$$= \frac{(k+1)(k+2)}{2}$$

$$= 1/2\,(k + 1)(k + 2)$$

So P is true for all $n \geq k$

Example 3:

Prove the following proposition (for $n \geq 0$):

$$P(n) : 1 + 2 + 2^2 + 2^3 + \ldots +2^n = 2^{n+1} - 1$$

solution :

(i) P(0) : left side =1

Right side $=2^1-1=1$

(ii) Assuming P(k) is true ; n=k

$$P(k) : 1 + 2 + 2^2 + 2^3 + \ldots +2^k = 2^{k+1} - 1$$

We add 2k+1 to both sides of P(k), obtaining

$$1 + 2 + 2^2 + 2^3 + \ldots +2^k + 2^{k+1} = 2^{k+1} - 1+ 2^{k+1}$$
$$= 2(2^{k+1}) - 1 = 2^{k+2} - 1$$

which is P(k +1). That is, P(k +1) is true whenever P(k) is true. By the principle of induction, P(n) is true for all n.

Homework:

Prove by induction:

1) $2 + 4 + 6 + \ldots\ldots + 2n = n (n + 1)$
2) $1 + 4 + 7 + \ldots\ldots + (3n - 2) = 1/2\, n (3n - 1)$

Relations

Binary relation:

There are many relations in mathematics :"less than" , "is parallel to ","is a subset of", etc. These relations consider the existence or nonexistence of a certain connection between pairs of objects taken in a definite order. We define a relation simply in terms of ordered pairs of objects.

Product sets:

Consider two arbitrary sets A and B. The set of all ordered pairs (a,b) where $a \in A$ and $b \in B$ is called the product, or cartesian product, of A and B.

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

Example: Let A = {1,2} and B = {a ,b ,c} then

$$A \times B = \{(1,a), (1,b),(1,c),(2,a),(2,b),(2,c)\}$$

Also, $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

- The order in which the sets are considered is important, so $A \times B \neq B \times A$.

Let A and B be sets. A binary relation, R, from A to B is a subset of $A \times B$. If $(x,y) \in R$, we say that x is R-related to y and denote this by xRy

if $(x,y) \notin R$, we write $x \not R y$ and say that x is not R-related to y .

if R is a relation from A to A ,i.e. R is a subset of $A \times A$, then we say that R is a relation on A.

The **domain** of a relation R is the set of all first elements of the ordered pairs which belong to R, and the **range** of R is the set of second elements.

Example 1:

Let A = {1, 2, 3, 4}. Define a relation R on A by writing $(x, y) \in R$ if $x < y$. Then

$$R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}.$$

Example 2:

let A = {1,2,3} and R = {(1,2),(1,3),(3,2)}. Then R is a relation on A since it is a subset of $A \times A$ with respect to this relation:

1R2, 1R3, 3R2 but $(1,1) \notin R$ & $(2,1) \notin R$

The domain of R is {1,3} and

The range of R is {2,3}

Example 3:

Let A = {1, 2, 3}. Define a relation R on A by writing $(x, y) \in R$, such that $a \geq b$, list the element of R

$$aRb \leftrightarrow a \geq b , a,b \in A$$

$\therefore R = \{(1,1),(2,1), (2,2), (3,1), (3,2), (3,3)\}$.

Example 4:

A relation on the set Z of integers is “m divides n.” A common notation for this relation is to write $m|n$ when m divides n. Thus $6 | 30$ but $7 \nmid 25$.

Representation of relations:

1) By language
2) By ordered pairs
3) By arrow form
4) By matrix form