Computer Graphics

Two Dimension Transformation

Fundamental to all computer graphics systems is the ability to simulate the movement and the manipulation of objects in the plane. These processes are described in terms of :

1-Translation
2-Scaling
3-Rotation
4-Reflection
5-Shearing

Our object is to describe these operations in mathematical form suitable for computer processing.

There are two complementary points of view for describing object movement:

1- Geometric transformation: the object itself is moved relative to a stationary coordinate system or background. Geometric transformation is applied to each point of the object.

2- Coordinate transformation: the object is held stationary while the coordinate system is moved relative to the object, for example the motion of a car in a scene, we can keep the car fixed while moving the background scenery.

Geometric transformation

1- Translation

In translation, an object is displaced a given distance and direction from its original position. A point in the XY plane can be translated by adding translation amount to the coordinates of the point. For each point P(X,Y) which is to be moved by TX unites parallel to the X- axis and by TY units parallel to the Y-axis to the new point P2(X2,Y2) we use the equations:

$$X2=X+TX$$
$$Y2=Y+TY$$

If TX is positive then the point moves to right

If TX is negative then the point moves to left

If TY is positive then the point moves up (in PC moves down)

If TY is negative then the point moves down (in PC moves up)

The transformation of Translation can be represented by (3*3) matrix:

1	0	0
0	1	0
TX	TY	1

Example : Move the line (-4,3) , (9,-6) 3 units in the X direction and 2 units in the Y direction

Solution:

TX=3 , TY=2

First point

X1new= -4 + 3= -1

Y1new= 3 + 2= 5

Second point

X2new= 9 + 3= 12

Y2new= -6 + 2= - 4

By using the matrix representation

$$\begin{bmatrix} X1new & Y1new & 1 \\ X2new & Y2new & 1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & 1 \\ 9 & -6 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

2-Scaling

Scaling is the process of expanding or compressing the dimensions of an object (changing the size of an object). The size of an object can be change by multiplying the points of an object by scaling factor.

If SF (scale factor) > 1 then the object is enlarged
If SF (scale factor) < 1 then the object is compressed
If SF (scale factor) = 1 then the object is unchanged

SX is the scale factor in the X direction
SY is the scale factor in the Y direction

To scale a point P(X,Y) we use the equations:

Xnew = X * SX
Ynew = Y * SY

If SX and SY have the same value (SX=SY) then the scaling is said to be *homogeneous* (or *balanced*).
By using the matrix representation

$$\begin{bmatrix} Xnew & Ynew & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} * \begin{bmatrix} SX & 0 & 0 \\ 0 & SY & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 1: Scale the rectangle (12,4),(20,4),(12,8),(20,8) with SX=2,SY=2

Solution : (By using the equations)

For the point (12,4)
X1new= 12 * SX =12 * 2 = 24
Y1new= 4 * SY = 4 * 2 = 8

For the point (20,4)
X2new= 20 * SX =20 * 2 = 40
Y2new= 4 * SY = 4 * 2 = 8

For the point (12,8)
X3new= 12 * SX =12 * 2 = 24
Y3new= 8 * SY = 8 * 2 = 16

For the point (20,8)
X4new= 20 * SX = 20 * 2 =40
Y4new= 8 * SY = 8 * 2 = 16

Solution : (By using matrices)

X1new	Y1new	1
X2new	Y2new	1
X3new	Y3new	1
X4new	Y4new	1

=

12	4	1
20	4	1
12	8	1
20	8	1

*

2	0	0
0	2	0
0	0	1

=

24	8	1
40	8	1
24	16	1
40	16	1

Notice that after a scaling transformation is performed, the new object is located at a different position relative to the origin. In fact, in scaling transformation the only point that remains fixed is the origin.

If we want to let one point of an object that remains at the same location (fixed), scaling can be performed by three steps:

1- Translate the fixed point to the origin, and all the points of the object must be moved the same distance and direction that the fixed point moves.

2- Scale the translated object from step one

3- Back translate the scaled object to its original position

Example 2: Scale the rectangle (12,4),(20,4),(12,8),(20,8) with SX=2,SY=2 so the point (12,4) being the fixed point.

Solution:

1- Translate the object with TX= -12 and TY= -4 so the point (12,4) lies on the origin

(12,4) ===> (0,0)
(20,4) ===> (8,0)
(12,8) ===> (0,4)
(20,8) ===> (8,4)

2- Scale the object by SX=2 and SY=2

(0,0) ===> (0 , 0)
(8,0) ===> (16 , 0)
(0,4) ===> (0 , 8)
(8,4) ===> (16 , 8)

3- Back translate the scaled object with TX= 12 and TY= 4

(0 , 0) ===> (12 , 4)
(16 , 0) ===> (28 , 4)
(0 , 8) ===> (12 ,12)
(16 , 8) ===> (28 ,12)

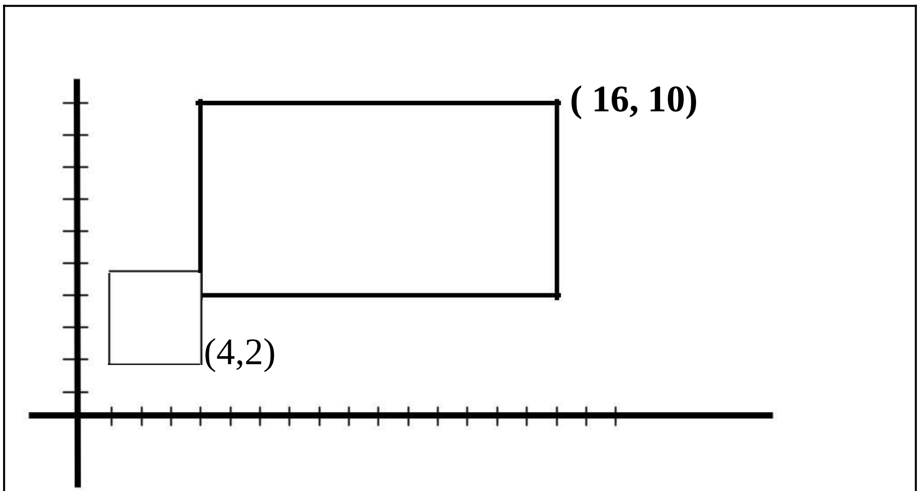
Example 3: Scale the triangle (80,40),(40,80),(120,80) by 0.25

Example 4: Scale the square (1,2) , (4,2) , (1,5) , (4,5) with 4 units in the X-axis ,and 2 units in the Y-axis

Solution:

$$\begin{bmatrix} X1new & Y1new & 1 \\ X2new & Y2new & 1 \\ X3new & Y3new & 1 \\ X4new & Y4new & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 1 \\ 1 & 5 & 1 \\ 4 & 5 & 1 \end{bmatrix} * \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 1 \\ 16 & 4 & 1 \\ 4 & 10 & 1 \\ 16 & 10 & 1 \end{bmatrix}$$



3-Rotation

In rotation, the object is rotated Ǿ about the origin. The convention is that the direction of rotation is counterclockwise if Ǿ is a positive angle and clockwise if Ǿ is a negative angle.

3-1 Rotation about the origin

The rotation matrix to rotate an object about the origin in anticlockwise direction is :

Cos Ǿ	Sin Ǿ	0
- Sin Ǿ	Cos Ǿ	0
0	0	1

Or, in equation :

Xnew= X * CosǾ - Y * SinǾ
Ynew= Y *Cos Ǿ + X * Sin Ǿ

When Ǿ=90, the matrix that cause a rotation through an angle of 90 (Π/2) is

0	1	0
-1	0	0
0	0	1

When Ǿ=180

-1	0	0
0	-1	0
0	0	1

When Ǿ=270

0	-1	0
1	0	0
0	0	1

When Ǿ=360

1	0	0
0	1	0
0	0	1

Example 1: rotate the line P1(1,4) and P2(3,1) anticlockwise 90 degree.

Solution:

1	4	1
3	1	1

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0	1	0
-1	0	0
0	0	1

=

-4	1	1
-1	3	1

3-2 Rotate about a specific point (XP,YP)

We need three steps:

First: translate the points (and the object) so that the point (XP,YP) lies on the origin

XP1= X - XP

YP1= Y - YP

Second: rotate the translated point (and the translated object) by Ǿ degree about the origin to obtain the new point (XP2,YP2)

XP2=XP1 * Cos Ǿ - YP1 * Sin Ǿ

YP2=YP1 * Cos Ǿ + XP1 * Sin Ǿ

Third : Back translation

XP3= XP2 + XP

YP3= YP2 + YP

Note:: Rotation in clockwise direction :

In order to rotate in clockwise direction we use a negative angle, and because :

Cos (-Ǿ)= Cos Ǿ

Sin (-Ǿ)= - sin Ǿ

So the matrix will be (for clockwise rotation):

Cos Ǿ	-Sin Ǿ	0
Sin Ǿ	Cos Ǿ	0
0	0	1

In equations: Xnew=X * Cos Ǿ + Y * Sin Ǿ

Ynew= Y * Cos Ǿ - X * Sin Ǿ

Example 2: Rotate the square (2,1),(4,1),(2,3),(4,3) counterclockwise with Ǿ=45 around the point (2,1)

Solution : First: translate the square by TX=-2 and TY=-1

(2,1) ===> (0,0)
(4,1) ===> (2,0)
(2,3) ===> (0,2)
(4,3) ===> (2,2)

Second: Rotate by Ǿ=45

(0,0) ===> (0 , 0)
(2,0) ===> (1.414 , 1.414)
(0,2) ===> (-1.414 , 1.414)
(2,2) ===> (0 , 2.828)

Third: Back translation by TX= 2 and TY= 1

(0 , 0) ===> (2 , 1)
(1.414 , 1.414) ===> (3.414 , 2.414)
(-1.414 , 1.414) ===> (0.586 , 2.414)
(0 , 2.828) ===> (2 , 3.828)