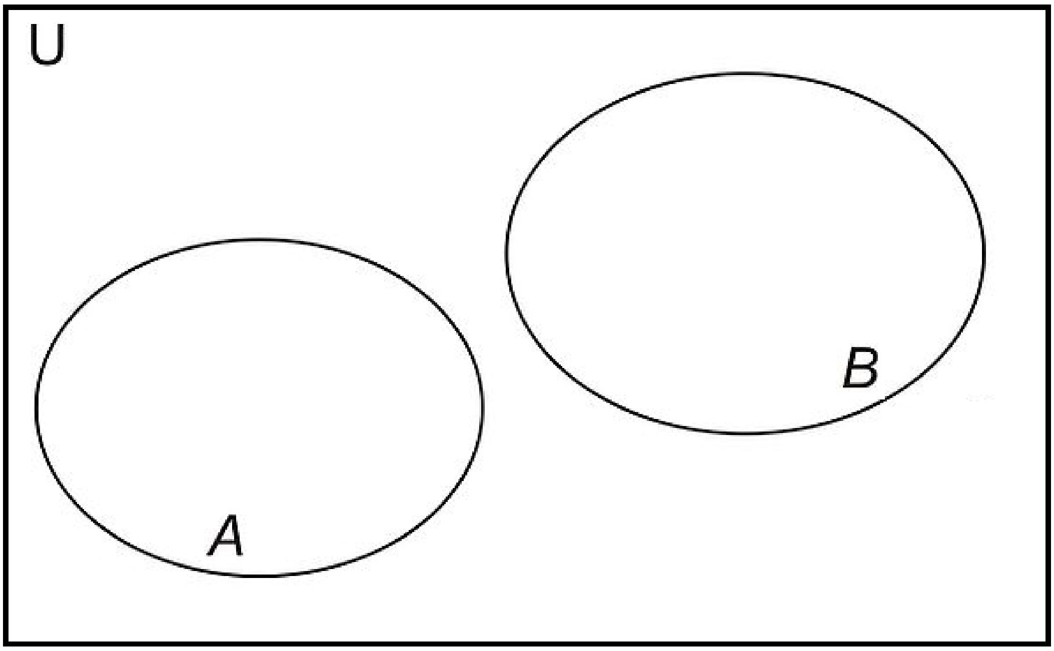


A and *B* are disjoint or ($A \cap B = \varnothing$) so we could write $A \not\subset B$ and $B \not\subset A$.

Set of numbers:

Several sets are used so often, they are given special symbols.

N = the set of *natural numbers* or positive integers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

Z = the set of all integers: . . . ,−2,−1, 0, 1, 2, . . .

$$\mathbb{Z} = \mathbb{N} \cup \{\ldots, -2, -1\}$$

Q = the set of rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\ldots, -1/3, -1/2, 1/2, 1/3, \ldots, 2/3, 2/5, \ldots\}$$

Where $Q = \{ a/b : a , b \in Z, b \neq 0\}$

R = the set of real numbers

$$\mathbb{R} = \mathbb{Q} \cup \{\ldots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \ldots\}$$

C = the set of complex numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2} + \pi i, \ldots\}$$

Where $C = \{ x + iy ; x , y \in R; i = \sqrt{-1}\}$

Observe that $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$.

Theorem 1:

For any set A, B, C:

1- $\emptyset \subset A \subset U$.

2- $A \subset A$.

3- If $A \subset B$ and $B \subset C$, then $A \subset C$.

4- $A = B$ if and only if $A \subset B$ and $B \subset A$.

Set operations:

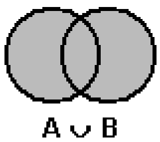
1) UNION:

The *union* of two sets *A* and *B*, denoted by $A \cup B$, is the set of all elements which belong to *A* or to *B*;

$$A \cup B = \{ x : x \in A \text{ or } x \in B\}$$

Example

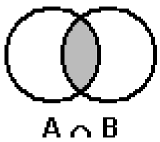
A={1,2,3,4,5} B={5,7,9,11,13}
$A \cup B$ = {1,2,3,4,5,7,9,11,13}



2) **INTERSECTION**

The *intersection* of two sets *A* and *B*, denoted by $A \cap B$, is the set of elements which belong to both *A* and *B*;

$$A \cap B = \{ x : x \in A \text{ and } x \in B\}.$$



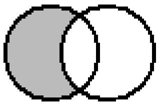
Example 1

A={1,3,5,7,9} B={2,3,4,5,6}
The elements they have in common are 3 and 5
$A \cap B$ = {3,5}

Example 2
A={The English alphabet} B={vowels}
So $A \cap B$ = {vowels}

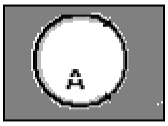
Example 3
A={1,2,3,4,5}   B={6,7,8,9,10}
In this case A and B have nothing in common. $A \cap B = \varnothing$

3) THE DIFFERENCE:
The difference of two sets A\B or A-B is those elements which belong to A but which do not belong to B.
$A \setminus B = \{x : x \in A, x \notin B\}$



A − B

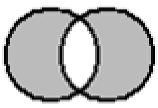
4) COMPLEMENT OF SET: Complement of set A^c or A' , is the set of elements which belong to U but which do not belong to A .
$A^c = \{x : x \in U, x \notin A\}$



U

Example: let   A={1,2,3}
B={3,4}
U={1,2,3,4,5,6}
Find:
$A \cup B = \{1, 2, 3, 4\}$
$A \cap B = \{3\}$
$A - B = \{1, 2\}$
$A^c = \{4, 5, 6\}$

5) Symmetric difference of sets
The symmetric difference of sets A and B, denoted by $A \oplus B$, consists of those elements which belong to A or B but not to both. That is,
$A \oplus B = (A \cup B)\setminus(A \cap B)$ or $A \oplus B = (A\setminus B) \cup (B\setminus A)$



$A \oplus B$

Example: Suppose U = N = {1, 2, 3, . . .} is the universal set.
Let A = {1, 2, 3, 4}, B= {3, 4, 5, 6, 7}, C= {2, 3, 8, 9}, E= {2, 4, 6, 8,. . .}
Then:
A^c = {5, 6, 7, . . .}, B^c = {1, 2, 8, 9, 10, . . .}, C^c ={1,4,5,6,7,10,…} E^c ={1, 3, 5, 7, ...}
A\B = {1, 2}, A\C = {1, 4}, B\C = {4, 5, 6, 7}, A\E = {1, 3},
B\A = {5, 6, 7}, C\A = {8, 9}, C\B = {2, 8, 9}, E\A = {6, 8, 10, 12, . . .}.
Furthermore:
$A \oplus B = (A\setminus B) \cup (B\setminus A) = \{1, 2, 5, 6, 7\}$, $B \oplus C = \{2, 4, 5, 6, 7, 8, 9\}$,
$A \oplus C = (A\setminus C) \cup (B\setminus C) = \{1, 4, 8, 9\}$, $A \oplus E = \{1, 3, 6, 8, 10, . . .\}$.

Theorem 2 :
$A \subset B$, $A \cap B = A$, $A \cup B = B$ are equivalent

Theorem 3: (Algebra of sets)
Sets under the above operations satisfy various laws or identities which are listed below:

1- $A \cup A = A$
$A \cap A = A$

2- $(A \cup B) \cup C = A \cup (B \cup C)$ Associative laws
$(A \cap B) \cap C = A \cap (B \cap C)$

3- $A \cup B = B \cup A$ Commutativity
$A \cap B = B \cap A$

4- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5- $A \cup \varnothing = A$ Identity laws
$A \cap U = A$

6- $A \cup U = U$ Identity laws
$A \cap \varnothing = \varnothing$

7- $(A^c)^c = A$ Double complements

8- $A \cup A^c = U$ Complement intersections and unions

$A \cap A^c = \varnothing$

9- $U^c = \varnothing$
$\varnothing^c = U$

10- $(A \cup B)^c = A^c \cap B^c$ De Morgan's laws
$(A \cap B)^c = A^c \cup B^c$

We discuss two methods of proving equations involving set operations. The first is to break down what it means for an object x to be an element of each side, and the second is to use Venn diagrams.
For example, consider the first of De Morgan's laws:

$$(A \cup B)^c = A^c \cap B^c$$

We must prove: 1) $(A \cup B)^c \subset A^c \cap B^c$
2) $A^c \cap B^c \subset (A \cup B)^c$

We first show that $(A \cup B)^c \subset A^c \cap B^c$

Let's pick an element at random $x \in (A \cup B)^c$. We don't know anything about x, it could be a number, a function. All we do know about x, is that:

$x \in (A \cup B)^c$, so
$x \notin A \cup B$

because that's what complement means. Therefore

$x \notin A$ and $x \notin B$,

by pulling apart the union. Applying complements again we get

$x \in A^c$ and $x \in B^c$

Finally, if something is in 2 sets, it must be in their intersection, so

$x \in A^c \cap B^c$

So, any element we pick at random from: $(A \cup B)^c$ is definitely in, $A^c \cap B^c$, so by definition

$(A \cup B)^c \subset A^c \cap B^c$

Next we show that $(A^c \cap B^c) \subset (A \cup B)^c$.

This follows a very similar way. Firstly, we pick an element at random from the first set, $x \in (A^c \cap B^c)$

Using what we know about intersections, that means

$x \in A^c$ and $x \in B^c$

Now, using what we know about complements,

$x \notin A$ and $x \notin B$.

If something is in neither A nor B, it can't be in their union, so

$x \notin A \cup B$,

And finally

$\therefore \quad x \in (A \cup B)^c$

We have prove that every element of $(A \cup B)^c$ belongs to $A^c \cap B^c$ and that every element of $A^c \cap B^c$ belongs to $A \cup B)^c$. Together, these inclusions prove that the sets have the same elements, i.e. that $(A \cup B)^c = A^c \cap B^c$