Let's pick an element at random $x \in (A \cup B)^c$. We don't know anything about x, it could be a number, a function. All we do know about x, is that:

$x \in (A \cup B)^c$, so
$x \notin A \cup B$

because that's what complement means. Therefore

$x \notin A$ and $x \notin B$,

by pulling apart the union. Applying complements again we get

$x \in A^c$ and $x \in B^c$

Finally, if something is in 2 sets, it must be in their intersection, so

$x \in A^c \cap B^c$

So, any element we pick at random from: $(A \cup B)^c$ is definitely in, $A^c \cap B^c$, so by definition

$(A \cup B)^c \subset A^c \cap B^c$

Next we show that $(A^c \cap B^c) \subset (A \cup B)^c$.

This follows a very similar way. Firstly, we pick an element at random from the first set, $x \in (A^c \cap B^c)$

Using what we know about intersections, that means

$x \in A^c$ and $x \in B^c$

Now, using what we know about complements,

$x \notin A$ and $x \notin B$.

If something is in neither A nor B, it can't be in their union, so

$x \notin A \cup B$,

And finally

$\therefore \quad x \in (A \cup B)^c$

We have prove that every element of $(A \cup B)^c$ belongs to $A^c \cap B^c$ and that every element of $A^c \cap B^c$ belongs to $A \cup B)^c$. Together, these inclusions prove that the sets have the same elements, i.e. that $(A \cup B)^c = A^c \cap B^c$

Power set

The power set of some set S, denoted P(S), is the set of all subsets of S (including S itself and the empty set)

Example 1: Let A = { 1,2 3}

Power set of set A = P(A) = [{1},{2},{3},{1,2},{1,3},{2,3},{},A]

Example 2: P({0,1})={{},{0},{1},{0,1}}

Classes of sets: Collection of subset of a set with some properties

Example: Suppose A = { 1,2 3} , let X be the class of subsets of A which contain exactly two elements of A. Then

class X = [{1,2},{1,3},{2,3}]

Cardinality

The cardinality of a set S, denoted **|S|**, is simply the number of elements a set has. So |{a,b,c,d**}| =** 4, and so on. The cardinality of a set need not be finite: some sets have infinite cardinality.

The cardinality of the power set

Theorem: If |A| = n then |P(A)| = 2^n (Every set with n elements has 2^n subsets)

Problem set

write the answers to the following questions.

1. |{1,2,3,4,5,6,7,8,9,0}|

2. |P({1,2,3})|

3. P({0,1,2})

4. P({1})

Answers

1. 10
2. 2^3=8
3. {{},{0},{1},{2},{0,1},{0,1,2},{0,2},{1,2}}
4. {{},{1}}

The Cartesian product

The Cartesian Product of two sets is the set of all tuples made from elements of two sets. We write the Cartesian Product of two sets A and B as A × B. It is defined as:

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

It may be clearer to understand from examples;

$$\{0,1\} \times \{2,3\} = \{(0,2),(0,3),(1,2),(1,3)\}$$
$$\{a,b\} \times \{c,d\} = \{(a,c),(a,d),(b,c),(b,d)\}$$
$$\{0,1,2\} \times \{4,6\} = \{(0,4),(0,6),(1,4),(1,6),(2,4),(2,6)\}$$

Example: If A = {1, 2, 3} and B = {x, y} then
A . B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}
B . A = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

$$|A \times B| = |A||B|$$

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*, A = { 0, 1 } and B = { 2, 3 }, the grid is

<table>
<tr><th colspan="2" rowspan="2">×</th><th colspan="2">A</th></tr>
<tr><th>0</th><th>1</th></tr>
<tr><th rowspan="2">B</th><th>2</th><td>(0,2)</td><td>(1,2)</td></tr>
<tr><th>3</th><td>(0,3)</td><td>(1,3)</td></tr>
</table>

Problem set

Answer the following questions:

1. {2,3,4}×{1,3,4}

2. {0,1}×{0,1}

3. |{1,2,3}×{0}|
4. |{1,1}×{2,3,4}|

Answers

1. {(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)}
2. {(0,0),(0,1),(1,0),(1,1)}
3. 3
4. 6