Chapter One

Introduction to Plasticity

1.1 Introduction

The theory of linear elasticity is useful for modelling materials which undergo small deformations and which return to their original configuration upon removal of load.

Almost all real materials will undergo some **permanent** deformation, which remains after removal of load. With metals, significant permanent deformations will usually occur when the stress reaches some critical value, called the **yield stress**, a material property.

Elastic deformations are termed **reversible;** the energy expended in deformation is stored as elastic strain energy and is completely recovered upon load removal. Permanent deformations involve the dissipation of energy; such processes are termed **irreversible**, in the sense that the original state can be achieved only by the expenditure of more energy.

The **classical theory of plasticity** grew out of the study of metals in the late nineteenth century. It is concerned with materials which initially deform elastically, but which deform **plastically** upon reaching a yield stress. In metals and other crystalline materials the occurrence of plastic deformations at the micro-scale level is due to the motion of dislocations and the migration of grain boundaries on the micro-level. In sands and other granular materials plastic flow is due both to the irreversible rearrangement of individual particles and to the irreversible crushing of individual particles. Similarly, compression of bone to high stress levels will lead to particle crushing. The deformation of microvoids and the development of micro-cracks is also an important cause of plastic deformations in materials such as rocks.

There are two broad groups of metal plasticity problem which are of interest to the engineer and analyst. The first involves relatively small plastic strains, often of the same order as the elastic strains which occur. Analysis of problems involving small plastic strains allows one to design structures optimally, so that they will not fail when in service, but at the same time are not stronger than they really need to be. In this sense, plasticity is seen as a material **failure**.

The second type of problem involves very large strains and deformations, so large that the elastic strains can be disregarded. These problems occur in the analysis of metals manufacturing and forming processes, which can involve extrusion, drawing, forging, rolling and so on. In these latter-type problems, a simplified model known as **perfect plasticity** is usually employed (see below), and use is made of special **limit theorems** which hold for such models.

Plastic deformations are normally **rate independent,** that is, the stresses induced are independent of the rate of deformation (or rate of loading). This is in marked contrast to classical **Newtonian fluids** for example, where the stress levels are governed by the *rate* of deformation through the viscosity of the fluid.

Materials commonly known as "plastics" are not plastic in the sense described here.

They, like other polymeric materials, exhibit **viscoelastic** behaviour where, as the name suggests, the material response has both elastic and viscous components. Due to their viscosity, their response is, unlike the plastic materials, **rate-dependent**.

Further, although the viscoelastic materials can suffer irrecoverable deformation, they do not have any critical yield or threshold stress, which is the characteristic property of plastic behaviour. When a material undergoes plastic deformations, i.e. irrecoverable and at a critical yield stress, and these effects *are* rate dependent, the material is referred to as being **viscoplastic**.

Plasticity theory began with Tresca in 1864, when he undertook an experimental program into the extrusion of metals and published his famous yield criterion discussed later on.

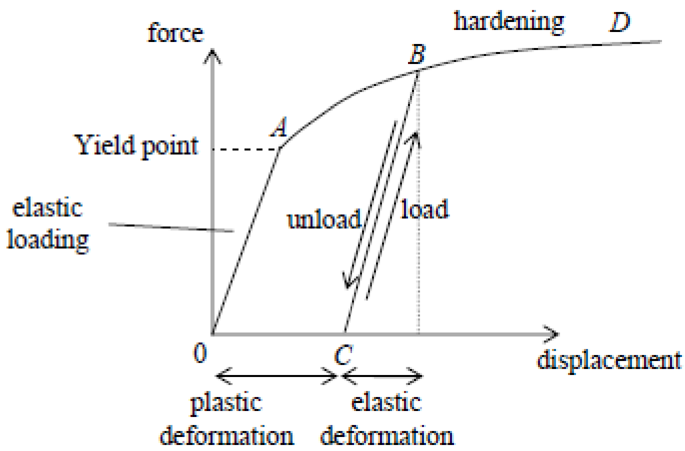
Further advances with yield criteria and plastic flow rules were made in the years which followed by Saint-Venant, Levy, Von Mises, Hencky and Prandtl. The 1940s saw the advent of the classical theory; Prager, Hill, Drucker and Koiter amongst others brought together many fundamental aspects of the theory into a single framework. The arrival of powerful computers in the 1980s and 1990s provided the impetus to develop the theory further, giving it a more rigorous foundation based on thermodynamics principles, and brought with it the need to consider many numerical and computational aspects to the plasticity problem.

1.2 Observations from Standard Tests

The Tension Test

Consider the following key experiment, the **tensile test**, in which a small, usually cylindrical, specimen is gripped and stretched, usually at some given rate of stretching

The force required to hold the specimen at a given stretch is recorded, see Figure. If the material is a metal, the deformation remains elastic up to a certain force level, the yield point of the material. Beyond this point, permanent plastic deformations are induced. On unloading only the elastic deformation is recovered and the specimen will have undergone a permanent elongation (and consequent lateral contraction).



In the elastic range the force-displacement behaviour for most engineering materials (metals, rocks, plastics, but not soils) is linear. After passing the elastic limit (point *A*), the material “gives” and is said to undergo plastic **flow**. Further increases in load are usually required to maintain the plastic flow and an increase in displacement; this initial plastic flow and hardening, the force-displacement curve decreases, as in some soils; the material is said to be **softening**. If the specimen is unloaded from a plastic state (*B*) it will return along the path *BC* shown, parallel to the original elastic line. This is **elastic recovery**. The strain which remains upon unloading is the permanent plastic deformation. If the material is now loaded again, the force-displacement curve will retrace the unloading path *CB* until it again reaches the plastic state. Further increases in stress will cause the curve to follow *BD*.

Two important observations concerning the above tension test (on most metals) are the following:

(1) after the onset of plastic deformation, the material will be seen to undergo negligible volume change, that is, it is **incompressible**.

(2) the force-displacement curve is more or less the same regardless of the rate at which the specimen is stretched (at least at moderate temperatures).

Nominal and True Stress and Strain

There are two different ways of describing the force *F* which acts in a tension test. First, normalising with respect to the *original* cross sectional area of the tension test specimen A_0, one has the **nominal stress** or **engineering stress**,

$$\sigma_0 = \frac{F}{A_0} \; \dots\dots\dots (1)$$

Alternatively, one can normalise with respect to the *current* cross-sectional area *A*,

leading to the **true stress**,

$$\sigma = \frac{F}{A} \dots\dots\dots (2)$$

in which *F* and *A* are both changing with time. For very small elongations, within the elastic range say, the cross-sectional area of the material undergoes negligible change and both definitions of stress are more or less equivalent.

Similarly, one can describe the deformation in two alternative ways. Denoting the original specimen length by l_0 and the current length by *l*, one has the **engineering strain**

$$e = \frac{l - l_0}{l_0} \dots\dots\dots (3)$$

Alternatively, the **true strain** is based on the fact that the "original length" is continually changing; a small change in length *dl* leads to a **strain increment** $d\varepsilon = dl\ /\ l$ and the total strain is *defined* as the accumulation of these increments:

$$\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = ln\frac{l}{l_0} \dots\dots\dots (4)$$

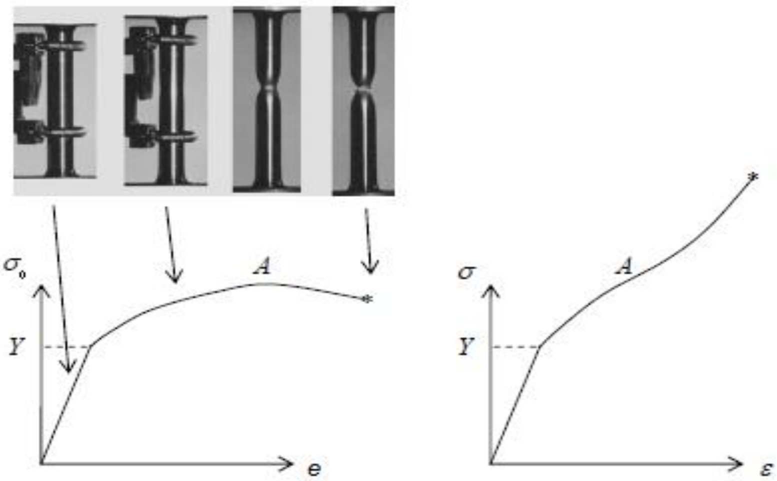
The true strain is also called the **logarithmic strain** or **Hencky strain**. Again, at small deformations, the difference between these two strain measures is negligible. The true strain and engineering strain are related through:

$$\varepsilon = \ln(1+e) \dots\dots\dots (5)$$

Using the assumption of constant volume, $Al=A_0l_0$ for plastic deformation and ignoring the very small elastic volume changes, one has also:

$$\sigma = \sigma_0 \frac{l}{l_0} \dots\dots\dots (6)$$

The stress-strain diagram for a tension test can now be described using the true stress/strain or nominal stress/strain definitions, as in the figure below. The shape of the nominal stress/strain diagram, is of course the same as the graph of force versus displacement (change in length) . *A* here denotes the point at which the maximum force the specimen can withstand has been reached. The *nominal stress* at *A* is called the **Ultimate Tensile Strength** (UTS) of the material. After this point, the specimen "necks", with a very rapid reduction in cross-sectional area somewhere about the centre of the specimen until the specimen ruptures, as indicated by the asterisk.



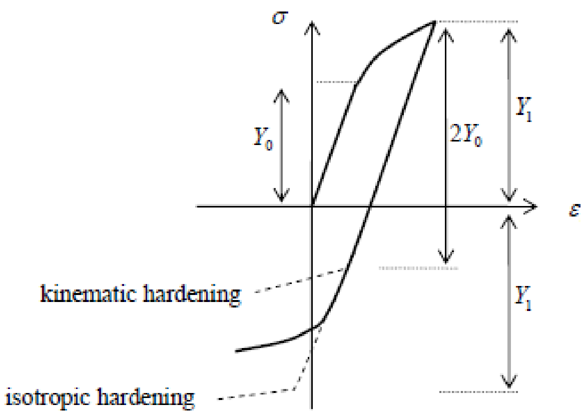
Note that, during loading into the plastic region, *the yield stress increases*. For example, if one unloads and re-loads , the material stays elastic up until a stress higher than the original yield stress *Y*. In this respect, the stress-strain curve can be regarded as a yield stress versus strain curve.

Compression Test

A compression test will lead to similar results as the tensile stress. The yield stress in compression will be approximately the same as (the negative of) the yield stress in tension. If one plots the true stress versus true strain curve for both tension and compression (absolute values for the compression), the two curves will more or less coincide. This would indicate that the behavior of the material under compression is broadly similar to that under tension. If one were to use the nominal stress and strain, then the two curves would not coincide; this is one of a number of good reasons for using the *true* definitions.

The Bauschinger Effect

If one takes a virgin sample and loads it in tension into the plastic range, and *then* unloads it and continues on into compression, one finds that the yield stress in compression is *not* the same as the yield strength in tension, as it would have been if the specimen had not first been loaded in tension. In fact the yield point in this case will be significantly *less* than the corresponding yield stress in tension. This reduction in yield stress is known as the **Bauschinger effect**. The effect is illustrated in the figure. The solid line depicts the response of a real material. The dotted lines are two extreme cases which are used in plasticity models; the first is the **isotropic hardening** model, in which the yield stress in tension and compression are maintained equal, the second being **kinematic hardening**, in which the total elastic range is maintained constant throughout the deformation.



The presence of the Bauschinger effect complicates any plasticity theory. However, it is not an issue provided there are no reversals of stress in the problem under study.

Hydrostatic Pressure

Careful experiments show that, for metals, the yield behavior is independent of hydrostatic pressure. That is, a stress state $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p$ has negligible effect on the yield stress of a material, right up to very high pressures. Note however that this is not true for soils or rocks.

1.3 Assumptions of Plasticity Theory

Regarding the above test results then, in formulating a basic plasticity theory with which to begin, the following assumptions are usually made:

(1) the response is independent of rate effects

(2) the material is incompressible in the plastic range

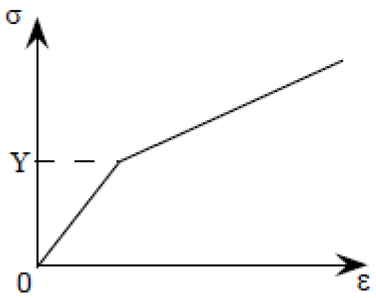
(3) there is no Bauschinger effect

(4) the yield stress is independent of hydrostatic pressure

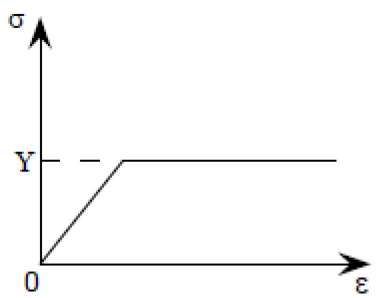
(5) the material is isotropic

The first two of these will usually be very good approximations, the other three may or may not be, depending on the material and circumstances. For example, most metals can be regarded as isotropic. After large plastic deformation however, for example in rolling, the material will have become anisotropic: there will be distinct material directions and asymmetries.

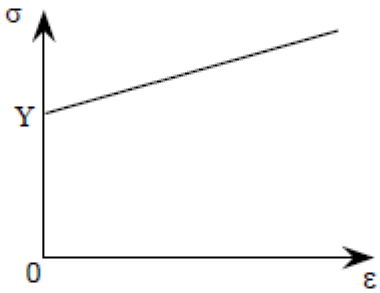
Together with these, assumptions can be made on the type of hardening and on whether elastic deformations are significant. For example, consider the



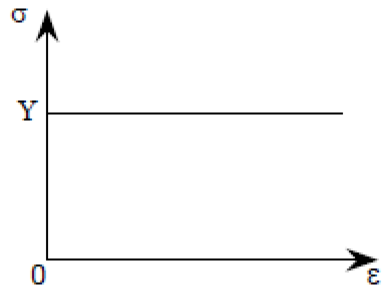
(a) Linear Elastic-Plastic



(b) Elastic/Perfectly-Plastic



(c) Rigid/Linear Hardening



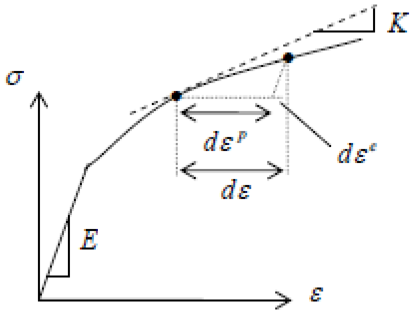
(d) Rigid-Perfectly-Plastic

hierarchy of models illustrated in the figure , commonly used in theoretical analyses. In (a) both the elastic and plastic curves are assumed linear. In (b) work-hardening is neglected and the yield stress is constant after initial yield. Such **perfectly-plastic** models are particularly appropriate for studying processes where the metal is worked at a high temperature – such as hot rolling – where work hardening is small. In many areas of applications the strains involved are large, e.g. in metal working processes such as extrusion, rolling or drawing, where up to 50% reduction ratios are common. In such cases the elastic strains can be neglected altogether as in the two models (c) and (d). The **rigid/perfectly-plastic** model (d) is the crudest of all – and hence in many ways the most useful. It is widely used in analyzing metal forming processes, in the design of steel and concrete structures and in the analysis of soil and rock stability.

1.4 The Tangent and Plastic Modulus

Stress and strain are related through $\sigma = E\varepsilon$ in the elastic region, E being the Young's modulus.

The **tangent modulus** K is the slope of the stress-strain curve in the plastic region and will in general change during a deformation. At any instant of strain, the *increment* in stress $d\sigma$ is related to the *increment* in strain $d\varepsilon$ through



$$d\sigma = K d\varepsilon \ldots\ldots\ldots (7)$$

After yield, the strain increment consists of both elastic, $d\varepsilon^e$ and plastic, $d\varepsilon^p$ strains:

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \ldots\ldots\ldots (8)$$

The stress and plastic strain increments are related by the **plastic modulus** H:

$$d\sigma = H d\varepsilon^p \ldots\ldots\ldots (9)$$

and it follows that:

$$\frac{1}{K} = \frac{1}{E} + \frac{1}{H} \ldots\ldots\ldots (10)$$

1.5 Friction Block Models

Some additional insight into the way plastic materials respond can be obtained from friction block models. The rigid perfectly plastic model can be simulated by a Coulomb friction block, Figure. No strain occurs until σ reaches the yield stress Y. Then there is movement – although the amount of movement or plastic strain cannot be determined without more