4-Reflection

If either the X or Y axis is treated as a mirror, the object has a mirror image or reflection. The reflected point Pnew is located the same distance from the mirror (the axis) as the original point P.

4-1:Reflection on the X axis

1	0	0
0	-1	0
0	0	1

OR Xnew= X

Ynew= -Y

4-2:Reflection on the Y axis

-1	0	0
0	1	0
0	0	1

OR Xnew= - X

Ynew= Y

4-3:Reflection on the origin

-1	0	0
0	-1	0
0	0	1

OR Xnew= - X

Ynew= -Y

4-4:Reflection on the line Y=X

0	1	0
1	0	0
0	0	1

OR Xnew= Y

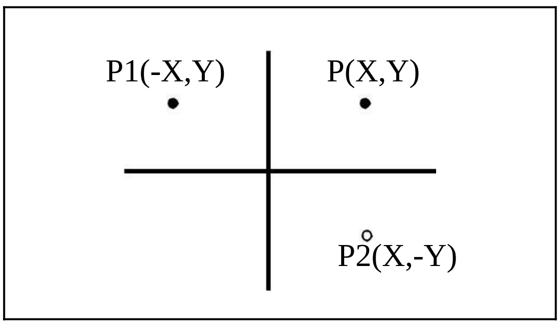
Ynew= X

4-5:Reflection on the line Y=-X

0	-1	0
-1	0	0
0	0	1

OR Xnew= -Y

Ynew= - X



Example 1: Reflect the point P(3,2) in :: a- X axis; b- Y axis; c-origin; d-line Y=X;

Solution : a-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$$

b-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & 1 \end{bmatrix}$$

c-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

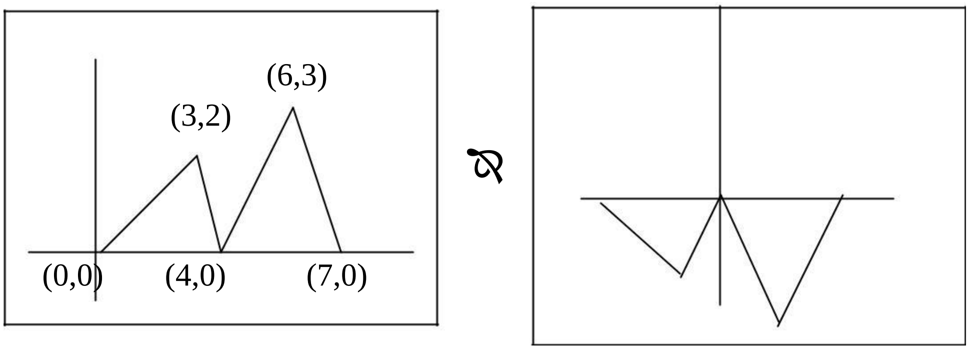
$$= \begin{bmatrix} -3 & -2 & 1 \end{bmatrix}$$

d-

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix}$$

Example 2: What (3*3) matrix will change the center of the scene to the origin, and reflect the mountains in the lake? [the center of the scene is (4,0)]



Solution:

First: Translate by TX= -4

1	0	0
0	1	0
-4	0	1

Second: Reflection on X axis

1	0	0
0	-1	0
0	0	1

Know multiply the two matrices :

1	0	0
0	1	0
-4	0	1

*

1	0	0
0	-1	0
0	0	1

The single matrix that perform Translation and Reflection is

1	0	0
0	-1	0
-4	0	1

0	0	1
3	2	1
4	0	1
6	3	1
7	0	1

*

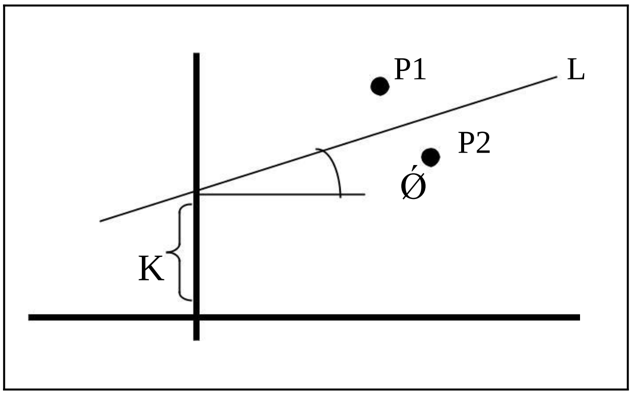
1	0	0
0	-1	0
-4	0	1

=

-4	0	1
-1	-2	1
0	0	1
2	-3	1
3	0	1

4-6:Reflection on an arbitrary line

To reflect an object on a line that does not pass through the origin, which is the general case:



As shown in the figure, let the line L intercept with Y axis in the point (0,K) and have an angle of inclination Ǿ degree with respect to the positive direction of X axis . To reflect the point P1 on the line L, we follow the following steps:

1- Move all the points up or down (in the direction of Y axis) so that L pass through the origin

T=

1	0	0
0	1	0
0	-K	1

2- Rotate all the points through (-Ǿ) degree about the origin making L lie along the X axis

R=

Cos Ǿ	-Sin Ǿ	0
Sin Ǿ	Cos Ǿ	0
0	0	1

3- Reflect the point P1 on the X axis

RefX=

1	0	0
0	-1	0
0	0	1

4- Rotate back the points by (-Ǿ) degree so that L back to its original orientation

R^{-1} =

Cos Ǿ	Sin Ǿ	0
- Sin Ǿ	Cos Ǿ	0
0	0	1

5- Shift in the direction of Y axis so that L is back in its original position

T^{-1} =

1	0	0
0	1	0
0	K	1

The sequence of matrices needed to perform this non-standard reflection is :

$$S = T * R * RefX * R^{-1} * T^{-1}$$

S=

Cos 2Ǿ	Sin 2Ǿ	0
Sin 2Ǿ	- Cos 2Ǿ	0
- K Sin 2Ǿ	K+K Cos 2Ǿ	1

Example 3: Find the single matrix that causes all the points in the plane to be reflected in the line with equation *Y=0.5X+2* , then apply this matrix to reflect the triangle with vertices at A(2,4), B(4,6), C(2,6) in the line.

Solution:

The Cartesian equation of a line in 2D is *Y= M * X +b* where b is the intersection of the lint with the Y axis and M is gradient of the line

M= ΔY / ΔX = Tan Ǿ

So the line ***Y=0.5 X + 2*** has gradient *M= 0.5* and intersect with the Y axis at the point where *y=2*
So K=2 , Tan Ǿ =0.5 ⇒ Ǿ=26.57
2Ǿ=53.13 , Cos 2Ǿ=0.6 , Sin 2Ǿ=0.8

S=

0.6	0.8	0
0.8	-0.6	0
-1.6	3.2	1

To reflect the triangle on the line :

2	4	1
4	6	1
2	6	1

*

0.6	0.8	0
0.8	-0.6	0
-1.6	3.2	1

=

2.8	2.4	1
5.6	2.8	1
4.4	1.2	1