LET US SUM UP

- If a function f(x) approaches l when x approches a, we say that l is the limit of f (x). Symbolically, it is written as

$$\lim_{x \to a} f(x) = \ell$$

- If $\lim_{x \to a} f(x) = \ell$ and $\lim_{x \to a} g(x) = m$, then

(i) $\lim_{x \to a} k f(x) = k \lim_{x \to a} f(x) = k\ell$

(ii) $\lim_{x \to a} \left[f(x) \pm g(x)\right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = \ell \pm m$

(iii) $\lim_{x \to a} \left[f(x) g(x)\right] = \lim_{x \to a} f(x) \lim_{x \to a} g(x) = \ell m$

(iv) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{\ell}{m}$, provided $\lim_{x \to a} g(x) \neq 0$

- **LIMIT OF IMPORTANT FUNCTIONS**

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \sin x = 0$

(iii) $\lim_{x \to 0} \cos x = 1$

(iv) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(v) $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

(vi) $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

(vii) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

SUPPORTIVE WEB SITES

- *http://www.wikipedia.org*
- *http://mathworld.wolfram.com*

TERMINAL EXERCISE

Evaluate the following limits :

1. $\lim\limits_{x\to 1} 5$

2. $\lim\limits_{x\to 0} \sqrt{2}$

3. $\lim\limits_{x\to 1} \dfrac{4x^5+9x+7}{3x^6+x^3+1}$

4. $\lim\limits_{x\to 2} \dfrac{x^2+2x}{x^3+x^2-2x}$

5. $\lim\limits_{x\to 0} \dfrac{(x+k)^4-x^4}{k(k+2x)}$

6. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

7. $\lim\limits_{x\to -1} \left[\dfrac{1}{x+1}+\dfrac{2}{x^2-1}\right]$

8. $\lim\limits_{x\to 1} \dfrac{(2x-3)\sqrt{x}-1}{(2x+3)(x-1)}$

9. $\lim\limits_{x\to 2} \dfrac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$

10. $\lim\limits_{x\to 1} \left[\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right]$

11. $\lim\limits_{x\to \pi} \dfrac{\sin x}{\pi-x}$

12. $\lim\limits_{x\to a} \dfrac{x^2-(a+1)x+a^2}{x^2-a^2}$

Find the left hand and right hand limits of the following functions :

13. $f(x)=\begin{cases} -2x+3 & \text{if } x\le 1 \\ 3x-5 & \text{if } x>1 \end{cases}$ as $x\to 1$

14. $f(x)=\dfrac{x^2-1}{|x+1|}$ as $x\to 1$

Evaluate the following limits :

15. $\lim\limits_{x\to 1^-} \dfrac{|x+1|}{x+1}$

16. $\lim\limits_{x\to 2^+} \dfrac{|x-2|}{x-2}$

17. $\lim\limits_{x\to 2^-} \dfrac{x-2}{|x-2|}$

18. If $f(x)=\dfrac{(x+2)^2-4}{x}$, prove that $\lim\limits_{x\to 0} f(x)=4$ though $f(0)$ is not defined.

19. Find k so that $\lim\limits_{x\to 2} f(x)$ may exist where $f(x)=\begin{cases} 5x+2, & x\le 2 \\ 2x+k, & x>2 \end{cases}$

20. Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 7x}{2x}$

21. Evauate $\lim\limits_{x\to 0} \left[\dfrac{e^x+e^{-x}-2}{x^2}\right]$

22. Evaluate $\lim\limits_{x \to 0} \dfrac{1-\cos 3x}{x^2}$ 23. Find the value of $\lim\limits_{x \to 0} \dfrac{\sin 2x + 3x}{2x + \sin 3x}$

24. Evaluate $\lim\limits_{x \to 1} (1-x)\tan\dfrac{\pi x}{2}$ 25. Evaluate $\lim\limits_{\theta \to 0} \dfrac{\sin 5\theta}{\tan 8\theta}$

Examine the continuity of the following :

26. $f(x)\begin{cases} 1+3x \text{ if } x > -1 \\ 2 \text{ if } x \le -1 \end{cases}$

at $x = -1$

27. $f(x) = \begin{cases} \dfrac{1}{x} - x, 0 < x < \dfrac{1}{2} \\ \dfrac{1}{2}, x = \dfrac{1}{2} \\ \dfrac{3}{2} - x, \dfrac{1}{2} < x < 1 \end{cases}$

at $x = \dfrac{1}{2}$

28. For what value of k, will the function

$$f(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} \text{ if } x \ne 4 \\ k \text{ if } x = 4 \end{cases}$$

be continuous at $x = 4$?

29. Determine the points of discontinuity, if any, of the following functions :

(a) $\dfrac{x^2 + 3}{x^2 + x + 1}$ (b) $\dfrac{4x^2 + 3x + 5}{x^2 - 2x + 1}$

(b) $\dfrac{x^2 + x + 1}{x^2 - 3x + 1}$ (d) $f(x) = \begin{cases} x^4 - 16, & x \ne 2 \\ 16, & x = 2 \end{cases}$

30. Show that the function $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos, & x \ne 0 \\ 2, & x = 0 \end{cases}$ is continuous at $x = 0$

31. Determine the value of 'a', so that the function f (x) defined by

$$f(x) = \begin{cases} \dfrac{a\cos x}{\pi - 2x}, & x \ne \dfrac{\pi}{2} \\ 5, & x = \dfrac{\pi}{2} \end{cases}$$ is continuous.