Ch.7: Curves Fitting

When we have many data from any laboratory (Fluid, Heat Transfer, strength of material ... etc.) for any test, we should find a function which governed this data.

So, we know there are many types of function such as: Linear Equ. Exponential Equ., Power Equ., polynomial Equ., Non-linear Equ....

We will study in this chapter

1- Linear Regression → Linear Equ.
   → Exponential Equ.
   → Power Equ.

2- Polynomial Regression → Polynomial Equ.

This equs. we will study in this chapter.

Also, there are many of equations such as:

* Multiple Linear Regression.
* Non-Linear Regression.

* we will study application of curve fitting on "Microsoft office Excel 2003"

1- Linear Regression

A- Linear Equation :-

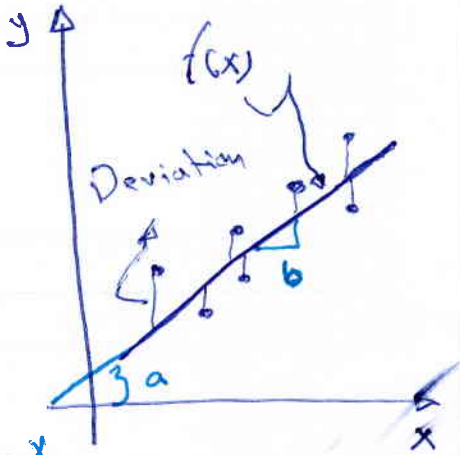
This is the simplest method to find Linear equ. for many points by this formula :-

$$y = a + bx$$

(a, b) are intersection with y-axes & slope of line respectively.

$$n.a + b\sum x_n = \sum y_n \quad \text{—— (1)}$$

$$a.\sum x_n + b.\sum x_n^2 = \sum (x_n.y_n) \quad \text{—— (2)}$$



n: No. parts or points which given in problems.

So, we must find $\sum x_n$, $\sum y_n$, $\sum x_n^2$, & $\sum y_n.x_n$ by using table.

n	x_n	y_n	x_n^2	$x_n.y_n$
1	✓	✓	✓	✓
2	✓	✓	✓	✓
3	✓	✓	✓	✓
4	⋮	⋮	⋮	⋮
⋮				
$\sum$	✓	✓	✓	✓

* Accuracy of Solution :- (R^2)

1- The sum of value squares linear Equ.

$$S_r = \sum_{n=1}^{i} (y_n - a - bx_n)^2$$

2- The mean value

$$y' = \frac{\sum y_n}{n}$$

3- Total sum of the squares of the residual between the data points and the mean value

$$S_t = \sum_{n=1}^{i} (y_n - y')^2$$

4- Accuracy of solution → $R^2 = \dfrac{S_t - S_r}{S_t}$ (Cofficient of determination)

Ch.7: Curves Fitting

Ex.1: Find the linear function that fit the data given below:-

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

& find the accuracy of solution (R^2)

Sol. the linear function is: $y = a + bx$

$$na + b\sum x_n = \sum y_n$$

$$a\sum x_n + b\sum x_n^2 = \sum x_n \cdot y_n \quad ; n = 8$$

So, we must find the values of:-

$\sum x_n, \sum y_n, \sum x_n^2, \sum x_n \cdot y_n$ $\xrightarrow{\text{by this table}}$

n	x_n	y_n	x_n^2	$x_n \cdot y_n$
1	1	1	1	1
2	3	2	9	6
3	4	4	16	16
4	6	4	36	24
5	8	5	64	40
6	9	7	81	63
7	11	8	121	88
8	14	9	196	126
$\sum$	56	40	524	364

($\sum x_n$, $\sum y_n$, $\sum x_n^2$, $\sum x_n \cdot y_n$)

$8a + 56b = 40 \rightarrow a = 5 - 7b$ —— ①

$a(56) + b(524)^2 = 364$ —— ②

sub. ① in ② $\rightarrow$

$(5 - 7b)56 + b(524) = 364 \rightarrow$

$b = 0.6363$ sub. in ① $\rightarrow$ $a = 0.5454$

$\therefore$ $y = 0.5454 + 0.6363x$ Linear Equ.

$S_r = \sum_{n=1} (y_n - a - bx_n)^2 \rightarrow$

$y' = \frac{\sum y_n}{n} = \frac{40}{8} \rightarrow y' = 5$

$S_t = \sum_{n=1} (y_n - y')^2 \longrightarrow$

n	$S_r = ( )^2$	$S_t = (y_n - y')^2$
1	0.0330	16
2	0.2063	9
3	0.8270	1
4	0.1319	1
5	0.4042	0
6	0.5298	4
7	0.2073	9
8	0.2057	16
$\sum$	2.5454	56

$R^2 = \frac{S_t - S_r}{S_t} = \frac{56 - 2.5454}{56}$

$\therefore R^2 = 0.9545$

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Ex.2: For given data below; Find the linear function.

x	1.1	3.2	5.3	7.2	9.4	11.6	13.7
y	3.2	5.1	6.4	7.5	7.9	8.4	8.5

& find accuracy of solution R^2

Sol: the linear Equ. is: $y = a + bx$

$$na + b\sum x_n = \sum y_n$$

$$a\sum x_n + b\sum x_n^2 = \sum x_n \cdot y_n \; ; n = 7$$

So, we should find the values:-

$\sum x_n, \sum y_n, \sum x_n^2, \sum x_n y_n$ $\xrightarrow{\text{by this table}}$

n	x_n	y_n	x_n^2	$x_n \cdot y_n$
1	1.1	3.2	1.21	3.52
2	3.2	5.1	10.24	16.32
3	5.3	6.4	28.09	33.92
4	7.2	7.5	51.84	54
5	9.4	7.9	88.36	74.26
6	11.6	8.4	134.56	97.44
7	13.7	8.5	187.69	116.45
$\sum$	51.5	47	501.99	395.19

$$7a + 51.5b = 47 \rightarrow a = \frac{1}{7}(47 - 51.5b) \quad \text{--- (1)}$$

$$51.5a + 501.99b = 395.19 \quad \text{--- (2)}$$

Sub. (1) in (2) $\rightarrow$

$$\frac{51.5}{7}(47 - 51.5b) + 501.99b = 395.19 \rightarrow$$

$b = 0.4013$ sub. in (1) $\rightarrow$ $a = 3.7615$

$\therefore$ $y = 3.7615 + 0.4013x$ Linear Equation

$$S_r = \sum_{n=1}^{7} (y_n - a - bx_n)^2 \longrightarrow$$

$$y' = \frac{\sum y_n}{n} = \frac{47}{7} \rightarrow y' = 6.7142$$

$$S_t = \sum_{n=1}^{7} (y_n - y')^2 \longrightarrow$$

n	$S_r = (\quad)^2$	$S_t = (y_n - y')^2$
1	1.00587	12.3496
2	0.002953	2.6056
3	0.26174	0.09872
4	0.72104	0.61748
5	0.13416	1.40612
6	0.000275	2.84192
7	0.57655	3.18908
$\sum$	2.7026	23.1085

$$R^2 = \frac{S_t - S_r}{S_t} = \frac{23.1085 - 2.7026}{23.1085} \rightarrow$$

$R^2 = 0.8831$

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B- Exponential Equation :-

The governing equation in this method is :-

$$y = a\, e^{b.x} \quad ; (a \,\&\, b) \text{ are constants}$$

In this case, we should change above equ. to Linear equ.

$$\ln y = \ln (a \cdot e^{b.x})$$

$$= \ln a + b.x \ln e$$

$$\ln y = \ln a + b.x \quad ; \ln y = z, \quad \ln a = c$$

$$z = c + b x \longrightarrow \text{Linear Equ.}$$

$$n.c + b \sum x_n = \sum z_n \quad \text{—①}$$

$$\sum x_n . z_n = c . \sum x_n + b \sum x_n^2 \quad \text{—②}$$

So, we must find $\sum x_n, \sum y_n, \sum x_n . y_n, \sum x_n^2$ by using table.

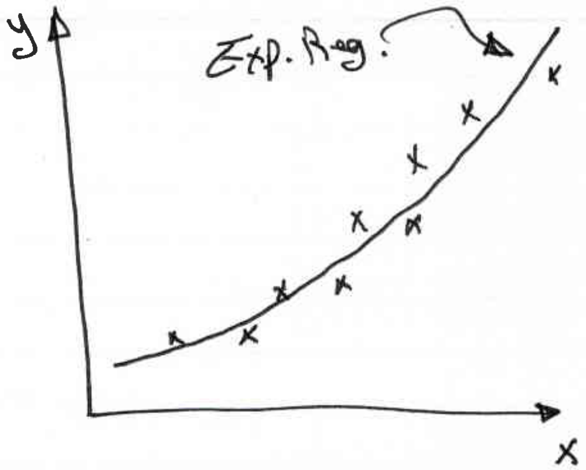
* Accuracy of Solution (R^2) : It's same for Linear equ. (P.2)

$$S_r = \sum_{n=1}^{i} (z_n - a - b x_n)^2$$

$$S_t = \sum_{n=1}^{i} (z_n - y')^2$$

$$y' = \frac{\sum y_n}{n}$$

$$\therefore R^2 = \frac{S_t - S_r}{S_t}$$



Ch.7: Curves Fitting

Ex.3: Find the equation which fit the following data, Using Exp. Equ.

X	1	2	3	4	5	6
y	15	20	27	36	49	65

& find "R^2" H.W.

Sol: $y = a e^{bx}$

$\ln y = \ln(a \cdot e^{b.x})$

$= \ln a + b.x \ln e$

$\ln y = \ln a + b x$; $z = \ln y$, $c = \ln a$

$z = c + b x$

$\sum z_n = n.c + b \sum x_n$

$\sum x_n z_n = c . \sum x_n + b \sum x_n^2$

$20.646 = 6 * c + 21 * b \rightarrow c = \frac{1}{6}(20.646 - 21b)$

$77.414 = 21 * c + b * 91$ — (2) sub. in (2)

$77.414 = 21 \frac{1}{6}(20.646 - 21b) + b * 91 \rightarrow$

n	x_n	y_n	x_n^2	$z = \ln y$	$x_n z_n$
1	1	15	1	2.708	2.708
2	2	20	4	2.995	5.991
3	3	27	9	3.295	9.885
4	4	36	16	3.538	14.332
5	5	49	25	3.891	19.455
6	6	65	36	4.174	25.044
$\sum$	21	212	91	20.646	77.414

$b = 0.2944$ & $c = 2.4104 = \ln a$

$a = 11.138$

$\therefore \left[y = 11.138 \, e^{0.2944 x} \right]$ Exponential Equation

C- Power Equation :-

The governing equation in this method is :-

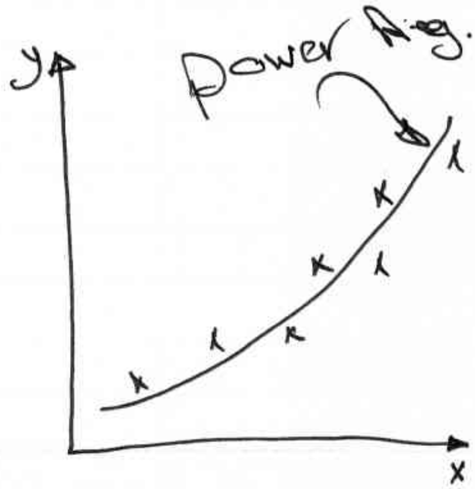
$y = a\,x^b$; (a, b) are Constants

In this method, we should change above equ. to Linear equ.

$\ln y = \ln(a\,x^b)$

$= \ln a + \ln x^b$

$\ln y = \ln a + b \ln x$; $z = \ln y$, $\ln a = c$, $\ln x = W$



$Z = C + b.W$ → Linear Equ.

$\sum Z_n = C.n + b \sum W_n$ —— ①

$\sum Z_n.W_n = C \sum W_n + b \sum W_n^2$ —— ②

Also, we should find, $\sum W_n$, $\sum Z_n$, $\sum W_n^2$, $\sum Z_n.W_n$ by using table

* Accuracy of (R^2): It's Same for linear equ. (P.2)

$S_r = \sum_{n=1}^{i} (Z_n - a - bW)^2$

$S_t = \sum_{n=1}^{i} (Z_n - \bar{Z})^2$

$\bar{Z} = \frac{\sum Z_n}{n}$ $\therefore R^2 = \frac{S_t - S_r}{S_t}$

Ex. 4: In Lab. of Heat Transfer for forced Convection test, we obtain on this data, Estimate the governing equ. by using power Equ.

n	1	2	3	4	5
x = Re	100	200	400	1000	2000
y = Nu	15.1	17.7	19	26	32

& find "R^2" H.W.

Sol: $y = a x^b \Longleftrightarrow Nu = a Re^b$

$\ln y = \ln(a \cdot x^b)$

$\ln y = \ln a + b \ln x$, $Z = \ln y$, $c = \ln a$, $w = \ln x$

$Z = c + bW$

$\sum Z_n = n \cdot c + b \sum W_n$

$\sum Z_n \cdot W_n = n \sum W_n + b \sum W_n^2$

n	x_n	y_n	W_n	W_n^2	Z_n	$Z_n \cdot W_n$
1	100	15.1	4.6051	21.2075	2.7146	12.5016
2	200	17.1	5.2983	28.0721	2.8735	15.2250
3	400	19	5.9914	35.8976	2.9444	17.6415
4	1000	26	6.9077	47.7170	3.2580	22.5061
5	2000	32	7.6009	57.7737	3.4657	26.3427
$\sum$	/	/	30.4036	190.668	15.256	94.217

now, we should find the value of:-

$\sum Z_n, \sum W_n, \sum W_n^2, \sum Z_n W_n$

$15.2565 = 5C + 30.4036\, b \longrightarrow C = 3.0513 - 6.0807 b$ —— ①

sub. in ②

$94.2170 = 30.4036\, C + 190.6682\, b$ —— ②

$\boxed{b = 0.2497}$ sub. in ① $\longrightarrow c = 3.0513 - 6.0807(0.2497) \longrightarrow$

$c = 1.5329$

$c = \ln a$.

$1.5329 = \ln a \longrightarrow a = e^{1.5329}$

$\boxed{a = 4.6318}$

$\therefore y = 4.6318\, x^{0.2497} \Longleftrightarrow \boxed{Nu = 4.6318\, Re^{0.2497}}$

Power Equation

2- Polynomial Equation :-

The governing equ. in this method is :-

$$y = C_0 + C_1 X + C_2 X^2 \,/\, + C_3 X^3 + \dots + C_m X^m$$

(cutting)

m = order of polynomial equ.

n = No. data given in problem.

$$\sum y_n = C_0 \cdot n + C_1 \sum X_n + C_2 \sum X_n^2 \quad \text{—— (1)}$$

$$\sum y_n \cdot X_n = C_0 \sum X_n + C_1 \sum X_n^2 + C_2 \sum X_n^3 \quad \text{—— (2)}$$

$$\sum y_n \cdot X_n^2 = C_0 \sum X_n^2 + C_1 \sum X_n^3 + C_2 \sum X_n^4 \quad \text{—— (3)}$$

For Second order polynomial equ. $m = 2$

Now; by three equs., we can find three constants (C_0, C_1, C_2).

Also, we must find the value of :-

$\sum y_n$, $\sum X_n$, $\sum X_n^2$, $\sum y_n \cdot X_n$, $\sum X_n^3$, $\sum y_n \cdot X_n^2$, $\sum X_n^4$

* Accuracy of solution :-

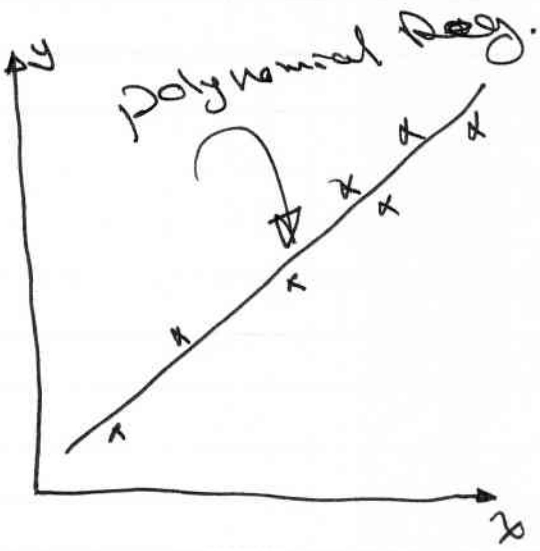
It's same for linear equ. (P.2) except the sum of value square linear equ. will become :-

$$S_r = \sum_{n=1}^{i} (y_n - C_0 - C_1 X_n - C_2 X_n^2)^2$$

$$S_t = \sum_{n=1}^{i} (y_n - y')^2$$

$$y' = \frac{\sum y_n}{n}$$

$$R^2 = \frac{S_t - S_r}{S_t}$$



Ex.5: Fit a second order polynomial for the following:

X	0	1	2	3	4	5
y	2.1	7.7	13.6	27.2	40.9	61.1

& find "R^2" H.W.

Sol: a second-order polynomial equ. is:-

$y = C_0 + C_1 x + C_2 x^2$, $n = 6$

$m = 2$

n	X_n	y_n	X_n^2	X_n^3	X_n^4	$X_n \cdot y_n$	$X_n^2 \cdot y_n$
1	0	2.1	0	0	0	0	0
2	1	7.7	1	1	1	7.7	7.7
3	2	13.6	4	8	16	27.2	54.4
4	3	27.2	9	27	81	81.6	244.8
5	4	40.9	16	64	256	163.6	654.4
6	5	61.1	25	125	625	305.5	1527.5
Σ	15	152.6	55	225	979	585.6	2488.8

$\Sigma y_n = C_0 + C_1 \Sigma x_n + C_2 \Sigma x_n^2 \rightarrow 152.6 = C_0 + 15 C_1 + 55 C_2$ —(1)

$\Sigma y_n \cdot x_n = C_0 \Sigma x_n + C_1 \Sigma x_n^2 + C_2 \Sigma x_n^3 \rightarrow 585.6 = 15 C_0 + 55 C_1 + 225 C_2$ —(2)

$\Sigma y_n \cdot x_n^2 = C_0 \Sigma x_n^2 + C_1 \Sigma x_n^3 + C_2 \Sigma x_n^4 \rightarrow 2488.8 = 55 C_0 + 225 C_1 + 979 C_2$ —(3)

Now, make a matrix to solve this three equ. to find C_0, C_1, C_2 →

$C_0 = 2.4785$, $C_1 = 2.3592$, $C_2 = 1.8607$

$\therefore$ [$y = 2.4785 + 2.3592\, x + 1.8607\, x^2$] polynomial Equ.