Chapter
8

Flood Routing

8.1 INTRODUCTION

The flood hydrograph discussed in Chap. 6 is in fact a wave. The stage and discharge hydrographs represent the passage of waves of the river depth and discharge respectively. As this wave moves down the river, the shape of the wave gets modified due to various factors, such as channel storage, resistance, lateral addition or withdrawal of flows, etc. When a flood wave passes through a reservoir, its peak is attenuated and the time base is enlarged due to the effect of storage. Flood waves passing down a river have their peaks attenuated due to friction if there is no lateral inflow. The addition of lateral inflows can cause a reduction of attenuation or even amplification of a flood wave. The study of the basic aspects of these changes in a flood wave passing through a channel system forms the subject matter of this chapter.

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir design and spillway design invariably include flood routing. In these applications two broad categories of routing can be recognised. These are:

1. Reservoir routing, and
2. Channel routing.

In *Reservoir routing* the effect of a flood wave entering a reservoir is studied. Knowing the volume-elevation characteristic of the reservoir and the outflow-elevation relationship for the spillways and other outlet structures in the reservoir, the effect of a flood wave entering the reservoir is studied to predict the variations of reservoir elevation and outflow discharge with time. This form of reservoir routing is essential (i) in the design of the capacity of spillways and other reservoir outlet structures, and (ii) in the location and sizing of the capacity of reservoirs to meet specific requirements.

In *Channel routing* the change in the shape of a hydrograph as it travels down a channel is studied. By considering a channel reach and an input hydrograph at the upstream end, this form of routing aims to predict the flood hydrograph at various sections of the reach. Information on the flood-peak attenuation and the duration of high-water levels obtained by channel routing is of utmost importance in flood-forecasting operations and flood-protection works.

A variety of routing methods are available and they can be broadly classified into two categories as: (i) hydrologic routing, and (ii) hydraulic routing. Hydrologic-routing methods employ essentially the equation of continuity. Hydraulic methods, on the other hand, employ the continuity equation together with the equation of motion of unsteady flow. The basic differential equations used in the hydraulic routing, known as St. Venant equations afford a better description of unsteady flow than hydrologic methods.

8.2 BASIC EQUATIONS

The passage of a flood hydrograph through a reservoir or a channel reach is an unsteady-flow phenomenon. It is classified in open-channel hydraulics as gradually varied unsteady flow. The equation of continuity used in all hydrologic routing as the primary equation states that the difference between the inflow and outflow rate is equal to the rate of change of storage, i.e.

$$I - Q = \frac{dS}{dt} \tag{8.1}$$

where I = inflow rate, Q = outflow rate and S = storage. Alternatively, in a small time interval Δt the difference between the total inflow volume and total outflow volume in a reach is equal to the change in storage in that reach

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S \tag{8.2}$$

where $\bar{I}$ = average inflow in time Δt, $\bar{Q}$ = average outflow in time Δt and ΔS = change in storage. By taking $\bar{I} = (I_1 + I_2)/2$, $\bar{Q} = (Q_1 + Q_2)/2$ and $\Delta S = S_2 - S_1$ with suffixes 1 and 2 to denote the beginning and end of time interval Δt, Eq. (8.2) is written as

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1 \tag{8.3}$$

The time interval Δt should be sufficiently short so that the inflow and outflow hydrographs can be assumed to be straight lines in that time interval. Further Δt must be shorter than the time of transit of the flood wave through the reach.

In the differential form the equation of continuity for unsteady flow in a reach with no lateral flow is given by

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0 \tag{8.4}$$

where T = top width of the section and y = depth of flow.

The equation of motion for a flood wave is derived from the application of the momentum equation as

$$\frac{\partial y}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_0 - S_f \tag{8.5}$$

where V = velocity of flow at any section, S_0 = channel bed slope and S_f = slope of the energy line. The continuity equation [Eq. (8.4)] and the equation of motion [Eq. (8.5)] are believed to have been first developed by A.J.C. Barrè de Saint Venant (1871) and are commonly known as St. Venant equations. Hydraulic-flood routing involves the numerical solution of St.Venant equations. Details about these equations, such as their derivations and various forms are available in Ref. 9.

8.3 HYDROLOGIC STORAGE ROUTING (LEVEL POOL ROUTING)

A flood wave $I(t)$ enters a reservoir provided with an outlet such as a spillway. The outflow is a function of the reservoir elevation only, i.e. $Q = Q(h)$. The storage in the reservoir is a function of the reservoir elevation, $S = S(h)$. Further, due to the passage of the flood wave through the reservoir, the water level in the reservoir changes with time, $h = h(t)$ and hence the storage and discharge change with time (Fig. 8.1). It is required to find the variation of S, h and Q with time, i.e. find $S = S(t)$, $Q = Q(t)$ and $h = h(t)$ given $I = I(t)$.

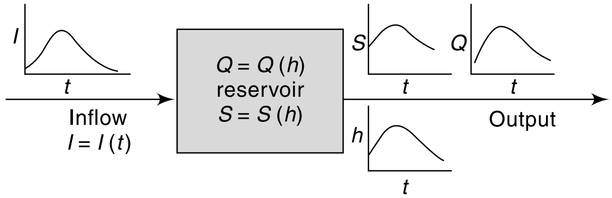


Fig. 8.1 Storage routing (Schematic)

If an uncontrolled spillway is provided in a reservoir, typically

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L_e H^{3/2} = Q(h)$$

where H = head over the spillway, L_e = effective length of the spillway crest and C_d = coefficient of discharge. Similarly, for other forms of outlets, such as gated spillways, sluice gates, etc. other relations for Q(h) will be available.

For reservoir routing, the following data have to be known:

- Storage volume *vs* elevation for the reservoir;
- Water-surface elevation *vs* outflow and hence storage *vs* outflow discharge;
- Inflow hydrograph, $I = I(t)$; and
- Initial values of S, I and Q at time $t = 0$.

There are a variety of methods available for routing of floods through a reservoir. All of them use Eq. (8.2) but in various rearranged manners. As the horizontal water surface is assumed in the reservoir, the storage routing is also known as *Level Pool Routing*.

Two commonly used semi-graphical methods and a numerical method are described below.

Modified Pul's Method

Equation (8.3) is rearranged as

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1 \Delta t}{2}\right) = \left(S_2 + \frac{Q_2 \Delta t}{2}\right) \tag{8.6}$$

At the starting of flood routing, the initial storage and outflow discharges are known. In Eq. (8.6) all the terms in the left-hand side are known at the beginning of a time step Δt. Hence the value of the function $\left(S_2 + \frac{Q_2 \Delta t}{2}\right)$ at the end of the time step is calculated by Eq. (8.6). Since the relation $S = S(h)$ and $Q = Q(h)$ are known, $\left(S + \frac{Q \Delta t}{2}\right)_2$ will enable one to determine the reservoir elevation and hence the discharge at the end of the time step. The procedure is repeated to cover the full inflow hydrograph.

For practical use in hand computation, the following semigraphical method is very convenient.

1. From the known storage-elevation and discharge-elevation data, prepare a curve of $\left(S + \frac{Q \Delta t}{2}\right)$ *vs* elevation (Fig. 8.2). Here Δt is any chosen interval, approximately 20 to 40% of the time of rise of the inflow hydrograph.
2. On the same plot prepare a curve of outflow discharge *vs* elevation (Fig. 8.2).
3. The storage, elevation and outflow discharge at the starting of routing are known.

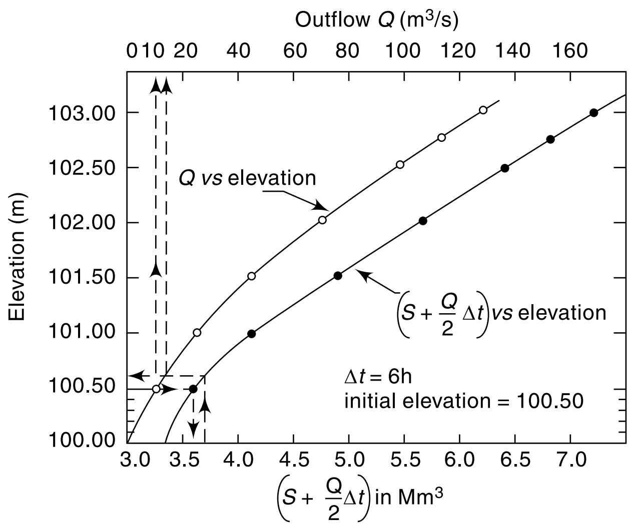


Fig. 8.2 Modified Pul's method of storage routing

For the first time interval Δt, $\left(\frac{I_1 + I_2}{2}\right)\Delta t$ and $\left(S_1 + \frac{Q_1\,\Delta t}{2}\right)$ are known and hence by Eq. (8.6) the term $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ is determined.

4. The water-surface elevation corresponding to $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ is found by using the plot of step (1). The outflow discharge Q_2 at the end of the time step Δt is found from plot of step (2).
5. Deducting $Q_2\,\Delta t$ from $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ gives $\left(S - \frac{Q\Delta t}{2}\right)_1$ for the beginning of the next time step.
6. The procedure is repeated till the entire inflow hydrograph is routed.

Example 8.1 *A reservoir has the following elevation, discharge and storage relationships:*

Elevation (m)	Storage (10^6 m^3)	Outflow discharge (m^3/s)
100.00	3.350	0
100.50	3.472	10
101.00	3.380	26
101.50	4.383	46
102.00	4.882	72
102.50	5.370	100
102.75	5.527	116
103.00	5.856	130

When the reservoir level was at 100.50 m, the following flood hydrograph entered the reservoir.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m^3/s)	10	20	55	80	73	58	46	36	55	20	15	13	11

Route the flood and obtain (*i*) *the outflow hydrograph and* (ii) *the reservoir elevation vs time curve during the passage of the flood wave.*

SOLUTION: A time interval Δt = 6 h is chosen. From the available data the elevation-discharge – $\left(S + \frac{Q\Delta t}{2}\right)$ table is prepared.

$$\Delta t = 6 \times 60 \times 60 = 0.0216 \times 10^6 \ s$$

Elevation (m)	100.00	100.50	101.00	101.50	102.00	102.50	102.75	103.00
Dischange Q (m^3/s)	0	10	26	46	72	100	116	130
$\left(S + \frac{Q\Delta t}{2}\right)$ (Mm^3)	3.35	3.58	4.16	4.88	5.66	6.45	6.78	7.26

A graph of Q *vs* elevation and $\left(S + \frac{Q\Delta t}{2}\right)$ *vs* elevation is prepared (Fig. 8.2). At the start of routing, elevation = 100.50 m, Q = 10.0 m^3/s, and $\left(S - \frac{Q\Delta t}{2}\right)$ = 3.362 Mm^3. Starting from this value of $\left(S - \frac{Q\Delta t}{2}\right)$, Eq. (8.6) is used to get $\left(S + \frac{Q\Delta t}{2}\right)$ at the end of first time step of 6 h as

$$\left(S + \frac{Q\Delta t}{2}\right)_2 = (I_1 + I_2)\frac{\Delta t}{2} + \left(S - \frac{Q\Delta t}{2}\right)_1 = (10 + 20) \times \frac{0.0216}{2} + (3.362) = 3.686 \text{ Mm}^3.$$

Looking up in Fig. 8.2, the water-surface elevation corresponding to $\left(S + \frac{Q\Delta t}{2}\right)$ = 3.686 Mm^3 is 100.62 m and the corresponding outflow discharge Q is 13 m^3/s. For the next step, Initial value of $\left(S - \frac{Q\Delta t}{2}\right)$ = $\left(S + \frac{Q\Delta t}{2}\right)$ of the previous step – $Q\,\Delta t$

$$= (3.686 - 13 \times 0.0216) = 3.405 \text{ Mm}^3$$

The process is repeated for the entire duration of the inflow hydrograph in a tabular form as shown in Table 8.1.

Using the data in columns 1, 8 and 7, the outflow hydrograph (Fig. 8.3) and a graph showing the variation of reservoir elevation with time (Fig. 8.4) are prepared.

Sometimes a graph of $\left(S - \frac{Q\Delta t}{2}\right)$ *vs* elevation prepared from known data is plotted in Fig. 8.2 to aid in calculating the items in column 5. Note that the calculations are sequential in nature and any error at any stage is carried forward. The accuracy of the method depends upon the value of Δt; smaller values of Δt give greater accuracy.

Table 8.1 Flood Routing through a Reservoir—Modified Pul's method—Example 8.1

$\Delta t = 6\text{ h} = 0.0216\text{ Ms},\ \bar{I} = (I_1 + I_2)/2$

Time (h)	Inflow I (m^3/s)	$\bar{I}$ (m^3/s)	$\bar{I}\cdot\Delta t$ (Mm^3)	$S - \frac{\Delta t Q}{2}$ (Mm^3)	$S + \frac{\Delta t Q}{2}$ (Mm^3)	Elevation (m)	Q (m^3/s)
1	2	3	4	5	6	7	8
0	10					100.50	10
		15.00	0.324	3.362	3.636		
6	20					100.62	13
		37.50	0.810	3.405	4.215		
12	55					101.04	27
		67.50	1.458	3.632	5.090		
18	80					101.64	53
		76.50	1.652	3.945	5.597		
24	73					101.96	69
		65.50	1.415	4.107	5.522		
30	58					101.91	66
		52.00	1.123	4.096	5.219		
36	46					101.72	57
		41.00	0.886	3.988	4.874		
42	36					101.48	48
		31.75	0.686	3.902	4.588		
48	27.5					101.30	37
		23.75	0.513	3.789	4.302		
54	20					100.10	25
		17.50	0.378	3.676	4.054		
60	15					100.93	23
		14.00	0.302	3.557	3.859		
66	13					100.77	18
		12.00	0.259	3.470	3.729		
72	11					100.65	14
				3.427			

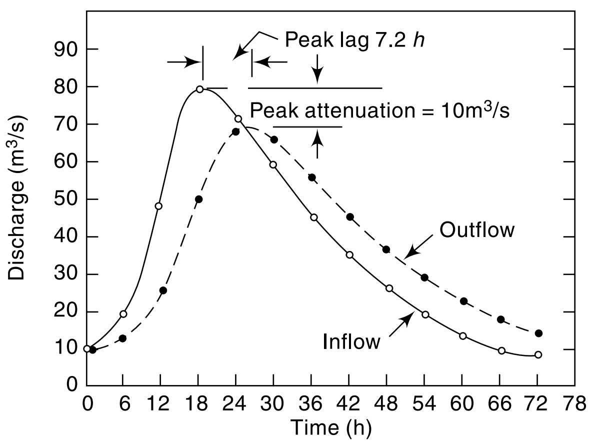


Fig. 8.3 Variation of inflow and outflow discharges—Example 8.1

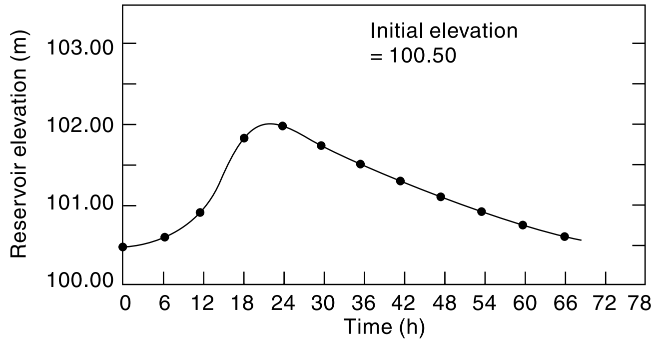


Fig. 8.4 Variation of reservoir elevation with time—Example 8.1

Goodrich Method

Another popular method of hydrologic reservoir routing, known as Goodrich method utilizes Eq. (8.3) rearranged as

$$I_1 + I_2 - Q_1 - Q_2 = \frac{2S_2}{\Delta t} - \frac{2S_1}{\Delta t}$$

where suffixes 1 and 2 stand for the values at the beginning and end of a time step Δt respectively. Collecting the known and initial values together,

$$(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right) \quad (8.7)$$

For a given time step, the left-hand side of Eq. 8.7 is known and the term $\left(\frac{2S}{\Delta t} + Q\right)_2$ is determined by using Eq. (8.7). From the known storage-elevation-discharge data, the function $\left(\frac{2S}{\Delta t} + Q\right)_2$ is established as a function of elevation. Hence, the discharge, elevation and storage at the end of the time step are obtained. For the next time step,

$$\left[\left(\frac{2S}{\Delta t} + Q\right)_2 - 2Q_2\right] \text{ of the previous time step}$$

$$= \left(\frac{2S}{\Delta t} - Q\right)_1 \text{ for use as the initial values}$$

The procedure is illustrated in Example 8.2.

Example 8.2 *Route the following flood hydrograph through the reservoir of Example 8.1 by the Goodrich method:*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	10	30	85	140	125	96	75	60	46	35	25	20

The initial conditions are: when $t = 0$, the reservoir elevation is 100.60 m.

SOLUTION: A time increment Δt = 6 h = 0.0216 Ms is chosen. Using the known storage-elevation-discharge data, the following table is prepared.

A graph depicting Q *vs* elevation and $\left(\frac{2S}{\Delta t}+Q\right)$ *vs* elevation is prepared from this data (Fig. 8.5).

Elevation (m)	100.00	100.50	101. 00	101.50	102.00	102.50	102.75	103.00
Outflow Q (m^3/s)	0	10	26	46	72	100	116	130
$\left(\frac{2S}{\Delta t}+Q\right)$ (m^3/s)	310.2	331.5	385.3	451.8	524.0	597.2	627.8	672.2

At t = 0, Elevation = 100.60 m, from Fig. 8.5, Q = 12 m^3/s and

$$\left(\frac{2S}{\Delta t}+Q\right) = 340 \text{ m}^3/\text{s}$$

$$\left(\frac{2S}{\Delta t}-Q\right)_1 = 340 - 24 = 316 \text{ m}^3/\text{s}$$

For the first time interval of 6 h,

$$I_1 = 10,\ I_2 = 30,\ Q_1 = 12, \text{ and}$$

$$\left(\frac{2S}{\Delta t}+Q\right)_2 = (10 + 30) + 316 = 356 \text{ m}^3/\text{s}$$

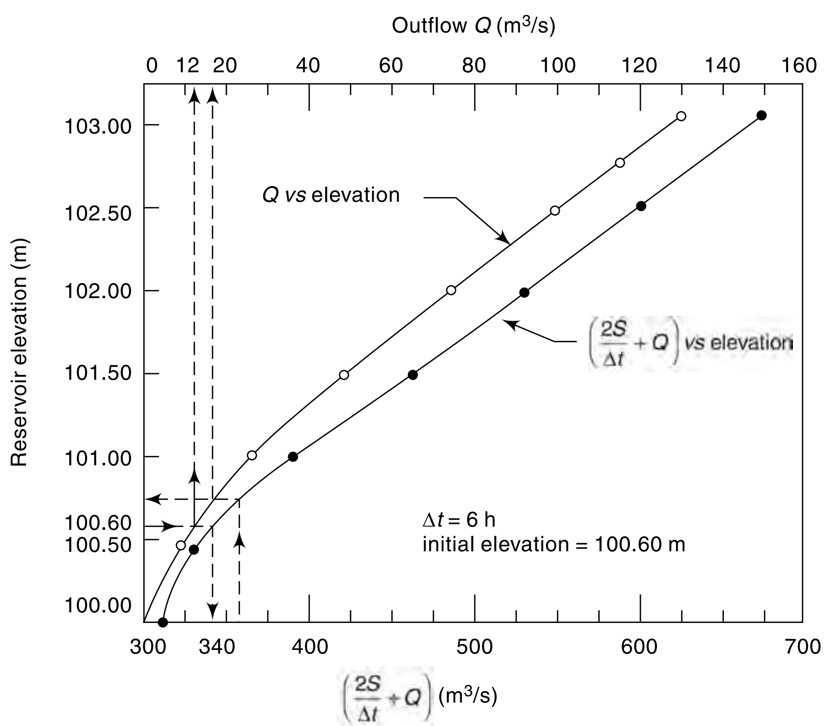


Fig. 8.5 Goodrich method of storage routing—Example 8.2

From Fig. 8.5 the reservoir elevation for this $\left(\frac{2S}{\Delta t}+Q\right)_2$ is 100.74 m

For the next time increment

$$\left(\frac{2S}{\Delta t}-Q\right)_1 = 356 - 2 \times 17 = 322 \text{ m}^3\text{/s}$$

The procedure is repeated in a tabular form (Table 8.2) till the entire flood is routed.

Using the data in columns 1, 7 and 8, the outflow hydrograph and a graph showing the variation of reservoir elevation with time (Fig. 8.6) are plotted.

In this method also, the accuracy depends upon the value of Δt chosen; smaller values of Δt give greater accuracy.

Table 8.2 Reservoir Routing—Goodrich Method—Example 8.2

Δt = 6.0 h = 0.0216 Ms

Time (h)	I (m³/s)	$(I_1 + I_2)$	$\left(\frac{2S}{\Delta t}-Q\right)$ (m³/s)	$\left(\frac{2S}{\Delta t}+Q\right)$ (m³/s)	**Elevation** (m)	**Discharge Q** (m³/s)
1	**2**	**3**	**4**	**5**	**6**	**7**
0	10			(340)	100.6	12
		40	316	356		
6	30				100.74	17
		115	322	437		
12	85				101.38	40
		225	357	582		
18	140				102.50	95
		265	392	657		
24	125				102.92	127
		221	403	624		
30	96				102.70	112
		171	400	571		
36	75				102.32	90
		135	391	526		
42	60				102.02	73
		106	380	486		
48	46				101.74	57
		81	372	453		
54	35				101.51	46
		60	361	421		
60	25				101.28	37
		45	347	392		
66	20				101.02	27
			335			

STANDARD FOURTH-ORDER RUNGE-KUTTA METHOD (SRK)

The Pul's method and Goodrich method of level pool routing are essentially semi-graphical methods. While they can be used for writing programs for use in a computer, a more efficient computation procedure can be achieved by use of any of the Runge-

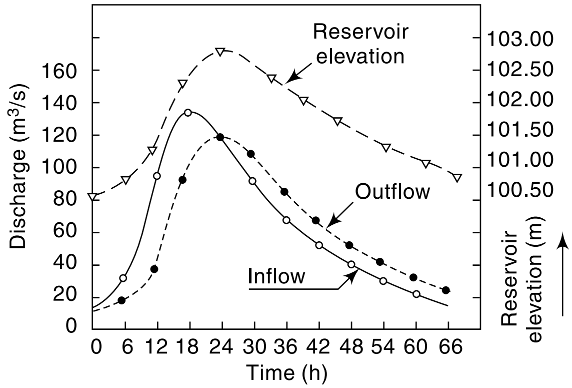


Fig. 8.6 Results of reservoir routing—Example 8.2

Kutta methods. The standard fourth-order Runge-Kutta method (SRK) is the most accurate one.

Designating

S = storage at a water surface elevation H in the reservoir = $S(H)$

A = area of the reservoir at elevation H = function of $H = A(H)$

Q = outflow from the reservoir = function of $H = Q(H)$

$$dS = A(H) \cdot dH \tag{8.8}$$

By continuity equation

$$\frac{dS}{dt} = I(t) - Q(H) = A(H)\frac{dH}{dt}$$

$$\frac{dH}{dt} = \frac{I(t) - Q(H)}{A(H)} = \text{Function of } (t, H) = F(t, H) \tag{8.9}$$

If the routing is conducted from the initial condition, (at $t = t_0$ and $I = I_0$; $Q = Q_0$, $H = H_0$, $S = S_0$) in time steps Δt, the water surface elevation H at $(i + 1)$th step is given in SRK method as

$$H_{i+1} = H_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)\,\Delta t \tag{8.10}$$

where $K_1 = F(t_i, H_i)$

$$K_2 = F\left(t_i + \frac{\Delta t}{2}, H_i + \frac{1}{2}K_1\Delta t\right)$$

$$K_3 = F\left(t_i + \frac{\Delta t}{2}, H_i + \frac{1}{2}K_2\Delta t\right)$$

$$K_4 = F(t_i + \Delta t, H_i + K_3\,\Delta t)$$

In Eq. (8.10) the suffix i denotes the values at the ith step, and suffix $(i + 1)$ denotes the values at the $(i + 1)$th step. At $i = 1$ the initial conditions I_0, Q_0, S_0 and H_0 prevail. Starting from the known initial conditions and knowing Q *vs* H and A *vs* H relationships, a given hydrograph $I = I(t)$ is routed by selecting a time step Δt. At any time $t = (t_0 + i\,\Delta t)$, the value of H_i is known and the coefficients K_1, K_2, K_3, K_4 are determined by repeated appropriate evaluation of the function $F(t, H)$. It is seen that the SRK method directly determines H_{i+1} by four evaluations of the function $F(t, H)$.

Knowing the values of H at various time intervals, i.e. $H = H(t)$, the other variables $Q(H)$ and $S(H)$ can be calculated to complete the routing operation.

Developing a computer program for level pool routing by using SRK is indeed very simple.

Other Methods In addition to the above two methods, there are a large number of other methods which depend on different combinations of the parameters of the basic continuity equation [Eq. (8.3)]. A third order Runge-Kutta method for level pool routing is described in Ref. 3.

8.4 ATTENUATION

Figures 8.3 and 8.6 show the typical result of routing a flood hydrograph through a reservoir. Owing to the storage effect, the peak of the outflow hydrograph will be smaller than that of the inflow hydrograph. This reduction in the peak value is called *attenuation*. Further, the peak of the outflow occurs after the peak of the inflow; the time difference between the two peaks is known as *lag*. The attenuation and lag of a flood hydrograph at a reservoir are two very important aspects of a reservoir operating under a flood-control criterion. By judicious management of the initial reservoir level at the time of arrival of a critical flood, considerable attenuating of the floods can be achieved. The storage capacity of the reservoir and the characteristics of spillways and other outlets controls the lag and attenuation of an inflow hydrograph.

In Figs. 8.3 and 8.6 in the rising part of the outflow curve where the inflow curve is higher than the outflow curve, the area between the two curves indicate the accumulation of flow as storage. In the falling part of the outflow curve, the outflow curve is higher than the inflow curve and the area between the two indicate depletion from the storage. When the outflow from a storage reservoir is uncontrolled, as in a freely operating spillway, the peak of the outflow hydrograph will occur at the point of intersection of the inflow and outflow curves (Figs. 8.3 and 8.6), as proved in Example 8.3.

Example 8.3 *Show that in the level pool routing the peak of the outflow hydrograph must intersect the inflow hydrograph.*

Solution: S = a function of water surface elevation in the reservoir = $S(H)$

$$\frac{dS}{dt} = A\frac{dH}{dt}$$

where A = area of the reservoir at elevation H.
Outflow Q = function of $H = Q(H)$

At peak outflow $\frac{dQ}{dt} = 0$, hence $\frac{dS}{dt} = 0$

Also, when $\frac{dH}{dt} = 0$, $\frac{dS}{dt} = 0$

By continuity equation $I - Q = \frac{dS}{dt}$

When $\frac{dS}{dt} = 0$, $I = Q$

Hence, when the peak outflow occurs, $I = Q$ and thus the peak of the outflow hydrograph must intersect the inflow hydrograph (Figs. 8.3 and 8.6).

8.5 HYDROLOGIC CHANNEL ROUTING

In reservoir routing presented in the previous sections, the storage was a unique function of the outflow discharge, $S = f(Q)$. However, in channel routing the storage is a function of both outflow and inflow discharges and hence a different routing method is needed. The flow in a river during a flood belongs to the category of gradually varied unsteady flow. The water surface in a channel reach is not only not parallel to the channel bottom but also varies with time (Fig. 8.7). Considering a channel reach having a flood flow, the total volume in storage can be considered under two categories as

1. Prism storage
2. Wedge storage

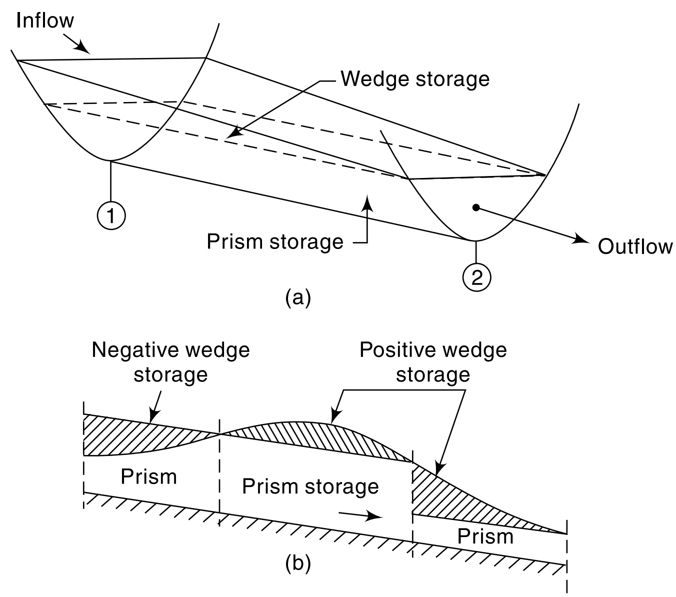


Fig. 8.7 Storage in a channel reach

Prism Storage

It is the volume that would exist if the uniform flow occurred at the downstream depth, i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge Storage

It is the wedge-like volume formed between the actual water surface profile and the top surface of the prism storage.

At a fixed depth at a downstream section of a river reach, the prism storage is constant while the wedge storage changes from a positive value at an advancing flood to a negative value during a receding flood. The prism storage S_p is similar to a reservoir and can be expressed as a function of the outflow discharge, $S_p = f(Q)$. The wedge storage can be accounted for by expressing it as $S_w = f(I)$. The total storage in the channel reach can then be expressed as

$$S = K[x\,I^m + (1 - x)Q^m] \tag{8.11}$$

where K and x are coefficients and m = a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to a value of about 1.0 for natural channels.

Muskingum Equation

Using $m = 1.0$, Eq. (8.11) reduces to a linear relationship for S in terms of I and Q as

$$S = K\,[x\,I + (1 - x)\,Q] \tag{8.12}$$

and this relationship is known as the *Muskingum equation*. In this the parameter x is known as *weighting factor* and takes a value between 0 and 0.5. When $x = 0$, obviously the storage is a function of discharge only and Eq. (8.12) reduces to

$$S = KQ \tag{8.13}$$

Such a storage is known as *linear storage* or *linear reservoir*. When $x = 0.5$ both the inflow and outflow are equally important in determining the storage.

The coefficient K is known as *storage-time constant* and has the dimensions of time. It is approximately equal to the time of travel of a flood wave through the channel reach.

Estimation of K and x

Figure 8.8 shows a typical inflow and outflow hydrograph through a channel reach. Note that the outflow peak does not occur at the point of intersection of the inflow and outflow hydrographs. Using the continuity equation [Eq. (8.3)],

$$(I_1 + I_2)\frac{\Delta t}{2} - (Q_1 + Q_2)\frac{\Delta t}{2} = \Delta S$$

the increment in storage at any time t and time element Δt can be calculated. Summation of the various incremental storage values enable one to find the channel storage S *vs* time t relationship (Fig. 8.8).

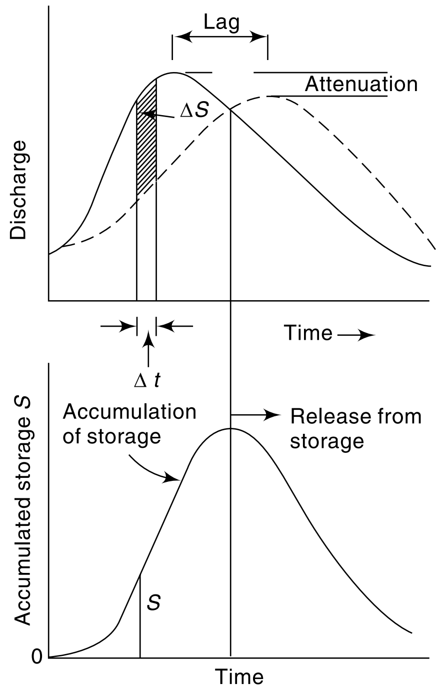


Fig. 8.8 Hydrographs and storage in channel routing

If an inflow and outflow hydrograph set is available for a given reach, values of S at various time intervals can be determined by the above technique. By choosing a trial value of x, values of S at any time t are plotted against the corresponding $[x\,I + (1 - x)\,Q]$ values. If the value of x is chosen correctly, a straight-line relationship as given by Eq. (8.12) will result. However, if an incorrect value of x is used, the plotted points will trace a looping curve. By trial and error, a value of x is so chosen that the data very nearly describe a straight line (Fig 8.9). The inverse slope of this straight line will give the value of K.

Normally, for natural channels, the value of x lies between 0 to 0.3. For a given reach, the values of x and K are assumed to be constant.

EXAMPLE 8.4 *The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of K and x applicable to this reach for use in the Muskingum equation.*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

SOLUTION: Using a time increment $\Delta t = 6$ h, the calculations are performed in a tabular manner as in Table 8.3. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units [(m^3/s).h] for storage terms.

As a first trial $x = 0.30$ is selected and the value of $[x\,I + (1 - x)\,Q]$ evaluated (column 8) and plotted against S in Fig. 8.9. Since a looped curve is obtained, further trials are performed with $x = 0.35$ and 0.25. It is seen from Fig. 8.9 that for $x = 0.25$ the data very nearly describe a straight line and as such $x = 0.25$ is taken as the appropriate value for the reach. From Fig. 8.9, $K = 13.3$ h

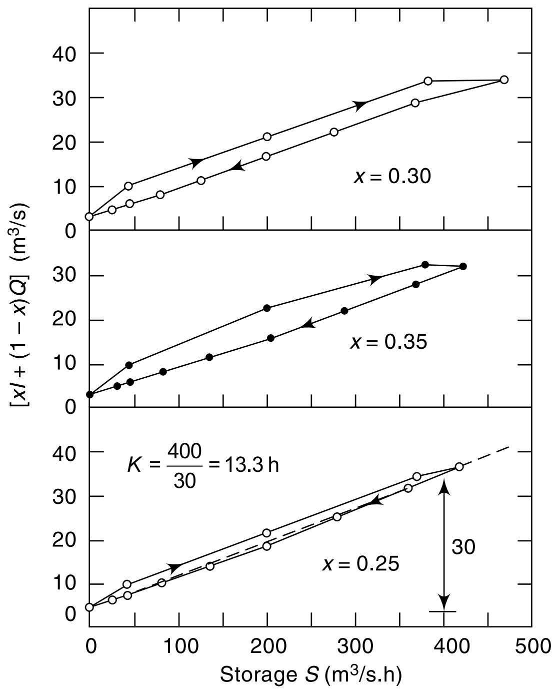


Fig. 8.9 Determination of K and x for a channel reach

MUSKINGUM METHOD OF ROUTING

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage is

$$S_2 - S_1 = K[x\,(I_2 - I_1) + (1 - x)\,(Q_2 - Q_1)] \quad (8.14)$$

where suffixes 1 and 2 refer to the conditions before and after the time interval Δt. The continuity equation for the reach is

Table 8.3 Determination of K and x—Example 8.4

Δt = 6 h, Storage in $(m^3/s) \cdot h$

Time (h)	I (m^3/s)	Q (m^3/s)	$(I-Q)$	Average $(I-Q)$	ΔS = Col. 5 × Δt $(m^3/s \cdot h)$	$S = \Sigma \Delta S$ $(m^3/s \cdot h)$	$[xI + (1-x)Q]$ (m^3/s) x = 0.35	x = 0.30	x = 0.25
1	2	3	4	5	6	7	8	9	10
0	5	5	0			0	5.0	5.0	5.0
				7.0	42				
6	20	6	14			42	10.9	10.2	9.5
				26.0	156				
12	50	12	38			198	25.3	23.4	21.5
				29.5	177				
18	50	29	21			375	36.4	35.3	34.3
				7.5	45				
24	32	38	–6			420	35.9	36.2	36.5
				–9.5	–57				
30	22	35	–13			363	30.5	31.1	31.8
				–13.5	–81				
36	15	29	–14			282	24.1	24.8	25.5
				–13.5	–81				
42	10	23	–13			201	18.5	19.1	19.8
				–11.5	–69				
48	7	17	–10			132	13.5	14.0	14.5
				–9.0	–54				
54	5	13	–8			78	10.2	10.6	11.0
				–6.0	–36				
60	5	9	–4			42	7.6	7.8	8.0
				–3.0	–18				
66	5	7	–2			24	6.3	6.4	6.5

$$S_2 - S_1 = \left(\frac{I_2 + I_1}{2}\right)\Delta t - \left(\frac{Q_2 + Q_1}{2}\right)\Delta t \tag{8.15}$$

From Eqs (8.14) and (8.15), Q_2 is evaluated as

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \tag{8.16}$$

where

$$C_0 = \frac{-Kx + 0.5\,\Delta t}{K - Kx + 0.5\,\Delta t} \tag{8.16a}$$

$$C_1 = \frac{Kx + 0.5\,\Delta t}{K - Kx + 0.5\,\Delta t} \tag{8.16b}$$

$$C_2 = \frac{K - Kx - 0.5\,\Delta t}{K - Kx + 0.5\,\Delta t} \tag{8.16c}$$

Note that $C_0 + C_1 + C_2 = 1.0$, Eq. (8.16) can be written in a general form for the n^{th} time step as

$$Q_n = C_0 I_n + C_1 I_{n-1} + C_2 Q_{n-1} \tag{8.16A}$$

Equation (8.16) is known as *Muskingum Routing Equation* and provides a simple linear equation for channel routing. It has been found that for best results the routing interval Δt should be so chosen that $K > \Delta t > 2Kx$. If $\Delta t < 2Kx$, the coefficient C_0 will be negative. Generally, negative values of coefficients are avoided by choosing appropriate values of Δt.

To use the Muskingum equation to route a given inflow hydrograph through a reach, the values of K and x for the reach and the value of the outflow, Q_1, from the reach at the start are needed. The procedure is indeed simple.

- Knowing K and x, select an appropriate value of Δt
- Calculate C_0, C_1 and C_2.
- Starting from the initial conditions I_1, Q_1 and known I_2 at the end of the first time step Δt calculate Q_2 by Eq. (8.16).
- The outflow calculated in step (c) becomes the known initial outflow for the next time step. Repeat the calculations for the entire inflow hydrograph.

The calculations are best done row by row in a tabular form. Example 8.5 illustrates the computation procedure. Spread sheet (such as MS Excel) is ideally suited to perform the routing calculations and to view the inflow and outflow hydrographs.

Example 8.5 *Route the following flood hydrograph through a river reach for which K = 12.0 h and x = 0.20. At the start of the inflow flood, the outflow discharge is 10 m^3/s.*

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	10	20	50	60	55	45	35	27	20	15

Solution: Since K = 12 h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, Δt should be such that 12 h $> \Delta t >$ 4.8 h. In the present case Δt = 6 h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (8. 16-a, b & c) the coefficients C_0, C_1 and C_2 are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 h,

$$I_1 = 10.0 \qquad C_1 I_1 = 4.29$$

$$I_2 = 20.0 \qquad C_0 I_2 = 0.96$$

$$Q_1 = 10.0 \qquad C_2 Q_1 = 5.23$$

From Eq. (8.16) $\quad Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \qquad = 10.48 \text{ m}^3/\text{s}$

For the next time step, 6 to 12 h, Q_1 = 10.48 m^3/s. The procedure is repeated for the entire duration of the inflow hydrograph. The computations are done in a tabular form as shown in Table 8.4. By plotting the inflow and outflow hydrographs the attenuation and peak lag are found to be 10 m^3/s and 12 h respectively.

Alternative Form of Eq. (8. 16): Equations (8.14) and (8.15) can be combined in an alternative form of the routing equation as

$$Q_2 = Q_1 + B_1 (I_1 - Q_1) + B_2(I_2 - I_1) \tag{8.17}$$

Table 8.4 Muskingum Method of Routing—Example 8.5

$\Delta t = 6$ h

Time (h)	I (m³/s)	0.048 I_2	0.429 I_1	0.523 Q_1	Q (m³/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2.88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04

where $B_1 = \dfrac{\Delta t}{K(1-x) + 0.5\,\Delta t}$ $\qquad B_2 = \dfrac{0.5\,\Delta t - Kx}{K(1-x) + 0.5\,\Delta t}$

The use of Eq. (8.17) is essentially the same as that of Eq. (8.16).

8.6 HYDRAULIC METHOD OF FLOOD ROUTING

The hydraulic method of flood routing is essentially a solution of the basic St Venant equations [Eqs (8.4) and (8.5)]. These equations are simultaneous, quasi-linear, first order partial differential equations of the hyperbolic type and are not amenable to general analytical solutions. Only for highly simplified cases can one obtain the analytical solution of these equations. The development of modern, high-speed digital computers during the past two decades has given rise to the evolution of many sophisticated numerical techniques. The various numerical methods for solving St Venant equations can be broadly classified into two categories:

1. Approximate methods
2. Complete numerical methods.

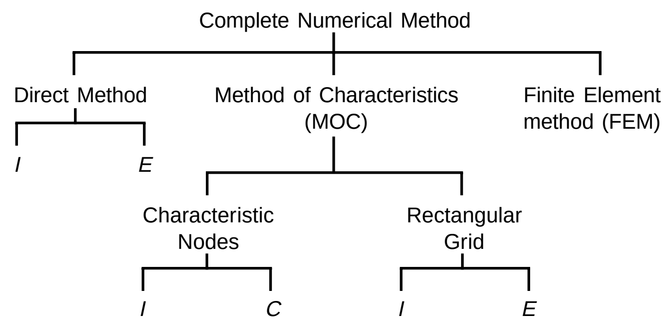
Approximate Methods

These are based on the equation of continuity only or on a drastically curtailed equation of motion. The hydrological method of storage routing and Muskingum channel routing discussed earlier belong to this category.

Other methods in this category are diffusion analogy and kinematic wave models.

Complete Numerical Methods

These are the essence of the hydraulic method of routing and are classified into many categories as mentioned below:



I = Implicit method, *E* = Explicit method

In the direct method, the partial derivatives are replaced by finite differences and the resulting algebraic equations are then solved. In the method of characteristics (MOC) St Venant equations are converted into a pair of ordinary differential equations (i.e. characteristic forms) and then solved by finite difference techniques. In the finite element method (FEM) the system is divided into a number of elements and partial differential equations are integrated at the nodal points of the elements.

The numerical schemes are further classified into explicit and implicit methods. In the explicit method the algebraic equations are linear and the dependent variables are extracted explicitly at the end of each time step. In the implicit method the dependent variables occur implicitly and the equations are nonlinear. Each of these two methods have a host of finite-differencing schemes to choose from. Details of hydraulic flood routing and a bibliography of relevant literature are available in Refs. 6, 8 and 9.

8.7 ROUTING IN CONCEPTUAL HYDROGRAPH DEVELOPMENT

Even though the routing of floods through a reservoir or channel discussed in the previous section were developed for field use, they have found another important use in the conceptual studies of hydrographs. The routing through a reservoir which gives attenuation and channel routing which gives translation to an input hydrograph are treated as two basic modifying operators. The following two fictitious items are used in the studies for development of synthetic hydrographs through conceptual models

1. *Linear reservoir:* a reservoir in which the storage is directly proportional to the discharge ($S = KQ$). This element is used to provide attenuation to a flood wave.
2. *Linear channel:* a fictitious channel in which the time required to translate a discharge Q through a given reach is constant. An inflow hydrograph passes through such a channel with only translation and no attenuation.

Conceptual modelling for IUH development has undergone rapid progress since the first work by Zoch (1937). Detailed reviews of various contributions to this field are available in Refs. 2 and 4 and the details are beyond the scope of this book. However, a simple method, viz., Clark's method (1945) which utilizes the Muskingum method of routing through a linear reservoir is indicated below as a typical example of the use of routing in conceptual models. Nash's model which uses routing through a cascade of linear reservoirs is also presented, in Sec. 8.9, as another example of a conceptual model.

8.8 CLARK'S METHOD FOR IUH

Clark's method, also known as *Time–area histogram* method aims at developing an IUH due to an instantaneous rainfall excess over a catchment. It is assumed that the rainfall excess first undergoes pure translation and then attenuation. The translation is achieved by a travel time–area histogram and the attenuation by routing the results of the above through a linear reservoir at the catchment outlet.

TIME—AREA CURVE

Time here refers to the time of concentration. As defined earlier in Sec. 7.2, the time of concentration t_c is the time required for a unit volume of water from the farthest point of catchment to reach the outlet. It represents the maximum time of translation of the surface runoff of the catchment. In gauged areas the time interval between the end of the rainfall excess and the point of inflection of the resulting surface runoff (Fig. 8.10) provides a good way of estimating t_c from known rainfall- runoff data. In ungauged areas the empirical formulae Eq. (7.3) or (7.4) can be used to estimate t_c.

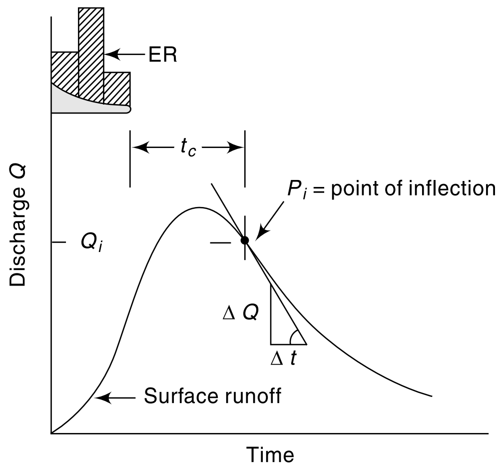


Fig. 8.10 Surface Runoff of a Catchment

The total catchment area drains into the outlet in t_c hours. If points on the area having equal time of travel, (say t_1 *h* where $t_1 < t_c$), are considered and located on a map of the catchment, a line joining them is called an *Isochrone* (or *runoff isochrone*). Figure (8.11) shows a catchment being divided into $N(=8)$ subareas by isochrones having an equal time interval. To assist in drawing isochrones, the longest water course is chosen and its profile plotted as elevation *vs* distance from the outlet; the distance is

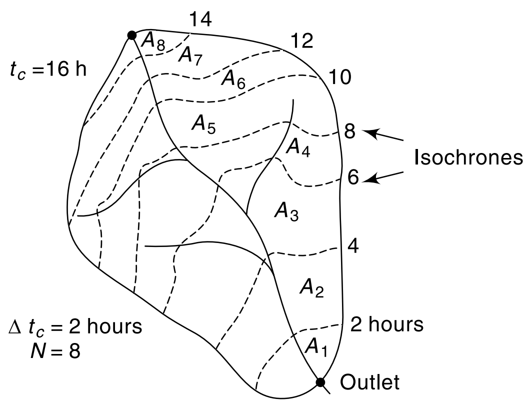


Fig. 8.11 Isochrones in a Catchment

then divided into N parts and the elevations of the subparts measured on the profile transferred to the contour map of the catchment.

The inter-isochrone areas A_1, A_2, …, A_N are used to construct a travel time-area histogram (Fig. 8.12). If a rainfall excess of 1 cm occurs instantaneously and uniformly over the catchment area, this time-area histogram represents the sequence in which the volume of rainfall will be moved out of the catchment and arrive at the outlet. In Fig. 8.12, a subarea A_r km^2 represent a volume of A_r km^2. cm = $A_r \times 10^4$ (m^3) moving out in time $\Delta t_c = t_c/N$ hours. The hydrograph of outflow obtained by this figure while properly accounting for the sequence of arrival of flows, do not provide for the storage properties of the catchment. To overcome this deficiency, Clark assumed a linear reservoir to be hypothetically available at the outlet to provide the requisite attenuation.

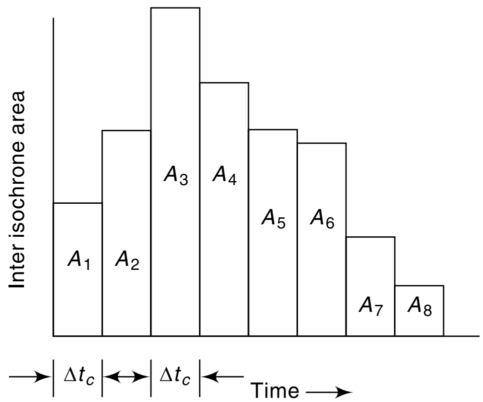


Fig. 8.12 Time-area Histogram

Routing

The linear reservoir at the outlet is assumed to be described by $S = KQ$, where K is the storage time constant. The value of K can be estimated by considering the point of inflection P_i of a surface runoff hydrograph (Fig. 8.10). At this point the inflow into the channel has ceased and beyond this point the flow is entirely due to withdrawal from the channel storage. The continuity equation

$$I - Q = \frac{dS}{dt}$$

becomes $$-Q = \frac{dS}{dt} = K\frac{dQ}{dt} \quad \text{(by Eq. 8.13)}$$

Hence $$K = -Q_i/(dQ/dt)_i \quad (8.\ 18)$$

where suffix i refers to the point of inflection, and K can be estimated from a known surface runoff hydrograph of the catchment as shown in Fig. 8.10. The constant K can also be estimated from the data on the recession limb of a hydrograph (Sec. 6.3).

Knowing K of the linear reservoir, the inflows at various times are routed by the Muskingum method. Note that since a linear reservoir is used $x = 0$ in Eq. (8.12). The inflow rate between an inter-isochrone area A_r km^2 with a time interval Δt_c (h) is

$$I = \frac{A_r \times 10^4}{3600\,\Delta t_c} = 2.78\frac{A_r}{\Delta t_c} \ (\text{m}^3/\text{s})$$

The Muskingum routing equation would now be by Eq. (8.16),

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \quad (8.19)$$

where $C_0 = (0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$ $\quad C_1 = (0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$

$C_2 = (K - 0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$

i.e. $C_0 = C_1$. Also since the inflows are derived from the histogram $I_1 = I_2$ for each interval. Thus Eq. (8.19) becomes

$$Q_2 = 2\,C_1 I_1 + C_2 Q_1 \tag{8.20}$$

Routing of the time-area histogram by Eq. (8.20) gives the ordinates of IUH for the catchment. Using this IUH any other *D*-h unit hydrograph can be derived.

EXAMPLE 8.6 *A drainage basin has the following characteristics:*
Area = 110 km², time of concentration = 18 h, storage constant = 12 h and inter-isochrone area distribution as below:

Travel time t (h)	0–2	2–4	4–6	6–8	8–10	10–12	12–14	14–16	16–18
Inter-Isochrone area (km²)	3	9	20	22	16	18	10	8	4

Determine the IUH for this catchment.

SOLUTION:

$$K = 12 \text{ h}, \quad t_c = 18 \text{ h}, \quad \Delta t_c = 2 \text{ h}$$

$$C_1 = \frac{0.5 \times 2}{12 + 0.5 \times 2} = 0.077$$

$$C_2 = \frac{12 - 0.5 \times 2}{12 + 0.5 \times 2} = 0.846$$

Equation (8.20) becomes $Q_2 = 0.154 I_1 + 0.846\,Q_1$ = Ordinate of IUH

At $t = 0$, $Q_1 = 0$

$$I_1 = 2.78\,A_r/2 = 1.39\,A_r \text{ m}^3/\text{s}$$

The calculations are shown in Table 8.5.

Table 8.5 Calculations of IUH—Clark's Method—Example 8.6

Time (h)	Area A_r (km²)	I (m³/s)	0.154 I_1	0.846 Q_1	Ordinate of IUH (m³/s)
1	**2**	**3**	**4**	**5**	**6**
0	0	0	0	0	0
2	3	4.17	0.64	0	0.64
4	9	12.51	1.93	0.54	2.47
6	20	27.80	4.28	2.09	6.37
8	22	30.58	4.71	5.39	10.10
10	16	22.24	3.42	8.54	11.96
12	18	25.02	3.85	10.12	13.97
14	10	13.90	2.14	11.82	13.96
16	8	11.12	1.71	11.81	13.52
18	4	5.56	0.86	11.44	12.30
20	0	0	0	10.40	10.40
22				8.80	8.80

(Contd.)

(*Contd.*)

24				7.45	7.45
26				6.30	6.30
28				5.30	5.30
				⋮	⋮
				so on	so on

8.9 NASH'S CONCEPTUAL MODEL

Nash[7] (1957) proposed the following conceptual model of a catchment to develop an equation for IUH. The catchment is assumed to be made up of a series of *n* identical linear reservoirs each having the same storage constant *K*. The first reservoir receives a unit volume equal to 1 cm of effective rain from the catchment instantaneously. This inflow is routed through the first reservoir to get the outflow hydrograph. The outflow from the first reservoir is considered as the input to the second; the outflow from the second reservoir is the input to the third and so on for all the *n* reservoirs. The conceptual cascade of reservoirs as above and the shape of the outflow hydrographs from each reservoir of the cascade is shown in Fig. 8.13. The outflow hydrograph from the *n*th reservoir is taken as the IUH of the catchment.

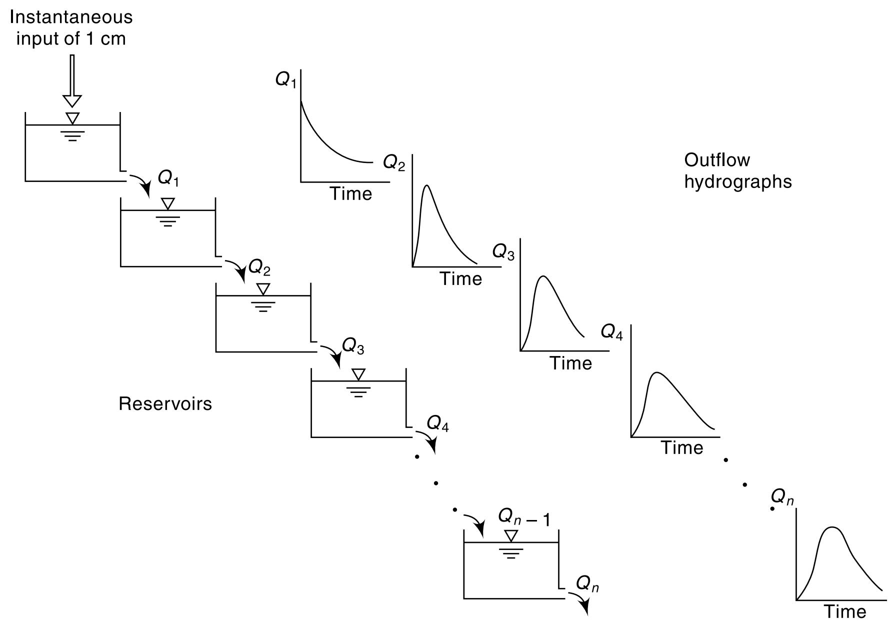


Fig. 8.13 Nash Model: Cascade of Linear Reservoirs

From the equation of continuity $I - Q = \dfrac{dS}{dt}$ (8.1)

For a linear reservoir $S = K\,Q$ and hence $\dfrac{dS}{dt} = K\dfrac{dQ}{dt}$ (8.21)

Substituting in Eq. (8.1) and rearranging,

$$K\frac{dQ}{dt} + Q = I \qquad (8.22)$$

and the solution of this differential equation, where Q and I are functions of time t, is

$$Q = \frac{1}{K} e^{-t/K} \int e^{t/K} I\, dt \tag{8.23}$$

Now for the first reservoir, the input is applied instantaneously. Hence for $t > 0$, $I = 0$. Also at $t = 0$, $\int I\, dt$ = instantaneous volume inflow = 1 cm of effective rain. Hence for the first reservoir Eq. (8.23) becomes,

$$Q_1 = \frac{1}{K} e^{-t/K} \tag{8.24}$$

For the second reservoir $Q_2 = \frac{1}{K} e^{-t/K} \int e^{t/K} I\, dt$

Here I = input = Q_1 given by Eq. (8.24). Thus,

$$Q_2 = \frac{1}{K} e^{-t/K} \int e^{t/K} \frac{1}{K} e^{-t/K} dt = \frac{1}{K^2} t\, e^{-t/K} \tag{8.25}$$

For the third reservoir in Eq. (8.23)

$$I = Q_2 \text{ and } Q_3 \text{ is obtained as } Q_3 = \frac{1}{2} \frac{1}{K^3} t^2 e^{-t/K} \tag{8.26}$$

Similarly, for the hydrograph of outflow from the n^{th} reservoir Q_n is obtained as

$$Q_n = \frac{1}{(n-1)!K^n} t^{n-1} e^{-t/K} \tag{8.27}$$

As the outflow from the nth reservoir was caused by 1 cm of excess rainfall falling instantaneously over the catchment Eq. (8.27) describes the IUH of the catchment. Using the notation $u(t)$ to represent the ordinate of the IUH, Eq. (8.27) to represent the IUH of a catchment is written as

$$u(t) = \frac{1}{(n-1)!K^n} t^{n-1} e^{-t/K} \tag{8.28}$$

Here, if t is in hours, $u(t)$ will have the dimensions of cm/h; K and n are constants for the catchment to be determined by effective rainfall and flood hydrograph characteristics of the catchment.

It should be remembered that Eq. (8.28) is based on a conceptual model and as such if n for a catchment happens to be a fraction, it is still alright. To enable $(n - 1)!$ to be determined both for integer and fractional values of n, the gamma function $\Gamma(n)$ is used to replace $(n - 1)!$ so that

$$u(t) = \frac{1}{K\Gamma(n)} (t/K)^{n-1} e^{-t/K} \tag{8.29}$$

When n is an integer, $\Gamma(n) = (n - 1)!$ which can be evaluated easily. However, when n is not an integer, the value of $\Gamma(n)$ is obtained from Gamma Tables[10] (Table 8.6).

Table 8.6 Gamma Function $\Gamma(n)$

n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$
1.00	1.000000	1.34	0.892216	1.68	0.905001
1.02	0.988844	1.36	0.890185	1.70	0.908639
1.04	0.978438	1.38	0.888537	1.72	0.912581

(Contd.)

(*Contd.*)

1.06	0.968744	1.40	0.887264	1.74	0.916826
1.08	0.959725	1.42	0.886356	1.76	0.921375
1.10	0.951351	1.44	0.885805	1.78	0.926227
1.12	0.943590	1.46	0.885604	1.80	0.931384
1.14	0.936416	1.48	0.885747	1.82	0.936845
1.16	0.929803	1.50	0.886227	1.84	0.942612
1.18	0.923728	1.52	0.887039	1.86	0.948687
1.20	0.918169	1.54	0.888178	1.88	0.955071
1.22	0.913106	1.56	0.889639	1.90	0.961766
1.24	0.908521	1.58	0.891420	1.92	0.968774
1.26	0.904397	1.60	0.893515	1.94	0.976099
1.28	0.900718	1.62	0.895924	1.96	0.983743
1.30	0.897471	1.64	0.898642	1.98	0.991708
1.32	0.894640	1.66	0.901668	2.00	1.000000

Note: Use the relation $\Gamma(n + 1) = n\,\Gamma(n)$ to evaluate $\Gamma(n)$ for any n.

EXAMPLE: (a) To find $\Gamma(0.6)$: $\Gamma(1.6) = \Gamma(0.6 + 1) = 0.6\,\Gamma(0.6)$

$$\text{thus}\quad \Gamma(0.6) = \frac{\Gamma(1.6)}{0.6} = \frac{0.8935}{0.6} = 1.489$$

(b) To find $\Gamma(4.7)$: $\Gamma(4.7) = \Gamma(3.7+1) = 3.7\,\Gamma(3.7)$

$= 3.7 \times 2.7\,\Gamma(2.7) = 3.7 \times 2.7 \times 1.7 \times \Gamma(1.7)$

$= 3.7 \times 2.7 \times 1.7 \times 0.9086 = 15.431$

Determination of *n* and *K* of Nash's model

From the property of the IUH given by Eq. (8.28), it can be shown that the first moment of the IUH about the origin $t = 0$ is given by

$$M_1 = nK \tag{8.30}$$

Also the second moment of the IUH about the origin $t = 0$ is given by

$$M_2 = n\,(n + 1)\,K^2 \tag{8.31}$$

Using these properties the values of n and K for a catchment can be determined adequately if the ERH and a corresponding DRH are available. If

M_{Q1} = first moment of the DRH about the time origin divided by the total direct runoff, and

M_{I1} = first moment of the ERH about the time origin divided by the total effective rainfall,

then,
$$M_{Q1} - M_{I1} = nK \tag{8.32}$$

Further, if

M_{Q2} = second moment of DRH about the time origin divided by total direct runoff, and

M_{I2} = second moment of ERH about the time origin divided by total excess rainfall,

then,
$$M_{Q2} - M_{I2} = n\,(n + 1)\,K^2 + 2nK\,M_{I1} \tag{8.33}$$

Knowing M_{I1}, M_{I2}, M_{Q1} and M_{Q2}, values of K and n for a given catchment can be calculated by Eqs. (8.32) and (8.33).

Example 8.7 illustrates the method of determining n and K of the Nash's model. Example 8.8 describes the computation of IUH and a D-hour UH when the values of n and K are known.

EXAMPLE 8.7 *For a catchment the effective rainfall hyetograph of an isolated storm and the corresponding direct runoff hydrograph is given below. Determine the coefficients n and K of Nash model IUH.*

Coordinates of ERH:

Time from start of storm (h)	Effective rainfall intensity (cm/s)
0 to 1.0	4.3
1.0 to 2.0	3.2
2.0 to 3.0	2.4
3.0 to 4.0	1.8

Coordinates of DRH:

Time from start of storm (h)	Direct runoff (m^3/s)	Time from start of storm (h)	Direct runoff (m^3/s)
0	0	9	32.7
1	6.5	10	23.8
2	15.4	11	16.4
3	43.1	12	9.6
4	58.1	13	6.8
5	68.2	14	3.2
6	63.1	15	1.5
7	52.7	16	0
8	41.9		

SOLUTION: The ERH is shown in Fig. 8.14(a) as a histogram. Each block has the total rainfall in a time interval of 1 hour marked on it.

M_{I1} = first moment of the ERH about the time origin divided by the total rainfall excess.

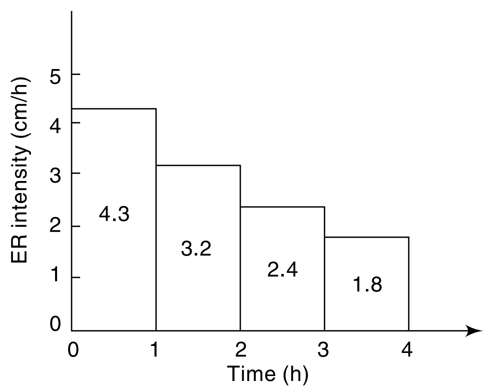


Fig. 8.14(a) Excess rainfall hyetograph of Example 8.7

$$M_{I1} = \frac{\sum(\text{Incremental area of ERH} \times \text{moment arm})}{\text{total area of ERH}}$$

M_{I2} = second moment of the ERH about the time origin divided by the total rainfall excess.

$$= \left\{\frac{1}{\text{total area of ERH}}\right\}\left\{\begin{array}{l}\sum[\text{incremental area} \times (\text{moment arm})^2] \\ +\sum[\text{second moment of the incremental} \\ \quad \text{area about its own centroid}]\end{array}\right\}$$

The calculations of M_{I1} and M_{I2} are shown in Table 8.7(a)

Table 8.7(a) Calculation of M_{I1} and M_{I2} : Example 8.7

1	2	3	4	5	6	7	8
Time (h)	**Excess rainfall in Δt (cm)**	**Interval Δt (h)**	**Incre. area**	**moment arm**	**First moment**	**Second moment part (a)**	**Second moment part (b)**
0	0	0	0	0	0	0	0
1	4.3	1	4.3	0.5	2.15	1.08	0.358
2	3.2	1	3.2	1.5	4.8	*7.20*	0.267
3	2.4	1	2.4	2.5	6.0	15.00	0.200
4	1.8	1	1.8	3.5	6.3	22.05	0.150
Sum			**11.7**		**19.25**	**45.325**	**0.975**

In Table 8.7(a)

Col. 6 = first moment of the incremental area about the origin = (Col. 4 × Col. 5)

Col. 7 = Col. 4 × (Col. 5)^2

Col. 8 = second moment of the incremental area about its own centroid

$$= \frac{1}{12} \times (\Delta t)^3\,(ER) = \frac{1}{12} \times (\text{Col. }3)^3 \times (\text{Col. }2)$$

From the data of Table 8.7(a):

M_{I1} = (sum of Col. 6)/(sum of Col. 4) = 19.25/11.7 = 1.645

M_{I2} = {(sum of Col. 7) + (sum of Co1. 8)}/(sum of Col. 4)

= (45.325 + 0.975)}/11.75 = 3.957

The DRH is shown plotted in Fig. 8.14(b). A time interval of Δt = 1 hour is chosen and considering the average DR in this interval the DRH is taken to be made up of large number of rectangular blocks.

For the DRH

M_{Q1} = first moment of the DRH about the time origin divided by the total direct runoff

$$= \frac{\sum(\text{Incremental area of DRH} \times \text{moment arm})}{\text{total area of DRH}}$$

M_{Q2} = second moment of the DRH about the time origin divided by the total direct runoff

$$= \left\{\frac{1}{\text{total area of DRH}}\right\}\left\{\begin{array}{l}\sum[\text{incremental area} \times (\text{moment arm})^2] \\ +\sum[\text{second moment of the incremental} \\ \quad \text{area about its own centroid}]\end{array}\right\}$$

The calculations of M_{Q1} and M_{Q2} are shown in Table 8.7(b).

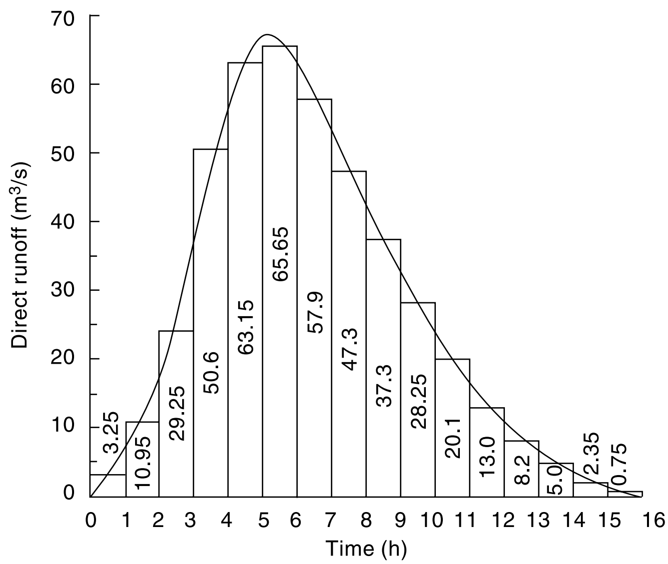


Fig. 8.14(b) Direct runoff hydrograph of Example 8.7

Table 8.7(b) Calculation of M_{Q1} and M_{Q2}—Example 8.7

1	2	3	4	5	6	7	8	9
Time (h)	**Ord. of DRH (m^3/s)**	**Average DR rate in Δt (m^3/s)**	**Interval Δt (*h*)**	**Increment area**	**Moment arm**	**First Moment**	**Second Moment part (a)**	**Second Moment part (b)**
0	0	0.00	0	0.00	0	0.00	0.00	0.00
1	6.5	3.25	1	3.25	0.5	1.63	0.81	0.27
2	15.4	10.95	1	10.95	1.5	16.43	24.64	0.91
3	43.1	29.25	1	29.25	2.5	73.13	182.81	2.44
4	58.1	50.60	1	50.60	3.5	177.10	619.85	4.22
5	68.2	63.15	1	63.15	4.5	284.18	1278.79	5.26
6	63.1	65.65	1	65.65	5.5	361.08	1985.91	5.47
7	52.7	57.90	1	57.90	6.5	376.35	2446.28	4.83
8	41.9	47.30	1	47.30	7.5	354.75	2660.63	3.94
9	32.7	37.30	1	37.30	8.5	317.05	2694.93	3.11
10	23.8	28.25	1	28.25	9.5	268.38	2549.56	2.35
11	16.4	20.10	1	20.10	10.5	211.05	2216.03	1.68
12	9.6	13.00	1	13.00	11.5	149.50	1719.25	1.08
13	6.8	8.20	1	8.20	12.5	102.50	1281.25	0.68
14	3.2	5.00	1	5.00	13.5	67.50	911.25	0.42
15	1.5	2.35	1	2.35	14.5	34.08	494.09	0.20
16	0	0.75	1	0.75	15.5	11.63	180.19	0.06
Sum				**443.00**		**2806.30**	**21246.25**	**36.92**

In Table 8.7(b):

Col. 7 = first moment of the incremental area of DRH about the origin = (Col. 4 × Col. 5)

Col. 8 = Col. 5 × (Col. 6)^2

Col. 9 = second moment of the incremental area about its own centroid

$$= \frac{1}{12} \times (\text{Col. } 4)^3 \times (\text{Col. } 3)$$

From the data of Table 8.7(b):

M_{Q1} = (sum of Col. 7)/(sum of Col. 5) = 2806.3/443 = 6.33

M_{Q2} = {(sum of Col. 8) + (sum of Col. 9)}/(sum of Col. 5)

= (21246.25 + 36.92)/443 = 48.04

[*Note* that in the calculation of M_{I2} and M_{Q2}, for small values of Δt the second term in the bracket, *viz*. second moment part (b) = Σ [second moment of incremental area about its own centroid], is relatively small in comparison with the first term [part(a)] and can be neglected without serious error.]

By Eq. (8.30) $\quad nK = M_{Q1} - M_{I1} = 6.335 - 1.645 = 4.690$

By Eq. (8.31) $\quad M_{Q2} - M_{I2} = n(n+1)K^2 + 2nKM_{I1} = (nK)^2 + (nK)K + 2(nK)M_{I1}$

Substituting for nK, M_{Q2}, M_{I2} and M_{I1}

$$48.04 - 3.96 = (4.69)^2 + (4.69)K + 2(4.69)(1.645)$$

$$K = 6.654/4.69 = 1.42 \text{ hours}$$

$$n = nK/K = 4.69/1.42 = 3.30$$

Example 8.8 *For a catchment of area 300 km^2 the values of the Nash model coefficients are found to have values of n = 4.5 and K = 3.3 hours. Determine the ordinates of (a) IUH and (b) 3-h unit hydrograph of the catchment.*

Solution: The ordinates of IUH by Nash model are given by

$$u(t) = \frac{1}{K\Gamma(n)} (t/K)^{n-1} e^{(t/K)}$$

In the present case $\quad n = 4.5$, $K = 3.3$ hours and $u(t)$ is in cm/h.

$$\Gamma(n) = \Gamma(4.5) = 3.5\,\Gamma(3.5) = 3.5 \times 2.5\,\Gamma(2.5)$$

$$= 3.5 \times 2.5 \times 1.5 \times \Gamma(1.5)$$

From Table 8.6, $\quad \Gamma(1.5) = 0.886227$

Hence $\quad \Gamma(4.5) = 3.5 \times 2.5 \times 1.5 \times 0.886227 = 11.632$

$$u(t) = \frac{1}{3.3 \times 11.632} (t/3.5)^{3.5} e^{-(t/3.3)} = 0.02605\,(t/3.3)^{3.5} e^{-(t/3.3)}$$

Values of $u(t)$ for various values of t are calculated as shown in Table 8.8. An interval of one hour is chosen. In Table 8.8, Col. 3 gives the ordinates of $u(t)$ in cm/h. Multiplying these values by (2.78 × A) where A = area of the catchment in km^2 gives the values of $u(t)$ in m^3/s, (Col. 4).

Thus Col. 4 = (Col. 3) × 2.78 × 300 = (Col. 3) × 834

Col. 5 is the ordinate of $u(t)$ [i.e. Col. 4] lagged by one hour

Col. 6 = (Col. 4 + Co1. 6)/2 = ordinate of 1-h UH by Eq. (6.26)

The S-curve technique is used to derive the 3-h UH from the 1-h UH obtained in Col. 6.

Col. 7 = S_1–curve addition.

Col. 8 = S_1–curve ordinates

Col. 9 = S_1–curve ordinates lagged by 3 hours

Col. 10 = (Col. 8 – Col. 9) = ordinates of a DRH of 3 cm occurring in 3 hours.

Col. 11 = (Col. 10)/3 = ordinates of 3-h UH

Table 8.8 Calculation of 3-Hour UH by Nash Method—Example 8.8

$K = 3.3\ h$ $n = 4.5$ $\Gamma(n) = 11.632$ Area of the catchment = 300 km^2

1	2	3	4	5	6	7	8	9	10	11
Time *t* in hours	(*t*/*K*)	*u*(*t*) (cm/h)	*u*(*t*) (m^3/s)	*u*(*t*) lagged by 1 hour	1-h UH (m^3/s)	S_1– Curve addition	Ordinate of S_1-Curve	3-h lagged S_1-Curve	DRH of 3 cm in 3 hours (m^3/s)	Ord. of 3-h UH (m^3/s)
0	0.000	0.0000	0.000		0.000	0.000	0.000		0.000	0.00
1	0.303	0.0003	0.246	0.000	0.123	0.000	0.123		0.123	0.40
2	0.606	0.0025	2.054	0.246	1.150	0.123	1.273		1.273	0.42
3	0.909	0.0075	6.271	2.054	4.162	1.273	5.435	0.000	5.435	1.81
4	1.212	0.0152	12.676	6.271	9.473	5.435	14.909	0.123	14.786	4.93
5	1.515	0.0245	20.444	12.676	16.560	14.909	31.469	1.273	30.196	10.07
6	1.818	0.0343	28.583	20.444	24.513	31.469	55.982	5.435	50.547	16.85
7	2.121	0.0434	36.209	28.583	32.396	55.982	88.378	14.909	73.469	24.49
8	2.424	0.0512	42.676	36.209	39.442	88.378	127.820	31.469	96.351	32.12
9	2.727	0.0571	47.600	42.676	45.138	127.820	172.958	55.982	116.975	38.99
10	3.030	0.0610	50.834	47.600	49.217	172.958	222.175	88.378	133.797	44.60
11	3.333	0.0628	52.411	50.834	51.623	222.175	273.798	127.820	145.978	48.66
12	3.636	0.0629	52.490	52.411	52.451	273.798	326.248	172.958	153.291	51.10
13	3.939	0.0615	51.303	52.490	51.897	326.248	378.145	222.175	155.970	51.99
14	4.242	0.0589	49.112	51.303	50.207	378.145	428.353	273.798	154.555	51.52
15	4.545	0.0554	46.180	49.112	47.646	428.353	475.998	326.248	149.750	49.92
16	4.848	0.0513	42.751	46.180	44.466	475.998	520.464	378.145	142.319	47.44
17	5.152	0.0468	39.039	42.751	40.895	520.464	561.359	428.353	133.006	44.34
18	5.455	0.0422	35.219	39.039	37.129	561.359	598.488	475.998	122.489	40.83
19	5.758	0.0377	31.431	35.219	33.325	598.488	631.812	520.464	111.348	37.12
20	6.061	0.0333	27.779	31.431	29.605	631.812	661.417	561.359	100.058	33.35
21	6.364	0.0292	24.337	27.779	26.058	661.417	687.475	598.488	88.988	29.66
22	6.667	0.0254	21.153	24.337	22.745	687.475	710.221	631.812	78.408	26.14
23	6.970	0.0219	18.253	21.153	19.703	710.221	729.924	661.417	68.507	22.84
24	7.273	0.0188	15.647	18.253	16.950	729.924	746.875	687.475	59.399	19.80
25	7.576	0.0160	13.332	15.647	14.489	746.875	761.364	710.221	51.143	17.05
26	7.879	0.0135	11.295	13.332	12.313	761.364	773.677	729.924	43.753	14.58
27	8.182	0.0114	9.520	11.295	10.408	773.677	784.085	746.875	37.210	12.40
28	8.485	0.0096	7.986	9.520	8.753	784.085	792.838	761.364	31.474	10.49
29	8.788	0.0080	6.669	7.986	7.328	792.838	800.166	773.677	26.488	8.83
30	9.091	0.0067	5.546	6.669	6.108	800.166	806.273	784.085	22.188	7.40
31	9.394	0.0055	4.594	5.546	5.070	806.273	811.344	792.838	18.505	6.17
32	9.697	0.0045	3.792	4.594	4.193	811.344	815.537	800.166	15.371	5.12
33	10.000	0.0037	3.119	3.792	3.456	815.537	818.992	806.273	12.719	4.24
34	10.303	0.0031	2.558	3.119	2.838	818.992	821.831	811.344	10.487	3.50
35	10.606	0.0025	2.091	2.558	2.324	821.831	824.155	815.537	8.618	2.87
36	10.909	0.0020	1.704	2.091	1.897	824.155	826.052	818.992	7.060	2.35
37	11.212	0.0017	1.385	1.704	1.545	826.052	827.597	821.831	5.766	1.92
38	11.515	0.0013	1.123	1.385	1.254	827.597	828.851	824.155	4.696	1.57
39	11.818	0.0011	0.909	1.123	1.016	828.851	829.867	826.052	3.815	1.27
40	12.121	0.0009	0.733	0.909	0.821	829.867	830.688	827.597	3.091	1.03

8.10 FLOOD CONTROL

The term *flood control* is commonly used to denote all the measures adopted to reduce damages to life and property by floods. Currently, many people prefer to use the term *flood management* instead of *flood control* as it reflects the activity more realistically. As there is always a possibility, however remote it may be, of an extremely large flood occurring in a river the complete control of the flood to a level of zero loss is neither physically possible nor economically feasible. The flood control measures that are in use can be classified as:

1. Structural measures:
   - Storage and detention reservoirs
   - Levees (flood embankments)
   - Flood ways (new channels)
   - Channel improvement
   - Watershed management
2. Non-structural methods:
   - Flood plain zoning
   - Flood forecast/warning
   - Evacuation and relocation
   - Flood insurance

STRUCTURAL METHODS

STORAGE RESERVOIRS Storage reservoirs offer one of the most reliable and effective methods of flood control. Ideally, in this method, a part of the storage in the reservoir is kept apart to absorb the incoming flood. Further, the stored water is released in a controlled way over an extended time so that downstream channels do not get flooded. Figure 8.15 shows an ideal operating plan of a flood control reservoir. As most of the present-day storage reservoirs have multipurpose commitments, the manipulation of reservoir levels to satisfy many conflicting demands is a very difficult and complicated task. It so happens that many storage reservoirs while reducing the floods and flood damages do not always aim at achieving optimum benefits in the flood-control aspect. To achieve complete flood control in the entire length of the river, a large number of reservoirs at strategic locations in the catchment will be necessary.

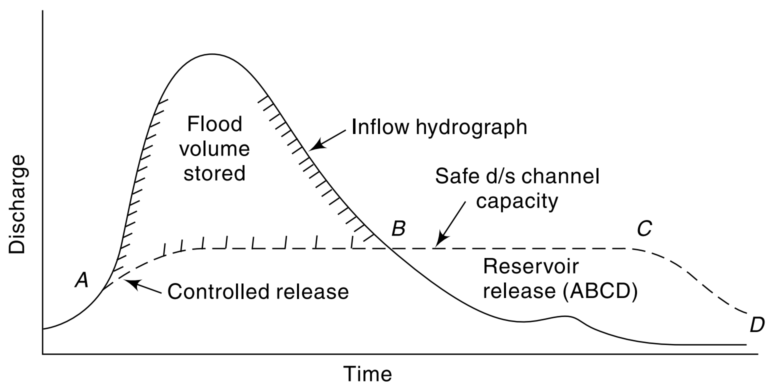


Fig. 8.15 Flood control operation of a reservoir

The Hirakud and Damodar Valley Corporation (DVC) reservoirs are examples of major reservoirs in the country which have specific volumes earmarked for flood absorption.

Detention Reservoirs A detention reservoir consists of an obstruction to a river with an uncontrolled outlet. These are essentially small structures and operate to reduce the flood peak by providing temporary storage and by restriction of the outflow rate. These structures are not common in India.

Levees Levees, also known as *dikes or flood embankments* are earthen banks constructed parallel to the course of the river to confine it to a fixed course and limited cross-sectional width. The heights of levees will be higher than the design flood level with sufficient free board. The confinement of the river to a fixed path frees large tracts of land from inundation and consequent damage (Fig. 8.16).

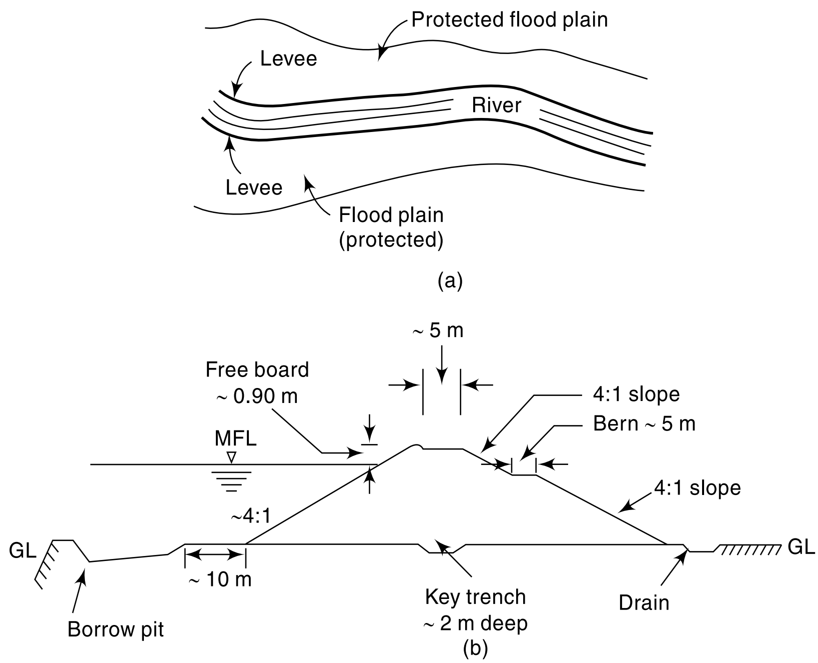


Fig. 8.16 A typical levee: (a) Plan (schematic), (b) Cross-section

Levees are one of the oldest and most common methods of flood-protection works adopted in the world. Also, they are probably the cheapest of structural flood-control measures. While the protection offered by a levee against food damage is obvious, what is not often appreciated is the potential damage in the event of a levee failure. The levees, being earth embankments require considerable care and maintenance. In the event of being overtopped, they fail and the damage caused can be enormous. In fact, the sense of protection offered by a levee encourages economic activity along the embankment and if the levee is overtopped the loss would be more than what would have been if there were no levees. Confinement of flood banks of a river by levees to a narrower space leads to higher flood levels for a given discharge. Further, if the bed levels of the river also rise, as they do in aggrading rivers, the top of the levees have to be raised at frequent time intervals to keep up its safety margin.

The design of a levee is a major task in which costs and economic benefits have to be considered. The cross-section of a levee will have to be designed like an earth dam for complete safety against all kinds of saturation and drawdown possibilities. In many instances, especially in locations where important structures and industries are to be protected, the water side face of levees are protected by stone or concrete revetment. Regular maintenance and contingency arrangements to fight floods are absolutely necessary to keep the levees functional.

Masonry structures used to confine the river in a manner similar to levees are known as *flood walls*. These are used to protect important structures against floods, especially where the land is at a premium.

FLOODWAYS Floodways are natural channels into which a part of the flood will be diverted during high stages. A floodway can be a natural or man-made channel and its location is controlled essentially by the topography. Generally, wherever they are feasible, floodways offer an economical alternative to other structural flood-control measures. To reduce the level of the river Jhelum at Srinagar, a supplementary channel has been constructed to act as a floodway with a capacity of 300 m^3/s. This channel is located 5 km upstream of Srinagar city and has its outfall in lake Wullar. In Andhra Pradesh, a floodway has been constructed to transfer a part of the flood waters of the river Budamaru to river Krishna to prevent flood damages to the urban areas lying on the downstream reaches of the river Budamaru.

CHANNEL IMPROVEMENT The works under this category involve:

- Widening or deepening of the channel to increase the cross-sectional area
- Reduction of the channel roughness, by clearing of vegetation from the channel perimeter
- Short circuiting of meander loops by cutoff channels, leading to increased slopes.

All these three methods are essentially short-term measures and require continued maintenance.

WATERSHED MANAGEMENT Watershed management and land treatment in the catchment aims at cutting down and delaying the runoff before it gets into the river. Watershed management measures include developing the vegetative and soil cover in conjunction with land treatment words like Nalabunds, check dams, contour bunding, zing terraces etc. These measures are towards improvement of water infiltration capacity of the soil and reduction of soil erosion. These treatments cause increased infiltration, greater evapotranspiration and reduction in soil erosion; all leading to moderation of the peak flows and increasing of dry weather flows. Watershed treatment is nowadays an integral part of flood management. It is believed that while small and medium floods are reduced by watershed management measures, the magnitude of extreme floods are unlikely to be affected by these measures.

NON-STRUCTURAL METHODS

The flood management strategy has to include the philosophy of *Living with the floods*. The following non-structural measures encompass this aspect.

FLOOD PLAIN ZONING When the river discharges are very high, it is to be expected that the river will overflow its banks and spill into flood plains. In view of the increasing pressure of population this basic aspects of the river are disregarded and there are greater encroachment of flood plains by man leading to distress.

Flood plain management identifies the flood prone areas of a river and regulates the land use to restrict the damage due to floods. The locations and extent of areas

Zone	Flood Return Period	Example of Uses
1	100 Years	Residential houses, Offices, Factories, etc.
2	25 Years	Parks
3	Frequent	No construction/Encroachments

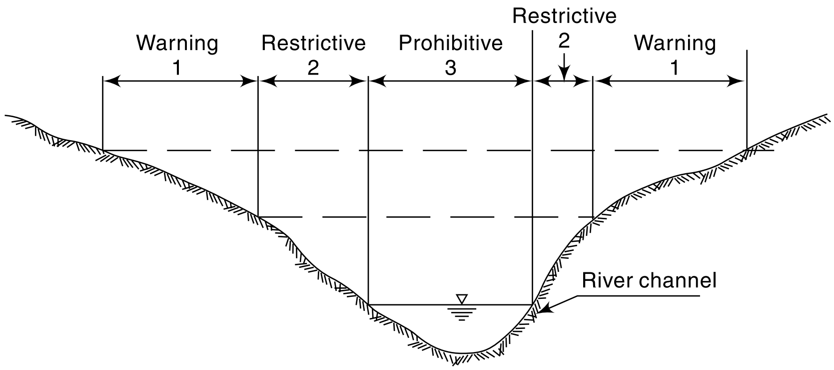


Fig. 8.17 Conceptual Zoning of a Flood Plain

likely to be affected by floods of different return periods are identified and development plans of these areas are prepared in such a manner that the resulting damages due to floods are within acceptable limits of risk. Figure 8.17 shows a conceptual zoning of a flood prone area.

FLOOD FORECASTING AND WARNING Forecasting of floods sufficiently in advance enables a warning to be given to the people likely to be affected and further enables civil authorities to take appropriate precautionary measures. It thus forms a very important and relatively inexpensive non-structural flood management measure. However, it must be realised that a flood warning is meaningful only if it is given sufficiently in advance. Further, erroneous warnings will cause the populace to lose confidence and faith in the system. Thus the dual requirements of reliability and advance notice are the essential ingredients of a flood-forecasting system.

The flood forecasting techniques can be broadly divided into three categories:

(i) Short range forecasts
(ii) Medium range forecasts
(iii) Long range forecasts.

Short-Range Forecasts In this the river stages at successive stations on a river are correlated with hydrological parameters, such as rainfall over the local area, antecedent precipitation index, and variation of the stage at the upstream base point during the travel time of a flood. This method can give advance warning of 12-40 hours for floods. The flood forecasting used for the metropolitan city of Delhi is based on this technique.

Medium-Range Forecasts In this method rainfall–runoff relationships are used to predict flood levels with warning of 2–5 days. Coaxial graphical correlations of runoff, with rainfall and other parameters like the time of the year, storm duration and antecedent wetness have been developed to a high stage of refinement by the US Weather Bureau.

Long-Range Forecasts Using radars and meteorological satellite data, advance information about critical storm-producing weather systems, their rain potential and time of occurrence of the event are predicted well in advance.

Evacuation and Relocation Evacuation of communities along with their live stocks and other valuables in the chronic flood affected areas and relocation of them in nearby safer locations is an area specific measure of flood management. This would be considered as non-structural measure when this activity is a temporary measure confined to high floods. However, permanent shifting of communities to safer locations would be termed as *structural measure*. Raising the elevations of buildings and public utility installations above normal flood levels is termed as *flood proofing* and is sometimes adopted in coastal areas subjected to severe cyclones.

Flood Insurance Flood insurance provides a mechanism for spreading the loss over large numbers of individuals and thus modifies the impact of loss burden. Further, it helps, though indirectly, flood plain zoning, flood forecasting and disaster preparedness activities.

8.11 FLOOD CONTROL IN INDIA

In India the Himalayan rivers account for nearly 60% of the flood damage in the country. Floods in these rivers occur during monsoon months and usually in the months of August or September. The damages caused by floods are very difficult to estimate and a figure of Rs 5000 crores as the annual flood damage in the country gives the right order of magnitude.

During 1953–2000, the average number of human lives and cattle lost due to floods in the country were 1595 and 94,000 respectively. It is estimated that annually, on an average about 40 M ha of land is liable to flooding and of this about 14 M ha have some kind of flood-control measure. At the beginning of the current millennium, in the country, as a part of flood control measure there were about 15800 km of levees and about 32000 km of drainage channels affording protection from floods.

On an average about 7.5 M ha land is affected by floods annually. Out of this, about 3.5 M ha are lands under crops. Similarly, annually about 3.345 lakhs of people are affected and about 12.15 lakhs houses are damaged by floods. On an average, about 60 to 80% of flood damages occur in the states of U.P., Bihar, West Bengal, Assam and Orissa.

Flood forecasting is handled by CWC in close collaboration with the IMD which lends meteorological data support. Nine flood Met offices with the aid of recording raingauges provide daily synoptic situations, actual rainfall amounts and rainfall forecasts to CWC. The CWC has 157 flood-forecasting stations, of which 132 stations are for river stage forecast and 25 for inflow forecast, situated in various basins to provide a forecasting service.

A national program for flood management was launched in 1954 and an amount of 3165 crores was spent till 1992. The ninth plan (1997–2002) had an outlay of 2928 crores for flood management. These figures highlight the seriousness of the flood problem and the efforts made towards mitigating flood damages. The experience gained in the flood control measures in the country are embodied in the report of the Rashtriya Barh Ayog (RBA) (National Flood Commission) submitted in March 1980. This report, containing a large number of recommendations on all aspects of flood control forms the basis for the evolution of the present national policy on floods. According to the national water policy (1987), while structural flood control measures will continue to be necessary, the emphasis should be on non-structural methods so as to reduce the recurring expenditure on flood relief.

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Revision Questions

8.1 Distinguish between:
(a) Hydraulic and hydrologic method of flood routing
(b) Hydrologic storage routing and hydrologic channel routing
(c) Prism storage and wedge storage

8.2 What are the basic equations used for flood routing by
(a) Hydrologic method, and (b) Hydraulic method

8.3 Define the problem of level pool routing. Describe a commonly used method of reservoir routing.

8.4 Describe a numerical method of hydrologic reservoir routing.

8.5 What is the basic premise in the Muskingum method of flood routing? Describe a procedure for estimating the values of the Muskingum coefficients K and x for a stream reach.

8.6 Describe the Muskingum method of routing an inflow hydrograph through a channel reach. Assume the values of the coefficients K and x for the reach are known.

8.7 Explain briefly
(a) Isochrone (b) Time of concentration
(c) Linear reservoir (d) Linear channel

8.8 Explain briefly the basic principles involved in the development of IUH by
(a) Clark's method, and (b) Nash's model.

8.9 Describe the various structural methods adopted for management of floods.

8.10 Describe the various non-structural measures of flood management.

8.11 Describe the problem of floods and their management with special reference to Indian scene.

Problems

8.1 The storage, elevation and outflow data of a reservoir are given below:

Elevation (m)	Storage 10^6 m^3	Outflow discharge (m^3/s)
299.50	4.8	0
300.20	5.5	0
300.70	6.0	15
301.20	6.6	40
301.70	7.2	75
302.20	7.9	115
302.70	8.8	160

The spillway crest is at elevation 300.20 m. The following flood flow is expected into the reservoir.

Time (h)	0	3	6	9	12	15	18	21	24	27
Discharge (m^3/s)	10	20	52	60	53	43	32	22	16	10

If the reservoir surface is at elevation 300.00 m at the commencement of the inflow, route the flood to obtain (a) the outflow hydrograph and (b) the reservoir elevation *vs* time curve.

8.2 Solve Prob. 8.1 if the reservoir elevation at the start of the inflow hydrograph is at 301.50 m.

8.3 A small reservoir has the following storage elevation relationship.

Elevation (m)	55.00	58.00	60.00	61.00	62.00	63.00
Storage (10^3 m^3)	250	650	1000	1250	1500	1800

A spillway provided with its crest at elevation 60.00 m has the discharge relationship $Q = 15\,H^{3/2}$, where H = head of water over the spillway crest. When the reservoir elevation is at 58.00 m a flood as given below enters the reservoir. Route the flood and determine the maximum reservoir elevation, peak outflow and attenuation of the flood peak.

Time (h)	0	6	12	15	18	24	30	36	42
Inflow (m^3/s)	5	20	40	60	50	32	22	15	10

8.4 The storage-elevation-discharge characteristic of a reservoir is as follows:

Elevation (m)	100.00	100.50	101.00
Discharge (m^3/s)	12	18	25
Storage (10^3 m^3)	400	450	550

When the reservoir elevation is at 101.00 m the inflow is at a constant rate of 10 m^3/s. Find the time taken for the water surface to drop to the elevation 100.00 m.

8.5 A small reservoir has a spillway crest at elevation 200.00 m. Above this elevation, the storage and outflow from the reservoir can be expressed as

Storage: $S = 36000 + 18000\,y$ (m^3)
Outflow: $Q = 10\,y$ (m^3/s)
where y = height of the reservoir level above the spillway crest in m.

Route an inflow flood hydrograph which can be approximated by a triangle as

$I = 0$ at $t = 0$ h
$I = 30$ m^3/s at $t = 6$ h (peak flow)
$I = 0$ at $t = 26$ h (end of inflow).

Assume the reservoir elevation as 200.00 m at $t = 0$ h.
Use a time step of 2 h.

8.6 A detention reservoir was found to have a linear storage discharge relationship, $Q = KQ$

(a) Show that the storage routing equation of an inflow hydrograph through this reservoir is $Q_2 = C_1 \bar{I}_1 + C_2 Q_2$ where C_1 and C_2 are constants and $\bar{I}_1 = (I_1 + I_2)/2$. Determine the values of C_1 and C_2 in terms of K and the routing time step Δt.

(b) If $K = 4.0$ h and $\Delta t = 2$ h, route the following inflow hydrograph through this reservoir. Assume the initial condition that at $t = 0$, $I_1 = Q_1 = 0$.

Time (h)	0	2	4	6	8	10	12	14	16	18
Inflow (m^3/s)	0	20	60	100	80	60	40	30	20	10

8.7 Observed values of inflow and outflow hydrographs at the ends of a reach in a river are given below. Determine the best values of K and x for use in the Muskingum method of flood routing.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	20	80	210	240	215	170	130	90	60	40	28	16
Outflow (m^3/s)	20	20	50	150	200	210	185	155	120	85	55	23

8.8 Route the following flood through a reach for which $K = 22$ h and $x = 0.25$. Plot the inflow and outflow hydrographs and determine the peak lag and attenuation. At $t = 0$ the outflow discharge is 40 m^3/s.

Time (h)	0	12	24	36	48	60	72	84	96	108	120	132	144
Inflow (m^3/s)	40	65	165	250	240	205	170	130	115	85	70	60	54

8.9 The storage in the reach of a stream has been studied. The values of x and K in Muskingum equation have been identified as 0.28 and 1.6 days. If the inflow hydrograph to the reach is as given below, compute the outflow hydrograph. Assume the outflow from the reach at $t = 0$ as 3.5 m^3/s.

Time (h)	0	6	12	18	24	30
Inflow (m^3/s)	35	55	92	130	160	140

8.10 Route the following flood hydrograph through a river reach for which Muskingum coefficient $K = 8$ h and $x = 0.25$.

Time (h)	0	4	8	12	16	20	24	28
Inflow (m^3/s)	8	16	30	30	25	20	15	10

The initial outflow discharge from the reach is 8.0 m^3/s.

8.11 A stream has a uniform flow of 10 m^3/s. A flood in which the discharge increases linearly from 10 m^3/s to a peak of 70 m^3/s in 6 h and then decreases linearly to a value of 10 m^3/s in 24 h from the peak arrives at a reach. Route the flood through the reach in which $K = 10$ h and $x = 0$

8.12 A drainage basin has area = 137 km^2, storage constant $K = 9.5$ h and time of concentration = 7 h. The following inter-isochrone area distribution data are available:

Time (h)	0–1	1–2	2–3	3–4	4–5	5–6	6–7
Inter-isochrone area (km^2)	10	38	20	45	32	10	2

Determine (a) the IUH and (b) the 1-h unit hydrograph for the catchment.

8.13 Solve Prob. 8.11 K = 10 h and x = 0.5. Determine the peak lag and attenuation and compare with the corresponding values of Prob. 8.11.

8.14 Show that the reservoir routing equation for a linear reservoir is

$$\frac{dQ}{dt} + \alpha Q = \alpha I$$

where α is a constant. Obtain the outflow from such a reservoir due to an inflow $I = I_0 + \beta t$ occurring from $t = 0$ to t_0 with the boundary condition $Q = 0$ at $t = 0$.

8.15 Given that $n = 4.0$ and $K = 6.0$ are the appropriate values of the coefficients in the Nash model for IUH of a catchment, determine the ordinates of IUH in cm/h at 3 hours interval. If the catchment area is 500 km^2, determine the ordinates of the IUH in m^3/s.

8.16 Show that in the IUH obtained by using the Nash model the peak flow occurs at a time

$$t_p = K\,(n - 1)$$

and the magnitude of the peak flow is

$$u(t)_p = \frac{1}{K\Gamma(n)}\, e^{(1-n)}\,(n-1)^{n-1}$$

8.17 For a sub-basin in lower Godavari catchment, with an area of 250 km^2 the following values of Nash model coefficients were found appropriate: n = 3.3 and K = 1.69 h. Determine the co-ordinates of (a) IUH at 1-h interval and (b) 1-hour UH at 1-h interval.

8.18 For a catchment X of area 100 km^2, an ERH of an isolated storm and its corresponding DRH were analysed to determine the first and second moments relative to the total area of the respective curves and the following values were obtained:

(1) (First moment of the curve)/(total area of the curve):

ERH = 11.0 h DRH = 25.0 h

(2) (Second moment of the curve)/(total area of the curve):

ERH = 170 h^2 DRH = 730 h^2

Determine the IUH with ordinates at 2 hour interval for catchment X by using Nash model.

8.19 For a catchment the effective rainfall hyetograph due to an isolated storm is given in Table 8.9(a). The direct runoff hydrograph resulting from the above storm is given in Table 8.9(b). Determine the values of Nash model IUH coefficients n and K for the above catchment.

Table 8.9(a) ERH

Time (h)	0 to 6	6 to 12	12 to 18	18 to 24
ERH ordinates (cm/s)	4.3	2.8	3.9	2.7

Table 8.9(b) DRH

Time (h)	**DR m^3/s**	**Time (h)**	**DR m^3/s**
0	0	36	160
6	20	42	75
12	140	48	30
18	368	54	10
24	380	60	0
30	280		

Objective Questions

8.1 The hydrologic flood-routing methods use
(a) Equation of continuity only (b) Both momentum and continuity equations
(c) Energy equation only (d) Equation of motion only

8.2 The hydraulic methods of flood routing use
(a) Equation of continuity only
(b) Both the equation of motion and equation of continuity
(c) Energy equation only
(b) Equation of motion only

8.3 The St Venant equations for unsteady open-channel flow are
(a) continuity and momentum equations
(b) momentum equation in two different forms
(c) momentum and energy equations
(d) energy and continuity equations.

8.4 The prism storage in a river reach during the passage of a flood wave is
(a) a constant (b) a function of inflow and outflow
(c) function of inflow only (d) function of outflow only

8.5 The wedge storage in a river reach during the passage of a flood wave is
(a) a constant (b) negative during rising phase
(c) positive during rising phase (d) positive during falling phase

8.6 In routing a flood through a reach the point of intersection of inflow and outflow hydrographs coincides with the peak of outflow hydrograph
(a) in all cases of flood routing
(b) when the inflow is into a reservoir with an uncontrolled outlet
(c) in channel routing only
(d) in all cases of reservoir routing.

8.7 Which of the following is a proper reservoir-routing equation?

(a) $\frac{1}{2}(I_1 - I_2)\,\Delta t + \left(S_1 + \frac{Q_1\,\Delta t}{2}\right) = \left(S_2 - \frac{Q_2\,\Delta t}{2}\right)$

(b) $(I_1 + I_2)\,\Delta t + \left(\frac{2\,S_1}{\Delta t} - Q_1\right) = \left(\frac{2\,S_2}{\Delta t} + Q_2\right)$

(c) $\frac{1}{2}\,(I_1 + I_2)\,\Delta t + \left(S_2 - \frac{Q_2\,\Delta t}{2}\right) = \left(S_1 + \frac{Q_1\,\Delta t}{2}\right)$

(d) $(I_1 + I_2) + \left(\frac{2\,S_1}{\Delta t} - Q_1\right) = \left(\frac{2\,S_2}{\Delta t} + Q_2\right)$

8.8 The Muskingum method of flood routing is a
(a) form of reservoir routing method
(b) hydraulic routing method
(c) complete numerical solution of St Venant equations
(d) hydrologic channel-routing method.

8.9 The Muskingum method of flood routing assumes the storage S is related to inflow rate I and outflow rate Q of a reach as $S =$
(a) $K[xI - (1 - x)Q]$ (b) $K[xQ + (1 - x)I]$
(c) $K[xI + (1 - x)Q]$ (d) $Kx[I - (1 - x)Q]$

8.10 The Muskingum method of flood routing gives $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$. The coefficients in this equation will have values such that
(a) $C_0 + C_1 = C_2$ (b) $C_0 - C_1 - C_2 = 1$
(c) $C_0 + C_1 + C_2 = 0$ (d) $C_0 + C_1 + C_2 = 1$.

8.11 The Muskingum channel routing equation is written for the outflow from the reach Q in terms of the inflow I and coefficients C_0, C_1 and C_2 as

(a) $Q_2 = C_0 I_0 + C_1 Q_1 + C_2 I_2$ (b) $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_2$
(c) $Q_2 = C_0 I_0 + C_1 I_1 + C_2 I_2$ (d) $Q_2 = C_0 Q_0 + C_1 Q_1 + C_2 I_2$

8.12 In the Muskingum method of channel routing the routing equation is written as $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$, If the coefficients $K = 12$ h and $x = 0.15$ and the time step for routing $\Delta t = 4$ h, the coefficient C_0 is

(a) 0.016 (b) 0.048 (c) 0.328 (d) 0.656

8.13 In the Muskingum method of channel routing the weighing factor x can have a value

(a) between –0.5 to 0.5 (b) between 0.0 to 0.5
(c) between 0.0 to 1.0 (d) between –1.0 to +1.0

8.14 In the Muskingum method of channel routing if $x = 0.5$, it represents an outflow hydrograph

(a) that has reduced peak
(b) with an amplified peak
(c) that is exactly the same as the inflow hydrograph
(d) with a peak which is exactly half of the inflow peak

8.15 If the storage S, inflow rate I and outflow rate Q for a river reach is written as

$$S = K\,[x\,I^n + (1 - x)\,Q^n]$$

(a) $n = 0$ represents storage routing through a reservoir
(b) $n = 1$ represents the Muskingum method
(c) $n = 0$ represents the Muskingum method
(d) $n = 0$ represents a linear channel.

8.16 A linear reservoir is one in which the

(a) volume varies linearly with elevation
(b) storage varies linearly with the outflow rate
(c) storage varies linearly with time
(d) storage varies linearly with the inflow rate.

8.17 An isochrone is a line on the basin map

(a) joining raingauge stations with equal rainfall duration
(b) joining points having equal standard time
(c) connecting points having equal time of travel of the surface runoff to the catchment outlet
(d) that connects points of equal rainfall depth in a given time interval.

8.18 In the Nash model for IUH given by

$$u(t) = \frac{1}{K\Gamma(n)}\,(t/K)^{n-1}\,(e)^{-t/K}$$

the usual units of $u(t)$, n and K are, respectively;

(a) cm/h, h, h (b) h^{-1}, *h*, *h*
(c) h^{-1}, dimensionless number, h (d) cm/h, dimensionless number, h

8.19 The peak ordinate of the IUH of a catchment was obtained from Nash model as 0.03 cm/h. If the area of the catchment is 550 km^2 the value of the peak ordinate in m^3/s is

(a) 16.5 (b) 45.83 (c) 30.78 (d) 183.3

8.20 If the Gamma function Γ (1.5) = 0.886, the value of Γ (0.5) is

(a) 0.5907 (b) 1.329 (c) –0.886 (d) 1.772

8.21 In the Nash model for IUH, if M_{I1} = the first moment of ERH about the time origin divided by the total effective rainfall and M_{Q1} = the first moment of DRH about the time origin divided by the total direct runoff, then

(a) $M_{Q1} - M_{I1} = nK$ (b) $M_{I1} - M_{Q1} = nK^2$
(c) $M_{Q1} - M_{I1} = n\,(n + 1)\,K$ (d) $M_{I1} - M_{Q1} = 2\,nK$

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Chapter 9

GROUNDWATER

9.1 INTRODUCTION

In the previous chapters various aspects of surface water hydrology that deal with surface runoff have been discussed. Study of subsurface flow is equally important since about 30% of the world's fresh water resources exist in the form of groundwater. Further, the subsurface water forms a critical input for the sustenance of life and vegetation in arid zones. Due to its importance as a significant source of water supply, various aspects of groundwater dealing with the exploration, development and utilization have been extensively studied by workers from different disciplines, such as geology, geophysics, geochemistry, agricultural engineering, fluid mechanics and civil engineering and excellent treatises are available, (Ref. 1, 2 and 4 through 10). This chapter confines itself to only an elementary treatment of the subject of groundwater as a part of engineering hydrology.

9.2 FORMS OF SUBSURFACE WATER

Water in the soil mantle is called *subsurface water* and is considered in two zones (Fig. 9.1):

1. Saturated zone, and
2. Aeration zone.

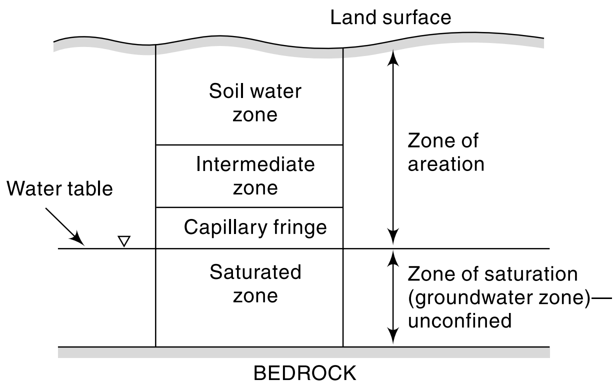

Fig. 9.1 Classification of Subsurface Water

SATURATED ZONE

This zone, also known as *groundwater zone,* is the space in which all the pores of the soil are filled with water. The water table forms its upper limit and marks a free surface, i.e. a surface having atmospheric pressure.

ZONE OF AERATION

In this zone the soil pores are only partially saturated with water. The space between the land surface and the water table marks the extent of this zone. The zone of aeration has three subzones.