Chapter
6

HYDROGRAPHS

6.1 INTRODUCTION

While long-term runoff concerned with the estimation of yield was discussed in the previous chapter, the present chapter examines in detail the short-term runoff phenomenon. The storm hydrograph is the focal point of the present chapter.

Consider a concentrated storm producing a fairly uniform rainfall of duration, *D* over a catchment. After the initial losses and infiltration losses are met, the rainfall excess reaches the stream through overland and channel flows. In the process of translation a certain amount of storage is built up in the overland and channel-flow phases. This storage gradually depletes after the cessation of the rainfall. Thus there is a time lag between the occurrence of rainfall in the basin and the time when that water passes the gauging station at the basin outlet. The runoff measured at the stream-gauging station will give a typical hydrograph as shown in Fig. 6.1. The duration of the rainfall is also marked in this figure to indicate the time lag in the rainfall and runoff. The hydrograph of this kind which results due to an isolated storm is typically single-peaked skew distribution of discharge and is known variously as *storm hydrograph*, *flood hydrograph* or simply *hydrograph*. It has three characteristic regions: (i) the rising limb *AB*, joining point *A*, the starting point of the rising curve and point *B*, the point of inflection, (ii) the crest segment *BC* between the two points of inflection with a peak *P* in between, (iii) the falling limb or *depletion curve CD* starting from the second point of inflection *C*.

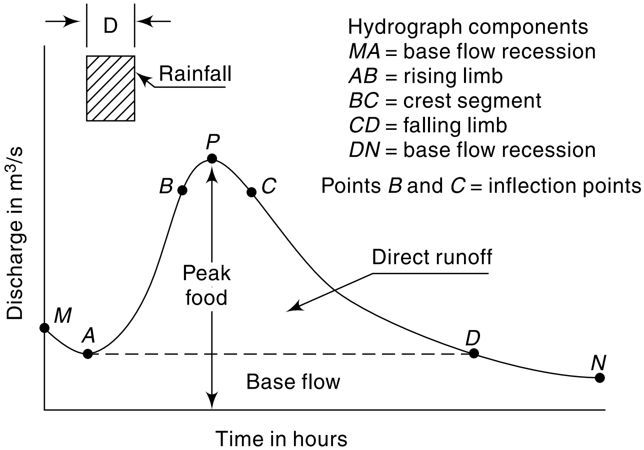

Fig. 6.1 Elements of a Flood Hydrograph

The hydrograph is the response of a given catchment to a rainfall input. It consists of flow in all the three phases of runoff, viz. surface runoff, interflow and base flow, and embodies in itself the integrated effects of a wide variety of catchment and rainfall parameters having complex interactions. Thus two different storms in a given catchment produce hydrographs differing from each other. Similarly, identical storms in two catchments produce hydrographs that are different. The interactions of various storms and catchments are in general extremely complex. If one examines the record of a large number of flood hydrographs of a stream, it will be found that many of them will have kinks, multiple peaks, etc. resulting in shapes much different from the simple single-peaked hydrograph of Fig. 6.1. These complex hydrographs are the result of storm and catchment peculiarities and their complex interactions. While it is theoretically possible to resolve a complex hydrograph into a set of simple hydrographs for purposes of hydrograph analysis, the requisite data of acceptable quality are seldom available. Hence, simple hydrographs resulting from isolated storms are preferred for hydrograph studies.

6.2 FACTORS AFFECTING FLOOD HYDROGRAPH

The factors that affect the shape of the hydrograph can be broadly grouped into climatic factors and physiographic factors. Each of these two groups contains a host of factors and the important ones are listed in Table 6.1. Generally, the climatic factors control the rising limb and the recession limb is independent of storm characteristics, being determined by catchment characteristics only. Many of the factors listed in Table 6.1 are interdependent. Further, their effects are very varied and complicated. As such only important effects are listed below in qualitative terms only.

Table 6.1 Factors Affecting Flood Hydrograph

Physiographic factors	Climatic factors
1. Basin characterstics: (a) Shape (b) size (c) slope (d) nature of the valley (e) elevation (f) drainage density 2. Infiltration characteristics: (a) land use and cover (b) soil type and geological conditions (c) lakes, swamps and other storage 3. Channel characteristics: cross-section, roughness and storage capacity	1. Storm characterstics: precipitation, intensity, duration, magnitude and movement of storm. 2. Initial loss 3. Evapotranspiration

Shape of the Basin

The shape of the basin influences the time taken for water from the remote parts of the catchment to arrive at the outlet. Thus the occurrence of the peak and hence the shape of the hydrograph are affected by the basin shape. Fan-shaped, i.e. nearly semi-circular shaped catchments give high peak and narrow hydrographs while elongated catchments give broad and low-peaked hydrographs. Figure 6.2 shows schematically the hydrographs from three catchments having identical infiltration characteristics due to identical rainfall over the catchment. In catchment *A* the hydrograph is skewed to the left, i.e. the peak occurs relatively quickly. In catchment *B*, the hydrograph is skewed to the right, the peak occurring with a relatively longer lag. Catchment *C* indicates the complex hydrograph produced by a composite shape.

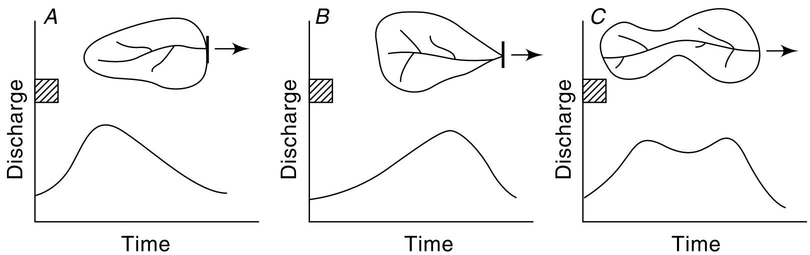

Fig. 6.2 Effect of Catchment Shape on the Hydrograph

Size

Small basins behave different from the large ones in terms of the relative importance of various phases of the runoff phenomenon. In small catchments the overland flow phase is predominant over the channel flow. Hence the land use and intensity of rainfall have important role on the peak flood. On large basins these effects are suppressed as the channel flow phase is more predominant. The peak discharge is found to vary as A^n where A is the catchment area and n is an exponent whose value is less than unity, being about 0.5. The time base of the hydro-graphs from larger basins will be larger than those of corresponding hydrographs from smaller basins. The duration of the surface runoff from the time of occurrence of the peak is proportional to A^m, where m is an exponent less than unity and is of the order of magnitude of 0.2.

Slope

The slope of the main stream controls the velocity of flow in the channel. As the recession limb of the hydrograph represents the depletion of storage, the stream channel slope will have a pronounced effect on this part of the hydrograph. Large stream slopes give rise to quicker depletion of storage and hence result in steeper recession limbs of hydrographs. This would obviously result in a smaller time base.

The basin slope is important in small catchments where the overland flow is relatively more important. In such cases the steeper slope of the catchment results in larger peak discharges.

Drainage Density

The drainage density is defined as the ratio of the total channel length to the total drainage area. A large drainage density creates situation conducive for quick disposal of runoff down the channels. This fast response is reflected in a pronounced peaked discharge. In basins with smaller drainage densities, the overland flow is predominant and the resulting hydrograph is squat with a slowly rising limb (Fig. 6.3).

Land Use

Vegetation and forests increase the infiltration and storage capacities of the soils. Further, they cause considerable retardance to the overland flow. Thus the vegetal cover reduces the peak flow. This effect is usually very pronounced in small catchments of area less than 150 km^2. Further, the effect of the vegetal cover is prominent in small storms. In general, for two catchments of equal area, other factors being identical, the peak discharge is higher for a catchment that has a lower density of forest cover. Of the various factors that control the peak discharge, probably the only factor that can be manipulated is land use and thus it represents the only practical means of exercising long-term natural control over the flood hydrograph of a catchment.

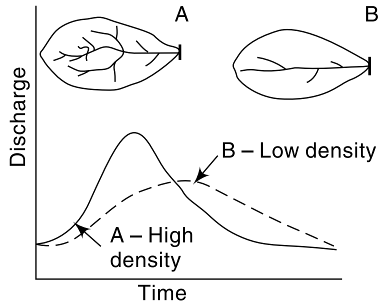

Fig. 6.3 Role of Drainage Density on the Hydrograph

Climatic Factors

Among climatic factors the intensity, duration and direction of storm movement are the three important ones affecting the shape of a flood hydrograph. For a given duration, the peak and volume of the surface runoff are essentially proportional to the intensity of rainfall. This aspect is made use of in the unit hydrograph theory of estimating peak-flow hydrographs, as discussed in subsequent sections of this chapter. In very small catchments, the shape of the hydrograph can also be affected by the intensity.

The duration of storm of given intensity also has a direct proportional effect on the volume of runoff. The effect of duration is reflected in the rising limb and peak flow. Ideally, if a rainfall of given intensity i lasts sufficiently long enough, a state of equilibrium discharge proportional to iA is reached.

If the storm moves from upstream of the catchment to the downstream end, there will be a quicker concentration of flow at the basin outlet. This results in a peaked hydrograph. Conversely, if the storm movement is up the catchment, the resulting hydrograph will have a lower peak and longer time base. This effect is further accentuated by the shape of the catchment, with long and narrow catchments having hydrographs most sensitive to the storm-movement direction.

6.3 COMPONENTS OF A HYDROGRAPH

As indicated earlier, the essential components of a hydrograph are: (i) the rising limb, (ii) the crest segment, and (iii) the recession limb (Fig. 6.1). A few salient features of these components are described below.

Rising Limb

The rising limb of a hydrograph, also known as *concentration curve* represents the increase in discharge due to the gradual building up of storage in channels and over the catchment surface. The initial losses and high infiltration losses during the early period of a storm cause the discharge to rise rather slowly in the initial periods. As the storm continues, more and more flow from distant parts reach the basin outlet. Simultaneously the infiltration losses also decrease with time. Thus under a uniform storm over the catchment, the runoff increases rapidly with time. As indicated earlier, the basin and storm characteristics control the shape of the rising limb of a hydrograph.

Crest Segment

The crest segment is one of the most important parts of a hydrograph as it contains the peak flow. The peak flow occurs when the runoff from various parts of the catchment simultaneously contribute amounts to achieve the maximum amount of flow at the basin outlet. Generally for large catchments, the peak flow occurs after the cessation of rainfall, the time interval from the centre of mass of rainfall to the peak being essentially controlled by basin and storm characteristics. Multiple-peaked complex hydrographs in a basin can occur when two or more storms occur in succession. Estimation of the peak flow and its occurrence, being important in flood-flow studies are dealt with in detail elsewhere in this book.

Recession Limb

The recession limb, which extends from the point of inflection at the end of the crest segment (point *C* in Fig. 6.1) to the commencement of the natural groundwater flow (point *D* in Fig. 6.1) represents the withdrawal of water from the storage built up in the basin during the earlier phases of the hydrograph. The starting point of the recession limb, i.e. the point of inflection represents the condition of maximum storage. Since the depletion of storage takes place after the cessation of rainfall, the shape of this part of the hydrograph is independent of storm characteristics and depends entirely on the basin characteristics.

The storage of water in the basin exists as (i) surface storage, which includes both surface detention and channel storage, (ii) interflow storage, and (iii) groundwater storage, i.e. base-flow storage. Barnes (1940) showed that the recession of a storage can be expressed as

$$Q_t = Q_0 K_r^t \tag{6.1}$$

in which Q_t is the discharge at a time t and Q_0 is the discharge at $t = 0$; K_r is a recession constant of value less than unity. Equation (6.1) can also be expressed in an alternative form of the exponential decay as

$$Q_t = Q_0 e^{-at} \tag{6.1a}$$

where $a = -\ln K_r$.

The recession constant K_r can be considered to be made up of three components to account for the three types of storages as

$$K_r = K_{rs} \cdot K_{ri} \cdot K_{rb}$$

where K_{rs} = recession constant for surface storage, K_{ri} = recession constant for interflow and K_{rb} = recession constant for base flow. Typically the values of these recession constants, when time t is in days, are

$$K_{rs} = 0.05 \text{ to } 0.20 \qquad K_{ri} = 0.50 \text{ to } 0.85 \qquad K_{rb} = 0.85 \text{ to } 0.99$$

When the interflow is not significant, K_{ri} can be assumed to be unity.

If suffixes 1 and 2 denote the conditions at two time instances t_1 and t_2, from Eq. (6.1) $$\frac{Q_1}{Q_2} = K_r^{(t_1 - t_2)} \quad (6.2)$$

or from Eq. (6.1a) $$\frac{Q_1}{Q_2} = e^{-a(t_1 - t_2)} \quad (6.2a)$$

Equation 6.1 (and also 6.1a) plots as a straight line when plotted on a semi-log paper with discharge on the log–scale. The slope of this line represents the recession constant. Using this property and using Eq. 6.2 (or 6.2a) the value of K_r for a basin can be estimated by using observed recession data of a flood hydrograph. Example 6.1 explains the procedure in detail.

The storage S_t remaining at any time t is obtained as

$$S_t = \int_t^{\infty} Q_t\, dt = \int_t^{\infty} Q_0 e^{-at}\, dt = \frac{Q_t}{a} \quad (6.3)$$

EXAMPLE 6.1 *The recession limb of a flood hydrograph is given below. The time is indicated from the arrival of peak. Assuming the interflow component to be negligible, estimate the base flow and surface flow recession coefficients. Also, estimate the storage at the end of day-3.*

Time from peak (day)	Discharge (m^3/s)	Time from Peak (day)	Discharge (m^3/s)
0.0	90	3.5	5.0
0.5	66	4.0	3.8
1.0	34	4.5	3.0
1.5	20	5.0	2.6
2.0	13	5.5	2.2
2.5	9.0	6.0	1.8
3.0	6.7	6.5	1.6
		7.0	1.5

SOLUTION: The data are plotted on a semi-log paper with discharge on the log-scale. The data points from $t = 4.5$ days to 7.0 days are seen to lie on straight line (line AB in Fig. 6.4). This indicates that the surface flow terminates at $t = 4.5$ days. The best fitting exponential curve for this straight-line portion (obtained by use of MS Excel) is

$$Q_t = 11.033e^{-0.2927t} \text{ with } R^2 = 0.9805.$$

The base flow recession coefficient K_{rb} is obtained as

$\ln K_{rb} = -0.2927$ and as such $K_{rb} = 0.746$.

[Alternatively, by using the graph, the value of K_{rb} could be obtained by selecting two points 1 and 2 on the straight line AB and using Eq. (6.2)].

The base flow recession curve is extended till $t \approx 1$ day as shown by line ABM Fig. 6.4. The Surface runoff depletion is obtained by subtracting the base flow from the given recession limb of the flood hydrograph. The computations are shown in the Table given on the next page.

The surface flow values (Col. 4 of Table above) are plotted against time as shown in Fig. 6.4. It is seen that these points lie on a straight line, *XY*. The best fitting exponential curve for this straight-line portion *XY* (obtained by use of MS Excel) is

$$Q_t = 106.84e^{-1.3603t} \text{ with } R^2 = 0.9951$$

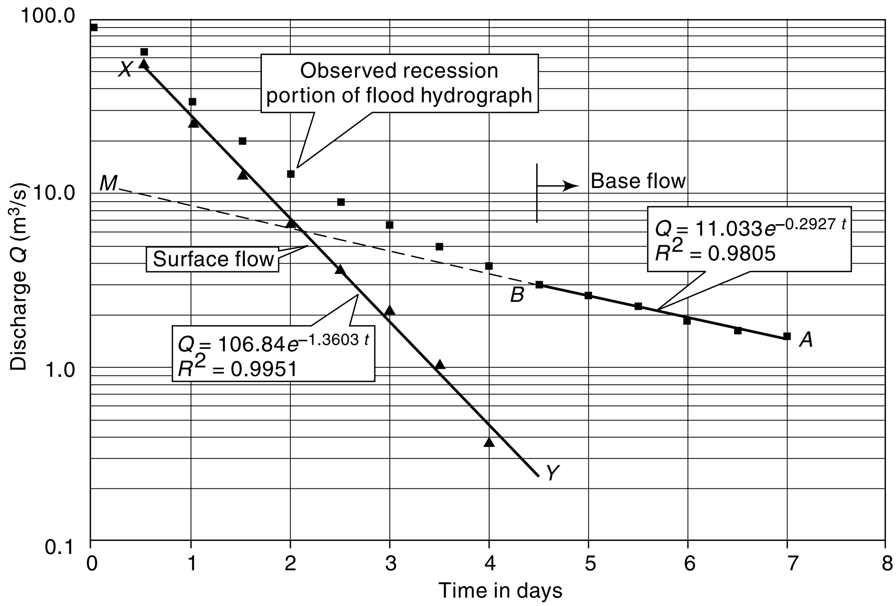

Fig. 6.4 Storage Recession Curve—Example 6.1

Time from peak (days)	Recession Limb of given flood hydrograph (m^3/s)	Base flow (Obtained by using K_{rb} = 0.746)	Surface runoff (m^3/s)
0.0	90.0		
0.5	66.0	10.455	55.545
1.0	34.0	7.945	26.055
1.5	20.0	6.581	13.419
2.0	13.0	5.613	7.387
2.5	9.0	4.862	4.138
3.0	6.7	4.249	2.451
3.5	5.0	3.730	1.270
4.0	3.8	3.281	0.519
4.5	3.0	2.884	
5.0	2.6	2.530	
5.5	2.2	2.209	
6.0	1.8	1.917	
6.5	1.6	1.647	
7.0	1.5	1.398	

The *Surface flow recession coefficient* K_{rs} is obtained as
ln K_{rs} = –1.3603 and as such K_{rs} = 0.257.
[Alternatively, by using the graph, the value of K_{rs} could be obtained by selecting two points 1 and 2 on the straight line *XY* and using Eq. (6.2)].

The storage available at the end of day-3 is the sum of the storages in surface flow and groundwater recession modes and is given by

$$S_{t3} = \left(\frac{Q_{s3}}{-\ln K_{rs}} + \frac{Q_{b3}}{-\ln K_{rb}} \right)$$

For the surface flow recession using the best fit equation:

$$Q_{s3} = 106.84e^{-1.3603 \times 3} = 1.8048; \; -\ln K_{rs} = 1.3603$$

$$\frac{Q_{s3}}{-\ln K_{rs}} = \frac{1.8048}{1.3603} = 1.3267 \text{ cumec-days}$$

Similarly for the base flow recession:

$$Q_{b3} = 11.033e^{-0.2927 \times 3} = 4.585; \; -\ln K_{rb} = 0.2927$$

$$\frac{Q_{b3}}{-\ln K_{rb}} = \frac{4.585}{0.2927} = 15.665 \text{ cumec-days}$$

Hence, total storage at the end of 3 days $= S_{t3} = 1.3267 + 15.665$
$= 16.9917$ cumec. days $= 1.468$ Mm^3

6.4 BASE FLOW SEPARATION

In many hydrograph analyses a relationship between the surface-flow hydrograph and the effective rainfall (i.e. rainfall minus losses) is sought to be established. The surface-flow hydrograph is obtained from the total storm hydrograph by separating the quick-response flow from the slow response runoff. It is usual to consider the interflow as a part of the surface flow in view of its quick response. Thus only the base flow is to be deducted from the total storm hydrograph to obtain the surface flow hydrograph.

There are three methods of base-flow separation that are in common use.

Methods of Base-flow Separation

Method I—Straight-Line Method In this method the separation of the base flow is achieved by joining with a straight line the beginning of the surface runoff to a point on the recession limb representing the end of the direct runoff. In Fig. 6.5 point *A* represents the beginning of the direct runoff and it is usually easy to identify in view of the sharp change in the runoff rate at that point.

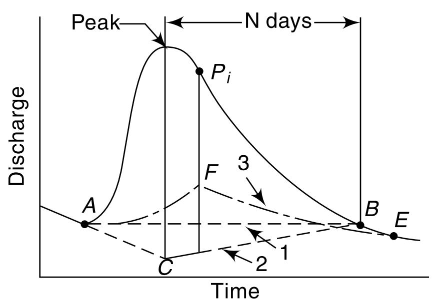

Fig. 6.5 Base Flow Seperation Methods

Point *B*, marking the end of the direct runoff is rather difficult to locate exactly. An empirical equation for the time interval *N* (days) from the peak to the point *B* is

$$N = 0.83A^{0.2} \tag{6.4}$$

where A = drainage area in km^2 and N is in days. Points *A* and *B* are joined by a straight line to demarcate to the base flow and surface runoff. It should be realised that the value of *N* obtained as above is only approximate and the position of *B* should be decided by considering a number of hydrographs for the catchment. This method of base-flow separation is the simplest of all the three methods.

Method 2 In this method the base flow curve existing prior to the commencement of the surface runoff is extended till it intersects the ordinate drawn at the peak (point *C* in Fig. 6.5). This point is joined to point *B* by a straight line. Segment *AC* and *CB* demarcate the base flow and surface runoff. This is probably the most widely used base-flow separation procedure.

METHOD 3 In this method the base flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line *EF* in Fig. 6.5). Points *A* and *F* are joined by an arbitrary smooth curve. This method of base-flow separation is realistic in situations where the groundwater contributions are significant and reach the stream quickly.

It is seen that all the three methods of base-flow separation are rather arbitrary. The selection of anyone of them depends upon the local practice and successful predictions achieved in the past. The surface runoff hydrograph obtained after the base-flow separation is also known as *direct runoff hydrograph* (DRH).

6.5 EFFECTIVE RAINFALL (ER)

Effective rainfall (also known as *Excess rainfall*) (ER) is that part of the rainfall that becomes direct runoff at the outlet of the watershed. It is thus the total rainfall in a given duration from which abstractions such as infiltration and initial losses are subtracted. As such, ER could be defined as that rainfall that is neither retained on the land surface nor infiltrated into the soil.

For purposes of correlating DRH with the rainfall which produced the flow, the hyetograph of the rainfall is also pruned by deducting the losses. Figure 6.6 shows the hyetograph of a storm. The initial loss and infiltration losses are subtracted from it. The resulting hyetograph is known as *effective rainfall hyetograph* (ERH). It is also known as *excess rainfall hyetograph*.

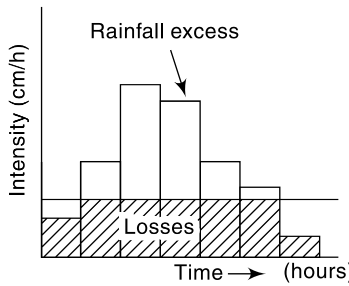

Fig. 6.6 Effective Rainfall Hyetograph (ERH)

Both DRH and ERH represent the same total quantity but in different units. Since ERH is usually in cm/h plotted against time, the area of ERH multiplied by the catchment area gives the total volume of direct runoff which is the same as the area of DRH. The initial loss and infiltration losses are estimated based on the available data of the catchment.

EXAMPLE 6.2 *Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km^2 produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and ϕ index.*

Time from start of rainfall (h)	–6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

SOLUTION: The hydrograph is plotted to scale (Fig. 6.7). It is seen that the storm hydrograph has a base-flow component. For using the simple straight-line method of base-flow separation, by eg. (6.4)

$$N = 0.83 \times (27)^{0.2} = 1.6 \text{ days} = 38.5 \text{ h}$$

However, by inspection, DRH starts at $t = 0$, has the peak at $t = 12$ h and ends at $t = 48$ h (which gives a value of $N = 48 - 12 = 36$ h). As $N = 36$ h appears to be more satisfactory

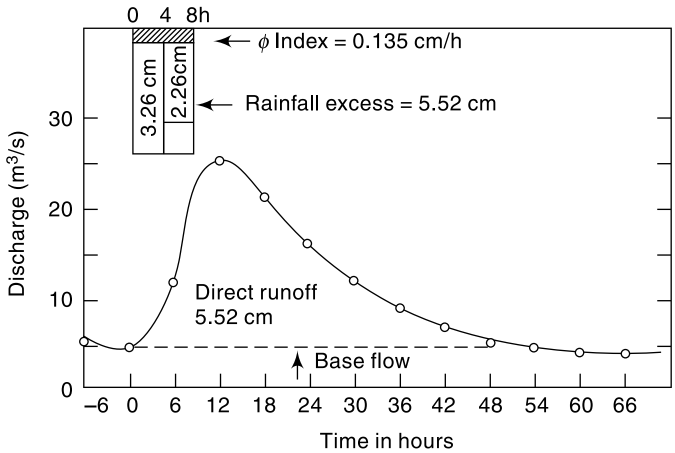

Fig. 6.7 Base Flow Separation—Example 6.2

than $N = 38.5$ h, in the present case DRH is assumed to exist from $t = 0$ to 48 h. A straight line base flow separation gives a constant value of 5 m^3/s for the base flow.

$$\text{Area of DRH} = (6 \times 60 \times 60)\left[\frac{1}{2}(8) + \frac{1}{2}(8+21) + \frac{1}{2}(21+16) + \frac{1}{2}(16+11) + \frac{1}{2}(11+7) + \frac{1}{2}(7+4) + \frac{1}{2}(4+2) + \frac{1}{2}(2)\right]$$

$$= 3600 \times 6 \times (8 + 21 + 16 + 11 + 7 + 4 + 2) = 1.4904 \times 10^6 \text{ m}^3$$

= Total direct runoff due to storm

$$\text{Runoff depth} = \frac{\text{runoff volume}}{\text{catchment area}} = \frac{1.4904 \times 10^6}{27 \times 10^6} = 0.0552 \text{ m}$$

= 5.52 cm = rainfall excess

Total rainfall = 3.8 + 2.8 = 6.6 cm

Duration = 8 h

$$\phi \text{ index} = \frac{6.6 - 5.52}{8} = 0.135 \text{ cm/h}$$

Example 6.3 *A storm over a catchment of area 5.0 km² had a duration of 14 hours. The mass curve of rainfall of the storm is as follows:*

Time from start of storm (h)	0	2	4	6	8	10	12	14
Accumulated rainfall (cm)	0	0.6	2.8	5.2	6.6	7.5	9.2	9.6

If the ϕ index for the catchment is 0.4 cm/h, determine the effective rainfall hyetograph and the volume of direct runoff from the catchment due to the storm.

Solution: First the depth of rainfall in a time interval $\Delta t = 2$ hours, in total duration of the storm is calculated, (col. 4 of Table 6.2).

Table 6.2 Calculation for Example 6.3

Time from start of storm, *t* (h)	Time interval Δt (h)	Accumulated rainfall in time *t* (cm)	Depth of rainfall in Δt (cm)	$\varphi \Delta t$ (cm)	ER (cm)	Intensity of ER (cm/h)
1	2	3	4	5	6	7
0	—	0	—	—	—	—
2	2	0.6	0.6	0.8	0	0
4	2	2.8	2.2	0.8	1.4	0.7
6	2	5.2	2.4	0.8	1.6	0.8
8	2	6.7	1.5	0.8	0.7	0.35
10	2	7.5	0.8	0.8	0	0
12	2	9.2	1.7	0.8	0.9	0.45
14	2	9.6	0.4	0.8	0	0

In a given time interval Δt, effective rainfall (ER) is given by

$$\text{ER} = (\text{actual depth of rainfall} - \phi \, \Delta \, t)$$

or $$\text{ER} = 0, \text{ whichever is larger.}$$

The calculations are shown in Table 6.2. For plotting the hyetograph, the intensity of effective rainfall is calculated in col. 7.

The effective rainfall hyetograph is obtained by plotting ER intensity (col. 7) against time from start of storm (col. 1), and is shown in Fig. 6.8.

Total effective rainfall = Direct runoff due to storm = area of ER hyetograph = $(0.7 + 0.8 + 0.35 + 0.45) \times 2 = 4.6$ cm

Volume of Direct runoff = $\dfrac{4.6}{1000} \times 5.0 \times (1000)^2 = 23000 \text{ m}^3$

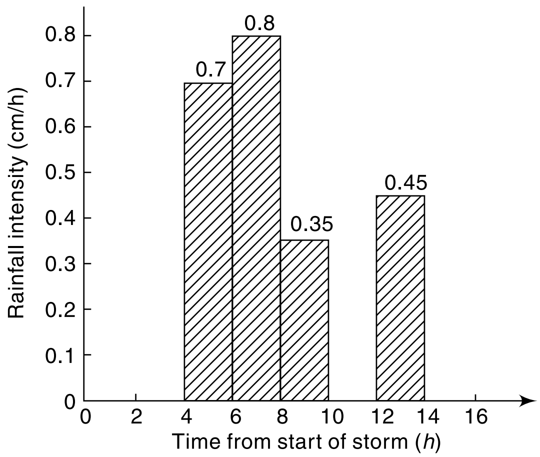

Fig. 6.8 ERH of Storm—Example 6.3

6.6 UNIT HYDROGRAPH

The problem of predicting the flood hydrograph resulting from a known storm in a catchment has received considerable attention. A large number of methods are proposed to solve this problem and of them probably the most popular and widely used method is the *unit-hydrograph method.* This method was first suggested by Sherman in 1932 and has undergone many refinements since then.

A *unit hydrograph* is defined as the hydrograph of direct runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours). The term unit here refers to a unit depth of rainfall excess which is usually taken as 1 cm. The duration, being a very important characteristic, is used as a prefix to a specific unit hydrograph. Thus one has a 6-h unit hydrograph, 12-h unit hydrograph, etc. and in general a *D-h* unit hydrograph applicable to a given catchment. The definition of a unit hydrograph implies the following:

- The unit hydrograph represents the lumped response of the catchment to a unit rainfall excess of D-h duration to produce a direct-runoff hydrograph. It relates only the direct runoff to the rainfall excess. Hence the volume of water contained in the unit hydrograph must be equal to the rainfall excess. As 1 cm depth of rainfall excess is considered the area of the unit hydrograph is equal to a volume given by 1 cm over the catchment.
- The rainfall is considered to have an average intensity of *excess rainfall* (ER) of $1/D$ cm/h for the duration D-h of the storm.
- The distribution of the storm is considered to be uniform all over the catchment.

Figure 6.9 shows a typical 6-h unit hydrograph. Here the duration of the rainfall excess is 6 h.

Area under the unit hydrograph = 12.92×10^6 m^3

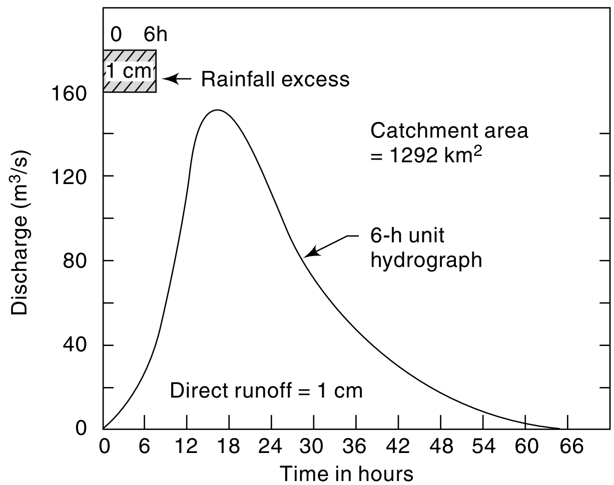

Fig. 6.9 Typical 6-h Unit Hydrograph

Hence

Catchment area of the basin = 1292 km^2

Two basic assumptions constitute the foundations for the unit-hydrograph theory. These are: (i) the time invariance and (ii) the linear response.

Time Invariance

This first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time-invariant. This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

Linear Response

The direct-runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit-hydrograph theory. Linear response means that if an input $x_1(t)$ causes an output $y_1(t)$ and an input $x_2(t)$ causes an output $y_2(t)$, then an input $x_1(t) + x_2(t)$ gives an output $y_1(t) + y_2(t)$. Consequently, if $x_2(t) = r\, x_1(t)$, then $y_2(t) = r\,y_1(t)$. Thus, if the rainfall excess in a duration D is r times the unit depth, the resulting DRH will have ordinates bearing ratio r to those of the corresponding D-h unit hydrograph. Since the area of the resulting DRH should increase by the ratio r, the base of the DRH will be the same as that of the unit hydrograph.

The assumption of linear response in a unit hydrograph enables the method of superposition to be used to derive DRHs. Accordingly, if two rainfall excess of D-h duration each occur consecutively, their combined effect is obtained by superposing the respective DRHs with due care being taken to account for the proper sequence of events. These aspects resulting from the assumption of linear response are made clearer in the following two illustrative examples.

Example 6.4 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 h.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m^3/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

Solution: The desired ordinates of the DRH are obtained by multiplying the ordinates of the unit hydrograph by a factor of 3.5 as in Table 6.3. The resulting DRH as also the unit hydrograph are shown in Fig. 6.10 (a). Note that the time base of DRH is not changed and remains the same as that of the unit hydrograph. The intervals of coordinates of the unit hydrograph (shown in column 1) are not in any way related to the duration of the rainfall excess and can be any convenient value.

Table 6.3 Calculation of DRH Due to 3.5 ER—Example 6.4

Time (h)	Ordinate of 6-h unit hydrograph (m^3/s)	Ordinate of 3.5 cm DRH (m^3/s)
1	2	3
0	0	0
3	25	87.5
6	50	175.0
9	85	297.5
12	125	437.5
15	160	560.0
18	185	647.5
24	160	560.0
30	110	385.0
36	60	210.0
42	36	126.0
48	25	87.5
54	16	56.0
60	8	28.0
69	0	0

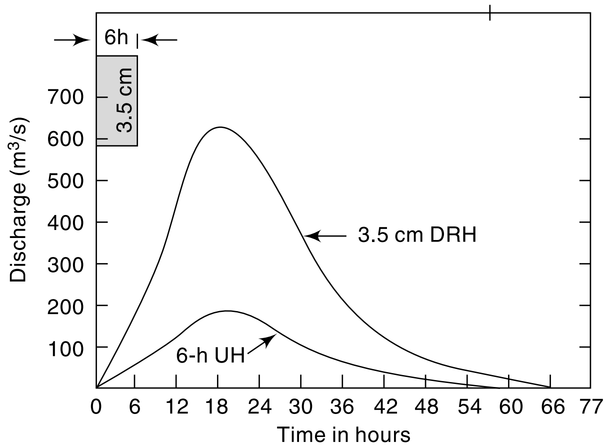

Fig. 6.10(a) 3.5 cm DRH derived from 6-h Unit Hydrograph—Example 6.4

EXAMPLE 6.5 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.4. Calculate the resulting DRH.*

SOLUTION: First, the DRHs due to 3.0 and 2.0 cm ER are calculated, as in Example 6.3 by multiplying the ordinates of the unit hydrograph by 3 and 2 respectively. Noting that the 2-cm DRH occurs after the 3-cm DRH, the ordinates of the 2-cm DRH are lagged by 6 hrs as shown in column 4 of Table 6.4. Columns 3 and 4 give the proper sequence of the two DRHs. Using the method of superposition, the ordinates of the resulting DRH are obtained by combining the ordinates of the 3- and 2-cm DRHs at any instant. By this process the ordinates of the 5 cm DRH are obtained in column 5. Figure 6.10(b) shows the component 3- and 2-cm DRHs as well as the composite 5-cm DRH obtained by the method of superposition.

Table 6.4 Calculation of DRH by method of Superposition—Example 6.5

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH (col. 2) × 3	Ordinate of 2-cm DRH (col. 2 lagged by 6 h) × 2	Ordinate of 5-cm DRH (col. 3 + col. 4) (m^3/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	

(*Contd.*)

(*Contd.*)

(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

Note: 1. The entries in col. 4 are shifted by 6 h in time relative to col. 2.
2. Due to unequal time interval of ordinates a few entries have to be interpolated to complete the table. These interpolated values are shown in parentheses.

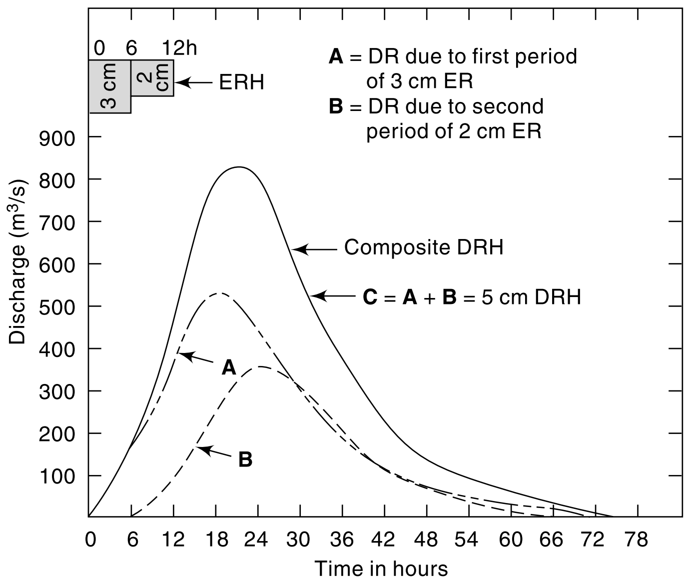

Fig. 6.10(b) Principle of Superposition—Example 6.5

Application of Unit Hydrograph

Using the basic principles of the unit hydrograph, one can easily calculate the DRH in a catchment due to a given storm if an appropriate unit hydrograph was available. Let it be assumed that a *D-h* unit-hydrograph and the storm hyetograph are available. The initial losses and infiltration losses are estimated and deducted from the storm hyetograph to obtain the ERH (Sec. 6.5). The ERH is then divided into *M* blocks of

D-h duration each. The rainfall excess in each *D*-h duration is then operated upon the unit hydrograph successively to get the various DRH curves. The ordinates of these DRHs are suitably lagged to obtain the proper time sequence and are then collected and added at each time element to obtain the required net DRH due to the storm.

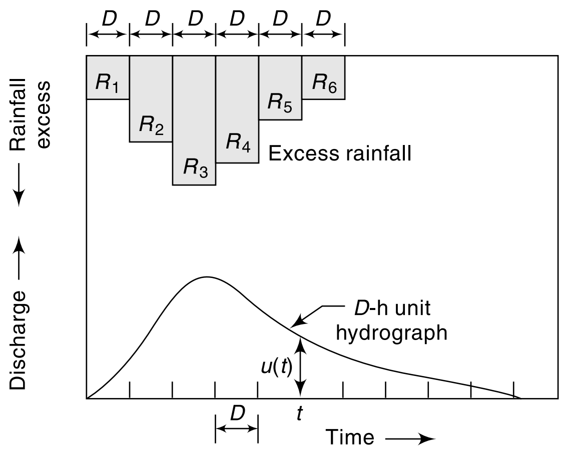

Fig. 6.11 DRH due to an ERH

Consider Fig. 6.11 in which a sequence of M rainfall excess values $R_1, R_2, \ldots, R_i, \ldots R_m$ each of duration *D*-h is shown. The line $u[t]$ is the ordinate of a *D*-h unit hydrograph at t h from the beginning.

The direct runoff due to R_1 at time t is

$$Q_1 = R_1 \cdot u[t]$$

The direct runoff due to R_2 at time $(t - D)$ is

$$Q_2 = R_2 \cdot u[t - D]$$

Similarly, $$Q_i = R_i \cdot u[t - (i - 1)D]$$

and $$Q_m = R_m \cdot u[t - (M - 1)D]$$

Thus at any time t, the total direct runoff is

$$Q_t = \sum_{i=1}^{M} Q_i = \sum_{i=1}^{M} R_i \cdot u[t - (i-1)D] \tag{6.5}$$

The arithmetic calculations of Eq. (6.5) are best performed in a tabular manner as indicated in Examples 6.5 and 6.6. After deriving the net DRH, the estimated base flow is then added to obtain the total flood hydrograph.

Digital computers are extremely useful in the calculations of flood hydrographs through the use of unit hydrograph. The electronic spread sheet (such as MS Excel) is ideally suited to perform the DRH calculations and to view the final DRH and flood hydrographs.

Example 6.6 *The ordinates of a 6-hour unit hydrograph of a catchment is given below.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48
Ordinate of 6-h UH	0	25	50	85	125	160	185	160	110	60	36	25
Time (h)	54	60	69									
Ordinate of 6-h UH	16	8	0									

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

The storm loss rate (ϕ– index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s at the beginning and increasing by 2.0 m^3/s for every 12 hours till the end of the direct-runoff hydrograph.

SOLUTION: The effective rainfall hyetograph is calculated as in the following table. The direct runoff hydrograph is next calculated by the method of superposition as indicated in Table 6.5. The ordinates of the unit hydrograph are multiplied by the ER values successively. The second and third set of ordinates are advanced by 6 and 12 h respectively and the ordinates at a given time interval added. The base flow is then added to obtain the flood hydrograph shown in Col 8, Table 6.6.

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	(11.0 – 3.5) = 7.5	(16.5 – 11.0) = 5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

Table 6.5 Calculation of Flood Hydrograph due to a known *ERH – Example 6.6*

Time	Ordinates of UH	DRH due to 2 cm ER Col. 2 × 2.0	DRH due to 2 cm ER Col. 2 × 6.0 (Advanced by 6 h)	DRH due to 4 cm ER Col. 2 × 4.0 (Advanced by 12 h)	Ordinates of final DRH (Col. 3 + 4 + 5)	Base flow (m^3/s)	Ordinates of flood hydrograph (m^3/s) (Col. 6 + 7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	]5	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	640	1945	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

Note: Due to the unequal time intervals of unit hydrograph ordinates, a few entries, indicated in parentheses have to be interpolated to complete the table.

6.7 DERIVATION OF UNIT HYDROGRAPHS

A number of isolated storm hydrographs caused by short spells of rainfall excess, each of approximately same duration [0.90 to 1.1 *D* h] are selected from a study of the continuously gauged runoff of the stream. For each of these storm hydrographs, the base flow is separated by adopting one of the methods indicated in Sec. 6.4.

The area under each DRH is evaluated and the volume of the direct runoff obtained is divided by the catchment area to obtain the depth of ER. The ordinates of the various DRHs are divided by the respective ER values to obtain the ordinates of the unit hydrograph.

Flood hydrographs used in the analysis should be selected to meet the following desirable features with respect to the storms responsible for them.

- The storms should be isolated storms occurring individually.
- The rainfall should be fairly uniform during the duration and should cover the entire catchment area.
- The duration of the rainfall should be 1/5 to 1/3 of the basin lag.
- The rainfall excess of the selected storm should be high. A range of ER values of 1.0 to 4.0 cm is sometimes preferred.

A number of unit hydrographs of a given duration are derived by the above method and then plotted on a common pair of axes as shown in Fig. 6.12. Due to the rainfall variations both in space and time and due to storm departures from the assumptions of the unit hydrograph theory, the various unit hydrographs thus developed will not be identical. It is a common practice to adopt a mean of such curves as the unit hydrograph of a given duration for the catchment. While deriving the mean curve, the average of peak flows and time to peaks are first calculated. Then a mean curve of best fit, judged by eye, is drawn through the averaged peak to close on an averaged base length. The volume of DRH is calculated and any departure from unity is corrected by adjusting the value of the peak. The averaged ERH of unit depth is customarily drawn in the plot of the unit hydrograph to indicate the type and duration of rainfall causing the unit hydrograph.

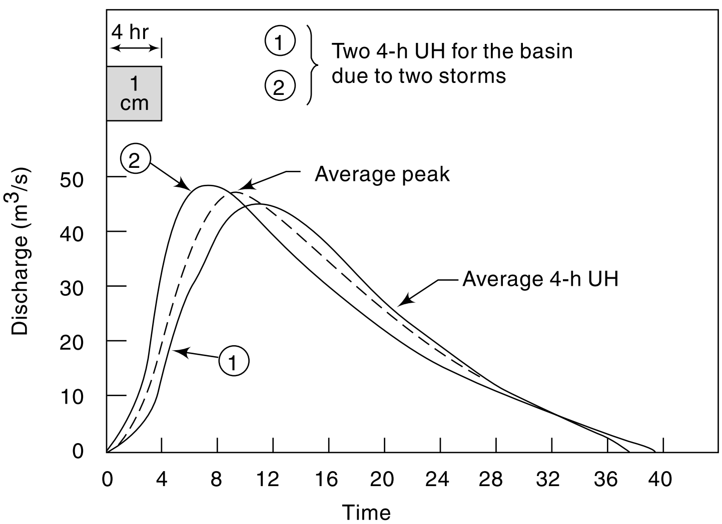

Fig. 6.12 Derivation of an Average Unit Hydrograph

By definition the rainfall excess is assumed to occur uniformly over the catchment during duration *D* of a unit hydrograph. An. ideal duration for a unit hydrograph is one wherein small fluctuations in the intensity of rainfall within this duration do not have any significant effect on the runoff. The catchment has a damping effect on the fluctuations of the rainfall intensity in the runoff-producing process and this damping is a function of the catchment area. This indicates that larger durations are admissible for larger catchments. By experience it is found that the duration of the unit hydrograph should not exceed 1/5 to 1/3 basin lag. For catchments of sizes larger than 250 km^2 the duration of 6 h is generally satisfactory.

Example 6.7 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km^2 due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment*

Time from start of storm (h)	–6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

Solution: The flood hydrograph is plotted to scale (Fig. 6.13). Denoting the time from beginning of storm as *t*, by inspection of Fig. 6.12,

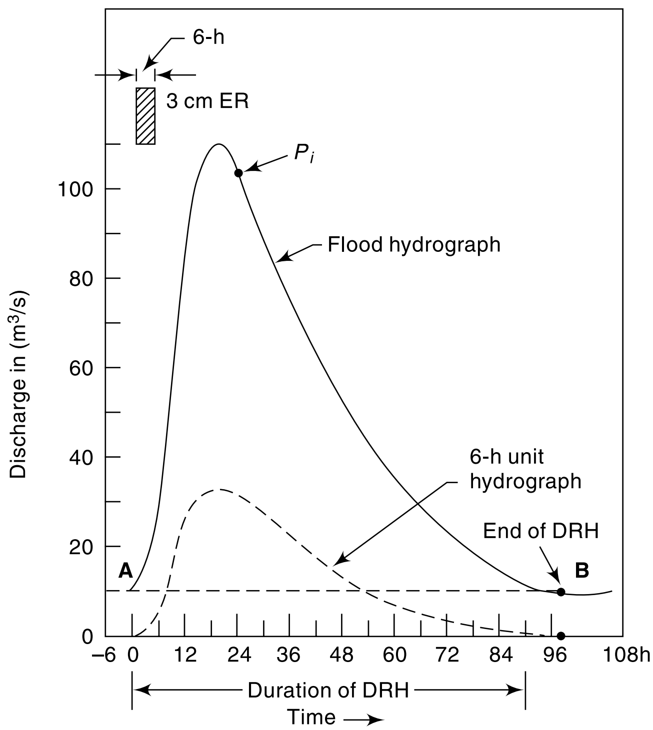

Fig. 6.13 Derivation of Unit Hydrograph from a flood Hydrograph

A = beginning of DRH $t = 0$
B = end of DRH $t = 90$ h
P_m = peak $t = 20$ h

Hence

$$N = (90 - 20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\,(423)^{0.2} = 2.78 \text{ days}$$

However, $N = 2.91$ days is adopted for convenience. A straight line joining A and B is taken as the divide line for base-flow separation. The ordinates of DRH are obtained by subtracting the base flow from the ordinates of the storm hydrograph. The calculations are shown in Table 6.6.

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$
$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm.}$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph (see Table 6.6).

Table 6.6 Calculation of the Ordinates of a 6-H Unit Hydrograph—Example 6.7

Time from beginning of storm (h)	Ordinate of flood hydrograph (m^3/s)	Base Flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4)/3
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0
54	39.0	11.5	27.5	9.2
60	31.5	11.5	20.0	
66	26.0	12.0	14.0	
72	21.5	12.0	9.5	
78	17.5	12.0	5.5	
84	15.0	12.5	2.5	
90	12.5	12.5	0	
96	12.0	12.0	0	
102	12.0	12.0	0	

Example 6.8 *(a) The peak of flood hydrograph due to a 3-h duration isolated storm in a catchment is 270 m^3/s. The total depth of rainfall is 5.9 cm. Assuming an average infiltration loss of 0.3 cm/h and a constant base flow of 20 m^3/s, estimate the peak of the 3-h unit hydrograph (UH) of this catchment.*
(b) If the area of the catchment is 567 km^2 determine the base width of the 3-h unit hydrograph by assuming it to be triangular in shape.

Solution:

(a) Duration of rainfall excess = 3 h — Loss @ 0.3 cm/h for 3 h = 0.9 cm
Total depth of rainfall = 5.9 cm — Rainfall excess = 5.9–0.9 = 5.0 cm

Peak flow:
Peak of flood hydrograph = 270 m^3/s — Peak of DRH = 250 m^3/s
Base flow = 20 m^3/s

$$\text{Peak of 3-h unit hydrograph} = \frac{\text{peak of DRH}}{\text{rainfall excess}} = \frac{250}{5.0} = 50 \text{ m}^3/\text{s}$$

(b) Let B = base width of the 3-h UH in hours.
Volume represented by the area of UH = volume of 1 cm depth over the catchment

Area of UH = (Area of catchment × 1 cm)

$$\frac{1}{2} \times B \times 60 \times 60 \times 50 = 567 \times 10^6 \times \frac{1}{100}$$

$$B = \frac{567 \times 10^4}{9 \times 10^4} = 63 \text{ hours.}$$

Unit Hydrograph from a Complex Storm

When suitable simple isolated storms are not available, data from complex storms of long duration will have to be used in unit-hydrograph derivation. The problem is to decompose a measured composite flood hydrograph into its component DRHs and base flow. A common unit hydrograph of appropriate duration is assumed to exist. This problem is thus the inverse of the derivation of flood hydrograph through use of Eq. (6.5). Consider a rainfall excess made up of three consecutive durations of D-h and ER values of R_1, R_2 and R_3. Figure 6.14 shows the ERR. By base flow separation of the resulting composite flood hydrograph a composite DRH is obtained (Fig. 6.14). Let the ordinates of the composite DRH be drawn at a time interval of D-h. At various time intervals $1D, 2D, 3D, \ldots$ from the start of the ERH, let the ordinates of the unit hydrograph be $u_1, u_2, u_3, \ldots$ and the ordinates of the composite DRH be $Q_1, Q_2, Q_3, \ldots,$
Then

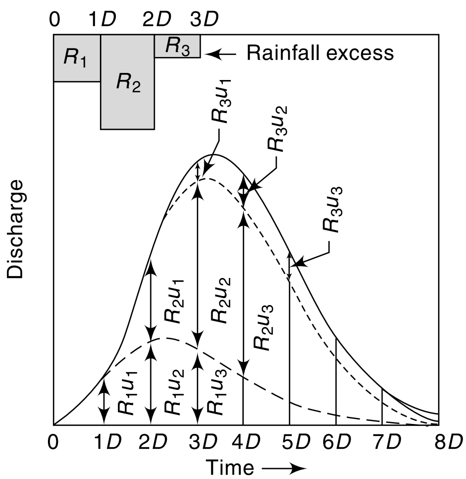

Fig. 6.14 Unit hydrograph from a Complex Storm

$$Q_1 = R_1 u_1$$
$$Q_2 = R_1 u_2 + R_2 u_1$$
$$Q_3 = R_1 u_3 + R_2 u_2 + R_3 u_1$$

$$Q_4 = R_1 u_4 + R_2 u_3 + R_3 u_2$$
$$Q_5 = R_1 u_5 + R_2 u_4 + R_3 u_3$$
$$\ldots\ldots\ldots\ldots \tag{6.6}$$

so on.

From Eq. (6.6) the values of $u_1, u_2, u_3, \ldots$ can be determined. However, this method suffers from the disadvantage that the errors propagate and increase as the calculations proceed. In the presence of errors the recession limb of the derived *D-h* unit hydrograph can contain oscillations and even negative values. Matrix methods with optimisation schemes are available for solving Eq. (6.6) in a digital computer.

6.8 UNIT HYDROGRAPHS OF DIFFERENT DURATIONS

Ideally, unit hydrographs are derived from simple isolated storms and if the duration of the various storms do not differ very much, say within a band of ± 20% *D*, they would all be grouped under one average duration of *D*-h. If in practical applications unit hydrographs of different durations are needed they are best derived from field data. Lack of adequate data normally precludes development of unit hydrographs covering a wide range of durations for a given catchment. Under such conditions a *D* hour unit hydrograph is used to develop unit hydrographs of differing durations *nD.* Two methods are available for this purpose.

- Method of superposition
- The *S*-curve

These are discussed below.

Method of Superposition

If a *D*-h unit hydrograph is available, and it is desired to develop a unit hydrograph of *nD* h, where *n* is an integer, it is easily accomplished by superposing *n* unit hydrographs with each graph separated from the previous on by *D*-h. Figure 6.15 shows three 4-h unit hydrographs *A*, *B* and *C*. Curve *B* begins 4 h after *A* and *C* begins 4-h, after *B*. Thus the combination of these three curves is a DRH of 3 cm due to an ER of 12-h duration. If the ordinates of this DRH are now divided by 3, one obtains a 12-h unit hydrograph. The calculations are easy if performed in a tabular form (Table 6.7).

Example 6.9 *Given the ordinates of a 4-h unit hydrograph as below derive the ordinates of a 12-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4-h UH	0	20	80	130	150	130	90	52	27	15	5	0

Solution: The calculations are performed in a tabular form in Table 6.7. In this

Column 3 = ordinates of 4-h UH lagged by 4-h
Column 4 = ordinates of 4-h UH lagged by 8-h
Column 5 = ordinates of DRH representing 3 cm ER in 12-h
Column 6 = ordinates of 12-h UH = (Column 5)/3

The 12-h unit hydrograph is shown in Fig. 6.15.

The S-Curve

If it is desired to develop a unit hydrograph of duration *mD*, where *m* is a fraction, the method of superposition cannot be used. A different technique known as the *S*-curve method is adopted in such cases, and this method is applicable for rational values of *m.*

Table 6.7 Calculation of a 12-h Unit Hydrograph from a 4-H Unit Hydrograph—Example 6.9

Time (h)	Ordinates of 4-h UH (m^3/s) *A*	*B* Lagged by 4-h	*C* Lagged by 8-h	DRH of 3 cm in 12-h (m^3/s) (Col. 2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

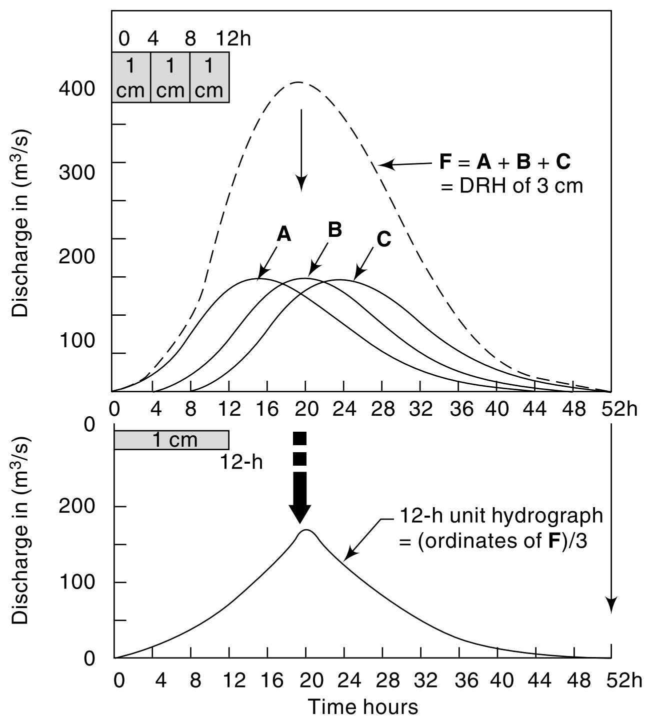

Fig. 6.15 Construction of a 12-h Unit Hydrograph from a 4-h Unit Hydrograph—Example 6.9

The *S-curve*, also known as *S-hydrograph* is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of *D*-h unit hydrographs spaced *D*-h apart. Figure 6.16 shows such a series of *D*-h hydrograph arranged with their starting points *D*-h apart. At any given time the ordinates of the various curves occurring at that time coordinate are summed up to obtain ordinates of the *S*-curve. A smooth curve through these ordinates result in an *S*-shaped curve called *S*-curve.

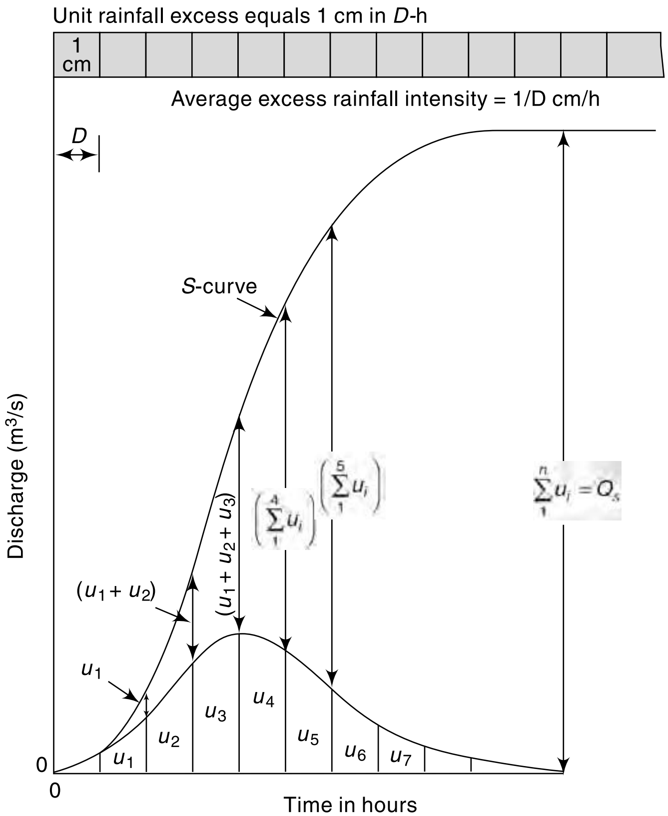

Fig. 6.16 *S*-curve

This *S*-curve is due to a *D*-h unit hydrograph. It has an initial steep portion and reaches a maximum equilibrium discharge at a time equal to the time base of the first unit hydrograph. The average intensity of ER producing the *S*-curve is 1/*D* cm/h and the equilibrium discharge,

$$Q_s = \left(\frac{A}{D} \times 10^4\right) \text{m}^3/\text{h},$$

where A = area of the catchment in km^2 and D = duration in hours of ER of the unit hydrograph used in deriving the *S*-curve. Alternatively

$$Q_s = 2.778 \frac{A}{D} \text{ m}^3/\text{s} \qquad (6.7)$$

where A is the km^2 and D is in h. The quantity Q_s represents the maximum rate at which an ER intensity of 1/D cm/h can drain out of a catchment of area A. In actual construction of an S-curve, it is found that the curve oscillates in the top portion at around the equilibrium value due to magnification and accumulation of small errors in the hydrograph. When it occurs, an average smooth curve is drawn such that it reaches a value Q_s at the time base of the unit hydrograph.

[***Note:*** It is desirable to designate the S-curve due to D-hour unit hydrograph as S_D-curve to give an indication that the average rainfall excess of the curve is $(1/D)$ cm/h. It is particularly advantageous when more than one S-curve is used as in such cases the curves would be designated as S_{D1}, S_{D2}, … etc. to avoid possible confusion and mistakes.]

Construction of S–curve By definition an S-curve is obtained by adding a string of D-h unit hydrographs each lagged by D-hours from one another. Further, if T_b = base period of the unit hydrograph, addition of only T_b/D unit hydrographs are sufficient to obtain the S-curve. However, an easier procedure based on the basic property of the S-curve is available for the construction of S-curves.

i.e. $U(t) = S(t) - S(t-D)$

or $$S(t) = U(t) + S(t-D) \tag{6.8}$$

The term $S(t-D)$ could be called S-curve addition at time t so that

Ordinate of S-curve at any time t = Ordinate of D-h unit hydrograph at time t + S-curve addition at time t

Noting that for all $t \le D$, $S(t-D) = 0$, Eq. (6.8) provides a simple recursive procedure for computation of S-curve ordinates. The procedure is explained in Example 6.10.

Example 6.10 *Derive the S-curve for the 4-h unit hydrograph given below.*

Time (h)	0	4	8	12	16	20	24	28
Ordinate of 4-h UH (m^3/s)	0	10	30	25	18	10	5	0

Solution: Computations are shown in Table 6.8. In this table col. 2 shows the ordinates of the 4-h unit hydrograph. col. 3 gives the S-curve additions and col. 4 gives the ordinates of the S-curve. The sequence of entry in col. 3 is shown by arrows. Values of entries in col. 4 is obtained by using Eq. (6.8), i.e. by summing up of entries in col. 2 and col. 4 along each row.

Table 6.8 Construction of S-curve—Example 6.10

Time in hours	Ordinate of 4-h UH	S-curve addition (m^3/s)	S_4-curve ordinate (m^3/s). (col. 2 + col. 3)
1	2	3	4
0	0		0
4	10	0	10
8	30	10	40
12	25	40	65
16	18	65	83
20	10	83	93
24	5	93	98
28	0	98	98

At $i = 4$ hours; Ordinate of 4-hUH = 10 m^3/s.

S-curve addition = ordinate of 4-h UH @$\{t = (4–4) \neq 0$ hours$\} = 0$

Hence S-curve ordinate Eq. (6.8) = 10 + 0 = 10 m^3/s,

At $t = 8$ hours; Ordinate of 4-hUH = 30 m^3/s.

S-curve addition = ordinate of 4-hUH @$\{t = (8–4) = 4$ hours$\}$ = 10 m^3/s

Hence S-curve ordinate by Eq. (6.8) = 30 + 10 = 40 m^3/s.

At $t = 12$ hours; Ordinate of 4-hUH = 25 m^3/s.

S-curve addition = ordinate of 4-hUH @$\{t = (12–4) = 48$ hours$\}$ = 40 m^3/s

Hence S-curve ordinate by Eq. (6.8) = 25 + 40 = 65 m^3/s.

This calculation is repeated for all time intervals till t = base width of UH = 28 hours. Plots of the 4-h UH and the derived S-curve are shown in Fig. 6.17.

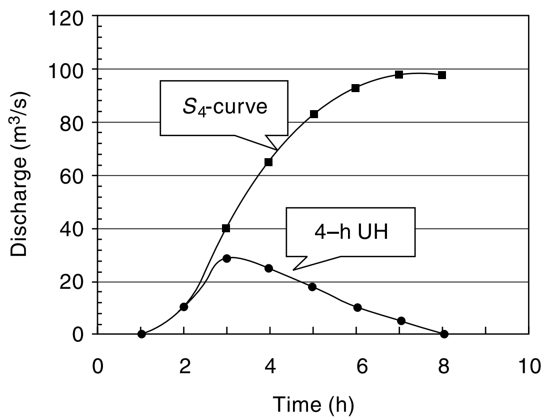

Fig. 6.17 Construction of S_4-curve—(Example 6.10)

DERIVATION OF T-HOUR UNIT HYDROGRAPH

Consider two D-h S-curves A and B displaced by T-h (Fig. 6.18). If the ordinates of B are subtracted from that of A, the resulting curve is a DRH produced by a rainfall

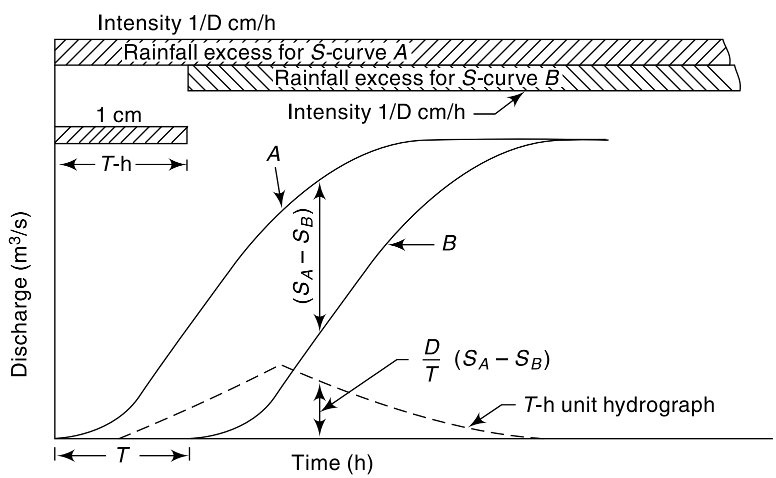

Fig. 6.18 Derivation of a T-h Unit Hydrograph by S-curve Lagging Method

excess of duration T-h and magnitude $\left(\frac{1}{D}\times T\right)$ cm. Hence if the ordinate differences of A and B, i.e. $(S_A - S_B)$ are divided by T/D, the resulting ordinates denote a hydrograph due to an ER of 1 cm and of duration T-h, i.e. a T-h unit hydrograph. The derivation of a T-h unit hydrograph as above can be achieved either by graphical means or by arithmetic computations in a tabular form as indicated in Example 6.11.

Example 6.11 *Solve Example 6.9 by the S-Curve method.*

Solution: Computations are shown in Table 6.9. Column 2 shows the ordinates of the 4-h unit hydrograph. Column 3 gives the S-curve additions and Column 4 the S-curve ordinates. The sequence of additions are shown by arrows. At $t = 4$ h, ordinate of the 4-h UH = ordinate of the S-curve. This value becomes the S-curve addition at $t = 2 \times 4 = 8$ h. At this $t = 8$ h, the ordinate of UH (80) + S-curve addition (20) = S-curve ordinate (100). The S-curve addition at $3 \times 4 = 12$ h is 100, and so on. Column 5 shows the S-curve lagged by 12 h. Column 6 gives the ordinate of DRH of (T/D) = 3 cm. Ordinates shown in Column 6 are divided by (T/D = 3) to obtain the ordinates of the 12-h unit hydrograph shown in Column 7.

Table 6.9 Determination of a 12-H Unit Hydrograph by *S*-Curve Method—Example 6.11

Time (h)	Ordinate of 4-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (Col. 2 + Col. 3)	S-curve lagged by 12 h (m^3/s)	(Col. 4– Col. 5)	$\frac{\text{Col. 6}}{(12/4)}$ = 12-h UH ordinates (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
4	20	0	20	—	20	6.7
8	80	20	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52			699	699	0	0

Example 6.12 *Ordinates of a 4-h unit hydrograph are given. Using this derive the ordinates of a 2-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate or 4-h UH (m^3/s)	0	20	80	130	150	130	90	52	27	15	5	0

Solution: In this case the time interval of the ordinates of the given unit hydrograph should be at least 2 h. As the given ordinates are at 4-h intervals, the unit-hydrograph is plotted and its ordinates at 2-h intervals determined. The ordinates are shown in column 2 of Table 6.10. The *S*-curve additions and *S*-curve ordinates are shown in columns 3 and 4 respectively. First, the *S*-curve ordinates corresponding to the time intervals equal to successive durations of the given unit hydrograph (in this case at 0, 4, 8, 12 … *h*) are determined by following the method of Example 6.11. Next, the ordinates at intermediate intervals (viz. at t = 2, 6, 10, 14 … h) are determined by having another series of *S*-curve additions. The sequence of these are shown by distinctive arrows in Table 6.9. To obtain a 2-h unit hydrograph the *S*-curve is lagged by 2 h (column 5) and this is subtracted from column 4 and the results listed in column 6. The ordinates in column 6 are now divided by $T/D = 2/4 = 0.5$, to obtain the required 2-h unit hydrograph ordinates, shown in column 7.

Table 6.10 Determination of 2-h Unit Hydrograph from A 4-h Unit Hydrograph—Example 6.12

Time (h)	Ordinate of 4-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (Col. (2) + (3)) (m^3/s)	S-curve lagged by 2 h	(Col. (4) – Col. (5)) DRH of $\left(\frac{2}{4}\right)$ = 0.5 cm	2-h UH ordinates $\frac{\text{Col. (6)}}{(2/4)}$ (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
2	8	—	8	0	8	16
4	20	0	20	8	12	24
6	43	8	51	20	31	62
8	80	20	100	51	49	98
10	110	51	161	100	61	122
12	130	100	230	161	69	138
14	146	161	307	230	77	154
16	150	230	380	307	73	146
18	142	307	449	380	69	138
20	130	380	510	449	61	122
22	112	449	561	510	51	102
24	90	510	600	561	39	78
26	70	561	631	600	31	62
28	52	600	652	631	21	42
30	38	631	669	652	17	34
32	27	652	679	669	10	20
34	20	669	689	679	10	(20)15
36	15	679	694	689	5	(10)10
38	10	689	699	694	5	(10)6
40	5	694	699	699	(0)	(0)3
42	2	699	701	699	(2)	(4)0
44	0	699	699	701	(–2)	(–4)0

Final adjusted values are given in col. 7.
Unadjusted values are given in parentheses.

The errors in interpolation of unit hydrograph ordinates often result in oscillation of *S*-curve at the equilibrium value. This results in the derived *T-h* unit hydrograph having an abnormal sequence of discharges (sometimes even negative values) at the tail end. This is adjusted by fairing the *S*-curve and also the resulting *T*-h unit-hydrograph by smooth curves. For example, in the present example the 2-h unit hydrograph ordinates at time > 36-h are rather abnormal. These values are shown in parentheses. The adjusted values are entered in column 7.

6.9 USE AND LIMITATIONS OF UNIT HYDROGRAPH

As the unit hydrographs establish a relationship between the ERH and DRH for a catchment, they are of immense value in the study of the hydrology of a catchment. They are of great use in (i) the development of flood hydrographs for extreme rainfall magnitudes for use in the design of hydraulic structures, (ii) extension of flood-flow records based on rainfall records, and (iii) development of flood forecasting and warning systems based on rainfall.

Unit hydrographs assume uniform distribution of rainfall over the catchment. Also, the intensity is assumed constant for the duration of the rainfall excess. In practice, these two conditions are never strictly satisfied. Non-uniform areal distribution and variation in intensity within a storm are very common. Under such conditions unit hydrographs can still be used if the areal distribution is consistent between different storms. However, the size of the catchment imposes an upper limit on the applicability of the unit hydrograph. This is because in very large basins the centre of the storm can vary from storm to storm and each can give different DRHs under otherwise identical situations. It is generally felt that about 5000 km^2 is the upper limit for unit-hydrograph use. Flood hydrographs in very large basins can be studied by dividing them into a number of smaller subbasins and developing DRHs by the unit-hydrograph method. These DRHs can then be routed through their respective channels to obtain the composite DRH at the basin outlet.

There is a lower limit also for the application of unit hydrographs. This limit is usually taken as about 200 ha. At this level of area, a number of factors affect the rainfall-runoff relationship and the unit hydrograph is not accurate enough for the prediction of DRH.

Other limitations to the use of unit hydrographs are:

- Precipitation must be from rainfall only. Snow-melt runoff cannot be satisfactory represented by unit hydrograph.
- The catchment should not have unusually large storages in terms of tanks, ponds, large flood-bank storages, etc. which affect the linear relationship between storage and discharge.
- If the precipitation is decidedly nonuniform, unit hydrographs cannot be expected to give good results.

In the use of unit hydrographs very accurate reproduction of results should not be expected. Variations in the hydrograph base of as much as ±20% and in the peak discharge by ±10% are normally considered acceptable.

6.10 DURATION OF THE UNIT HYDROGRAPH

The choice of the duration of the unit hydrograph depends on the rainfall records. If recording raingauge data are available any convenient time depending on the size of the basin can be used. The choice is not much if only daily rainfall records are available. A rough guide for the choice of duration D is that it should not exceed the least of (i) the time of rise, (ii) the basin lag, and (iii) the time of concentration. A value of D equal to about 1/4 of the basin lag is about the best choice. Generally, for basins with areas more than 1200 km^2 a duration D = 12 hours is preferred.

6.11 DISTRIBUTION GRAPH

The distribution graph introduced by Bernard (1935) is a variation of the unit hydrograph. It is basically a D-h unit hydrograph with ordinates showing the percentage of the surface runoff occurring in successive periods of equal time intervals of D-h. The duration of the rainfall excess (D-h) is taken as the unit interval and distribution-graph ordinates are indicated at successive such unit intervals. Figure 6.19 shows a typical 4-h distribution graph. Note the ordinates plotted at 4-h intervals and the total area under the distribution graph adds up to 100%. The use of the distribution graph to generate a DRH for a known ERH is exactly the same as that of a unit hydrograph (Example 6.13). Distribution graphs are useful in comparing the runoff characteristics of different catchments.

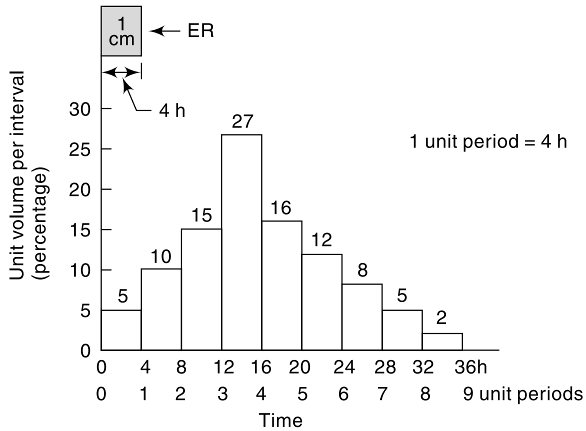

Fig. 6.19 Four-hour Distribution Graph

EXAMPLE 6.13 *A catchment of 200 hectares area has rainfalls of 7.5 cm, 2.0 cm and 5.0 cm in three consecutive days. The average ϕ index can be assumed to be 2.5 cm/day. Distribution-graph percentages of the surface runoff which extended over 6 days for every rainfall of 1-day duration are 5, 15, 40, 25, 10 and 5. Determine the ordinates of the discharge hydrograph by neglecting the base flow.*

SOLUTION: The calculations are performed in a tabular form in Table 6.11.

Table 6.11 Calculation of DRH using Distribution Graph—Example 6.13

Time interval (days)	Rainfall (cm)	Infiltration loss (cm)	Effective rainfall (cm)	Average distribuition ratio (per-cent)	Distributed runoff for rainfall excess of			Runoff	
					5 cm	0	2.5 cm	cm	$m^3/s \times 10^{-2}$
0–1	7.5	2.5	5.0	5	0.250	0		0.250	5.79
1–2	2.0	2.5	0	15	0.750	0	0	0.750	17.36
2–3	5.0	2.5	2.5	40	2.000	0	0.125	2.750	49.19

(*Contd.*)

(*Contd.*)

3–4				25	1.250	0	0.375	2.125	37.62
4–5				10	0.500	0	1.000	1.625	34.72
5–6				5	0.250	0	0.625	1.500	20.25
6–7				0	0	0	0.250	0.875	5.79
7–8						0	0.125	0.250	2.89
8–9							0	0.125	0

[Runoff of 1 cm in 1 day = $\frac{200 \times 100 \times 100}{86400 \times 100}$ m^3/s for 1 day = 0.23148 m^3/s for 1 day]

(The runoff ordinates are plotted at the mid-points of the respective time intervals to obtain the DRH)

6.12 SYNTHETIC UNIT HYDROGRAPH

Introduction

To develop unit hydrographs to a catchment, detailed information about the rainfall and the resulting flood hydrograph are needed. However, such information would be available only at a few locations and in a majority of catchments, especially those which are at remote locations, the data would normally be very scanty. In order to construct unit hydrographs for such areas, empirical equations of regional validity which relate the salient hydrograph characteristics to the basin characteristics are available. Unit hydrographs derived from such relationships are known as *synthetic-unit hydrographs*. A number of methods for developing synthetic-unit hydrographs are reported in literature. It should, however, be remembered that these methods being based on empirical correlations are applicable only to the specific regions in which they were developed and should not be considered as general relationships for use in all regions.

Snyder's Method

Snyder (1938), based on a study of a large number of catchments in the Appalachian Highlands of eastern United States developed a set of empirical equations for synthetic-unit hydrographs in those areas. These equations are in use in the USA, and with some modifications in many other countries, and constitute what is known as *Snyder's synthetic-unit hydrograph.*

The most important characteristic of a basin affecting a hydrograph due to a storm is *basin lag.* While actually basin lag (also known as *lag time*) is the time difference between the centroid of the input (rainfall excess) and the output (direct runoff hydrograph), because of the difficulty in determining the centroid of the direct runoff hydrograph (DRH) it is defined for practical purposes as the elapsed time between the centroid of rainfall excess and peak of DRH. Physically, lag time represents the mean time of travel of water from all parts of the watershed to the outlet during a given storm. Its value is determined essentially on the topographical features, such as the size, shape, stream density, length of main stream, slope, land use and land cover. The modified definition of basin time is very commonly adopted in the derivation of synthetic unit hydrographs for a given watershed.

The first of the Snyder's equation relates the basin lag t_p, defined as the time interval from the mid-point of rainfall excess to the peak of the unit hydrograph (Fig. 6.20), to the basin characteri-stics as

$$t_p = C_t(LL_{ca})^{0.3} \tag{6.9}$$

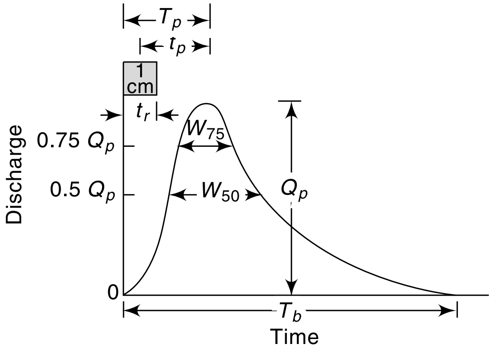

Fig. 6.20 Elements of a Synthetic Unit Hydrograph

where t_p = basin lag in hours

L = basin length measured along the water course from the basin divide to the gauging station in km

L_{ca} = distance along the main water course from the gauging station to a point opposite to the watershed centroid in km

C_t = a regional constant representing watershed slope and storage effects.

The value of C_t in Snyder's study ranged from 1.35 to 1.65. However, studies by many other investigators have shown that C_t depends upon the region under study and wide variations with the value of C_t ranging from 0.3 to 6.0 have been reported[6].

Linsley et al.[5] found that the basin lag t_p is better correlated with the catchment parameter $\left(\frac{LL_{ca}}{\sqrt{S}}\right)$ where S = basin slope. Hence, a modified form of Eq. (6.9) was suggested by them as

$$t_p = C_{tL}\left(\frac{LL_{ca}}{\sqrt{S}}\right)^n \tag{6.10}$$

where C_{tL} and n are basin constants. For the basins in the USA studied n by them n was found to be equal to 0.38 and the values of C_{tL} were 1.715 for mountainous n drainage areas, 1.03 for foot-hill drainage areas and 0.50 for valley drainage areas.

Snyder adopted a standard duration t_r hours of effective rainfall given by

$$t_r = \frac{t_p}{5.5} \tag{6.11}$$

The peak discharge Q_{ps} (m^3/s) of a unit hydrograph of *standard duration* t_r h is given by Snyder as

$$Q_{ps} = \frac{2.78\,C_p\,A}{t_p} \tag{6.12}$$

where A = catchment area in km^2 and C_p = a regional constant. This equation is based on the assumption that the peak discharge is proportional to the average discharge of $\left(\frac{1\text{ cm} \times \text{catchment area}}{\text{duration of rainfall excess}}\right)$. The values of the coefficient C_p range from 0.56 to 0.69 for Snyder's study areas and is considered as an indication of the retention and storage capacity of the watershed. Like C_t, the values of C_p also vary quite considerably depending on the characteristics of the region and values of C_p in the range 0.31 to 0.93 have been reported.

If a non-standard rainfall duration t_R h is adopted, instead of the standard value t_r to derive a unit hydrograph the value of the basin lag is affected. The modified basin lag is given by

$$t_p' = t_p + \frac{t_R - t_r}{4} = \frac{21}{22} t_p + \frac{t_R}{4} \tag{6.13}$$

where t_p' = basin lag in hours for an effective duration of t_R h and t_p is as given by Eq. (6.9) or (6.10). The value of t_p' must be used instead of t_p in Eq. (6.11). Thus the peak discharge for a nonstandard ER of duration t_R is in m^3/s

$$Q_p = 2.78\, C_p\, A/t_p' \tag{6.12a}$$

Note that when $t_R = t_r$

$$Q_p = Q_{ps}$$

The time base of a unit hydrograph (Fig. 6.20) is given by Synder as

$$T_b = 3 + \frac{t_p'}{8} \text{ days} = (72 + 3\,t_p') \text{ hours} \tag{6.14}$$

where T_b = time base. While Eq. (6.14) gives reasonable estimates of T_b for large catchments, it may give excessively large values of the time base for small catchments. Taylor and Schwartz[1] recommend

$$T_b = 5\left(t_p' + \frac{t_R}{2}\right) \text{ hours} \tag{6.15}$$

with t_b (given in h) taken as the next larger integer value divisible by t_R, i.e. T_b is about five times the time-to-peak.

To assist in the sketching of unit hydrographs, the widths of unit hydrographs at 50 and 75% of the peak (Fig. 6.20) have been found for US catchments by the US Army Corps of Engineers. These widths (in time units) are correlated to the peak discharge intensity and are given by

$$W_{50} = \frac{5.87}{q^{1.08}} \tag{6.16}$$

and

$$W_{75} = W_{50}/1.75 \tag{6.17}$$

where

W_{50} = width of unit hydrograph in h at 50% peak discharge

W_{75} = width of unit hydrograph in h at 75% peak discharge

$q = Q_p/A$ = peak discharge per unit catchment area in m^3/s/km^2

Since the coefficients C_t and C_p vary from region to region, in practical applications it is advisable that the value of these coefficients are determined from known unit hydrographs of a meteorologically homogeneous catchment and then used in the basin under study. This way Snyder's equations are of use in scaling the hydrograph information from one catchment to another similar catchment.

Example 6.14 *Two catchments A and B are considered meteorologically similar. Their catchment characteristics are given below.*

Catchment A	Catchment B
$L = 30$ km $L_{ca} = 15$ km $A = 250$ km^2	$L = 45$ km $L_{ca} = 25$ km $A = 400$ km^2

For catchment A, a 2-h unit hydrograph was developed and was found to have a peak discharge of 50 m^3/s. The time to peak from the beginning of the rainfall excess in this unit hydrograph was 9.0 h. Using Snyder's method, develop a unit hydrograph for catchment B.

SOLUTION: ***For Catchment A:***

$$t_R = 2.0 \text{ h}$$

Time to peak from beginning of ER

$$T_p = \frac{t_R}{2} + t_p' = 9.0 \text{ h}$$

$$\therefore \quad t_p' = 8.0 \text{ h}$$

From Eq. (6.13),

$$t_p' = \frac{21}{22} t_p + \frac{t_R}{4} = \frac{21}{22} t_p + 0.5 = 8.0$$

$$t_p = \frac{7.5 \times 22}{21} = 7.857 \text{ h}$$

From Eq. (6.9),

$$t_p = C_t (L\,L_{ca})^{0.3} \qquad 7.857 = C_t (30 \times 15)^{0.3} \qquad C_t = 1.257$$

From Eq. (6.12a),

$$Q_p = 2.78\, C_p\, A / t_p' \qquad 50 = 2.78 \times C_p \times 250/8.0 \qquad C_p = 0.576$$

For Catchment B: Using the values of $C_t = 1.257$ and $C_p = 0.576$ in catchment B, the parameters of the synthetic-unit hydrograph for catchment B are determined. From Eq. (6.9),

$$t_p = 1.257\,(45 \times 25)^{0.3} = 10.34 \text{ h}$$

By Eq. (6.11),

$$t_r = \frac{10.34}{5.5} = 1.88 \text{ h}$$

Using $t_R = 2.0$ h, i.e. for a 2-h unit hydrograph, by Eq. (6.12),

$$t_p' = 10.34 \times \frac{21}{22} + \frac{2.0}{4} = 10.37 \text{ h}$$

By Eq. (6.12a),

$$Q_p = \frac{2.78 \times 0.576 \times 400}{10.37} = 61.77 \text{ m}^3\text{/s, say } 62 \text{ m}^3\text{/s}$$

From Eq. (6.16),

$$W_{50} = \frac{5.87}{(62/400)^{1.08}} = 44 \text{ h}$$

By Eq. (6.17),

$$W_{75} = \frac{44}{1.75} = 25 \text{ h}$$

Time base: From Eq. (6.14), $T_b = 72 + (3 \times 10.37) = 103$ h
From Eq. (6.14), $T_b = 5\,(10.37 + 10) \approx 58$ h

Considering the values of W_{50} and W_{75} and noting that the area of catchment *B* is rather small, $T_b \approx 58$ h is more appropriate in this case.

Finalizing of Synthetic-Unit Hydrograph After obtaining the values of Q_p, t_R, t'_p, W_{75}, W_{50} and T_b from Snyder's equations, a tentative unit hydrograph is sketched and *S*-curve is then developed and plotted. As the ordinates of the unit hydrograph are tentative, the *S*-curve thus obtained will have kinks. These are then smoothened and a logical pattern of the *S*-curve is sketched. Using this *S*-curve t_R hour unit hydrograph is then derived back. Further, the area under the unit hydrograph is checked to see that it represents 1 cm of runoff. The procedure of adjustments through the *S*-curve is repeated till satisfactory results are obtained. It should be noted that out of the various parameters of the synthetic unit hydrograph the least accurate will be the time base T_b and this can be changed to meet other requirements.

SCS Dimensionless Unit Hydrograph

Dimensionless unit hydrographs based on a study of a large number of unit hydrographs are recommended by various agencies to facilitate construction of synthetic unit hydrographs. A typical dimensionless unit hydrograph developed by the US Soil Conservation Services (SCS) is shown in Fig. 6.21(a). In this the ordinate is (Q/Q_p) which is the discharge *Q* expressed as a ratio to the peak discharge Q_p, and the abscissa is (t/T_p), which is the time *t* expressed as a ratio of the time to peak T_p. By definition, $Q/Q_p = 1.0$ when $t/T_p = 1.0$. The coordinates of the SCS dimensionless unit hydrograph is given in Table 6.12 for use in developing a synthetic unit hydrograph in place of Snyder's equations (6.14) through (6.17).

Table 6.12 Coordinates of SCS Dimensionless Unit Hydrograph[4]

t/T_p	Q/Q_p	t/T_p	Q/Q_p	t/T_p	Q/Q_p
0.0	0.000	1.10	0.980	2.80	0.098
0.1	0.015	1.20	0.92	3.00	0.074
0.2	0.075	1.30	0.840	3.50	0.036
0.3	0.160	1.40	0.750	4.00	0.018
0.4	0.280	1.50	0.660	4.50	0.009
0.5	0.430	1.60	0.560	5.00	0.004
0.6	0.600	1.80	0.420		
0.7	0.770	2.00	0.320		
0.8	0.890	2.20	0.240		
0.9	0.970	2.40	0.180		
1.0	1.000	2.60	0.130		

SCS Triangular Unit Hydro-graph The value of Q_p and T_p may be estimated using a simplified model of a triangular unit hydrograph (Fig. 6.21(b)) suggested by SCS. This triangular unit hydrograph has the same percentage of volume on the rising side as the dimensionless unit hydrograph of Fig. 6.21(a).

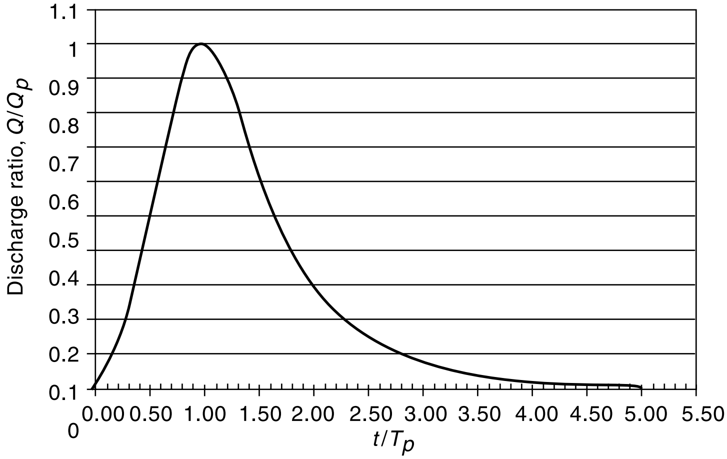

Fig. 6.21(a) Dimensionless SCS Unit Hydrograph

In Fig. 6.21(b),

Q_p = peak discharge in m^3/s

t_r = duration of effective rainfall

T_p = time of rise = time to peak = $(t_r/2) + t_p$

t_p = lag time

T_b = base length

SCS suggests that the time of recession = $(T_b - T_p) = 1.67\ T_p$

Thus $T_b = 2.67\ T_p$

Since the area under the unit hydrograph is equal to 1 cm,

If A = area of the watershed in km^2,

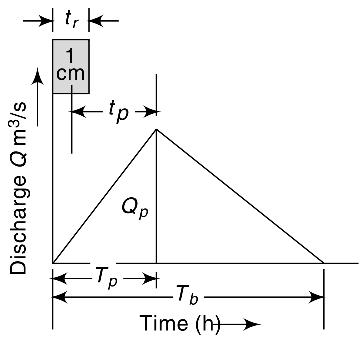

Fig. 6.21(b) SCS Triangular Unit Hydrograph

$$\frac{1}{2} Q_F \times (2.67 T_p) \times (3600) = \frac{1}{100} \times A \times 10^6$$

$$Q_p = \frac{2A \times 10^4}{3600 \times 2.67 T_p} = 2.08 \frac{A}{T_p} \tag{6.18}$$

Further on the basis of a large number of small rural watersheds, SCS found that $t_p \approx 0.6\ t_c$, where t_c = time of concentration (described in detail in Sec. 7.2, Chapter 7).

Thus $$T_p = \left(\frac{t_r}{2} + 0.6 t_c\right) \tag{6.19}$$

The SCS triangular unit hydrograph is a popular method used in watershed development activities, especially in small watersheds.

Example 6.15 *Develop a 30 minute SCS triangular unit hydrograph for a watershed of area 550 ha and time of concentration of 50 minutes.*

Solution: $A = 550 \text{ ha} = 5.5 \text{ km}^2$

$t_r = 30 \text{ min} = 0.50 \text{ h} \qquad t_c = 50 \text{ min} = 0.833 \text{ h}$

lag time $\quad t_p = 0.6\, t_c = 0.6 \times 0.833 = 0.50 \text{ h}$

$$T_p = \left(\frac{t_r}{2} + t_p\right) = 0.25 + 0.50 = 0.75 \text{ h}$$

$$Q_p = 2.08 \frac{A}{T_p} = 2.08 \times \frac{5.5}{0.75} = 15.25 \text{ m}^3/\text{s}$$

$$T_b = 2.67\, T_p = 2.67 \times 0.75 = 2.00 \text{ h}$$

The derived triangular unit hydrograph is shown in Fig. 6.22

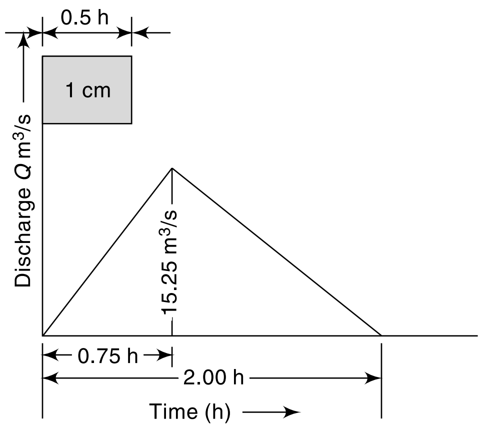

Fig. 6.22 Triangular Unit Hydrograph—Example 6.15

The Indian Practice

Two approaches (short term plan and long term plan) were adopted by CWC to develop methodologies for estimation of design flood discharges applicable to small and medium catchments (25–1000 ha) of India.

Under the *short-term plan*, a quick method of estimating design flood peak has been developed[2] as follows:

The peak discharge of a D-h unit hydrograph Q_{pd} in m³/s is

$$Q_{pd} = 1.79 A^{3/4} \qquad \text{for } S_m > 0.0028 \tag{6.20}$$

and

$$Q_{pd} = 37.4 A^{3/4} S_m^{2/3} \qquad \text{for } S_m < 0.0028 \tag{6.21}$$

where A = catchment area in km² and S_m = weighted mean slope given by

$$S_m = \left[\frac{L_{ca}}{(L_1/S_1)^{1/2} + (L_2/S_2)^{1/2} + \cdots + (L_n/S_n)^{1/2}}\right]^2 \tag{6.22}$$

in which L_{ca} = distance along the river from the gauging station to a point opposite to the centre of gravity of the area.

$L_1, L_2, \ldots L_n$ = length of main channel having slopes $S_1, S_2, \ldots S_n$ respectively, obtained from topographic maps.

The lag time in hours (i.e. time interval from the mid-point of the rainfall excess to the peak) of a 1-h unit hydrograph, t_{p1} is given by

$$t_{P1} = \frac{1.56}{[Q_{pd}/A]^{0.9}} \tag{6.23}$$

For design purposes the duration of rainfall excess in hours is taken as

$$D = 1.1\, t_{p1} \tag{6.24}$$

Equations (6.20) through (6.22) enable one to determine the duration and peak discharge of a design unit hydrograph. The time to peak has to be determined separately by using Eq. (6.9) or (6.10).

Under the *long-term plan*, a separate regional methodology has been developed by CWC. In this, the country is divided into 26 hydrometeorologically homogeneous subzones. For each subzone, a regional synthetic unit hydrograph has been developed. Detailed reports containing the synthetic unit hydrograph relations, details of the computation procedure and limitations of the method have been prepared, [e.g. CWC Reports No. CB/11/1985 and GP/10/1984 deal with flood estimation in Kaveri Basin (Sub-zone – 3i) and Middle Ganga Plains (Sub-zone – 1f) respectively.]

6.13 INSTANTANEOUS UNIT HYDROGRAPH (IUH)

The unit-hydrograph concept discussed in the preceding sections considered a D-h unit hydrograph. For a given catchment a number of unit hydrographs of different durations are possible. The shape of these different unit hydrographs depend upon the value of D. Figure 6.23 shows a typical variation of the shape of unit hydrographs for different values of D. As D is reduced, the intensity of rainfall excess being equal to $1/D$ increases and the unit hydrograph becomes more skewed. A finite unit hydrograph is indicated as the duration $D \to 0$. The limiting case of a unit hydrograph of zero duration is known as *instantaneous unit hydrograph* (IUH). Thus IUH is a fictitious, conceptual unit hydrograph which represents the surface runoff from the catchment due to

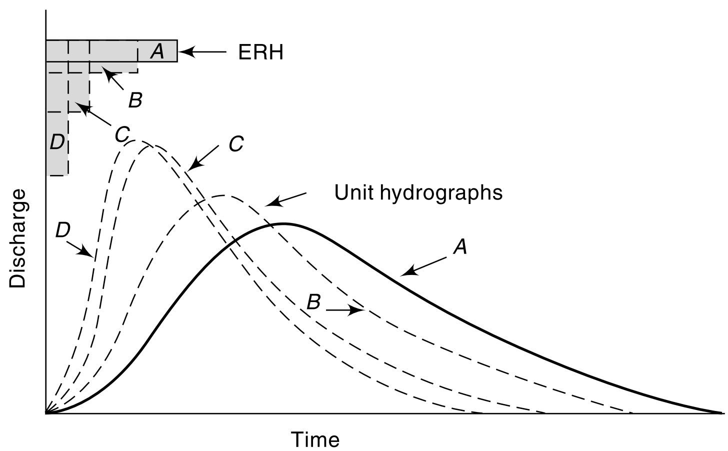

Fig. 6.23 Unit Hydrographs of Different Durations

an instantaneous precipitation of the rainfall excess volume of 1 cm. IUH is designated as $u(t)$ or sometimes as $u(0, t)$. It is a single-peaked hydrograph with a finite base width and its important properties can be listed as below:

1. $0 \le u(t) \le$ a positive value, for $t > 0$;
2. $u(t) = 0$ for $t \le 0$;
3. $u(t) \rightarrow = 0$ as $t \rightarrow \infty$;
4. $\int_0^\infty u(t)\,dt =$ unit depth over the catchment; and
5. time to the peak time to the centroid of the curve.

Consider an effective rainfall $I(\tau)$ of duration t_0 applied to a catchment as in Fig. 6.24. Each infinitesimal element of this ERH will operate on the IUH to produce a DRH whose discharge at time t is given by

$$Q(t) = \int_0^{t'} u(t-\tau)\, I(\tau)\, d\tau \tag{6.25}$$

where $t' = t$ when $t < t_0$ and $t' = t_0$ when $t \ge t_0$

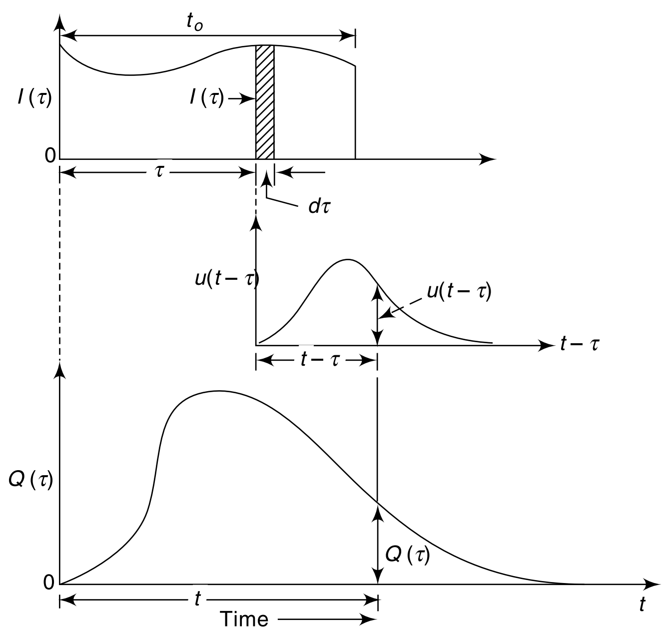

Fig. 6.24 Convolution of $I(\tau)$ and IUH

Equation (6.25) is called the *convolution integral* or *Duhamel integral.* The integral of Eq. (6.25) is essentially the same as the arithmetical computation of Eq. (6.5).

The main advantage of IUH is that it is independent of the duration of ERH and thus has one parameter less than a *D*-h unit hydrograph. This fact and the definition of IUH make it eminently suitable for theoretical analysis of rainfall excess-runoff relationship of a catchment. For a given catchment IUH, being independent of rainfall characteristics, is indicative of the catchment storage characteristics.

Derivation of IUH

Consider an S-curve, designated as S_1, derived from a D-h unit hydrograph. In this the intensity of rainfall excess, $i = 1/D$ cm/h. Let S_2 be another S-curve of intensity i cm/h. If S_2 is separated from S_1 by a time interval dt and the ordinates are subtracted, a DRH due to a rainfall excess of duration dt and magnitude $i\,dt = dt/D$ h is obtained. A unit hydrograph of dt hours is obtained from this by dividing the above DRH by $i\,dt$. Thus the dt-h unit hydrograph will have ordinates equal to $\left(\dfrac{S_2 - S_1}{i\,dt}\right)$. As dt is made smaller and smaller, i.e. as $dt \to 0$, an IUH results. Thus for an IUH, the ordinate at any time t is

$$u(t) = \lim_{dt \to 0}\left(\frac{S_2 - S_1}{i\,dt}\right) = \frac{1}{i}\frac{dS}{dt} \tag{6.26}$$

If $i = 1$, then $u(t) = dS'/dt$, (6.27)

where S' represents a S-curve of intensity 1 cm/h. Thus the ordinate of an IUH at any time t is the slope of the S-curve of intensity 1 cm/h (i.e. S-curve derived from a unit hydrograph of 1-h duration) at the corresponding time. Equation (6.26) can be used in deriving IUH approximately.

IUHs can be derived in many other ways, notably by (i) harmonic analysis (ii) Laplace transform, and (iii) conceptual models. Details of these methods are beyond the scope of this book and can be obtained from Ref. 3. However, two simple models viz., Clark's model and Nash's model are described in Chapter 8 (Sections 8.8 and 8.9).

Derivation of D-hour Unit Hydrograph from IUH For simple geometric forms of IUH, Eq. (6.25) can be used to derive a D-hour unit hydrograph. For complex shaped IUHs the numerical computation techniques used in deriving unit hydrographs of different durations (Sec. 6.7) can be adopted.

From Eq. 6.27, $dS' = u(t)\,dt$

Integrating between two points 1 and 2

$$S_2' - S_1' = \int_{t_1}^{t_2} u(t)\,dt \tag{6.28}$$

If $u(t)$ is essentially linear within the range 1–2, then for small values of $\Delta t = (t_2 - t_1)$, by taking

$$u(t) = \bar{u}(t) = \frac{1}{2}[u(t_1) + u(t_2)]$$

$$S_2' - S_1' = \frac{1}{2}[u(t_1) + u(t_2)]\,(t_2 - t_1) \tag{6.29}$$

But $(S_2' - S_1')/(t_2 - t_1)$ = ordinate of a unit hydrograph of duration $D_1 = (t_2 - t_1)$. Thus, in general terms, for small values of D_1, the ordinates of a D_1-hour unit hydrograph are obtained by the equation

$$(D_1\text{-hour UH})_t = \frac{1}{2}[(\text{IUH})_t + (\text{IUH})_{t-D_1}] \tag{6.30}$$

Thus if two IUHs are lagged by D_1–hour where D_1 is small and their corresponding ordinates are summed up and divided by two, the resulting hydrograph will be a D_1-hour UH. After obtaining the ordinates of a D-hour unit hydrograph from

Eq. (6.30), the ordinates of any *D*-hour UH can be obtained by the superposition method or *S*-curve method described in Sec. 6.7. From accuracy considerations, unless the limbs of IUH can be approximated as linear, it is desirable to confine D_1 to a value of 1-hour or less.

EXAMPLE 6.16 *The coordinates of the IUH of a catchment are given below. Derive the direct runoff hydrograph (DRH) for this catchment due to a storm of duration 4 hours and having a rainfall excess of 5 cm.*

Time (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12
IUH ordinate $u(t)$ (m^3/s)	0	8	35	50	47	40	31	23	15	10	6	3	0

SOLUTION: The calculations are performed in Table 6.13.

1. First, the ordinates of 1-h UH are derived by using Eq. (6.30)
   In Table 6.13, Col. 2 = ordinates of given IUH = $u(t)$
   Col. 3 = ordinates of IUH lagged by 1-h
   Col. 4 = $\frac{1}{2}$ (Col. 2 + Col. 3) = ordinates of 1-h UH by Eq. (6.30)
2. Using the 1-hour UH, the *S*-curve is obtained and lagging it by 4 hours the ordinates of 4-h UH are obtained.
   In Table 6.12, Col. 5 = *S*-curve additions
   Col. 6 = (Col. 4 + Col. 5) = *S*-curve ordinates
   Col. 7 = Col. 6 lagged by 4 hours = *S*-curve ordinates lagged by 4-h.
   Col. 8 = (Col. 6 – Col. 7) = Ordinates of a DRH due to 4 cm of ER in 4 hours.
   Col. 9 = (Col. 8)/4 = Ordinates of 4-hour UH
3. The required DRH ordinates due to 5.0 cm ER in 4 hours are obtained by multiplying the ordinates of 4-h UH by 5.0
   In Table 6.12, Co1. 10 = (Col. 9) × 5.0 = ordinates of required DRH
   [***Note:*** Calculation of 4-hour UH directly by using D_1 = 4-h in Eq. (6.30) will lead to errors as the assumptions of linearity of $u(t)$ during D_1 may not be satisfied.]

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REVISION QUESTIONS

6.1 List the factors affecting a flood hydrograph. Discuss the role of these factors.
6.2 Describe the analysis of the recession limb of a flood hydrograph.
6.3 Explain the term Rainfall Excess (ER). How is ERH of a storm obtained?
6.4 Why is base flow separated from the flood hydrograph in the process of developing a unit hydrograph?
6.5 What is a unit hydrograph? List the assumptions involved in the unit hydrograph theory.

Table 6.13 Determination of DRH from IUH—Example 6.16

1	2	3	4	5	6	7	8	9	10
Time (h)	$u(t)$ (m³/s)	$u(t)$ lagged by 1 hour	Ordinate of 1-h UH (m³/s)	S-Curve addition (m³/s)	S-Curve ordinate (m³/s)	S-Curve lagged by 4 hours (m³/s)	DRH of 4 cm in 4 hours	Ordinate of 4-h UH (m³/s)	DRH due to 5 cm ER in 4 hours (m³/s)
			([2] + [3])/2		[4] + [5]		[6]–[7]	[8]/4	[9] × 5
0	0		0		0		0.0	0.00	0.00
1	8	0	4.0	0	4.0		4.0	1.00	5.00
2	35	8	21.5	4.0	25.5		25.5	6.38	31.88
3	50	35	42.5	25.5	68.0		68.0	17.00	85.00
4	47	50	48.5	68.0	116.5	0.0	116.5	29.13	145.63
5	40	47	43.5	116.5	160.0	4.0	156.0	39.00	195.00
6	31	40	35.5	160.0	195.5	25.5	170.0	42.50	212.50
7	23	31	27.0	195.5	222.5	68.0	154.5	38.63	193.13
8	15	23	19.0	222.5	241.5	116.5	125.0	31.25	156.25
9	10	15	12.5	241.5	254.0	160.0	94.0	23.50	117.50
10	6	10	8.0	254.0	262.0	195.5	66.5	16.63	83.13
11	3	6	4.5	262.0	266.5	222.5	44.0	11.00	55.00
12	0	3	1.5	266.5	268.0	241.5	26.5	6.63	33.13
13		0	0.0	268.0	268.0	254.0	14.0	3.50	17.50
14		0	0.0	268.0	268.0	262.0	6.0	1.50	7.50
15		0	0.0	268.0	268.0	266.5	1.5	0.38	1.88
16		0	0.0	268.0	268.0	268.0	0.0	0.00	0.00

6.6 Describe briefly the procedure of preparing a *D*-hour unit hydrograph for a catchment.

6.7 Explain the procedure of using a unit hydrograph to develop the flood hydrograph due to a storm in a catchment.

6.8 Describe the *S*-curve method of developing a 6-h UH by using 12-h UH of the catchment.

6.9 Explain a procedure of deriving a synthetic unit hydrograph for a catchment by using Snyder's method.

6.10 What is an IUH? What are its characteristics?

6.11 Explain a procedure of deriving a *D*-h unit hydrograph from the IUH of the catchment.

6.12 Distinguish between

(a) Hyetograph and hydrograph (b) *D*-h UH and IUH

Problems

6.1 The flood hydrograph of a small stream is given below. Analyse the recession limb of the hydrograph and determine the recession coefficients. Neglect interflow.

Time (days)	Discharge (m^3/s)	Time (days)	Discharge (m^3/s)	Time (days)	Discharge (m^3/s)
0	155	2.0	9.0	4.0	1.9
0.5	70.0	2.5	5.5	5.0	1.4
1.0	38.0	3.0	3.5	6.0	1.2
1.5	19.0	3.5	2.5	7.0	1.1

Estimate the groundwater storage at the end of 7th day from the occurrence of peak.

6.2 On June 1, 1980 the discharge in a stream was measured as 80 m^3/s. Another measurement on June 21, 1980 yielded the stream discharge as 40 m^3/s. There was no rainfall in the catchment from April 15, 1980. Estimate the (a) recession coefficient, (b) expected stream flow and groundwater storage available on July 10, 1980. Assume that there is no further rainfall in the catchment up to that date.

6.3 If $Q(t) = Q_0 K^t$ describes the base flow recession in a stream, prove that the storage $S(t_1)$ left in the basin at any time for supplying base flow follows the linear reservoir model, viz. $S(t_1) = C\,Q\,(t_1)$, where C is a constant.

[**Hint:** Use the boundary condition: at $t = \infty$, $S_\infty = 0$ and $Q_\infty = 0$]

6.4 A 4 -hour storm occurs over an 80 km^2 watershed. The details of the catchment are as follows.

Sub Area (km^2)	ϕ-Index (mm/hour)	Hourly Rain (mm)			
		1st hour	2nd hour	3rd hour	4th hour
15	10	16	48	22	10
25	15	16	42	20	8
35	21	12	40	18	6
5	16	15	42	18	8

Calculate the runoff from the catchment and the hourly distribution of the effective rainfall for the whole catchment.

6.5 Given below are observed flows from a storm of 6-h duration on a stream with a catchment area of 500 km^2

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Observed flow (m^3/s)	0	100	250	200	150	100	70	50	35	25	15	5	0

Assuming the base flow to be zero, derive the ordinates of the 6-h unit hydrograph.

6.6 A flood hydrograph of a river draining a catchment of 189 km^2 due to a 6 hour isolated storm is in the form of a triangle with a base of 66 hour and a peak ordinate of 30 m^3/s occurring at 10 hours from the start. Assuming zero base flow, develop the 6-hour unit hydrograph for this catchment.

6.7 The following are the ordinates of the hydrograph of flow from a catchment area of 770 km^2 due to a 6-h rainfall. Derive the ordinates of the 6-h unit hydrograph. Make suitable assumptions regarding the base flow.

Time from beginning of storm (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m^3/s)	40	65	215	360	400	350	270	205	145	100	70	50	42

6.8 Analysis of the surface runoff records of a 1-day storm over a catchment yielded the following data:

Time (days)	0	1	2	3	4	5	6	7	8	9
Discharge (m^3/s)	20	63	151	133	90	63	44	29	20	20
Estimated base flow (m^3/s)	20	22	25	28	28	26	23	21	20	20

Determine the 24-h distribution graph percentages. If the catchment area is 600 km^2, determine the depth of rainfall excess.

6.9 The ordinates of a hydrograph of surface runoff resulting from 4.5 cm of rainfall excess of duration 8 h in a catchment are as follows:

Time (h)	0	5	13	21	28	32	35	41	45	55
Discharge (m^3/s)	0	40	210	400	600	820	1150	1440	1510	1420
Time (h)	61	91	98	115	138					
Discharge (m^3/s)	1190	650	520	290	0					

Determine the ordinates of the 8-h unit hydrograph for this catchment.

6.10 The peak of a flood hydrograph due to a 6-h storm is 470 m^3/s. The mean depth of rainfall is 8.0 cm. Assume an average infiltration loss of 0.25 cm/h and a constant base flow of 15 m^3/s and estimate the peak discharge of the 6-h unit hydrograph for this catchment.

6.11 Given the following data about a catchment of area 100 km^2, determine the volume of surface runoff and peak surface runoff discharge corresponding to a storm of 60 mm in 1 hour.

Time (h)	**0**	**1**	**2**	**3**	**4**	**5**
Rainfall (mm)	0	40	0	0	0	0
Runoff (m^3/s)	300	300	1200	450	300	300

6.12 The ordinates of a 6-h unit hydrograph are given.

Time (h)	0	3	6	9	12	18	24	30	36	42	48	54	60	66
6-h UH ordinate (m^2/s)	0	150	250	450	600	800	700	600	450	320	200	100	50	0

A storm had three successive 6-h intervals of rainfall magnitude of 3.0, 5.0 and 4.0 cm, respectively. Assuming a ϕ index of 0.20 cm/h and a base flow of 30 m^3/s, determine and plot the resulting hydrograph of flow.

6.13 The ordinates of a 6-h unit hydrograph are as given below:

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
ordinate of 6-h UH (m^3/s)	0	20	60	150	120	90	66	50	32	20	10	0

If two storms, each of 1-cm rainfall excess and 6-h duration occur in succession, calculate the resulting hydrograph of flow. Assume base flow to be uniform at 10 m^3/s.

6.14 Using the 6-h unit hydrograph of Prob. 6.13 derive a 12-h unit hydrograph for the catchment.

6.15 The ordinates of the 2-h unit hydrograph of a basin are given:

Time (h)	0	2	4	6	8	10	12	14	16	18	20	22
2-h UH ordinate (m^3/s)	0	25	100	160	190	170	110	70	30	20	6	0

Determine the ordinates of the S_2-curve hydrograph and using this S_2-curve, determine the ordinates of the 4-h unit hydrograph of the basin.

6.16 The 6-hour unit hydrograph of a catchment is triangular in shape with a base width of 64 hours and a peak ordinate of 30 m^3/s. Calculate the equilibrium discharge of the S_6-curve of the basin.

6.17 Ordinates of the one hour unit hydrograph of a basin at one-hour intervals are 5, 8, 5, 3 and 1 m^3/s. Calculate the

(i) watershed area represented by this unit hydrograph. (ii) S_1-curve hydrograph.

(iii) 2-hour unit hydrograph for the catchment.

6.18 Using the ordinates of a 12-h unit hydrograph given below, compute the ordinates of the 6-h unit hydrograph of the basin.

Time (h)	**Ordinate of 12-h UH (m^3/s)**	**Time (h)**	**Ordinate of 12-h UH (m^3/s)**	**Time (h)**	**Ordinate of 12-h UH (m^3/s)**
0	0	54	130	108	17
6	10	60	114	114	12
12	37	66	99	120	8
18	76	72	84	126	6
24	111	78	71	132	3
30	136	84	58	138	2
36	150	90	46	144	0
42	153	96	35		
48	146	106	25		

[Note that the tail portion of the resulting 6-h UH needs fairing.]

6.19 The 3-h unit hydrograph for a basin has the following ordinates. Using the S-curve method, determine the 9-h unit hydrograph ordinates of the basin.

Time (h)	0	3	6	9	12	15	18	21	24	27	30
3-h UH ordinates (m^3/s)	0	12	75	132	180	210	183	156	135	144	96
Time (h)	33	36	39	42	45	48	51	54	57	60	
3-h UH ordinates (m^3/s)	87	66	54	42	33	24	18	12	6	6	

6.20 Using the given 6-h unit hydrograph derive the flood hydrograph due to the storm given below.

UH:

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
6-h UH ordinates (m^3/s)	0	20	60	150	120	90	66	50	32	20	10	0

Storm:

Time from beginning of the storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	4	5	10

The φ index for the storm can be assumed to be 0.167 cm/h. Assume the base flow to be 20 m^3/s constant throughout.

6.21 The 6-hour unit hydrograph of a basin is triangular in shape with a peak of 100 m^3/s occurring at 24-h from the start. The base is 72-h.

(a) What is the area of the catchment represented by this unit hydrograph?

(b) Calculate the flood hydrograph due to a storm of rainfall excess of 2.0 cm during the first 6 hours and 4.0 cm during the second 6 hours interval. The base flow can be assumed to be 25 m^3/s constant throughout.

6.22 The 6-h unit hydrograph of a catchment of area 1000 km^2 can be approximated as a triangle with base of 69 h. Calculate the peak ordinate of this unit hydrograph.

6.23 The 4-h, distribution graph of a catchment of 50 km^2 area has the following ordinates:

Unit periods (4-h units)	1	2	3	4	5	6
Distribution (percentage)	5	20	40	20	10	5

If the catchment has rainfalls of 3.5. 2.2 and 1.8 cm in three consecutive 4-h periods, determine the resulting direct runoff hydrograph by assuming the φ-index for the storm as 0.25 cm/h.

6.24 The 6-h unit hydrograph of a catchment of area 259.2 km^2 is triangular in shape with a base width of 48 hours. The peak occurs at 12 h from the start. Derive the coordinates of the 6-h distribution graph for this catchment.

6.25 The one-hour unit hydrograph of a small rural catchment is triangular in shape with a peak value of 3.6 m^3/s occurring at 3 hours from the start and a base time of 9 hours. Following urbanisation over a period of two decades, the infiltration index φ has decreased from 0.70 cm/h to 0.40 cm/h. Also the one-hour unit hydrograph has now a peak of 6.0 m^3/s at 1.5 hours and a time base of 6 hours. If a design storm has intensities of 4.0 cm/h and 3.0 cm/h for two consecutive one hour intervals, estimate the percentage increase in the peak storm runoff and in the volume of flood runoff, due to urbanisation.

6.26 The following table gives the ordinates of a direct-runoff hydrograph resulting from two successive 3-h durations of rainfall excess values of 2 and 4 cm, respectively. Derive the 3-h unit hydrograph for the catchment.

Time (h)	0	3	6	9	12	15	18	21	24	27	30
Direct runoff (m^3/s)	0	120	480	660	460	260	160	100	50	20	0

6.27 Characteristics of two catchments *M* and *N* measured from a map are given below:

Item	Catchment *M*	Catchment *N*
L_{ca}	76 km	52 km
L	148 km	106 km
A	2718 km^2	1400 km^2

For the 6-h unit hydrograph in catchment *M*, the peak discharge is at 200 m^3/s and occurs at 37 h from the start of the rainfall excess. Assuming the catchments *M* and *N* are meteorologically similar, determine the elements of the 6-h synthetic unit hydrograph for catchment *N* by using Snyder's method.

6.28 A basin has an area of 400 km^2, and the following characteristics:
L = basin length = 35 km
L_{ca} = Length up to the centroid of the basin = 10 km
Snyder's coefficients: $C_t = 1.5$ and $C_p = 0.70$.
Develop synthetically the 3-h synthetic-unit hydrograph for this basin using Snyder's method.

6.29 Using the peak discharge and time to peak values of the unit hydrograph derived in Prob. 6.27, develop the full unit hydrograph by using the SCS dimensionless-unit hydrograph.

6.30 The rainfall excess of a storm is modelled as

$I(t) = 6$ cm/s for $0 \le t \le 4$ h
$I(t) = 0$ for $t \ge 4$ h

The corresponding direct runoff hydrograph is expressed in terms of depth over unit catchment area per hour (cm/h) as

$Q(t) = 6.0\,t$ cm/h for $0 \le t \le 4$ h
$Q(t) = 48 - 6.0\,t$ cm/h for $8 > t \ge 4$ h
$Q(t) = 0$ for $t > 8$

where t is in hours. Determine the (i) 4-h unit hydrograph of the catchment and corresponding *S*-curve of the catchment (ii) 3-h unit hydrogen of the catchment.

6.31 A 2-h unit hydrograph is given by

$U(t) = 0.5$ cm/h for $0 \le t \le 2$ h
$U(t) = 0$ for $t \ge 4$ h

(i) Determine the *S*-curve corresponding to the given 2-h UH
(ii) Using the *S*-curve developed above, determine the 4-h unit hydrograph

6.32 A 1-h unit hydrograph is rectangular in shape with a base of 3 hours and peak of 100 m^3/s. Develop the DRH due to an ERH given below:

Time since start (h)	1	2	3
Excess Rainfall (cm)	3	0	5

6.33 A 750 ha watershed has a time of concentration of 90 minutes.
(i) Derive the 15-minute unit hydrograph for this watershed by using SCS triangular unit hydrograph method.
(ii) What would be the DRH for a 15-minute storm having 4.0 cm of rainfall?

6.34 The IUH of a catchment is triangular in shape with a base of 36 h and peak of 20 m^3/s occurring at 8 hours from the start. Derive the 2-h unit hydrograph for this catchment.

6.35 The coordinates of the IUH of a catchment are as below:

Time (h)	0	1	2	3	4	5	6	8	10	12	14	16	18	20
Ordinates (m^3/s)	0	11	37	60	71	75	72	60	45	33	21	12	6	0

(a) What is the areal extent of the catchment?
(b) Derive the 3-hour unit hydrograph for this catchment.

Objective Questions

6.1 The recession limb of a flood hydrograph can be expressed with positive values of coefficients, as $Q_t/Q_0 =$

(a) K_c^{at} (b) $a\,K_t^{-at}$ (c) a^{-at} (d) e^{-at^2}

6.2 For a given storm, other factors remaining same,
(a) basins having low drainage density give smaller peaks in flood hydrographs
(b) basins with larger drainage densities give smaller flood peaks
(c) low drainage density basins give shorter time bases of hydrographs
(d) the flood peak is independent of the drainage density.

6.3 Base-flow separation is performed
(a) on a unit hydrograph to get the direct-runoff hydrograph
(b) on a flood hydrograph to obtain the magnitude of effective rainfall
(c) on flood hydrographs to obtain the rainfall hyetograph
(d) on hydrographs of effluent streams only.

6.4 A direct-runoff hydrograph due to a storm was found to be triangular in shape with a peak of 150 m^3/s, time from start of effective storm to peak of 24 h and a total time base of 72 h. The duration of the storm in this case was
(a) < 24 h (b) between 24 to 72 h
(c) 72 h (d) > 72 h.

6.5 A unit hydrograph has one unit of
(a) peak discharge (b) rainfall duration
(c) direct runoff (d) the time base of direct runoff.

6.6 The basic assumptions of the unit-hydrograph theory are
(a) nonlinear response and time invariance
(b) time invariance and linear response
(c) linear response and linear time variance
(d) nonlinear time variance and linear response.

6.7 The *D*-hour unit hydrograph of a catchment may be obtained by dividing the ordinates of a single peak direct runoff hydrograph (DRH) due to a storm of *D* hour duration by the
(a) Total runoff volume (in cm) (b) Direct runoff volume (in cm)
(c) Duration of DRH (d) Total rainfall (in cm)

6.8 A storm hydrograph was due to 3 h of effective rainfall. It contained 6 cm of direct runoff. The ordinates of DRH of this storm
(a) when divided by 3 give the ordinates of a 6-h unit hydrograph
(b) when divided by 6 give the ordinates of a 3-h unit hydrograph

(c) when divided by 3 give the ordinates of a 3-h unit hydrograph
(d) when divided by 6 give the ordinates of a 6-h unit hydrograph.

6.9 A 3-hour storm over a watershed had an average depth of 27 mm. The resulting flood hydrograph was found to have a peak flow of 200 m^3/s and a base flow of 20 m^3/s. If the loss rate could be estimated as 0.3 cm/h, a 3-h unit hydrograph for this watershed will have a peak of
(a) 66.7 m^3/s (b) 100 m^3/s (c) 111.1 m^3/s (d) 33.3 m^3/s

6.10 A triangular DRH due to a storm has a time base of 80 hrs and a peak flow of 50 m^3/s occurring at 20 hours from the start. If the catchment area is 144 km^2, the rainfall excess in the storm was
(a) 20 cm (b) 7.2 cm (c) 5 cm (d) none of these.

6.11 The 12-hr unit hydrograph of a catchment is triangular in shape with a base width of 144 hours and a peak discharge value of 23 m^3/s. This unit hydrograph refers to a catchment of area
(a) 756 km^2 (b) 596 km^2 (c) 1000 km^2 (d) none of these.

6.12 The 6-h unit hydrograph of a catchment is triangular in shape with a base width of 64 h and peak ordinate of 20 m^3/s. If a 0.5 cm rainfall excess occurs in 6 h in that catchment, the resulting surface-runoff hydrograph will have
(a) a base of 128 h (b) a base of 32 h
(c) a peak of 40 m^3/s (d) a peak of 10 m^3/s

6.13 A 90 km^2 catchment has the 4-h unit hydrograph which can be approximated as a triangle. If the peak ordinate of this unit hydrograph is 10 m^3/s the time base is
(a) 120 h (b) 64 h (c) 50 h (d) none of these.

6.14 A triangular DRH due to a 6-h storm in a catchment has a time base of 100 h and a peak flow of 40 m^3/s. The catchment area is 180 km^2. The 6-h unit hydrograph of this catchment will have a peak flow in m^3/s of
(a) 10 (b) 20 (c) 30 (d) none of these.

6.15 The 3-hour unit hydrograph U_1 of a catchment of area 250 km^2 is in the form of a triangle with peak discharge of 40 m^3/s. Another 3-hour unit hydrograph U_2 is also triangular in shape and has the same base width as U_1 but with a peak flow of 80 m^3/s. The catchment which U_2 refers to has an area of
(a) 125 km^2 (b) 250 km^2 (c) 1000 km^2 (d) 500 km^2

6.16 U_c is the 6-h unit hydrograph for a basin representing 1 cm of direct runoff and U_m is the direct runoff hydrograph for the same basin due to a rainfall excess of 1 mm in a storm of 6 hour duration.
(a) Ordinates of U_m are 1/10 the corresponding ordinates of U_c
(b) Base of U_m is 1/10 the base of U_c
(c) Ordinates of U_m are 10 times the corresponding ordinates of U_c
(d) Base of U_m is 10 times the base of U_c

6.17 A basin with an area of 756 km^2 has the 6-h unit hydrograph which could be approximated as a triangle with a base of 70 hours. The peak discharge of direct runoff hydrograph due to 5 cm of rainfall excess in 6 hours from that basin is
(a) 535 m^3/s (b) 60 m^3/s (c) 756 m^3/s (d) 300 m^3/s

6.18 The peak flow of a flood hydrograph caused by isolated storm was observed to be 120 m^3/s. The storm was of 6 hours duration and had a total rainfall of 7.5 cm. If the base flow and the φ-index are assumed to be 30 m^3/s and 0.25 cm/h respectively, the peak ordinate of the 6-h unit hydrograph of the catchment is
(a) 12.0 m^3/s (b) 15.0 m^3/s (c) 16.0 m^3/s (d) 20.0 m^3/s

6.19 The peak ordinate of 4-h unit hydrograph a basin is 80 m^3/s. An isolated storm of 4-hours duration in the basin was recorded to have a total rainfall of 7.0 cm. If it is assumed that the base flow and the φ-index are 20 m^3/s and 0.25 cm/h respectively, the peak of the flood discharge due to the storm could be estimated as

(a) 580 m^3/s (b) 360 m^3/s (c) 480 m^3/s (d) 500 m^3/s

6.20 The peak flow of a flood hydrograph caused by isolated storm was observed to be 100 m^3/s. The storm had a duration of 8.0 hours and the total depth of rainfall of 7.0 cm. The base flow and the φ-index were estimated as 20 m^3/s and 0.25 cm/h respectively. If in the above storm the total rainfall were 9.5 cm in the same duration of 8 hours, the flood peak would have been larger by

(a) 35.7% (b) 40% (c) 50% (d) 20%

6.21 For a catchment with an area of 360 km^2 the equilibrium discharge of the *S*-curve obtained by summation of 4-h unit hydrograph is

(a) 250 m^3/s (b) 90 m^3/s (c) 278 m^3/s (d) 360 m^3/s

6.22 For a catchment of area *A* an *S*-curve has been derived by using the *D*-hour unit hydrograph which has a time base *T*. In this *S*-curve

(a) the equilibrium discharge is independent of *D*
(b) the time at which the *S*-curve attains its maximum value is equal to *T*
(c) the time at which the *S*-curve attains its maximum value is equal to *D*
(d) the equilibrium discharge is independent of *A*

6.23 An IUH is a direct runoff hydrograph of

(a) of one cm magnitude due to rainfall excess of 1-h duration
(b) that occurs instantaneously due to a rainfall excess of 1-h duration
(c) of unit rainfall excess precipitating instantaneously over the catchment
(d) occurring at any instant in long duration

6.24 An instantaneous unit hydrograph is a hydrograph of

(a) unit duration and infinitely small rainfall excess
(b) infinitely small duration and of unit rainfall excess
(c) infinitely small duration and of unit rainfall excess of an infinitely small area
(d) unit rainfall excess on infinitely small area

Chapter
7

FLOODS

7.1 INTRODUCTION

A flood is an unusually high stage in a river, normally the level at which the river overflows its banks and inundates the adjoining area. The damages caused by floods in terms of loss of life, property and economic loss due to disruption of economic activity are all too well known. Thousands of crores of rupees are spent every year in flood control and flood forecasting. The hydrograph of extreme floods and stages corresponding to flood peaks provide valuable data for purposes of hydrologic design. Further, of the various characteristics of the flood hydrograph, probably the most important and widely used parameter is the flood peak. At a given location in a stream, flood peaks vary from year to year and their magnitude constitutes a hydrologic series which enable one to assign a frequency to a given flood-peak value. In the design of practically all hydraulic structures the peak flow that can be expected with an assigned frequency (say 1 in 100 years) is of primary importance to adequately proportion the structure to accommodate its effect. The design of bridges, culvert waterways and spillways for dams and estimation of scour at a hydraulic structure are some examples wherein flood-peak values are required.

To estimate the magnitude of a flood peak the following alternative methods are available:

1. Rational method
2. Empirical method
3. Unit-hydrograph technique
4. Flood-frequency studies

The use of a particular method depends upon (i) the desired objective, (ii) the available data, and (iii) the importance of the project. Further the *rational formula* is only applicable to small-size (< 50 km^2) catchments and the unit-hydrograph method is normally restricted to moderate-size catchments with areas less than 5000 km^2.

7.2 RATIONAL METHOD

Consider a rainfall of uniform intensity and very long duration occurring over a basin. The runoff rate gradually increases from zero to a constant value as indicated in Fig. 7.1. The runoff increases as more and more flow from remote areas of the catchment reach the outlet. Designating the time taken for a drop of water from the farthest part of the catchment to reach the outlet as t_c = time of concentration, it is obvious that if the rainfall continues beyond t_c, the runoff will be constant and at the peak value. The peak value of the runoff is given by

$$Q_p = C\,A\,i; \text{ for } t \geq t_c \qquad (7.1)$$