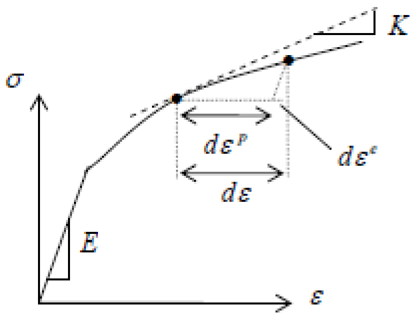
1.4 The Tangent and Plastic Modulus

Stress and strain are related through σ=*Eε* in the elastic region, *E* being the Young's modulus.

The **tangent modulus** *K* is the slope of the stress-strain curve in the plastic region and will in general change during a deformation. At any instant of strain, the *increment* in stress *dσ* is related to the *increment* in strain *dε* through



$$d\sigma = Kd\varepsilon \ldots\ldots\ldots (7)$$

After yield, the strain increment consists of both elastic, $d\varepsilon^e$ and plastic, $d\varepsilon^p$ strains:

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p \ldots\ldots\ldots (8)$$

The stress and plastic strain increments are related by the **plastic modulus** *H*:

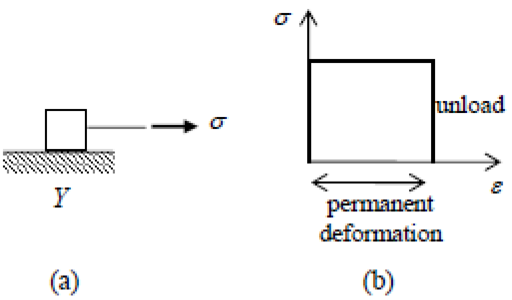
$$d\sigma = Hd\varepsilon^p \ldots\ldots\ldots (9)$$

and it follows that:

$$\frac{1}{K} = \frac{1}{E} + \frac{1}{H} \ldots\ldots\ldots (10)$$

1.5 Friction Block Models

Some additional insight into the way plastic materials respond can be obtained from friction block models. The rigid perfectly plastic model can be simulated by a Coulomb friction block, Figure. No strain occurs until *σ* reaches the yield stress *Y*. Then there is movement – although the amount of movement or plastic strain cannot be determined without more

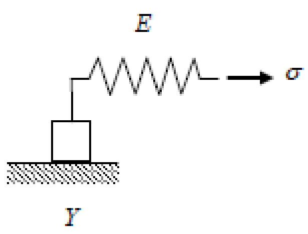


information being available. The stress cannot exceed the yield stress in this model:

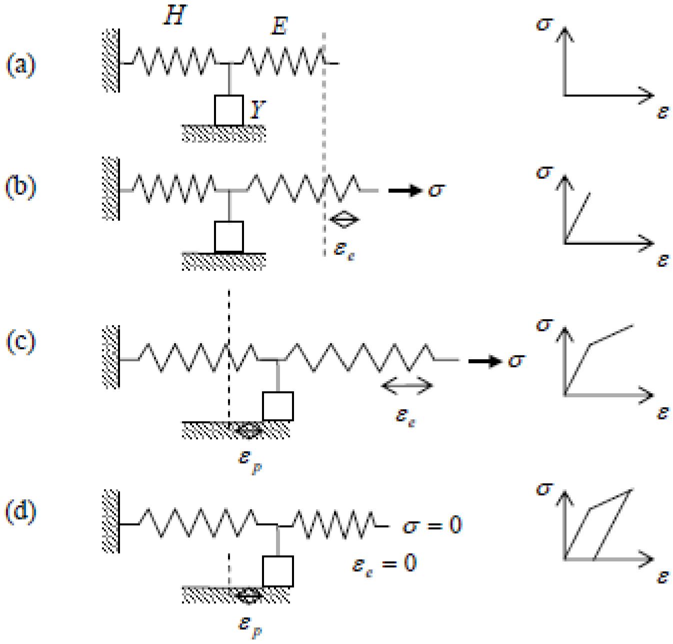
$$|\sigma| \leq Y \dots\dots\dots (11)$$

If unloaded, the block stops moving and the stress returns to zero, leaving a permanent strain, Fig.(b).

The linear elastic perfectly plastic model incorporates a free spring with modulus *E* in series with a friction block, Figure. The spring stretches when loaded and the block also begins to move when the stress reaches *Y*, at which time the spring stops stretching, the maximum possible stress again being *Y*. Upon unloading, the block stops moving and the spring contracts.



The linear elastic plastic model with linear strain hardening incorporates a second, hardening, spring with stiffness *H*, in parallel with the friction block, Figure. Once the yield stress is reached, an ever increasing stress needs to be applied in order to keep the block moving – and elastic strain continues to occur due to further elongation of the free spring. The stress is then split into the yield stress, which is carried by the moving block, and an **overstress** *σ*–*Y* carried by the hardening spring.



Upon unloading, the block “locks” – the stress in the hardening spring remains constant whilst the free spring contracts. At zero stress, there is a negative stress taken up by the friction block, equal and opposite to the stress in the hardening spring.

The slope of the elastic loading line is *E*. For the plastic hardening line,

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + \frac{\sigma - Y}{H} \ldots\ldots\ldots (12)$$

It can be seen that *H* is the plastic modulus.

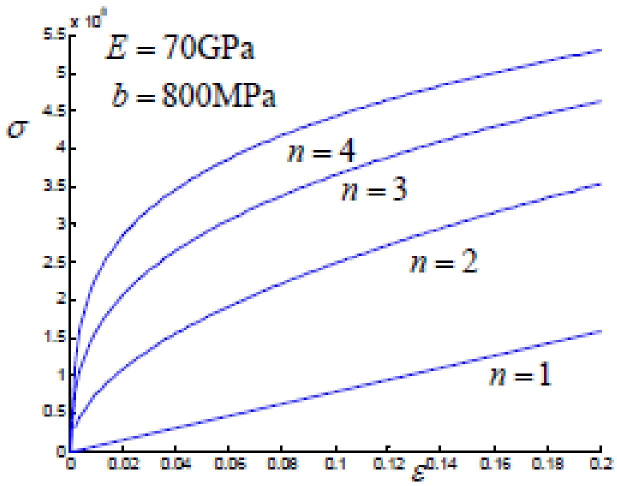
1.6 Problems

1. A test specimen of initial length 0.01m is extended to length 0.0101m. What is the percentage difference between the engineering and true strains (relative to the engineering strain)? What is this difference when the specimen is extended to length 0.015m?

2. The **Ramberg-Osgood** model of plasticity is given by:

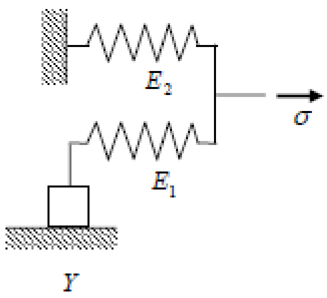
$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{\sigma}{E} + \left(\frac{\sigma}{b}\right)^n$$

where *E* is the Young's modulus and *b* and *n* are model constants (material parameters) obtained from a curve-fitting of the uniaxial stress-strain curve.



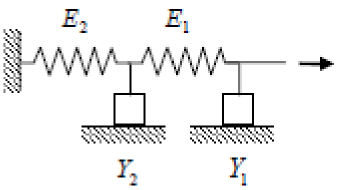
(i) Find the tangent and plastic moduli in terms of plastic strain ε^p (and the material constants).

(ii) A material with model parameters *n* =4 , *E*=70GPa and *b*=800MPa is strained in tension to ε^p =0.02 and is subsequently unloaded and put into



compression. Find the stress at the initiation of compressive yield assuming isotropic hardening. [Note that the yield stress is actually zero in this model, although the plastic strain at relatively low stress levels is small for larger values of *n*.]

3. Consider the plasticity model shown.
(i) What is the elastic modulus?
(ii) What is the yield stress?
(iii) What are the tangent and plastic moduli?
Draw a typical loading and unloading curve.



4. Draw the stress-strain diagram for a cycle of loading and unloading to the rigid - plastic model shown here. Take the maximum load reached to be $\sigma_{max} = 4Y_1$ and $Y_2=2Y_1$. What is the permanent deformation after complete removal of the load?
[Hint: split the cycle into the following regions:
(a) $0 \le \sigma \le Y_1$, (b) $Y_1 \le \sigma \le 3Y_1$, (c) $3Y_1 \le \sigma \le 4Y_1$
, then unload, (d) $4Y_1 \le \sigma \le 3Y_1$, (e) $3Y_1 \le \sigma \le 2Y_1$, (f) $2Y_1 \le \sigma \le 0$.]

Stress Analysis for Plasticity

2.1 The Stress–Strain Behavior

The standard tensile test is unsuitable for obtaining the stress–strain curve of metals up to large values of the strain, since the specimen begins to neck when the rate of hardening decreases to a critical value. At this stage, the increase in load due to strain-hardening is exactly balanced by the decrease in load caused by the diminution of the area of cross section. Consequently, the load attains a maximum at the onset of necking. The longitudinal load at any stage is $P = \sigma A$, where A is the current cross-sectional area and σ the current stress, and the corresponding volume of the specimen is $l\,A$, where l is the current length. Using the constancy of volume, the maximum load condition $dP = 0$ may be written as:

$$\frac{d\sigma}{\sigma} = -\frac{dA}{A} = \frac{dl}{l} = d\varepsilon$$

Thus, the condition for the onset of necking becomes:

$$\frac{d\sigma}{d\varepsilon} = \sigma \;\ldots\ldots\ldots (1)$$

When the true stress–strain curve is given, the point on the curve that corresponds to the tensile necking can be located graphically from the fact