Micromechanical Analysis of a Lamina

LECTURE 5: CHAPTER TWO

Print · April 2020.

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Chapter Two: Objectives

• Develop concepts of volume and weight fraction (mass fraction) of fiber and matrix, density, and void fraction in composites.

• Find the nine mechanical and four hygrothermal constants: four elastic moduli, five strength parameters, two coefficients of thermal expansion, and two coefficients of moisture expansion of a unidirectional lamina from the individual properties of the fiber and the matrix, fiber volume fraction, and fiber packing.

• Discuss the experimental characterization of the nine mechanical and four hygrothermal constants. The two main processes are called chemical and mechanical processes.

2.1 Introduction

The stress–strain relationships, engineering constants, and failure theories for an angle lamina were developed using **four elastic moduli**, **five strength parameters, two coefficients of thermal expansion (CTE)**, and **two coefficients of moisture expansion (CME) for a unidirectional lamina**. These **13** parameters can be found experimentally by conducting several tension, compression, shear, and hygrothermal tests on unidirectional lamina (laminates).

However, unlike in isotropic materials, experimental evaluation of these parameters is quite costly and time consuming because they are functions of several variables: the individual constituents of the composite material, fiber volume fraction, packing geometry, processing, etc. Thus, the need and motivation for developing analytical models to find these parameters are very important.

In this chapter, we will develop simple relationships for the these parameters in terms of the stiffnesses, strengths, coefficients of thermal and moisture expansion of the individual constituents of a composite, fiber volume fraction, packing geometry, etc. An understanding of this relationship, called micromechanics of lamina, helps the designer to select the constituents of a composite material for use in a laminated structure. Because this text is for a first course in composite materials, details will be explained only for the simple models based on the mechanics of materials approach and the semi-empirical approach. Results from other methods based on advanced topics such as elasticity are also explained for completeness. As mentioned in a previous chapter, a unidirectional lamina is not homogeneous. However, one can assume the lamina to be homogeneous by focusing on the average response of the lamina to mechanical and hygrothermal loads (Figure 3.1).

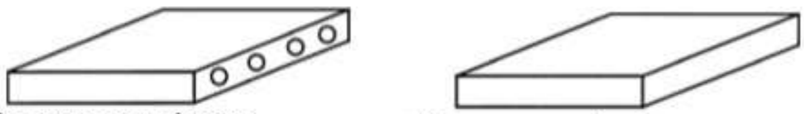

FIGURE 3.1
A nonhomogeneous lamina with fibers and matrix approximated as a homogeneous lamina.

The lamina is simply looked at as a material whose properties are different in various directions, but not different from one location to another. Also, the chapter focuses on a unidirectional continuous fiber-reinforced lamina. This is because it forms the basic building block of a composite structure, which is generally made of several unidirectional laminae placed at various angles. The modeling in the evaluation of the parameters is discussed first. This is followed by examples and experimental methods for finding these parameters.

2.2 Volume and Mass Fractions, Density, and Void Content

Before modeling the 13 parameters of a unidirectional composite, we introduce the concept of relative fraction of fibers by volume. This concept is critical because theoretical formulas for finding the stiffness, strength, and hygrothermal properties of a unidirectional lamina are a function of fiber volume fraction. Measurements of the constituents are generally based on their mass, so fiber mass fractions must also be defined. Moreover, defining the density of a composite also becomes necessary because its value is used in the experimental determination of fiber volume and void fractions of a composite. Also, the value of density is used in the definition of specific modulus and specific strength in Chapter 1.

2.2.1 Volume Fractions

Consider a composite consisting of fiber and matrix. Take the following symbol notations:

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively
$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction V_f and the matrix volume fraction V_m as

$$V_f = \frac{v_f}{v_c},$$

and

$$V_m = \frac{v_m}{v_c}. \tag{3.1a, b}$$

Note that the sum of volume fractions is

$$V_f + V_m = 1,$$

from Equation (3.1) as

$$v_f + v_m = v_c.$$

2.2.2 Mass Fractions

Consider a composite consisting of fiber and matrix and take the following symbol notation: $w_{c,f,m}$ = mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (W_f) and the matrix (W_m) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}. \tag{3.2a, b}$$

Note that the sum of mass fractions is

$$W_f + W_m = 1 ,$$

from Equation (3.2) as

$$w_f + w_m = w_c .$$

From the definition of the density of a single material,

$$w_c = r_c v_c ,$$

$$w_f = r_f v_f , \text{ and} \tag{3.3a–c}$$

$$w_m = r_m v_m .$$

Substituting Equation (3.3) in Equation (3.2), the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f , \text{ and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m , \tag{3.4a, b}$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\dfrac{\rho_f}{\rho_m}}{\dfrac{\rho_f}{\rho_m} V_f + V_m} V_f ,$$

$$W_m = \frac{1}{\dfrac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m . \qquad (3.5a, b)$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Based on Equation (3.4), it is evident that volume and mass fractions are not equal and that the mismatch between the mass and volume fractions increases as the ratio between the density of fiber and matrix differs from one.

2.2.3 Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite w_c is the sum of the mass of the fibers w_f and the mass of the matrix w_m as

$$w_c = w_f + w_m. \tag{3.6}$$

Substituting Equation (3.3) in Equation (3.6) yields

$$\rho_c v_c = \rho_f v_f + \rho_m v_m,$$

and

$$\rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c}. \tag{3.7}$$

Using the definitions of fiber and matrix volume fractions from Equation (3.1),

$$\rho_c = \rho_f V_f + \rho_m V_m. \tag{3.8}$$

Now, consider that the volume of a composite v_c is the sum of the volumes of the fiber v_f and matrix (v_m):

$$v_c = v_f + v_m. \tag{3.9}$$

The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}. \tag{3.10}$$

Example 2.1: A glass/epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy from Table 3.1* and Table 3.2, respectively, to determine:

1. Density of lamina

2. Mass fractions of the glass and epoxy

3. Volume of composite lamina if the mass of the lamina is 4 kg

4. Volume and mass of glass and epoxy in part (3)

TABLE 3.1

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	μm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	μm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

TABLE 3.2

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	μm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

Solution:

1. From Table 3.1, the density of the fiber is

$$\rho_f = 2500\ kg/m^3.$$

From Table 3.2, the density of the matrix is

$$\rho_m = 1200\ kg/m^3.$$

Using Equation (3.8), the density of the composite is

$$\rho_c = (2500)(0.7) + (1200)(0.3)$$
$$= 2110\ kg/m^3.$$

2. Using Equation (3.4), the fiber and matrix mass fractions are

$$W_f = \frac{2500}{2110} \times 0.3$$
$$= 0.8294$$

$$W_m = \frac{1200}{2110} \times 0.3 \ .$$
$$= 0.1706$$

Note that the sum of the mass fractions,

$$W_f + W_m = 0.8294 + 0.1706$$
$$= 1.000.$$

3. The volume of composite is

$$v_c = \frac{w_c}{\rho_c}$$

$$= \frac{4}{2110}$$

$$= 1.896 \times 10^{-3} m^3 \ .$$

4. The volume of the fiber is

$$v_f = V_f v_c$$

$$= (0.7)(1.896 \times 10^{-3})$$

$$= 1.327 \times 10^{-3} m^3 \ .$$

The volume of the matrix is

$$v_m = V_m v_c$$

$$=(0.3)(0.1896 \times 10^{-3})$$

$$= 0.5688 \times 10^{-3}\ m^3 .$$

The mass of the fiber is

$$w_f = \rho_f v_f$$

$$= (2500)(1.327 \times 10^{-3})$$

$$= 3.318\ kg\ .$$

The mass of the matrix is

$$w_m = \rho_m v_m$$

$$= (1200)(0.5688 \times 10^{-3})$$

$$= 0.6826\ kg\ .$$

An E-glass fiber reinforced vinyl ester composite has a fiber volume fraction of 40%. Experimentally measured density of E-glass fiber and vinyl ester is 2.54 g/cm^3 and 1.3 g/cm^3, respectively. What is the weight fraction of fibers?

Solution:

$V_f = 40\%$; $V_m = (1 - V_f) = 60\%$ $\rho_f = 2.54$ g/cm^3; $\rho_m = 1.3$ g/cm^3

According to Equation (7.1)

$$W_f = \frac{V_f \rho_f}{V_f \rho_f + V_m \rho_m} = \frac{0.4 \times 2.54}{0.4 \times 2.54 \times 0.6 \times 1.3} = \frac{1.016}{1.016 + 0.78} = 57\%$$

The glass vinyl ester composite has a fiber weight fraction of 57%.

A carbon fiber reinforced polyphenylene sulfide (PPS) composite has a resin weight fraction of 48%. What is the volume fraction of carbon fiber in this composite? The density of carbon fiber is 1.8 g/cm^3 and the density of PPS is 1.48 g/cm^3.

Solution:

$W_m = 48\%$; $W_f = (1 - W_m) = 52\%$; $\rho_f = 1.8$ g/cm^3; $\rho_m = 1.48$ g/cm^3

According to Equation (7.4)

$$V_f = \frac{W_f / \rho_f}{W_f / \rho_f + W_m / \rho_m + W_a / \rho_a} = \frac{0.52 / 1.8}{0.52 / 1.8 + 0.48 / 1.48} = \frac{0.289}{0.289 + 0.324} = 47\%$$

The fiber volume fraction of the composite is 47%.