

not one to one & not onto

### Graph of a function:

By a *real polynomial function*, we mean a function  $f: \mathbf{R} \rightarrow \mathbf{R}$  of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

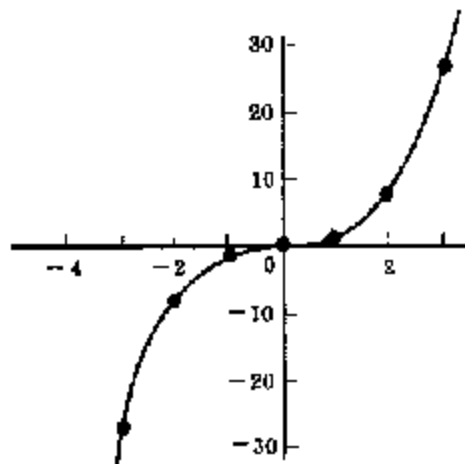
where the  $a_i$  are real numbers. Since  $\mathbf{R}$  is an infinite set, it would be impossible to plot each point of the graph. However, the graph of such a function can be approximated by first plotting some of its points and then drawing a smooth curve through these points. The table points are usually obtained from a table where various values are assigned to  $x$  and the corresponding value of  $f(x)$  computed.

Example 1 : let  $f: \mathbf{R} \rightarrow \mathbf{R}$  and  $f(x) = x^3$ , find  $f(x)$

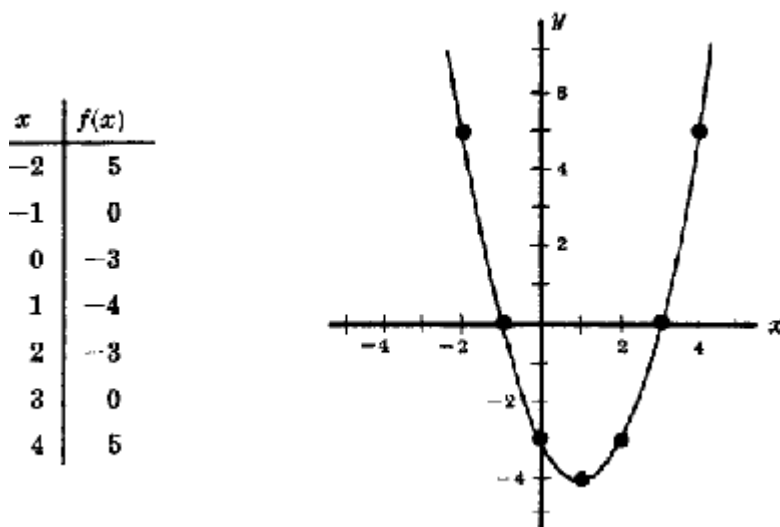
$$f(3) = 3^3 = 27$$

$$f(-2) = (-2)^3 = -8$$

$x$	$f(x)$
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27



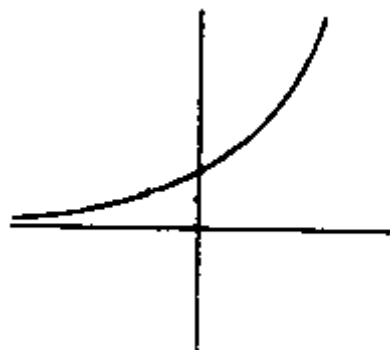
Example 2: let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $f(x) = x^2 - 2x - 3$ , find  $f(x)$



### Geometrical Characterization of One-to-One and Onto Functions

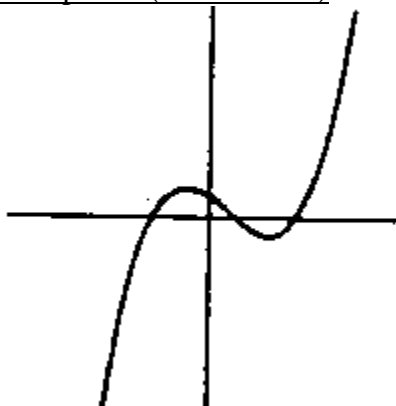
For the functions of the form  $f: \mathbb{R} \rightarrow \mathbb{R}$ , the graphs of such functions may be plotted in the Cartesian plane and functions may be identified with their graphs, so the concepts of being one-to-one and onto have some geometrical meaning :

(1)  $f: \mathbb{R} \rightarrow \mathbb{R}$  is said to be one-to-one if there are no 2 distinct pairs  $(a_1, b)$  and  $(a_2, b)$  in the graph one-to-one or if each horizontal line intersects the graph of  $f$  in at most one point.



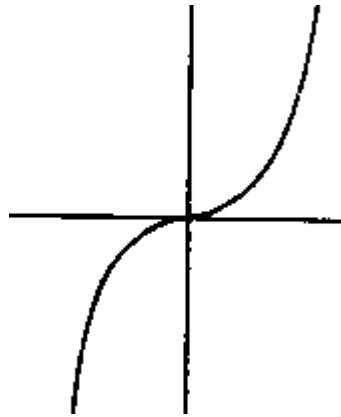
$$f_2(x) = 2^x$$

(2)  $f: \mathbb{R} \rightarrow \mathbb{R}$  is an onto function if each horizontal line intersects the graph of  $f$  at one or more points (at least once)

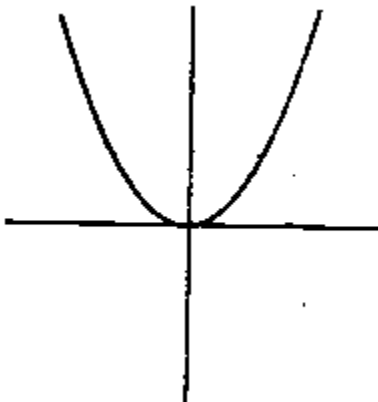


$$f_3(x) = x^3 - 2x^2 - 5x + 6$$

(3) if  $f$  is both one-to-one and onto, i.e. invertible, then each horizontal line will intersect the graph of  $f$  at exactly one point.



$$f_4(x) = x^3$$



$$f_1(x) = x^2$$

$f(x)$  NOT (ONE-TO-ONE) & NOT (ONTO)

### Composition of function:

Let  $f:A \rightarrow B$  and  $g:B \rightarrow C$ , to find the composition function  $\text{gof}:A \rightarrow C$

$$(\text{gof})(a) = g(f(a)) = g(y) = t$$

$$(\text{gof})(b) = g(f(b)) = g(x) = s$$

$$(\text{gof})(c) = g(f(c)) = g(y) = t$$

