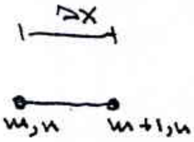
y', y'', y''', y'''' → Forward, Backword, Center Expression (m,n) [Review of F.D.]

* Forward → $T_{m+1} = T_m + \Delta x T' + \frac{\Delta x^2}{2!} T'' + \dots$ cutting

$y' - x$

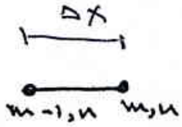
$$T_{m+1} - T_m = \Delta x T' \rightarrow \boxed{T' = \frac{T_{m+1} - T_m}{\Delta x}}$$



* Backword → $T_{m-1} = T_m - \Delta x T' + \frac{\Delta y^2}{2!} T'' + \dots$ Cutting

$y' - x$

$$T_{m-1} = T_m - \Delta x T' \rightarrow \boxed{T' = \frac{T_m - T_{m-1}}{\Delta x}}$$

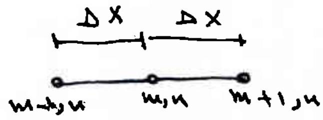


* Center Exp. → $T_{m+1} = \cancel{T_m} + \Delta x T'$

$y' - x$

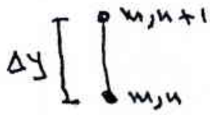
طرح $T_{m-1} = \cancel{T_m} - \Delta x T'$

$$T_{m+1} - T_{m-1} = 2\Delta x T' \rightarrow \boxed{T' = \frac{T_{m+1} - T_{m-1}}{2\Delta x}}$$



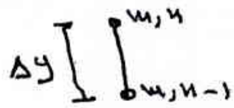
* Forward → $T_{n+1} = T_n + \Delta y T' + \frac{\Delta y^2}{2!} T''$ cutting $\rightarrow \boxed{T' = \frac{T_{n+1} - T_n}{\Delta y}}$

$y' - y$



* Backword → $T_{n-1} = T_n - \Delta y T' + \frac{\Delta y^2}{2!} T''$ cutting $\rightarrow \boxed{T' = \frac{T_n - T_{n-1}}{\Delta y}}$

$y' - y$

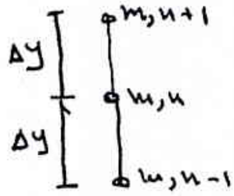


* Center Exp → $T_{n+1} = \cancel{T_n} + \Delta y T'$

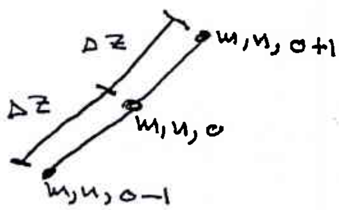
$y' - y$

طرح $T_{n-1} = \cancel{T_n} - \Delta y T'$

$$T_{n+1} - T_{n-1} = 2\Delta y T' \rightarrow \boxed{T' = \frac{T_{n+1} - T_{n-1}}{2\Delta y}}$$

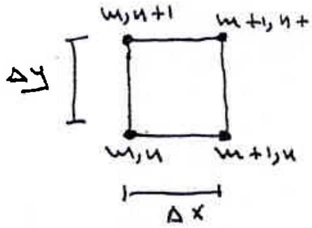


H.W. Solve y' in Z-direction for forword, Backword, & Center Expression



Forward $\rightarrow \dfrac{\partial^2 T}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial T}{\partial y}\right) = \dfrac{\partial \psi}{\partial x} = \dfrac{\psi_{m+1,n} - \psi_{m,n}}{\Delta x}$ — ①

$y'' - x, y$



$$\psi_{m,n} = \frac{\partial T}{\partial y}\Big)_{m,n} = \frac{T_{m,n+1} - T_{m,n}}{\Delta y} \quad — ②$$

$$\psi_{m+1,n} = \frac{\partial T}{\partial y}\Big)_{m+1,n} = \frac{T_{m+1,n+1} - T_{m+1,n}}{\Delta y} \quad — ③$$

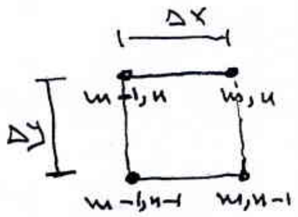
② , ③ sub in ① $\rightarrow$

$$\frac{\partial^2 T}{\partial x \partial y} = \frac{\dfrac{T_{m+1,n+1} - T_{m+1,n}}{\Delta y} - \dfrac{T_{m,n+1} - T_{m,n}}{\Delta y}}{\Delta x}$$

$$\boxed{\frac{\partial^2 T}{\partial x \partial y} = \frac{T_{m+1,n+1} - T_{m,n+1} - T_{m,n+1} + T_{m,n}}{\Delta x \Delta y}}$$

Backword $\rightarrow \dfrac{\partial^2 T}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial T}{\partial y}\right) = \dfrac{\partial \psi}{\partial x} = \dfrac{\psi_{m,n} - \psi_{m-1,n}}{\Delta x}$ — ①

$y'' - x, y$



$$\psi_{m,n} = \frac{\partial T}{\partial y}\Big)_{m,n} = \frac{T_{m,n} - T_{m,n-1}}{\Delta y} \quad — ②$$

$$\psi_{m-1,n} = \frac{\partial T}{\partial y}\Big)_{m-1,n} = \frac{T_{m-1,n} - T_{m-1,n-1}}{\Delta y} \quad — ③$$

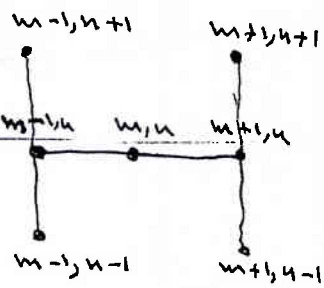
② , ③ sub in ① $\rightarrow$

$$\frac{\partial^2 T}{\partial x \partial y} = \frac{\dfrac{T_{m,n} - T_{m,n-1}}{\Delta y} - \dfrac{T_{m-1,n} - T_{m-1,n-1}}{\Delta y}}{\Delta x}$$

$$\boxed{\frac{\partial^2 T}{\partial x \partial y} = \frac{T_{m,n} - T_{m,n-1} - T_{m-1,n} + T_{m-1,n-1}}{\Delta x \Delta y}}$$

---

* Center Exp. $\rightarrow \dfrac{\partial^2 T}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial T}{\partial y}\right) = \dfrac{\partial \psi}{\partial x} = \dfrac{\psi_{m+1,n} - \psi_{m-1,n}}{\Delta x}$ — ①



$$\psi_{m+1,n} = \frac{\partial T}{\partial y}\Big)_{m+1,n} = \frac{T_{m+1,n+1} - T_{m+1,n-1}}{\Delta y} \quad — ②$$

$$\psi_{m-1,n} = \frac{\partial T}{\partial y}\Big)_{m-1,n} = \frac{T_{m-1,n+1} - T_{m+1,n-1}}{\Delta y} \quad — ③$$

$$\boxed{\frac{\partial^2 T}{\partial x \partial y} = \frac{T_{m+1,n+1} - T_{m+1,n-1} - T_{m-1,n+1} + T_{m+1,n-1}}{\Delta x \Delta y}}$$

H.W $\rightarrow$ Solve $\dfrac{\partial^2 T}{\partial y \partial x}$ for forward, Backword & Center F.D.

* Center $\rightarrow \frac{\partial^4 T}{\partial x^4} = \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_{m+1} + \psi_{m-1} - 2\psi_m}{\Delta x^2}$ — ①

$y^{IV} - x$

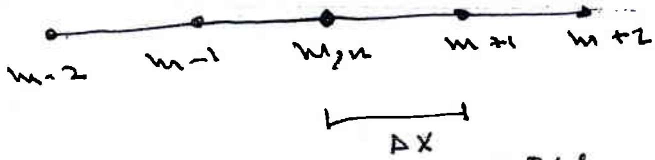
$\psi_m = \left.\frac{\partial^2 T}{\partial x^2}\right)_m = \frac{\qquad\qquad}{\Delta x^2}$ — ②

$\psi_{m+1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m+1} = \frac{\qquad\qquad}{\Delta x^2}$ — ③

$\psi_{m-1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m-1} = \frac{\qquad\qquad}{\Delta x^2}$ — ④

Sub.

H.W.



* Backword $\rightarrow \frac{\partial^4 T}{\partial x^4} = \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_m + \psi_{m-2} - 2\psi_{m-1}}{\Delta x^2}$ — ①

$y^{IV} - x$

Complete above Equ. (H.W.)

* Forword $\rightarrow \frac{\partial^4 T}{\partial x^4} = \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_{m+2} + \psi_m - 2\psi_{m+1}}{\Delta x^2}$ — ①

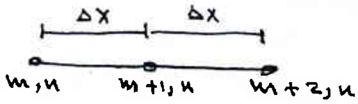
$y^{IV} \rightarrow x$

H.W. $\frac{\partial^4 T}{\partial y^4}$ forword, Backword, & Center Exp.

*Forword $\rightarrow T'' = \frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial \Psi}{\partial x} = \frac{\Psi_{m+1} - \Psi_m}{\Delta x}$ — ①

$y'' - x$

$$\Psi_m = \left.\frac{\partial T}{\partial x}\right)_m = \frac{T_{m+1} - T_m}{\Delta x} \quad \text{—②}$$



$$\Psi_{m+1} = \left.\frac{\partial T}{\partial x}\right)_{m+1} = \frac{T_{m+2} - T_{m+1}}{\Delta x} \quad \text{—③}$$

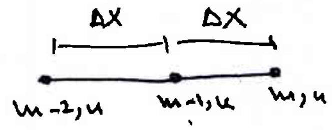
Now, sub. ② & ③ in ① $\rightarrow \frac{\partial \Psi}{\partial x} = \frac{\partial^2 T}{\partial x} = \dfrac{\frac{T_{m+2} - T_{m+1}}{\Delta x} - \frac{T_{m+1} - T_m}{\Delta x}}{\Delta x}$

$$\boxed{T'' = \frac{T_{m+2} + T_m - 2\,T_{m+1}}{\Delta x^2}}$$

Backword $\rightarrow T'' = \frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial \Psi}{\partial x} = \frac{\Psi_m - \Psi_{m-1}}{\Delta x}$ — ①

$y'' - x$

$$\Psi_m = \left.\frac{\partial T}{\partial x}\right)_m = \frac{T_m - T_{m-1}}{\Delta x} \quad \text{—②}$$



$$\Psi_{m-1} = \left.\frac{\partial T}{\partial x}\right)_{m-1} = \frac{T_{m-1} - T_{m-2}}{\Delta x} \quad \text{—③}$$

Now, sub ② & ③ in ① $\rightarrow \frac{\partial \Psi}{\partial x} = \frac{\partial^2 T}{\partial x^2} = \dfrac{\frac{T_m - T_{m-1}}{\Delta x} - \frac{T_{m-1} - T_{m-2}}{\Delta x}}{\Delta x}$

$$\boxed{T'' = \frac{T_{m-2} + T_m - 2\,T_{m-1}}{\Delta x^2}}$$

Center Exp. $\rightarrow T_{m+1} = T_m + \cancel{\Delta x\, T'} + \frac{\Delta x^2}{2!} T'' + \frac{\Delta x^3}{3!} T''' + \cdots$

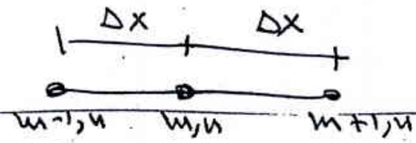
$y'' - x$

$$T_{m-1} = T_m - \cancel{\Delta x\, T'} + \frac{\Delta x^2}{2!} T'' - \frac{\Delta x^3}{3!} T''' + \cdots$$

Σ — cutting

$$T_{m+1} + T_{m-1} = 2T_m + \Delta x^2 T''$$

$$\boxed{T'' = \frac{T_{m+1} + T_{m-1} - 2T_m}{\Delta x^2}}$$



H.W. Solve T'' in y-direction for Forword, Backword, & Center Expression

H.W. Solve T'' in Z-direction for Forword, Backword, & Center Exp.

* Forword → $\frac{\partial^3 T}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial \psi}{\partial x} = \frac{\psi_{m+1} - \psi_m}{\Delta x}$ — ①

$y'''-x$

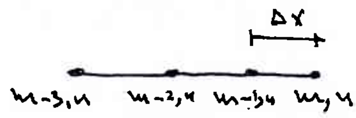
$\psi_m = \left.\frac{\partial^2 T}{\partial x^2}\right)_m = \frac{T_{m+2} + T_m - 2T_{m+1}}{\Delta x^2}$ — ② sub.

$\psi_{m+1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m+1} = \frac{T_{m+3} + T_{m+1} - 2T_{m+2}}{\Delta x^2}$ — ③

$$\boxed{\frac{\partial^3 T}{\partial x^3} = \frac{T_{m+3} + 3T_{m+1} - 3T_{m+2} - T_m}{\Delta x 3}}$$

*Backword → $\frac{\partial^3 T}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial \psi}{\partial x} = \frac{\psi_m - \psi_{m-1}}{\Delta x}$ — ①

$y'''-x$

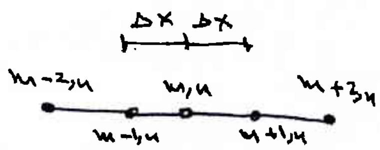


$\psi_m = \left.\frac{\partial^2 T}{\partial x^2}\right)_m = \frac{T_{m-2} + T_m - 2T_{m-1}}{\Delta x^2}$ — ② sub.

$\psi_{m-1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m-1} = \frac{T_{m-3} + T_{m-1} - 2T_{m-2}}{\Delta x^2}$ — ③

$$\boxed{\frac{\partial^3 T}{\partial x^3} = \frac{3T_{m-2} + T_m - 3T_{m-1} - T_{m-3}}{\Delta x^3}}$$

Center Exp. → $\frac{\partial^3 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial^2 T}{\partial x^2}\right) = \frac{\partial \psi}{\partial x} = \frac{\psi_{m+1} - \psi_{m-1}}{\Delta x}$ — ①



$\psi_{m+1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m+1} = \frac{T_{m+2} + T_m - 2T_{m+1}}{\Delta x 2}$ — ② sub.

$\psi_{m-1} = \left.\frac{\partial^2 T}{\partial x^2}\right)_{m-1} = \frac{T_{m-2} + T_m - 2T_{m-1}}{\Delta x 2}$ ③

$$\boxed{\frac{\partial^3 T}{\partial x^3} = \frac{T_{m+2} - 2T_{m+1} - T_{m-2} + 2T_{m-1}}{\Delta x 3}}$$

H.W. Solve T''' in y-direction for forword, Backword & Center Expression

H.W. $\frac{\partial^3 T}{\partial x \partial y^2}$, $\frac{\partial^3 T}{\partial y^2 \partial x}$