

WEEK- 6

Chapter 2

Data Representation in Computer Systems

2.1 Introduction.

- **Bit**: Is the most basic unit of information in a computer.
 - It is a state of “on” or “off” in a digital circuit.
 - Sometimes these states are “high” or “low” voltage instead of “on” or “off.”
- **Byte**: Is a group of eight bits.
 - A byte is the smallest possible *addressable* unit of computer storage.
 - The term, “addressable,” means that a particular byte can be retrieved according to its location in memory.
- **Word**: Is a contiguous group of bytes.
 - Words can be any number of bits or bytes.
 - Word sizes of 16, 32, or 64 bits are most common.
 - In a word-addressable system, a word is the smallest addressable unit of storage.
- A group of four bits is called a *nibble*.

Bytes, therefore, consist of two nibbles:
a “high-order nibble,” and a “low-order nibble”.

2.2 Decimal to Binary Conversions.

- Bytes store numbers when the position of each bit represents a power of 2.
 - The binary system is also called the base-2 system.
 - Our decimal system is the base-10 system. It uses powers of 10 for each position in a number.
 - Any integer quantity can be represented exactly using any base (or *radix*).

Positional Numbering Systems

- The decimal number 947 in powers of 10 is:

$$9 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

- The decimal number 5836.47 in powers of 10 is:

$$5 \times 10^3 + 8 \times 10^2 + 3 \times 10^1 + 6 \times 10^0 \\ + 4 \times 10^{-1} + 7 \times 10^{-2}$$

- The binary number 11001 in powers of 2 is:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 16 + 8 + 0 + 0 + 1 = 25$$

- When the radix of a number is something other than 10, the base is denoted by a subscript.

Sometimes, the subscript 10 is added for emphasis:

$$(11001)_2 = (25)_{10}$$

$$\begin{aligned} 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ = 16 + 8 + 0 + 0 + 1 \\ = 25 \end{aligned}$$

- Because binary numbers are the basis for all data representation in digital computer systems, it is important that you become proficient with this radix system.
- Your knowledge of the binary numbering system will enable you to understand the operation of all computer components as well as the design of instruction set architectures.
- Every integer value can be represented exactly using any radix system.
- You can use either of two methods for radix conversion:
 - ❖ **The subtraction method.**
 - ❖ **The division remainder method.**
- Another method of converting integers from decimal to some other radix uses division.
- This method is mechanical and easy.
- It employs the idea that successive division by a base is equivalent to successive subtraction by powers of the base.
- Fractional values of other radix systems have nonzero digits to the right of the *radix point*.

- Numerals to the right of a radix point represent negative powers of the radix:

$$\begin{aligned}
 0.47_{10} &= 4 \times 10^{-1} + 7 \times 10^{-2} \\
 0.11_2 &= 1 \times 2^{-1} + 1 \times 2^{-2} \\
 &= \frac{1}{2} + \frac{1}{4} \\
 &= 0.5 + 0.25 = 0.75
 \end{aligned}$$

- As with whole-number conversions, you can use either of two methods:
a subtraction method and an easy multiplication method.
- We always start with the largest value first, $n - 1$, where n is our radix, and work our way along using larger negative exponents.

- **Converting 0.8125 to binary.**

- Using the multiplication method to convert the decimal 0.8125 to binary, we multiply by the radix 2.
 - The first product carries into the unit's place.

-Ignoring the value in the unit's place at each step, continue multiplying each fractional part by the radix.

$$\begin{array}{r}
 .8125 \\
 \times 2 \\
 \hline
 1.6250 \\
 \\
 .6250 \\
 \times 2 \\
 \hline
 1.2500 \\
 \\
 .2500 \\
 \times 2 \\
 \hline
 0.5000
 \end{array}$$

- You are finished when the product is zero, or until you have reached the desired number of binary places.

The image shows a series of handwritten calculations on a light blue background, illustrating the doubling method for converting a decimal fraction to binary. A yellow highlight is placed over the first four steps. Each step shows a decimal fraction multiplied by 2, with the integer part of the result being the next binary digit (1 for non-zero, 0 for zero), and the fractional part being carried over to the next step.

.8125	×	2	
1.6250			1
.6250	×	2	
1.2500			1
.2500	×	2	
0.5000			0
.5000	×	2	
1.0000			1

– Our result, reading from top to bottom is:

$$(0.8125)_{10} = (0.1101)_2$$

This method also works with any base. Just use the target radix as the multiplier.

- The binary numbering system is the most important radix system for digital computers.
- However, it is difficult to read long strings of binary numbers and even a modestly-sized decimal number becomes a very long binary number.

For example:

$$(11010100011011)_2 = (13595)_{10}$$

- For compactness and ease of reading, binary values are usually expressed using the hexadecimal, or base-16, numbering system.
- The hexadecimal numbering system uses the numerals 0 through 9 and the letters A through F.
 - The decimal number 12 is $(B)_{16}$.
 - The decimal number 26 is $(1A)_{16}$.
- It is easy to convert between base 16 and base 2, because $16 = 2^4$.
- Thus, to convert from binary to hexadecimal, all we need to do is group the binary digits into groups of four.

A group of four binary digits is called a hextet.

- Using groups of hextets, the binary number

$(11010100011011)_2 = (13595)_{10}$ in hexadecimal is:

0011	0101	0001	1011
3	5	1	B

Octal (base 8) values are derived from binary by using groups of three bits ($8 = 2^3$):

011	010	100	011	011
3	2	4	3	3

Octal was very useful when computers used six-bit words