LECTUER 2 :Homogeneous Equations

Homogeneous Function

f (x,y) is called homogenous of degree *n* if : $f(\lambda x,\lambda y)=\lambda^n f(x,y)$

Examples:

$f(x,y)=x^4-x^3y$ → homogeneous of degree 4

$$f(\lambda x,\lambda y)=(\lambda x)^4-(\lambda x)^3(\lambda y)$$
$$=\lambda^4\left(x^4-x^3y\right)=\lambda^4 f(x,y)$$

$f(x,y)=x^2+\sin x\cos y$ → non-homogeneous

$$f(\lambda x,\lambda y)=(\lambda x)^2+\sin(\lambda x)\cos(\lambda y)$$
$$=\lambda^2x^2+\sin(\lambda x)\cos(\lambda y)$$

The homogeneous differential equation

$$\frac{dy}{dx}=f(x,y)$$

having the property $f(tx, ty) = f(x, y)$ (see Chapter One) can be transformed into a separable equation by making the substitution

$$y = xv \tag{2.6}$$

along with its corresponding derivative

$$\frac{dy}{dx}=v+x\frac{dv}{dx} \tag{2.7}$$

The resulting equation in the variables v and x is solved as a separable differential equation; the required solution to Equation 2.5 is obtained by back substitution.

Alternatively, the solution to 2.5 can be obtained by rewriting the differential equation as

$$\frac{dx}{dy}=\frac{1}{f(x,y)} \tag{2.8}$$

and then substituting

$$x = yu \tag{2.9}$$

and the corresponding derivative

$$\frac{dx}{dy} = u + y\frac{du}{dy}$$

into Equation 2.8. After simplifying, the resulting differential equation will be one with variables (this time, *u* and *y*) separable.

Ordinarily, it is immaterial which method of solution is used. Occasionally, however, one of the substitutions 2.6 or 2.9 is definitely superior to the other one. In such cases, the better substitution is usually apparent from the form of the differential equation itself.

Example: Find G.S. for D. E. $y' = \frac{y+x}{x}$

This differential equation is not separable. Instead it has the form $y' = f(x, y)$, with

$$f(x, y) = \frac{y+x}{x}$$

where

$$f(tx, ty) = \frac{ty+tx}{tx} = \frac{t(y+x)}{tx} = \frac{y+x}{x} = f(x, y)$$

so it is homogeneous. Substituting Equations 2.6 and 2.7 into the equation, we obtain

$$v + x\frac{dv}{dx} = \frac{xv+x}{x}$$

which can be algebraically simplified to

$$x\frac{dv}{dx} = 1 \quad \text{or} \quad \frac{1}{x}dx - dv = 0 \qquad (2.26)$$

This last equation is separable; its solution is

$$\int \frac{1}{x}dx - \int dv = c$$

which, when evaluated, yields $v = \ln|x| - c$, or

$$v = \ln|kx|$$

where we have set $c = -\ln|k|$ and have noted that $\ln|x| + \ln|k| = \ln|xk|$.

Finally, substituting $v = y / x$ back into 2.26, we obtain the solution to the given differential equation as $y = x \ln | kx |$.

LECTUER 3: NON HOMOGENOUES EQUTION:

$$\frac{dy}{dx} = \frac{a_1 x_1 + b_1 y_1 - c_1}{a_2 x_2 + b_2 y_2 - c_2} \quad \text{------ 1 non Homogenous}$$

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$$\frac{dy}{dx} \quad \frac{a_1 x_1 + b_1 y_1}{a_2 x_2 + b_2 y_2} \qquad \text{Homogenous}$$

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$$\begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array} \qquad a_1 b_2 \neq a_2 b_1$$

$$\left.\begin{aligned} Y &= \nu + k \\ X &= u + h \end{aligned}\right] \text{----2}$$

بتعويض المعادلة (2) في معادلة (1)

$$\frac{dy}{dx} = \frac{a_1 u + b_1 \nu + a_1 h + b_1 k + c_1}{a_2 u + b_2 \nu + a_2 h + b_2 k + c_2}$$

4- $\nu = z u$

$$\frac{d\nu}{du} = z + u \frac{dz}{du}$$ الاولى * مشتقة الثانية + الثانية * مشتقة الاولى

5- sprat ion   6- direct integration

Ex :- Find the G.S for following D.E

$$\frac{dy}{dx} = \frac{x+y-3}{x-y-1} \quad \text{-----} \ *$$ معاملة غير متجانسة والمستقيمان متقاطعان لان اشارة الثوابت متشابهة ( ) , ( )

$$x + \cancel{y} - 3 = 0 \ \text{-----} \ 1$$
بالجمع $$x - \cancel{y} - 1 = 0 \ \text{------} \ 2$$
$$2x - 4 = 0$$

نعوض في معادلة (1) $2x = 4 \Longrightarrow x = 2$

$2 + y - 3 = 0 \Longrightarrow y = 1$ $(h , k) = (2 , 1)$

$$\left.\begin{aligned} Y = \nu + k &\Longrightarrow y = \nu + 1 \\ X = u + h &\Longrightarrow x = u + 2 \end{aligned}\right] \text{-----} \ 3$$

نعوض معادلة (3) في معادلة (*)

$$\frac{dy}{dx} = \frac{u+2+\nu+1-3}{u+2-\nu-1-1} \qquad \frac{dy}{dx} = \frac{u+\nu}{u-\nu}$$

$$\left(\nu = z\,u \ , \quad \frac{d\nu}{du} = z + \frac{u\,dz}{du} \right)$$

$$\frac{dy}{dx} = \frac{u+z\,u}{u-z\,u} \qquad :- \ \frac{dy}{dx} = \frac{d\nu}{du}$$

$$\frac{u + z u}{u - z u} = z + \frac{u d z}{du} \qquad \frac{\cancel{u}(1 + z)}{\cancel{u}(1 - z)} = z + \frac{u d z}{du}$$

$$\frac{(1 + z)}{(1 - z)} = z + \frac{u d z}{du}$$

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$$\frac{1 + \cancel{z} - \cancel{z} + z^2}{1 - z} = \frac{u d z}{du}$$

$$\frac{1 + z^2}{1 - z} = \frac{u d z}{du} \qquad u d z = \frac{1 + z^2}{1 - z}\, du$$

$$u (1 - z)\, dz = 1 + z^2\, du$$

$$\int \frac{1 - z}{1 + z^2}\, d z = \int \frac{du}{u} \Rightarrow \int \frac{1}{1 + z^2}\, d z - \int \frac{z}{1 + z^2}\, d z = \int \frac{du}{u}$$

$$\tan^{-1} z - \frac{1}{2} \ln \left|1 + z^2\right| = \ln \left|u\right| + c \text{ --- } 3$$

$$:- \nu = z u \Rightarrow z = \frac{\nu}{U} \text{ ---- } 4$$

نعوض معادلة (4) في معادلة (3)

$$\tan^{-1} \frac{\nu}{u} - \frac{1}{2} \ln \left|1 + \left(\frac{\nu}{u}\right)^2\right| = \ln \left|u\right| + c \text{ ---- } 5$$

$$\nu = y - 1 \text{ --- } 6 \qquad u = x - 2 \text{ ---- } 7$$

نعوض معادلة (6 , 7) في معادلة (5)

$$\tan^{-1} \frac{y - 1}{x - 2} - \frac{1}{2} \ln \left|1 + \left(\frac{y - 1}{x - 2}\right)^2\right| = \ln \left|x - 2\right| + c$$