DEFORMATION OF STRUCTURES

Conjugate Beam Method

Properties of Conjugate Beam

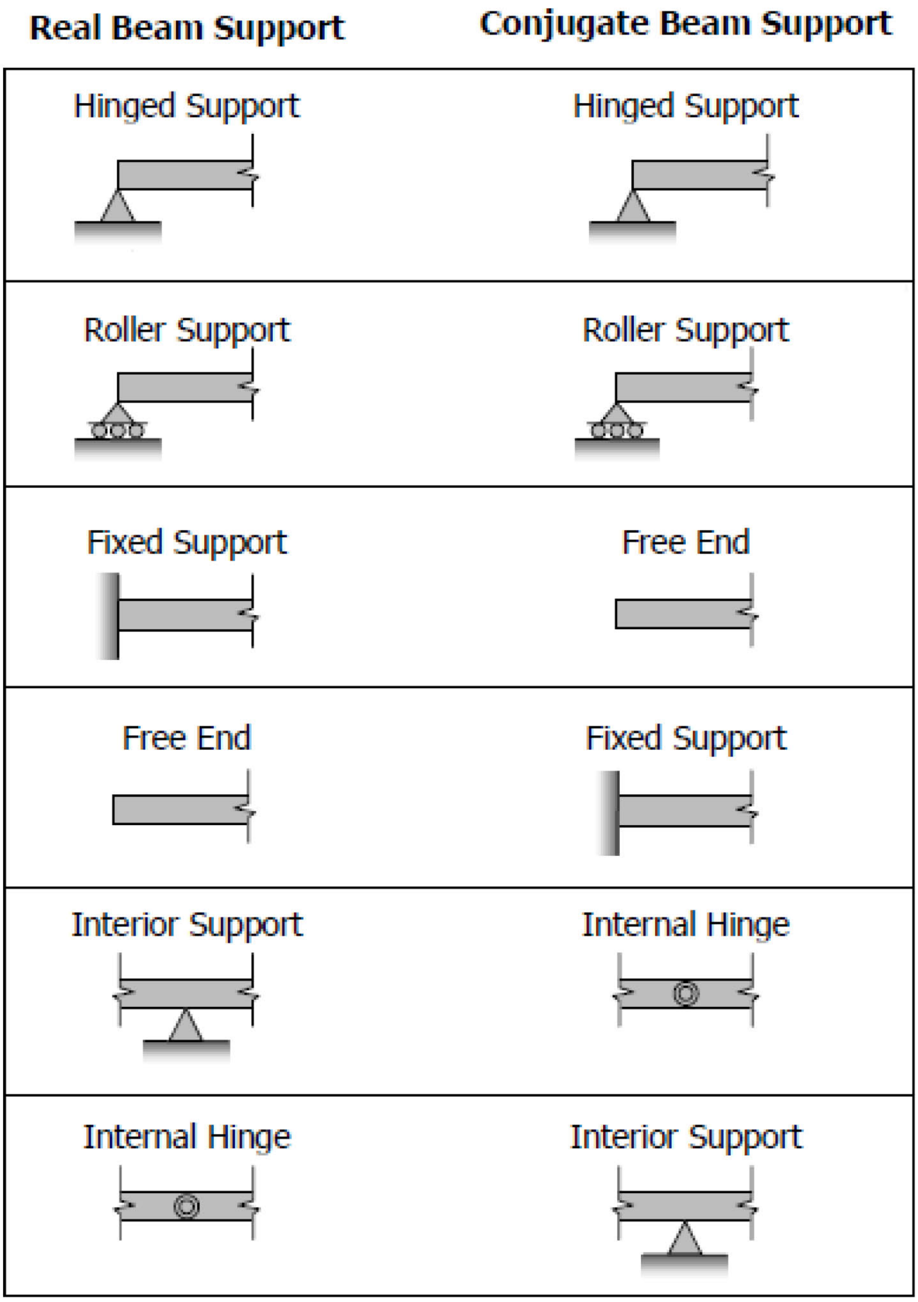
1. The length of a conjugate beam is always equal to the length of the actual beam.
2. The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
3. A simple support for the real beam remains simple support for the conjugate beam.
4. A fixed end for the real beam becomes free end for the conjugate beam.
5. The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.
6. The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.

Slope on real beam = Shear on conjugate beam

Deflection on real beam = Moment on conjugate beam

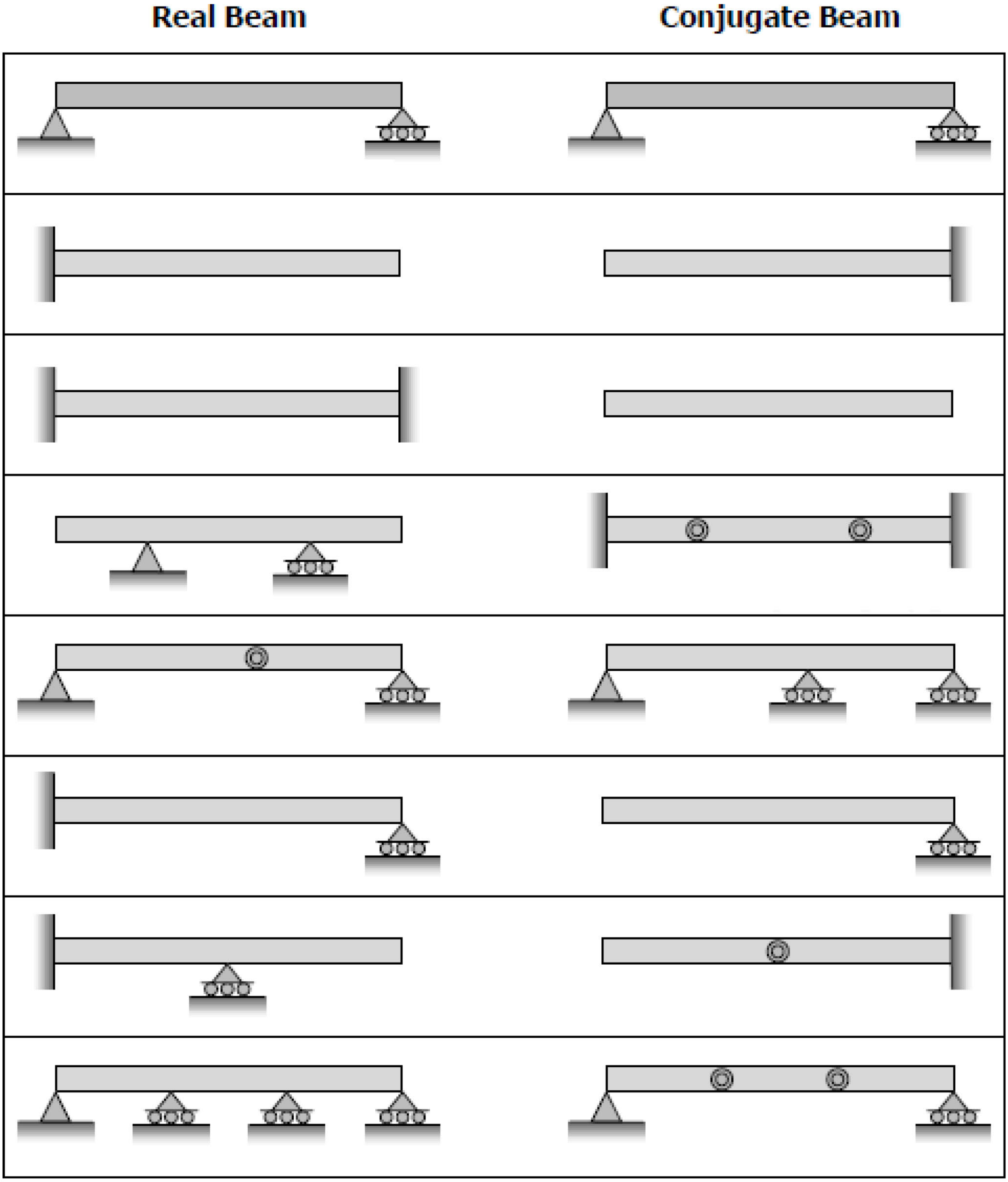
Supports of Conjugate Beam

Knowing that the slope on the real beam is equal to the shear on conjugate beam and the deflection on real beam is equal to the moment on conjugate beam, the shear and bending moment at any point on the conjugate beam must be consistent with the slope and deflection at that point of the real beam. Take for example a real beam with fixed support; at the point of fixed support there is neither slope nor deflection, thus, the shear and moment of the corresponding conjugate beam at that point must be zero. Therefore, the conjugate of fixed support is free end.



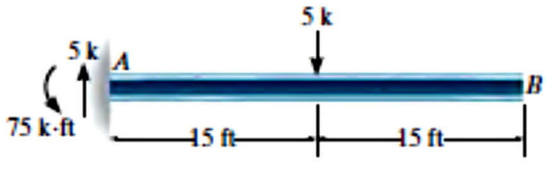
Examples of Beam and its Conjugate

The following are some examples of beams and its conjugate. Loadings are omitted.



Example 1

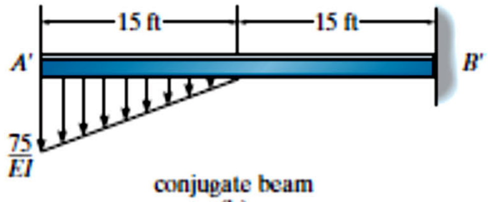
Determine the slope and deflection at point ***B*** of the steel beam shown in **Fig. a**. The reactions have been computed. $\boldsymbol{E = 29(10^3)}$ **ksi,** $\boldsymbol{I = 800}$ **in**^{4}.



real beam
(a)

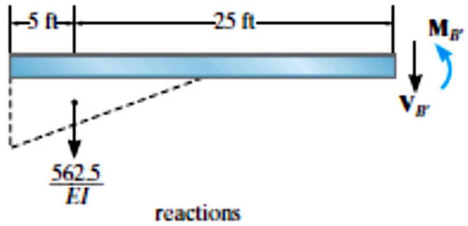
Solution

Conjugate Beam. The conjugate beam is shown in **Fig. b**. The supports at ***A'*** and ***B'*** correspond to supports ***A*** and ***B*** on the real beam. The ***M/EI*** diagram is *negative,* so the distributed load acts *downward,* i.e., away from the beam.



conjugate beam
(b)

Equilibrium. Since θ_B and Δ_B are to be determined, we must compute $V_{B'}$ and $M_{B'}$ in the conjugate beam, **Fig. c**.



reactions
(c)

$$+\uparrow \sum F_y = 0; \quad -\frac{562.5\ \text{k.ft}^2}{EI} - V_{B'} = 0$$

$$\theta_B = V_{B'} = -\frac{562.5\ \text{k.ft}^2}{EI}$$

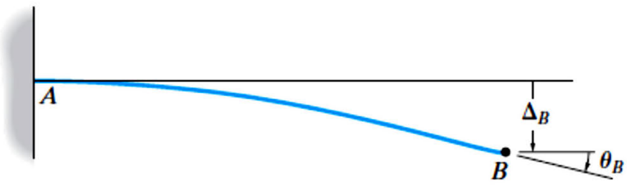
$$= \frac{-562.5\ \text{k.ft}^2}{29(10^3)\text{k/in}^2(144\ \text{in}^2/\text{ft}^2)\ 800\ \text{in}^4(1\text{ft}^4/(12)^4\,\text{in}^4)} = -0.00349\ \text{rad}$$

$\circlearrowleft +$

$$\sum M_{B'} = 0; \qquad \frac{562.5\ \text{k.ft}^2}{EI}(25\ \text{ft}) + M_{B'} = 0$$

$$\Delta_B = M_{B'} = \frac{14062.5\ \text{k.ft}^3}{EI} = -0.0873\ \text{ft} = -1.05\ \text{in}$$

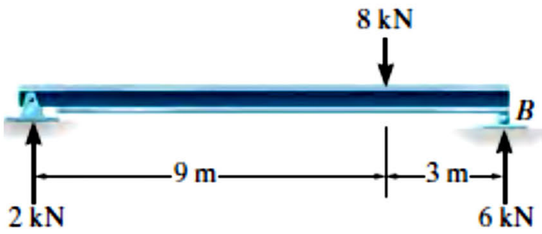
The negative signs indicate the slope of the beam is measured clockwise and the displacement is downward, **Fig. d**.



(d)

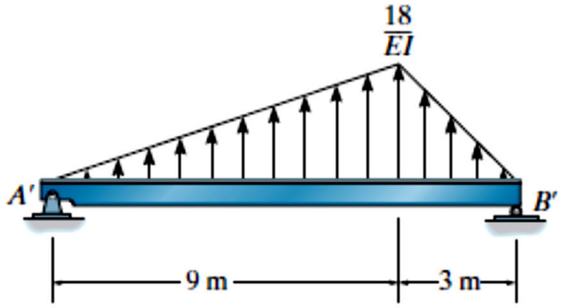
Example 2

Determine the maximum deflection of the steel beam shown in **Fig. a**. The reactions have been computed. ***E* = 200 GPa, *I* = 60(10^6) mm^4**.



Solution

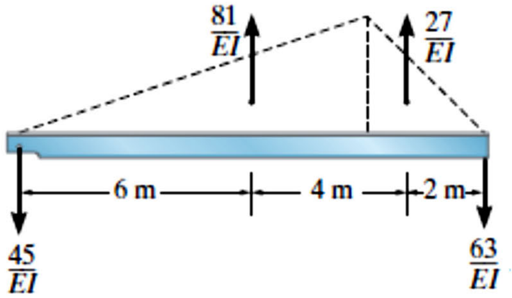
Conjugate Beam. The conjugate beam loaded with the ***M/EI*** diagram is shown in **Fig. b**. Since the ***M/EI*** diagram is positive, the distributed load acts upward (away from the beam).



conjugate beam

(b)

Equilibrium. The external reactions on the conjugate beam are determined first and are indicated on the free-body diagram in Fig. c. ***Maximum deflection*** of the real beam occurs at the point where the ***slope*** of the beam is ***zero***. This corresponds to the same point in the conjugate beam where the ***shear*** is ***zero***.



external reactions

(c)

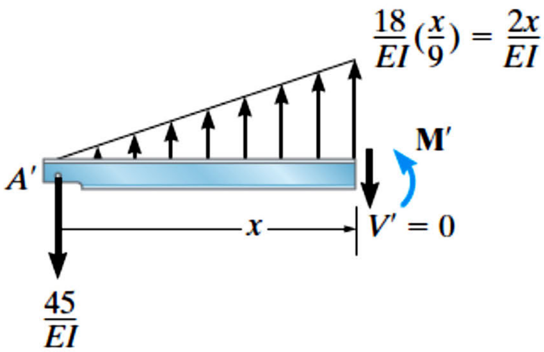
Assuming this point acts within the region **0 ≤ *x* ≥ 9 m** from ***A′***. we can isolate the section shown in Fig. d. Note that the peak of the distributed loading was determined from proportional triangles, that is,

w/x = (18/EI)/9

We require ***V′* = 0** so that,

$$+\uparrow\sum F_y = 0; \quad -\frac{45}{EI}+\frac{1}{2}\left(\frac{2x}{EI}\right)x=0$$

$$x = 6.71 \text{ m} \qquad (0 \le x \ge 9 \text{ m}) \text{ OK}$$



internal reactions

(d)

Using this value for ***x***, the maximum deflection in the real beam corresponds
to the moment ***M′***. Hence,

$+\circlearrowleft$

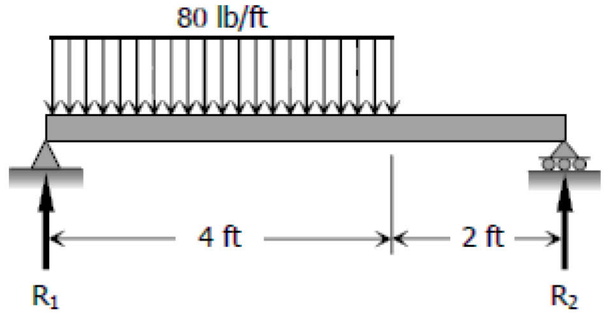
$$\sum M = 0; \quad \frac{45}{EI}(6.71)-\left[\frac{1}{2}\left(\frac{2(6.71)}{EI}\right)6.71\right]\frac{1}{3}(6.71)+M'$$

$$\Delta_{max} = M' = -\frac{201.2 \text{ kN.m}^3}{EI} = -0.0168 \text{ m} = -16.8 \text{ mm}$$

The negative sign indicates the deflection is downward.

Example 3

For the beam in the figure find the value of ***EIδ*** at **2 *ft*** from ***R*₂**



Solution

Solving for reactions

$\Sigma M_{R2} = 0$

$6R_1 = 80(4)(4)$

$R1 = 213.33\,\text{lb}$

$\Sigma M_{R1} = 0$

$6R_2 = 80(4)(2)$

$R_2 = 106.67\,\text{lb}$

From the conjugate beam

$\Sigma M_A = 0$

$$6F_2 + \frac{1}{3}(4)(640)\left[\frac{3}{4}(4)\right] =$$

$$\frac{1}{2}(4)(853.33)\left[23(4)\right] + \frac{1}{2}(2)(213.33)\left[4 + \frac{1}{3}(2)\right]$$

$F_2 = 497.77\,\text{lb.ft}^2$

$$M_B = \frac{1}{2}(2)(213.33\left[\frac{1}{3}(2)\right] - 2F_2$$

$$M_B = \frac{1}{2}(2)(213.33\left[\frac{1}{3}(2)\right] - 2(497.77)$$

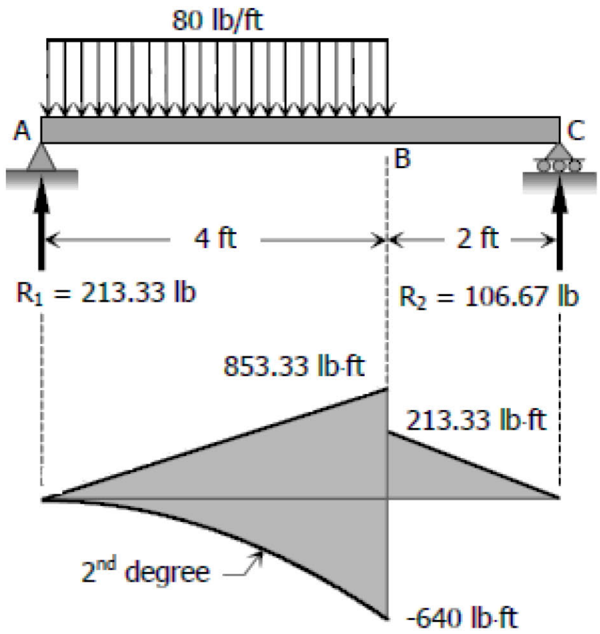
$M_B = -853.32\ \text{lb.ft}^3$

Thus, the deflection at B is

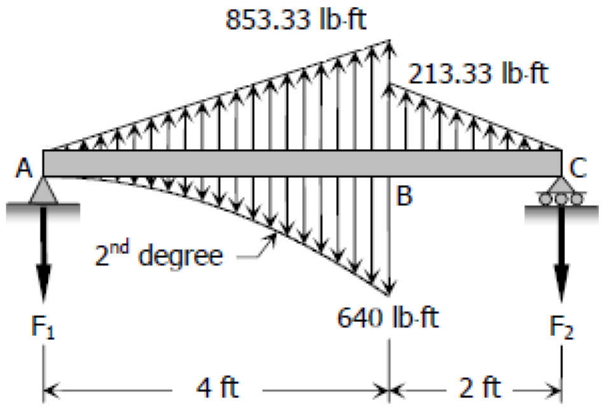
$EI\,\delta_B = M_B$

$EI\,\delta_B = -853.32\ \text{lb.ft}^3$

$EI\,\delta_B = 853.32\ \text{lb.ft}^3$ downward



Moment Diagram by Parts



Conjugate Beam Loaded with M/EI Diagram