CHAPTER 8
GENERATOR OPERATION

In generator operation, a dc machine is driven by a prime mover and supplies an electrical load. This chapter explains the operating characteristics of dc generator and the affect them.

8.1 The Voltage Equation

In generator operation, we are interested in the voltage supplied at the output terminals. From KVL and the information of the previous chapters, the terminal voltage of a dc generator can be written:

$$V = E_A - (\sum IR + V_b) \quad (8.1a)$$

$$= K_e n \Phi_r - (\sum IR + V_b) \quad (8.1b)$$

$$= E_{Aoc} - (\sum IR + V_b + \Delta E) \quad (8.1c)$$

$$= K_e n \Phi_m - (\sum IR + V_b + \Delta E) \quad (8.1d)$$

Where $E_A = K_e n \Phi_r$ is the induced emf in the armature, and Φ_r is the actual (i.e. resultant) flux per pole; $E_{Aoc} = K_e n \Phi_m$ is the induced emf on open circuit (no armature current), and Φ_m is the flux per pole due to the main field. Φ_r may be somewhat less than Φ_m due to the demagnetizing effect of armature reaction (chapter 5); ΔE represents the corresponding reaction in induced emf ($\Delta E = E_{Aoc} - E_A$, section 5.3). The difference between the induced emf E_A and the terminal voltage V is the sum of series resistive drops $\sum IR$ (in the armature, series field wdg, commutating wdg, and compensating wdg) and the brush contact drop V_b. Eqn. 8.1d tells us that the terminal voltage is determined primarily by the speed n and the main field flux Φ_m, with some reduction due to series voltage drops and armature reaction.

8.2 Speed of Rotation

The speed is set at the prime mover, not the generator itself. Of course the generator is a mechanical load on the prime mover, and hence affects its operation :as the electrical load on the generator increases, the armature current I_A increases thus increasing the developed torque T_d=(K I_A Φ_r); if the prime mover torque does not increase to balance the increase in T_d, it will slow down (reducing E_A, hence V, hence I_A, hence T_d). However, in many applications, the prime mover is equipped with automatic control that maintains the speed almost constant (eg governor :as speed begins to fall, the governor enlarges the steam openings to the turbine).

8.3 Field Excitation

The main flux Φ_m is determined by the field mmf through the magnetization carves, or we say that E_{Aoc} is determined by the field excitation current through the OCC; see section 4.3. The shunt field excitation may be controlled by adding the variable resistance in series with the shunt field wdgs, and the series field excitation may be controlled by means of a small variable resistor (diverter)in parallel with the series field wdgs, see fig.8.2.

8.4 Voltage Drops

The series resistance drops $\sum IR$ and armature reaction drop ΔE increase with load (why?); the brush constant drop V_b is practically constant over the normal working range of I_A. The total drop is generally small (small wdg resistance, and small demagnetizing effect of armature reaction). For simplicity, we shall use the symbol ΔV for the total drop:

$$\Delta V = \sum IR + V_b + \Delta E \qquad (8.2)$$

So that equs. 8.1c and 8.1d become:

$$V = E_{Aoc} - \Delta V \qquad (8.3a)$$

$$= K_e n \Phi_m - \Delta V \qquad (8.3b)$$

8.5 Definitions

We shall need the terms and concepts defined below in our description of generator operation and the factors that affect it.

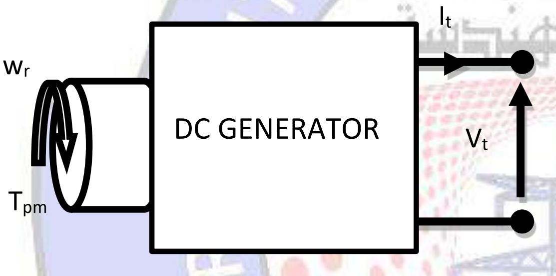


Fig. (8.1) Generator operation

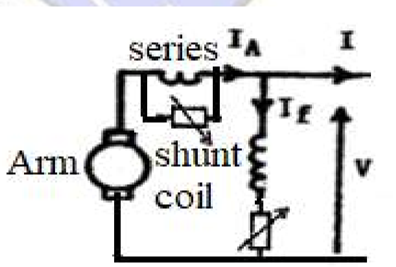
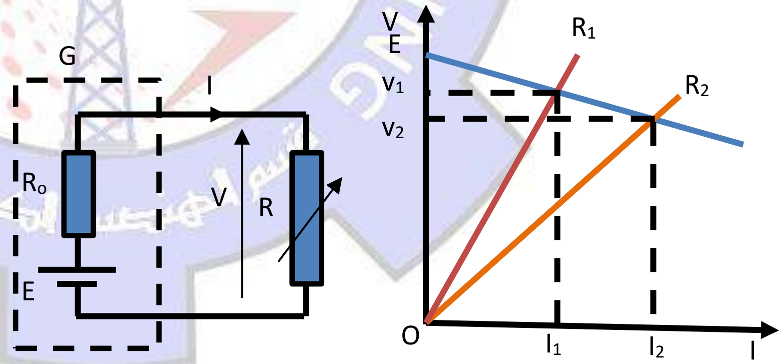


Fig. (8.2) DC machine with field control resistance

8.6 External Characteristic

The external characteristic of a dc generator is the curve relating terminal voltage V and terminal current I (i.e. load current). The curve shows now voltage change with load.

For a simple source circuit composed of constant emf E in series with constant internal resistance- see fig. 8.3a, the external characteristic is a straight line with negative slope, fig. 8.3b:



(a) circuit diagram (b) external characteristic

Fig. (8.3) Simple voltage source

$$V = E - IR_0 \qquad (8.4)$$

Comparing eqn. 8.4 with eqn. 8.1d we see that the external characteristic of a dc generator will be different from that of fig. 8.3b. We shall study the external characteristics of dc generators in section 8.3-8.6.

If a load resistance R_1 is connected across the terminals of the simple source of fig.3.3a, the terminal voltage and current will be:

$V_1 = \frac{E\,R_1}{R_o+R_1}$ and $I_1 = \frac{E}{R_o+R_1}$ (8.5)

The operating point (I_1, V_1) may also be found graphically by drawing the V-I characteristic of the load, and intersecting it with the characteristic of the source, fig. 8.3b; the intersection is the point that satisfies both characteristics at the same time. For the typical drooping characteristic shown, it is seen that decreasing the load resistance increases the load current and decreases the terminal voltage ($R_2 < R_1$, $I2 > I_1$, and $V_2 < V_1$).

Remark 1 :The graphical method can be used even when the external characteristic and the load V-I characteristic are nonlinear (not straight lines).

Remark 2 :The internal characteristic is the curve relating the emf and current. It is a horizontal line at E for the simple source of fig. 8.3a, but can be different in generators.

8.7 Voltage Control

A given external characteristic corresponds to a fixed speed and fixed settings of the field control resisters. If the setting of the field resisters is changed, Φ_m will also change (since the excitation currents are changed), and operation shifts to another curve. Therefore, the operating point may be moved from one curve to another by changing the field excitation, fig. 8.4. The terminal voltage may be kept approximately constant by automatic regulators that sense the terminal voltage and increase or decrease the excitation to keep the voltage at the set value.

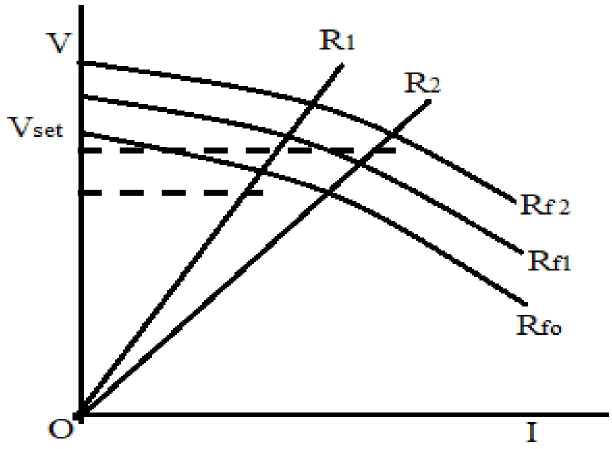


Fig. (8.4) Voltage control of DC generator with field control.

8.9 Voltage Regulation

The voltage regulation of a generator at a given load is defined by:

$$V_{reg} = \frac{V_{no}-V_{fl}}{V_{fl}} \quad (8.6)$$

It is a figure of merit that indicates how constant the terminal voltage is with load; a good voltage source should small voltage regulation. The voltage regulation of generators equipped with automatic voltage control is almost zero.

8.10 Separately Excited Generator

If the field current of a separately excited generator is kept constant, Φ_m and E_{Aoc} will be constant. The external characteristic is then as shown in fig. 8.5 :V is less than E_{Aoc} due to the armature circuit resistive drop $I_A R_A$ (linear with current), the demagnetizing effect of armature reaction ΔE (nonlinear function of I_A), and the brush contact drop V_b (constant-not shown in figure). The curvature of the characteristic comes from ΔE. Compare with the simple source of fig. 8.3.

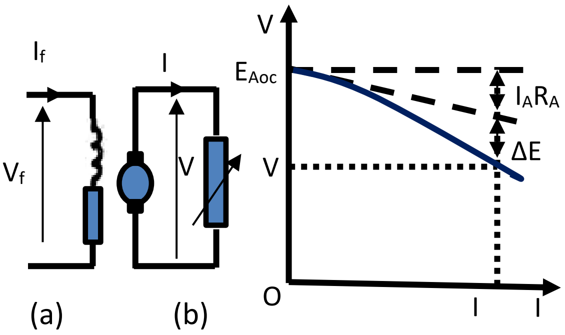


Fig. (8.5) Separately excited DC generator

8.11 Shunt Generator

The external characteristic of the shunt generator, fig. 8.6, is similar to that of the separately excited generator, but has an additional drop due to shunt field weakening:

$I_A\uparrow \rightarrow \Delta V\uparrow \rightarrow V\downarrow \rightarrow I_f\downarrow \rightarrow \Phi_m\downarrow \rightarrow E_{Aoc}\downarrow$

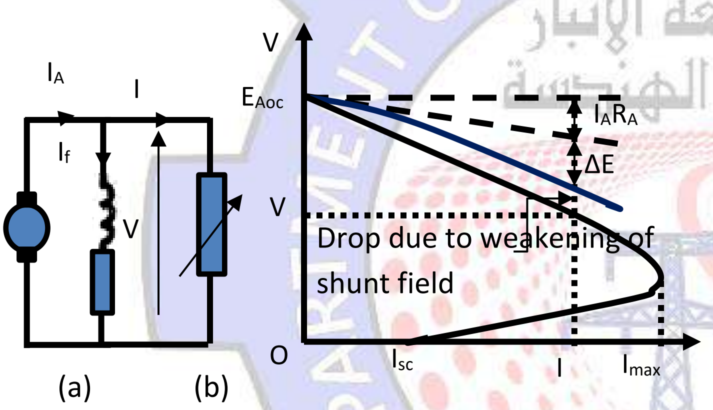


Fig. (8.6) Shunt excited DC generator

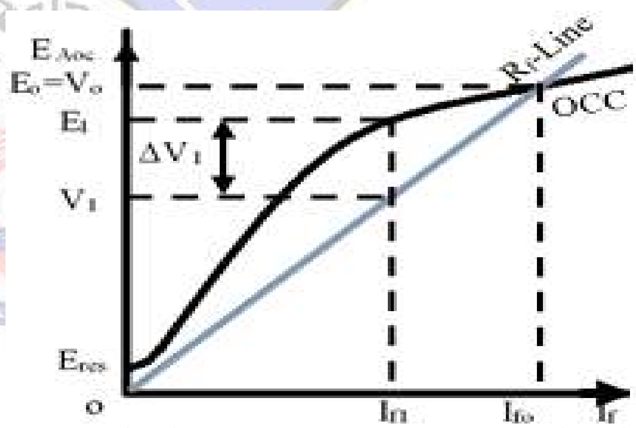


Fig. (8.7) Open Circuit Characteristics (OCC) of shunt excited DC generator.

To explain this process in more detail, we first draw the OCC as in fig. 8.7; note that the OCC is obtained with the shunt field wdg disconnected from the armature and fed from a separate source. Next, we draw the V-I characteristic for the shunt field resistance.

$$V = R_f I_f \qquad (8.7)$$

On the same graph. Now, for any field current If, the point (I_f, E_{Aoc}) must lie on the OCC, and the point (I_f, V) must lie on the R_f-line. At no-load the terminal current I is zero so that I_A is equal to 1_f which is small, so that we may neglect the drop ΔV; thus, the terminal voltage V is equal to the induced emf E_{Aoc}. This condition is satisfied only at the point of intersection of the R_f-line with the OCC; therefore, at no-load, we have:

$$I_f = I_{fo},\ E_{Aoc} = E_0,\ V = V_0, \text{ with } V_0 = E_0 \qquad (8.8)$$

Consider next the generator on load. I_A has increased so that the drop ΔV is now large enough to make V less than E_{Aoc}, eqn. 8.3a. Operation has to shift from the point of intersection :(I_f, E_{Aoc}) moves down the OCC, while (I_f, V) moves down the R_f-line. If will take up a position at which the difference between E_{Aoc} and V is equal to the drop ΔV:

$I_f = I_{f1}$, $E_{Aoc} = E_1$, $V = V_1$, with $V_1 = E_1 - \Delta V_1$ (8.9)

Comparing V_1 and $V_0 = (E_0)$, we see that the difference between them is the drop ΔV_1 plus an additional drop (E_0-E_1) due to the reduction of the induced emf E_{Aoc} from E_0 to E_1 corresponding to the reduction of the field current I_f from I_{f0} to I_{f1} as was stated at the beginning of this section.

At each value of load current, I, the field current I_f moves to a position that makes the difference between the OCC and the R_f-line equal to the drop ΔV that corresponds to that load current or, more precisely, to the armature current I_A. If you study the OCC and the R_f-line carefully, you will see that there is certain I_f at which the difference between them is maximum load current, I_{max} in fig.8.6; this is called the breakdown point.

The short circuit current of the shunt generator is inherently limited :at SC the terminal voltage V is zero so that $I_f = 0$; the emf is E_{res} (induced by the residual flux alone) which is very small. The resulting armature current is therefore small.

8.12 Voltage Build-Up

The preceding discussion helps us understand how the voltage of the shunt generator builds up. Assume that there is no load on the generator, and that there is an open switch in the field circuit so that $I_f=0$, $E_{Aoc}=E_{res}$, and $I_A=0$. If the switch is now closed, E_{res} is applied to R_f, and a small current If flows causing E_{Aoc} to climb up the OCC. But this increased value of E_{Aoc} is again applied to R_f and will increase I_f, which in turn increases E_{Aoc} some more. The process

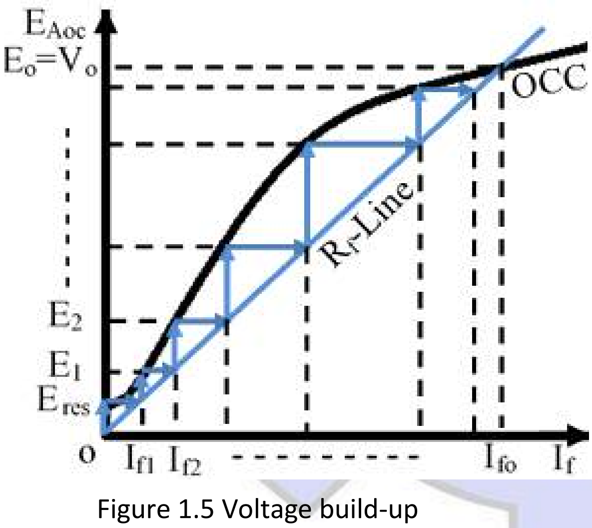


Figure 1.5 Voltage build-up

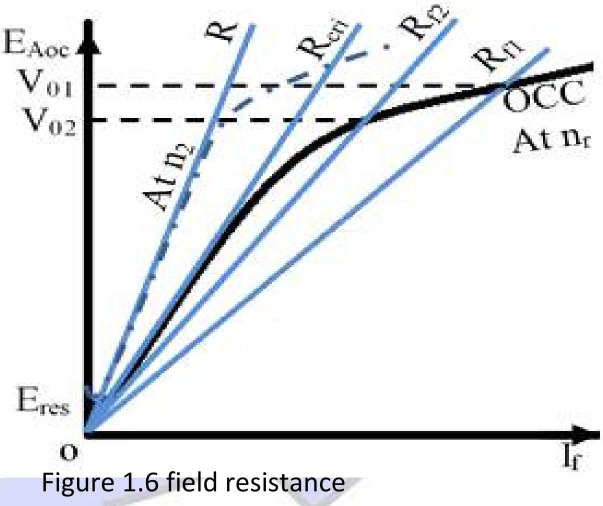


Figure 1.6 field resistance control

continues with (I_f, E_{Aoc}) climbing up the OCC and (I_f, V) climbing up the R_f-line until the two points coincide at the intersection point. We say that the shunt generator voltage V has 'build up' to V_0; what stops the build-up process from continuing indefinitely is the curvature of the OCC, i.e. saturation.

(Exercise :how does KVL apply to the circuit during build up?).

The process of voltage builds up requires the following conditions to succeed:

(1) There must be residual flux to start the process. A new generator, or one that has not been used for a long time, must be magnetized first. This is done by applying a separate dc source (for example a battery) to the field wdg for a short time; it is called 'flashing the field'.

(2) The flux produced by I_f should aid the residual flux. If the field wdg is connected in the reverse direction, the voltage will 'build-down' i.e. it becomes less than E_{res} (i.e. almost zero).

(3) The resistance of the field circuit R_f should be small enough to intersect the OCC in the saturation region. As R_f is made larger, the intersection moves down the OCC, fig. 8.8. The field resistance line that is tangent to the linear part of the OCC is called the critical field resistance, R_{crit} in fig. 8.8 :if R_f is increased further, there will be no build up. The critical resistance is higher for higher speeds (why?); the critical speed corresponding to a given field resistance is the speed at which the linear part of the OCC becomes tangent to that resistance. If the shunt generator fails to build-up at a certain speed due to large field resistance, it might build-up at a higher speed.

8.13 Compound Generator

A compound generator is essentially a shunt generator with additional mmf from the series wdg:

$MMF_{total} = N_f I_f \pm N_s I_s$ (8.10)

The compounding is cumulative if the series field aids the shunt field (plus sign in eqn 8.10); the compounding is differential if the series field opposes the shunt field (minus sign in eqn 8.10). For long shunt connection, fig. 4.3, $I_s = I_A$, and for short shunt connection $I_s = I_A - I_f \approx I_A$. If a diverter is used, fig. 8.2, I_s may be less than these values.

The OCC of a compound machine corresponds to separate excitation of shunt field alone. Dividing eqn 8.10 by the shunt field turns:

$$\frac{MMF_{total}}{N_f} = I_f \mp \frac{N_s}{N_f} I_s = I_{eq} \qquad (8.11)$$

The term $(N_s/N_f)I_s$ is the series field excitation referred to the shunt field circuit; I_{eq} represents total excitation in terms of shunt field amperes, and can be read off or projected directly on the horizontal axis of the OCC.

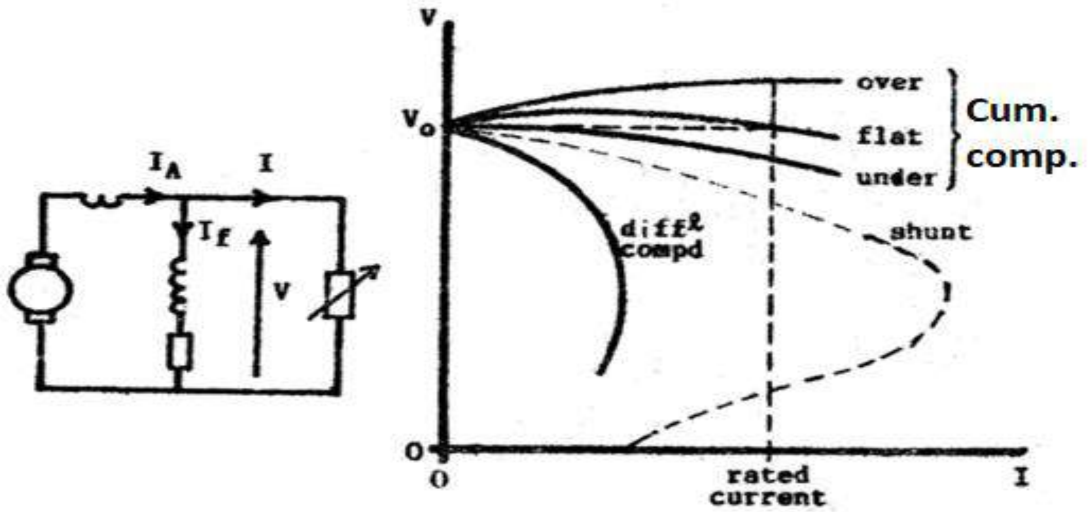


Fig. (8.9) Compound generator

8.13.1 Cumulative Compounding

As seen in section 8.4, the terminal voltage of a shunt generator due to ΔV and the reduction in E_{Aoc}. In a cumulative compound generator, the series field compensates for part or all of the drop.

The series field current changes with load (why?) so that the degree of compensation changes with load. The number of series turns N_s may be chosen such that the resulting series field compensates exactly for the drop at full load; the full load voltage is then equal to the no-load voltage (zero voltage regulation), and the machine is said to be flat-compounded (or level-compounded)-see fig. 8.9. If fewer series turns are used, we have under-compounding :the full load voltage is less than the no-load voltage (positive regulation), but still more than the shunt generator full load voltage. If more series turns are used, we have over-compounding :the full load voltage is greater than the no load voltage (negative regulation). It is also possible to choose N_s for over-compounding, and change the actual degree of compounding by means of a diverter, fig. 8.2.

8.13.2 Differential Compounding

When the series field opposes the shunt field, ineffectively increases the drops, the external characteristic is then below that of the shunt generator, fig. 8.9.

8.14 Series Generator

The OCC for a series generator, fig. 8.10, is obtained with the field supplied from a separate source (the armature is open circuited by definition). In normal operation, the field wdg is connected in series with the armature, and the terminal voltage V is less than the induced emf E_{Aoc} due to the drop ΔV; the external characteristic is thus below the OOC as shown in fig. 8.10. The rising part of the curve is not stable :a slight change of load resistance causes large changes in terminal voltage and current. In the saturation region, the OCC is almost horizontal, but ΔV continues to increase with I_A so that the curve is falling; the fall is sharp in series generator designed to have strong demagnetizing armature reaction.

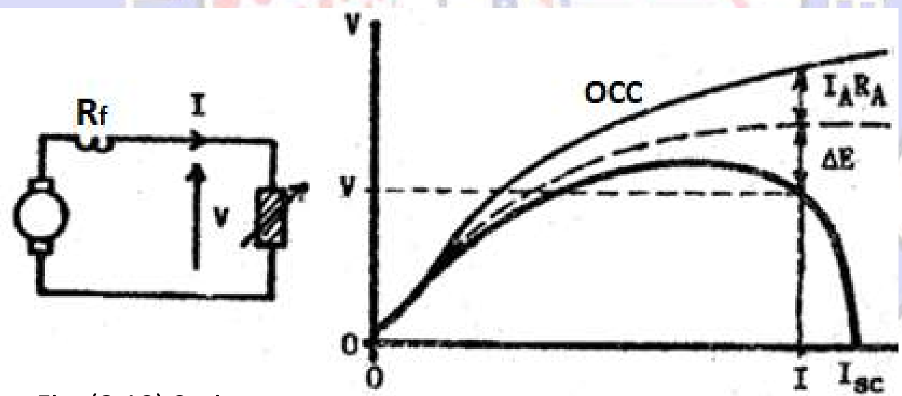


Fig. (8.10) Series generator

8.15 Applications

For dc power generation, the separately excited generator has acceptable voltage regulation, but has the disadvantage of requiring a separate source. The self-excited shunt generator does not require a separate source, but has poor regulation. The cumulative compound generator overcomes this problem; it can be designed to have zero regulation by suitable compounding. Modern generators are equipped with automatic voltage control, possibly solid-state, so that they have excellent regulation; the design of the control system is determined by the external characteristic of the generator. However, solid-state rectifiers are rapidly replacing dc generator in most applications; technological advances have made it possible to manufacture commercial solid-state components of high rating, i.e. components capable of passing high currents and withstanding high voltages. For example, the dc generator in the automobile has now been replaced by an ac generator (alternator)with rectifier.

The external characteristics of the series and differential compound generators make them unsuitable for dc power generation at constant voltage. The falling portions of their characteristics correspond to constant current operation over that range. The series generator has been used as a booster :it is connected in series with the line between a generator and its load; its rising characteristic compensates for the drop in the line.

8.16 Parallel Operation

Two dc generators, or a generator and a battery, may be operated in parallel to supply a common load. The over-all characteristic is obtained by graphical parallel addition of their external characteristics, fig. 8.11. The figure also shows how the two generators share the load current according to their individual characteristics.

When we intend to operate two generators in parallel, before closing the paralleling switch we must make sure that the voltages of the two generators are equal and have the same polarity; otherwise large currents may circulate between them.

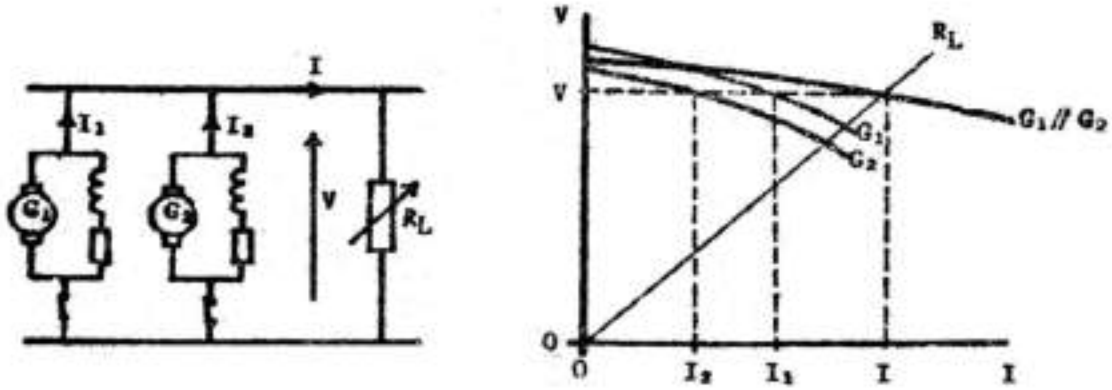


Fig. (8.11) parallel operation of shunt generators

8.17 EXERCISE

The table represents the OCC of a dc machine at 850 rpm. The machine has a shunt field wdg of 600 turns and 20 Ω. The armature circuit resistance is 0.25 Ω.

NB unless otherwise stated, you may neglect the demagnetizing effect of armature reaction and the brush constant drop.

1. Separate excitation

The field wdg is connected in series with a variable resistor to a 330 v separate source.

1.1 The field control resistor (FCR)is set to 10 Ω.

Plot the external characteristic up to a load current of 300 A. find the load current and voltage regulation when the terminal voltage is 220 V. (112 A, 12.7%)

I_f (A)	E (V)
0	20
0.5	24
1	40
2	80
3	119
4	150
5	174
6	193
7	209
8	222
9	233
10	241
11	248
12	253
13	258
14	262
16	268
18	272
20	275

1.2 Repeat 1.1 (on the same sheet) for a FCR of 20. (20 A, 2.3%)

1.3 Find FCR to make the terminal voltage 220 V at a load current of 180 A. (2.3 Ω)

1.4 Find the maximum possible load current at a terminal voltage of 220 V. (196 A)

1.5 Find the terminal voltage when the speed is changed to 1100 rpm, FCR is set to 10 Ω, and the load current is 200 A. (270.9 V)

2. **Shunt**

2.1 for a speed of 850 rpm:

A. What is the no load voltage when FCR is shorted out? Set to 10 Ω? Set to 30 Ω?

B. Find the critical resistance and short circuit current?

C. What is the critical speed when FCR is set to 10 Ω? 30 Ω?

D. What is the minimum speed at which the generator can build up? (257.5 V, 208 V, 24 V; 40 Ω, 80 A; 637.5 rpm, 1062.5 rpm; 425 rpm)

2.2 for a speed of 1100 rpm:

A. Fined the critical resistance and short circuit current?

B. Find the no load voltage when FCR is shorted out? (51.76 Ω, 103.5 A; 351.5 V)

2.3 At 850 rpm and FCR shorted out:

A. Plot the external characteristic.

B. Find the load current and VR when the terminal voltage is 220 V.

C. Find the terminal voltage, load current and VR when the load resistance is 1.5 Ω.

D. What is the breakdown current?

(101A,17%; 207V, 138A, 24.4%; 291A)

2.4 Estimate the value of FCR needed to make the terminal voltage 220V at a load current of 80A when the speed is 850 rpm. (1.7Ω)

2.5 The speed is 350 rpm and FCR is shorted out. V_b is 2 V, and armature reaction demagnetization is <u>not</u> negligible. Find ΔE and ΔI when the terminal voltage is 220 V at a load current of 56A. (9.25V, 1.3A).

3. **Compound**

The machine is connected in short shunt for cumulative compounding. The speed is 850 rpm.

3.1 With FCR shorted out, it is required to make the terminal voltage 220V when the load current is 150A. how many series turns are needed? What is the voltage regulation? Assume negligible series field resistance. (10 turns, 17%)

3.2 The series field winding has 15 turns and a resistance of 0.05Ω; FCR is shorted out. V_b is 2V and the demagnetizing effect of armature reaction demagnetization is <u>not</u> negligible. when the terminal voltage is 220 V at a load current of 80A. Find the armature current, the induced emf, and ΔE. (91.2A, 259V, 10.2V)

3.3 The series field winding has 16 turns and a resistance of 0.05Ω; FCR is set at some unknown value. V_b is 2V, and the demagnetizing effect of armature reaction is negligible. The load current is found to be 150 A when the shunt field current 10A. Find the terminal voltage, the value of FCR, and VR. (212.5V, 2Ω, 17.6%)

Solution of chapter 8 (EXERCISE 1)

Q1/ N_f=600 R=20Ω R_A=0.25

1) Separate excitation

1-1- F_{CR}=10Ω , I_L=300 I_L=? , V_R=? , At V_t=220V

R_f=F_{CR}+R_f ➡ R_f=10+20=30Ω

$I_f = \frac{vsh}{Rf} = \frac{330}{30} = 11A$

E_A= 248, V_t=E_A-I_AR_A , $I_A = \frac{EA-Vt}{RA} = \frac{248-220}{0.25} = 112A$

$V_R = \frac{Vmt-VAt}{Vfc} = \frac{248-220}{220} * 100 = 12.72\%$

To draw external characteristic

V_t=E_A-I_AR_A

I_A=0 ➡ 300 A, V_t=E_A=248 when I_A=0

V_t=248-30*0.25=240.5 when I_A=30

I_A	V_t
0	248
30	240.5
60	233
90	225.5
120	218
150	210.5
180	203
210	159.5
300	173

1-2- F_{CR}=20, R_f=40Ω

$I_f = \frac{330}{40} = 8.25A$, so E_A=225v

$I_A = \frac{EA-Vt}{RA} = \frac{225-220}{0.25} = 20A$

$V_R = \frac{Vnt-Vt}{Vt} = \frac{225-220}{220} * 100\% = 2.27 \approx 2.3\%$

To draw external characteristic

V_t=E_A-I_AR_A

V_t=E_A=225 when I_A=0

V_t=225-30*0.25=217.5, When I_A=30A

I_A	V_t
0	225
30	217.5
60	210
90	202.5
120	195
150	187.5
180	180
210	172.5
240	165
270	157.5
300	150

1-3- F_{CR}=? V_t=220 I_A=180A

$V_t=E_A-I_AR_A$

E_A=220+180*0.25=265 V, $R_f=\frac{330}{14.78}=22.3$

F_{CR}=22.3-20=2.3Ω

1-4- V_t=220, Imax ? when, F_{CR}=zero

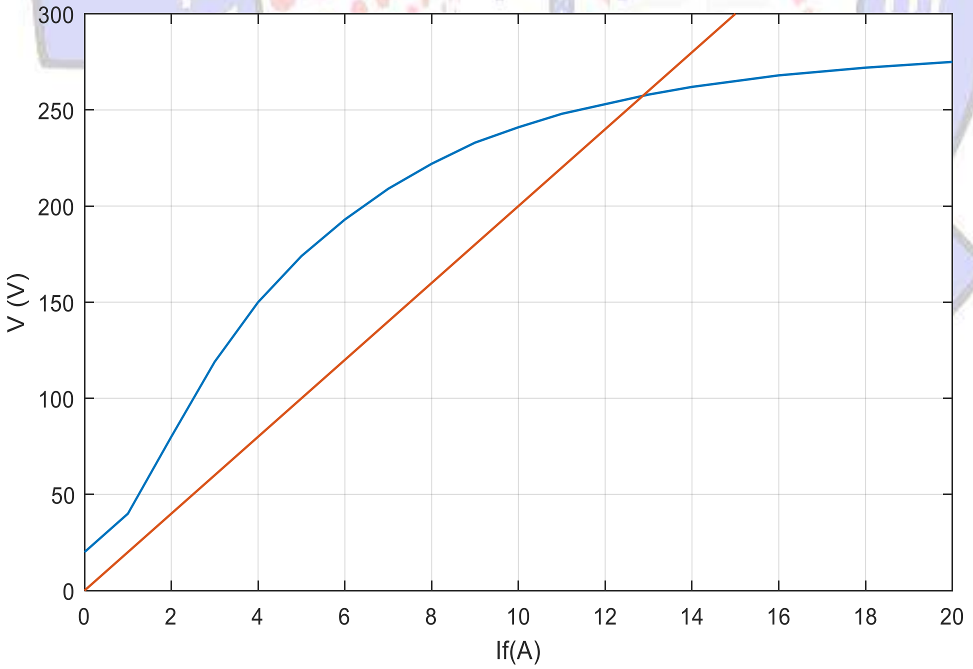
$I_f=\frac{Vf}{Rf}=\frac{330}{20}=16.5$, so E_A=269, $V_t=E_A-I_AR_A$, $I_A=\frac{EA-Vt}{RA}=\frac{269-220}{0.25}=196A$

1-5- N=1100Vpm F_{CR}=10Ω I_L=200A V_t=?

$I_f=\frac{Vf}{Rf}=\frac{330}{30}=11A$, so Ea=248 v at 850 rpm.

$$\frac{EA0}{EA1}=\frac{No}{N1}\rightarrow EA1=\frac{1100*248}{850}=320.9\,V \quad at \quad 1100Vpm$$

$V_t=E_A-I_AR_A$ =320.9-200.0.25=270.9 V



2-Shunt

2.1- for a speed of 850Vpm, Vno?

A) $F_{CR}=0$

To find Vno load we must draw the Rf= line with occ and the point of intersection represent.

I_f(A)	V_f (v)		
	$F_{CR}=0$	$F_{CR}=10$	$F_{CR}=30$
0	0	0	0
1	20	30	50
2	40	60	100
3	60	90	150
4	80	120	200
5	100	150	250
6	120	180	300
7	140	210	350
8	160	240	400
9	180	270	450
10	200	300	500
11	220	330	550
12	240	360	600
13	260	390	650
14	280	420	700
15	300	450	750
16	320	480	800
17	340	510	850
18	360	540	900
19	380	570	950

At $F_{CR}=0$ $R_f=20$, $V_f=I_fR_f$

$V_f=0*20=0$, $V_f=0.5*20=10$

$V_f=1*20=20$, $V_f=2*20=40$

From the point of intersection

$V_f=257.5V$

At $F_{CR}=10\Omega$, $R_{ft}= 30\ \Omega$

$V_f=30*0=0$, $30*1=30v$, $30*2=60v$

From the point of intersection

$V_n=208V$

At $F_{CR}=30\ \Omega$ $R_{ft}=30+20=50\ \Omega$

$V_f=50*0=0$, $5*1=50$

From the point of intersection

$V_n=24V$

B) $R_{cri}=?$ Isc=? $V_t=0$

To find R crictical, we draw a line tangent to the occ and take any point intersection between them and find Rcr from Rcr=$\frac{V}{I}$

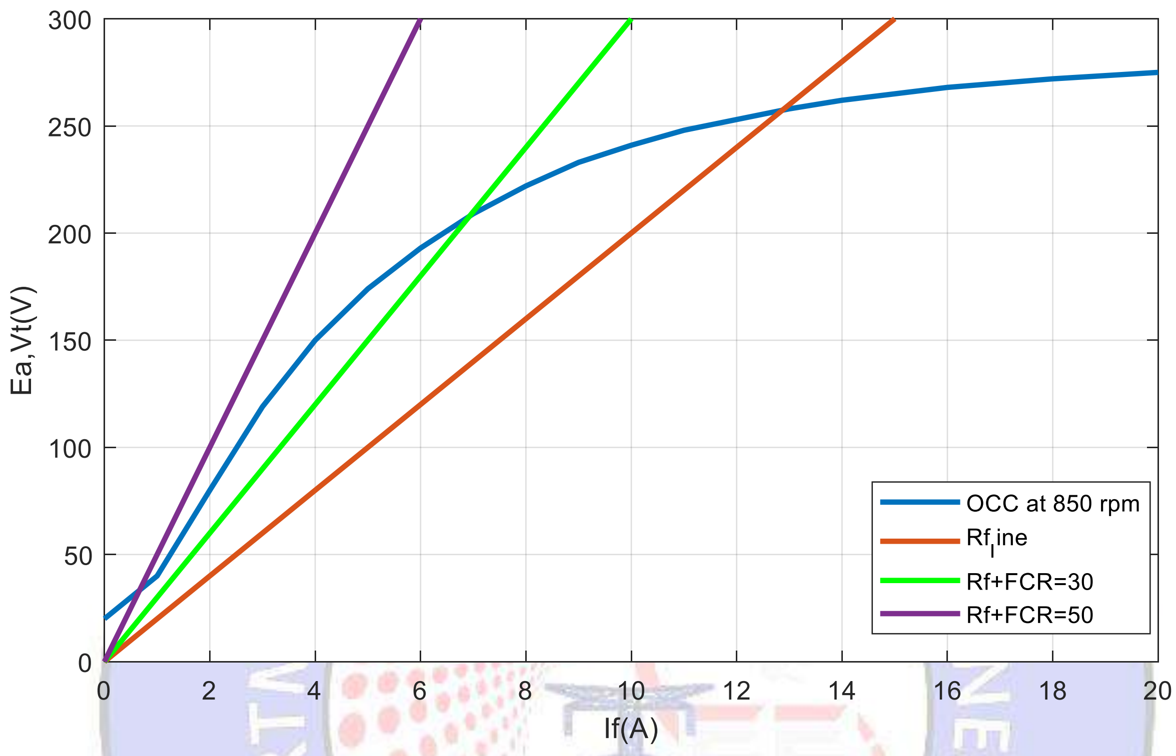
Rcr=$\frac{80}{2} = 40\Omega$

At Short cct $I_f=0$ $E_A=E_{res}$

$V_t=0$ $I_A=I_{sc}$

$V_t=E_A-I_AR_A$, $0=E_{res}-I_{s.c}\,R_A$

$I_{s.c}=\frac{Eres}{RA}=\frac{20}{0.25}=80A$



c)

1) when $F_{CR}=10$ $n_{cri}=?$

To find critical speed at $F_{CR}=10\Omega$. we draw the $R_{f\text{-line}}$ when $R_f=10+20=30\ \Omega$ take any value of (I_f) and alling it on occ and Rf-line $F_{CR}=10\ \Omega$ and find the value of E_1 from Rf line and E_0 from occ at new speed and applied :

$\frac{E1}{E0}=\frac{n1}{n0}$ When $I_F=I_A$ $E_0=40V$ from occ at 850Vpm

$E_1=30V$ from R_f line $F_{CR}=10\ \Omega$

$$\frac{E1}{E0}=\frac{n1}{n0}\rightarrow\frac{30}{40}=\frac{n1}{850}\rightarrow n1=637.5\ rpm$$

2) at $F_{CR}=30\ \Omega$, same procedure in (1) using $R_{f\text{-line}}$ when $R_f=30+20=50\ \Omega$, when $I_f=3$, $E_0=120V$, $E_1=150$

$$\frac{E1}{E0}=\frac{n1}{n0}\rightarrow\frac{150}{120}=\frac{n1}{850}\rightarrow n1=1062.5\,rpm$$

D)

At F_{CR}=0,to get minimum speed we take any value of (I_f) in the med points of R_f line and falling it on occ and F_{CR}=0 and find E_0 /E_1,At I_f=2

E_0=80 on occ, E_1=40 on R_f (F_{CR}=0 Ω)

$$\frac{E1}{E0} = \frac{n1}{n0} \rightarrow n1 = \frac{40 * 850}{80} = 425Vpm$$

2-2-

A) R_C=? And $I_{s.c}$=? at 1100 rpm, we take the point (80V,2A) from occ at 850Vpm

E_1=$\frac{1100*80}{850} = 103.52$ V, R_C=$\frac{E1}{If} = \frac{103.52}{2} = 51.76\Omega$

E_1=$\frac{1100*20}{850} = 25.88$ V

Is.c =$\frac{25.88}{0.25} = 103.52A$

The good solustion must draw the OCC at 1100 rpm and plot a tangent line to new OCC.

B) V_{nL}=? F_{CR}=0

We plot occ at N=1100 rpm

And plot the R_f line (F_{CR}=0)

R_f=20 850 Vpm

V_f=$I_F R_F$, V_f=0*20=0

V_f=20*0.5=10, V_f=20*1=20

E_A=$\frac{E0*1100}{850}$

E_A=$\frac{20*1100}{850} = 25.88$

E_A=$\frac{24*1100}{850} = 31.05$

The intersection point between the OCC and Rf-line get

V_n load=351.5 V

Vf(v) FCR(0)	EA at 1100Vpm	If	EA
0	25.88	0	20
20	51.76	1	40
40	103.52	2	80
60	154	3	119
80	194.11	4	150
100	225.17	5	174
120	249.76	6	193
140	270.47	7	209
160	287.29	8	222
180	301.529	9	233
200	311.88	10	241
220	320.94	11	248
240	327.41	12	253
260	333.88	13	258
280	339.05	14	262
320	346.88	16	268
360	352	18	272
400	355.88	20	275

2-3-

V_t=E_A-$I_A R_A$, The intersection point between the OCC and Rf-line get at F_{CR}=0

V_n lood=257.5

A)

$V_t=E_A=257.5$ V when $I_A=0$

$V_t=257.5-150*0.25=220$

B) $V_t=220V$ $I_L=?$ $V_r=?$

$E_A=V_t+I_AR_A$

$I_A=\frac{EA-Vt}{RA}=\frac{257.5-220}{0.25}=150A$

$I_L=I_A-I_f \rightarrow I_L=150-11=139A$

$V_R=\frac{VNL-Vt}{VT}=\frac{257.5-220}{220}*100\%=17\%$

C) $V_T=?$ $I_L=?$ $V_R=?$ $R_L=1.5\Omega$

$V_t=I_L*R_L$

$V_T=0*1.5=0 \rightarrow I_L=0$

$V_t=30*1.5=45$

$V_t=60*1.5=90$

from occ , $V_t=221V$, $I_L=148A$

$V_R=\frac{VNL-Vt}{VT}=\frac{257.5-221}{221}*100\%$

$V_R=16.5\%$

D) from the curves of OCC and rf-line

I break down = I max= $\frac{\Delta V}{R0}=\frac{193-120}{0.25}$

$\Delta V=73$ FCR=0

I max=$\frac{73}{0.25}=292A$

2-4-

$F_{CR}=?$ $V_t=220$ $I_L=80A$

Using external characteristic when IL=80A, Vt=227V

$E_A=V_t+I_AR_A$, $E_A=220+80.(0.25)=240$ V

When EA=240 V,from OCC so $I_f=10A$

So $R_{ft}=220/10=22\ \Omega$

So FCR=22-20=2 Ω

2-5-

$N=350$ Rpv $F_{CR}=0$ $V_b=2V$

$\Delta E=?$ $\Delta If=?$ $V_t=220$

$I_L=56A$, $I_f=\frac{220}{20}=11A$

From occ $E_0=248V$, $I_A=I_L+I_f=56+11=67$

$E_A=V_t+I_A\ R+V_b$

E_A=220+67(0.25)+2=238.75V,   From occ I_f*=9.7

Δ E=E_0-EA,   ΔE=248-238.75=9.25V

ΔI_f= I_f-I_f*   ΔI_f=11-9.7=1.3A

3-Compound short shunt, cumulative, n=850 rpm, Nf=600   Rf=20

3-1- F_{CR}=0 V_t=220 I_L=150 N_s=? V_r=?   Rs=0

I_f= $\frac{220}{20} = 11A$ $Is = IL = 150$

I_A=I_f+I_L ⇒ I_A=150+11=161A

E_A=V_t+$I_A R_A$ ⇒ E_A=220+161*0.25=260.25v

From occ   Ieq=13.5A

Ieq=I_f+$\frac{Ns}{Nf} * Is$

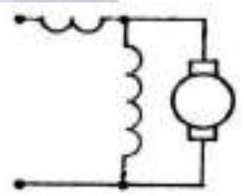
13.5=11+$\frac{Ns}{600} * 150$

2.5= $\frac{Ns}{600} * 150$ $Ns = 10turns$

V_n load =257.5V

V_r=$\frac{VnL-Vt}{Vt} * 100\%$

V_r=$\frac{257.5-220}{220} * 100\% = 17\%$



(e) compound - short shunt

3-2-

Ns=15   Rs=0.05 F_{CR}=0 V_b=2V

V_t=220V I_L=80A I_A=? E_A=?

KVL, V_{sh}=V_t+V_s, V_{sh}=220+0.05*80, V_{sh}=224

I_f= $\frac{Vsh}{Rf} = \frac{224}{20} = 11.2A$, I_A=I_f+I_L=80+11.2=91.2A

E_A=V_t+I_A R_A+ILRs+2

E_A=220+91.2*0.25+80*0.05)+2=248.8V

Ieq=11+$\frac{15}{600} * 80 = 13A$

E_0=258V,   ΔE=258-248.8=9.2 V

3-3-

Ns=16   Rs=0.05 I_L=150 I_f=10 V_b=2V

Fund F_{CR}=? V_r,V_t

I_L=I_s=150

IA=150+10=160A

$Ieq=I_f+\frac{Ns}{Nf}*Is$

$Ieq= 10+\frac{16}{600}*150=14A$

From occ E_A=262.5V

$V_t=E_A-I_A*RA-IL*Rs- V_b$

V_t=262.5-160*0.25-150*0.05-2=213V

$V_{sh}= V_f=V_t+V_s$

V_{sh}=213+150*0.05=220.5

$R_f=\frac{220.5}{10}=22.05$ $FCR=22.05-20=2.05\Omega$

$V_{sh}=R_f*I_f=0$ ⇒ I_f=0A, V_{sh}= 22.05*1=22.05V, V_{sh}=22.05*2=44.1V

From occ , V_n load =250V

$VR=\frac{250-213}{213}$*100%=17.4%

EXERCISES

Unless otherwise stated, assume that (a) winding resistances are given at the working temperatures, (b) the demagnetizing effect of armature reaction is negligible, and (c) the brush contact drop is 2V.

Answers obtained from graphical solutions are approximate and cannot be reproduced exactly. In some questions you have to use your judgment to make simplifying assumptions.

I_f(A)	E_A(v)
0.0	6
0.08	11
0.20	26
0.40	52
0.50	64.5
0.66	79
0.84	94
1.08	110
1.34	125
1.67	140
1.93	150
2.20	159
2.52	167
2.92	175
3.40	183

Questions 1-8 refer to **machine 1** which is a dc generator rated at 3 KW, 125 V, and 1150 rpm. The OCC is given in the adjacent table. The armature winding resistance is 0.38 Ω and the commutating winding resistance is 0.0716 Ω. The field winding has 1070 turns per pole and its resistance is 66.6Ω.

1. Machine 1 is separately excited from a 150 V source.
   a. Find the field current at rating; also find the setting of the field control resistor and the voltage regulation.
   b. The load resistance and field control resistance remain as in part (a), but the speed is raised to 1500 rpm. Find the terminal voltage and current, and the voltage regulation.
   c. The generator runs at 1500 rpm and delivers rated current at rated voltage. Find the field current, FCR, and VR.
   d. The machine is delivering rated current at rated at rated voltage with FCR set at 40 Ω. Find the speed and VR.
2. Machine 1 is separately excited from a 150 V source, but it has no interpoles. The brushes are shifted to improve commutation. The generator operates are rating with FCR set at 16Ω.
   a. Find the voltage regulation.
   b. Determine the demagnetizing effect of armature reaction in volts and in field amps.
3. Machine 1 is operated as a shunt generator.
   a. The generator is started with no external field resistance. Find the voltage to which it builds up at 1150 rpm, and find the minimum speed at which it can build up.
   b. repeat part (a) for a cold start (assume an ambient temperature of 20 ^{0}c).
   c. what is the maximum field circuit resistance that allows build-up at 1150 rpm? at 800 rpm? at 1500 rpm?

d. Find the field circuit resistance that allows the generator to build up to rated voltage at rated speed.

e. With no external field resistance, what is the maximum accelerating voltage during build-up at 1150 rpm? What is the corresponding field current?

f. The generator build –up to rated voltage with the external field resistance set to zero. Find the speed.

4. Machine 1 is operated as a shunt generator at 1150 rpm.

a. Find the short circuit current.

b. Find the breakdown current when the external field resistance is set to give rated voltage at no load.

c. Find the maximum breakdown current.

5. Machine 1 is operated as a shunt generator at 1150 rpm. The terminal voltage is at rated value.

a. Find the current and VR when FCR is shorted out.

b. Repeat part (a) for a field control resistor of 20Ω.

c. Find the value of the external field resistance when the generator delivers rated current; also find VR.

6. Machine 1 runs at 1150 rpm and supplies a load current of 20A with a field current of 1.58 A. find VR and. FCR for the following cases:

a. Separate excitation as in problem I.

b. Shunt connection.

c. Cumulative compound, long shunt connection. The series field winding has 18 turns/pole and its resistance is 0.069 Ω.

d. Cumulative compound, short shunt connection. Series field winding as in part c.

e. Differential compound, short shunt connection. Series field winding as in part c.

7. Machine 1 is connected in long shunt cumulative with a series field wdg of 18 turns/pole and 0.069 Ω resistance. It is operating at rating with a diverter across the series field wdg.

a. Estimate the diverter resistance when the shunt field external resistance is set at 20 Ω and the ratio of diverter resistance to series field resitance is 0.5 ; also find the voltage regulation.
b. Estimate the diverter resistance when VR is zero.

8. Machine 1 is connected in long shunt cumulative compound as in problem 7. There is no diverter and no interpoles. The brushes are shifted from the q-axis to improve commutation. Find the demagnetising effect of armature reaction in volts and in field amps.
9. The machine described in the exercises of pages 4.6 and 5.4 is runas a generator at 800 rpm. Armature current is 100 A. The no load voltage is 271 V, and the Armature reaction is not negligible. Find the load current, terminal voltage, and voltage regulation for the following cases:
   a) Separate excitation.
   b) Shunt connection.
   c) Cumulative compound long shunt connection. The series field wdg has 12 turns /pole and its resistance is 25 mΩ.
10. A series generator has the OCC given in the table. The armature resistance is 0.6Ω. The field wdg has 50 turns /pole, and its resistance is 0.1Ω. The brushes are shifted from the q-axis so that the demagnetizing armature reaction is 750 AT/pole when the load current is 60A.
   a) Plot the external characteristic up to a load current of 90 A.
   b) What is the load current and terminal voltage when the load resistance is 2Ω?
   c) What is the critical load resistance?
   d) Find the load current when the terminal voltage is 85 V?
   e) What is the maximum voltage?
   f) A diverter 0.15Ω is connected across the field winding; find the terminal voltage when the load current is 70A?

Amps	volts
0	5
4	20
8	39
10	49
15	69
20	87
26	107
30	117
35	126
40	131
45	134
50	136
60	140
80	144

Volte	Amp
15.5	0.0
15.4	2.0
15.2	3.8
14.9	5.7
14.7	6.8
14.5	7.8
14.0	9.8
13.5	11.5
13.0	13.0
12.2	14.6
11.9	15.8
10.5	19.0

11. A battery is rated at 12 V and 10 A. it has a constant internal resistance of 0.2 Ω.
    a) Find the load resistance and VR at rating.
    b) Find the voltage, current, and VR when the load resistance is 0.8Ω and when it is 1.6 Ω.
12. A dc generator has the external characteristic given in the adjacent table.
    a) Find the voltage, current, and VR when the load resistance is 0.8 Ω and when 'it is 1.6 Ω.
    b) Find the load current, load resistance, and VR when the terminal voltage is 12.5 V.
    c) Find the terminal ve1tage, load resistance, and VR when the load current is 18 A.
13. The battery of problem 11 and the generator of problem 12 are operated in parallel.
    a) Find the voltage, current; and VR when the load resistance is 0.8 Ωand when it is1.6Ω.
    b) Determine who the battery and generator share the load current sin port a.
    c) Find the voltage and currents when there is no external load. What does this case represent?
14. The adjacent table gives the (V-I) characteristics for two dc generators, G1 and G2 and for a nonlinear resistor.
    a) Find the voltage, current, voltage regulation (VR) when G1 is only loaded by (i) 1.5 Ω resistor (ii) R_n resistor (iii) 1.5Ω parallel with R_n.
    b) Repeat part (a) for G2 is only.
    c) Repeat part (a) when G1 and G2 are in parallel.
    d) Determine the current sharing of the two generators for part (c).

	volts		
Amp.	G1	G2	R_n
0	360	360	0
50	351	357	50
100	339	352	114
150	323	347	180
200	302	338	230
250	277	326	265
300	244	310	291
350	202	286	312
400	----	256	328
450	----	221	340
500	----	----	343

ANSWERS

Recall that answers obtained from graphical solutions are approximate and cannot be reproduced exactly. That is, your solution might be correct although your answer appears different from the one given here. An error is indicated only if the difference is rather large.

1. a. 1.61A, 26.6Ω, 10.2 %. b. 163.6 V, 31.4A, 9.8%.

c. 1.0A, 82 , 10.2%. d. 1235 rpm, 10.2%.

2. a. 16.8%, b. 9.9V, 0.24 A.
3. a. 167.5 V, 589 rpm. b. 181.5 V, 485 rpm. c. 130Ω, 90.5Ω, 170 Ω.
   d. 93 Ω, e. 39 V, 0.96 A. f. 968 rpm.
4. a. 8.86 A. b. 34.8 A. c. 81 A.
5. a . 45.7 A, 34%. b. 4.3 A, 9.2 %. c. 10Ω, 22.4%.]
6. a. 8.8%, 28.3Ω. b. 19.4%, 12.4Ω. c. 0.9%, 20.3Ω.
   d. 1.5%, 22.1Ω. e. 50.5%, 2.18Ω.
7. a. 73mΩ, 9.2%. b. 208mΩ.
8. 30 V, 0.42 A.
9. a. 100 A, 248.5 V, 9%. b. 94A, 240 V, 12.8%. c. 94A, 261 V, 3.8%.
10. a. volts: 30.5, 57, 78.5, 95,101.7, 102, 100.4, 98,95. b. 67.4A, 100.8V; 50.8A, 101.7V.
   c. 2.9Ω. d. 40A, 90A. e.102 V. f.67.8V.
11. a. 1.2, 16.7. b. 11.2 V, 14 A, 25%; 12.4V, 7.8A, 12.5%.
12. a. 12.2V, 15.2A, 27.4%; 14.3V, 0.61Ω, 42%. b. 14.3, 0.87Ω, 24%. C. 11V,0.61Ω, 42%.
13. a. 13.2V, 16.5A, 14%; 14.1V, 8.8A, 6.4%. b. 4.1A, 12.4A; - 0.6A, 9.4A. c. 15V, 5.1A.
14. a. 301V, 201A, 19.6%; 270.5V, 261A, 33.1%; 217.5V, 333A, 65.5%. b. 333V, 223A, 8.1%; 300V, 321A, 20%; 253.5V, 402A, 42%. C. 345V, 231A, 4.4%, 325.5V, 392 A, 10.6%; 301V, 522A, 19.6% d. 74A, 160A; 143A, 251A; 202A, 320A.

CHAPTER 9
MOTOR OPERATION

In motor operation, a dc machine is supplied electrically, and drives a mechanical load, fig. 9.1; compare with fig. 8.1 for the dc generator. This chapter explains the operating characteristics of a dc motors and the factors that affect them.

9.1 Governing Equations

In motor operation, we are interested in the output torque and shaft speed, and their influence on the current drawn by the motor. From previous chapters, the emf and torque equations for a dc machine are:

$$E_A = K_e n\Phi \tag{9.1}$$

And $$T_d = K I_A \Phi \tag{9.2}$$

Φ in these equations is the resultant useful flux per pole, i.e. it includes any demagnetization due to armature reaction. E_A is the actual induced emf, and T_d is the developed torque. From KVL, we have:

$$E_A = V - (\sum IR + V_b) \tag{9.3}$$

V is the voltage applied to the motor terminals, and $\sum IR$ is the total series resistive drop (armature wdg, commutating wdg, compensating wdg, series field wdg, and any additional series resistance). To simplify our study, we shall approximate this equation to:

$$E_A = V - I_A R \tag{9.4}$$

I.e. the brash voltage drop is ignored, and R includes all resistances in the path of the armature currant. Dividing eqn 9.4 by $K_e\Phi$, and substituting eqn 9.1, we get:

$$n = \frac{V - I_A R}{K_e \Phi} \tag{9.5}$$

This equation tells us that speed is determined primarily by the applied voltage V and the flux Φ, with some reduction due to the series voltage drop $I_A R$ (which depends on current, and hence on load torque).

The above equations allow us to understand motor operation.

Load :torque and current

T_d in eqn 9.2 is the developed torque; it is slightly greater than the load torque T_L due to rotational losses (see chapter 7)

$$T_d = T_L + T_{rot.\ loss} \tag{9.6}$$

The greater the load on the motor, the greater the current it draws from the supply, eqn 9.2; that is the load torque determines the current of the motor.

Actually, eqn 9.6 holds only under steady state conditions, i.e. when the speed is constant. Under transient (or dynamic) conditions, the two sides of the equation are not equal, and the difference between them produces acceleration:

$$J\frac{dw_r}{dt} = T_d - \left(T_L + T_{rot\_loss}\right) \qquad (9.7)$$

Where J is the moment of inertia of the rotating parts (rotor, shaft, and load); dw_r/dt is the angular acceleration. Eqn 9.7 is a development from Newton's law F =ma (i.e. it relates to the mechanics of the system).

Suppose that the motor is running at some constant speed so that eqn 9.6 holds (i.e. $dw_r/dt = 0$ in eqn 9.7). Now suppose the load on the motor suddenly increases :the motor will slow down according to eqn 9.7. But this causes the induced emf to decrease, eqn 9.1. The resulting increase in the difference between V and E_A must be balanced by an increase in the armature current I_A, eqn 9.4. The increase in current increases the developed torque T_d, eqn 9.2, and the initial increase in load torque T_L is thus met. The motor now operates at steady state again, but at a reduced speed. The process is summarized as follows:

$T_L\uparrow \rightarrow n\downarrow \rightarrow E_A\downarrow \rightarrow I_A\uparrow \rightarrow T_d\uparrow$

9.7 9.1 9.4 9.2

The reverse process is summarized by:

$T_L\downarrow \rightarrow n\uparrow \rightarrow E_A\uparrow \rightarrow I_A\downarrow \rightarrow T_d\downarrow$

Note that if the series resistance in eqn 9.4 is small, then only a slight change in speed is sufficient to cause large changes in armature current (and hence in developed torque).

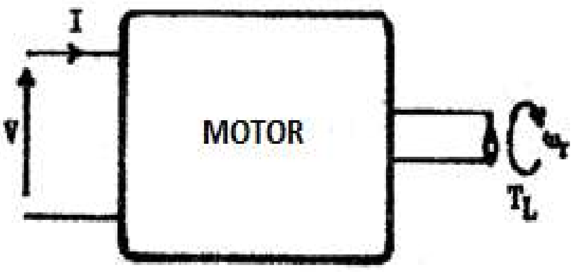


Fig. (9.1) Motor operation

After a disturbance (sudden change in load), the time it takes the motor to settle at a new speed is called the response time. It is determined by the electrical time constant of the motor and the mechanical time constants of the motor and connected load. In certain applications, particularly automatic control system, the response must be quick, and the motor is designed to have low inductance and low inertia.

$$n = \frac{V}{K_e\Phi} - \frac{R}{K_e K\Phi^2}T_d$$

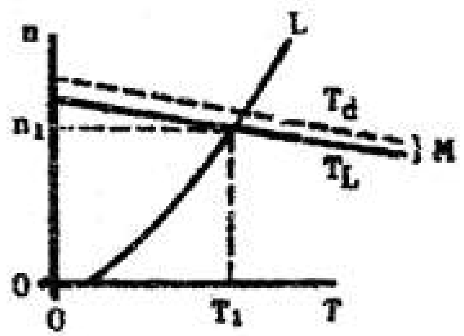
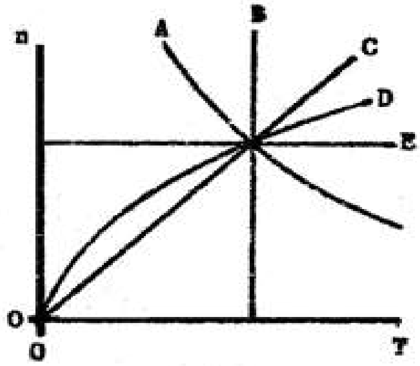


Fig. (9.2) intersection between mechanical characteristic and load



$A: T \propto \frac{1}{n}$ (constant power; traction)

$B: T = constant$ (traction; lifts; cranes)

$C: T = n$ (machine tools)

$D: T \propto n^2$ (fans; centerfugal pumps)

$E: n = constant$ (ac alternaters)

Fig. (9.3) Ideal load -speed / torque characteristic for typical mecancal load

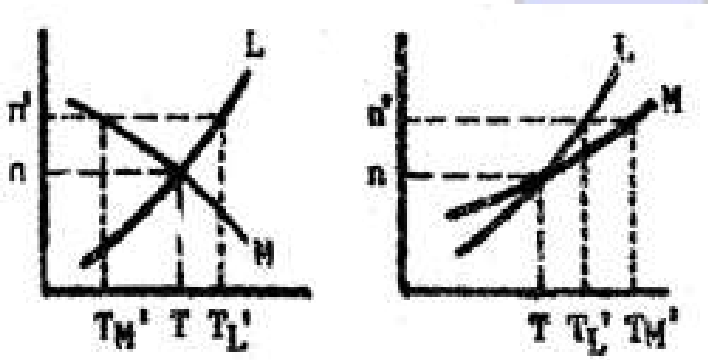


Fig. (9.4) Stability of operation point

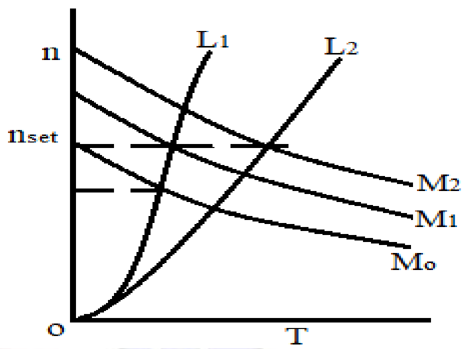


Fig. (9.5) Speed control of DC motor

9.2 Applied Voltage

The series resistance in the armature circuit is small, so that the induced emf E_A is approximately equal to the applied voltage V, eqn 9.4. Then, from eqn 9.1 we see that, for a given value of flux Φ, the speed is determined primarily by the applied voltage V.

9.3 Field Excitation

The flux Φ is determined primarily by the main field mmf (i.e. by field current), and may be controlled by field resistors as in fig. 8.2. The torque is directly proportional to flux, eqn 9.2, but the speed is inversely proportional to it, eqn 9.5. Thus, an increase in flux tends to decrease speed, and a decrease in flux tends to increase speed. Clearly, then, armature reaction tends to increase speed, while the series field in cumulative-compound motors tends to decrease speed.

9.4 Definitions

We shall need the terms and concepts defined below in our description of motor operation and the factors that affect it.

9.4.1 Mechanical characteristic

The mechanical characteristic of a dc motor is the curve relating the motor's two output variables, torque and speed; the curve shows how speed changes with load. Substituting for I_A from eqn 9.2 into eqn 9.5, we get:

$$n = \frac{V}{K_e \Phi} - \frac{R}{K_e K \Phi^2} T_d \qquad (9.8)$$

For constant V and Φ, eqn 9.8 represents a straight line with negative slope, fig. 9.2. The first term on the RHS gives the vertical intercept (no-load speed), and the coefficient of T_d in the second term gives the slope. The load torque T_L is a little less than the developed torque T_d, eqn 9.6, so that the relationship curves below the straight line. The shape of the curve may be further modified due to changes in the flux Φ (which affects both slope and intercept) as the motor load changes; the flux changes with load when there is a series field, when the demagnetizing effect of armature reaction is not negligible.

Note that eqn 9.5, and hence 9.8, are derived from KVL:

$$E_A = V - I_A R \qquad (9.9)$$

So that the straight line of fig. 9.2 is just a scaled representation of KVL.

When the motor is driving a mechanical load, the torque and speed are found from the motor mechanical characteristic and the load characteristic, fig. 9.2; that is, the operating point (T_1, n_1) is found graphically. Fig. 9.3 shows some typical characteristics of mechanical loads.

9.4.2 Stability

The operating point may or may not be stable. In fig. 9.4a it is stable :if the speed suddenly increases from n to n', the load torque T_L' will be greater than the motor torque T_M ', causing deceleration back to the operating point (n, T); the operating point will also be restored to (n, T) for a sudden decrease in speed (try it). In fig. 9.4b, the operating point is unstable :if the speed suddenly increases from n to n', the motor torque T_M' will be greater than the load torque T_L', causing acceleration and further increase in speed away from the operating point (n, T); a sudden decrease in speed may result in stall (zero speed). Clearly, then, the stability of the operating point depends on the relative shapes of the motor and load torque-speed characteristics.

9.4.3 Speed control

Eqn 9.5 indicates that the speed may be controlled by means of the applied voltage, main flux, and the series resistance; these parameters may be adjusted manually or automatically. Although the armature current I_A appears in the equation, and hence affects speed, it is not a proper controlling parameter because it cannot be adjusted as desired, but is determined by the mechanical load, eqn 9.2.

New a given mechanical characteristic corresponds to a particular setting of the applied voltage, field control resistor, and series resistance. If any of the settings is changed, operation shifts to another curve. Therefore, the operating point may be moved from one curve to another by changing the setting of one or more of the control parameters, fig. 9.5. The speed may be kept approximately constant by automatic regulators that sense the shaft speed and adjust one of the control parameters to keep it at the set value.

9.4.4 Speed regulation

The speed regulation of a motor at a given load is defined by:

$$SR = \frac{n_0 - n_L}{n_L} \qquad (9.10)$$

It is a figure of merit that indicates how constant the shaft speed is with load. For many applications, a good drive motor is one which maintains its speed constant over a wide range of loads. The speed regulation of motors equipped with automatic speed control is almost zero. Note the analogy between speed regulations in motor operation with voltage regulation in generator operation, section 8.2.

A low value of speed regulation is not always desirable. There are applications that require the motor to change its speed with load, for example to keep the torque or output power constant. A main feature of the dc motor is that its operation can be tailored to suit any type of load requirements.

9.5 Constant-Flux Motors (Permanent-Magnet; Separately-Excited; Shunt)

The difference between shunt and separately excited motors is that the field of a shunt motor is fed from the same source as the armature, while the field of a separately excited motor is fed from a different source, possibly at a different voltage. In both cases, constant field voltage and resistance result in constant field current (If does not change with load), and hence constant main field flux. Permanent magnet motors also operate with a constant main field.

If the demagnetizing effect of armature reaction is neglected, the developed torque Td will be directly proportional to the armature current IA, eqn 9.2, so that the two variables are related by the straight line shown dotted in figure 9.6. Armature reaction may reduce the flux Φ, and hence reduce Td, so that the actual relationship between Td and TL is curved slightly below the straight line. The load torque TL is less than the developed torque Td due to rotational losses, eqn 9.6 so that the TL curve is slightly below the Td curve, fig. 9.6. The relationship between torque and current is sometimes called the torque characteristic of the motor.

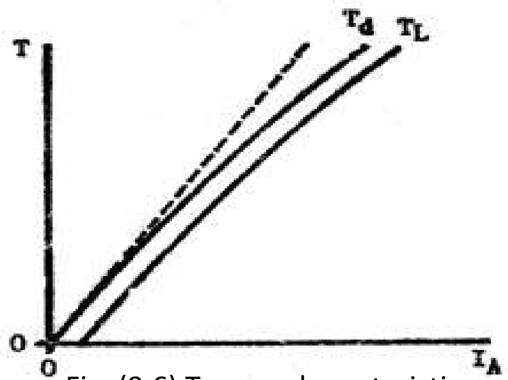


Fig. (9.6) Torque characteristic

For constant-flux motors, the mechanical characteristic is a straight line with a slight negative slope, eqn 9.3 and fig.9.7. Armature reaction may reduce useful flux and hence increase the speed, so that the mechanical characteristic curves slightly above the straight line. This upward curvature may lead to instability, section 9.2; it is avoided by designing the motor to have no demagnetizing armature reaction (by the use of interpoles), and by adding a weak series field to compensate for the reduction in flux (stabilized shunt motor).

The reduction in speed with load is very small for constant-flux motors. The mechanical characteristic is said to be hard, and the motors operate in an essentially constant speed mode.

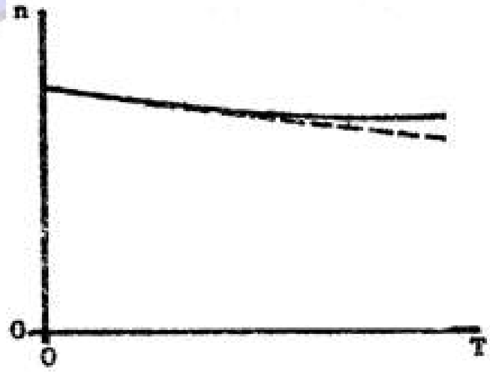


Fig. (9.7) mechanical characteristic

9.6 Series Motor

The main field flux of the series motor changes with load current according to the OCC; therefore, the series motor is characterized-by variable flux, as opposed to the constant flux motors of section 9.5.

At light loads, operation is on the linear part of the OCC, so that:

$\Phi \propto I_A$

And eqn 9.2 yields,

$T_d \propto I_A^2$

Thus, the torque characteristic follows a parabola at light loads, fig. 9.8. At heavy loads, the machine will be saturated so that the flux is almost constant, and operation approaches that of constant-flux motors

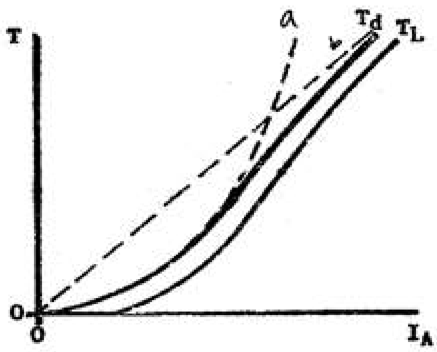


Fig. (9.8) Torque characteristic

Φ =constant → $T_d \propto I_A$

The torque characteristic approaches a straight line at heavy loads, fig.9.8.

Applying the same reasoning to the mechanical characteristic, we see that

At light loads: $n \approx \frac{k_1}{\sqrt{T_d}} - k_2$

And

At heavy Loads :$n \approx K_3 - K_4 T_d$ (similar to shunt)

The mechanical characteristic will then have the general shape shown in fig. 9.9. The change of speed with load is quite large; the mechanical characteristic is said to be soft, and the series motor operates in a variable speed mode. The motor has a high starting torque, but the torque quickly decreases as speed goes up. At no-load the speed becomes so high it can damage the motor; therefore, series motors are never run unloaded, and are always rigidly coupled to their loads (i.e. belts are never used).

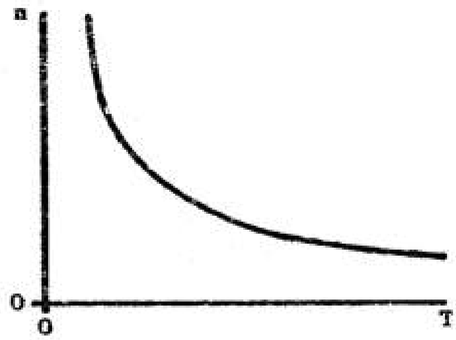


Fig. (9.9) Mechanical

9.7 Compound Motors

A compound motor has both shunt and series fields. For cumulative compounding, the motor characteristics will move from shunt characteristics in the direction of series characteristics as load increases (i.e. as the series field becomes stronger); see

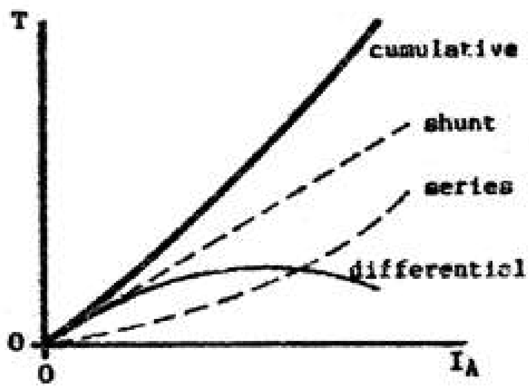


Fig. (9.10) Torque characteristic

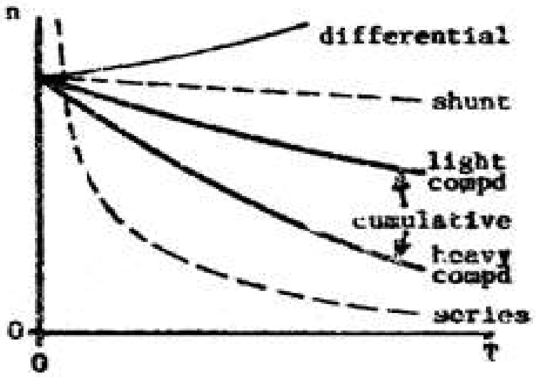


Fig. (9.11) Mechanical characteristic

figures 9.10 and 9.11. The actual shape of the mechanical characteristic is determined by the degree of compounding, i.e. by the ratio Ns/Nf, fig. 9.11. Differential compound motors have rising mechanical characteristics because of the reduction in main field flux with load, fig. 9.11; therefore, they are unstable (section 9.4), and are not used in practice.

9.8 Speed Control

According to eqn 9.5, the parameters that can be adjusted to control speed are :the voltage applied to the armature V, the series resistance R, and the field excitation I_f (which determines main field flux).

9.8.1 Armature Voltage Control

For constant flux motors, section 9.5, the torque-speed equation 9.8 describes a straight line, fig. 9.7. The first term on the RHS gives the intercept, and the coefficient of Td in the second term gives the slope, which is negative. In permanent-magnet and separately-excited motors, the voltage applied to the motor can be varied with the field remaining constant. Different voltages then give different intercepts, and we get a family of parallel (i.e. same slope) mechanical characteristics as in fig. 9.12. In the figure, Vr denotes rated voltage; as the applied voltage is decreased, the characteristic is shifted down. Similar downward shifts occur for the series motor, fig. 9.13; at heavy loads, the series motor approaches constant-flux operation, section 9.4, and the curves approach straight lines that are parallel as in fig. 9.12.

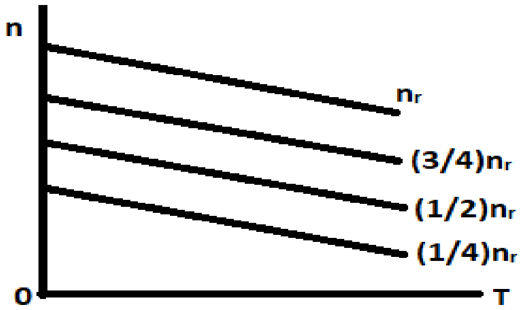


Fig. (9.12) voltage control for constant flux DC machine

$$n = \frac{\boldsymbol{V}}{K_e\Phi} - \frac{R}{K_eK\Phi^2}T_d$$

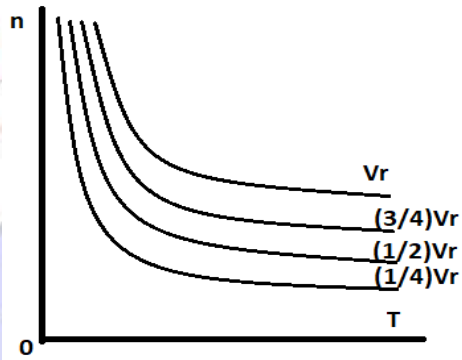


Fig. (9.13) voltage control for series DC machine

The simplest method of obtaining variable dc voltage is to use a voltage divider, out this method is impractical and uneconomical; it is used only for testing.

In modern applications, variable dc voltage for the armature is often obtained from a solid-state controlled rectifier, with the field fed from an uncontrolled rectifier, fig. 9.14. The firing angle of the controlled rectifier

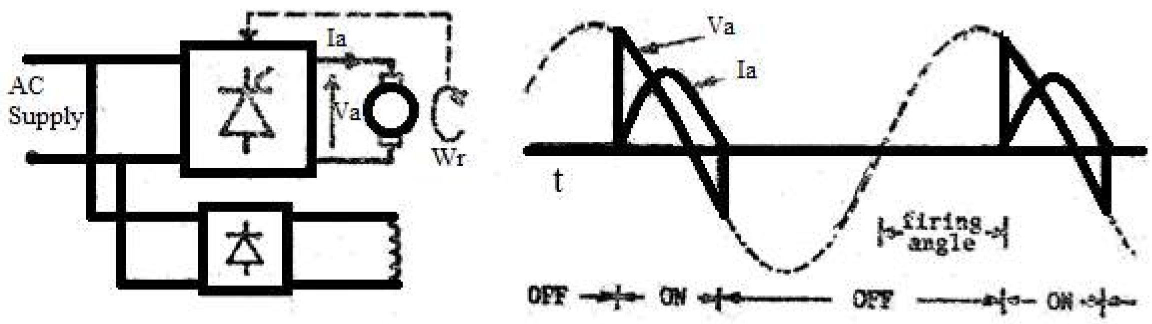
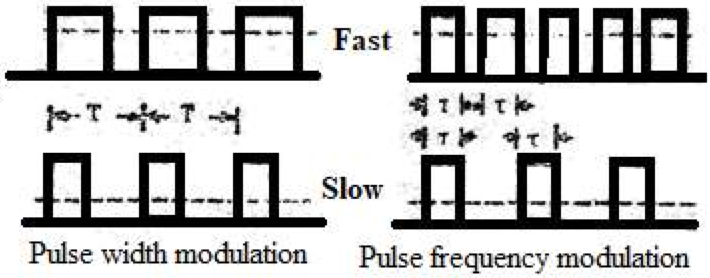


Fig. (9.14) Armature voltage control using solid- state controlled rectifier.

may be changed manually, but in practice it is adjusted automatically using a speed signal or armature current signal (i.e. load), or both for optimum control.

Fig. (9.15) Armature voltage control for series motor using chopper DC supply.



In road vehicles, the supply is itself dc, and hence needs no rectification. Voltage control is often obtained by an electronic chopper circuit, fig. 9.15. Choppers may use pulse-width modulation PWM at constant frequency, or pulse-frequency modulation PFM with constant pulse width.

Another effective method for obtaining smooth voltage control is the ward-Leonard system, fig. 9.16. The dc motor is fed from a dc generator driven by some prime-mover (eg ac motor or Diesel engine). By varying the field excitation of the generator, the armature voltage of the motor is varied (and can be even reversed). The motor field is fed from an exciter (smell dc generator) or rectifier at constant voltage. The Ward-Leonard system is generally more expensive then a solid-state drive, but has compensating advantages for certain applications.

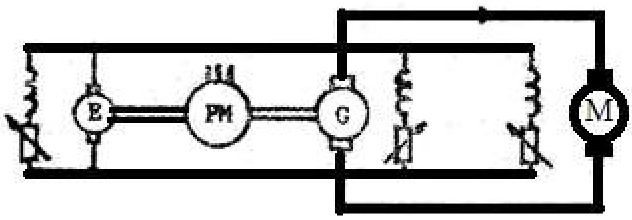
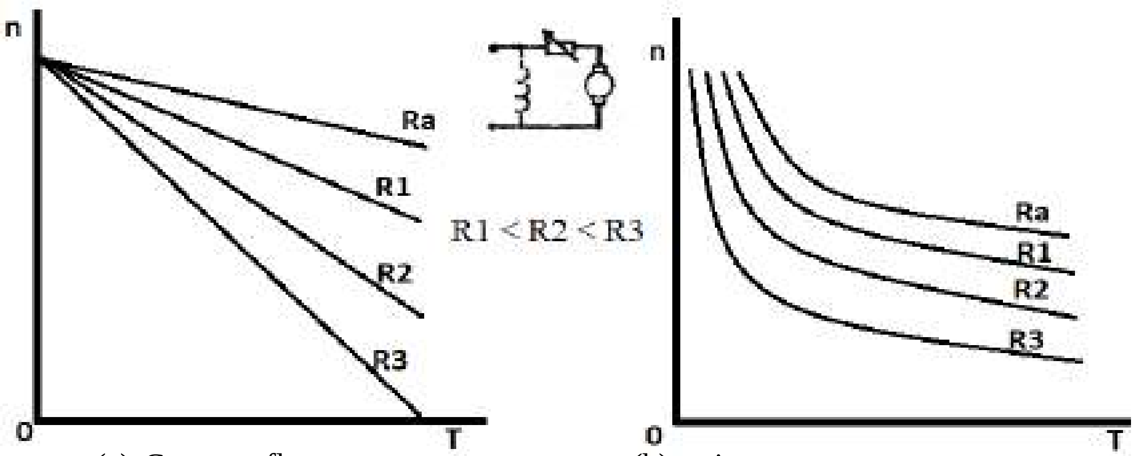


Fig. (9.16) Ward-Leonard system for armature voltage control

9.8.2 Armature Resistance Control

For a given load torque, and hence given current, placing an external resistance in series with the armature, fig. 9.17, reduces the emf and hence speed. The increasing value of resistance increases the slope of the mechanical



(a) Constant flux motor (b) series motor

Fig. (9.17) Armature resistance control

characteristic, eqn 9.8. Thus, the hard characteristic of constant-flux motor is seen to become softer with increasing resistance in fig. 9.17a, but the intercept remains unchanged. Armature resistance control may also be used with series motors giving the family of curves shown in fig. 9.17b; at heavy loads the machine is saturated (why?) and operation approaches that of constant-flux motors with slope increasing as resistance is increased.

$$n = \frac{V}{K_e \Phi} - \frac{R}{K_e K \Phi^2} T_d$$

Armature resistance control is inexpensive and simple to use with small motors, but it is impractical and wastes energy with large motors.

9.8.3 Field Control

This method of speed control may be used with shunt and separately excited motors. With no external resistance in the field circuit, and with rated voltage applied to the armature, the motor will operate at constant flux and follow a certain mechanical-characteristic; this is called the base case, fig. 9.18. If now the field circuit resistance is increased, the field current, and hence the main field, will be reduced, and the speed will increase according to eqn 9.5 or 9.8. We thus obtain a family of curves above the base case, fig. 9.18. The higher the field resistance, the higher the intercept and the greater the slope (i.e. the characteristic becomes softer).

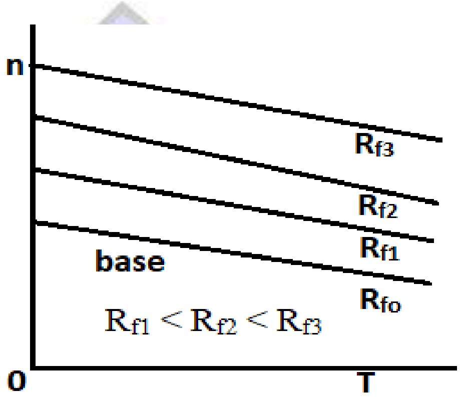


Fig. (9.18) Field control

The flux cannot be reduced indefinitely because the speed becomes too high and may damage the motor. Moreover, if the main field becomes too weak, the demagnetising effect of armature reaction becomes prominent (relatively large) which may lead to instability, section 9.4.

9.8.4 Comparison

Armature voltage control provides the best method of speed control of dc motors; indeed, it is the main reason for the continuing existence of large dc machines. The speed may be controlled from zero to maximum safe speed in either forward or reverse direction. The control can be manual, or automatic with speed or current sensors. The constant-speed feature of constant-flux motors is retained, giving adjustable speed drives. $P_{out} = w_r * T$, $T = P_{out}/w_r$

Armature resistance control is cheaper than armature voltage control, but it is wasteful and less versatile. As the armature resistance increases, the mechanical characteristic becomes softer, and the motor operates at variable-speed. This type of control is sometimes used with relatively small series or permanent magnet motors.

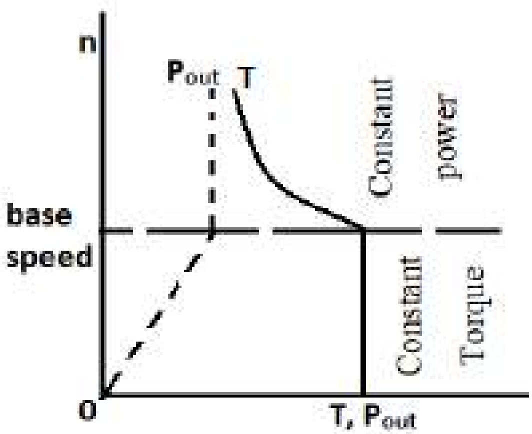


Fig. (9.19) Armature voltage control up to base speed, with field weakeaing to exteed speed

Shunt field control is also relatively cheap (and wasteful); but it does not compete with armature voltage control :it increases speed from base value, and cannot reduce speed to zero. The mechanical characteristic becomes softer as the field resistance is increased, and there is an upper limit on the speed that can be reached, usually around 2:1 (i.e. twice base speed).

In some applications, particularly traction, both armature voltage and field control are used, fig. 9.19 :armature voltage control is used up to base speed to maintain the torque constant; above base speed, field control is used to maintain output power constant (2πnT).

Small dc motors with fixed excitation are often used in automatic control systems. They are designed to have linear mechanical characteristics with accurate voltage control down to stall (standstill)as shown in fig. 9.20.

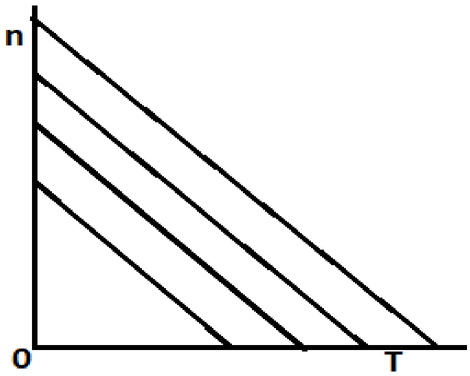


Fig. (9.20) Control motor characteristic.

9.9 Starting

At the moment the motor is switched on, it is at standstill, so that there is no induced emf. The entire line voltage is applied across the armature resistance since eqn 9.4 reduces to Ohm's law:

$$I_{start} = \frac{V}{R_A}$$

The starting current is therefore very high, especially for large motors which have very small armature circuit resistance. The starting current may be more than 20 times rated value, and would damage the motor unless some means is found to limit it.

Note that the problem exists only at starting because as soon as the motor begins to rotate, the emf begins to build up and thus reduce the current in accordance with eqn 9.4. Note also that the rate at which the emf builds up depends on the rate at which the motor accelerates from standstill which, in turn, depends on the starting torque; that is, a high starting torque is desirable for rapid initial acceleration (and hence rapid build-up of emf and hence rapid reduction of the high starting current).

9.9.1 Direct On-Line Starting (DOL)

DOL starting means simply connecting the motor to the supply through a switch. This method can be used only with small motors where (a)the armature resistance is high enough to limit the starting current, and (b)the rotor inertia is small enough to allow rapid acceleration (and hence rapid buildup of mmf leading to rapid reduction of current).

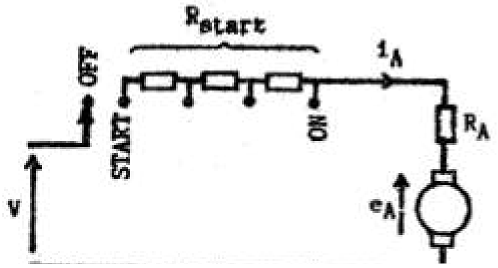
9.9.2 Variable Voltage Starting

Motors supplied from Ward-Leonard sets or controlled rectifiers can be started by raising the supply gradually from zero. The low initial voltage results in a reduced starting current.

9.9.3 Resistance Starting

This is the most common method of starting do motors. A specially designed variable resistor is connected in series with the armature, fig. 9.21. When the moving contact is moved from the OFF position to the START position, all sections of the starting resistor or in the circuit so that the starting current is limited to:

$$I_{start} = \frac{V}{R_A + R_{start}}$$



The value of Rstart is chosen to limit the starting current to a safe value, usually 1.5-2.5 times rated current. Although the starting current is still greater than the rated current, it is considered safe because it flows only for a short time. Moreover, a relatively high current is needed to obtain a high torque for rapid acceleration.

As the motor builds up speed (and hence emf), the starting resistance is cut out section by section until it is totally out of the circuit. During this process, the starting current i and the induced emf eA follow stepped curves of the forms shown in fig. 9.22. In principle, a given section is cut out when the current has fallen to some minimum value, say rated current; upon cutting out the section, the current will jump up again to a value limited by the sections remaining in the circuit (maximum safe value).

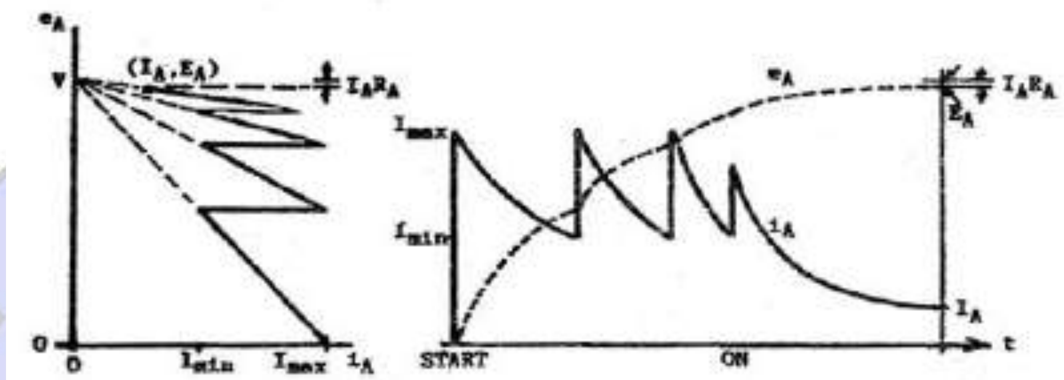


Fig. (9.22) Variation of armature current and induced emf during starting wit 3 switch starters.

During starting, full voltage must be applied to the shunt field winding to make the flux maximum; this maximizes starting torque, eqn 9.2, maximizes emf build-up, eqn 9.1, and prevents over speed (the high starting current may cause severe armature reaction, i.e. reduce the flux). For this reason, the starting resistor is connected in the armature circuit and not in the line, and the field control resistor is shorted out during starting.

9.10 Starters

A starter is a box that contains the starting resistor and other necessaries. Manual starters have a handle which the operator uses to cut out the starting resistor in steps. Fig. 9.23 shows a typical four-point starter. The studs (contacts)of the starting resistor are arranged in such a way as to ensure that the moving handle breaks contact with a stud only after it has contacted the next stud (why?). The starter of fig.9.23 has a number of protective functions that will be discussed later.

Automatic starters use relays instead of a handle; the contacts of the relays short out the sections of the starting resistor in sequence. The relays may be simple time delay relays, or they may be current sensitive relays (sections are cut out when the armature current falls below a preset value-Imin in fig. 9.22), or they may be voltage sensitive relays (sections are cut out when armature voltage rises above preset values).

The starter usually has additional relays, coils, and contactors to perform a number of functions. These include:

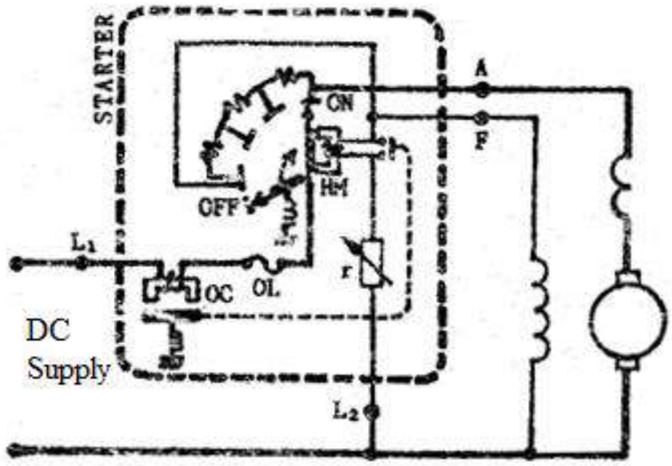


Fig. (9.23) Typical 4-point manual starter

Thermal overload protection (OL) :A temperature sensitive element set to disconnect the motor if the current exceeds rating for more than the permissible time; it works on an 'inverse-time' characteristic (i.e. it disconnects a slight overload after a long time, and a heavy overload after a short time).

Over-current protection (OC) :electromagnetic relay set to disconnect the motor instantaneously if the current begins to rise several times rated value (indicating short-circuits rather than mere overload).

No-volt trip : An electromagnet that returns the starting resistor to the OFF position when the supply voltage falls below a set value; this protects the motor from high starting current when the supply voltage is restored. In the starter of fig. 9.23, no-volt tripping is achieved by means of the holding magnet HM.

Loss-of field trip :electromagnetic relay set to disconnect the motor when the field current falls below a set value to prevent over speed.

Field discharge resistor :A resistor that is connected across the field winding at the instant the motor is switched off; it provides a path for the field current to decay safely (RL transient). Attempting to switch off the field current without a discharge resistor can produce a high voltage, Lfdif/dt, that may cause heavy sparking in the starter and electric shook to the operator.

9.11 Braking

When the electric supply to the motor is switched off, the rotation does not stop immediately, but continues until the kinetic energy of the rotating parts (rotor and load) is dissipated. But in many applications, such as electric trains, cranes, and lifts, the motor must be stopped quickly, so that some form of braking must be applied at switch-off.

9.11.1 External braking

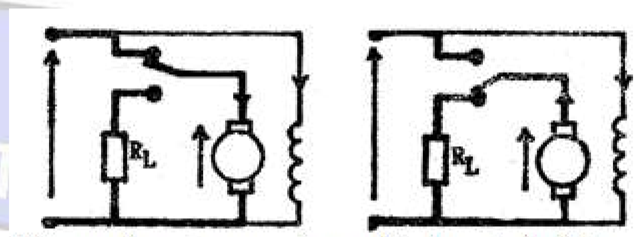
Quick braking can be achieved by an external friction brake mounted on the shaft and operated by a solenoid (electromagnet). At the instant the supply is switched off, the brake is applied to stop rotation. In effect, the kinetic energy of the rotating parts is dissipated quickly as heat in the brake pads.

The eddy-current brake is another type of external brake. It is made up of a conducting disc mounted on the shaft, and a set of stationary coils adjacent to the disc. At the instant the supply to the motor is switched off, the brake coils are energized to induce eddy currents in the rotating disc. The field of the coils and the currents of the disc produce a torque that opposes rotation (generator action) and hence slows the shaft rapidly. In effect, the kinetic energy of the rotating parts is dissipated as heat in the disc of the brake.

9.11.2 Electric Braking

Instead of using an external brake, it is sometimes possible to use the properties of the dc machine itself to achieve quick braking, or to assist in braking.

In **dynamic braking** (or **rheostat braking**), a resistor (possibly the starting resistor itself) is connected across the armature terminals at the instant it is disconnected from the supply, fig. 9.24. with the shunt field still excited, the machine acts as a generator loaded by the resistor; the armature current reverses, and the developed torque now opposes rotation. In effect, the kinetic energy is dissipated as heat (I^2R losses) mainly in the resistor, but also in the armature wdg. During braking, it is preferable to energies the field from the line and not from the armature; otherwise, braking action stops when the speed falls below; the critical value (see section 8.5).

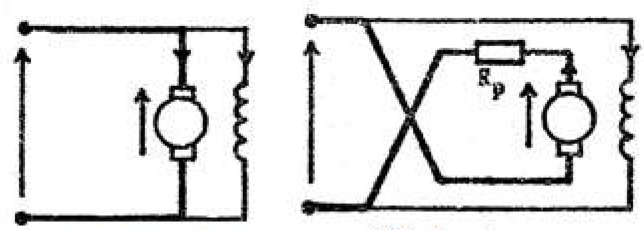


(a) normal motor operation (b) dynamic braking

Fig. (9.24) principle of dynamic braking

A dc motor is said to be **regenerating** when its **emf exceeds the applied voltage** so that the **armature current reverses** and the machine **becomes a generator** that returns electrical power to the supply; the source of the power is the kinetic energy of the rotating parts, and hence regeneration **slows the motor down**. **Regenerative braking** uses this principle to aid in stopping the motor or in slowing it down; regeneration is achieved by strengthening the field or by reducing the applied voltage. The main advantage of regenerative braking is the saving of energy, which is returned to the supply and not dumped as heat as in the other methods of braking. It is often used in **electric trains** to exploit downhill runs, and in cranes to exploit the descending part of the duty cycle. Regenerative braking can be used only if the electric supply is capable of accepting electrical energy from the motor (eg chargeable batteries or dc mains); standard controlled rectifiers cannot accept electrical power from the motor unless they ore modified for the purpose (eg by the inclusion of inverters). In regenerative braking, the braking action stops when the speed become low enough to reduce the emf below the terminal voltage.

A strong braking effect down to zero speed is obtained by **plugging (or counter-current braking)**. The supply connections to the armature are reversed, fig. 9.25, so that the supply end armature emf act as series sources aiding each other to circulate a heavy counter-current: $\boldsymbol{I_A = \frac{E_A+V}{R}}$



(a) normal operation (b) plugging

Fig. (9.25) principle of plugging

The machine operates **in the generating mode with a heavy current**, and hence with a strong braking torque. A series **limiting resistor, Rp,** is inserted in the circuit **to avoid damaging currents**; if the starting resistor is used as Rp, the plugging current will **be twice starting current**, (i.e. up to five times rated current). During plugging the kinetic energy of the rotating parts plus heavy power from the supply (VI_A)are dissipated in the armature winding and the limiting resistor Rp. **The supply must be disconnected from the motor at the instant the speed reaches zero**; otherwise the motor will run in the **reverse direction**. Plugging involves such **heavy currents and high mechanical stresses that it is used only with small motors.**

9.12 Modes of operation

The four-quadrant diagram of fig. 9.26 helps clarify the various modes of operation of a dc motor. The first quadrant corresponds to standard motor operation in one direction, while the third quadrant corresponds to motor operation in the reverse direction. The second and fourth quadrants correspond to generator operation.

Taking the first quadrant as reference, it is seen that motor operation in the reverse direction, third quadrant, requires reversal of either the applied voltage or the field current. If both are reversed at the same time, motor operation will continue in the same direction, first quadrant.

If initial operation is in the first quadrant, then the second quadrant corresponds to dynamic or regenerative braking. Plugging also shifts operation to the second quadrant, but attempts to continue to the third; the supply is disconnected when the operation point passes through zero speed. If initial operation is in the third quadrant, i.e. in the reverse direction, then dynamic braking, regenerative braking, and plugging occur in the fourth quadrant.

For a lift or crane, we have:

Quadrant I :motor raises load;

Quadrant III **:**motor lowers load;

Quadrant II :motor brakes upward (inertia)motion of load;

Quadrant IV :load moves down by its own weight while motor applies a braking torque to keep speed constant.

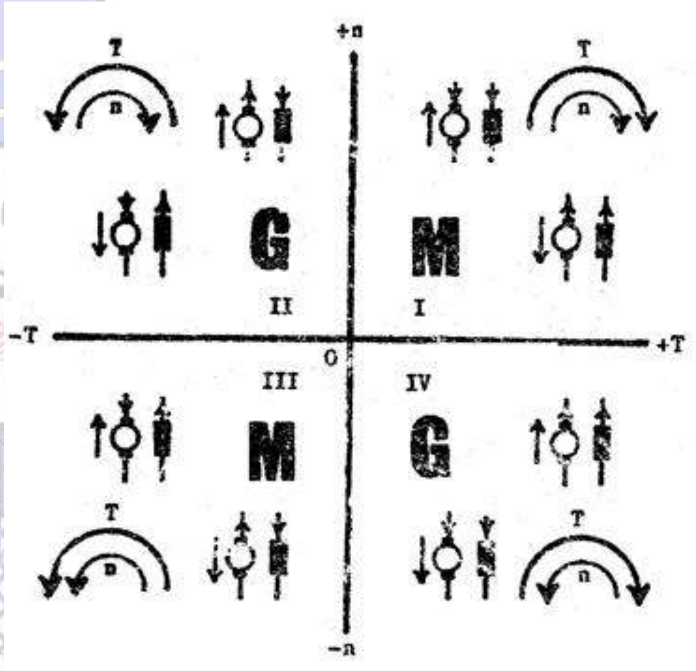


Fig. (9.26) Four-quadrant operation of DC motor.

9.13 Applications

DC motors are lass common than ac motors because ac motors are cheaper, more robust, and require less maintenance, and because standard mains are ac. However, there are two main types of application for which dc motors are more suitable than ac motors :battery-operated equipment, and applications requiring accurate or flexible control of speed or torque.

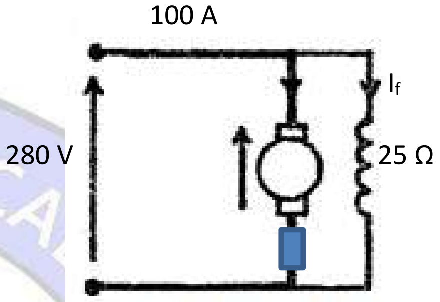
Battery-operated equipment includes small portable apparatus (cassette recorders, etc.), cordless tools, and toys, as well as electric drives in road vehicles. These are usually permanent-magnet or shunt (fixed-excitation) motors, but occasionally series motors. The high starting torque of the series motor makes it suitable for self-starter duty in cars. Electric vehicles employ permanent-magnet or series motors, with speed control by means of choppers (armature voltage control) or armature series resistance.

The accurate control of dc motors makes them suitable for servomotor duty in automatic control systems. The motors in such applications generally have small power rating (less than 1 KW), and are required to drive a load in accordance with a control signal applied to the armature (armature voltage control). They are usually constant field motors (PM or separately-excited) designed to have a low moment of inertia for quick response, and linear mechanical characteristics for accurate control, see fig. 9.20.

The flexibility of control of dc motors makes them suitable for certain heavy power applications such as lifts, cranes, and electric traction (electric trains), as well as certain drives in heavy industry. These app1icaticns can invo1ve frequent changes in speed, stops and starts, and possibly reversals. The hard characteristics of shunt motors with armature voltage control are ideal for adjustable1 speed drives, while the softer characteristics of compound and series motors are sometimes exploited in traction (locomotives)to do without different gear ratios.

Example 1: A shunt motor runs on 280 V. the armature circuit resistance is 0.07 ohms, and the field circuit resistance is 25 ohms. For a certain load, the motor rotates at 1500 rpm and draws 100 A, brush voltage is 2 V. (a) Find the developed torque when the motor current is 100 A. (b) find the motor current and shaft speed when the motor develops 300 Nm. (c)Estimate the no-load speed and the rate of change of speed with torque.

Solution:



(a) normal operation

a) $T_d = kI_a\varphi$, n=1500/60 = 25 rps
Ia = I_L-I_f , $I_f = \frac{V_t}{R_f} = \frac{280}{25} = 11.2\ A$

I_a=100 – 11.2 = 88.8 A, $E_a = k_e n\varphi$, $E_a = V_t - I_a R_a - V_b$

E_a = 280 – 88.8*0.07 – 2 = 271.8 V, $K_e\varphi = \frac{E_a}{n} = \frac{271.8}{25} = 10.87\frac{vs}{r}$

T_d =10.87*88.8*/2π =153.7 N

b) Ia = T_d *2*π / k_eφ =300*2*3.14 / 10.87 = 173.3A
I_L = Ia +I_f =173.3+11.2=184.5 A

E_a = 280 – 173.3*0.07 – 2 = 265.87 V

$n = \frac{E_a}{k_e\varphi}$ = 265.87/10.87=24.46 rps, n = 1467 rpm

c) At no load T_L = 0 , Ia = 0
Ea = 280-Vb =280-2 = 278 V, $n = \frac{E_a}{k_e\varphi} = \frac{278}{10.87}60 = 1534\ rpm$

$$\frac{\Delta n}{\Delta T_d} = \left|\frac{n_1 - n_2}{T_{d1} - T_{d2}}\right| = \left|\frac{1500 - 1534}{153.7 - 300}\right| = \frac{34}{146.3} = 0.225\ rpm/N$$

Example 2: Machine M1 in question sheet chapter 9, is first connected in shunt, and then in cumulative long shunt. In both cases, operation is at rated voltage and full field. (a) plot on one sheet the torque characteristics for the two cases (up to 75 A). (b) plot on one sheet mechanical characteristics for the two cases; plot on the same sheet the torque-speed curves of the machine tool load L1 and the fan load L2. (c)use your curves to obtain the no-load speed and no-load current for each motor connection. (d) use your curves to determine the torque, speed, and current for each of the three motor connections when loaded by each of the two loads (i.e. six operating points) also find the corresponding output horsepower, efficiency, and speed regulation.

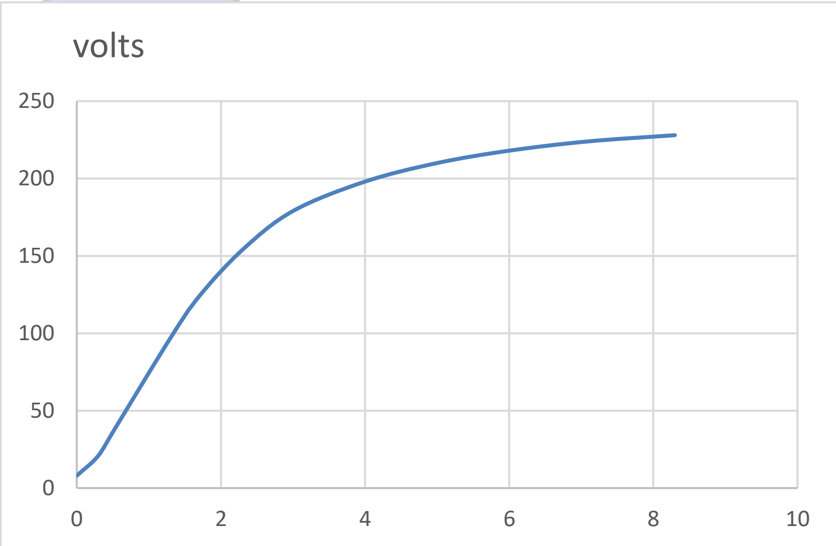
M1: v_t = 220 V, Ra =0.357Ω, Rsh = 55Ω, Ns =0.06Nsh, Rs = 0.233Ω, Pout = 13.5hp, n = 1600 rpm, OCC.

Solution: plot the OCC of the machine from table.

a) for shunt connection:

I_f =Vt / R_f = 220/ 55 = 4A, from OCC, we get Ea = 198 V,

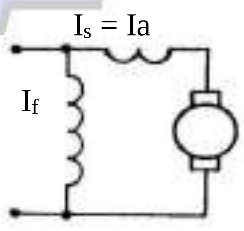
$$K_e\varphi = \frac{E_a}{n} = \frac{198}{1600} * 60 = 7.425\frac{vs}{r}$$



The torque characteristic is:

$T_d = kI_a\varphi$= 7.425*Ia /2π =1.182 Ia

To plot the torque characteristic, we used as in table,



(d) compound - long shunt

For long shunt cumulative compound: we must take care the flux will change with armature current Ia.

When Ia = 0,

$I_{eq} = I_f + \frac{N_s}{N_{sh}} I_s$= 4+0= 4 A, from OCC, we get Ea = 198 V,

$K_e\varphi = \frac{E_a}{n} = \frac{198}{1600} * 60 = 7.425\frac{vs}{r}$, $T_d = k\varphi\, I_a$, T_d = 0

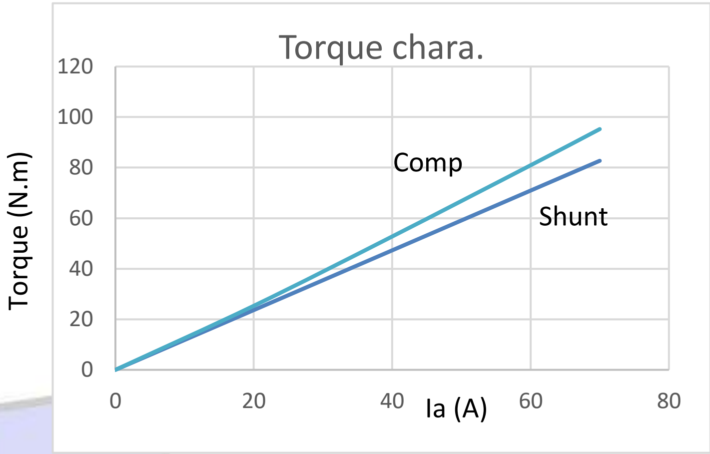
Ia = 20, $I_{eq} = 4 + 0.06 * 20 = 5.2\ A$, from OCC, we get Ea = 212 V, $K_e\varphi = \frac{E_a}{n} = \frac{212}{1600} * 60 = 7.95\frac{vs}{r}$,

$T_d = k\varphi\, I_a$, $T_d = \frac{7.95}{2\pi} * 20 = 25.3\ N.m$, by the same procedure we get the following table 1

Ia (A)	0	20	40	60	70
T_d (shunt) (N.m)	0	23.64	47.28	70.92	82.72
T_d (comp.) (N.m)	0	25.33	52.76	80.93	95.25

We plot this table in the same sheet to get the torque characteristic as in figure

b) to plot the mechanical characteristic, we must get table between the torque and speed using:



$$n = \frac{\boldsymbol{V}}{K_e\Phi} - \frac{R}{K_e K\Phi^2}T_d$$

for shunt: $K_e\Phi$ is constant = 7.425,

$$n = \frac{\mathbf{220-2}}{7.425} - \frac{0.357 * 2\pi}{7.425^2}T_d$$

$\underline{n = 29.36 - 0.0407\, T_d}$

When Td = 0 , then n = 29.36 *60=1761.6 rpm , Td = 70 ,

$n = (29.36 - 0.0407 * 25.33) * 60 = 1699.7$ rpm, as in table

For compounding, $K_e\Phi$ is changed when load current change so must calcolate $K_e\Phi$ evry time,

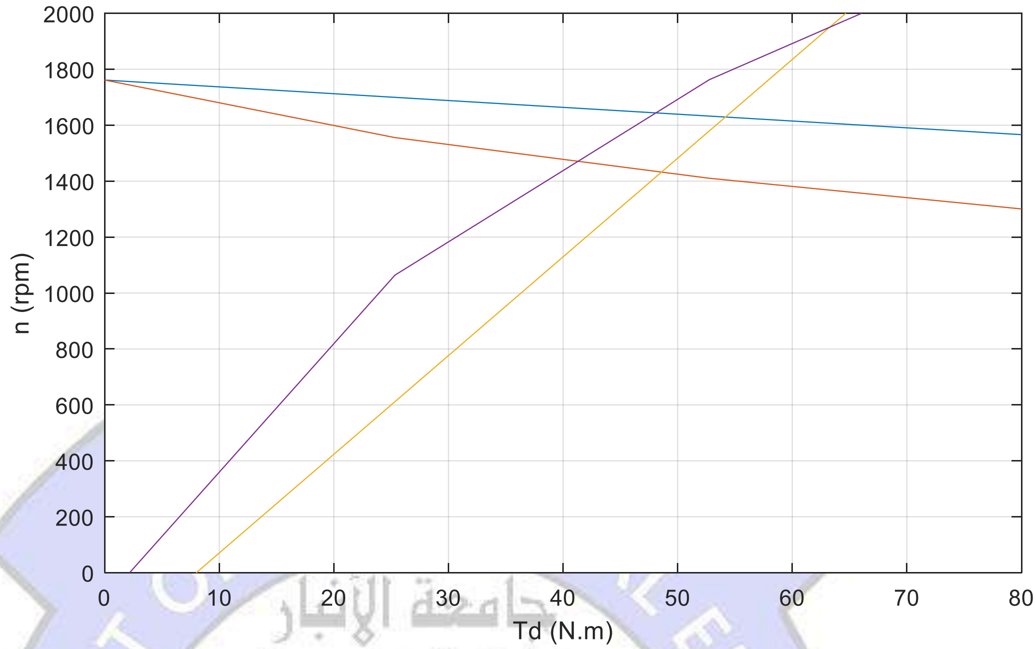
When Ia = 0 then $I_{eq} = I_f + \frac{N_s}{N_{sh}} I_s$= 4+0= 4 A, from OCC, we get Ea = 198 V, then $K_e\Phi$ = 7.425, Td = 0 then n = 29.36 *60=1761.6 rpm, when Ia = 20 , $I_{eq} = 4 + 0.06 * 20 = 5.2\, A$, from OCC, we get Ea = 212 V, $K_e\varphi = \frac{E_a}{n} = \frac{212}{1600} * 60 = 7.95\frac{vs}{r}$, $T_d = \frac{7.95}{2\pi} * 20 = 25.3\ N.m$

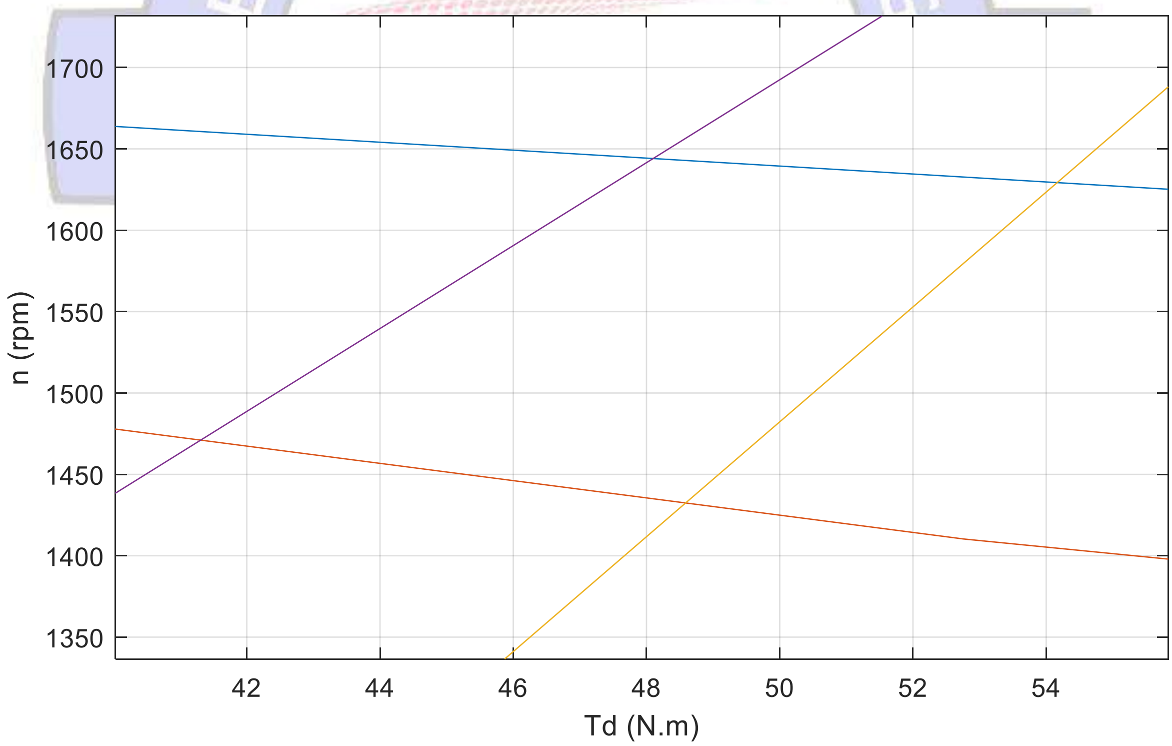
$n = \left(\frac{\mathbf{220-2}}{7.95} - \frac{(0.357+0.223)*2\pi}{7.95^2} * 25.337\right) * 60 = 1555.78$ rpm, so using the data in table 1 and sub in mechanical charactristic, we get the table 2.

For load 1: T = 8 + 1.7n, so, $n = \frac{T-8}{1.7}$, using the same value of torque Td in table2 to plot in the same graph, so when T=0 , $n = \frac{-8}{1.7} * 60 = -282.3$rpm, $n = \frac{25.33-8}{1.7} * 60$ =611.6 rpm

For load 2: T = 9.6 + 0.05n^2 , $n = \sqrt{\frac{T-9.6}{0.05}}$, $n = \sqrt{\frac{25.33-9.6}{0.05}} * 60 = 1064$rpm

Td (N.m)	0	25.33	52.76	80.93	95.25
n (shunt) rpm	1761.6	1699.7	1632.7	1564	1529
n (comp.) rpm	1761.6	1555.8	1410.3	1297	1245
n (load)1	-282.3	611.6	1579.7	2574	3079.4
n (load) 2	----	1064	1762.8	2266	2479.6


2000
1800
1600
1400
1200
1000
800
600
400
200
0
n (rpm)
0
10
20
30
40
50
60
70
80
Td (N.m)


1700
1650
1600
1550
1500
1450
1400
1350
n (rpm)
42
44
46
48
50
52
54
Td (N.m)

9.14 Sheet ch. 9

Unless otherwise stated, assume that (i) winding resistances are given at the working temperatures, (ii) the demagnetizing effect of armature reaction is negligible, and (iii)the brush contact drop is 2 volts. Some of the questions refer to the following machines and loads:

Table 1 OCC for M1at 1600 rpm.

Amps	volts
0.0	8
0.3	21
0.55	40
1.3	97
1.7	124
2.3	154
3.0	179
4.0	198
5.0	210
6.0	218
7.1	224
8.3	228

Table 2 OCC for M2 at 750 rpm.

Amps	Volts
0	0.0
3	2.0
5	3.5
7	4.7
10	6.45
15	8.45
20	10.1
25	11.35
30	12.35
35	12.9
40	13.4
50	13.92
60	14.2
75	14.3

Machine M1 is a dc motor rated at 220 v, 1600 rpm, and 13.5 hp. The armature and shunt field resistances are 0.357 ohms and 55 ohms. The OCC is given in table 1 . it is known from tests that the torque due to rotational losses is approximately proportional to speed. The motor also has a series winding having 6 % of the shunt field turns and a resistance of 0.223 ohms; unless otherwise stated, assume the series winding to be disconnected.

Mac**hine M2** is a series motor whose armature and field resistances are 40 and 10 milliohms respectively. the occ is given in table 2. The brush contact drop may be assumed constant at 1v, and the rotational losses are approximately equal to 3n, where n is the speed in rps.

Machine tool load L1 has a torque-speed characteristic given by:

$T = 8 + 1.7n$.

fan load L2 has a torque-speed characteristic given by $T = 9.6 + 0.05n^2$.

1. A shunt motor runs on 250 V. the armature circuit resistance is 0.07 ohms, and the field circuit resistance is 25 ohms. For a certain load, the motor rotates at 1200 rpm and draws 100 A. (a) Find the developed torque when the motor current is 100 A. (b) find the motor current and shaft speed when the motor develops 300 Nm. (c)Estimate the no-load speed and the rate of change of speed with torque.
2. A PM motor runs from a 60 V supply, and has an armature resistance of 0.2 ohms. At no-load it draws 1.5 A and rotates at 900 rpm. The brush contact drop is 1v. (a) what is the full-load current if the full-load speed is 750rpm? (b)find the speed regulation and developed torque when the current is 30 A. (c) A series resistor of 0.07 ohms is used to reduce speed. Find the speed regulation and current for the same developed torque obtained in part b.
3. A shunt motor runs on 400 V. the armature and field circuit resistances are 0.5 and 250 ohms. At no-load it draws 4.3 A and rotates at 1350 rpm. At full-load it draws 42.6 A. (a) Find $K_e\Phi$, the electrical losses, and the rotational losses at no-load. (b) Find the full-load speed, output horsepower, efficiency, speed regulation, and developed torque. (c)Repeat part b assuming armature reaction reduces the field by 25% with the field current kept constant (by decreasing the resistance in series with the shunt field winding). Neglecting armature reaction, find the motor speed and torque when it draws full-load current.
4. A series motor is fed from a 120 V line. its armature and field resistances are 0.3 and 0.1 ohms respectively. (a)the motor drives a certain load at 650 rpm and draws 78 A; find the developed torque. (b) with the developed torque unchanged from part a, the speed is reduced by means of a 0.6 ohm series resistor; find the speed. (c) for the same developed torque of parts a and b, what resistance is needed to reduce the speed to 450 rpm? (d) the load is changed so that the current becomes 50 A; find the developed torque and speed assuming this current is large enough to saturate the iron circuit of the

motor. (e)repeat part d assuming that at a load of 78 A operation is on the linear part of the magnetization curve of the motor.

5. Machine M1 is connected in shunt and operates at rating with no external resistor in the field circuit. (a) find the rated current of the motor; also find the efficiency. (b) find the torque due rotational losses, and hence write out the equation of this torque as a function of speed.
6. Machine M1 is driven from a 220 V source, with the field supplied with 5.5 A from a separate source. (a) find the developed torque and speed when the armature current is 40 A. (b)find the armature current & speed when the developed torque is 25 Nm. (c) find the armature current and the developed torque when the speed is 1400 rpm. (d) repeat part a when the series field winding is connected in the armature circuit cumulatively. (e) repeat part d for differential compounding.
7. Machine M1 is connected in shunt with full field excitation. It operates at rated voltage. (a) neglecting the torque due rotational losses, determine the equations for the torque characteristic and the mechanical characteristic; also find the speeds at (i) no load, (ii) rated motor current, and (iii) 25%overload current. (b) repeat part a with the torque due to rotational losses included; also find the no-load current. (c) the motor is loaded by machine tool load L1; find the torque, speed, and motor current with the rotational losses (i)neglected as in part a, and (ii) included as in part b . (d) repeat part c for the fun load L2.
8. Machine M1 is connected in long shunt. The applied voltage is 220 V, and there is no external resistance in the shunt field circuit. At a line current of 60 A, find the speed, developed torque, and output power when the compounding is : (a) cumulative with no diverter; (b) cumulative with a 0.4 ohm diverter; and (c) differential.
9. Machine M1 is connected first as a shunt motor with a field control resistor of 33 ohms, then as a long shunt cumulative compound motor with the same resistor in the field circuit, and finally as a series motor. Rated voltage is applied in all cases. (a) plot on one sheet the mechanical characteristics for the three cases. (b) for each case, find (i) the speed when the load torque is 50 Nm, (ii) the load torque when the speed is 1700 rpm, and (iii) the reduction in speed when the torque is increased from 30 Nm to 90Nm.
10. Machine M1 is first connected in shunt, and then in cumulative long shunt. In both cases, operation is at rated voltage and full field. (a) plot on one sheet the torque characteristics for the two cases (up to 75 A). (b) plot on one sheet mechanical characteristics for the two cases; plot on the same sheet the torque-speed curves of the machine tool load L1 and the fan load L2. (c)use your curves to obtain the no-load speed and no-load current for each motor connection. (d) use your curves to determine the torque, speed, and current for each of the three motor connections when loaded by each of the two loads (i.e. six operating points) also find the corresponding output horsepower, efficiency, and speed regulation.
11. The series motor M2 is driven from a 12 V source and draws 25A. (a) find the torque, speed, output horsepower, and efficiency. (b) repeat part a with a 0.015-ohm diverter across the series field winding.
12. Plot the torque and mechanical characteristics (up to 70 A) for machine M2 when driven from a 12V source. (a) find the current and speed at a torque of 7.5 Nm. (b) find the current and torque at a speed of 550 rpm.
13. The field winding of machine M1 is connected in series with a 20-ohm resistor and fed from a separate 150 V source. The motor is required to drive the fan load L2 at 1050 rpm. (a) if speed control is by armature voltage control, what voltage must be applied? (b) if speed control is by armature resistance control, what resistance must be connected when the supply voltage is 220 V? (c) compute the electrical losses for parts (a) and (b) and compare.
14. Machine M1 is connected in shunt to a 220 V supply. It is required to drive the fan load L2 at 2000 rpm. What resistance must be added in series with the shunt field winding?
15. Machine M1 is connected in cumulative long shunt to a 220 V supply. It is required to drive the fan load L2 at 1600 rpm. What resistance must be added in series with the shunt field winding?

16. Series motor M2 is to develop the same torque as in question 11,part (a), but at half the speed. (a) if speed control is by voltage control what voltage must be applied to the motor? (b) if speed control is by armature resistance control, what resistance must be connected when the supply voltage is 12 V? (c) compute the electrical losses for parts (a) and (b) and compare.
17. Machine M1 is separately excited. Plot curves for speed control as follows: (a) Armature voltage control at field current 4A; draw your curves in steps of 40V from 220V to 60. (b) field control with the armature supplied the rated voltage; plot curves for the field currents are 4, 3, 2, 1.5 and 1A on the same sheet as part (a). (c)Armature resistance control with rated voltage and 4A field current; plot curves for control resistance values of 0,1,2,3,4, and 5 ohms.
18. Apply the 3 methods of speed control of question 17 to the machine tool load L1; that is use your curves to determine the speed at each setting of the control parameters.
19. Machine M1 is required to operate a load at a constant torque of 60 Nm up to a base speed of 1600 rpm, and at a constant power at higher speeds (see fig. 9.19). Constant torque operation is obtained by armature voltage control at constant field current 4A, while constant power operation is obtained by field control at constant armature voltage 220V. (a) What is the output horsepower at constant power operation? (b) plot the required torque-speed curve on the control curves of question 18, parts a and b. (c) determine the armature voltage and field current settings at 1000 rpm; also give the torque and output horsepower. (d) Determine the speed and torque when the field current is 2A.
20. Machine M1 is connected in shunt at rated voltage and full field.(a) Find the starting current and torque with no external resistor in the armature cot. (b) design a starter to limit maximum armature current to 120A, and set minimum armature currant to 60A.(c) Find the starting torque when the starter of part b is used. (d) Estimate the speed at each switching operation of the starter.
21. Motor M2 is driven from a battery. The battery has an open –circuit voltage of 14 V, and is rated at 12V and 15 A. find the torque, current, and terminal voltage at starting; also find power dissipation in motor and battery.
22. Machine M1 is connected in shunt at rated voltage and full field, and drives fan load L2.(a) if the motor is stopped by dynamic braking using a load resistor of 3.5 ohms, find the current, braking torque, and ohmic power dissipation at the first instant of braking; also find the braking torque when the speed has gone down to 1000rpm. (b) Repeat part (a) for plugging with a 1.5 ohm current –limiting series resistor; also find the armature current at the instant rotation stops. (c) repeat part (a) for regenerative braking with the terminal voltage reduced to 150 V; also find the speed at which the electrical braking of the motor ceases.

Answers:

1. (a) 173 Nm (b)166A, 1177rpm (c)1231rpm, 0.18 rpm/Nm.

2. (a)50.4A (b) 10.7%, 18.7Nm (c) 83.4%, 30A.

3. (a)17.6Wb/rev, 649 W, 1.07 KW (b) 1235 rpm, 19.3 hp, 85%, 5%, 115Nm. (c)1367 rpm, 19.3 hp, 85%, -1.2%, 108 Nm (d) 944 rpm, 115 Nm.

4. (a) 99.5Nm, (b)300 rpm, (c) 0.342 ohms (d) 64 Nm, 734 rpm (e)41Nm, 1145 rpm.

5. (a) 60A, 76% (b)6 Nm, 0.225n.

6. (a) 51Nm,1520 rpm (b) 19.5 A,1574 rpm (c)85 A, 109 Nm(d)54 Nm, 1373 rpm (e) 43 Nm,1717 rpm.

7. (a)T=1.18i , n=29.36-0.0407T, 1762 rpm, 1600 rpm, 1557 rpm (b)T=1.19i-6.6,n=29.09-0.0403T,1746rpm,1600 rpm,1557rpm, 9.54A (c) (i) 54 Nm, 1629 rpm, 50 A (ii) 54 Nm, 1616 rpm, 55A. (d) (i) 47 Nm, 1646 rpm, 44A (ii) 47 Nm, 1633 rpm, 49 A.

8. rpm: 1318, 1386, 6184; Nm: 75, 73, 16; hp: 13,13, -6.2.

9. (b) rpm: 1938,1457, 1660; Nm:116,18,47; rpm: 218, 318,627.

10. (c) rpm:1746, 1700; A: 9.5, 9.3 (d) A: 55, 49, 44, 38; Nm: 54, 47, 48, 39; rpm: 1615, 1632, 1408, 1450; hp:12, 11 , 9, 8; % eff: 76, 74, 73, 71; %SR: 8, 7, 21, 17.

11. (a)3.1Nm, 644 rpm,0.28hp, 71% (b) 2.2 Nm, 874 rpm, .27 hp, 68%.

12. (a)45.5 A,475 rpm (b) 33.7 A, 5 Nm.

13.(a) 106V (b) 3.3 ohms (c) 0.79 KW, 4.72KW.

14. 43.7 ohms.

15. 178.2 ohms.

16 (a) 7.13V (b) 0.195ohms (c) 56 W. 178 W.

18. voltage control: 1615, 1320, 1010, 710, 415 rpm; field control: 1615, 1745, 2110,2450,2880 rpm; resistance control:1615, 1300, 1075, 905, 775, 675 rpm.

19. (a) 13.5 hp (c) 145 V, 4 A, 60 Nm, 8.42 hp (d) 2230 rpm, 43 Nm.

20.(a) 620 A, 728 Nm (b) 0.917, 0.458, 0.101 ohms (c) 142 Nm (d) 873, 1317, 1589 rpm.

21.13.9Nm,76.4A,3.82V,292W,777W.

22. (a) 52 A ,114 Nm, 0.96 KW, 9.42 KW, 65 Nm (b) 226 A, 320 Nm, 18.3KW, 76.8KW,245Nm,117A(c) 140A, 218Nm, 7KW, 21KWregenerated, -66 Nm, 1212rpm

CHAPTER 10
BRUSHLESS DC MOTORS

10.1 Introduction

Conventional dc motors are highly efficient and their characteristics make them suitable for use as servomotors. However, their only drawback is that they need a commutator and brushes which are subject to wear and require maintenance. When the functions of commutator and brushes were implemented by solid-state switches, maintenance-free motors were realized. These motors are now known as brushless dc motors.

In this chapter, the basic structures, drive circuits, fundamental principles, steady state characteristics, and applications of brushless dc motors will be discussed.

10.2 Structures and Drive Circuits

10.2.1 Basic structures

The construction of modern brushless motors is very similar to the ac motor, known as the permanent magnet synchronous motor. Fig.1 illustrates the structure of a typical three-phase brushless dc motor. The stator windings are similar to those in a polyphase ac motor, and the rotor is composed of one or more permanent magnets. Brushless dc motors are different from ac synchronous motors in that the former incorporates some means to detect the rotor position (or magnetic poles) to produce signals to control the electronic switches as shown in Fig.2. The most common position/pole sensor is the Hall element, but some motors use optical sensors.

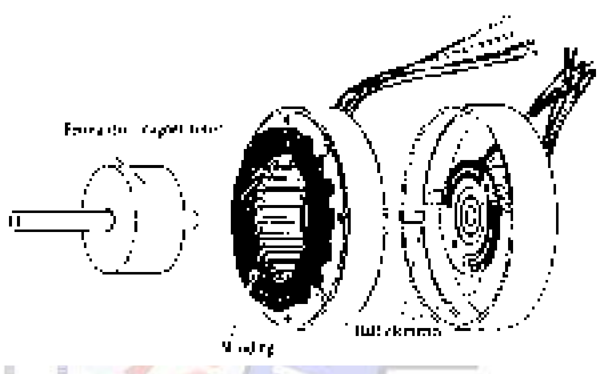


Fig.10.1 Disassembled view of a brushless dc motor (from Ref.[1] p58 Fig.4.1)

Although the most orthodox and efficient motors are three-phase, two-phase brushless dc motors are also very commonly used for the simple construction and drive circuits. Fig.3 shows the cross section of a two-phase motor having auxiliary salient poles

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10.2.2 Comparison of conventional and brushless dc motors

Although it is said that brushless dc motors and conventional dc motors are similar in their static characteristics, they

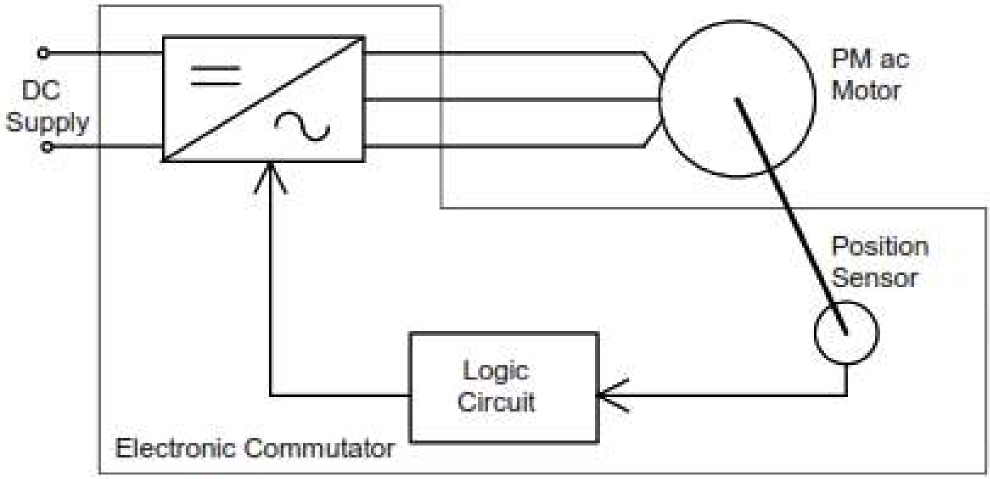


Fig.10.2 Brushless dc motor = Permanent magnet ac motor + Electronic commutator

actually have remarkable differences in some aspects. When we compare both motors in terms of present-day technology, a discussion of their differences rather than their similarities can be more helpful in understanding their proper applications. Table 1 compares the advantages and disadvantages of these two types of motors. When we discuss the functions of electrical motors, we should not forget the significance of windings and commutation.

Commutation refers to the process which converts the input direct current to alternating current and properly distributes it to each winding in the armature. In a conventional dc motor, commutation is undertaken by brushes and commutator; in contrast, in a brushless dc motor it is done by using semiconductor devices such as transistors.

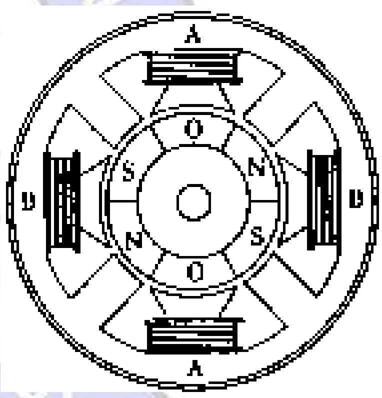


Fig.10.3 Two-phase motor having auxiliary salient poles (from Ref.[1] p95 Fig.5.22)

10.3 Drive Circuits

10.3.1 Unipolar Drive

Table 1. Comparison of conventional and brushless DC motors

	Conventional motors	Brushless motors
Mechanical structure	Field magnets on the stator	Field magnets on the rotor Similar to AC synchronous motor
Distinctive features	Quick response and excellent controlability	Long-lasting Easy maintenance (usually no maintenance required)
Winding connections	Ring connection The simplest: Δ connection	The highest grade: Δ or Y-connected three-phase connection Normal: Y-connected three-phase winding with grounded neutral point, or four-phase connection The simplest: Two-phase connection
Commutation method	Mechanical contact between brushes and commutator	Electronic switching using transistors
Detecting method of rotor's position	Automatically detected by brushes	Hall element, optical encoder, etc.
Reversing method	By a reverse of terminal voltage	Rearranging logic sequencer

Fig.4 illustrates a simple three-phase unipolar-operated motor that uses optical sensors (phototransistors) as position detectors. Three phototransistors PT1, PT2, and PT3 are placed on the end-plate at 120o intervals, and are exposed to light in sequence through a revolving shutter coupled to the motor shaft.

As shown in Fig.4, the north pole of the rotor now faces the salient pole P2 of the stator, and the phototransistor PT1 detects the light and turns transistor Tr1 on. In this state, the south pole which is created at the salient pole P1 by the electrical current flowing through the winding W1 is attracting the north pole of the rotor to move it in the direction of the arrow. When the north pole comes to the position to face the salient pole P1, the shutter, which is coupled to the shaft, will shade PT1, and PT2 will be exposed to the light and a current will flow through the transistor Tr2. When a current flow through the winding W2, and creates a south pole on salient pole P2, then the north pole in the rotor will revolve in the direction of the arrow and face the salient pole P2. At this moment, the shutter shades PT2, and the phototransistor PT3 is exposed to the light. These actions steer the current from the winding W2 to W3. Thus, salient pole P2 is de- energized, while the salient pole P3 is energized and creates the south pole. Hence the north pole on the rotor further travels from P2 to P3 without stopping. By repeating such a switching action in sequence given in Fig.5, the permanent magnet rotor revolves continuously.

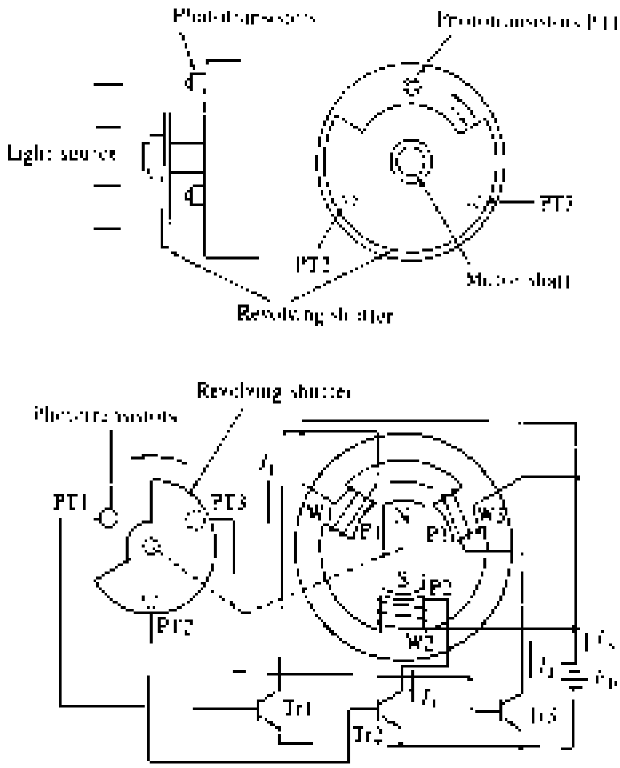


Fig.10.4 Three-phase unipolar-driven brushless dc motor (from Ref. [1] p59

10.3.2 Bipolar Drive

When a three-phase (brushless) motor is driven by a three-phase bridge circuit, the efficiency, which is the ratio of the mechanical output power to the electrical input power, is the highest, since in this drive an alternating current flow through each winding as an ac motor. This drive is often referred to as 'bipolar drive'. Here, 'bipolar' means that a winding is alternatively energised in the south and north poles.

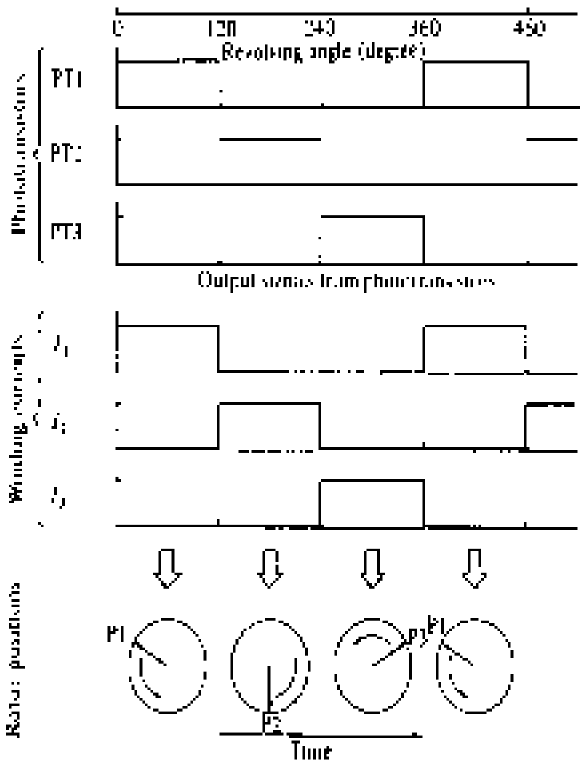


Fig.10.5 Switching sequence and rotation of stator's magnetic field (from Ref. [1] p60 Fig.4.3)

We shall now survey the principle of the three-phase bridge circuit of Fig.6. Here too, we use the optical method for detecting the rotor position; six p hototransistors are placed on the end-plate at equal intervals.

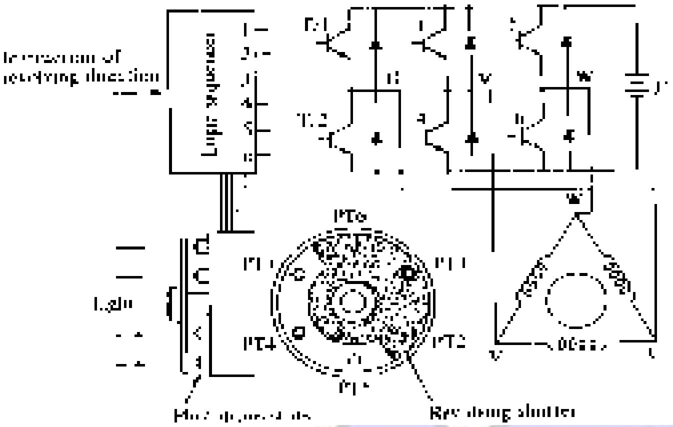


Fig.10.6 Three phase bipolar-driven brushless motor (from Ref. [1] p61, Fig.4.4)

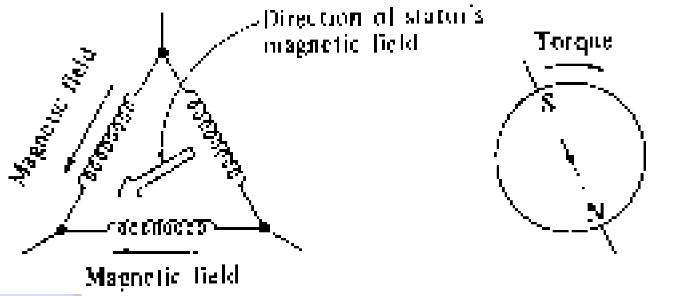


Fig.10.7 Stator's magnetic field in the shutter state of Fig.6, and the direction of torque (from Ref. [1] p62, Fig.4.5)

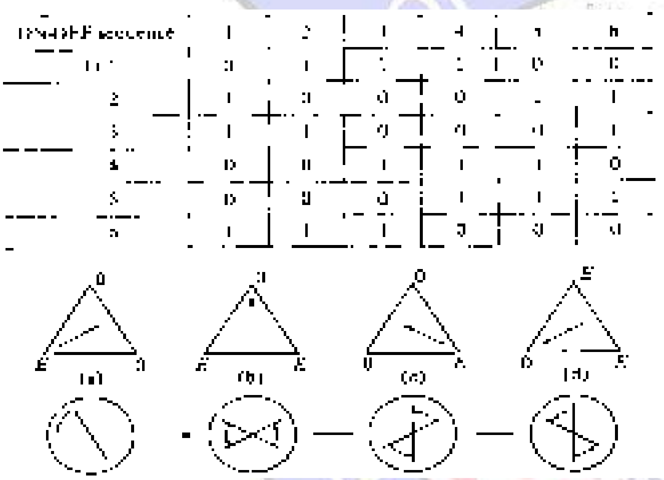


Fig.10.9 Counter-clockwise revolutions of the stator's magnetic field and rotor (from Ref. [1] p63 Fig.4.7)

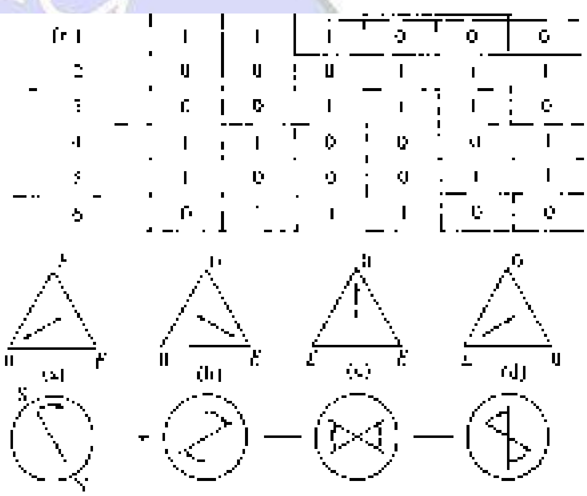


Fig.10.8 Clockwise revolutions of the stator's magnetic field and rotor (from Ref. [1] p63 Fig.4.6)

Since a shutter is coupled to the shaft, these photo elements are exposed in sequence to the light emitted from a lamp placed in the left of the figure. Now the problem is the relation between the ON/OFF state of the transistors and the light detecting phototransistors. The simplest relation is set when the logic sequencer is arranged in such a way that when a phototransistor marked with a certain number is exposed to light, the transistor of the same number turns ON. Fig.6 shows that electrical currents flow through Tr1, Tr4, and Tr5, and terminals U and W have the battery voltage, while terminal V has zero potential. In this state, a current will flow from terminal U to V, and another current from W to V as illustrated in Fig.7. We may assume that the solid arrows in this figure indicate the directions of the magnetic fields generated by the currents in each phase. The fat arrow in the centre is the resultant magnetic field in the stator. The rotational direction may be reversed by arranging the logic sequencer in such a way that when a photodetector marked with a certain number is exposed to light, the transistor of the same number is turned OFF. On the other hand, when a phototransistor is not exposed to light, the transistor of the same number is turned ON.

In the positional state of Fig.6, Tr2, 3, and 6 are ON, and the battery voltage E appears at terminal V, while U and W have zero electric potential. Then, as shown in Fig.9(a), the magnetic field in the stator is reversed, and the rotor's torque is counter-clockwise. After the motor revolves about 30^{o}, Tr2 turns OFF and Tr1 ON. At this point, the field has revolved 60^{o} and becomes as shown in (b). As the rotor produces another counter-clockwise torque, the counter-clockwise motion continues and the field becomes as shown in (c). This action is replaced in the sequence of (a)-(b)-(c)-(d) to produce a continuous counter-clockwise motion.

The motor discussed above has -connected windings, but it may also have Y-connected windings. Fig.10(a) shows a practical circuit which is used in a laser-beam printer or a hard-disc drive. As shown in Fig.10(b), three Hall elements are placed at intervals of 60^{o} for detection of the rotor's magnetic poles. Because this motor has four magnetic poles, a mechanical angle of 60^{o} corresponds to an electrical angle of 120^{o}.

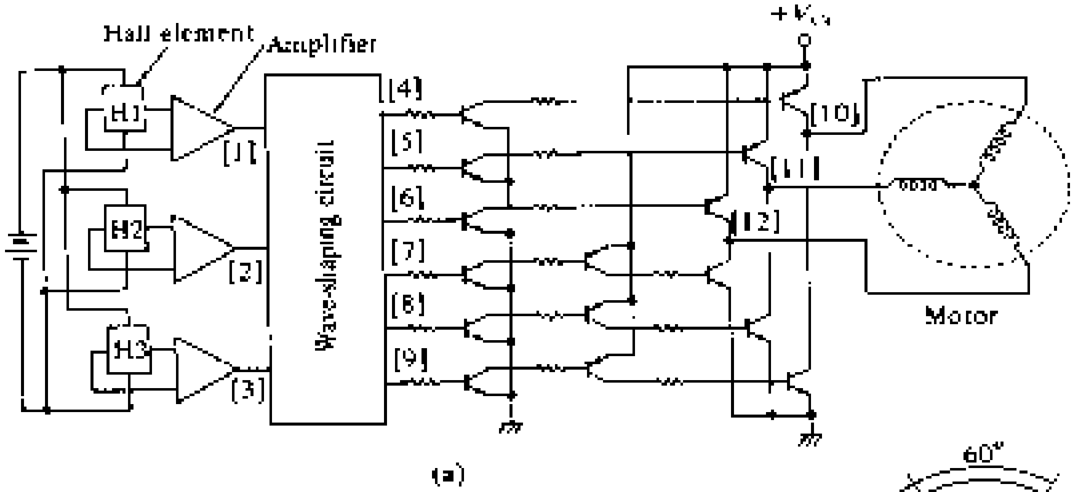
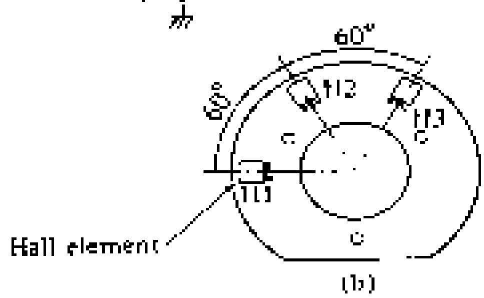


Fig.10.10 Practical circuit for a three-phase bipolar-driven motor, and arrangement of Hall elements (from Ref.[1] p80 Fig.5.1)



10.4 Equivalent Circuit and General Equations

The per phase equivalent circuit is shown in Fig.11 as following, where λ_m is the flux linkage of stator winding per phase due to the permanent magnet.

For steady state conditions, assuming *v* and e are sinusoidal at frequency ω, the equivalent circuit becomes the one shown in Fig.12, where X= ω L, and V, I, E, and λ_m are phasors with rms amplitudes. The steady state circuit equation can be written as

V = E + (R + jwL) I (10.1)

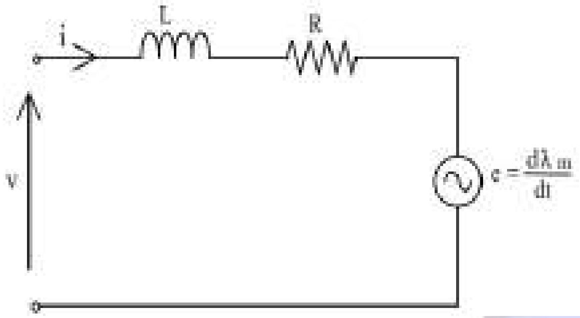


Fig.10.11 Dynamic per phase equivalent circuit of brushless dc

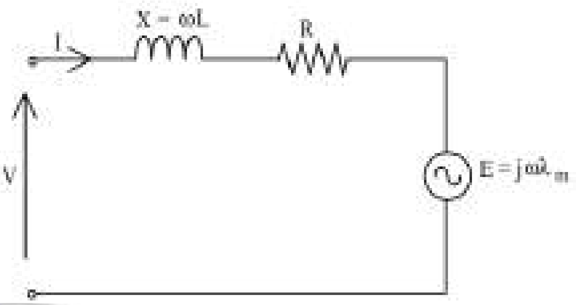


Fig.10.12 Steady state per phase equivalent circuit of brushless dc

For a maximum mechanical power at a given speed, *I* and *E* are in phase. This also gives maximum torque/ampere (minimum current/Nm). A brushless dc motor has position feedback from the rotor via Hall devices, optical devices, encoder etc. to keep a particular angle between *V* and *E*, since *E* is in phase with rotor position, and *V* is determined by the inverter supply to the motor. Assuming that *wL<<R*, when *I* is in phase with *E*, *V* will also be in phase with *E*. Thus, the circuit can be analyzed using magnitudes of *E*, *V*, and *I* as if it were a dc circuit.

But first note that when *E* and *I* are in phase, the motor mechanical power output (before friction, windage, and iron losses) i.e. the electromagnetic output power is

$$P_{em} = m\,|E|\,|I| = mw|\lambda_m|\,|I| \qquad (10.2)$$

where *m* is the number of phases, $|E|$, $|I|$, and $|\lambda_m|$ are the amplitudes of phasor *E*, *I*, and λ_m, and the electromagnetic torque is

$$T_{em} = \frac{P_{em}}{w_r} = \frac{mw|\lambda_m|\,|I|}{w_r} \qquad (10.3)$$

where $w_r = 2w/p$ is the rotor speed in Rad/s, and *p* the number of poles

$$T_{em} = \frac{mP|\lambda_m|\,|I|}{2} \qquad (10.4)$$

The actual shaft output torque is

$$T_{load} = T_{em} - T_{losses} \qquad (10.5)$$

where T_{losses} is the total torque due to friction, windage, and iron losses. Dropping the amplitude (modulus) signs, we have

$$T_{em} = \frac{mP}{2}\lambda_m I \qquad (10.6)$$

and in terms of rotor speed

$$E = \frac{P}{2}\lambda_m w_r \qquad (10.7)$$

10.5 Performance of Brushless DC Motors

10.5.1 Speed-Torque (T~w) curve

Still assuming $wL<<R$ and position feed back keeps V and E (and hence I) in phase, the voltage equation can be simplified in algebraic form as

$$V = E + RI \tag{10.8}$$

Substituting relations of E~w_r and T~I, we obtain

$$V = \frac{P}{2}\lambda_m w_r + \frac{2R}{mp\lambda_m}T_{em} \tag{10.9}$$

So

$$w_r = \frac{V}{p\lambda_m/_2} - \frac{R}{m(^{p\lambda_m}/_2)^2}T_{em} \tag{10.10}$$

The corresponding T~w curve is shown in Fig.13 for a constant voltage.

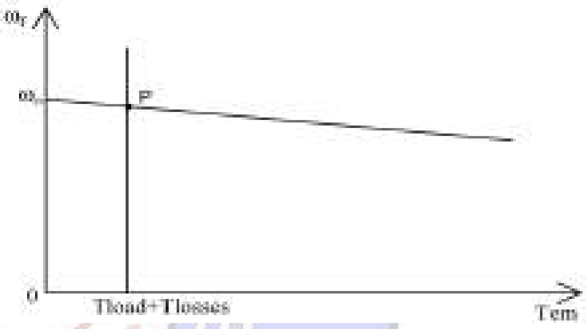


Fig.10.13 *T~w* curve of a brushless dc motor with a constant voltage supply

10.5.2 Efficiency

Efficiency is defined as the ratio of output power and input power, i.e

$$\zeta = \frac{P_{out}}{P_{in}} \tag{10.11}$$

where $P_{in} = mVI$, and $P_{out} = T_{load}\, w_r$. In term of the power flow

$$P_{in} = P_{cu} + P_{Fe} + P_{mec} + P_{out} \tag{10.12}$$

where $P_{cu} = mRI^2$ is the copper loss due to winding resistance, P_{Fe} the iron loss due to hysteresis and eddy currents, and P_{mec} the mechanical loss due to windage and friction.

10.6 Applications

Brushless dc motors are widely used in various applications. Two examples of them are illustrated in the following.

10.6.1 Laser Printer

In a laser printer, a polygon mirror is coupled directly to the motor shaft and its speed is controlled very accurately in the range from 5000 to 40,000 rpm. When an intensity- modulated laser beam strikes the revolving polygon mirror, the reflected beam travels in different direction according to the position of the rotor at that moment. Therefore, this reflected beam can be used for scanning as shown in Fig.14. How an image is produced is explained, using Fig.15 and the following statements:

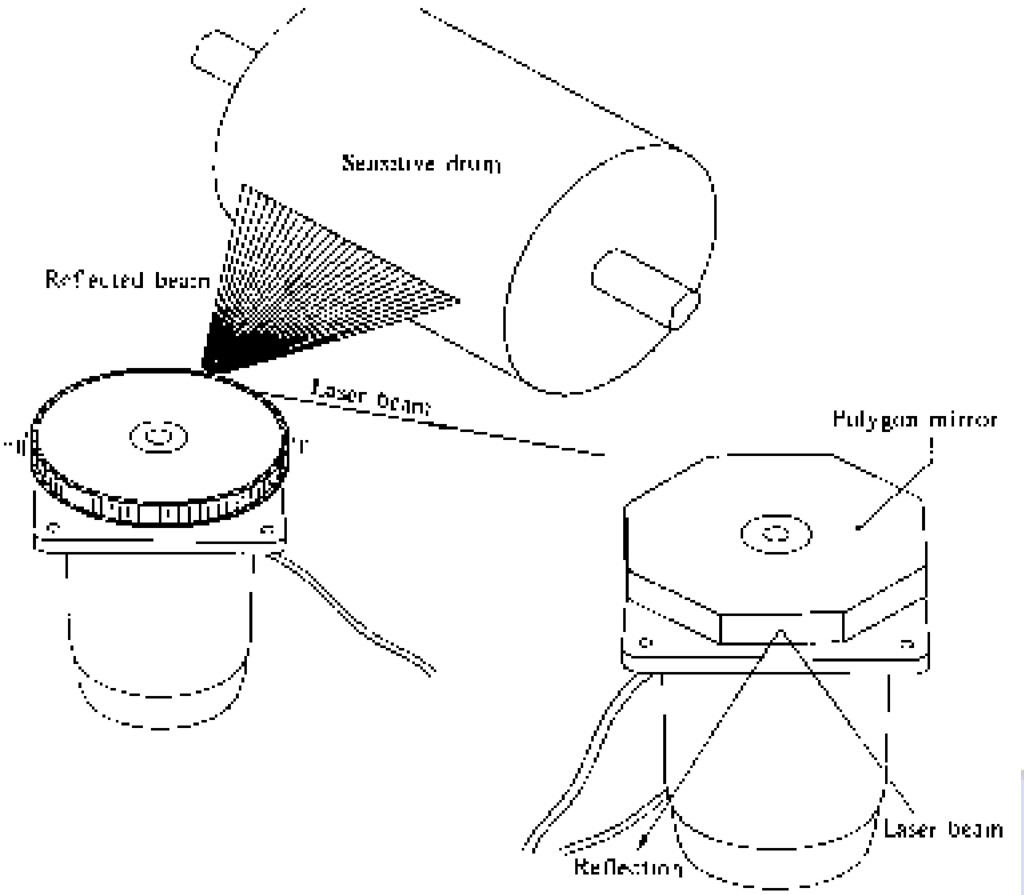


Fig.10.14 Role of motors for laser printers; (right) a brushless dc motor driving a polygon mirror, and (above) how to scan laser beams (from Ref. [1] p82 Fig.5.3)

(1) The drum has a photoconductive layer (e.g. Cds) on its surface, with photosensitivity of the layer being tuned to the wavelength of the laser. The latent image of the information to be printed formed on the drum surface by the laser and then developed by the attracted toner.

(2) The developed image is then transferred to normal paper and fixed using heat and pressure.

(3) The latent image is eliminated.

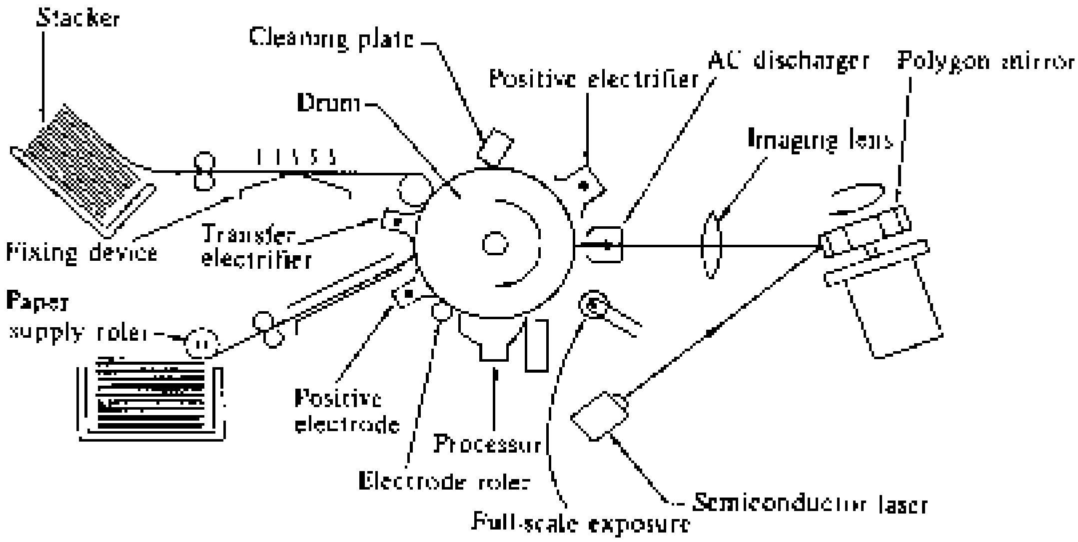


Fig.10.15 Principles of laser printers (from Ref. [1] p82 Fig.5.4)

A recent brushless dc motor designed for a laser printer is shown in Fig.16, and its characteristic data are given in Table 2.

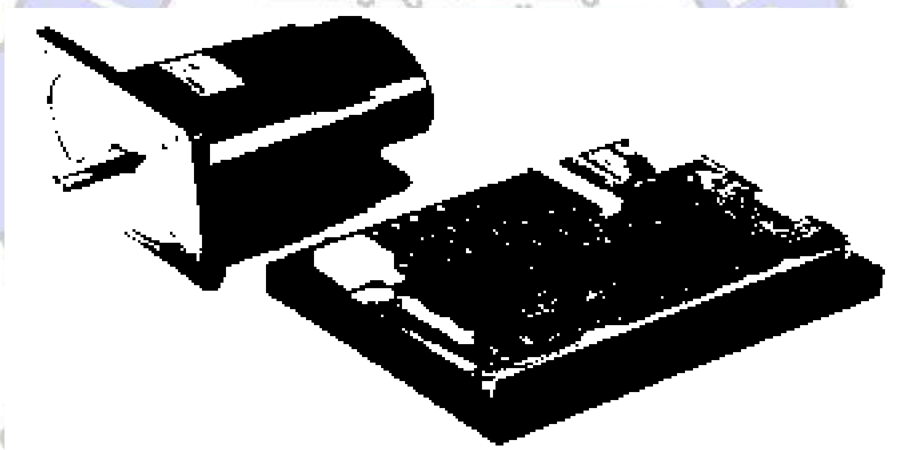


Fig.10.16 Brushless dc motor for a laser printer (from Ref. [1] p83 Fig.5.5

Table 2. Characteristics of three-phase bipolar type brushless motors

Item	Manufacturer Model	Nippon Densan Corporation 09PF8E4036
Voltage	V	±24 ± 1.2
Output	W	36
Rated torque	10^{-2} N m	0.294
Starting torque	10^{-2} N m	0.588
Starting time	s	3 (at non-inertial load)*
Rated speed	r.p.m.	6000, 9000, 12 000 selection
Rated current	A	3.5
Temperature	°C	5 – 45
Stability	per cent	±0.01
		Three-phase Δ connection

* A non-inertial load is a load applied by using a pulley and a weight

10.6.2 Hard Disk Drive

As the main secondary memory device of the computer, hard disks provide a far greater information storage capacity and shorter access time than either a magnetic tape or floppy disk. Formerly, ac synchronous motors were used as the spindle motor in floppy or hard disk drives. However, brushless dc motors which are smaller and more efficient have been developed for this application and have contributed to miniaturization and increase in memory capacity in computer systems. Table 3 compares a typical ac synchronous motor with a brushless dc motor when they are used as the spindle motor in an 8-inch hard disk drive. As is obvious from the table, the brushless dc motor is far superior to the ac synchronous motor. Although the brushless dc motor is a little complicated structurally because of the Hall elements or ICs mounted on the stator, and its circuit costs, the merits of the brushless dc motor far outweigh the drawbacks.

	AC synchronous motor	Brushless DC motor
Power supply: direct current, low voltage (for extension and interchangeability)	Inverter required	Direct current, low voltage (12–24 V)
Speed adjustment	Since speed depends on the frequency, regional adaptability is low	Adjustable independent of frequency
Adjustment of starting time	Adjustment not possible	Adjustment possible
Temperature rise	High	Low
Efficiency	Low (approx 30 per cent)	High (40–50 per cent)
Output to volume ratio	Small (bad)	Large (good)
Speed control	Fixed	Feedback control
Structure/cost	Simple, low cost	Slightly complicated, control circuit is not so expensive by the use of ICs

The hard disk drive works as follows (see Fig.17): The surface of the aluminium disk is coated with a film of magnetic material. Data is read/written by a magnetic head floating at a distance of about 0.5 μm from the disk surface due to

Item	Manufacturer	Nippon Densan Corporation	
	Model	09FH9C4018	09FH9C4022
Voltage	V	24 ± 2.4	24 ± 2.4
Output	W	18	22
Rated torque	10^{-1} N m	0.490	0.588
Starting torque	10^{-1} N m	1.47	1.96
Starting time	s	1.35	1.55
Rated speed	r.p.m.	3600	3600
Rated current	A	2.0	2.4
Temperature	°C	0 ~ 50	
Stability	per cent	± 1.0	
Inertia	10^{-6} kg m^2	1380	1670
Braking method		Electromagnetic method	
Number of disks		2	4

Table 3 Comparison of an ac synchronous motor and a brushless dc motor for an 8-inch hard disk drive.

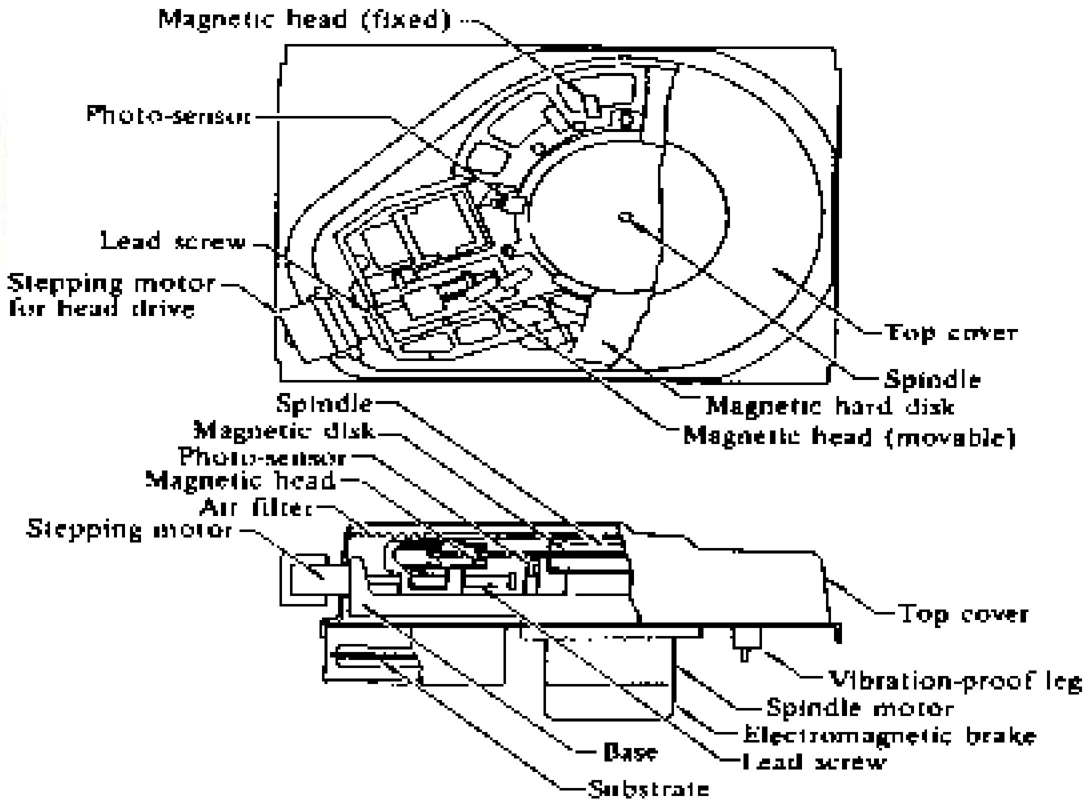


Fig.10.17 An example of hard disk drive (single disk type) (from Ref. [1] p86 Fig.5.9).

the airflow caused by the rotating disk, and this maintains a constant gap. Therefore, when the disk is stopped or slowed down, the head may touch the disk and cause damage to the magnetic film. To prevent this, this spindle motor must satisfy strict conditions when starting the stopping. Table 4 lists the basic characteristic data of brushless dc motors used in 8-inch hard disk drives (Fig.18).

REFERENCES

[1] T. Kenjo, "Permanent magnet and brushless dc motors", Oxford, 1985

[2] T.J.E. Miller, "Brushless permanent magnet and reluctance motor drive", Oxford, 1989

EXERCISES

1. Describe the essential features of a brushless dc motor (alternatively called a self- synchronous motor).

2. What additional features would be required for a brushless dc servomotor with torque and

Table ... indle drive
in a ha

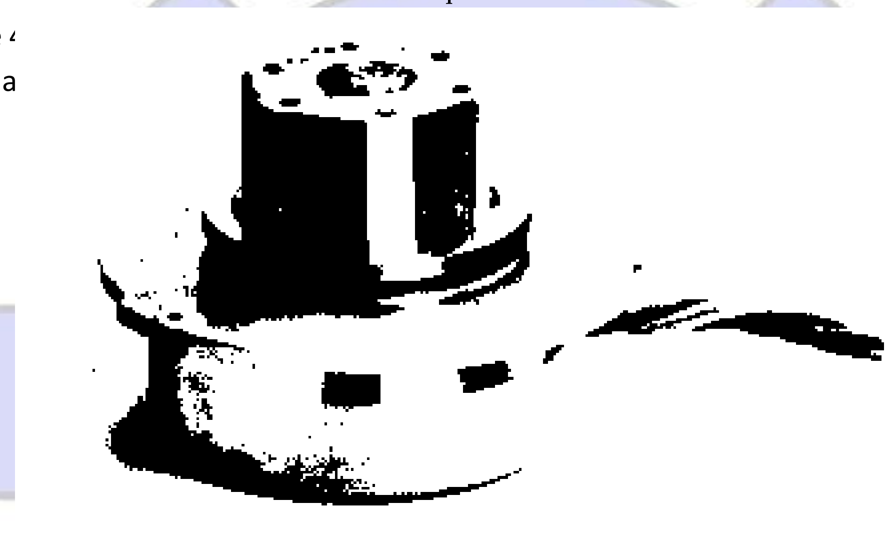


Fig.10.18 A brushless dc motor used for 8-inch hard disk drives
(from Ref. [1] p87 Fig.5.10)

position control?

3. Sketch the power circuit for a 3-phase brushless dc motor.

4. Calculate the supply frequency required for a twelve-pole motor to rotate at
(a) 360 rpm, and (b) 3600 rpm.

5. A brushless dc motor has 3 phases and 4 poles. The generated emf is 220 V rms sinusoidal at 1000 rpm (open circuit voltage when tested as generator with a drive motor). Calculate
(a) the emf constant (V/Rad/s);

(b) the torque constant (Nm/A) with optimum position feedback angle;

(c) the speed/torque curve, if the resistance per phase is 4 ;

(d) the supply frequency at 1000 rpm;

(e) curves of input power, output power and efficiency against torque, assuming friction and iron losses are zero;

(f) the frequency and speed at which X= L is equal to the resistance R, if the phase inductance is 5 mH;

(g) what is the effect of (f) on the speed/torque curve i.e. the effect of L>0 and L>R as speed increases?

6. A brushless dc motor has 3 phases and 6 poles. The electromagnetic torque is 4 Nm with a current of 0.5 A rms. Friction and iron losses produce a constant retarding torque of 0.1 Nm. The resistance and inductance per phase are 70 and 50 mH. Assume optimum position feedback. Calculate

(a) the torque and emf constants;

(b) the emf generated for a speed of 600 rpm;

(c) the speed of the motor for a supply voltage of 200 V (ac rms per phase) with no external load;

(d) the speed, current and efficiency for an external load of 4 Nm and a supply voltage of 200 V ac rms;

(e) the supply frequency for (d), and check wL<R.

Chapter 10

BLDCM

7. A brushless dc motor has 3 phases and 4 poles. The generated emf is 220 V rms sinusoidal at 1000 rpm (open circuit voltage when tested as generator with a drive motor). Calculate

(a) the emf constant (V/Rad/s);

(b) the torque constant (Nm/A) with optimum position feedback angle;

(c) the speed/torque curve, if the resistance per phase is 4 ;

(d) the supply frequency at 1000 rpm;

(e) curves of input power, output power and efficiency against torque, assuming friction and iron losses are zero;

(f) the frequency and speed at which X= wL is equal to the resistance R, if the phase inductance is 5 mH;

(g) what is the effect of (f) on the speed/torque curve i.e. the effect of L>0 and L>R as speed increases?

Solution:

a) $E = \frac{P}{2}\lambda_m w_r$,   220= $\frac{P}{2}\lambda_m$ (2*(22/7)*1000/60)

$\frac{P}{2}\lambda_m$= 2.1 V/Rad/Sec

b) $T_{em} = \frac{mP}{2}\lambda_m I$, $\frac{mP}{2}\lambda_m$= 3*2.1=6.3 Nm/A

c) $w_r = \frac{V}{p\lambda_m/2} - \frac{R}{m(p\lambda_m/2)^2}T_{em}$

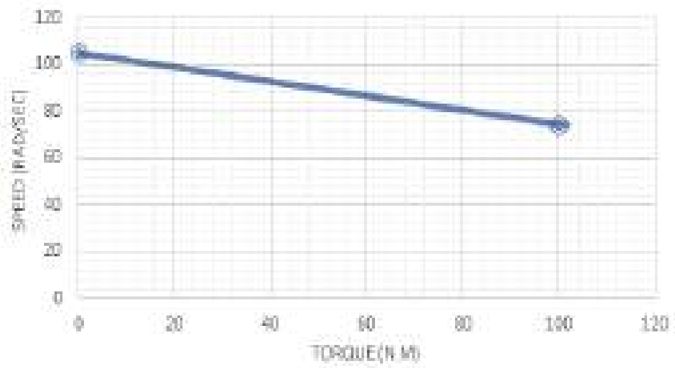
$w_r = \frac{220}{2.1} - \frac{4}{3(2.1)^2}T_{em}$,

$\boldsymbol{w_r = 104.76 - 0.302T_{em}}$

Make table between speed and torque take two values of torque

When T_{em}=0, wr=104.76 rad/sec

When T_{em}=100   ,   wr=74.56 rad/sec



So the curve as in Figure

d) w_r = 2w/p, w=wr*p/2=2*(22/7)*1000/60)*4/2=209.5 Hz

e) $T_{em} = \frac{P_{em}}{w_r}$, **$P_{out}$** = p_{em}=T_{em}*w_r,   **=104.76 T_{em}** (1)

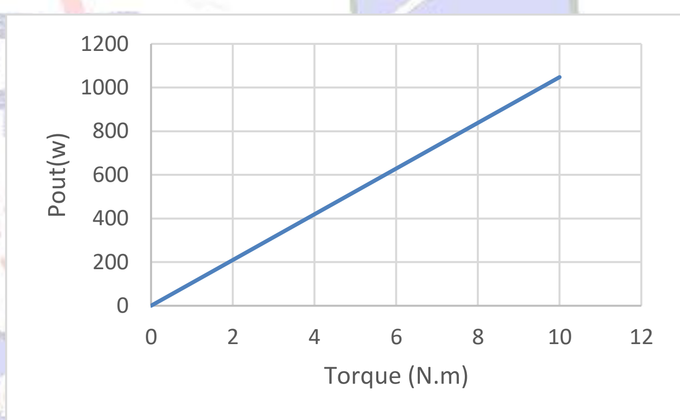
use T_{em}= 0, then P_{out} =0 watt, T_{em}=10, then p_{out}=1047.6 watt

T_{em}	Pout
0	0
2	209.5
5	523.8
8	838
10	1047.6

$P_{in} = P_{cu} + P_{Fe} + P_{mec} + P_{out}$, $P_{in} = P_{cu} + P_{out}$

Pin = mRI^2 + T_{em}*w_r, $I = \frac{2*T_{em}}{mP\lambda_m}$ sub in P_{in}

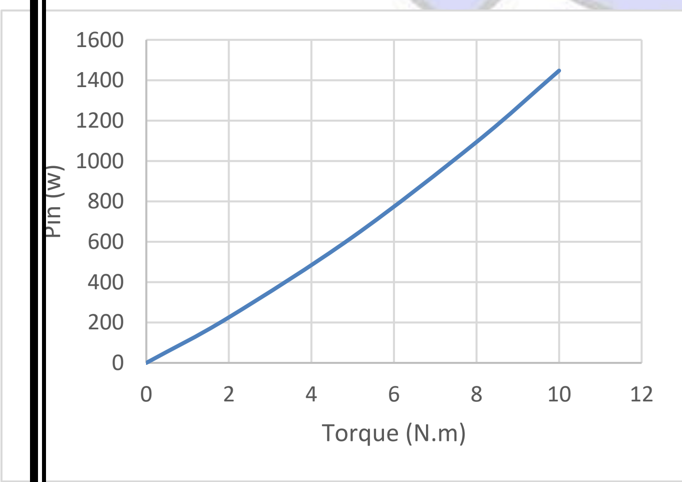
P_{in} = mR$\frac{2*T_{em}}{mP\lambda_m}^2$ + T_{em}*w_r, P_{in} = 4$\frac{T_{em}}{2.1}^2$ + T_{em}*104.76,

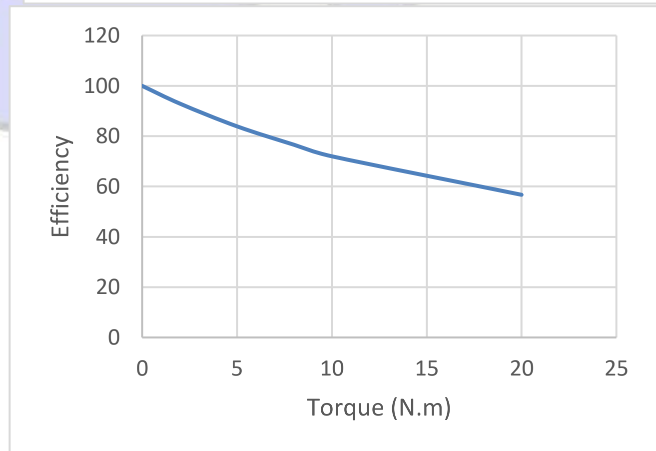


P_{in} = $0.302T_{em}^2$ + 104.76 T_{em}   (2)

T=0, then p_{in}=0,   T=2, p_{in}=225.5

Other calculation in table





$$\eta = \frac{P_{out}}{P_{in}} = \frac{\text{Tem} * \text{wr}}{0.302T_{em}^2 \, + \, 104.76 \text{ Tem}}$$

$$\boldsymbol{\eta} = \frac{\mathbf{104.76}}{\mathbf{4\ Tem+\ 104.76}}\mathbf{x100\%} \qquad (3)$$

T=0, $\eta = 100\%$, T=2, Type equation here.

T_{em}	P_{in}
0	0
2	225.5
5	623.8
8	1094
10	1447.6

T_{em}	Effe
0	100
2	
5	98.5
8	
10	
20	
100	

f) X=wL=R, w=R/L
W=4/(0.005)=800 Hz
Wr=2w/p=2*800/4=400 rad /sec

g) $w_r = \frac{V}{p\lambda_m/_2} - (\frac{R}{m\left(p\lambda_m/_2\right)^2} + j\frac{wL}{m\left(p\lambda_m/_2\right)^2})T_{em}$

When L increased the drop-in speed will increased at constant torque while the torque will decreased also when L increased at constant speed