Ch.1: Solution of Non-Linear Equations

1- Simple Iterative Method
2- Bisection Method
3- Secant Method
4- Newton-Raphson Method

1- Simple Iterative Method

It's simple, easy and can be applied for most Sciences & Engineering problems.

Solution Steps

1- Find the first (initial) root if It's **not** given in question by change the function from $f(x)=0$ to $x=F(x)$ and then signal test of $F(x)$.
2- Make a table to determine the new value of (x_{n+1}) as a function of (x_n).
3- The solution is continued to find the right root when $|x_{n+1} - x_n| \le \epsilon$.

Ex.1: Find the real root of the following Equ. [$f(x)= e^{-x} - x$]the initial value (0.1), Є=0% and work to (4) decimal places. **Use** Simple Iterative Method.

Sol.

$f(x)= e^{-x} - x=0 \rightarrow x=e^{-x}=F(x)$

$F(0.1) = e^{-0.1}=0.9048$
$F(0.9048) = e^{-0.9048}=0.4046$
$F(0.4046) = e^{-0.4046}=0.6672$
$F(0.6672) = e^{-0.6672}=0.5131$
$F(0.5131) = e^{-0.5131}=0.5986$
$F(0.5986) = e^{-0.5986}=0.5495$
$F(0.5495) = e^{-0.5495}=0.5772$
$F(0.5772) = e^{-0.5772}=0.5614$
$F(0.5614) = e^{-0.5614}=0.5703$
$F(0.5703) = e^{-0.5703}=0.5653$
$F(0.5653) = e^{-0.5653}=0.5681$
$F(0.5681) = e^{-0.5681}=0.5665$
$F(0.5665) = e^{-0.5665}=0.5674$
$F(0.5674) = e^{-0.5674}=0.5669$
$F(0.5669) = e^{-0.5669}=0.5672$
$F(0.5672) = e^{-0.5672}=0.5670$
$F(0.5670) = e^{-0.5670}=0.5671$
$F(0.5671) = e^{-0.5671}=0.5671$

n	x_n	$F(x_n)$	Є
0	0.1	0.9048	---
1	0.9048	0.4046	0.5002
2	0.4046	0.6672	0.2626
3	0.6672	0.5131	0.1541
4	0.5131	0.5986	0.0855
5	0.5986	0.5495	0.0491
6	0.5495	0.5772	0.0277
7	0.5495	0.5614	0.0119
8	0.5772	0.5703	0.0069
9	0.5614	0.5653	0.0039
10	0.5703	0.5681	0.0022
11	0.5653	0.5665	0.0012
12	0.5681	0.5674	0.0007
13	0.5665	0.5669	0.0004
14	0.5674	0.5672	0.0002
15	0.5669	0.5670	0.0001
16	0.5672	0.5671	0.0001
17	0.5671	**0.5671**	0.0000

So, the real root = 0.5671 for Є=0

Ex.2: Find the real root of the following Equ. [$f(x)= x^3- x- 1$], the initial value (1), Є=0% and work to (4) decimal places. **Use** Simple Iterative Method.

Sol.

$f(x)= x^3- x- 1=0 \rightarrow x^3= x+1 \rightarrow x=\sqrt[3]{x+1}= F(x)$

$F(1) = \sqrt[3]{1+1}=1.2599$

$F(1.2599) = \sqrt[3]{1.2599+1}=1.3122$

$F(1.3122) = \sqrt[3]{1.3122+1}=1.3246$

$F(1.3246) = \sqrt[3]{1.3246+1}=1.3223$

$F(1.3223) = \sqrt[3]{1.3223+1}=1.3242$

$F(1.3242) = \sqrt[3]{1.3242+1}=1.3246$

$F(1.3246) = \sqrt[3]{1.3246+1}=1.3246$

n	x_n	$F(x_n)$	Є
0	1	1.2599	---
1	1.2599	1.3122	0.0523
2	1.3122	1.3223	0.0101
3	1.3223	1.3242	0.0019
4	1.3242	1.3246	0.0004
5	1.3246	**1.3246**	0.0000

In that case, the real root =1.3246 for Є=0

Ex.3: Determine the smallest positive root for [$x^3+4x= 2$] correct to the level (0%) and work to (4D). **Use** Simple Iterative Method.

Sol. Firstly, we must to find the initial root (x_o). $F(x)= x^3+4x- 2$

x	–2	–1	0	1
f(x)	–	–	–	+

The initial root (x_o)=0

There are two ways:

First way:

$F(x)= x^3+4x- 2=0$

$\rightarrow x^3=2- 4x$

$\rightarrow x=\sqrt[3]{2-4x}$

n	x_n	$F(x_n)$	Є
0	0	1.2599	---
1	1.2599	–1.4485	0.1886
2	–1.4485	1.9827	0.5342
3	1.9827	–1.8101	0.1726
4	–1.8101	2.0984	0.2883

Hence, the solution is **Diverged**

Second way:

$F(x)= x^3+4x- 2=0$

$\rightarrow 4x=2- x^3$

$\rightarrow x=\frac{2-x^3}{4}$

n	x_n	$F(x_n)$	Є
0	1	0.2500	---
1	0.2500	0.4960	0.246
2	0.4960	0.4694	0.0266
3	0.4694	0.4741	0.0047
4	0.4741	0.4733	0.0008
5	0.4733	0.4734	0.0001
6	0.4734	**0.4734**	0.0000

So, the solution is **Converged**
the real root =0.4734 for Є=0

H.W. The following Equ. by using **Simple Iterative Method** to find the lowest positive root, correct to the level (0%) and work to (4D).

1- $2x3-7x=-2$, $x_o=0$

2- $f(x)= \cos(x) - x +1$, $x_o=1$

3- $f(x)= \ln(x) + x -2$, $x_o=1$

4- $e^{-x}.\tan(x)=1$, $x_o=0.6$

Ex.4: The velocity of falling parachutist is given by $v=(m.g/c).[1-e^{-(c.t/m)}]$, (g =980cm/s^2), (mass=7500kg), Use **Simple Iterative Method** to compute the drag coefficient (c), so that (v=3600cm/s) at time (6 seconds), start your approximation at (1400)? Work to (3DP), (0.1%).

Sol.

First way:

$$v=(m.g/c).[1-e^{-(c.t/m)}] \mapsto c=(m.g/v).[1-e^{-(c.t/m)}]$$

$$c_n = f(c_n)=(m.g/v).[1-e^{-(c_n.t/m)}]$$

n	c_n	$F(c_n)$	Є
0	1400	1375.512	---
1	1375.512	1362.333	0.13179
2	1362.333	1355.133	0.072
3	1355.133	1351.167	0.03966
4	1351.167	1348.973	0.02194
5	1348.973	1347.756	0.01217
6	1347.756	1347.080	0.00676
7	1347.080	1346.704	0.00376
8	1346.704	1346.495	0.00209
9	1346.495	1346.379	0.00116
10	1346.379	**1346.314**	0.00065

The real root =1346.314

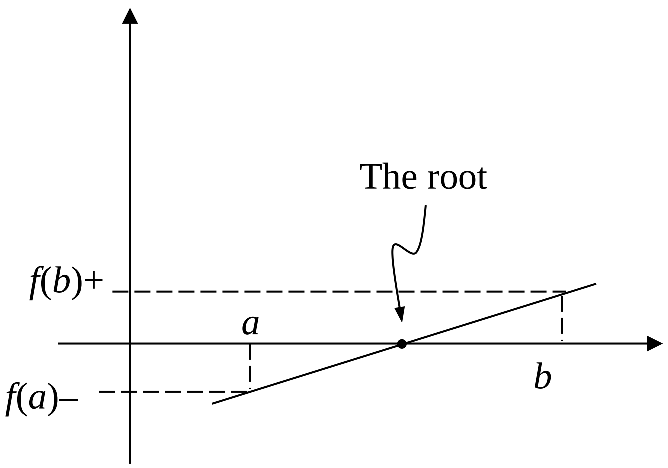
Second way: **(H.W.)**

2- Bisection Method (Halving interval)

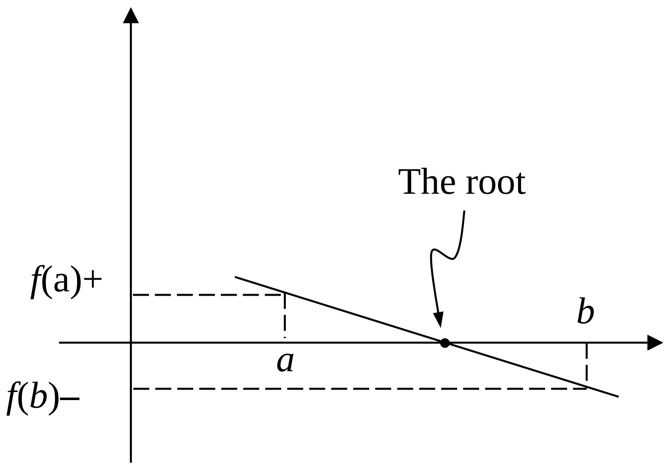
It is impossible always satisfy the convergence by using Simple Iterative Method; therefore we used "**Bisection Method**" that leads to the solution convergence.

Lets *f(x)* to be a function which is defined in *[a, b]*, $f(a)$ & $f(b)$ are less than zero "$f(a)$, $f(b) < 0$" and then $f(x)$ has a root at *[a, b]*, this root is an approximation root call (r), where $r = (a+b)/2$.

This method is need to two roots; *[a, b]*.



$f(a) < 0$



$f(b) < 0$

Solution Steps

1- Find the first (initial) root if It's **not** given in question by signal test of the given function $f(x)$.
2- Find the value (r) by $r = (a+b)/2$.
3- Make a table to determine the value of $f(a)$ & $f(r)$.
4- If the signal of $f(a)$ is **similar to** the signal of $f(r)$, the value of (r) is replaced by (a) & find new (r).
5- If the signal of $f(a)$ is **different** the signal of $f(r)$, the value of (r) is replaced by (b) & find new (r).
6- The solution is continued to find the right root when $|r_{n+1} - r_n| \le \epsilon$.

Ex.5: Using **Bisection Method** to determine the smallest positive root for Equ. [$e^{-x} - x = 0$], correct to the level (0.4%) for the interval (0, 1).

Sol.

$$f(x) = e^{-x} - x$$

n	*a*	b	r	*f(a)*	*f(r)*	Є
0	0	1	0.5	+	+	----
1	0.5	1	0.75	+	–	25%
2	0.5	0.75	0.625	+	–	12.5%
3	0.5	0.625	0.5625	+	+	6.25%
4	0.5625	0.625	0.59375	+	–	3.125%
5	0.5625	0.59375	0.5781	+	–	1.565%
6	0.5625	0.5781	0.5703	+	–	0.8%
7	0.56641	0.57032	**0.56837**	+	–	0.193%

Thus, the smallest positive root = (0.56837)

Ex.6: Determine the smallest positive root for [$f(x)= x^3 - x-1$] correct to the level (0.5%).**Use Halving interval Method.**

Sol. Firstly, must signal test of $f(x)= x^3 - x-1$

x_o	0	1	2
$f(x_o)$	–	–	+

So, the interval values (1, 2)

n	*a*	b	r	*f*(*a*)	*f*(*r*)	Є
0	1	2	1.5	–	+	----
1	1	1.5	1.25	–	–	25%
2	1.25	1.5	1.375	–	+	12.5%
3	1.25	1.375	1.3125	–	–	6.25%
4	1.3125	1.375	1.34375	–	+	3.125%
5	1.3125	1.34375	1.32818	–	+	1.56%
6	1.3125	1.32818	1.32032	–	–	0.78%
7	1.32032	1.32818	**1.32423**	–	–	0.391%

Then, the smallest positive root (1.32423)

Ex.7: Determine the lowest positive root for [$cos(x) - x= -1$] correct to the level (0.3%). **Use Halving interval Method.**

Sol. Initially, must signal test of $f(x) = cos(x) - x+1$

x_o	0	1	2
$f(x_o)$	+	+	–

So, the interval values (1, 2)

n	*a*	b	f	*f*(*a*)	*f*(*r*)	Є
0	1	2	1.5	+	+	----
1	1.5	2	1.75	+	+	25%
2	1.75	2	1.875	+	+	12.5%
3	1.875	2	1.9375	+	+	6.25%
4	1.9375	2	1.9687	+	+	3.13%
5	1.9687	2	1.9843	+	+	1.56%
6	1.9843	2	1.9921	+	+	0.78%
7	1.9921	2	1.9960	+	+	0.39%
8	1.9960	2	**1.9980**	+	+	0.2%

Therefore, the smallest positive root (1.998)

H.W. Using **Bisection Method** to evaluate the smallest positive root for the following Equs. Correct to the level (0%) and work to (4D).

1- $f(x)= x^3 + x-1$, (1%).

2- $x^3 - x=1$, (0%).

3- $x\ ln(x) =1$ (1%).

4- $f(x)= e^x. Sin(x) - 1$ (1%).

5- $f(x)= x^6 + x-1$, (0.5%).

Ex.8: The equation (*tan(x) – x = 0.01*) arises in the motion of helical gears. Evaluate the smallest positive root of this equation which is between (0.2, 0.4), correct to (three DP), (0.5%). Using **Bisection Method.**

Sol. $f(x) = \tan(x) - x - 0.01$

n	*a*	b	r	*f*(*a*)	*f*(*r*)	Є
0	0.200	0.400	0.300	–	–	----
1	0.300	0.400	0.350	–	–	0.05
2	0.350	0.400	0.375	–	–	0.025
3	0.375	0.400	0.387	–	–	0.013
4	0.387	0.400	0.393	–	–	0.007
5	0.393	0.400	**0.396**	–	–	0.003

Therefore, the smallest positive root (**0.396**)

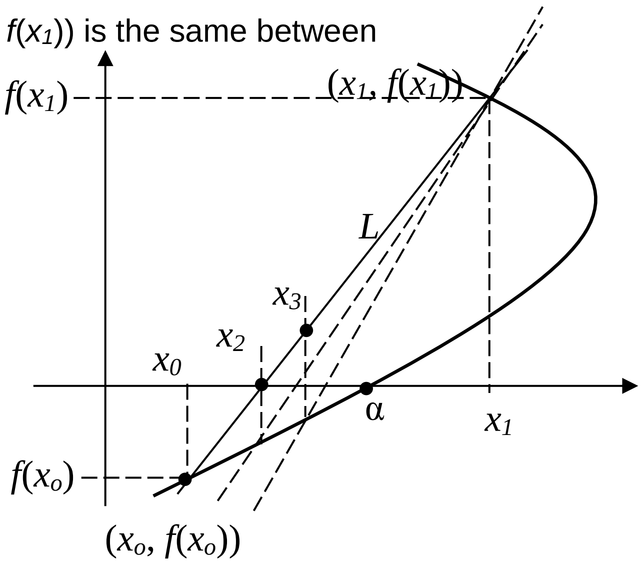
H.W. 1- The velocity of falling parachutist is given by $v = (m.g/c).[1 - e^{-(c.t/m)}]$, (g =980cm/s^2), (mass=68100kg), Use Simple Iterative Method to compute the drag coefficient (c), so that (v=4000cm/s) at time (7 seconds), start your approximation between (10000, 15000)? Work to (3DP), (0.1%). Using **Bisection Method.**

2- The equ. [*tan(x)=0.01*] arises in the motion of helical gears. Find the smallest positive root of this equ. which is between (0.2, 0.4) correct to (3D).

3- The alternative equ. of state for gases is given by: $(p + a/v^2)(v - b) = RT$ where $v = RT / p$, *(a=12.02, b=0.8407)* are constants empirical for "Ethyl alcohol gas" at a temp. (500K), (700K) & pressure (10atm),(100atm). Find by numerical method the molar volume (*v*).*R=0.08205L atm/mol K*

3- Secant Method

Let $f(x)$ be a continuous at two points (a, b), or (x_o, x_1) and the equation has a root (α). Let (L) be a secant to the curve $f(x)$ at points (x_o, $f(x_o)$) & (x_1, $f(x_1)$).
Drive the secant formula:-
Slope = dy/dx , the slope between (x_o, $f(x_o)$) & (x_1, $f(x_1)$) is the same between (x_2, $f(x_2)$) & (x_1, $f(x_1)$) :



$$\frac{f(x_1)-f(x_0)}{x_1-x_0}=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$x_2-x_1=\frac{x_1-x_0}{f(x_1)-f(x_0)}.[-f(x_1)]$$

For General

$$x_{n+1}=x_n-\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}.\,f(x_n)$$

Solution Steps

1- Find interval values (a, b) when it's **not** given in question by signal test of the given function $f(x)$.
2- Make a table to determine (x_{n-1}), ($f(x_{n-1})$), (x_n), ($f(x_n)$), (x_{n+1}) & (Є).
3- The solution is continued to find the right root when $|\Delta x_{n+1}| \le$ Є.

Ex.9: Using **Secant Method** to determine the lowest positive root for the following Equ. [$e^{-x} - x=0$], correct to the level (0.5%) and work to (5D), at (0, 1)

Sol. $x_o=0$, $x_1=1$,

n	x_{n-1}	$f(x_{n-1})$	x_n	$f(x_n)$	x_{n+1}	Є
1	0	1	1	-0.63212	0.6127	---
2	0.6127	-0.07081	1	-0.63212	0.5638	0.0489
3	0.5638	0.00518	1	-0.63212	**0.56717**	0.0033

Therefore, the smallest positive root (0.56717)

Ex.10: Determine the smallest positive root for [$f(x)= \ln x + x - 2$], correct to the level (0.1 %) and work to (4D), at (1, 2), Using **Secant Method.**
Sol. $x_o=1$, $x_1=2$, $f(x)= \ln x + x - 2$

n	x_{n-1}	$f(x_{n-1})$	x_n	$f(x_n)$	x_{n+1}	Є
1	1	-1	2	0.6931	1.5906	---
2	1.5906	0.0547	2	0.6931	1.5555	0.035
3	1.5555	-0.0027	2	0.6931	1.5572	0.0017
4	1.5572	0.0005	2	0.6931	**1.5577**	0.0005

Hence, the smallest positive root (1.5577)

Ex.11: Evaluate the real root of Equ. [$\cos(t)-4t= -1$] for the interval rate (0, 1) &(0.2%).

Sol. $x_o=0$, $x_1=1$, $f(x)= \cos(t)-4t+1$

n	x_{n-1}	$f(x_{n-1})$	x_n	$f(x_n)$	x_{n+1}	Є
1	0	2	1	-2	0.5	---
2	0.5	0	1	-2	**0.5**	0

Therefore, the real root (0.5).

H.W. Apply **Secant Method** to evaluate the smallest positive root for the following Equs. correct to the level (0.2%) and work to (4D).

1- *ln x=5*,
2- *z ln z-1=0*, (1, 2)
3- *f(t)=* t^3*-t-1*, (1, 2)
4- $p^3-p^2-3p-3=0$, (1, 2)

H.W. 1- The velocity of falling parachutist is given by $v=(m.g/c).[1-e^{-(c.t/m)}]$, (g =980cm/s^2), (mass=68100kg), Use Simple Iterative Method to compute the drag coefficient (c), so that (v=4000cm/s) at time (7 seconds), start your approximation between (10000, 15000)? Work to (3DP), (0.1%). Using **Secant Method.**

2- For fluid flow in pipes, friction is described by a dimensionless number ass following $f^{-0.2}=4\log_{10}\left(\mathrm{Re}\sqrt{f}\right)-0.4$,solve for (*f*) are(0.001)to(0.01)&Re=2500 by **Bisection & Secant Method.** (Є =0.00005).

3- The equ. [*tan(x-x)=0.01*] arises in the motion of helical gears. Find the smallest positive root of this equ. which is between (0.2, 0.4) correct to (3D).

4- The alternative equ. of state for gases is given by: $\left(p+a/v^2\right)(v-b)=RT$ where $v=RT/p$, *(a=12.02, b=0.8407)*are constants empirical for "Ethyl alcohol gas" at a temp.(400K)& .(500K) pressure(2.5atm), (4atm). By numerical method, Find the molar volume (*v*).*R=0.08205L atm/mol K*

4- Newton-Raphson Method

This method is one of set of the most widely methods used for solving the type non-linear equs.

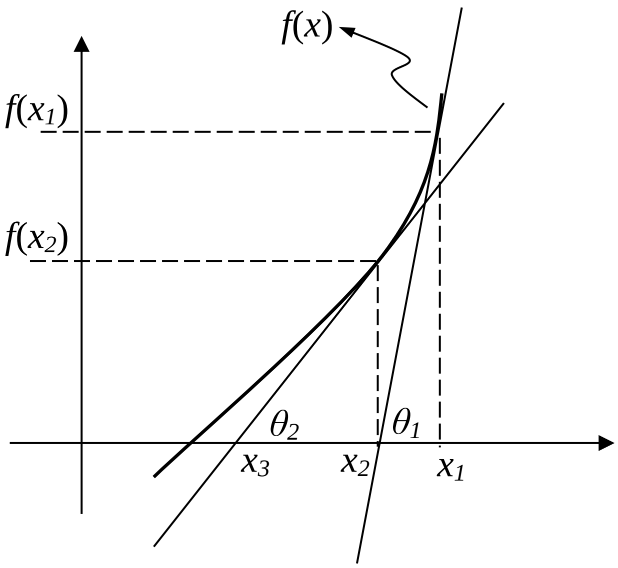
$\tan\theta_1 = slope = \overline{f}(x_1) = \dfrac{f(x_1)}{x_1 - x_2}$

$$x_2 - x_1 = \frac{f(x_1)}{\overline{f}(x_1)}, \quad x_2 = x_1 - \frac{f(x_1)}{\overline{f}(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{\overline{f}(x_2)}, \quad x_4 = x_3 - \frac{f(x_3)}{\overline{f}(x_3)}$$

For general

$$x_{n+1} = x_n - \frac{f(x_n)}{\overline{f}(x_n)}$$



Solution Steps

1- Find initial value (x_o) when it's **not** given in question by signal test of the given function $f(x)$.

2- Make a table to determine (x_n), ($f(x_o)$), $\overline{f}(x_n)$, (x_{n+1}) & (Є).

3- The solution is continued to find the right root when $|\Delta x_{n+1}| \le$ Є.

Ex.12: Using **Newton-Raphson Method** to determine the lowest positive root for the following Equ. [$e^{-x} - x = 0$], correct to the level (0.5%) and work to (5D), at (1)

Sol. $x_o=1$, $f(x) = e^{-x} - x$, $\overline{f}(x_n) = -e^{-x} - 1$

n	x_n	$f(x_n)$	$\overline{f}(x_n)$	x_{n+1}	Є
0	1	-0.63212	-1.3678	0.53788	---
1	0.53788	0.04610	-1.58398	0.56698	0.0291
2	0.56698	0.00256	-1.56723	**0.56714**	0.00016

Therefore, the real root (0.56714)

Ex.13: Determine the lowest positive root for the following Equ. [$x^2=8$], correct to the level (0.2 %) and work to (4D), by **Newton-Raphson Method**

Sol. We must find initial value (x_o) by test signal, $f(x) = x^2 - 8$

x_o	0	1	2	3
$f(x_o)$	–	–	–	+

So, $x_o=2$,

$f(x) = x^2 - 8$, $\overline{f}(x_n) = 2x$

Hence,

n	x_n	$f(x_n)$	$\overline{f}(x_n)$	x_{n+1}	Є
0	2	-4	4	3	---
1	3	1	6	2.8333	0.1666
2	2.8333	0.02759	5.6666	2.8284	0.0048
3	2.8284	-0.000153	5.6568	**2.8284**	0

the real root (2.8284)

Ex.14: Calculate the lowest positive root for the following Equ. [$x^3 - x = 4$], correct to the level (0.2 %) and work to (4D), by **N.R. Method**

Sol. We must find initial value (x_o) by test signal, there are **two** ways:

Firstly, $f(x) = x^3 - x - 4$

So,

x_o	0	1	2
$f(x_o)$	–	–	+

$x_o = 1$, $f(x) = x^3 - 4 \rightarrow \overline{f}(x_n) = 3x^2 - 1$

Hence,

n	x_n	$f(x_n)$	$\overline{f}(x_n)$	x_{n+1}	Є
0	1	-4	2	3	---
1	3	20	26	2.2307	0.7692
2	2.2307	4.8693	13.9280	1.8810	0.3261
3	1.8810	0.7742	6.07632	1.7535	0.1274
4	1.7535	-0.3616	8.2242	1.7974	0.0439
5	1.7974	0.009364	8.6919	**1.7963**	0.0011

the real root (1.7963)

H.W. Apply **N.R. Method** to evaluate the smallest positive root for the following functions, Correct to the level (0.5 %) and work to (4D).

1- $x^4 - x = 10$,

2- $3x - \cos(x) = 1$, $x_o = 0.5$

3- $f(x) = x^6 - x - 1$,

H.W. 1- For fluid flow in pipes, friction is described by a dimensionless number ass following $f^{-0.5} = 4\log_{10}\left(\mathrm{Re}\sqrt{f}\right) - 0.4$,solve for ($f$) are (0.001)to(0.01) & Re=2500 by **N.R. Method**.
(**Є** =0.00005).

2- The alternative equ. of state for gases is given by: $\left(p + a/v^2\right)(v - b) = RT$ where $v = RT/p$, *(a=12.02, b=0.8407)*are constants empirical for "Ethyl alcohol gas" at a temp.(500K), (700K)& pressure(10atm), (100atm). By numerical method, Find the molar volume(v).*R=0.08205L atm/mol K*

3- The displacement of a structure is define by $y = 9e^{-kt}\cos(wt)$ where k=0.7, w=4. Find the root when The displacement = 3.5 when t =(0.2, 0.3)

Special Cases for Newton-Raphson Method

1- Determine the Root of any Order

Ex.15: Find the square root of (8) by **Newton-Raphson Method,** beginning with (2) as an initial value.

Sol. Let $f(x) = x^r = 8 = C \rightarrow f(x) = x^r - C \rightarrow \overline{f}(x) = r.x^{r-1}$

By **Newton-Raphson Method**

$$x_{n+1} = x_n - \frac{f(x_n)}{\overline{f}(x_n)} = x_n - \frac{x_n^r - C}{r.x_n^{r-1}} = x_n - \frac{x_n^r}{r.\frac{x_n^r}{x_n}} + \frac{C}{r.\frac{x_n^r}{x_n}} = x_n - \frac{1}{r}x_n + \frac{C}{r}.\frac{x_n}{x_n^r}$$

$$= x_n - \frac{1}{r}.x_n + \frac{C}{r}.x_n^{1-r} \Rightarrow x_{n+1} = x_n[1-\frac{1}{r}] + \frac{C}{r}.x_n^{1-r}$$

n	x_n	x_{n+1}	Є
0	2	3	1
1	3	2.8333	0.1666
2	2.8333	2.8284	0.005
3	2.8284	2.8284	0.000

The square root of (8) = 2.8284
$f(x) = x_n^2 = 8 = C = 2.8284$

Ex.16: Find the value of ($\sqrt[3]{7}$) by **Newton-Raphson Method,** beginning with (1.5) as an initial value, work to (5D).

Sol. Let $f(x) = x^r = 7 = C \rightarrow f(x) = x^r - C \rightarrow \overline{f}(x) = r.x^{r-1}$, C & $r > 0$

By **Newton-Raphson Method**

$$x_{n+1} = x_n - \frac{f(x_n)}{\overline{f}(x_n)} = x_n - \frac{x_n^r - C}{r.x_n^{r-1}} = x_n - \frac{x_n^r}{r.\frac{x_n^r}{x_n}} + \frac{C}{r.\frac{x_n^r}{x_n}} = x_n - \frac{1}{r}x_n + \frac{C}{r}.\frac{x_n}{x_n^r}$$

$$= x_n - \frac{1}{r}.x_n + \frac{C}{r}.x_n^{1-r} \Rightarrow x_{n+1} = x_n[1-\frac{1}{r}] + \frac{C}{r}.x_n^{1-r}$$

N	x_n	x_{n+1}	Є
0	1.5	2.03704	0.537
1	2.03704	1.92034	0.116
2	1.92034	1.91296	0.0073
3	1.91296	1.91231	0.00026
4	1.91231	1.91231	0.0000

Therefore, $\sqrt[3]{7}$ = 1.91293

H.W. Find 1- $x^4=98$, $x_o=4$
2- $x^2= 82$, $x_o=8$
3- $x^3= 125$, $x_o=4$

2- Determine the Reciprocal of any Number

Ex.17: Find the **Reciprocal** of (3) by **Newton-Raphson Method,** beginning with (0.1) as an initial value, work to (5D).

Sol. Let $f(x) = x = 1/3 = 1/C$ → $f(x) = x - (1/C)$ → $\overline{f}(x) = -\frac{1}{x^2}$, $C > 0$

By **Newton-Raphson Method**

$$x_{n+1} = x_n - \frac{f(x_n)}{\overline{f}(x_n)} = x_n - \frac{(1/x_n) - C}{(-1/x_n^2)} = x_n + \frac{(1/x_n)}{(1/x_n^2)} - \frac{C}{(1/x_n^2)} = x_n + x_n - C.x_n^2 \Rightarrow$$

$$x_{n+1} = x_n(2 - C.x_n)$$

n	x_n	x_{n+1}	є
0	0.1000	0.1700	----
1	0.1700	0.2533	0.0833
2	0.2533	0.3141	0.0608
3	0.3141	0.3322	0.0181
4	0.3322	0.3333	0.0011
5	0.3333	0.3333	0.0000

Summary
Solution of Non-Linear Equation

1- Simple Iterative Method

Make a table to find

n	x_n	$f(x_n)$	Є
0	---	---	---
1	---	---	---

It's need to **one** initial value (x_n)

2- Bisection Method (Halving interval)

Make a table to find

n	*a*	b	r	*f*(*a*)	*f*(*r*)	Є
0	---	---	---	---	---	---
1	---	---	---	---	---	---

It's need to **two** initial value (***a***, ***b***)

3- Secant Method

Make a table to find

n	x_{n-1}	$f(x_{n-1})$	x_n	$f(x_n)$	x_{n-1}	Є
1	---	---	---	---	---	---
2	---	---	---	---	---	---

It's need to **two** initial value (x_{n-1}, x_n)

$$x_{n+1} = x_n . \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} . f(x_n)$$

4- Newton-Raphson Method

Make a table to find

n	x_n	$f(x_n)$	$\overline{f}(x_n)$	x_{n+1}	Є
0	---	---	---	---	---
1	---	---	---	---	---

It's need to **one** initial value (x_n)

$$x_{n+1} = x_n - \frac{f(x_n)}{\overline{f}(x_n)}$$