Function:

Function is an important class of relation.

Definition:

Let A,B be two nonempty sets, a function F: A→B is a rule which associates with **each** element of A a **unique** element in B.

The set A is called the **domain** of the function, and the set B is called the **range** of the function.

Example 1:

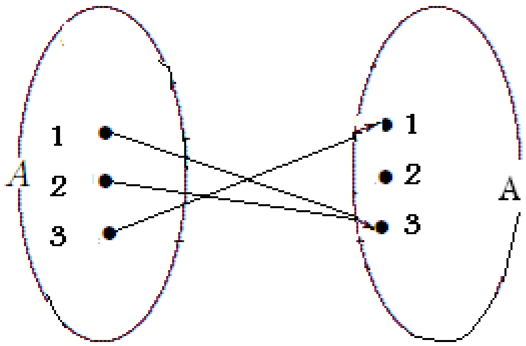
Consider the function $f(x) = x3$, i.e., f assigns to each real number its cube. Then the image of 2 is 8, and so we may write $f(2) = 8$.

Example2 :

consider the following relation on the set A={1,2,3}

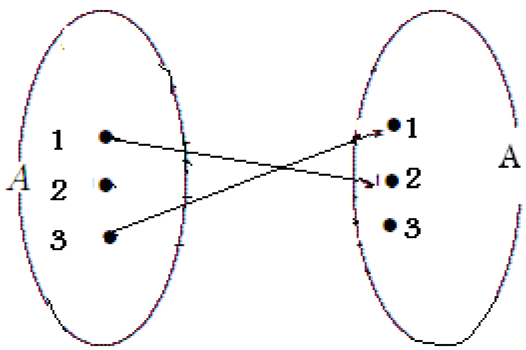
F = {(1,3),(2,3),(3,1)}

F is a function



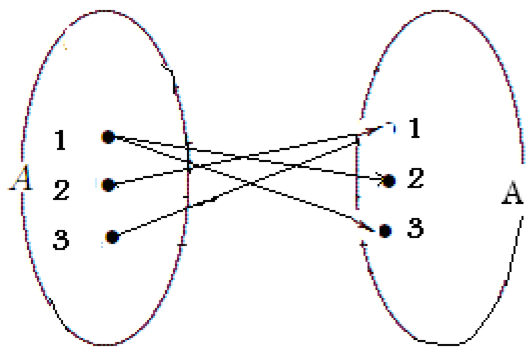
G = {1,2},(3,1)}

G is not a function from A to A



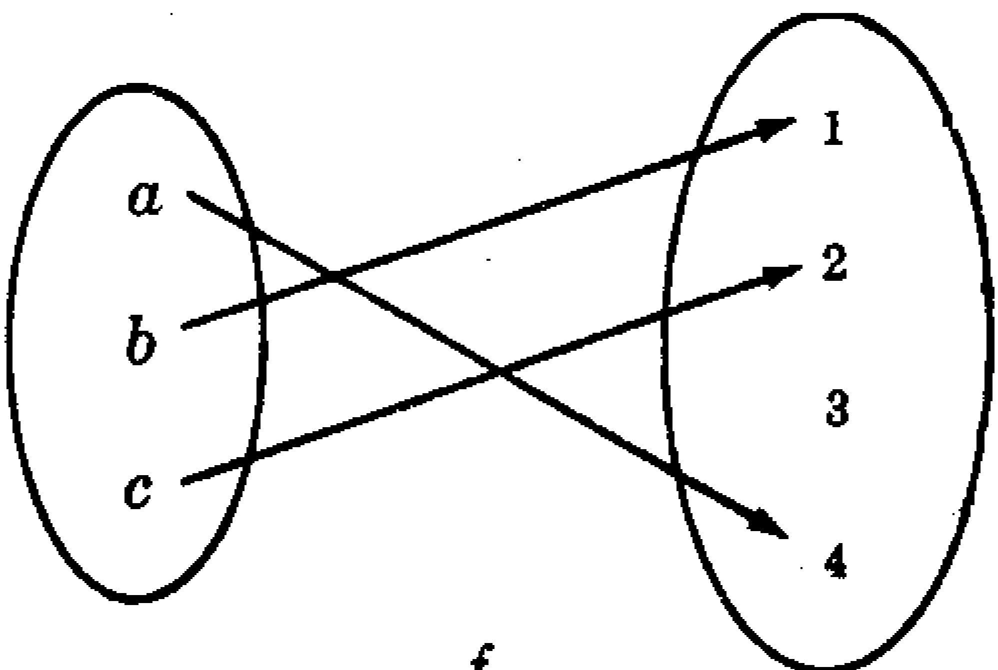
H = {(1,3),(2,1),(1,2),(3,1)}

H is not a function



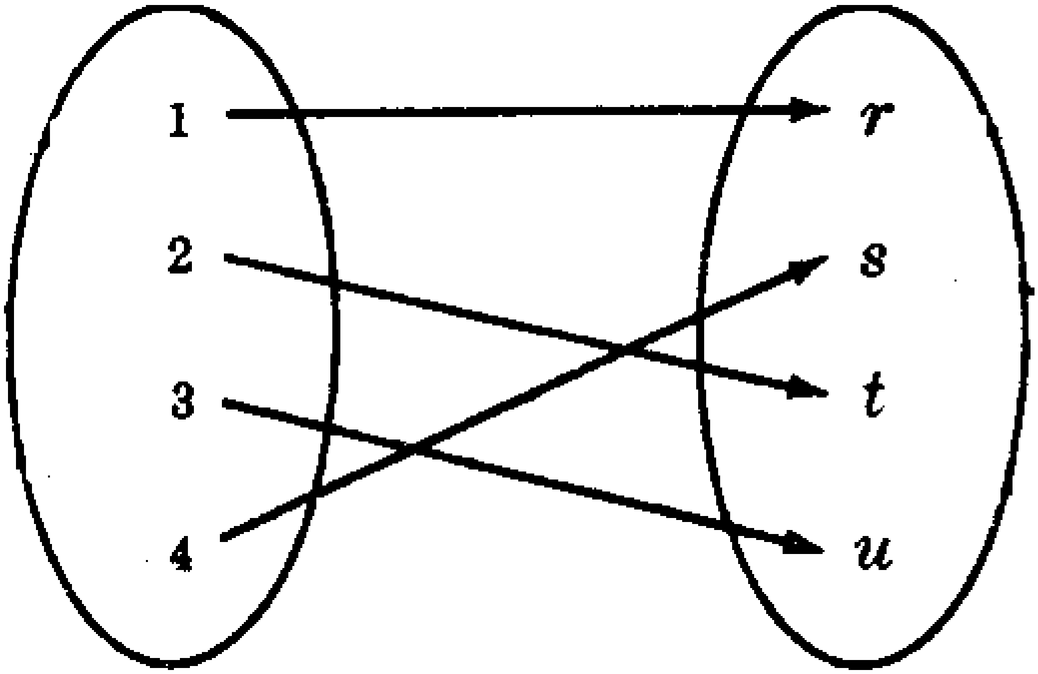
One-to-one ,onto and invertible functions :

1) One –to-one : a function $F:A\rightarrow B$ is said to be one-to-one if different elements in the domain (A) have distinct images.
   Or If $F(a) = F(a')$ $\Rightarrow$ $a = a'$
2) Onto : $F:A\rightarrow B$ is said to be an onto function if each element of B is the image of some element of A.
   $\forall\ b\in B \quad \exists \quad a \in A : F(a) = b$
3) Invertible (One-to-one correspondence)
   $F:A\rightarrow B$ is invertible if its inverse relation f^{-1} is a function $F:B\rightarrow A$
   $F:A\rightarrow B$ is invertible if and only if F is **both** one-to-one and onto
   $F^{-1} : \{(b,a)\ \forall\ (a,b) \in F\}$

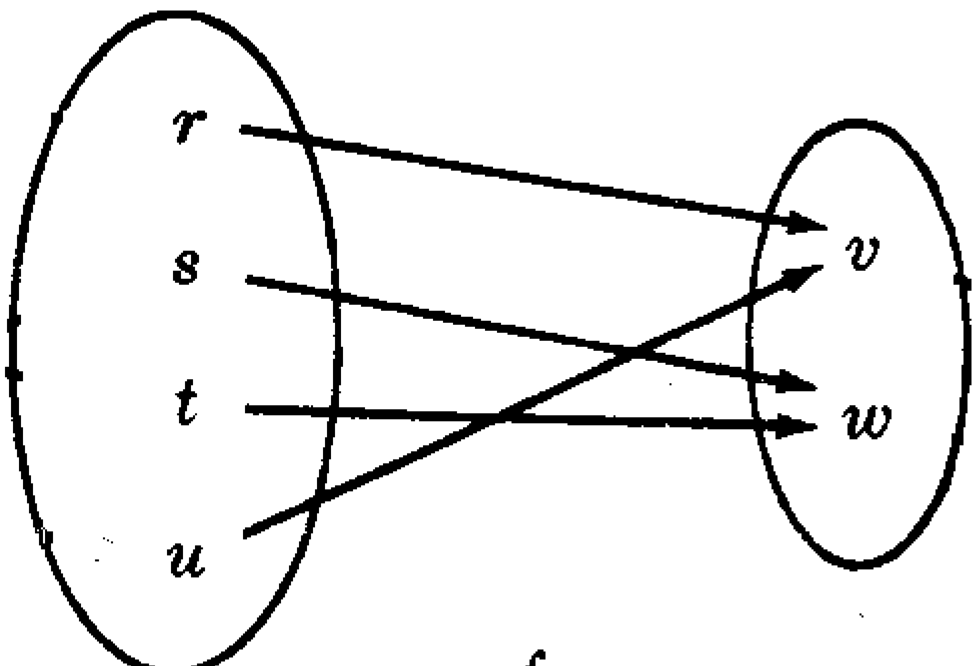


f_1

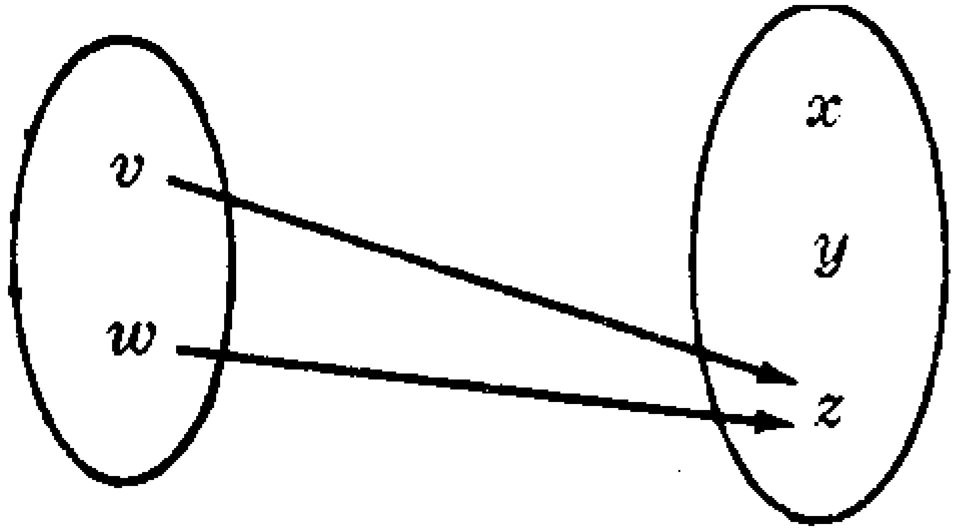
one to one but not onto ($3\in B$ but it is not the image under f1)



both one to one & onto
(or one to one correspondence between A and B)



f_3
not one to one & onto



not one to one & not onto

Graph of a function:

By a *real polynomial function*, we mean a function *f*: **R** → **R** of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the *ai* are real numbers. Since **R** is an infinite set, it would be impossible to plot each point of the graph. However, the graph of such a function can be approximated by first plotting some of its points and then drawing a smooth curve though these points. The table points are usually obtained from a table where various values are assigned to x and the corresponding value of f(x) computed.

Example 1 : let f:R→R and $f(x)= x^3$, find f(x)

$f(3) = 3^3 = 27$

$f(-2) = (-2)^3 = -8$

x	$f(x)$
−3	−27
- 2	8
1	−1
0	0
1	1
2	8
3	27