[redacted]ph of f (x) = |x|.

(ii) Discusss the continuity of f (x) at x = 0.

Solution : We know that for $x \geq 0, |x| = x$ and for $x < 0, |x| = -x$. Hence f (x) can be written as.

$$f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

(i) The graph of the function is given in Fig 20.9

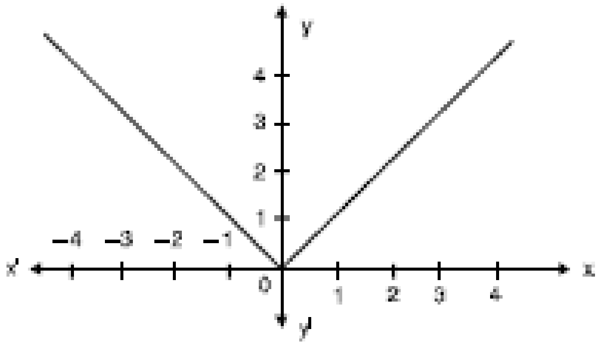

Fig. 20.9

(ii) Left hand limit $= \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0}(-x) = 0$

Right hand limit $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0} x = 0$

Thus, $\lim_{x \to 0} f(x) = 0$

Also, f (0) = 0

$\therefore$ $\lim_{x \to 0} f(x) = f(0)$

Hence the function f (x) is continuous at x =0.

Examine the continuity of $f(x) = |x - b|$ at $x = b$.

[redacted]e $f(x) = |x - b|$. This function can be written as

$$f(x) = \begin{cases} -(x - b), & x < b \\ (x - b), & x \geq b \end{cases}$$

Left hand limit $= \lim_{x \to b^-} f(x) = \lim_{h \to 0} f(b - h)$

$$= \lim_{h \to 0}[-(b - h - b)]$$

$$= \lim_{h \to 0} h = 0 \qquad \text{.....(i)}$$

Right hand limit $= \lim_{x \to b^+} f(x) = \lim_{h \to 0} f(b+h)$

$$= \lim_{h \to 0}[(b+h) - b]$$

$$= \lim_{h \to 0} h = 0 \qquad \text{.....(ii)}$$

Also, $f(b) = b - b = 0$ .....(iii)

From (i), (ii) and (iii), $\lim_{x \to b} f(x) = f(b)$

Thus, f(x) is continuous at x = b.

$$\text{If } f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

[redacted] continuous at x = 0 or not.

Solution : Here $f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

Left hand limit $= \lim_{x \to 0^-} \dfrac{\sin 2x}{x}$

$$= \lim_{h \to 0} \frac{\sin 2(0-h)}{0-h}$$

$$= \lim_{h \to 0} \frac{-\sin 2h}{-h}$$

$$= \lim_{h \to 0}\left(\frac{\sin 2h}{2h} \times \frac{2}{1}\right)$$

$$= 1 \times 2 = 2 \qquad \text{.....(i)}$$

Right hand limit $= \lim_{x \to 0^+} \dfrac{\sin 2x}{x}$

$$= \lim_{h \to 0} \frac{\sin 2(0+h)}{0+h}$$

$$= \lim_{h \to 0} \frac{\sin 2h}{2h} \times \frac{2}{1}$$

$$= 1 \times 2 = 2 \qquad \text{(ii)}$$

Also $\qquad f(0) = 2 \qquad$ (Given) $\qquad$ (iii)

From (i) to (iii),

$$\lim_{x \to 0} f(x) = 2 = f(0)$$

Hence f(x) is continuous at x = 0.

[redacted] If $f(x) = \dfrac{x^2 - 1}{x - 1}$ for $x \neq 1$ and $f(x) = 2$ when $x = 1$, show that the function $f(x)$ is continuous at $x = 1$.

Solution : Here $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & , x \neq 1 \\ 2 & , x = 1 \end{cases}$

Left hand limit $\lim\limits_{x \to 1^-} f(x)$

$$= \lim_{h \to 0} f(1 - h)$$

$$= \lim_{h \to 0} \frac{(1 - h)^2 - 1}{(1 - h) - 1}$$

$$= \lim_{h \to 0} \frac{1 - 2h + h^2 - 1}{-h}$$

$$= \lim_{h \to 0} \frac{h(h - 2)}{-h}$$

$$= \lim_{h \to 0} -(h - 2)$$

$$= 2$$

Right hand limit $= \lim\limits_{x \to 1^+} f(x)$ $\qquad$ .....(i)

$$= \lim_{h \to 0} f(1 + h)$$

$$= \lim_{h \to 0} \frac{(1 + h)^2 - 1}{(1 + h) - 1}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{h(h + 2)}{h}$$

$$= \lim_{h \to 0} (h + 2)$$

$$= 2 \qquad .....(ii)$$

Also $\qquad f(1) = 2$ (Given) $\qquad$ .....(iii)

$\therefore$ From (i) to (iii),

$$\lim_{x \to 1} f(x) = f(1)$$

Thus, f (x) is continuous at x = 1.

[redacted] Find whether f (x) is continuous at x = 0 or not, where

$$f(x)=\begin{cases}\dfrac{x}{x}, & x\neq 0\\ 2, & x=0\end{cases}$$

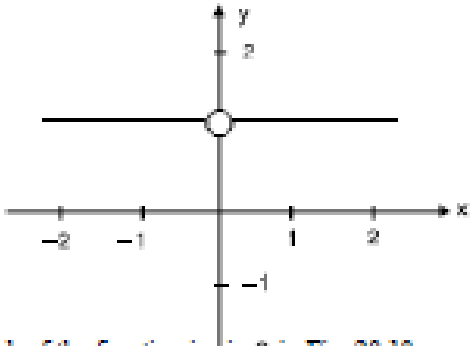

Fig. 20.10

Solution : $\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\dfrac{x}{x}$

$$=\lim_{x\to 0} 1 = 1$$

and $f(0) = 2$

$\therefore \quad \lim\limits_{x\to 0} f(x)\neq f(0)$

Hence f (x) is not continuous at x = 0. The graph of the function is given in Fig. 20.10.

Clearly, the point (0,1) does not lie on the graph. Therefore, the function is discontinuous at x =0

Signum Function : The function f (x)=sgn(x) (read as signum x) is defined as

$$f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$$

Find the left hand limit and right hand limit of the function from its graph given below:

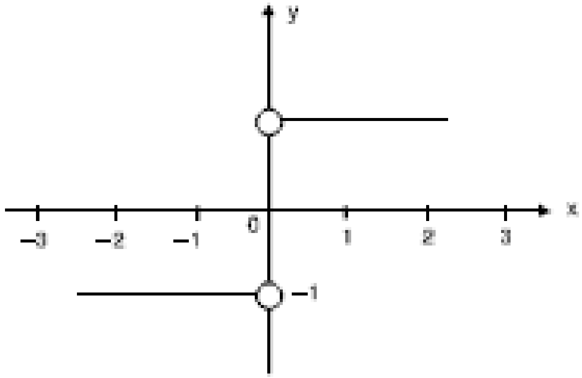

Fig. 20.11

From the graph, we see that as $x\to 0^+$, $f(x)\to 1$ and as $(x)\to 0^-$, $f(x)\to -1$

Hence, $\lim\limits_{x\to 0^+} f(x)=1$, $\lim\limits_{x\to 0^-} f(x)=-1$

As these limits are not equal, $\lim\limits_{x\to 0} f(x)$ does not exist. Hence f (x) is discontinuous at x =0.