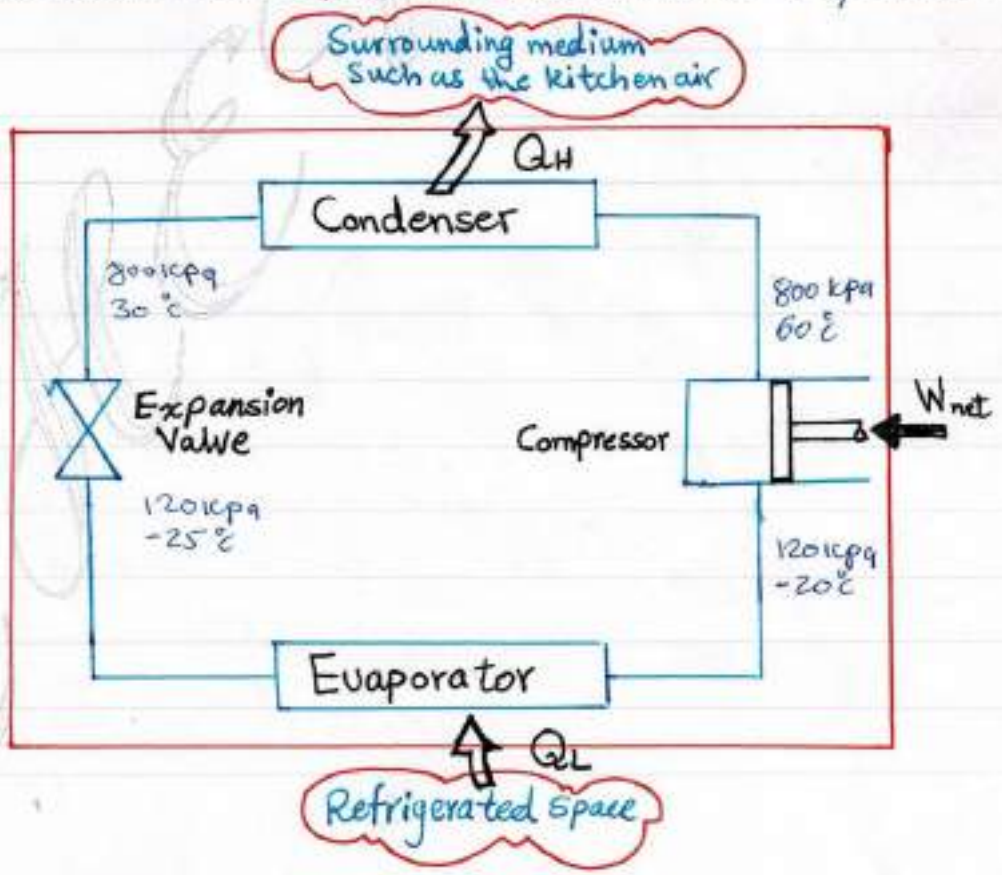
It is a closed System, from which heat is removed or to which heat is added. The temperature of the reservoir does not change. A heat reservoir may be a large heat source or a large heat sink.

• Refrigerators and Heat Pumps:

We know that the heat flows in the direction of decreasing temperature (from high-temperature media to low-temperature ones), this heat transfer process occurs in nature without requiring any devices.

The reverse process, cannot occure by itself. The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators**.

Refrigerators, like heat engines, are cyclic devices. The working fluid used in the refrigeration cycle is called a **Refrigerant**. This cycle which involves four main components: a compressor, a condenser, an expansion valve, and an evaporator.



• Coefficient of Performance:

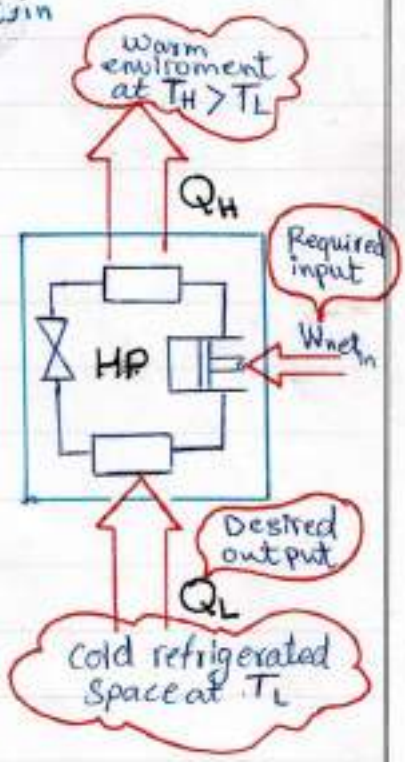
The efficiency of refrigerator is expressed in terms of the **coefficient of performance** (COP), denoted by COP_R. The objective of a refrigerator is to remove heat (Q_L) from the refrigerated Space. To accomplish this objective, it requires a work input of $W_{net,in}$. Then the COP of a refrigerator can be expressed as

$$COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{Q_L}{W_{net,in}}$$

$$W_{net,in} = Q_H - Q_L$$

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

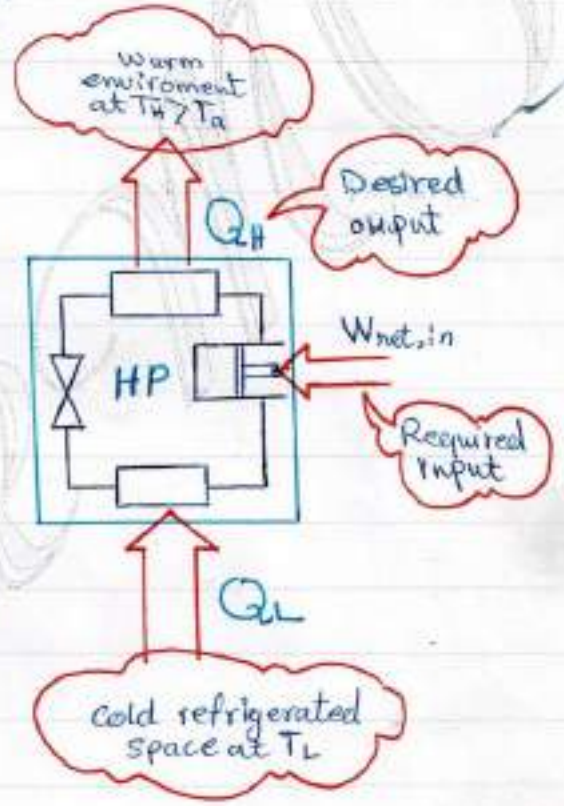
Notice that the value of COP_R can be greater than unity. That is, the amount of heat removed from the refrigerated Space can be greater than the amount of work input.



- The Heat Pumps:

It is another device that transfers heat from a low-temperature medium to a high-temperature one.

Refrigerators and heat pumps Operate on the same cycle but differ in their objectives.



The measure of performance of heat pump is also expressed in terms of the coefficient of Performance COP_{HP}.

$$COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{Q_H}{W_{net,in}}$$

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

$$(Q_H = Q_L + W_{net,in})$$

$$COP_{HP} = \frac{Q_L + W_{net,in}}{W_{net,in}}$$

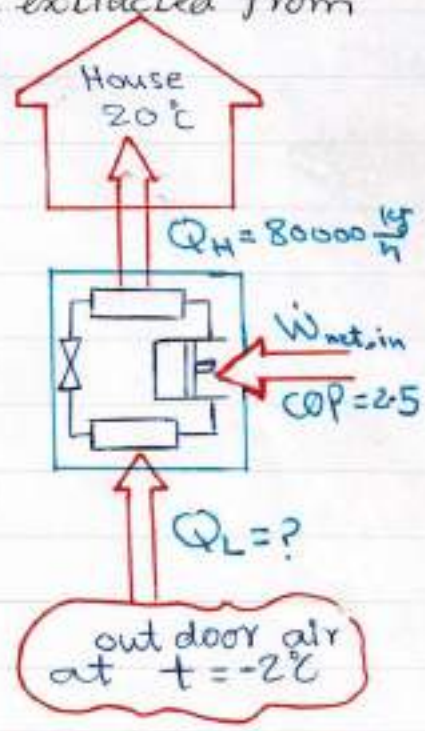
$$= \frac{Q_L}{W_{net,in}} + 1$$

$$COP_{HP} = COP_R + 1$$

2. The Clausius statement:

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower temperature body to a higher temperature body.

Ex: A heat pump is used to meet the heating requirements of a house and maintain it at 20°C. On a day when the outdoor air temperature drops to -2°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump under these conditions has a COP of 2.5, determine (a) the power consumed by the heat pump,
(b) the rate at which heat is extracted from cold outdoor air.



Sol:

a. $COP_{HP} = \dfrac{\dot{Q}_H}{\dot{W}_{net,in}}$

$$\dot{W}_{net,in} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{80000}{2.5}$$

$$= 32000 \text{ kJ/hr}$$

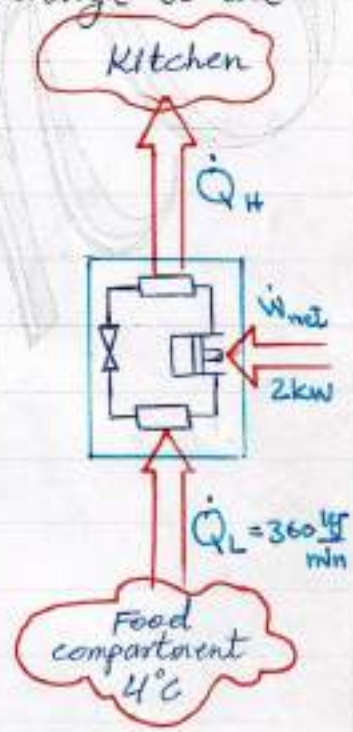
$$= 8.9 \text{ kW}$$

b. $\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,in}$

$$= 80\,000 - 32000$$

$$= 48000 \text{ kJ/h}$$

• Ex:- The food compartment of a refrigerator is maintained at 4°C by removing heat from it at rate of 360 kJ/min. If the required power input to the refrigerator is 2 kW, determine a. COP_R b. the rate of heat discharge to the room that houses the refrigerator.



Sol: a. $COP_R = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{360\ kJ/min}{2\ kW}\left(\frac{1\ kW}{60\ kJ/min}\right)$
$= 3$

b. $\dot{Q}_H = \dot{Q}_L + \dot{W}_{net}$
$= 360\ \frac{kJ}{min} + 2\ kW\left(\frac{60\ kJ/min}{1\ kW}\right)$
$= 480\ kJ/min$

Ex: Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

Sol:- $\dot{Q}_H = 80\ MW$, $\dot{Q}_L = 50\ MW$

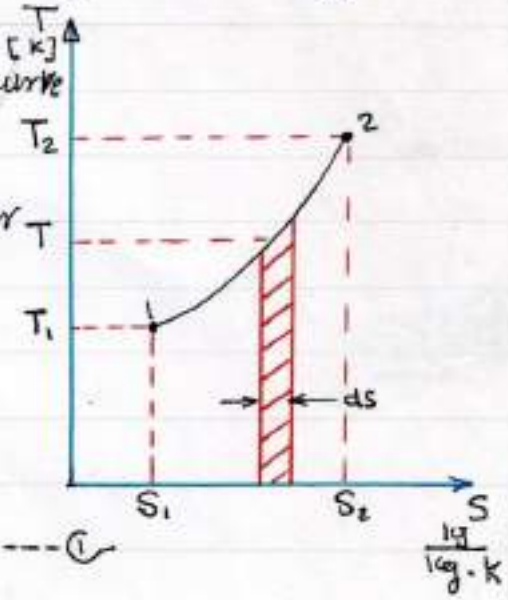
$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L$
$= 80 - 50 = 30\ MW$

$\eta_{thermal} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{30}{80} = 0.375$
$= 37.5\ \%$

ENTROPY

The term 'entropy' which literally means transformation, was first introduced by Clausius. It is an important thermodynamics property of a working substance, which increas withe the addition of heat, and decreases with its removal.

In a reversible process, over a small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working substance. Mathematically, heat absorbed by the working substance,



Heat transfer reversibly = Area under the curve

$$= T\,dS$$

Total heat transfer 1→2 = total area under the curve

$$= \sum_{S=S_1}^{S=S_2} T\,dS$$

in the limit as $ds \to 0$

Heat transfer reversibly

from $1 \to 2 = \int_{S_1}^{S_2} T\,dS = Q_{rev.}$ --- ①

Differentiating eq. ①

$$dQ_{rev.} = T.dS$$

$$\boxed{dS = \frac{dQ_{rev}}{T}} \quad \sim\sim * \; ②$$

This equation gives the relationship which must exist between S, T and Q_{rev} in order that the area of the graph shall be heat transferred reversibly.

Now it is possible to calculate the amount of heat transferred reversibly during a non-flow process. Thus, by using equation (2) changes in (S) can be determined. It is this function (S) which is called entropy.

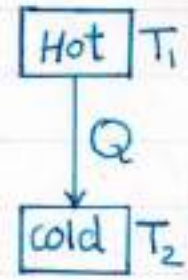
If heat is received, which makes Q_{rev} positive, then the entropy of the receiving substance has increased.

If heat is rejected, which makes Q_{rev} negative then the entropy of the rejecting substance is decreased.

Now consider a given quantity of heat energy Q. Let this quantity of heat be lost from a hot fluid at temperature T_1 and let it be received by a cooler fluid at temperature T_2.

$$S = \frac{Q}{T} = \frac{Q}{T_1}$$

$$T_1 > T_2 \Rightarrow \frac{Q}{T_2} > \frac{Q}{T_1}$$



∴ the gain of entropy to the cooler fluid is greater than the loss of entropy from the hot fluid.

* The entropy of vapours (two-phase system)

δ- Liquid entropy:

$$Q = m C_p (T_2 - T_1)$$

$$Q = m C_p \Delta T \quad , \text{ for 1kg mass}$$

$$Q = C_p \Delta T \quad , \text{ for Liquid}$$

$$Q = C_{p_L} \Delta T \Rightarrow \boxed{dQ = C_{p_L} dT} \quad \text{①}$$

C_{p_L} = Specific heat capacity of the liquid at constant pressure.
$= 4.187$ kJ/kg.K , for H_2O

Dividing eq.(1) throughout by T, then

$$\frac{dQ}{T} = C_{p_L} \frac{dT}{T} = ds \Rightarrow ds = C_{p_L} \frac{dT}{T}$$

$$\int_{s_1}^{s_2} ds = C_{p_L} \int_{T_1}^{T_2} \frac{dT}{T} \Rightarrow S_2 - S_1 = C_{p_L} \ln \frac{T_2}{T_1}$$

when $T_1 = 0°C = 273K \Rightarrow S_1 = 0$

$$S_2 - 0 = C_{p_L} \ln \frac{T_2}{273}$$

$$\therefore \boxed{S = C_{p_L} \cdot \ln \frac{T}{273}} \quad \text{Liquid Entropy}$$

for H_2O , $S = 4.187 \ln \frac{T}{273}$

Ex: Determine the value of the specific entropy of water at 100°C.

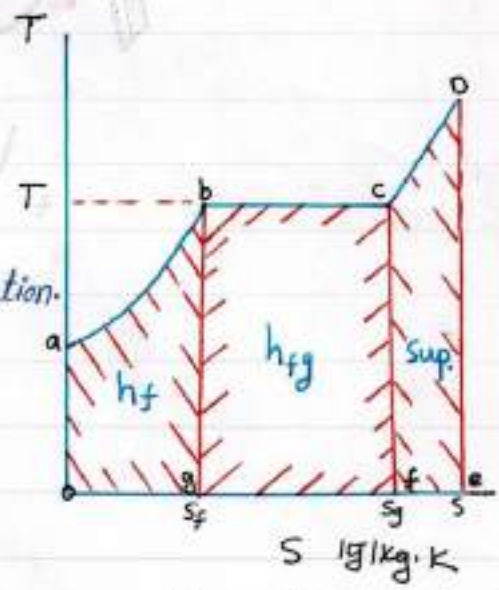
Sol: $S = 4.187 \ln\left(\frac{T}{273}\right)$

$S_f = 4.187 \ln \frac{373}{273} = 1.31$ kJ/kg.K

from Steam tables at 100°C, $S_f = 1.3068$ kJ/kg.K

Entropy of evaporation:

Heat transferred = Change of enthalpy



Area abgo = h_f = Specific liquid enthalpy

Area bcfg = h_{fg} = Specific enthalpy of evaporation.

Area cdef = sup = Specific Superheat.

Area bcfg = $h_{fg} = T_s(S_g - S_f)$

$S_g - S_f = \frac{h_{fg}}{T_s}$ = Specific entropy of evaporation = S_{fg}

for wet Steam

$S = S_f + x S_{fg}$, $S_f = 4.187 \ln \frac{T_s}{273}$ for H_2O

$$\boxed{S = C_{PL} \cdot \ln \frac{T_s}{273} + x \cdot \frac{h_{fg}}{T_s}}$$ for wet Steam

at $x = 1 \Rightarrow S = S_g$

Ex: Determine the value of the specific entropy of wet steam at a pressure of 2 Mpa and 0.8 dry.

a. by calculation b. by using values of entropy from steam tables.

Sol: a. $S = C_p \cdot \ln \frac{T_s}{273} + x \frac{h_{fg}}{T_s}$

from steam tables at $p = 2$ Mpa, $T_s = 212.42°C = 485.4$ K
$h_{fg} = 1890.74$

$\therefore S = 4.187 \ln \frac{485.4}{273} + 0.8 * \frac{1890.74}{485.4}$

$= 5.525$ kJ/kg.K

b. $S = S_f + x S_{fg}$, at 2 Mpa, $S_f = 2.4473$ kJ/kg.K
$S_{fg} = 3.8935$ kJ/kg.K

$= 2.4473 + 0.8 * 3.8935$

$= 5.562$ kJ/kg.K

Entropy of Superheated Vapour:

Area cdef $= C_{pv} \cdot dT$

C_{pv} = Specific heat capacity of superheated vapour at constant pressure.

$dQ = C_{pv} \cdot dT \quad \} \div T$

$\frac{dQ}{T} = C_{pv} \cdot \frac{dT}{T} \Rightarrow ds = C_{pv} \cdot \frac{dT}{T}$

$$\int_{s_g}^{s} ds = C_{pv} \cdot \int_{T_s}^{T} \frac{dT}{T} \Rightarrow s - s_g = C_{pv} \cdot \ln \frac{T}{T_s}$$

$$s = s_g + C_{pv} \ln \frac{T}{T_s}$$

$$\boxed{s = C_{pL} \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \ln \frac{T}{T_s}}$$

Ex: Determine the value of Specific entropy of Steam at 1.5 Mpa with a temperature of 300°C. $C_{pv} = 2.0934$ kJ/kg.K

a. by calculation b. from Steam tables

Sol: a. $s = C_{pL} \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \ln \frac{T}{T_s}$

from Steam tables at 1.5 Mpa, $T_s = 198.32°C = 471.32$ K

$h_{fg} = 1947.28$ kJ/kg

$$s = 4.187 \times \ln \frac{471.32}{273} + \frac{1947.28}{471.32} + 2.0934 \times \ln \frac{573}{471.32}$$

$$= 6.8268 \text{ kJ/kg.K}$$

b. from Superheated Steam tables at 1.5 Mpa & 300°C

$s = 6.9188$ kJ/kg.K

	1400 kPa	(1500)	1600 kPa
300°C	6.9533	s	6.8844

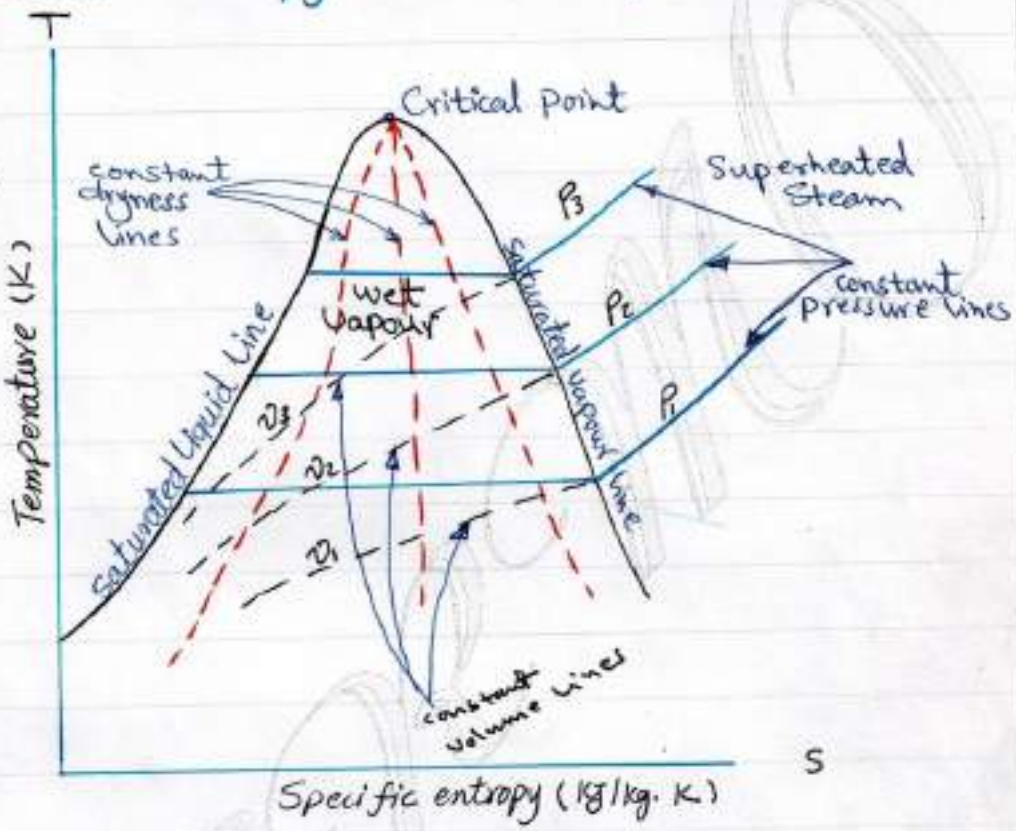
$$\frac{1500 - 1400}{1600 - 1400} = \frac{s - 6.9533}{6.8844 - 6.9533}$$

(The temperature - entropy chart for vapour)

$P_1 < P_2 < P_3$

$v_1 > v_2 > v_3$

$S_x = S_f + x S_{fg}$



Ex: 1 kg of Steam at 7 bar, entropy 6.5 kJ/kg.K is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied and show on T-s diagram the area which represent the heat flow.

Sol:

at 7 bar from Steam tables, $S_g = 6.708 \frac{kJ}{kg.K}$

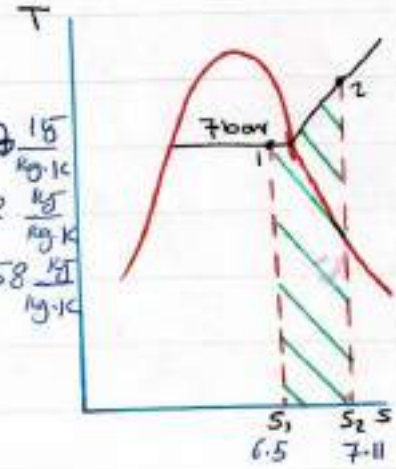
$S_1 < S_g \Rightarrow$ wet Steam, $S_f = 1.9922 \frac{kJ}{kg.K}$

$S_{fg} = 4.7158 \frac{kJ}{kg.K}$

$S_1 = S_f + x S_{fg}$

$6.5 = 1.9922 + x \cdot 4.7158$

$x = 0.9558$



$h_1 = h_f + x \cdot h_{fg}$, $h_f = 697.2$ kJ/kg , $h_{fg} = 2066.3$ kJ/kg

$h_1 = 697.2 + 0.9558 \times 2066.3 = 2672.169$ kJ/kg

at point ② $p = 7$ bar , $t = 250°C$

from Steam tables $t_s = 164.97°C$, $t > t_s \Rightarrow$ Superheated

from Superheated Steam tables , $h_2 = 2953.565$ kJ/kg

	600kPa	(700kPa)	800kPa
$h \to 250°C$	2957.16	h_2	2949.97
$S \to 250°C$	7.1816	S_2	7.0384

$Q = H_2 - H_1 = m(h_2 - h_1)$

$= 1(2953.565 - 2672.169)$

$= 281.396$ kJ

$S_2 = 7.11$ kJ/kg·K

Ex: A rigid Cylinder of volume $0.025\ m^3$ contains steam at 80bar and 350°C the Cylinder is cooled Until the pressure is 50bar calculate the state of the steam after cooling and amount of heat rejected by the steam. sketch the process on T-s diagram

Sol:

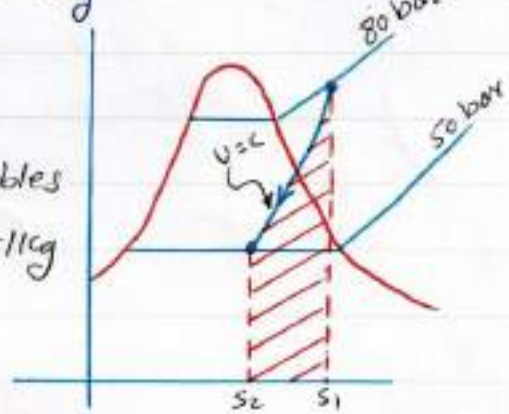
at $P = 80$ bar $t_s = 295.06°C$ $t > t_s \Rightarrow$ Superheated

from Steam table , $v_1 = 0.02995\ m^3/kg$

$v_1 = \frac{V_1}{m} \Rightarrow m = \frac{0.025}{0.02995} = 0.8347$ kg

at $P_1 = 80$ bar & 350°C from Steam tables

$h_1 = 2987.3$ kJ/kg , $u_1 = 2747.67$ kJ/kg



$v_1 = v_2$ (rigid cylinder)

$\therefore v_2 = 0.02995\ m^3/kg$, at $P_2 = 50\,bar$, $v_g = 0.03944\ \frac{m^3}{kg}$

$\because v_g > v_2 \Rightarrow$ wet Steam

$v_2 = x\, v_g \Rightarrow 0.02995 = x_2 * 0.03944 \Rightarrow x_2 = 0.759$

at 50 bar , $u_f = 1147.78\ kJ/kg$, $u_{fg} = 1449.34\ kJ/kg$

$u_2 = u_f + x\, u_{fg} = 1147.78 + 0.759 * 1449.34$

$= 2247.82\ kJ/kg$

$Q = \cancel{W}^{0} + \Delta U$, $V = c \Rightarrow W = 0$

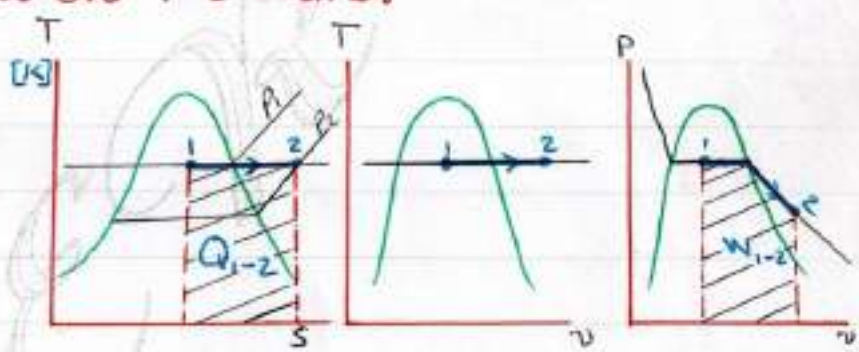
$= m(u_2 - u_1) = 0.8347(2247.82 - 2747.67)$

$= -417.22\ kJ$ (rejected heat)

* Isothermal process on T-s chart:

$Q = T(S_2 - S_1)$

$T_1 = T_2$

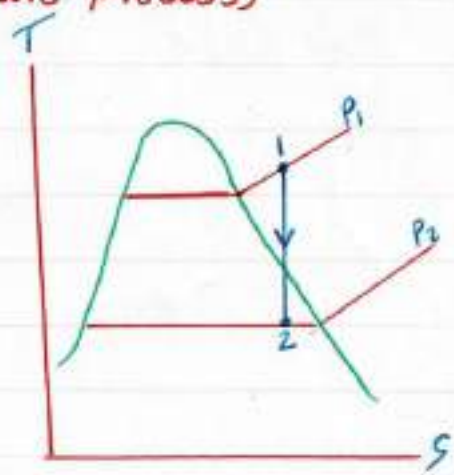


* Isentropic process (reversible adiabatic process)

$Q = 0$, $S_1 = S_2$

$dQ = T\,dS$, $S = constant$

$\therefore Q = 0$ $\therefore dS = 0$



* The Entropy of Ideal gas (single-phase system)

$dQ = dW + dU$, for non-flow process

for unit mass of gas and let its state change from P_1, T_1, v_1 to new state P_2, T_2, v_2

$$dQ = dW + dU \quad \text{for unit mass}$$

$$dq = du + dw$$

$$dq = C_v dT + P dv \;\}\div T \quad ----(1)$$

$$\frac{dq}{T} = C_v \frac{dT}{T} + \frac{P}{T} dv \quad ----(2)$$

$$Pv = RT \Rightarrow P/T = R/v \quad ----(3)$$

sub. eq (3) in (2)

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v}$$

$$\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} C_v \frac{dT}{T} + \int_{v_1}^{v_2} R \frac{dv}{v} \quad ----(4)$$

$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}} \quad ----(5)$$

eq (5) represent the relation between (S, T, v).

$C_p - C_v = R$, substituting this into eq. 5

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{v_2}{v_1} \quad ----(6)$$

$$S_2 - S_1 = C_v \left(\ln \frac{T_2}{T_1} - \ln \frac{v_2}{v_1}\right) + C_p \ln \frac{v_2}{v_1} \quad ----(7)$$

$$S_2 - S_1 = C_v \ln\left(\frac{T_2}{T_1}\cdot\frac{v_1}{v_2}\right) + C_p \ln\frac{v_2}{v_1} \quad ---- (8)$$

$$\because \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \Rightarrow \frac{T_2}{T_1}\cdot\frac{v_1}{v_2} = \frac{P_2}{P_1} \quad ---- (9)$$

Sub. eq. (9) in (8)

$$\boxed{S_2 - S_1 = C_v \ln\frac{P_2}{P_1} + C_p \ln\frac{v_2}{v_1}} \quad --- (10)$$

eq. (10) represent the relation between (S, P, v)

$C_v = C_p - R$, sub. in eq (5)

$$S_2 - S_1 = C_v \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} \quad ---- (5)$$

$$\therefore S_2 - S_1 = (C_p - R) \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1} \quad ---- (11)$$

$$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\left(\ln\frac{T_2}{T_1} - \ln\frac{v_2}{v_1}\right) \quad ---- (12)$$

$$S_2 - S_1 = C_p \cdot \ln\frac{T_2}{T_1} - R \ln\left(\frac{T_2}{T_1}\cdot\frac{v_1}{v_2}\right) \quad ---- (13)$$

$$\because \frac{T_2}{T_1}\cdot\frac{v_1}{v_2} = \frac{P_2}{P_1}$$

$$\boxed{S_2 - S_1 = C_p \cdot \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}} \quad ---- (14)$$

eq. (14) represent the relation between (S, T, P)

بسم الله الرحمن الرحيم
{ وما أوتيتم من العلم إلا قليلا }
صدق الله العظيم

- Isometric Process $V=C$

from eq (5) & (10)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{---- (5)}$$

$$v_2 = v_1 \Rightarrow v_2/v_1 = 1 \Rightarrow \ln \frac{v_2}{v_1} = \ln 1 = 0$$

$$\therefore S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$$

and

$$S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1} \quad \text{---- (10)}$$

$$v_2 = v_1 \Rightarrow \ln v_2/v_1 = 0$$

$$\therefore S_2 - S_1 = C_v \ln \frac{P_2}{P_1}$$

$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} = C_v \ln \frac{P_2}{P_1}}$$ for constant volume

- Isobaric process $P=C$

from eq (10) & (14)

$$S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1} \quad \text{---- (10)}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{---- (14)}$$

But $P_1 = P_2 \Rightarrow \ln (P_2/P_1) = 0$, Sub. in eq (10) & (14)

$$\boxed{S_2 - S_1 = C_p \ln \frac{v_2}{v_1} = C_p \ln \frac{T_2}{T_1}}$$ for constant pressure

- Isothermal process $T=C$

from eqs 5 & 14

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (5)}, \quad T_1 = T_2 \Rightarrow \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = R \ln \frac{v_2}{v_1}$$

and

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- (14)} \quad \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = -R \ln \frac{P_2}{P_1} = R \ln \frac{P_1}{P_2}$$

$$\boxed{S_2 - S_1 = R \ln \frac{v_2}{v_1} = R \ln \frac{P_1}{P_2}}$$ for constant temperature

- Polytropic Process $Pv^n = c$

Heat transferred $= \frac{\gamma - n}{\gamma - 1} \cdot$ Work done

$$dQ = \frac{\gamma - n}{\gamma - 1} P dv \quad \} \div T$$

$$\frac{dQ}{T} = ds = \frac{\gamma - n}{\gamma - 1} \frac{P dv}{T}$$

$$Pv = RT \Rightarrow P/T = R/v$$

$$ds = \frac{\gamma - n}{\gamma - 1} \cdot R \cdot \frac{dv}{v}$$

$$\int_{s_1}^{s_2} ds = \frac{\gamma - n}{\gamma - 1} R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} R \ln \frac{v_2}{v_1} \quad , \quad R = C_v(\gamma - 1)$$

$$S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} \cdot C_v(\gamma - 1) \ln \frac{v_2}{v_1}$$

$$= C_v(\gamma - n) \ln \frac{v_2}{v_1}$$

$$\boxed{S_2 - S_1 = C_v(\gamma - n) \ln \frac{v_2}{v_1}} \quad , \quad \frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$S_2 - S_1 = C_v(\gamma - n) \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$\boxed{S_2 - S_1 = C_v \frac{\gamma - n}{n - 1} \cdot \ln \frac{T_1}{T_2}}$$

$$S_2 - S_1 = C_v(\gamma - n) \ln \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$\boxed{S_2 - S_1 = C_v \frac{\gamma - n}{n} \ln \frac{P_1}{P_2}}$$

and

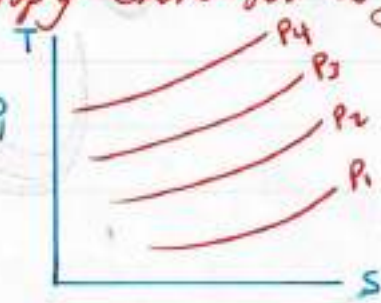
$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{P_2}{P_1} = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}}$$

- *Isentropic process* $S = C$

$$S_2 - S_1 = 0$$

* *The entropy chart for a gas (perfect gas)*

$P_4 > P_3 > P_2 > P_1$

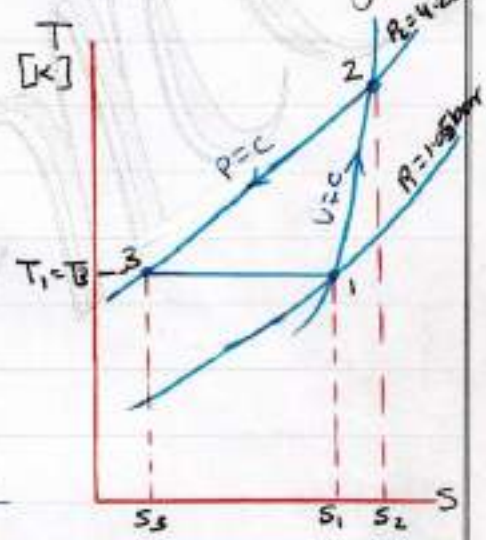


$v_1 > v_2 > v_3 > v_4$

slope v > Slope P

Ex: Air at 15°C and 1.05 bar occupies 0.02 m³. The air is heated at constant volume until the pressure is 4.2 bar, and then cooled at constant pressure back to the original temperature. Calculate the heat flow to or from the air and the net entropy change. Sketch the processes on T-S diagram.

Sol:



$R = 0.287$ kJ/kg·K

$C_v = 0.718$ kJ/kg·K , $C_p = 1.005$ kJ/kg·K

$P_1 V_1 = m R T_1$, $T_1 = 15 + 273 = 288$ K

$1.05 * 10^2 * 0.02 = m * 0.287 * 288$

$m = 0.0254$ kg

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow T_2 = \frac{P_2 \cdot T_1}{P_1} = \frac{4.2 * 288}{1.05} = 1152 \text{ K}$$

$$Q_{1-2} = m C_v (T_2 - T_1) = 0.0254 * 0.718 * (1152 - 288) = 15.75 \text{ kJ}$$

$$Q_{2\to3} = m C_p (T_3 - T_2) = 0.0254 * 1.005 * (288 - 1152) = -22.05 \text{ kJ}$$

$$Q_{1\to3} = Q_{1-2} + Q_{2-3} = 15.75 + (-22.05) = -6.3 \text{ kJ}$$

for $v = c$, $S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$ kJ/kg·K

$S_2 - S_1 = m C_v \ln \frac{T_2}{T_1}$ kJ/K

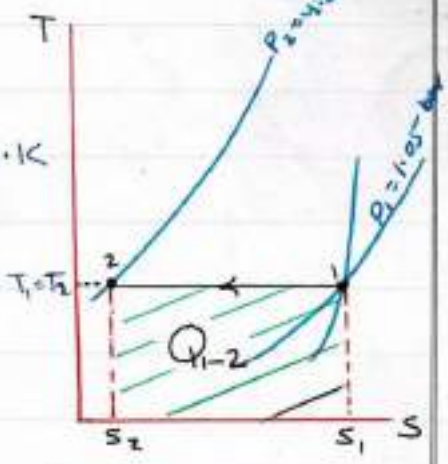
$$S_2 - S_1 = 0.0254 * 0.718 * \ln \frac{1152}{288} = 0.0253 \text{ kJ/K}$$

for $p=c$ from $2 \to 3$

$$S_3 - S_2 = m\,C_p \ln \frac{T_3}{T_2} = 0.0254 * 1.005 * \ln \frac{288}{1152}$$
$$= -0.0354 \text{ kJ/K}$$

$$\Delta S_{net} = \Delta S_{1-2} + \Delta S_{2-3}$$
$$= 0.0253 - 0.0354$$
$$= -0.0101 \text{ kJ/K}$$

Ex: $0.03\ m^3$ of nitrogen contained in a cylinder behined a piston is initially at 1.05 bar and 15°C. The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, the work done. Sketch the process on P-V & T-s diagram.



Sol: $R = R_o/M = 8.314/28 = 0.297$ kJ/kg·K

$P_1 V_1 = mRT_1$, $T_1 = 15 + 273 = 288$ K

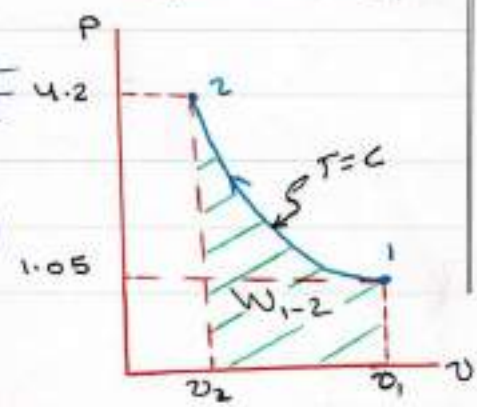
$1.05 * 10^2 * 0.03 = m * 0.297 * 288$

$m = 0.0368$ kg

$$S_2 - S_1 = m\,C_p \ln \cancel{\frac{T_2}{T_1}}^{\,0} + mR \ln \frac{P_1}{P_2}$$

$$\therefore S_2 - S_1 = mR \ln \frac{P_1}{P_2} = 0.0368 * 0.297 \ln \frac{1.05}{4.2}$$
$$= -0.01516 \text{ kJ/K}$$

Q_{1-2} = area under the line representing the process $1 \to 2$



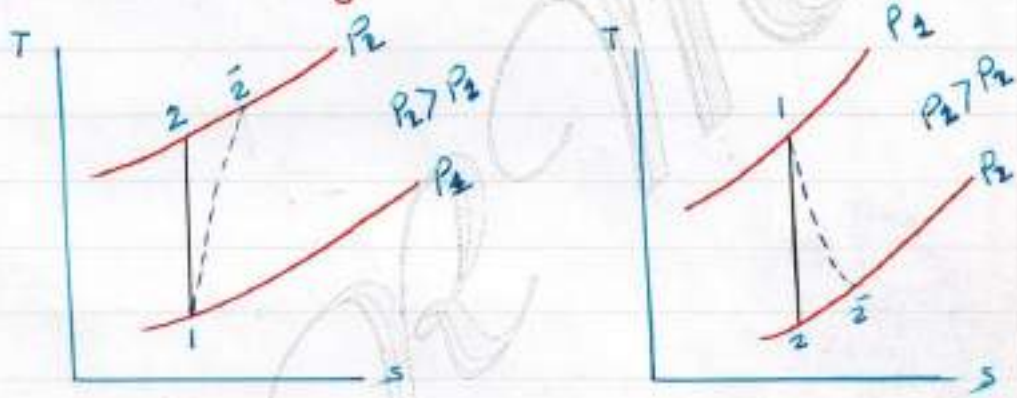
$Q = T_1(S_2 - S_1) = 288 * -0.01516 = -4.37$ kJ

$Q = W + \Delta U$, $T = C \Rightarrow \Delta U = 0$

$\therefore Q = W$

$\Rightarrow W = -4.37$ kJ

• Entropy & Irreversiblity

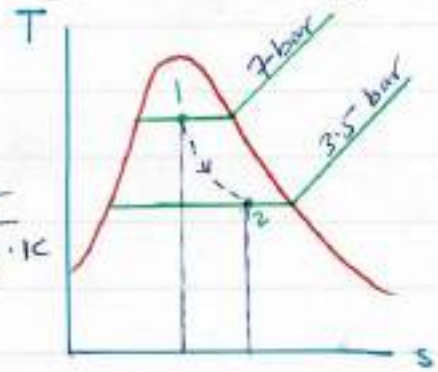


- A - - B -

fig. A represent the compression of fluid.
" B " " expansion " " .

Ex: Steam at 7 bar, dryness fraction 0.96, is throttled down to 3.5 bar, calculate the change of entropy per kg of Steam.

Sol:



at 7 bar, $S_1 = S_f + x S_{fg}$

$\therefore S_1 = 1.99 + 0.96 * 4.717 = 6.522 \frac{kJ}{kg \cdot K}$

throttling $\Rightarrow h_1 = h_2$

$h_1 = h_f + x h_{fg} = 697 + 0.96 * 2067$
$= 2682 \text{ kJ/kg}$

$\therefore h_2 = 2682 \text{ kJ/kg}$, from Steam tables at $P = 3.5$ bar
$h_g = 2732 \text{ kJ/kg}$, $h_f = 584 \text{ kJ/kg}$

$h_f < h_2 < h_g \Rightarrow$ wet Steam

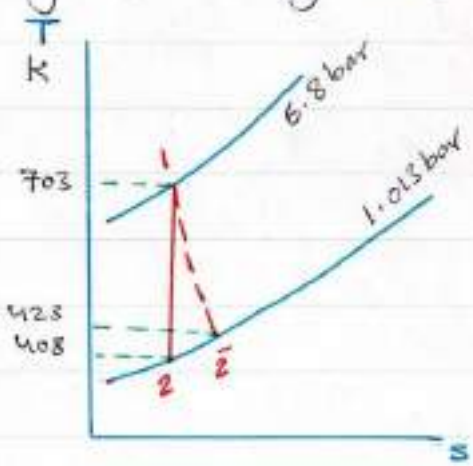
$h_2 = h_f + x h_{fg} \Rightarrow 2682 = 584 + x_2 * 2148 \Rightarrow x_2 = 0.977$

$S_2 = S_f + x_2 S_{fg}$, at $P = 3.5$ bar

$S_2 = 1.727 + 0.977 * 5.214$
$= 6.817 \text{ kJ/kg.K}$

Ex: In an air turbine the air expands from 6.8 bar and 430°C to 1.013 bar and 150°C. The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and Calculat the change of entropy. $\gamma = 1.4$

Sol:



since the process is with negligible heat loss.

$\therefore$ the process is adiabatic.

If it is adiabatic & reversible.

$\therefore$ it is isentropic

$T_1 = 430 + 273 = 703 \text{ K}$

$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{703}{T_2} = \left(\frac{6.8}{1.013}\right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 408 \text{ K} = 135°\text{C}$

the process is irreversible because $T_2 < T_{\bar{2}}$
(If $T_2 = T_{\bar{2}} \Rightarrow$ reversible process)

$S_1 - S_{\bar{2}} = S_2 - S_{\bar{2}}$, $S_1 = S_2$ (reversible adiabatic)

the process $2 \rightarrow \bar{2}$, $P = C$

$\therefore S_2 - S_{\bar{2}} = C_p \ln \frac{T_{\bar{2}}}{T_2} = 1.005 \ln \frac{423}{408} = 0.0355 \text{ kJ/kg.K}$

If ΔS_{total} → > 0 irreversible process
→ $= 0$ reversible process
→ < 0 impossible process
} for adiabatic process

H.W:

1. Calculate the change of entropy of 1kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°c to 1.05 bar. The index of expansion is 1.3.

2. 0.05 kg of carbone dioxide (M=44) is compressed from 1 bar & 15°c until the pressure is 8.3 bar and the volume is then 0.004 m³. Calculate the change of entropy.
Take: $C_p = 0.88 \frac{kJ}{kg.K}$.

(Sheet No. 6)

Q1:

A rigid cylinder containing $0.006 m^3$ of nitrogen at 1.04 bar, 15°C, is heated reversibly until the temperature is 90°C. Calculate the change of entropy and the heat supplied. Sketch the process on T-s diagram. Take $\gamma = 1.4$, assume nitrogen is a perfect gas. Ans. [$0.00125 \frac{kJ}{K}$, 0.407 kJ]

Q2:

$1 m^3$ of air is heated reversibly at constant pressure from 15°C to 300°C, and then cooled reversibly at constant volume back to the initial temperature, the initial pressure is 1.03 bar. Calculate the net heat flow and the overall change of entropy. Sketch the process on T-s diagram. Ans. [101.5 kJ , 0.246 kJ/K]

Q3:

1 kg of air is allowed to expand reversibly in a cylinder behind a piston in such away that the temperature remains constant at 260°C while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the overall change of entropy. Sketch the process on T-s diagram.

Ans. [$-161.9 \frac{kJ}{kg}$, $-0.497 \frac{kJ}{kg \cdot K}$

Q4: 1 kg of air at 1.02 bar, 20°C undergoes a process in which the pressure is raised to 6.12 bar, and the volume becomes $0.25 m^3$. Calculate the change of entropy. Ans. [0.087 kJ/kg·K]

Q5: Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated. to a pressure of 65 kpa. Assuming that the Cylinder is perfectly thermally insulated. Calculate the work done during the expansion per kg of Steam. Ans. [323.8 kJ/kg]

Q6: In a centrifugal Compressor the air is compressed through a pressure ratio of 4 to 1, and the temperature of the air increases by a factor of 1.65. Show that the process is irreversible and calculate the change of entropy per kg of air. Assume the process is adiabatic. Ans. [0.105 kJ/kg.K]

Q7: A turbine is supplied with Steam at 40 bar, 400°C, which expands through the turbine in Steady flow to an exit pressure of 0.2 bar and a dryness fraction of 0.93. The inlet velocity is negligible but the Steam leaves at high velocity through a duct of 0.14 m^3 cross-sectional area. If the mass flow rate is 3 kg/s, and the mechanical efficiency is 90%. Calculate the horse power output of the turbine. Show that the process is irreversible and calculate the change of entropy, negligible the heat losses from the turbine.

Ans. [2018 kW, 0.643 kJ/kg.K]

The Heat Engine Cycles

A system is said to have undergone a cycle if it returns to its initial state at the end of the process, and we have two types of cycles 1. Thermodynamic cycles 2. Mechanical cycles.

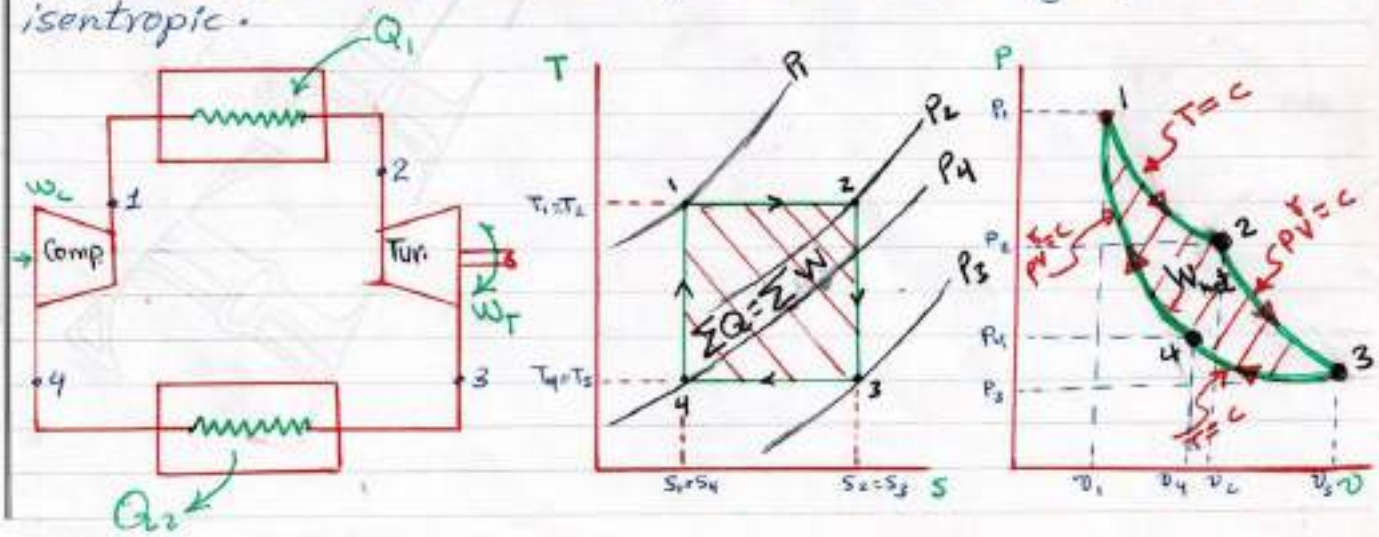
It can be shown that there is an ideal theoretical cycle which is the most efficient conceivable; this cycle is called the **Carnot** cycle. The highest thermal efficiency possible for a heat engine in practice is only about half that of the ideal theoretical carnot cycle, between the same temperature limits.

• The Carnot cycle:

It can be shown from the Second Law of Thermodynamics that no heat engine can be more efficient than a reversible heat engine working between the same temperature.

Carnot, a French engineer, showed that the most efficient possible cycle is one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature.

The cycle therefore consists of two isothermal processes joined by two adiabatic processes. Since all the process are reversible, then the adiabatic processes in the cycle are also isentropic.



* Carnot cycle on T-S diagram

- **Process 1→2 :** Isothermal process $T=C$

from non-flow energy equation

$$Q = W + \Delta U \quad , \text{ for gas } \Delta U = 0 \text{ when } T = C$$

$$\therefore \quad Q = W = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1}$$
$$= T(S_2 - S_1)$$

- **Process 2→3 :** reversible adiabatic expansion $PV^{\gamma} = C$

(isentropic $\Delta S = 0$, $S = C$)

$$Q = W + \Delta U \quad , \quad Q = 0$$

$$\therefore \quad W = -\Delta U = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1} = \frac{mR(T_2 - T_3)}{\gamma - 1}$$

- **Process 3→4 :** Isothermal process , heat rejection $T = C$

$$Q = W + \Delta U$$

for gas $\Delta U = 0$ when $T = C$

$$Q = W = P_3 V_3 \ln \frac{V_4}{V_3} = mRT_3 \ln \frac{V_4}{V_3} = T(S_3 - S_4)$$

- **Process 4→1 :** reversible adiabatic compression $PV^{\gamma} = C$

(isentropic $\Delta S = 0$, $S = C$)

$$Q = W + \Delta U \quad , \quad Q = 0$$

$$W = -\Delta U = \frac{P_4 V_4 - P_1 V_1}{\gamma - 1} = \frac{mR(T_4 - T_1)}{\gamma - 1}$$

Work done by Carnot Cycle.

$$W.D = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$= mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_2 - T_3)}{\gamma - 1} + mRT_3 \ln \frac{V_4}{V_3} + \frac{mR(T_4 - T_1)}{\gamma - 1}$$

Since $T_1 = T_2$ and $T_3 = T_4$

$$\therefore \quad (T_2 - T_3) = (T_1 - T_4)$$

$$W.D = mRT_1 \ln \frac{V_2}{V_1} + \cancel{\frac{mR(T_2 - T_3)}{\gamma - 1}} + mRT_3 \ln \frac{V_4}{V_3} - \cancel{\frac{mR(T_1 - T_4)}{\gamma - 1}}$$

$$\therefore \; W.D = mRT_1 \ln \frac{V_2}{V_1} + mRT_3 \ln \frac{V_4}{V_3}$$

Process $4 \to 1$ $\quad pV^{\gamma} = c$, Process $2 \to 3$ $\quad pV^{\gamma} = c$

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad , \quad \frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$$

since $\frac{T_1}{T_4} = \frac{T_2}{T_3} \Rightarrow \frac{V_4}{V_1} = \frac{V_3}{V_2} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$

$$\therefore \; W.D = mRT_1 \ln \frac{V_2}{V_1} - mRT_3 \ln \frac{V_2}{V_1}$$

$$\boxed{W.D = mR(T_1 - T_3) \ln \frac{V_2}{V_1}}$$

$$\boxed{\text{Work ratio} = \frac{\text{Work done}}{\text{gross work}}}$$

thermal efficiency. η_{th}

$$\eta_{th} = \frac{\sum dQ}{Q_{add}} = \frac{Q_1 - Q_2}{Q_1} = \frac{W.D}{Q_1}$$

$$\eta_{th} = \frac{mR(T_1 - T_3) \ln \frac{V_2}{V_1}}{mRT_1 \ln \frac{V_2}{V_1}} = \frac{T_1 - T_3}{T_1} \Rightarrow$$

$$\boxed{\eta_{th})_{carnot} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_{min}}{T_{max}}}$$

Carnot Cycle on T-s diagram:

1. Heat added $= mT_1(S_2 - S_1)$
2. Heat rejected $= mT_3(S_2 - S_1)$
3. Work done by the cycle $= \sum dw = \sum dQ = Q_1 - Q_2$

$$W.D = mT_1(S_2 - S_1) - mT_3(S_2 - S_1)$$
$$= m(T_1 - T_3)(S_2 - S_1) \quad \text{------} \quad (1)$$

the chang of entropy $(S_2 - S_1)$ occure at constant temperature.

$$Q = W + \Delta U$$
$$dQ = dW + dU = pdV + CvdT \;\} \div T$$

$$\frac{dQ}{T} = P\frac{dV}{T} + C_v\frac{dT}{T} = dS \quad , \text{since } T = C \Rightarrow dT = 0$$

$$dS = \frac{P}{T}\,dV \quad , PV = mRT \Rightarrow \frac{P}{T} = \frac{mR}{V}$$

$$dS = mR\ln\frac{V_2}{V_1}$$

$$ds = R\ln\frac{v_2}{v_1} \quad ---- \text{②}$$

Sub. eq ② in ①

$$\therefore W.D = mR(T_1 - T_3)\ln\frac{V_2}{V_1}$$

$$\eta_{th} = \frac{\sum dQ}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mT_3(S_2 - \cancel{S_1})}{mT_1(S_2 - \cancel{S_1})}$$

$$\boxed{\eta_{th} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_{min}}{T_{max}}}$$

r = volume ratio of adiabatic process

$$r = \frac{V_3}{V_2} = \frac{V_4}{V_1} \quad , \frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad , T_1 = T_2$$

$$\eta_{th} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_3}{T_2} \Rightarrow \boxed{\eta_{th} = 1 - \frac{1}{r^{\gamma-1}}}$$

- Types of volume ratio:

1. Volume ratio of adiabatic process:

$$r = \frac{V_3}{V_2} = \frac{V_4}{V_1} > 1$$

2. Volume ratio of isothermal process:

$$r = \frac{V_3}{V_4} = \frac{V_2}{V_1} > 1$$

3. Overall volume ratio:

$$r = \frac{V_3}{V_1} = \frac{\text{max. Volume in Cycle}}{\text{min. Volume in Cycle}}$$

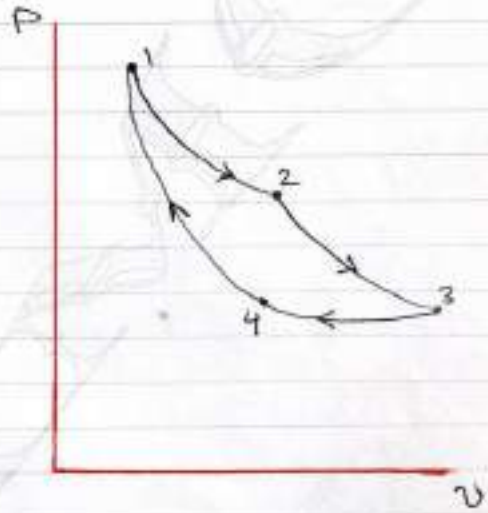
Ex: The overall volume ratio of a carnot Cycle is (15). The temperature limits of the cycle is 260°C and 21°C. Determine:

a. the volume ratio of isothermal and adiabatic process.

b. the thermal efficiency. (for air)

Sol:

$$\frac{V_3}{V_1} = 15$$



$$r = \frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}} \quad , T_1 = T_2$$

$$\therefore r = \left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}} = \left(\frac{260+273}{21+273}\right)^{1/(1.4-1)}$$

$$r = 4.425$$

$$\frac{V_2}{V_1} = \frac{V_2}{V_1} \cdot \frac{V_3}{V_3} = \frac{V_2}{V_3} \cdot \frac{V_3}{V_1}$$

$$= \frac{1}{4.425} \cdot 15 = 3.35$$

$$\eta_{th} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{(21+273)}{(260+273)}$$

$$= 0.448$$

$$\therefore \eta_{th} = 44.8\%$$

Ex: 1kg of air is at initial pressure and temperature of 1.73Mpa, 175°C respectively. It is expand isothermally to three times of the initial volume, and then further expand adiabaticaly to six times of the initial volume. Isothermal compression followed by adiabatic compression returen to its original state.

for this cycle:

a. name the cycle b. determine (P, T & V) at each corner of Cycle c. find η_{th} d. calculate the work done

Sol: a. the cycle is carnot cycle

b. point (1) $P_1 = 1.73 Mpa$, $T_1 = 175°C$

$$P_1V_1 = mRT_1 \Rightarrow V_1 = mRT_1/P_1 \Rightarrow V_1 = 0.075\ m^3$$

Point (2)

$$V_2 = 3V_1 \Rightarrow V_2 = 0.225\ m^3$$

$$T_1 = T_2 = 175°C \quad , (T = C)$$

$P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2}$

point (3) $\quad V_3 = 6V_1 \Rightarrow V_3 = 0.45\,m^3$

$$P_2V_2^{\gamma} = P_3V_3^{\gamma} \Rightarrow P_3 = \left(\frac{V_2}{V_3}\right)^{\gamma} \cdot P_2 \Rightarrow P_3 = 0.219\,Mpa$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} \Rightarrow T_3 = \left(\frac{P_3V_3}{P_2V_2}\right)T_2 \Rightarrow T_3 = 67.6^{\circ}C$$

point (4)

$$T_3 = T_4 = 67.6^{\circ}C \qquad (T=C)$$

$$\left(\frac{T_1}{T_4}\right)^{\frac{1}{\gamma-1}} = \frac{V_4}{V_1} \Rightarrow V_4 = 0.148\,m^3$$

$$\frac{P_1}{P_4} = \left(\frac{V_4}{V_1}\right)^{\gamma} \Rightarrow P_4 = 0.656\,Mpa$$

c. $\quad \eta_{th} = 1 - \frac{T_{min}}{T_{max}} \Rightarrow \eta_{th} = 0.242 = 24.2\%$

d. $\quad W.D = mR(T_1 - T_3)\ln\frac{V_2}{V_1}$
$= 34.2\,kg$

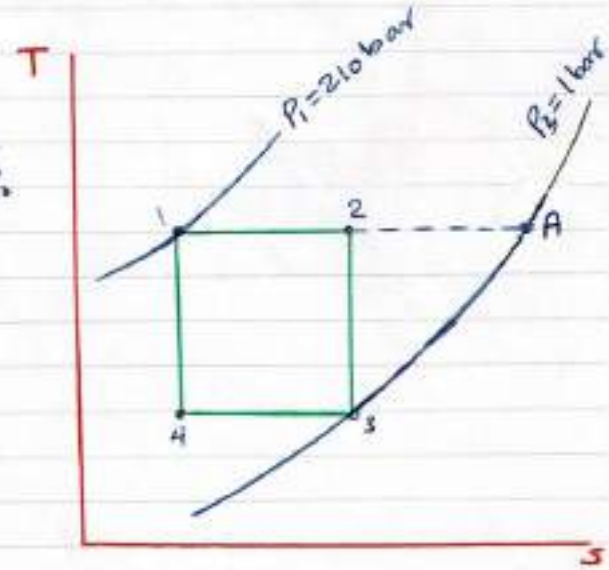
Ex: A hot reservoir at 800°C and cold reservoir at 15°C are available. Calculate the thermal efficiency and the work ratio of a carnot cycle using air as the working fluid. If the max & min pressure in the cycle are 210 bar & 1 bar.

Sol:

$$\eta_{carnot} = 1 - \frac{T_3}{T_2} = 1 - \frac{288}{1073} = 0.732 = 73.2\%$$

at $T = C$,

$$S_A - S_1 = R\ln\frac{P_1}{P_3}$$
$$= 0.287\ln\frac{210}{1}$$
$$= 1.535\,kJ/kg.K$$



at $p=c$

$$S_A - S_3 = C_p \ln \frac{T_2}{T_3} = 1.005 \ln \frac{1073}{288}$$
$$= 1.34 \text{ kJ/kg.K}$$

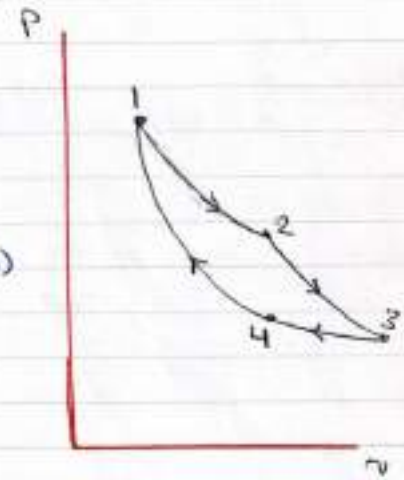
$$S_3 - S_1 = S_2 - S_1 = (S_A - S_1) - (S_A - S_3)$$
$$= 1.5353 - 1.321$$
$$= 0.214 \text{ kJ/kg.K}$$

$$\Sigma W = \Sigma Q = \text{area } 12341 = (T_2 - T_3)(S_2 - S_1) \;,\; T_2 = T_1$$

$$\Sigma W = (1073 - 288)(0.214)$$

$\therefore W_{net}$ (W.D) $= 168$ kJ/kg

$$W_{gross} = W_{1-2} + W_{2-3}$$



$$W_{1-2} = Q_{1-2} = T_1(S_2 - S_1) \;,\; T = c \Rightarrow \Delta U = 0$$
$$= 1073 * 0.214$$
$$= 229.6$$

$$W_{2-3} = U_2 - U_1 = C_v(T_2 - T_3) \;,\; (Q = 0 \;,\; \Delta S = 0)$$
$$= 0.718(1073 - 288)$$
$$= 563.6 \text{ kJ/kg}$$

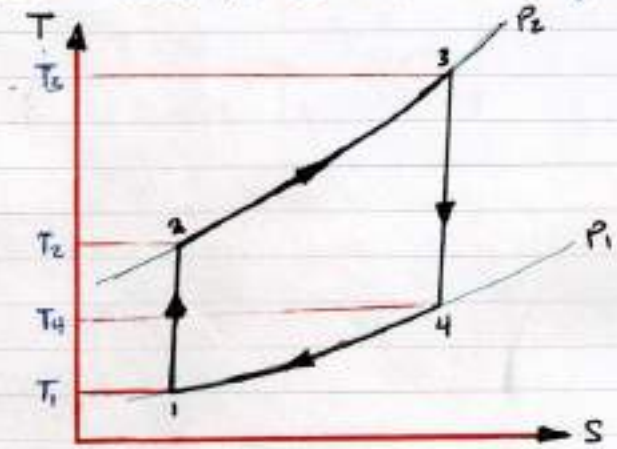
$$W_{gross} = 229.6 + 563.6$$
$$= 793.2 \text{ kJ/kg}$$

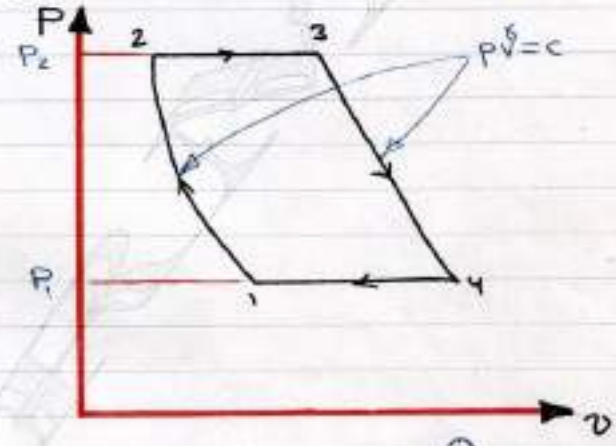
$$\text{Work ratio, (W.R.)} = \frac{W_{net}}{W_{gross}} = \frac{168}{793.2}$$
$$= 0.212$$

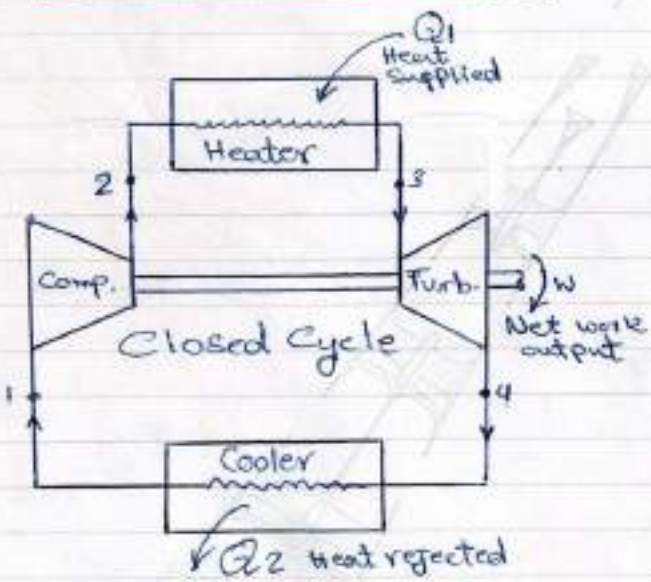
Joule or Brayton cycle

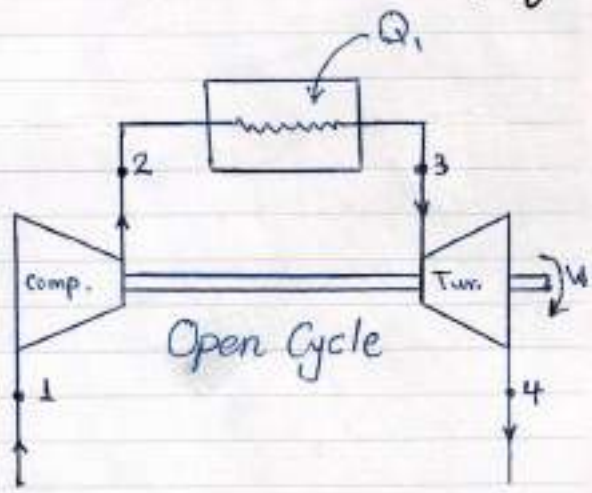
(constant pressure cycle)

In this cycle the heat supply and heat rejection processes occur reversibly at constant pressure. The expansion and compression processes are isentropic. This cycle was at one time used as the ideal basis for a hot-air reciprocating engine.









Process 1→2 (adiabatic reversible compression) $PV^{\gamma}=c$

$$gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w \quad ,$$

$$z_1 \simeq z_2 \quad , C_1 \simeq C_2 \quad , q = 0$$

$\therefore$ Work input to Compressor $= h_1 - h_2 = C_p(T_1 - T_2)$

Process 2 → 3 (Heater) $P = C$

$$gz_2 + h_2 + \frac{1}{2}C_2^2 + q = gz_3 + h_3 + \frac{1}{2}C_3^2 + w$$

$z_2 \simeq z_3$, $C_2 \simeq C_3$, $w = 0$

∴ heat supplied to heater $= h_3 - h_2 = C_p(T_3 - T_2)$

Process 3→4 (adiabatic reversible expansion) $PV^{\gamma} = C$

$$gz_3 + h_3 + \frac{1}{2}C_3^2 + q = gz_4 + h_4 + \frac{1}{2}C_4 + w$$

$z_3 \simeq z_4$, $C_3 \simeq C_4$, $q = 0$

∴ Work output from turbine $= h_3 - h_4 = C_p(T_3 - T_4)$

Process 4→1 (constant pressure heat rejection) $P = C$

$$gz_4 + h_4 + \frac{1}{2}C_4^2 + q = gz_1 + h_1 + \frac{1}{2}C_1^2 + w$$

$z_4 \simeq z_1$, $C_4 \simeq C_1$, $w = 0$

∴ heat rejected in cooler $= h_1 - h_4 = C_p(T_1 - T_4)$

for positive value

work comp. $= h_2 - h_1 = C_p(T_2 - T_1)$

heat rejected $= h_4 - h_1 = C_p(T_4 - T_1)$

$$\text{thermal efficiency} = \frac{\text{Net work}}{\text{heat added}}$$

$$= \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{----} \quad *$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad , \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\because \; P_1 = P_4 \;\&\; P_2 = P_3 \Rightarrow \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\therefore \; \frac{T_3}{T_4} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_3 = T_4 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

let $\frac{P_2}{P_1} = r_p$ = pressure ratio

$$T_3 = T_4 \cdot r_p^{\gamma-1/\gamma} \quad ---- (1)$$

$$T_2 = T_1 \cdot r_p^{\gamma-1/\gamma} \quad ---- (2)$$

Sub eq 1 & 2 in * eq.

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_4 \cdot r_p^{\gamma-1/\gamma} - T_1 r_p^{\gamma-1/\gamma}} = 1 - \frac{\cancel{(T_4 - T_1)}}{\cancel{(T_4 - T_1)}\, r_p^{\gamma-1/\gamma}}$$

$$\therefore \; \boxed{\eta_{th} = 1 - \frac{1}{r_p^{\gamma-1/\gamma}}}$$

$$\text{Work Ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{W_{Tur.} - W_{comp.}}{W_T}$$

$$W.R = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$T_2 = T_1 \cdot r_p^{\frac{\gamma-1}{\gamma}} \quad , \quad T_4 = T_3 \cdot \frac{1}{r_p^{\gamma-1/\gamma}}$$

$$W.R = 1 - \frac{T_1 r_p^{\frac{\gamma-1}{\gamma}} - T_1}{T_3 - T_3 \frac{1}{r_p^{\gamma-1/\gamma}}} = 1 - \frac{T_1 \left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)}{T_3 \left(1 - \frac{1}{r_p^{\gamma-1/\gamma}}\right)}$$

$$W.R = 1 - \frac{T_1}{T_3} \cdot \frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{\frac{r_p^{\gamma-1/\gamma} - 1}{r_p^{\gamma-1/\gamma}}}$$

$$\boxed{W.R = 1 - \frac{T_1}{T_3} \cdot r_p^{\frac{\gamma-1}{\gamma}}}$$

Ex: In a gas turbine unit air is drawn in at 1.02 bar and 15°c and compressed to 6.12 bar. Calculate the thermal efficiency and the work ratio of the ideal constant pressure cycle ~~when~~ the maximum temperature is 800°C.

Sol: $r_p = \frac{P_2}{P_1} = \frac{6.12}{1.02} = 6$

$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \Rightarrow \eta_{th} = 0.4$
$= 40\%$

$T_{min} = T_1 = 15°C$, $T_{max} = T_3 = 800°C$

$\text{Work ratio} = 1 - \frac{T_1}{T_3} \cdot r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow \text{Work ratio} = 0.552$

OR

$T_1 = 15 + 273 = 288K$, $T_3 = 800 + 273 = 1073K$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 481K$

$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 643K$

$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{1.67} = 0.4$
$= 40\%$

$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$

$= 1.005(1073 - 643) - 1.005(481 - 288)$
$= 238 \text{ kJ/kg}$

$W_{gross} = C_p(T_3 - T_4) = 1.005(1073 - 643)$
$= 432 \text{ kJ/kg}$

$\text{Work ratio} = \frac{W_{net}}{W_{gross}} = \frac{238}{432} = 0.552$

Ex: In an air-standard Brayton cycle the air enters the compressor at 0.1Mpa, 15°c. The pressure leaving the compressor is 1.0Mpa, and the maximum temperature in the cycle is 1100°C. Determine.

1. the pressure and temperature at each point in the cycle.
2. the compressor work, turbin work, heat added and η_{th}.

Sol:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = (15+273)\cdot\left(\frac{1}{0.1}\right)^{\frac{1.4-1}{1.4}} = 556\,K$$

$$W_{comp} = h_2 - h_1 = C_p(T_2 - T_1) = 1.005(556 - 288) = 269.34\ kJ/kg$$

$$W_{Turbine} = h_3 - h_4 = C_p(T_3 - T_4) \quad , \quad T_3 = 1100°C = 1373K$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 10^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 710.8\,K$$

$$W_T = 1.005(1373 - 710.8) = 665.5\ kJ/kg$$

$$W_{net} = W_T - W_c = 665.5 - 269.34 = 396.16\ kJ/kg$$

$$q_{add.} = h_3 - h_2 = C_p(T_3 - T_2) = 1.005(1373 - 556) = 821\ kJ/kg$$

$$\eta_{th} = \frac{W_{net}}{q_{add.}} = \frac{396.16}{821} = 0.482 = 48.2\%$$

or

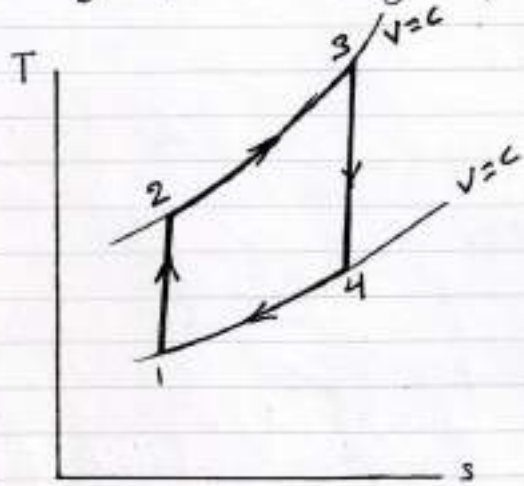
$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{10^{0.286}} = 48.2\%$$

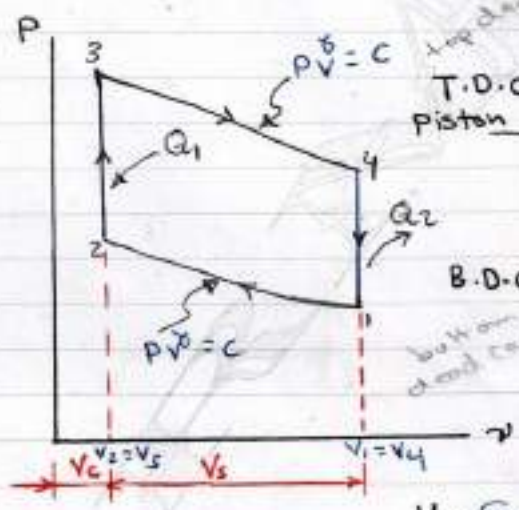
ولدتك أذ ولدتك أمك باكياً
والناس حولك يضحكون سروراً
فأعمل لنفسك أن تكون أذا بكوا
في يوم موتك ضاحكاً مسروراً

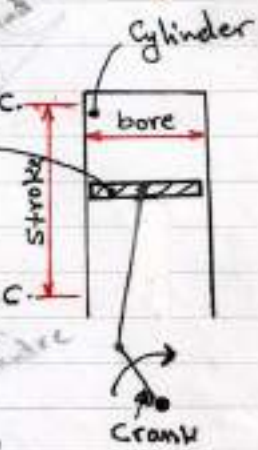
Otto cycle

(constant volume cycle)

The Otto cycle is the ideal air standard cycle for the petrol engine, gas engine, and the high-speed oil engine.







V_s = Swept volume
V_c = Clearance "

Process 1→2 Isentropic compression

$$Q = W + \Delta U \quad , \quad \text{for } m = 1\,kg \quad , \quad Q = 0$$
$$W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$$

Process 2→3 Reversible constant volume heat added

$$Q = W + \Delta U \quad , \quad W = 0$$
$$Q_1 = \Delta U = u_3 - u_2 = C_v(T_3 - T_2)$$

Process 3→4 Isentropic Expansion

$$Q = W + \Delta U \quad , \quad Q = 0$$
$$W = -\Delta U = -(u_4 - u_3) = C_v(T_3 - T_4)$$

Process 4→1 Reversible constant volume heat rejection

$$Q = W + \Delta U \quad , \quad W = 0$$
$$Q = \Delta U = u_1 - u_4 = C_v(T_1 - T_4)$$

for positive value.

$$W_{comp} = C_v(T_2 - T_1)$$
$$Q_{ref} = C_v(T_4 - T_1)$$

$$r_v = \frac{V_1}{V_2} = \frac{\text{Swept volume} + \text{Clearanc volume}}{\text{Clearance volume}}$$

$$\eta_{th} = \frac{\text{Net work}}{\text{heat added}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{C_v(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_3}{T_4} = r_v^{\gamma-1}$$

$$T_3 = T_4 \cdot r_v^{\gamma-1} \quad \& \quad T_2 = T_1 \cdot r_v^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_4 r_v^{\gamma-1} - T_1 \cdot r_v^{\gamma-1}} = 1 - \frac{(T_4 - T_1)}{(T_4 - T_1) r_v^{\gamma-1}}$$

$$\boxed{\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}}$$

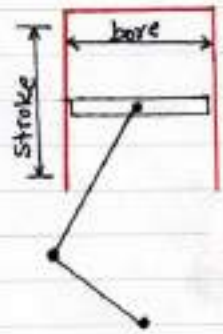
Ex: Calculate the ideal air standard thermal efficiency of the Otto cycle for a petrol engine with a cylinder bore of 50 mm and a stroke of 75 mm and clearance volume of 21.3 cm^3.

Sol: Swept Volume = area of bore * stroke

$$\therefore S.V. = \frac{\pi}{4}(5)^2 * 7.5 = 147.26 \ cm^3$$

$$r_v = \frac{147.26 + 21.3}{21.3} = 7.914$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} = 0.56$$



Ex: In an air standard Otto cycle the maximum and minimum temperature are 1400°C and 15°C. The heat supplied per kg of air is 800 kJ. Calculate the compression ratio and the thermal efficiency. Calculate also the ratio of maximum to minimum pressure in the cycle.

Sol:

$$Q_1 = mC_v(T_3 - T_2) \quad , T_3 = 1400°C = 1673K$$

$$800 = 1 * 0.717(1673 - T_2)$$

$$\therefore T_2 = 557.2\,K = 284.2°C \quad , T_1 = 15°C = 288\,K$$

$$\frac{T_2}{T_1} = r_v^{\gamma-1} \Rightarrow \frac{557.2}{288} = r_v^{1.4-1}$$

$$r_v = 5.2$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - \frac{1}{5.2^{1.4-1}} = 0.483$$

$$\frac{P_3}{P_1} = ? \quad (H.W) \qquad \frac{P_3}{P_1} = 30.24$$

$$\frac{P_3}{P_2} = \frac{P_3}{P_2} \cdot \frac{P_2}{P_1}$$

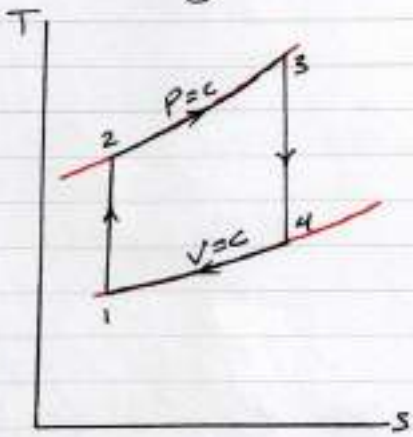
$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1} \Rightarrow \frac{P_2}{P_1} = \checkmark$$

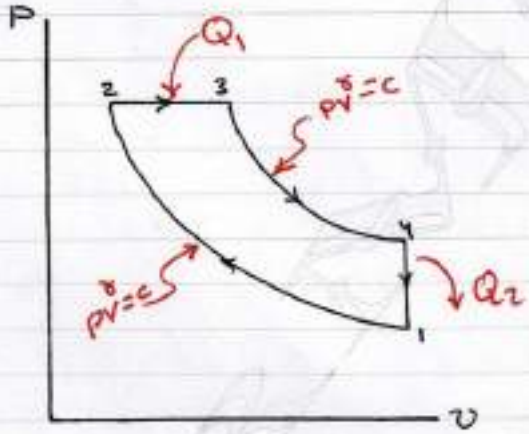
$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$$

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow \frac{P_3}{P_2} = \checkmark$$

Diesel cycle

This is an important cycle on which all the diesel engines work. It is also known as constant pressure cycle as heat is received at a constant pressure.





Process 1→2 , Isentropic compression

$$Q = W + \Delta U \quad , Q = 0 \quad , m = 1\,kg$$

$$W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$$

for positive value

$$W_c = C_v(T_2 - T_1)$$

Process 2→3 , Reversible constant pressure heating

$$Q = W + \Delta U$$
$$= C_p(T_3 - T_2)$$

Process 3→4 , Isentropic expansion

$$Q = W + \Delta U \quad , Q = 0$$

$$W = -\Delta U = -C_v(T_4 - T_3)$$

for positive value

$$W = C_v(T_3 - T_4)$$

Process 4→1, Reversible Constant Volume Cooling

$$Q = W + \Delta U \quad , \quad W = 0$$

$$Q = \Delta U = C_v (T_1 - T_4)$$

for positive value

$$Q = C_v (T_4 - T_1)$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v (T_4 - T_1)}{C_p (T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2}$$

Process 2→3 $p = c$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow T_3 = T_2 \cdot \frac{V_3}{V_2} \quad \text{----} \; ①$$

Process 3→4 isentropic

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 \cdot \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\because \; V_4 = V_1 \Rightarrow T_4 = T_3 \cdot \left(\frac{V_3}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_2}\right)^{\gamma-1} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \text{----} \; ②$$

Sub. eq ① in ②

$$T_4 = T_2 \cdot \left(\frac{V_3}{V_2}\right) \cdot \left(\frac{V_3}{V_2}\right)^{\gamma-1} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2 \left(\frac{V_3}{V_2}\right)^{\gamma} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow T_1 = T_2 \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{T_2 \cdot \left(\frac{V_3}{V_2}\right)^{\gamma} \cdot \left(\frac{V_2}{V_1}\right) - T_2\left(\frac{V_2}{V_1}\right)}{\gamma\left(T_2 \cdot \left(\frac{V_3}{V_2}\right) - T_2\right)}$$

let $r_v = \frac{V_1}{V_2}$ (compression ratio)

$\beta = \frac{V_3}{V_2}$ (Cutt off ratio)

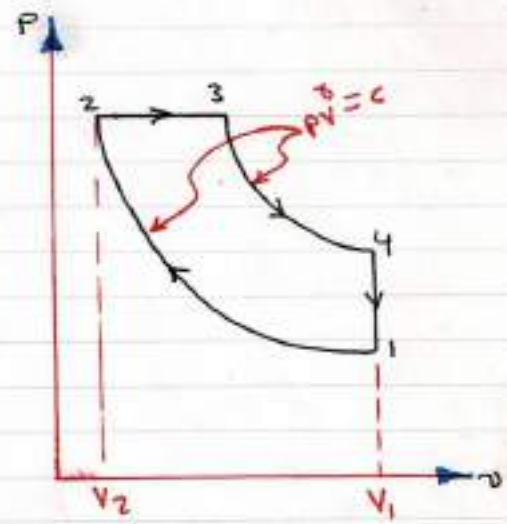
$$\therefore \quad \eta_{th} = 1 - \frac{\cancel{T_2}\left(\beta^{\gamma} \cdot \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}\right)}{\gamma \cancel{T_2}(\beta - 1)}$$

$$\eta_{th} = 1 - \frac{\beta^{\gamma} \cdot \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}}{\gamma(\beta - 1)}$$

$$\boxed{\eta_{th} = 1 - \frac{(\beta^{\gamma} - 1)}{(\beta - 1) \cdot \gamma \cdot r_v^{\gamma-1}}}$$

Ex: A diesel engine has an inlet temperature and pressure of 15°C and 1 bar respectively. The compression ratio is 12:1 and the maximum cycle temperature is 1100°C. Calculate the air standard thermal efficiency based on the Diesel Cycle.

Sol:



$T_1 = 15 + 273 = 288\ K$

$T_3 = 1100 + 273 = 1373\ K$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r_v)^{\gamma-1}, \quad \gamma = 1.4$$

$T_2 = 778\ K$

Process $2 \to 3$ $\quad \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$, $P_2 = P_3$

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1373}{728} = 1.765$$

$$\frac{V_4}{V_3} = \frac{V_4}{V_2} \cdot \frac{V_2}{V_3} = \frac{V_1}{V_2} \cdot \frac{1}{V_3/V_2}$$

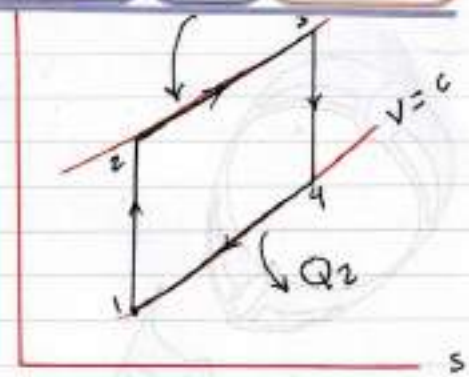
$$= 12 * \frac{1}{1.765} = 6.8$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \Rightarrow \frac{1373}{T_4} = (6.8)^{0.4} \Rightarrow T_4 = 638\ K$$

$Q_1 = C_p(T_3 - T_2) = 1.005(1373 - 778)$
$= 598$ kJ/kg

$Q_2 = C_v(T_4 - T_1) = 0.718(638 - 288)$
$= 251$ kJ/kg

$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{251}{598} = 0.58$
$= 58\%$



Ex: An engine uses air as a working substance, at the begining of compression the pressure is 90 kpa and the temperature is 40°c. During the adiabatic compression the volume is raduced to one-sixtenth of it's volume at the begining of compression stroke. Heat is then added at constant pressure until the temperature is 1400°c. The strok is completed by adiabatic expansion until the initial volume is reached, and the cycle is closed by constant volume process. Find: a. the temp. and pressure at all four corner.
b. thermal efficiency.

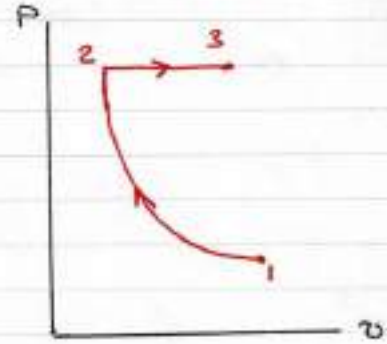
Sol:

* point -1-
$P_1 = 90$ kpa , $T_1 = 313$ K

* point -2-
$V_2 = \frac{1}{16} V_1$

$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot P_1 = (16)^{1.4} * 90$
$= 4365$ kpa

$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \cdot T_1 = (16)^{0.4} * 313 = 949$ K



* point -3-
$P_2 = P_3 = 4365$ kpa
$T_3 = 1400 + 273 = 1673$ K

* point -4-

$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1}$

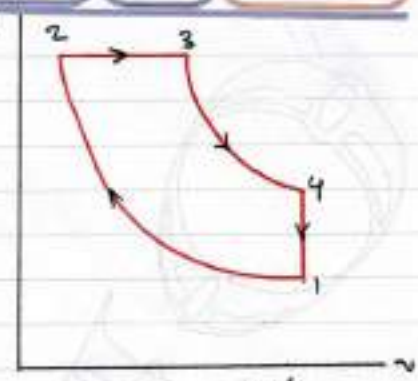
$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{T_3}{T_2}\cdot\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3\cdot\left(\frac{T_3}{T_2}\cdot\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = 1673\cdot\left(\frac{1673}{949}\cdot\frac{1}{16}\right)^{0.4} = 692\ K$$

$$P_4 = P_3\left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 4365\left(\frac{692}{949}\right)^{\frac{1.4}{0.4}}$$

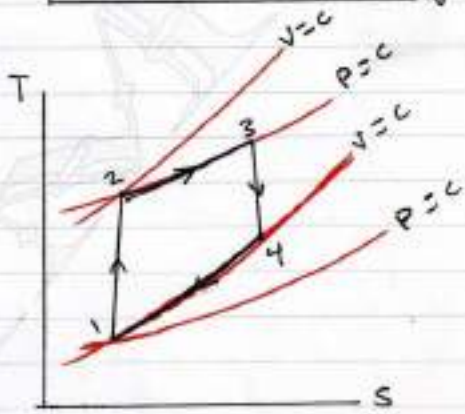
$$= 198.6\ kpa$$



$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \Rightarrow$$

$$\eta_{th} = 1 - \frac{1}{\gamma}\cdot\frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{692-313}{1.4(1673-949)}$$

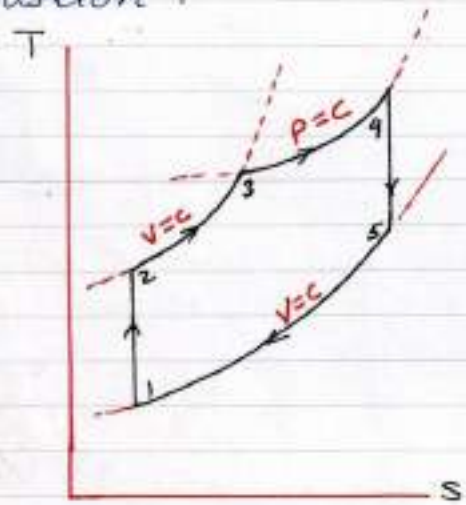
$$= 0.63 = 63\%$$

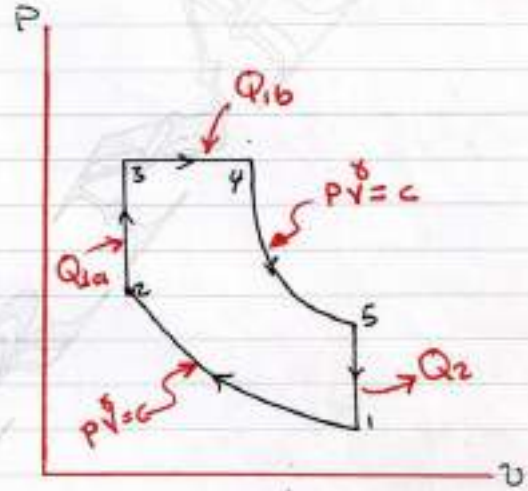


H.W.: find the Wnet from the shaded area on (P-V) diagram.

Dual combustion cycle

Modern oil engines althoug still called diesel engines are more closely derived from an engine invented by Ackroy-Stuar. The heat is supplied in two parts. the first part at constant volume and the remainder at constant pressure, hence the name "dual combustion".





Process 1→2 Adiabatic compression $PV^{\gamma}=C$

$$Q = W + \Delta U \quad , Q=0 \quad , m = 1\,kg$$

$$W = -\Delta U = -C_v(T_2 - T_1)$$

for positive value, $W = C_v(T_2 - T_1)$

Process 2→3 Reversible constant volume heating (Q_{1a})

$$v = c \Rightarrow W = 0$$

$$Q_{1a} = C_v(T_3 - T_2)$$

Process 3→4 Reversible constant pressure heating (Q_{1b})

$$P = C$$

$$Q = W + \Delta U$$

$$Q_{1b} = h_4 - h_3$$

$$Q_{1b} = C_p(T_4 - T_3)$$

Process 4→5 Adiabatic expansion $PV^{\gamma}=C$

$$Q = W + \Delta U \quad , Q=0$$

$$W = -\Delta U = -C_v(T_5 - T_4)$$

for positive value, $W = C_v(T_4 - T_5)$

Process 5→1 Reversible isobaric volume [illegible]

$V = C \Rightarrow W = 0$

$Q = W + \Delta U$

$Q_2 = \Delta U = C_v (T_1 - T_5)$

for positive value

$Q_2 = C_v (T_5 - T_1)$

* $r_v = \frac{V_1}{V_2}$ = Compression ratio

* $\alpha = \frac{P_3}{P_2}$ = ratio of pressure during heat adding at constant volume.

* $\beta = \frac{V_4}{V_3}$ = ratio of volume during heat adding at constant pressure.

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2}{Q_{1a} + Q_{1b}}$$

$$\eta_{th} = 1 - \frac{C_v (T_5 - T_1)}{C_v (T_3 - T_2) + C_p (T_4 - T_3)} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma (T_4 - T_3)}$$

$$\eta_{th} = 1 - \frac{(\alpha \beta^{\gamma} - 1)}{[(\alpha - 1) + \gamma \alpha (\beta - 1)]\, r_v^{\gamma - 1}}$$

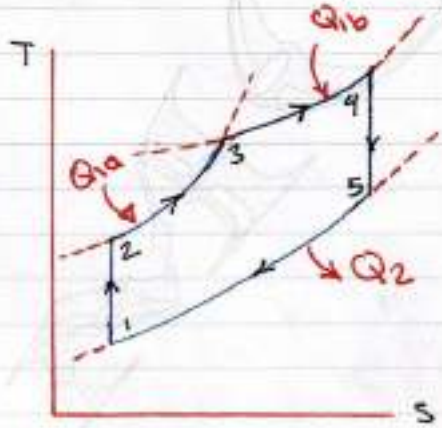
if $\alpha = 1$ (heat added only at $p = c$)

$$\eta_{th} = 1 - \frac{\beta^{\gamma} - 1}{r_v^{\gamma - 1} \cdot \gamma \cdot (\beta - 1)} \quad *$$

if $\beta = 1$ (heat added only at $v = c$)

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma - 1}} \quad *$$

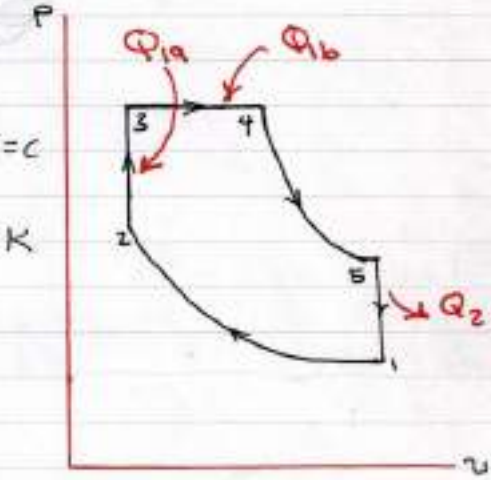
Ex: An oil engine takes at 1.01 bar, 20°C and the maximum pressure is 69 bar. The compression ratio is 18. Calculate the air standard thermal efficiency based on the dual combustion cycle. Assuming that the heat added at constant volume is equal to the heat added at constant pressure.



Sol:

$$\eta_{th} = 1 - \frac{Q_2}{Q_{1a} + Q_{1b}}$$

$$= 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$



point (1) $P_1 = 1.01\,bar$, $T_1 = 298\,K$

point (2) $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(18)^{1.4-1} = 931\,K$

$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot P_1 = 57.8\,bar$

Point (3) $P_3 = 69\,bar$, $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$, $V = c$

$T_3 = \frac{P_3}{P_2} \cdot T_2 = \frac{69}{57.8} \cdot 931 = 1112\,K$

Point (4) $P_4 = P_3 = 69\,bar$, $Q_{1a} = Q_{1b}$

$C_v(T_3 - T_2) = C_p(T_4 - T_3)$

$T_4 = \frac{1}{\gamma}(T_3 - T_2) + T_3 = 1241\,K$

Point (5) $\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$

$\frac{T_5}{T_4} = \left(\frac{V_4}{V_3} \cdot \frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$

$= \left(\frac{T_4}{T_3} \cdot 1 \cdot \frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1241}{1112} \cdot 1 \cdot \frac{1}{18}\right)^{0.4}$

so $T_5 = 408\,K$

$$\eta_{th} = 1 - \frac{408 - 293}{(1112 - 931) + 1.4(1241 - 1112)} = 0.681$$

$$= 68.1\%$$

* Mean effective pressure :

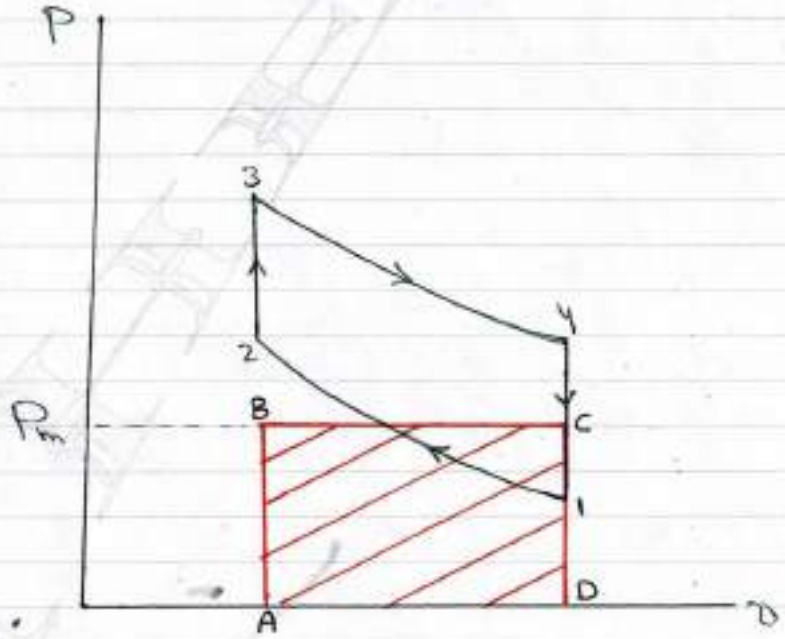
In internal-combustion engines the term work ratio is not such a useful concept, since the work done on and by the working fluid takes place inside one cylinder.

In order to compare reciprocating engines another term is defined called the **mean effective pressure**. The mean effective pressure is defined as the height of a rectangle having the same length and area as the cycle plotted on a p-v diagram.

for Otto cycle, the rectangle ABCDA is the same length as the cycle 12341, and area ABCDA is equal to area 12341, then the mean effective pressure, P_m is the height AB of the rectangle.

The work done per kg of air can therefore be written as

$$W = \text{area } ABCDA = P_m (v_1 - v_2)$$



Ex: Calculate the thermal efficiency and mean effective pressure of an air standard Diesel Cycle with a compression ratio 15/1 and max. & min. cycle temperature of 1650°C & 15°C respectively. The maximum pressure is 45 bar.

Sol:

$$\frac{V_1}{V_2} = 15$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 577.8°C$$

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow \frac{V_3}{V_2} = 2.26$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1}, \quad (V_4 = V_1)$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_1} \cdot \frac{V_2}{V_2}\right)^{\gamma-1} = T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = 628.9°C$$

$$\eta_{th} = 1 - \frac{901.9 - 288}{1.4(1923 - 850.8)}$$

$$= 0.59$$

$$= 59\%$$

$$Q_1 = C_p(T_3 - T_2) \Rightarrow Q_1 = 1077.36 \ \frac{kJ}{kg}$$

$$W = \eta_{th} \cdot Q_1 \Rightarrow W = 636.6 \ kJ/kg$$

$$V_1 = \frac{mRT_1}{P_1} \Rightarrow V_1 = 0.8319 \ m^3 \ \& \ V_2 = \frac{V_1}{15} = 0.0592 \ m^3$$

$$W = P_m(V_1 - V_2)$$

$$636.6 = P_m(0.8319 - 0.0592)$$

$$\therefore P_m = 830 \ kPa$$

Sheet No. -7-

* Sketch the processes on T-s & P-v diagram.

Q1:

What is the highest thermal efficiency possible for a heat engine operating between 800°C and 15°C. Ans - 73.2%

Q2:

In a carnot cycle operating between 307°C and 17°C the max. and min. pressure are 62.4 bar and 1.04 bar. Calculate the thermal efficiency and the work ratio. Ans - 50%, 0.287

Q3:

A closed cycle gas turbine unit operating with maximum and minimum temperature of 760°C and 20°C has a pressure ratio of 7/1. Calculate the ideal thermal efficiency and the work ratio.

Ans - 42.7%, 0.503

Q4:

In an air standard Otto cycle the maximum and minimum temperatures are 1400°C and 15°C. The heat supplied per kg of air is 800 kJ calculate the compression ratio and thermal efficiency. Calculate also the ratio of maximum to minimum pressure in the Cycle. Ans - 5.26/1, 48.6%, 30.5/1

Q5:

A four-cylinder petrol engine has a swept volume of 2000 cm^3 and the clearance volume in each cylinder is 60 cm^3. Calculate the air Standard thermal efficiency. If the induction conditions are 1 bar and 24°C, and the maximum cycle temperature is 1400°C, Calculate the mean effective pressure based on the air Standard Cycle. Ans - 59%, 5.27 bar

Q6:

In a dual combustion cycle the maximum temperature is 2000°C and the maximum pressure is 70 bar. Calculate the thermal efficiency and the mean effective pressure when the pressure and temperature at the start of compression are 1 bar and 17°C respectively. The compression ratio is 18/1.

Ans - 63.6%, 10.5 bar

Q7:

An air standard dual combustio cycle has a mean effective pressure of 10 bar. The minimum pressure and temperature are 1 bar and 17°C respectively, and the compression ratio is 16:1. Calculate the maximum cycle temperature when the thermal efficiency is 60%. The maximum cycle pressure is 60 bar.

Ans - 1959°C

Q8:

A gasoline engine has a volumatric compressio ratio of 9:1. the state before compression is 290 K, 90 kpa and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency.

Ans - 231.9 kpa, 58.4%, 461 kJ/kg

Q9:

In an ideal diesel cycle the compression ratio is 15:1 and the expansion ratio is 7.5:1. The pressure and temperature at the begining of compression are 98 kpa and 44°C, the pressure at the end of expansion is 288 kpa. Find: a. the maximum temperature b. the thermal efficiency.

Ans - 1597°C, 60.5%

العلم يحيي قلوب الميتين كما تحيا البلاد اذا ما مسها المطر

والعلم يجلي العمى عن قلب صاحبه كما يجلي سواد الظلمة القمر

THERMODYNAMICS

6

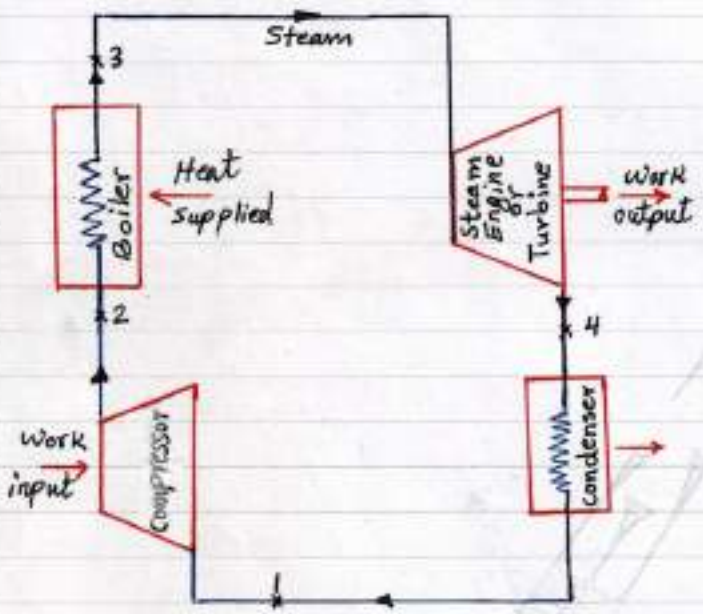
CHAPTER SIX

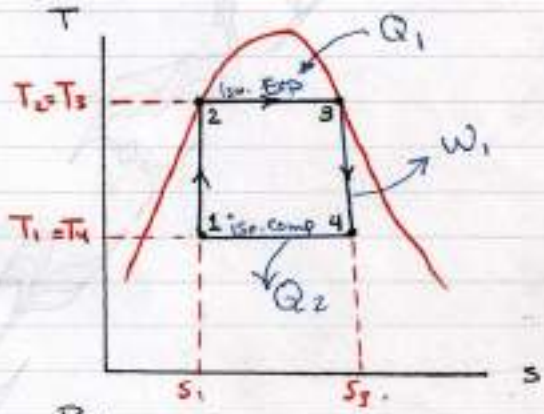
The Steam Cycles

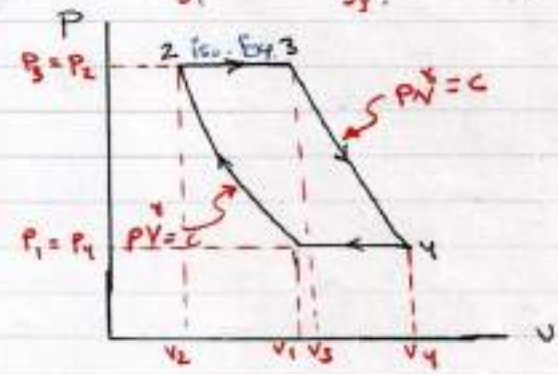
The Steam Cycles

In a vapour cycle, all the theory of air cycles remains the same, except the working substance, which is steam. The steam may be in any form, wet, dry saturated or superheated.

* Carnot Cycle with steam as working substance:







Process 1→2

Isentropic compression of wet steam in compressor the pressure and temperature rises, and no heat is absorbed or rejected during this process. $Q = 0$

Process 2→3

The saturated water at point (2) is isothermally converted into dry saturated steam, in a boiler, and the heat is absorbed at constant temperature.

$$q_{2\to3} = \text{change in entropy} \times \text{Absolute temperature}$$

$$q_{2\to3} = (S_3 - S_2) \cdot T_2 = (S_3 - S_2) \cdot T_3$$

Process 3→4

The dry steam now expands isentropically in a steam engine or turbine. the pressure and temperature falls from P_3 to P_4 and T_3 to T_4 respectively, and no heat is supplied or rejected during this process.

Process 4→1

The wet steam is now isothermally condensed in condenser and the heat is rejected at constant temperature and pressure.

$$q_{4\to1} = (S_4 - S_1)\cdot T_1 = (S_4 - S_1)\cdot T_4$$

$$\Sigma Q = \Sigma W$$

∴ Work done during the cycle = Heat absorbed − Heat rejected

$$W.D = Q_1 - Q_2 \quad , \text{ for } 1 \text{ kg}$$

$$W.D = q_{2\to3} - q_{4\to1}$$

$$W.D = (S_3 - S_2)T_2 - (S_4 - S_1).T_1$$

$$= (S_3 - S_2)(T_2 - T_1) \quad , \quad S_3 = S_4 \ , S_2 = S_1$$

or

$$W.D = (S_4 - S_1)(T_2 - T_1) \quad , \quad T_2 = T_3 \ , T_1 = T_4$$

or $$W.D = (S_4 - S_1)(T_3 - T_4)$$

or $$W.D = (S_3 - S_2)(T_3 - T_4)$$

$$\eta = \frac{\text{work done}}{\text{heat added}} = \frac{(S_3 - S_2)(T_2 - T_1)}{(S_3 - S_2)T_2} = \frac{T_2 - T_1}{T_2}$$

$$\boxed{\eta_{th} = 1 - \frac{T_1}{T_2}}$$

Notes:

1. Since the heat absorbed is at the highest temperature and rejected at the lowest temperature, the carnot cycle would give a maximum possible efficiency.

2. It may be noted that it is impossible to make a steam engine working on Carnot cycle. the isothermal expansion 2→3 and the isothermal compression 4→1 will have to be carried out extermely slow to ensure that the steam is always at constant temperature. But the isentropic expansion 3→4 and isentropic compression 1→2 should by carried out as quickly as possible in order to approach ideal isentropic condition. We know that sudden changes in the speed of an engine are not possible in actual practice, therefore it is impossible to realise Carnot's engine in actual pradice.

3. At state 1 the steam is wet at T_1, it is difficult to stop condensation at the point 1 and then compress it just to state 2. It is more convenient to allow the condensation process to proceed to completion.

4. The working fluid is water at the new state, and this can be conveniently pumped to boiler pressure, the pump has much smaller dimensions than it would have if it had to pump a wet vapour, the compression process is carried out more efficiently, and the equipment required is simpler and less expensive.

Ex: A power plant is supplied with dry saturated steam at a pressure of 16 bar and exhausts at 0.2 bar. find the efficiency of the Carnot cycle.

Sol: from Steam tables at 16 bar, $T_s = 201.4 + 273 = 474.4\,K = T_2$

from Steam tables, at 0.2 bar, $T_s = 60.1 + 273 = 333.1\,K = T_1$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{333.1}{474.4} = 0.298 \quad \text{or} \quad 29.8\%$$

Ex: In a carnot cycle, heat is supplied at 350°C and is rejected at 25°C. The working fluid is water, which while receiving heat, evaporates from liquid at 350°C to steam at 350°C. from Steam tables the entropy change for this process is 1.438 kJ/kg.K.

If the cycle operates on a stationary mass of 1 kg of water, find the heat supplied, work done and heat rejected per cycle. What is the pressure of water during heat reception.

Sol: the heat supplied $= (S_3 - S_2)\,T_2$, $T_2 = 350 + 273$
$= 1.438 * 623$ $= 623\,K$
$= 895.87\ kJ/kg$

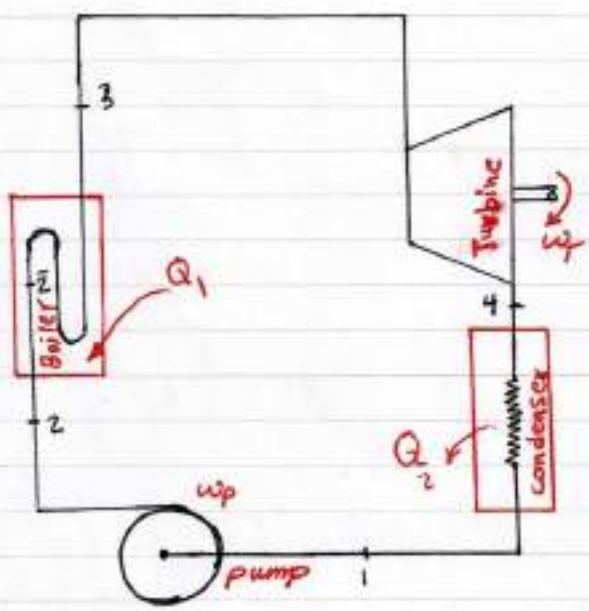
the work done $= (S_3 - S_2)(T_2 - T_1)$, $T_1 = 25 + 273$
$= 1.438 * (623 - 298)$ $= 298\,K$
$= 467.35\ kJ/kg$

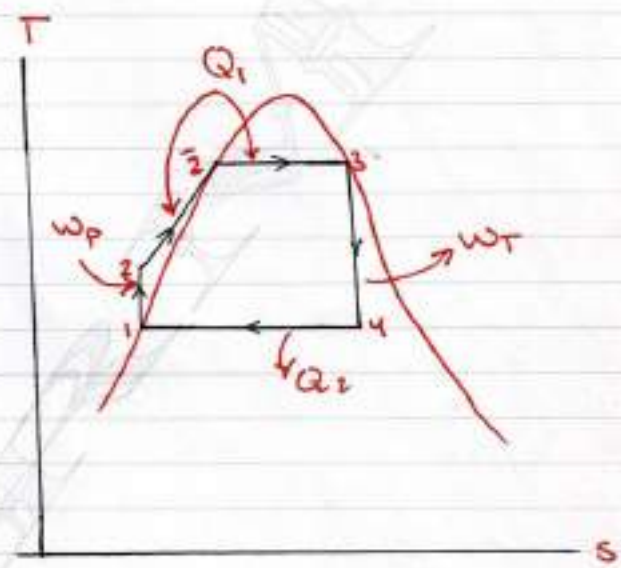
the heat rejected $= (S_4 - S_1) \cdot T_1$, $S_4 - S_1 = S_3 - S_2$
$= 1.438 * 298$
$= 428.52\ kJ/kg$

from Steam tables at $T_s = 350°C$, $P = 165.35$ bar

* Rankine Cycle :

The Rankine cycle is an ideal cycle for comparing the performance of Steam plants. It is modified form of carnot Cycle, in which the condensation process (4→1) is continued until the Steam is condensed into water.





- By the ~~non~~ flow Energy Equation ($\Delta Z \simeq 0$, $\Delta C \simeq 0$)

1. Boiler 2→3 (Heat added, Q_1)

$$h_2 + q = h_3 + w \quad , \quad w = 0 \text{ in the boiler}$$

$$\therefore q_{2\to3} = h_3 - h_2 \Rightarrow \boxed{Q_{2\to3} = m(h_3 - h_2)}$$

2. Turbine 3→4 (work output, W_T)

$$h_3 + q = h_4 + w \quad , \quad q = 0 \text{ , isentropic Exp.}$$

$$w_{3-4} = h_3 - h_4 \Rightarrow \boxed{W_{3\to4} = m(h_3 - h_4)}$$

3. Condenser $4 \to 1$ (Heat rejected, Q_2)

$$h_4 + q = h_1 + w \quad , \quad w = 0$$

$q_{4\to1} = h_1 - h_4$, for positiv value , $\boxed{Q_{4\to1} = m(h_4 - h_1)}$

4. Pump $1 \to 2$ (work input , W_p)

$$h_1 + q = h_2 + w \quad , \quad q = 0$$

$w_{1\to2} = h_1 - h_2$, for positive value , $\boxed{W_{1\to2} = m(h_2 - h_1)}$

or $W_{pump} = v_f(P_2 - P_1)$

Net work done $= W_{3\to4} - W_{1\to2}$
$= W_T - W_c$
$W.D = (h_3 - h_4) - (h_2 - h_1)$

if the feed pump work is neglected, $W_p = 0$

$$W.D = h_3 - h_4$$

Heat added $= q_1 = h_3 - h_2$

$$\text{Rankine efficiency, } \eta_R = \frac{W.D}{Q_{add.}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

If $w_p = 0$ $\eta_R = h_3 - h_4 / h_3 - h_1$

* **Efficiency ratio:** It is also known as relative efficiency. It is defined as the ratio of thermal efficiency (or actual cycle efficiency) to Rankine efficiency (or ideal cycle efficiency). Mathematically,

$$\boxed{\text{Efficiency ratio} = \frac{\text{Thermal efficiency}}{\text{Rankine efficiency}}}$$

* **Work ratio :** It is defined as the ratio of net work output to the gross (engine or turbine) output.

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross output}} = \frac{\text{Turbine work} - \text{Compressor Work}}{\text{Turbine work}}$$

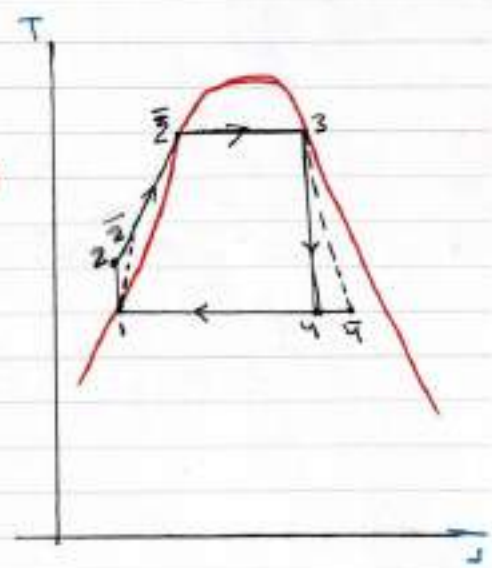
It may be noted that the Carnot cycle, despite of its high ideal thermal efficiency, has low work ratio. It is one of the reasone that Carnot cycle is not attempted. The higher value of work ratio also means a smaller size of the plant.

* **Specific Steam Consumption (S.S.C.) :** It is also known as Steam rate or specific rate of flow of Steam. It is defined as the mass of Steam that must be supplied to a Steam engine or turbine in order to develop a unit amount of work or power output. The amount of work or power output is usually expressed in kilowatt hour (kWh).

$$S.S.C. = \frac{1\,kWh}{W} = \frac{3600}{W} = \frac{3600}{h_3 - h_4} \quad kg/kW\cdot h \quad , W_p = 0$$

W = net work done

* **Isentropic efficiency:**



a. For expansion process: (Turbine, $3 \to \bar{4}$)

$$\eta_{ise.} = \frac{\text{actual W.D}}{\text{Isentropic W.D}} = \frac{h_3 - h_{\bar{4}}}{h_3 - h_4}$$

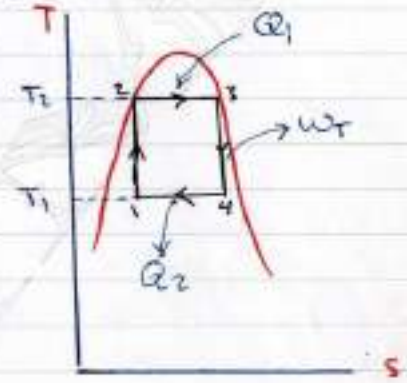
b. For compression process: (Pump)

$$\eta_{ise.} = \frac{\text{Isentropic W.D}}{\text{actual W.D}} = \frac{h_2 - h_1}{h_{\bar{2}} - h_1}$$

Ex: A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption:

a. for a carnot cycle using wet steam.
b. for a Rankine cycle with dry saturted steam at entry to the turbine.
c. for the Rankine cycle of (b), when the expansion process has an isentropic efficiency of 80%

Sol: from Steam tables
at $P = 42$ bar, $T_s = 253.2°C$
at $P = 0.035$ bar, $T_s = 26.7°C$



a

$$\eta_{carnot} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{26.7+273}{253.2+273}$$
$$= 0.432 \text{ or } 43.2\%$$

$$Q_1 = h_3 - h_2 = h_{g_3} - h_{f_2} \quad , \text{at } P = 42 \text{ bar}$$
$$= 2800 - 1105$$
$$= 1695 \text{ kJ/kg}$$

$$\text{Net work done} = \eta_{carnot} * Q_1$$
$$= 0.432 * 1695$$
$$= 732.3 \text{ kJ/kg}$$

$$W_T = h_3 - h_4 \quad , \quad S_3 = S_4 = S_g \text{ at } P = 42 \text{ bar}$$
$$= 6.05 \text{ kJ/kg}$$

$$S_4 = S_f + x_4 S_{fg} \quad , \text{at } P = 0.035 \text{ bar}$$

$$6.05 = 0.391 + x_4 * 8.13 \Rightarrow x_4 = 0.696$$

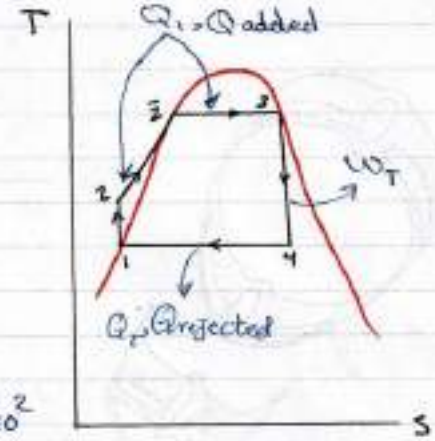
$$h_4 = h_f + x_4 \cdot h_{fg} \quad , \text{at } P = 0.035 \text{ bar}$$
$$= 112 + 0.696 * 2438 = 1808 \text{ kJ/kg}$$

$$W_T = h_3 - h_4 = 2800 - 1808 = 992 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{\text{net work}}{\text{gross work}} = \frac{732.3}{992} = 0.738$$

$$S.S.C. = 3600 / W_{net} = 3600 / 732.3 = 4.92 \text{ kg/kW.h}$$

b. $h_3 = 2800$ kJ/kg , $h_4 = 1808$ kJ/kg
$h_{\bar{2}} = 1105$ kJ/kg , $h_1 = 111.9$ kJ/kg
$v_1 = v_f = 0.001\ m^3/kg$



$$\eta_R = \frac{W.D}{Q_1} = \frac{(h_3-h_4)-(h_2-h_1)}{(h_3-h_2)}$$

$$\text{pump work} = h_2 - h_1 = v_1(P_2 - P_1)$$
$$= 0.001(42 - 0.035)\times 10^2$$
$$= 4.2\ kJ/kg$$

$$h_2 = h_1 + \text{Pump work} = 111.9 + 4.2$$
$$= 116.1\ kJ/kg$$

$$W_T = h_3 - h_4 = 2800 - 1808 = 992\ kJ/kg$$
$$W_P = 4.2\ kJ/kg$$

$$q_1 = h_3 - h_2 = 2800 - 116.1 = 2684\ kJ/kg$$

$$\text{work ratio} = \frac{W_{net}}{\text{gross work}} = \frac{992-4.2}{992} = 0.995$$

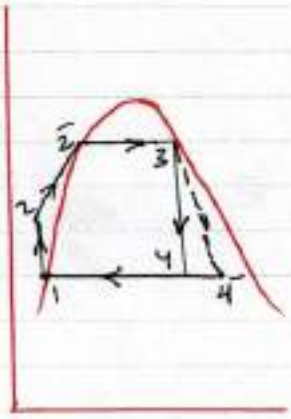
$$S.S.C. = \frac{3600}{W_{net}} = \frac{3600}{992-4.2} = 3.64\ kg/kW.h$$

$$\eta_R = \frac{W.D}{Q_1} = \frac{992-4.2}{2684} = 0.368 \text{ or } 36.8\%$$

c.

$$\eta_{ise.} = \frac{h_3 - h_{\bar{4}}}{h_3 - h_4} = 0.8 \Rightarrow 0.8(2800-1808) = 2800 - h_{\bar{4}}$$
$$\therefore h_{\bar{4}} = 2006.4\ kJ/kg$$

$$\eta_R = \frac{(h_3-h_4)-(h_2-h_1)}{(h_3-h_2)} = \frac{(2800-2006.4)-4.2}{2800-116.1}$$
$$= 0.294 \text{ or } 29.4\%$$

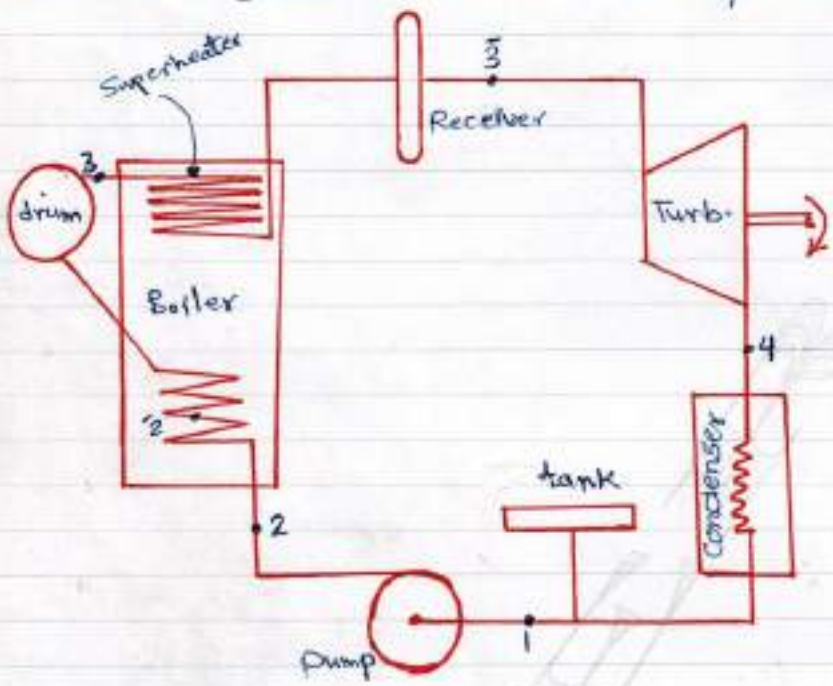


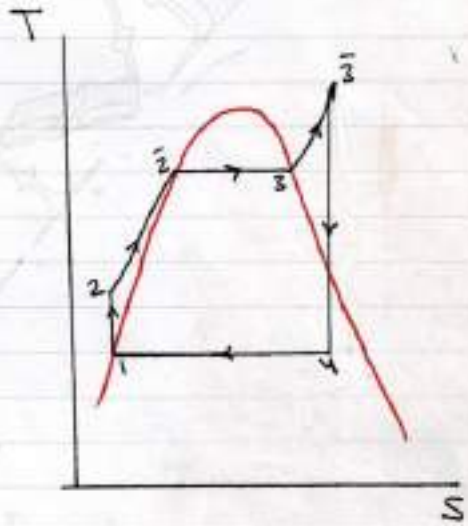
$$W.R = \frac{(2800-2006.4)-4.2}{2800-2006.4} = 0.9947$$

$$S.S.C = \frac{3600}{793.6-4.2} = 4.56\ kg/kW.h$$

* Rankine Cycle with Superheated Steam:

The average temperature at which heat is supplied in the boiler can be increased by superheating the steam. Usually the dry saturated steam from the boiler drum is passed through a second bank of smaller bore tubes within the boiler. This bank is situated such that it is heated by hot gases from the furnace until the steam reaches the required temperature.





$w_T = h_{\bar{3}} - h_4$

Assuming $w_p = 0$ (always)

$Q_1 = Q_{add} = h_{\bar{3}} - h_1$, $W.D = W_T = h_{\bar{3}} - h_4$

$$\eta_{th} = \frac{W.D}{Q_{add.}} = \frac{h_{\bar{3}} - h_4}{h_{\bar{3}} - h_1}$$

Ex: A steam turbine operating on the Rankine cycle received steam from the boiler at 3.5 Mpa and 350°C and exhaust to the condenser at 10 kpa. The condensate is then returned to the boiler by the feed pump. Calculate, neglecting all losses:

a. the energy supplied in the boiler per kg of steam,
b. the dryness fraction of steam entering the condenser,
c. the Rankine efficiency.

Sol: At 10 kpa, $h_1 = h_f = 192$ kJ/kg

at 3.5 Mpa & 350°C, $h_5 = 3104$ kJ/kg, $S_5 = 6.6579$ kJ/kg.K

$S_4 = S_f + x_4 S_{fg}$, at 10 kpa

$S_5 = S_4 \Rightarrow \quad 6.657 = 0.6493 + x_4 \cdot 7.5 \Rightarrow x_4 = 0.8$

$h_4 = h_f + x\, h_{fg} = 191.8 + 0.8 * 2392.8$, at 10 kpa

$= 2106$ kJ/kg

Energy Supplied $= h_5 - h_1$

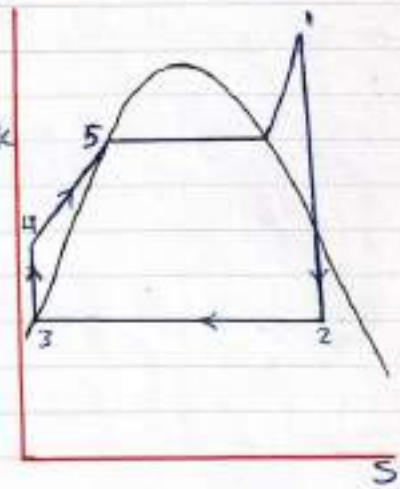
$= 3104 - 192$

$= 2912.2$ kJ/kg

$$\eta_R = \frac{h_5 - h_4}{h_5 - h_1} = \frac{3104 - 2106}{3104 - 191.8} = 0.3427 \text{ or } 34.27\%$$

Ex: Compare the Rankine cycle performance of example in page (211) with that obtained when the steam is superheated to 500°C, neglecting feed water work.

Sol: from Steam tables at 42 bar & 500°C



$h_1 = 3442.6$ kJ/kg, $S_1 = 7.066$ kJ/kg.K $= S_2$

$S_2 = S_f + x\, S_{fg}$

$7.066 = 0.391 + x_2 \cdot 8.13 \Rightarrow x_2 = 0.821$

$h_2 = h_f + x_2 h_{fg} = 112 + 0.821 * 2438$

$= 2113$ kJ/kg

$h_3 = h_f = 112$ kJ/kg

$w_T = h_1 - h_2 = 3442.6 - 2113 = 1329.6$ kJ/kg

Q_1 = heat Supplied $= h_1 - h_3 = 3442.6 - 112 = 3330.6$ kJ/kg

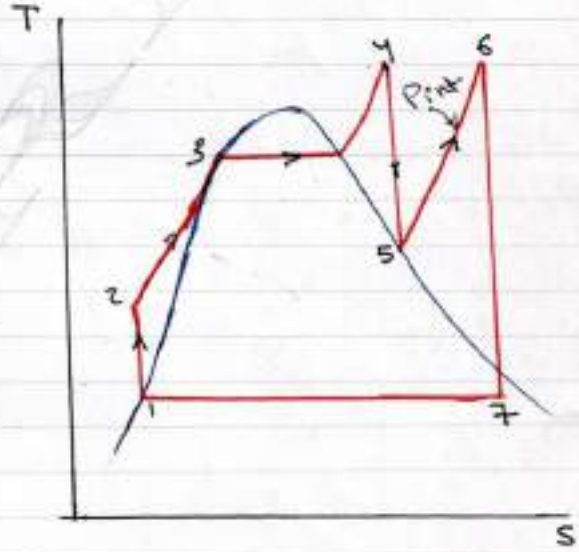
$$\eta_R = \frac{h_1 - h_2}{h_1 - h_3} = \frac{1329.6}{3330.6} = 0.399 \text{ or } 39.9\%$$

$$S.S.C = \frac{3600}{h_1 - h_2} = \frac{3600}{1329.6} = 2.71 \text{ kg/kW.h}$$

* the thermall efficiency has increased due to superheating and the improvement in S.S.C is even more marked

* Reheat Steam Cycle :

It is desirable to increase the average temperature at which heat is supplied to the steam, and also to keep the steam as dry as possible in the lower pressure stages of the turbine. The wetness at exhaust should be not greater than 10%. The high boiler pressures are required for high efficiency, but that expansion in one stage can result in exhaust steam which is wet. This is a condition which is improved by superheating the steam. The exhaust steam condition can be improved most effectively by reheating the steam, the expansion being carried out in two stages.



Heat Supplied $= Q_{234} + Q_{56}$

$W_p \simeq 0$

$Q_{234} = Q_{134} = h_4 - h_1$

$Q_{56} = h_6 - h_5$

Work output $= W_{45} + W_{67}$

$W_{45} = h_4 - h_5$

$W_{67} = h_6 - h_7$

$$\eta_{th} = \frac{(h_4 - h_5) + (h_6 - h_7)}{(h_4 - h_1) + (h_6 - h_5)}$$

Ex: Calculat new cycle efficiency if reheat is included in the plant of above example. The steam condition at inlet to the turbine are 42 bar and 500°c and condenser pressure is 0.035 bar as before. Assume that the steam is just dry saturated on leaving the first turbine, and reheated to the initial temperature. Neglect feed water work.

Sol: $h_1 = 3442.6$ kJ/kg , $S_1 = 7.066$ kJ/kg.K

$S_1 = S_2 = S_g \Rightarrow P_2 \simeq 2.4$ bar
$= P_{intermediate}$

for $P_{int.} = 2.4$ bar

$h_2 = h_g = 2715$ kJ/kg

$h_6 = 3487$ kJ/kg,
(at $P = 2.4$ & $500°C$)

$S_6 = S_7$

$h_7 = 2535$ kJ/kg

$h_3 = h_f$ at $P_{cond.} = 0.035$ bar
$= 112$ kJ/kg

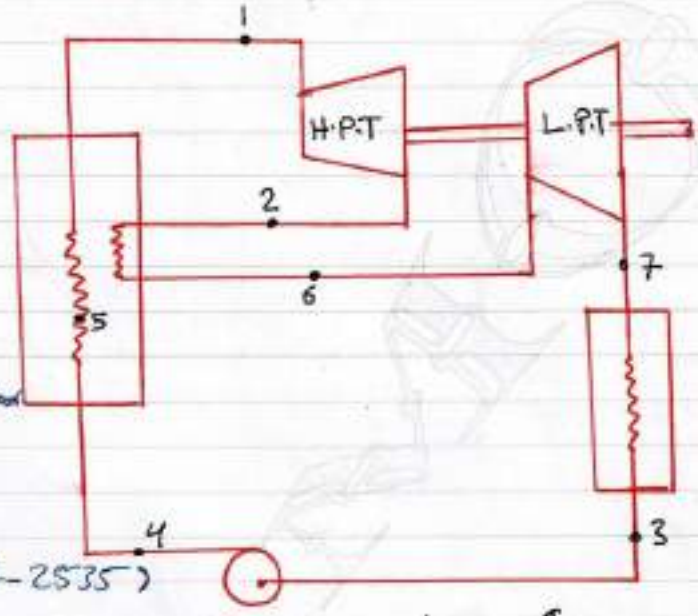
$W_{tur.} = (h_1 - h_2) + (h_6 - h_7)$
$= (3442.6 - 2715) + (3487 - 2535)$
$= 1688$ kJ/kg

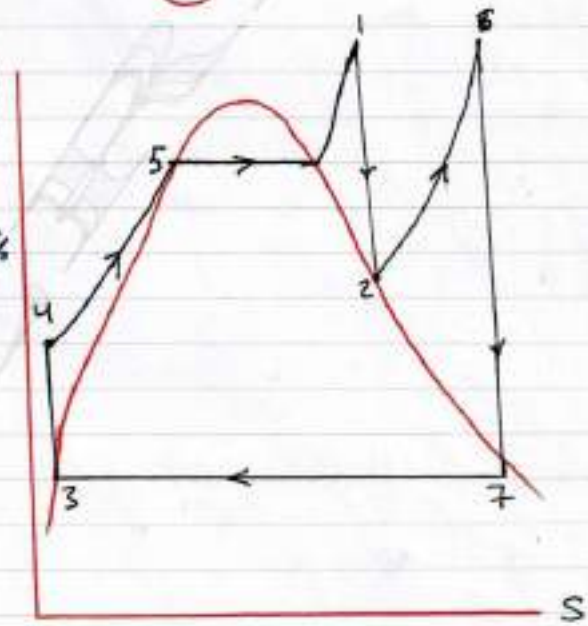
Heat supplied $= (h_1 - h_3) + (h_6 - h_2)$
$= 3104$ kJ/kg

$\eta_{th} = \frac{W_{net}}{Q_{sup}} = \frac{1682}{3104} = 0.41$ or 41%

$S.S.C. = \frac{3600}{1682} = 2.14$ kg/kW.h

* η_{th} & S.S.C. improved.





Sheet No. -8-

Q1: A. Steam is supplied, dry saturated at 40 bar to a turbine and the condenser pressure is 0.035. If the plant operates on the Rankine Cycle, Calculate per kg of Steam: **a.** the work output neglecting feed pump work, **b.** the work required for feed pump **c.** the heat transfered to the condenser cooling water, and the amount of cooling water required through the condenser if the temperature rise of water is assumed to be 5.5 K, **d.** the heat supplied **e.** the Rankine efficiency

Ans. [982.4 kJ, 4 kJ, 1706.6 kJ, 74.1 kg, 2685 kJ, 36.6 %]

B. for the Same Steam conditions calculate the efficiency for a carnot cycle operating with wet Steam.

Ans. [43 %]

Q2: Repeat problem (1-A) for a Steam supply condition of 40 bar and 350°C and the same condenser pressure.

Ans. [1125 kJ, 4 kJ, 1857 kJ, 80.5 kg, 2978 kJ, 37.8 %]

Q3: Steam is supplied to a two-Stage turbine at 40 bar and 350°C. It expands in the first turbine untile it is just dry saturated, then it is reheated to 350°C and expanded through the Second-Stage turbine, the condenser pressure is 0.035 bar. Calculate the work output and the heat supplied per kg of Steam for the plant, assuming ideal processes and neglecting the feed pump term, and Calculate the cycle efficiency.

Ans. [1290 kJ, 3362 kJ, 38.4 %]

Q4: If the expansion processes in the turbine of (Q.3) have isentropic efficiencies of 84% and 78% respectively in the 1st and 2nd Stages, Calculate the work output and the heat supplied per kg of Steam, the thermal efficiency and the specific Steam consumption.

1026 kJ, 3311 kJ, 31.1 %, 3.51 kg/kW.h

THERMODYNAMICS

7

CHAPTER SEVEN

Vapour Compression Refrigeration System

„Vapour Compression Refrigeration System„

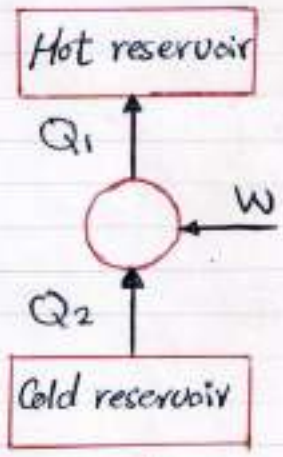
Refrigeration in the engineering sense, means maintaining a System at a temperature less than the temperature of the surroundings. This will not occur naturally, so adevice must be developed that will maintain this condition.

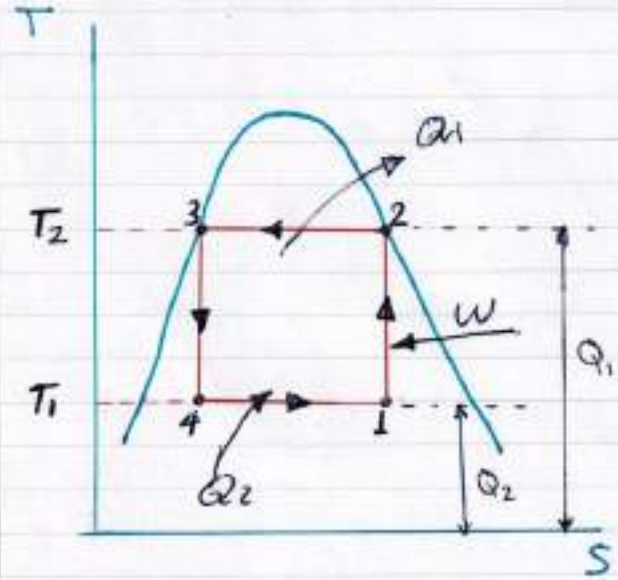
The vapour compression refrigeration system is now used for all purpose refrigeration. It is generally used for all industrial purposes from a small domestic refrigerator to a big air conditioning plant.

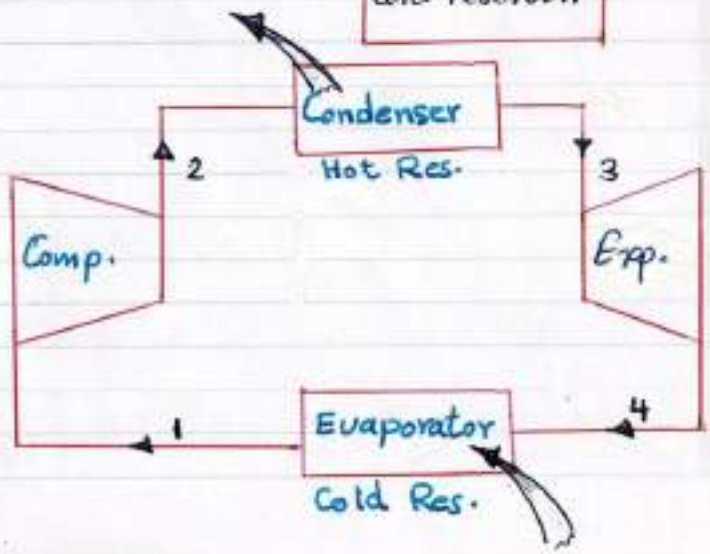
A reversed carnot engine will removed heat from a Law temperature reservoir and deliver this energy, plus work necessary to transfer the heat, to high temperature reservoir. The refrigerated System in this case is the low temperature reservoir.

* Reversed Carnot Cycle :-

In Refrigeration Carnot cycle the heat is absorbed from the cold reservoir and rejected to the hot reservoir.







* Processes of Refrigeration Carnot Cycle:-

1 → 2 : Work input to the compressor to compress to dry saturated vapour, rising its pressure and temperature to dry or superheated condition.

2 → 3 : heat rejected in condenser from the vapour to the ambient, changing the dry, or superheated vapour to saturated liquid.

3 → 4 : throttling the saturated liquid (expansion) changing it to wet vapour.

4 → 1 : heat absorbed from cold reservior, changing the wet vapour of low quality to wet vapour of high quality.

work input (process 1→2) = $Q_1 - Q_2$
Heat rejected (" 2→3) = $T_2(S_2 - S_3) = Q_1$
Heat absorbed (" 4→1) = $T_1(S_1 - S_4) = Q_2$

Since $S_3 = S_4$ & $S_2 = S_1 \Rightarrow Q_2 = T_1(S_2 - S_3)$

* Coefficient Of Performance (COP)

The performance ratio of refrigeration System is not the efficiency, but rather the Coefficient Of Performance, and define as the refrigeration effect (heat absorbed) divided by the net work done on the cycle (work input).

$$COP_R = \frac{Q_2}{W} = \frac{T_1(S_2 - S_3)}{T_2(S_2 - S_3) - T_1(S_2 - S_3)} = \frac{T_1}{T_2 - T_1}$$

$$COP_R = \frac{T_{Low}}{T_{high} - T_{low}}$$

It is more suitable to change the names of the processes of the reversed Carnot Cycle to:

Heat absorbed	to	Refrigeration effect	$Q_2 = T_1(S_2 - S_1)$
Heat rejected	to	Heat rejected in the Condenser	$Q_1 = T_2(S_2 - S_3)$
Work input	to	Work input to Compressor	$W = Q_1 - Q_2$

* Units of Refrigeration:

The practical unit of refrigeration is expreed in terms of « tonne of refrigeration » briefly written as "TR". A tonne of refrigeration is defined as the amount of refrigeration effect produced by the uniform melting of one tonne (1000 kg) of ice from and at 0°C in 24 hours. Since the latent heat of ice is 335 kJ/kg, therefor one tonne of refrigeration.

$$1\,TR = 1000 * 335\ kJ \quad \text{in 24 hours}$$
$$= \frac{1000 * 335}{60 * 24} = 232.6\ kJ/min$$

In actual practice, One tonne of refrigeration is taken as equivalent to 210 kJ/min or 3.5 kW (3.5 kJ/s).

Ex: A refrigerator has working temperature in the evaporator and condenser of -30°C and 32°C respectively, what is the maximum COP possible?, If the actual COP of 0.75 of the maximum COP, Calculate the refrigeration effect in kW per kW of power input.

Sol: $$COP = \frac{T_1}{T_2 - T_1} = \frac{-30 + 273}{(32 + 273) - (-30 + 273)} = 3.91$$

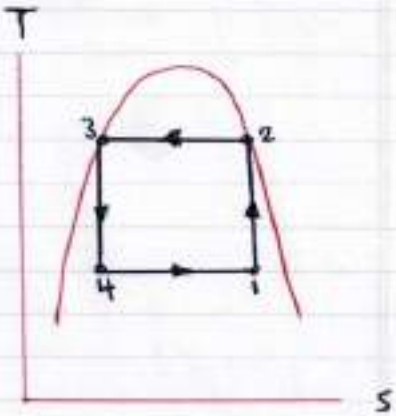
actual $COP_r = 0.75 * 3.91 = 2.939$

$$COP_r = \frac{Q_2}{W} \Rightarrow 2.939 = \frac{Q_2}{1}$$

$Q_2 = 2.939$ kW of refrigeration / kW of work input.

Ex: A machine working on a Carnot Cycle operates between 305K and 260K. Determine the COP, when it is operated as:
1. a refrigerating machine,
2. a heat pump,
3. a heat Engine.

Sol: 1. for refrigerating machine:



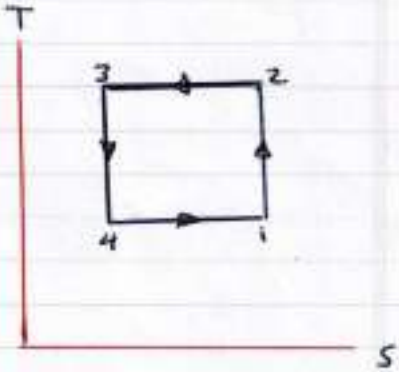
$$COP_R = \frac{T_1}{T_2 - T_1} = \frac{260}{305-260} = 5.78$$

2. for Heat pump:

$$COP_P = \frac{T_2}{T_2 - T_1} = \frac{305}{305-260} = 6.78$$

or $COP_P = COP_R + 1 = 5.78 + 1 = 6.78$

3. for Heat Engine:



$$COP_E = \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{min}}{T_{max}}$$

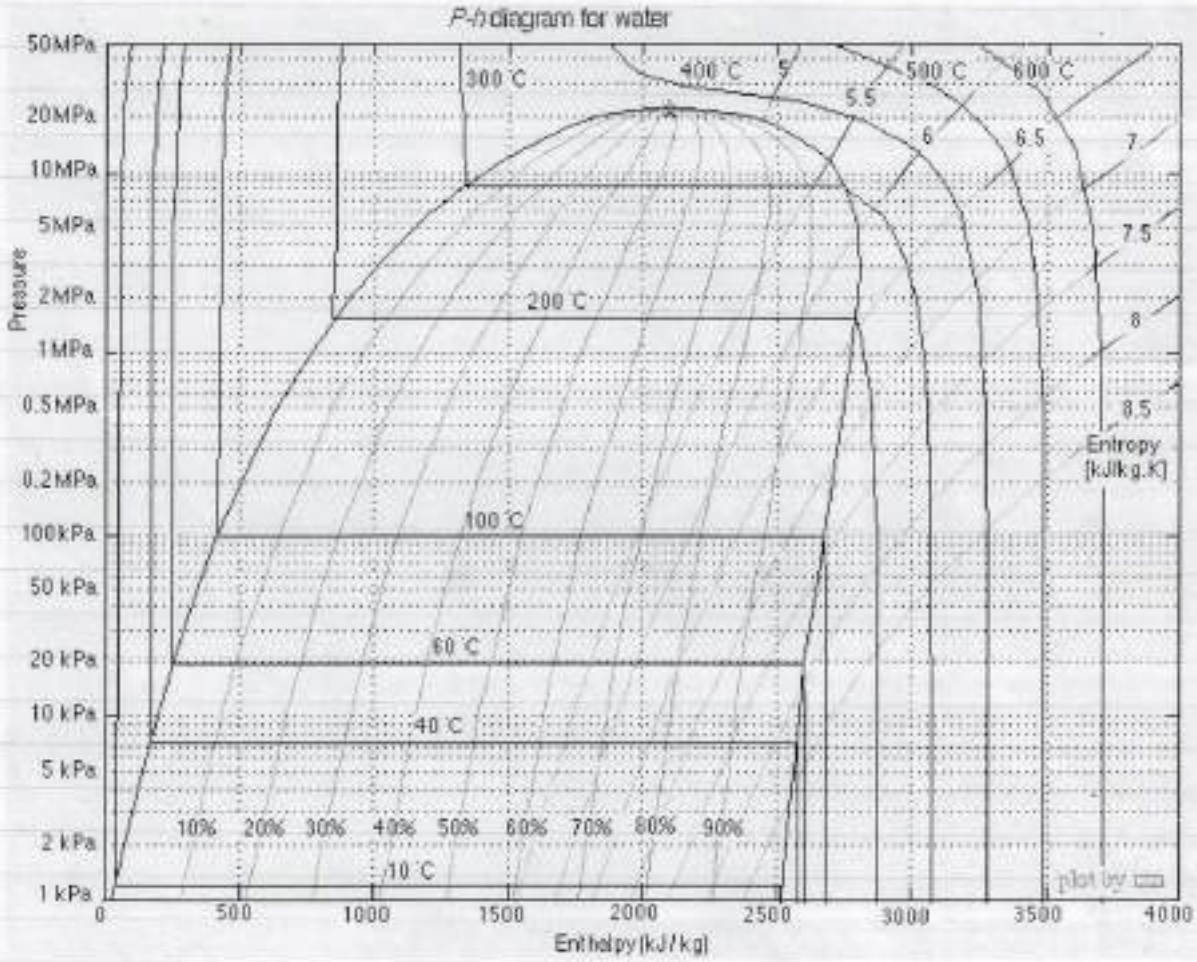
$$\therefore COP_E = 1 - \frac{260}{305} = 0.147$$

H.W: A carnot refrigeration cycle absorbs heat at -3°C and rejects it at 27°C.

1. Calculate the COP_R.
2. If the Cycle is absorbing 1130 kJ/min at -3°C, how many kJ of work is required per second?
3. If the carnot heat pump operates between the same temperature as the above refrigeration cycle, what is the COP_P?
4. How many kJ/min will the heat pump deliver at 27°C if it absorbs 1130 kJ/min at -3°C?

- Pressure - Enthalpy (p-h) Chart:

The most convenient chart for studing the behavior of a refrigerant is the p-h chart in which the vertical ordinates represent pressure and horizontal ordinates represent enthalpy.

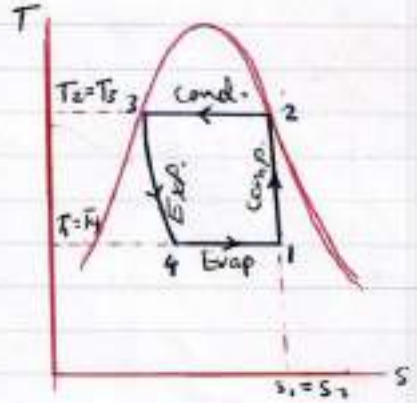


* Types of Vapour Compression Cycle:

We have already discussed that a vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles, yet the following are important from the subject point of view:

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

* 1 * Vapour Compression Cycly with dry Saturated Vapour after compression:



+ Process: 1→2: Compression process

$$W_{1-2} = h_2 - h_1 \quad \text{kJ/kg}$$

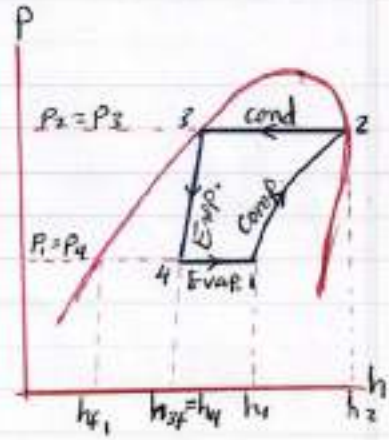
2→3: Condensing process

heat rejected $\equiv Q_1 = h_2 - h_3$, $T_2 = T_3$, $P_2 = P_3$
$= T_2(S_2 - S_3) = T_2 \cdot S_{fg}$

3→4: Expansion process (throttling process)

$$h_3 = h_4 \quad , \quad h_3 = h_f \text{ at } P_{cond.}$$

for Expansion process using Expansion valve or a capillary (small bore tube).



4→1: Vaporising process

$$R_E = h_1 - h_4 \quad , \quad h_4 = h_3$$
$$= T_1(S_1 - S_4)$$

$$COP = \frac{\text{Refrigerating Effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1}$$

Ex: The temperatur limits of an ammonia refrigerating System are 25°c and -10°C. If the gas is dry Saturated at the end of compression, calculate the coefficient of performanc of the cycle assuming no undercooling of the liquid ammonia. Use the following table for properties of ammonia:

Temperature (°C)	Liquid heat kJ/kg	Latent heat kJ/kg	Liquid entropy kJ/kg·K
25	298.9	1166.94	1.1242
-10	135.37	1297.68	0.5443

Sol: $S_1 = S_f + x_1 S_{fg}$, $h_{fg_{4-1}} = T_1 \cdot S_{fg} \Rightarrow S_{fg} = \frac{h_{fg}}{T_1}$

$$S_1 = 0.5443 + x_1 * \frac{1297.68}{263} = 0.5443 + 4.934 x_1$$

$S_2 = S_g$ at $T_{cond.}$, $S_2 - S_3 = S_g - S_f$

$$h_2 - h_3 = h_{fg_{cond.}} = T_2 * S_{fg_{cond.}} \Rightarrow S_2 = S_f + \frac{h_{fg_{cond}}}{T_2}$$

$$= 1.1242 + \frac{1166.94}{298}$$

$$= 5.04 \text{ kJ/kg·K} = S_1$$

$\therefore$ $5.04 = 0.5443 + 4.934 * x_1 \Rightarrow x_1 = 0.91$

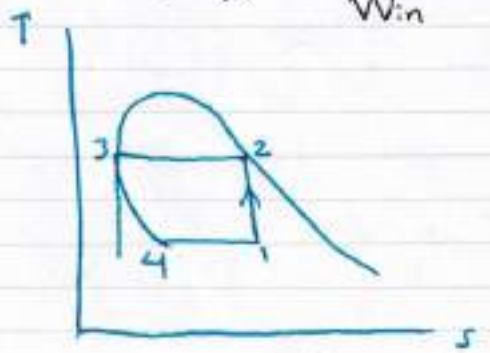
$$h_1 = h_{f_1} + x_1 h_{fg_1} = 135.37 + 0.91 * 1297.68 = 1316.26 \text{ kJ}$$

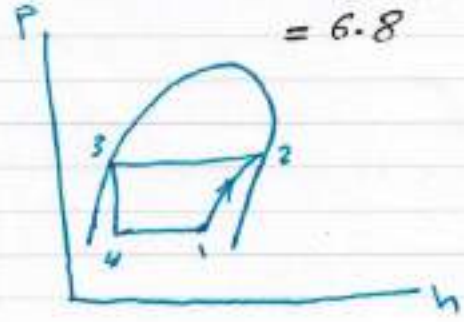
$$h_2 = h_g = h_{fg_2} + h_{f_2} = 298.9 + 1166.94 \text{ kJ/kg}$$

$$h_4 = h_3 = h_{f_{cond.}} = h_{f_3}$$

$$COP_R = \frac{R.E}{W_{in}} = \frac{h_1 - h_{f_3}}{h_2 - h_1} = \frac{1316.26 - 298.9}{1465.84 - 1316.26}$$

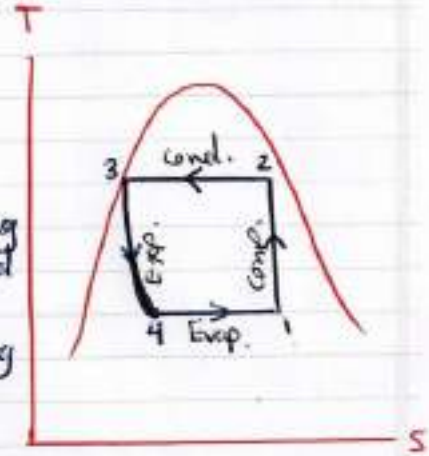
$$= 6.8$$





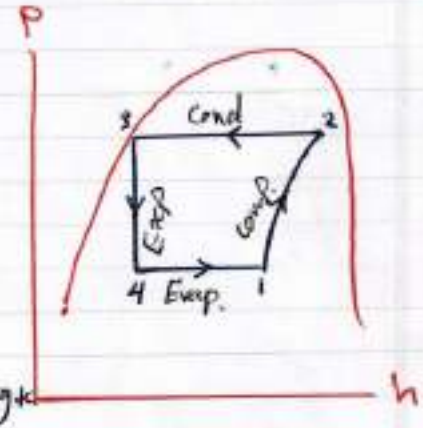
2. Vapour Compression Cycle with Wet Vapour after Compression:

$$COP = \frac{R.E.}{W_{in}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$



Ex: Fin the COP for a CO_2 machine working between the temperature range of 25°C and -5°C. The dryness fraction of CO_2 gas during the suction stroke is 0.6. Following properties of CO_2 are given:

T °C	h_f kJ/kg	h_{fg} kJ/kg	h_g kJ/kg	S_f kJ/kg·K	S_g kJ/kg·K
25	81.3	121.4	202.6	0.251	0.63
-5	-7.54	245.3	237	-0.042	0.84



Sol

$$S_1 = S_{f1} + x_1 S_{fg} \quad , \quad S_{fg_1} = \frac{h_{fg}}{T_1}$$

$$= -0.042 + \frac{0.6 * 245.3}{268} = 0.507 \text{ kJ/kg·K}$$

$$S_2 = S_{f2} + x_2 \cdot \frac{h_{fg2}}{T_2} = S_1$$

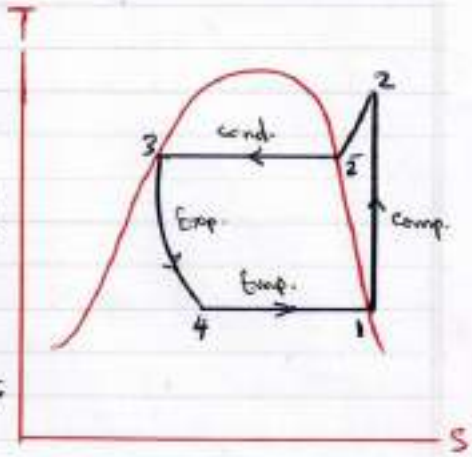
$$\therefore \; 0.507 = 0.251 + x_2 \cdot \frac{121.4}{298} \Rightarrow x_2 = 0.629$$

$$h_1 = h_{f1} + x_1 h_{fg_1} = -7.54 + 0.6 * 245.3 = 139.64 \text{ kJ/kg}$$

$$h_2 = h_{f2} + x_2 h_{fg_2} = 81.3 + 0.629 * 121.4 = 157.66 \text{ kJ/kg}$$

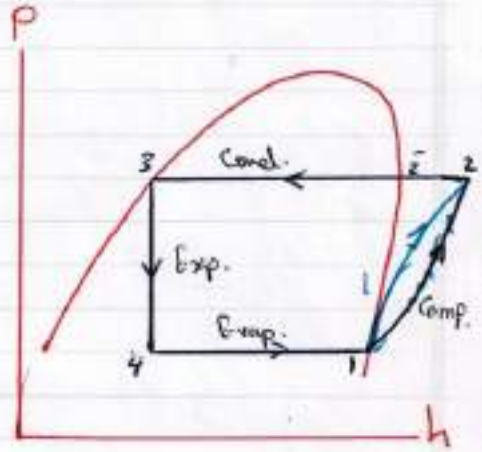
$$\therefore \; COP = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{139.64 - 81.3}{157.66 - 139.64} = 3.24$$

3. Vapour Compression Cycle with Superheated Vapour after Compression:



Ex: A vapour compression refrigerator uses methyl chloride (R40) and operating between temperature limits of −10°C and 45°C. At entry to the compressor, the refrigerant is dry saturated and after compression it acquires at temperature of 60°C. Find the COP of refrigerator.

$T_{sat.}$ (°C)	h_f kJ/kg	h_g kJ/kg	S_f kJ/kg·K	S_g kJ/kg·K
−10	45.4	460.7	0.183	1.637
45	133.0	483.6	0.485	1.587



Sol:

$$S_2 - S_{\bar{2}} = C_p \ln \frac{T_2}{T_{\bar{2}}}$$

$$S_2 = S_{\bar{2}} + C_p \ln \frac{T_2}{T_{\bar{2}}} = S_1$$

$$1.637 = 1.587 + C_p \ln \frac{333}{318}$$

$$\therefore C_{p_{avg}} = 1.09 \text{ kJ/kg·K}$$

$$h_2 = h_{\bar{2}} + C_p \times DOS \quad , DOS = \text{Degree of Superheat}$$

$$= h_{\bar{2}} + C_p (T_2 - T_{\bar{2}})$$

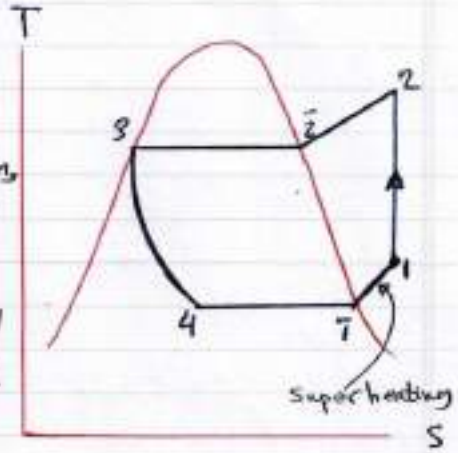
$$h_2 = 483.6 + 1.09(333 - 318) = 500 \text{ kJ/kg}$$

$$\therefore COP = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{460.7 - 133}{500 - 460.7} = 3.77$$

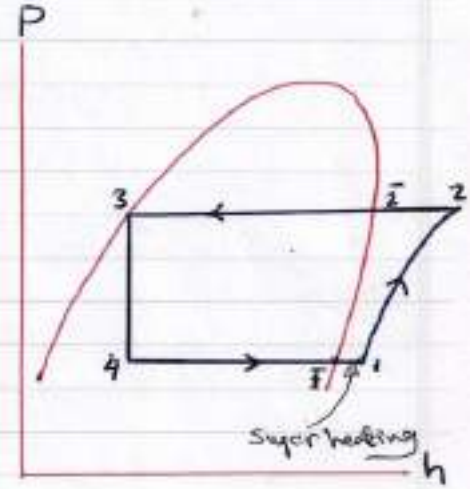
4. Vapour Compression Cycle with Superheated Vapour before Compression:

Ex: A vapour compression refrigeration plant works between pressure limits of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression, its temperature being 37°C. The vapour is superheated by 5°C before entering the compressor.

If the Specific heat of superheated vapour is 0.63 kJ/kg.K, find the COP of the plant. Use data given below:



Pressure (bar)	T_{sat} (°C)	h_f (kJ/kg)	h_{fg} (kJ/kg)
5.3	15.5	56.15	144.9
2.1	-14.0	25.12	158.7



Sol

$$h_1 = h_{\bar{1}} + C_{p_v}(T_1 - T_{\bar{1}}) \quad , \quad h_{\bar{1}} = h_g$$

$$h_{\bar{1}} = h_{fg} + h_f$$

$$\therefore h_1 = (25.12 + 158.7) + 0.63 \times (5)$$
$$= 186.97 \text{ kJ/kg}$$

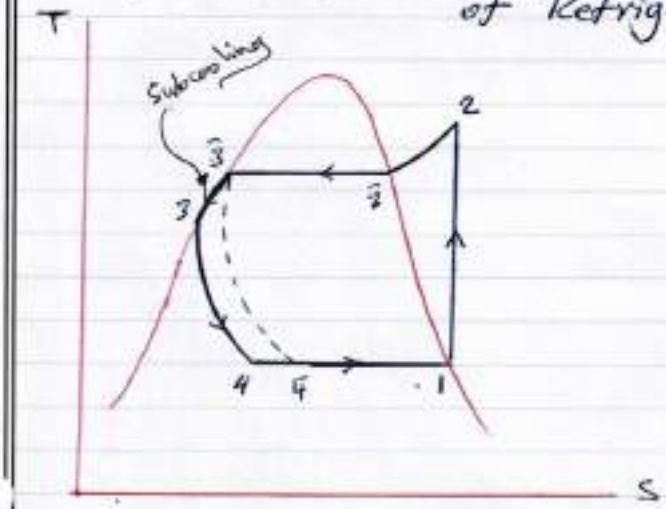
$$h_2 = h_{\bar{2}} + C_{p_v}(T_2 - T_{\bar{2}}) = (h_{f_{\bar{2}}} + h_{fg_{\bar{2}}}) + C_{p_v}(T_2 - T_{\bar{2}})$$

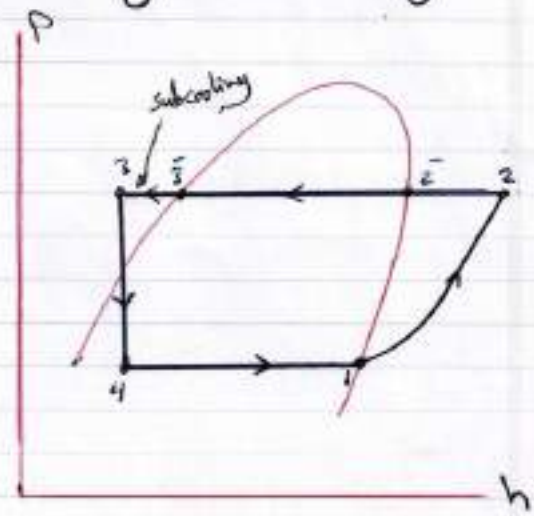
$$\therefore h_2 = (56.15 + 144.9) + 0.63 \times (310 - 288.5)$$
$$= 214.6 \text{ kJ/kg}.$$

$$COP = \frac{R.E}{W_{in}} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{186.97 - 56.15}{214.6 - 186.97}$$

$$\therefore COP = 4.735$$

5. Vapour Compression Cycle with undercooling or subcooling of Refrigerant





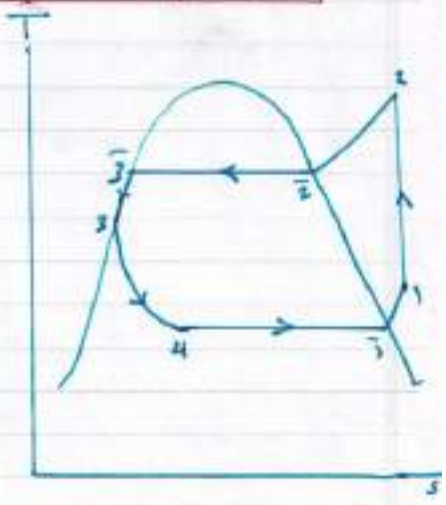
Ex: A food storage locker requires a refrigeration capacity of 12TR, and works between the evaporating temperature of -8°C and condensing temperature of 30°C. The refrigerant R12 is subcooled by 5°C before entry to expansion valve and the vapour is superheated to -2°C before leaving the evaporator coils. Determine: 1. COP 2. Power per TR.
Use the following data for R-12: $C_{p_L} = 1.235 \frac{kJ}{kg \cdot K}$, $C_{p_v} = 0.733$

Tsat (°C)	Pressure (bar)	Enthalpy kJ/kg		Entropy kJ/kg·K	
		h_f	h_g	S_f	S_g
-8	2.354	28.72	184.07	0.1149	0.7007
30	7.451	64.59	199.62	0.2400	0.6853

Sol 1. $S_1 = S_{\bar{1}} + C_{p_v} \ln \frac{T_1}{T_{\bar{1}}}$

$= 0.7007 + 0.733 \ln \frac{271}{265} \Rightarrow$

$S_1 = 0.7171$ kJ/kg·K

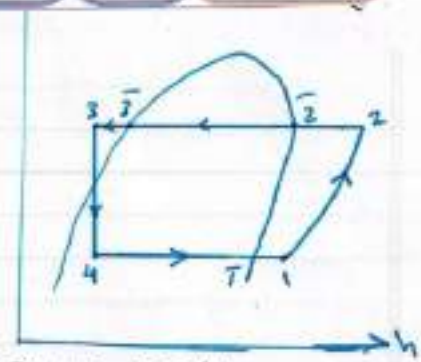


$S_2 = S_{\bar{2}} + C_{p_v} \ln \frac{T_2}{T_{\bar{2}}}$

$= 0.6853 + 0.733 \times \ln \frac{T_2}{303} = S_1$

$\therefore \quad 0.7171 = 0.6853 + 0.733 \ln \frac{T_2}{303}$

$\therefore \quad T_2 = 316.4\,K$ or $43.4^\circ C$



$h_1 = h_{\bar{1}} + C_{p_v}(T_1 - T_{\bar{1}})$

$= 184.07 + 0.733(271 - 265) = 188.47\ kJ/kg$

$h_2 = h_{\bar{2}} + C_{p_v}(T_2 - T_{\bar{2}})$

$= 199.62 + 0.733(316.4 - 303) = 209.44\ kJ/kg$

$Q_{5\to 3} = C_{p_L}(T_{\bar{3}} - T_3) = h_{\bar{3}} - h_3 \quad , \quad h_{\bar{3}} = h_f$

$h_3 = h_{f\bar{3}} - C_{p_L}(T_{\bar{3}} - T_3)$

$= 64.59 - 1.235 * 5$

$= 58.42\ kJ/kg = h_4$

$$COP = \frac{R.E}{W_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{188.47 - 58.42}{209.44 - 188.47} = 6.2$$

2. $R.E = h_1 - h_4 = 188.47 - 58.42 = 130.05\ kJ/kg$

Refrigerating Capacity $(Q) = 12\,TR$

$= 12 * 210 = 2520\ kJ/min$

or $Q = 12 * 3.5 = 42\ kW$

$$Q = \dot{m} * R.E \Rightarrow \dot{m} = \frac{Q}{R.E} = \frac{42}{130.05} = 0.32295\ kg/s$$

Power $= W_{in} = \dot{m}(h_2 - h_1) = 0.32295 * (209.44 - 188.47)$

$= 6.7723\ kW$

Power per tonne of Refrigeration $= \frac{W}{Q}$

$= \frac{6.7723}{12}$

$= 0.564\ \frac{kW}{TR}$

„ Sheet No. 9 „

Q1: The temperature in evaporator coils is $-6^\circ C$ and that in the condenser coil is $22^\circ C$. Assuming that the machine operates on the reversed carnot cycle. Calculate the COP, the Refrigeration effect per kW of input work, and the heat rejected to the condenser.

Ans. (9.54, 9.54, 10.4 kW)

Q2: A vapour compression refrigerator using R134a works between temperature limits of $-5^\circ C$ and $40^\circ C$. The refrigerant leaves the compressor dry saturated. Calculate the refrigeration effect and COP if, a- the refrigerant leaves the condenser saturated, and b- the refrigerant is subcooled to $20^\circ C$ before entering the throttle valve.

Ans. (a. 131.15 kJ/kg, 4.54, b. 163.08 kJ/kg, 5.65)

Q3: In a refrigerator, R134a is compressed isentropically from a Saturated State at $-5^\circ C$ to a pressure of 11.59 bar. The refrigerant is then cooled at constant pressure to $25^\circ C$, and is throttled down to a temperature of $-5^\circ C$ at which it is evaporated. Determine the temperature and enthalpy after compression in two ways: (a) by using the tables (b) by assuming the $C_{p_v} = 1.153$ kJ/kg.K. Sketch the cycle on T-S and p-h diagrams and Calculate the COP by each method.

Ans. (a. $50.66^\circ C$, 428.05 kJ/kg, 4.94, b. $50.62^\circ C$, 428.01, 4.95)

Q4: An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of $-10^\circ C$ and $30^\circ C$. The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at $30^\circ C$. If the actual coefficient of performance is 60% of the maximum, find the ice produced per kW hour at $0^\circ C$ from water at $10^\circ C$. The latent heat of ice is 335 kJ/kg. The ammonia has the following properties:

Temperature (°C)	h_f (kJ/kg)	h_{fg} kJ/kg	S_f	S_g
30	323.08	1145.79	1.2037	4.9842
-10	135.37	1297.68	0.5443	5.477

Ans. (33. kg/kW.h)