References

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SETS AND ELEMENTS

A set is a collection of objects called the elements or members of the set. The ordering of the elements is not important and repetition of elements is ignored, for example {1, 3, 1, 2, 2, 1} = {1, 2, 3}.
One usually uses capital letters, A,B,X, Y, . . . , to denote sets, and lowercase letters, a, b, x, y, . . ., to denote elements of sets.
Below you'll see just a sampling of items that could be considered as sets:

- The items in a store
- The English alphabet
- Even numbers

A set could have as many entries as you would like.
It could have one entry, 10 entries, 15 entries, infinite number of entries, or even have no entries at all!
For example, in the above list the English alphabet would have 26 entries, while the set of even numbers would have an infinite number of entries.

Each entry in a set is known as an **element or member**

Sets are written using curly brackets "{" and "}", with their elements listed in between.
For example the English alphabet could be written as
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}

and even numbers could be {0,2,4,6,8,10,...} (Note: the dots at the end indicating that the set goes on infinitely)

Principles:

- $\in$ belong to
- $\notin$ not belong to
- $\subseteq$ subset
- $\subset$ proper subset, For example, {a, b} is a proper subset of {a, b, c}, but {a, b, c} is not a proper subset of {a, b, c}.
- $\not\subset$ not subset

So we could replace the statement "a is belong to the alphabet" with
a $\in$ {alphabet} and replace the statement "3 is not belong to the set of even numbers" with 3 $\notin$ {Even numbers}

Now if we named our sets we could go even further.
Give the set consisting of the **alphabet** the name A, and give the set consisting of **even numbers** the name E.

We could now write
a $\in$ A
and
3 $\notin$ E.

Problem

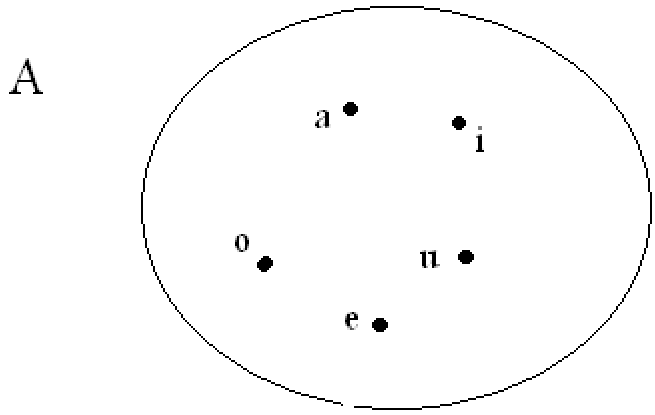
Let A = {2, 3, 4, 5} and C = {1, 2, 3, . . ., 8, 9}, Show that A is a proper subset of C.

Answer

Each element of A belongs to C so A $\subseteq$ C. On the other hand, 1 $\in$ C but 1 $\notin$ A. Hence A $\neq$ C. Therefore A is a proper subset of C.

There are three ways to specify a particular set:

1) By list its members separated by commas and contained in braces{ }, (if it is possible), for example, A= {a,e,i,o,u}
2) By state those properties which characterize the elements in the set, for example, A={x:x is a letter in the English alphabet, x is a vowel}
3) Venn diagram: (A graphical representation of sets).



Example (1)

A={x:x is a letter in the English alphabet, x is a vowel}
$e \in A$ (e is belong to A)
$f \notin A$ (f is not belong to A)

Example (2)
X is the set {1,3,5,7,9}
$3 \in X$ and $4 \notin X$

Example (3)
Let $E = \{x \mid x^2 - 3x + 2 = 0\}$ $\rightarrow (x-2)(x-1)=0$ $\rightarrow x=2$ & $x=1$
$E = \{2, 1\}$, and $2 \in E$

Universal set, empty set:

In any application of the theory of sets, the members of all sets under investigation usually belong to some fixed large set called the universal set. For example, in human population studies the universal set consists of all the people in the world. We will let the symbol U denotes the universal set.
The set with no elements is called the empty set or null set and is denoted by $\varnothing$ or {}

Subsets:

Every element in a set A is also an element of a set B, then A is called a subset of B. We also say that B contains A. This relationship is written:

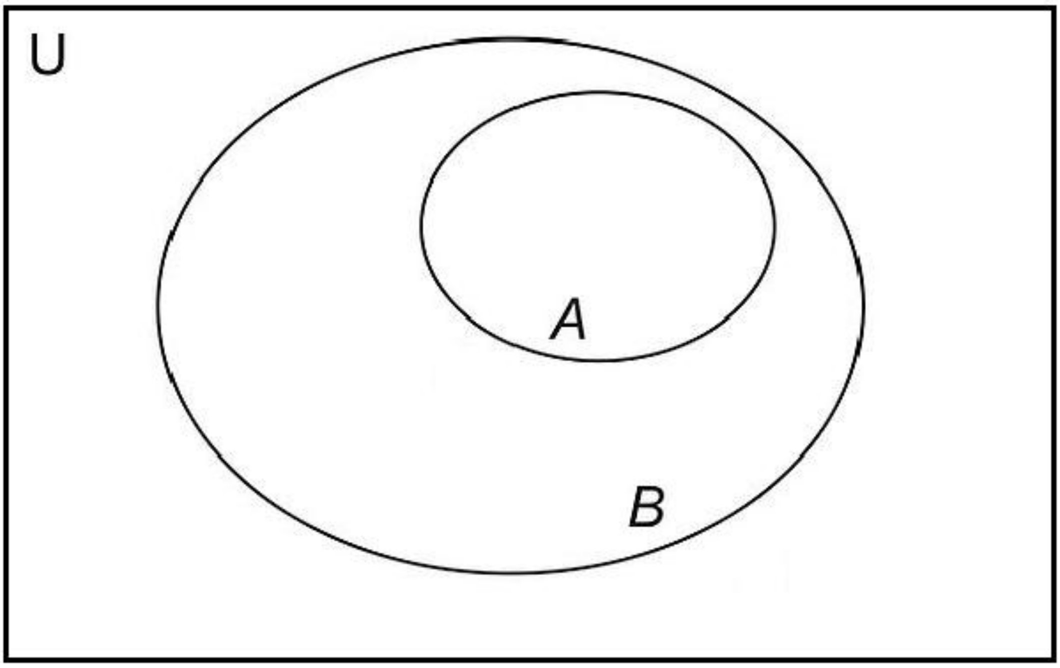
$A \subset B$ or $B \supset A$

If A is not a subset of B, i.e. if at least one element of A dose not belong to B, we write $A \not\subset B$.
Example 4:
Consider the sets.
A = {1,3,4,5,8,9} B = {1,2,3,5,7} and C ={1,5}
Then $C \subset A$ and $C \subset B$ since 1 and 5, the element of C, are also members of A and B. But $B \not\subset A$ since some of its elements, e.g. 2 and 7, do not belong to A. Furthermore, since the elements of A,B and C must also belong to the universal set U, we have that U must at least the set {1,2,3,4,5,7,8,9}.

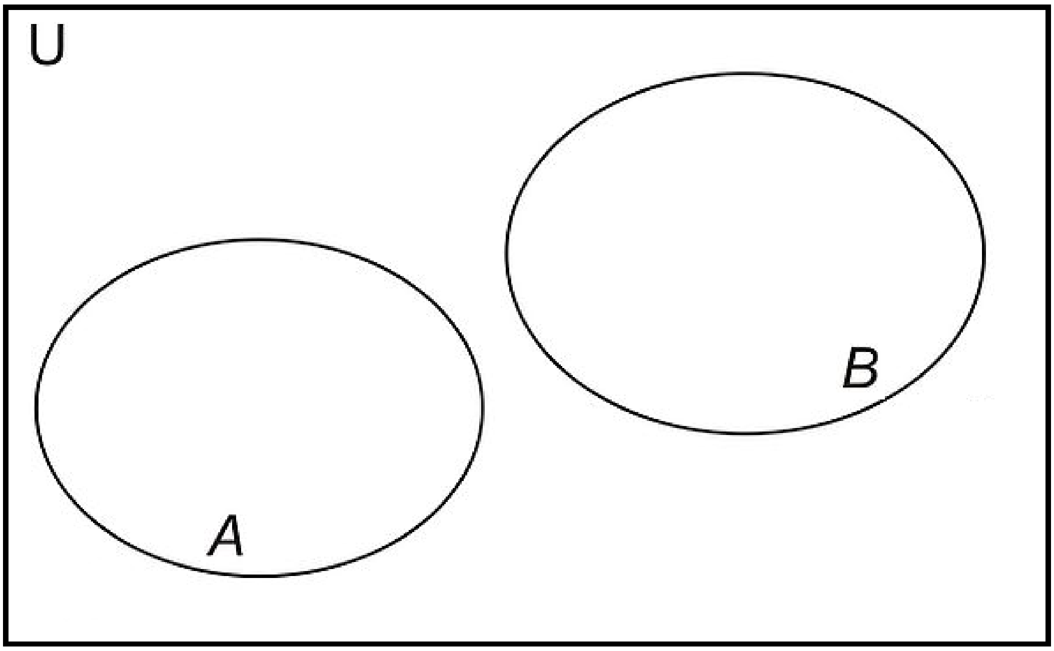
$A \subset B$: $\{\forall\ x \in A \quad \Rightarrow \quad x \in B$
$A \not\subset B$: $\{\exists\ x \in A \quad \text{but} \quad x \notin B$

$\forall$: For all لكل
$\exists$: There exists يوجد على الاقل

The notion of subsets is graphically illustrated below:



A is entirely within B so $A \subset B$.



A and *B* are disjoint or ($A \cap B = \varnothing$) so we could write $A \not\subset B$ and $B \not\subset A$.

Set of numbers:

Several sets are used so often, they are given special symbols.

N = the set of *natural numbers* or positive integers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Z = the set of all integers: . . . ,−2,−1, 0, 1, 2, . . .

$$\mathbb{Z} = \mathbb{N} \cup \{\ldots, -2, -1\}$$

Q = the set of rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\ldots, -1/3, -1/2, 1/2, 1/3, \ldots, 2/3, 2/5, \ldots\}$$

Where $Q = \{ a/b : a , b \in Z, b \neq 0\}$

R = the set of real numbers

$$\mathbb{R} = \mathbb{Q} \cup \{\ldots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \ldots\}$$

C = the set of complex numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2} + \pi i, \ldots\}$$

Where $C = \{ x + iy ; x , y \in R; i = \sqrt{-1}\}$

Observe that $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$.

Theorem 1:

For any set A, B, C: