$$T_3 - T_1 + T_2 = 0.5 \quad --- \ (2)$$

Node (3) $\quad \dfrac{T_4 - T_2}{2(0.25)} + 2T_3 = 2(0.75) \quad * 2(0.25)$

$X = 0.75$

$\Delta X = 0.25 \quad 50 - T_2 + 4(0.25)\,T_3 = 4(0.75)(0.25)$

$$T_3 - T_2 = 49.25 \quad --- \ (3)$$

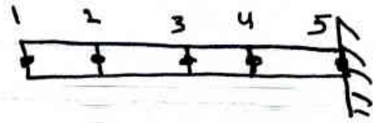
So,
$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 50.25 \\ 0.5 \\ 49.25 \end{bmatrix}$$

H.W. Solve this system Matrix.

---

Ex.2: Find profile Temp. in one-D. by using Finite difference for this Equ.

$$\frac{\partial T}{\partial x} + T = -200\,x$$



at B.C.

$$T(0) = 20^\circ C \ \& \ T(1), \ \frac{\partial T}{\partial x} = 0 \ \text{or} \ q = \text{zero}$$

Sol: Nodes (1), (2), (3), & (4) solve by F.D. Center Expression

$$y' = \frac{y_{x+\Delta x} - y_{x-\Delta x}}{2\Delta X} \ ; \text{ sub in P.D.E.} \rightarrow$$

$$\left[ \frac{T_{x+\Delta x} - T_{x-\Delta x}}{2\Delta X} + T_x = -200\,X \right] \text{G.F.} \ , \Delta X = \frac{L}{M-1} = \frac{1}{5-1} = 0.25$$

Node (1) $\quad \dfrac{T_3 - T_1}{2\Delta X} + T_2 = -200(0.2) \quad * 2\Delta X$

$X = 0.2$

$\Delta X = 0.2$

$$T_3 - T_1 + 2(0.2)T_2 = -200(0.2) * 2(0.2)$$

$$T_3 + 0.4\,T_2 = 4 \quad --- \ (1)$$

Node (3) $\quad \dfrac{T_4 - T_2}{2\Delta X} + T_3 = -200(0.4) \quad * 2\Delta X$

$X = 0.4$

$\Delta X = 0.2$

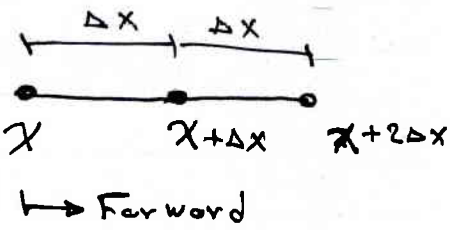
$$T_4 - T_2 + 2(0.2)T_3 = -200(0.4) * 2(0.2)$$

$$T_4 - T_2 + 0.4\,T_3 = -32 \quad --- \ (2)$$

To Find $\left(\frac{\partial^2 y}{\partial x^2}\right)$ forword, backword, & Center expression :-

* By forword expression :-

$$y'' = \frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) \quad \& \text{ Let } \psi = \frac{\partial y}{\partial x}$$




$$\frac{\partial \psi}{\partial x} = \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{x+\Delta x} - \psi_x}{\Delta x} \quad \text{---} (1)$$

$$\psi_x = \left.\frac{\partial y}{\partial x}\right)_x = \frac{y_{x+\Delta x} - y_x}{\Delta x} \quad \text{---} (2)$$

$$\psi_{x+\Delta x} = \left.\frac{\partial y}{\partial x}\right)_{x+\Delta x} = \frac{y_{x+2\Delta x} - y_{x+\Delta x}}{\Delta x} \quad \text{---} (3)$$

Now, sub. (2) & (3) in (1) →

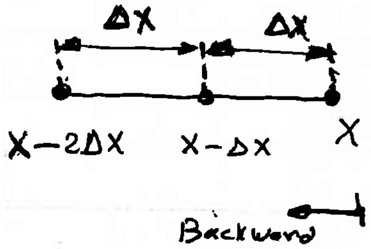
$$\left[\frac{\partial \psi}{\partial x} = \frac{\partial^2 y}{\partial x^2} = \frac{y_{x+2\Delta x} - 2y_{x+\Delta x} + y_x}{\Delta x^2}\right]$$

* By backword

$$\ddot{y} = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \quad \text{and let } \psi = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d\psi}{dx} = \frac{\psi_x - \psi_{x-\Delta x}}{\Delta x} \quad --- \; ①$$

$$\left.\frac{dy}{dx}\right)_x = \psi_x = \frac{y_x - y_{x-\Delta x}}{\Delta x} \quad --- \; ②$$




$$\psi_{x+\Delta x} = \left.\frac{dy}{dx}\right)_{x-\Delta x} = \frac{y_{x-\Delta x} - y_{x-2\Delta x}}{\Delta x}$$

Now; Sub. ③ & ② in ① ⇒

$$\left[\frac{d\psi}{dx} = \frac{d^2y}{dx^2} = \frac{y_{x-2\Delta x} - 2y_{x-\Delta x} + y_x}{\Delta x^2}\right]; \; E_0 \; \Delta x$$

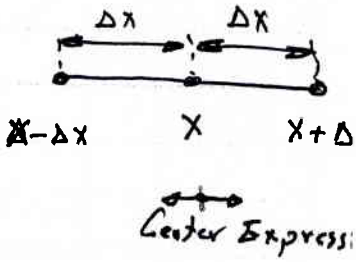
* By center expression

$$y_{x+\Delta x} = y_x + \Delta x\,\dot{y} + \frac{\Delta x^2}{2!}\ddot{y} + \frac{\Delta x^3}{3!}\dddot{y} + \cdots$$

cutting

$$y_{x-\Delta x} = y_x - \Delta x\,\dot{y} + \frac{\Delta x^2}{2!}\ddot{y} - \frac{\Delta x^3}{3!}\dddot{y} + \cdots$$

cutting




$$\sum$$

$$y_{x+\Delta x} + y_{x-\Delta x} = 2y_x + \Delta x^2\,\ddot{y}$$

$$\left[\ddot{y} = \frac{d^2y}{dx^2} = \frac{y_{x+\Delta x} - 2y_x + y_{x-\Delta x}}{\Delta x^2}\right]; \; E_0 \; \Delta x^2$$

Node ②

$x = 0.5$

$\Delta x = 0.25$

$$\frac{T_3 - 2T_2 + T_1}{(0.25)^2} + 100 = 0 \qquad * (0.25)^2$$

$$T_3 - 2T_2 + T_1 + (0.25)^2\, 100 = 0$$

$$T_3 - 2T_2 + T_1 = -6.25 \quad ---- \; ②$$

Node ③

$x = 0.75$

$\Delta x = 0.25$

$$\frac{T_4 - 2T_3 + T_2}{(0.25)^2} + 100 = 0 \qquad * (0.25)^2$$

$$100 - 2T_3 + T_2 + (0.25)^2\, 100 = 0$$

$$-2T_3 + T_2 = -106.25 \quad --- \; ③$$

So,

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} * \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -106.25 \\ -6.25 \\ -106.25 \end{bmatrix}$$

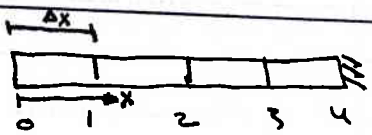
H.W. Solve when B.C.

$T(0) \rightarrow \frac{\partial T}{\partial x} = 0$ or $q = 0$

$T(1) \rightarrow \frac{\partial T}{\partial x} = 100\,°c/m$

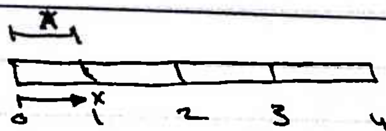
H.W. Solving the above system matrix by Matrix Inverse, Gauss-Elimination method, Gauss-Jordon Elimination method, Jacob's method & Gauss-Seidal method

H.W. ① Solve $\ddot{T} + T = -200x$ ; $T_0 = 200°c$ & $q_4 = 0$




(diffirent B.C.)

② Solve $\ddot{T} + 2T = 0$ ;




and then solve the system matrix by Gauss-Jordon Elimination & Gauss-Seidal method.

$T_0 = 0°c$ $\qquad$ $T_4 = 20°c$

(diffirent B.C.)

* By center expression

$$\frac{d^3y}{dx^3} = \frac{d\psi}{dx} = \frac{\psi_{x+\Delta x} - \psi_{x-\Delta x}}{2\,\Delta x} \quad --- (1)$$

$$\psi_{x+\Delta x} = \left.\frac{d^2y}{dx^2}\right)_{x+\Delta x} = \frac{y_x - 2y_{x+\Delta x} + y_{x+2\Delta x}}{\Delta x^2} \quad --- (2)$$

$$\psi_{x-\Delta x} = \left.\frac{d^2y}{dx^2}\right)_{x-\Delta x} = \frac{y_{x-2\Delta x} - 2y_{x-\Delta x} + y_x}{\Delta x^2} \quad --- (3)$$

Sub. (2) (3) in (1) $\Rightarrow$

$$\left[\frac{d\psi}{dx} = \frac{d^3y}{dx^3} = \frac{y_{x+2\Delta x} - 2y_{x+\Delta x} - y_{x-2\Delta x} + 2y_{x-\Delta x}}{2\,\Delta x^3}\right]; \; E \propto \Delta x^2$$

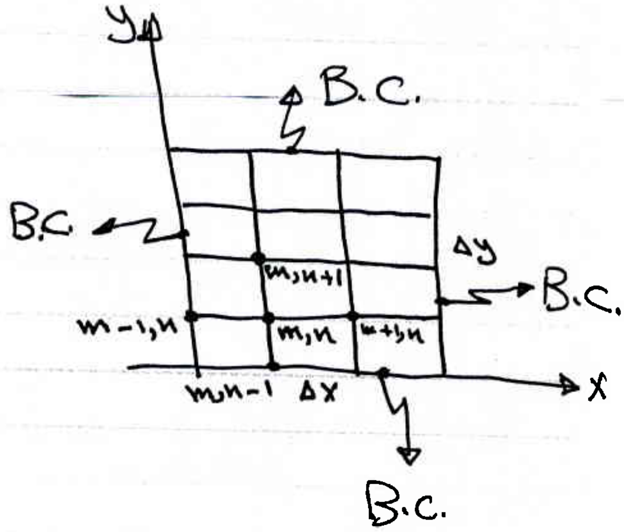
H.W. Find $\frac{d^4y}{dx^4}$ forword, backword, & center Expression?

A-2- Regular shapes, Linear Equs., Two dimensions, steady sta

Let $z = f(x,y)$

For Center expressions:-

$$\frac{\partial z}{\partial x} = \frac{z_{m+1,n} - z_{m-1,n}}{2\,\Delta x}$$




$$\frac{\partial z}{\partial y} = \frac{z_{m,n+1} - z_{m,n-1}}{2\,\Delta y}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{z_{m+1,n} - 2z_{m,n} + z_{m-1,n}}{\Delta x^2}$$

[Same or diffirent B.C

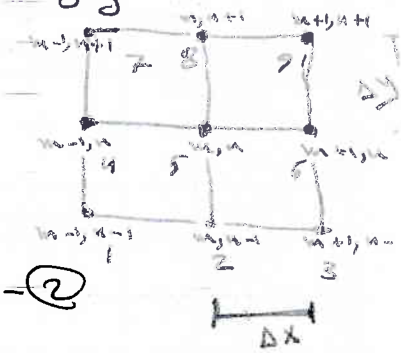
$$\frac{\partial^2 z}{\partial y^2} = \frac{z_{m,n+1} - 2z_{m,n} + z_{m,n-1}}{\Delta y^2}$$

**Ex,5;** write the following P.D.E. in Finite diffirence form

$$\frac{\partial^2 T}{\partial x \partial y} + 2X = 0 \quad ; \text{ for center expression.}$$

**Sol;**

$$\frac{\partial^2 T}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial \Psi}{\partial X} \quad , \quad \Psi = \frac{\partial T}{\partial y}$$




$$\frac{\partial \Psi}{\partial X} = \frac{\Psi_{m+1,n} - \Psi_{m-1,n}}{2\Delta X} \quad --- (1)$$

$$\Psi_{m+1,n} = \left.\frac{\partial T}{\partial y}\right)_{m+1,n} = \frac{T_{m+1,n+1} - T_{m+1,n-1}}{2\Delta y} \quad --- (2)$$

$$\Psi_{m-1,n} = \left.\frac{\partial T}{\partial y}\right)_{m-1,n} = \frac{T_{m-1,n+1} - T_{m-1,n-1}}{2\Delta y} \quad --- (3)$$

sub. (2) (3) in (1) $\Rightarrow$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial^2 T}{\partial x \partial y} = \frac{T_{m+1,n+1} - T_{m+1,n-1} - T_{m-1,n+1} + T_{m-1,n-1}}{4\Delta X \Delta y} \quad --- (4)$$

sub. (4) in P.D.E. $\Rightarrow$

$$\left[\frac{T_{m+1,n+1} - T_{m+1,n-1} - T_{m-1,n+1} + T_{m-1,n-1}}{4\Delta x \Delta y} + 2X = 0\right] \text{ F. D. Form}$$

---

**H.W:** write the following P.D.E $\rightarrow$ Finite diffirence for

I) $\frac{\partial^3 T}{\partial x \partial y^2} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial T}{\partial y} = 0$ ; for center expression.

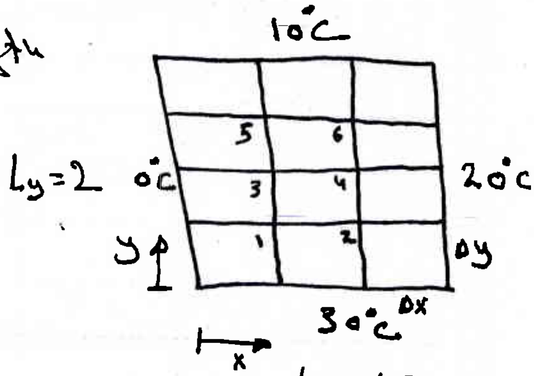
II) $\frac{\partial^3 T}{\partial x \partial y^2} + \frac{\partial^3 T}{\partial x^2 \partial y} = 0$ ; for center expression.

III) $\frac{\partial^2 P}{\partial x \partial y} + \frac{\partial^2 P}{\partial z^2} = \sin(wx)$ ; for center expression.

$$\begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -4 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -4 \end{bmatrix} * \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} -300 \\ -300 \\ -300 \\ -300 \end{bmatrix}$$

H.W. Solve when $\Delta x = \Delta y$ 0.25

**Ex.7:** Solve the poisson's equ $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -x^2 y$; which subject conditions are showed in fig.(2) when $\Delta x = \Delta y = 0.5$ unit length




Sol: By center expressions-

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{m,n+1} - 2T_{m,n} + T_{m,n-1}}{\Delta y^2}$$ So, Sub. in poisson's equ. $\Rightarrow$

$$\frac{T_{m+1,n} - 2T_{m,n} + T_{m-1,n}}{\Delta x^2} + \frac{T_{m,n+1} - 2T_{m,n} + T_{m,n-1}}{\Delta y^2} = -x^2 y \quad (* \Delta x^2)$$

$$T_{m+1,n} - 2T_{m,n} + T_{m-1,n} + T_{m,n+1} - 2T_{m,n} + T_{m,n-1} = -x^2 y \Delta x^2$$

$$\left[ T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = -x^2 y \Delta x^2 \right]$$ General formula

Node①: $T_2 + 0 + T_3 + 30 - 4T_1 = -(0.5)^2 (0.5)(0.5)^2$

$x = 0.5$
$\Delta x = 0.5$
$y = 0.5$

$$T_2 + T_3 - 4T_1 = -30.031 \quad ---①$$

Node②: $T_1 + 20 + T_4 + 30 - 4T_2 = -(1)^2 (0.5)(0.5)^2$

$x = 1$
$y = 0.5$
$\Delta x = 0.5$

$$T_1 + T_4 - 4T_2 = -50.125 \quad ---②$$

Node③: $T_4 + 0 + T_5 + T_1 - 4T_3 = -(0.5)^2 (1)(0.5)^2$

$x = 0.5$
$y = 1$

$$T_4 + T_5 + T_1 - 4T_3 = -0.0625 \quad ---③$$

Node ①: $M_2 + M_0 - 2M_1 = (0.25)^2 \sin(\pi * 0.25)$ 180°

$x = 0.25$

$$M_2 - 2M_1 = 0.0442 \quad --- ①$$

Node ②: $M_3 + M_1 - 2M_2 = \sin(\pi * 0.5) * (0.25)^2$

$x = 0.5$

$$M_3 + M_1 - 2M_2 = 0.0625 \quad --- ②$$

Node ③: $M_4 + M_2 - 2M_3 = (0.25)^2 \sin(\pi * 0.75)$

$x = 0.75$

$$M_2 - 2M_3 = 0.0442 \quad --- ③$$

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} 0.00442 \\ 0.0625 \\ 0.00442 \end{bmatrix}$$

H.W Solve this Matrix by Invers matrix

---

H.W I) Solve the Laplace's equ $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ ; which subject to B.C. as:

$T(x,0) \Rightarrow y = 0$ & $T(x,1) = 50°c$

$T(0,y) = 25°c$ & $T(1,y) = 50°c$

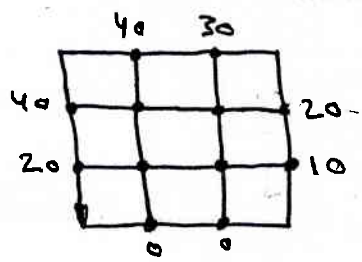
when $\Delta x = \Delta y = 1$ Unit.

II) Solve the poisson's equ. $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -xy$ ; which subject to B.C. as:

$T(x,0) = -10°c$ & $T(x,1) = -10$

$T(0,y) \Rightarrow y = 60$ & $T(x,0) \Rightarrow y = 0$ ; for $\Delta x = \Delta y = 1$ unit

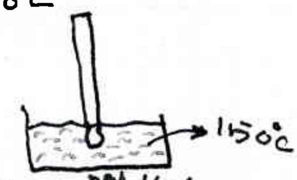
III) Solve the laplace's equ. $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ ; which subject to B.C. as showed in fig(4) for $\Delta x = \Delta y = 1$ unit




fig(4)

IV) Solve $\frac{\partial^2 T}{\partial y^2} + T = 0$ for a spoon is partialy drown in a pot as showed in fig.(5) when $y = 0 \rightarrow T = 150°c$ & $y = 1 \Rightarrow T = 25°C$ & $\Delta y = 1$ unit

V) write the following P.D.E. in Finite difference form: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial x \partial y^2} = 0$




fig(5)