First year/ 2nd Semester - 2018-2019- Chemical and Petroleum Engineering Department

By

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Lecture –Nine

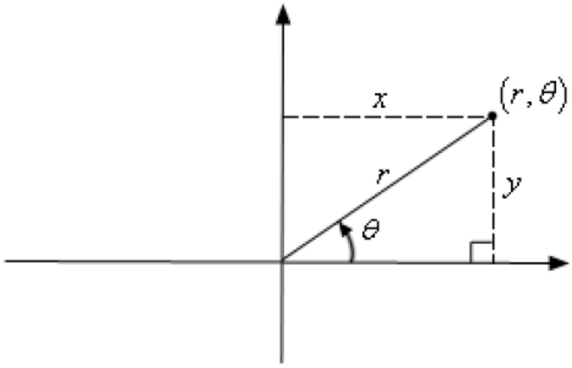
***Polar Coordinates* Technique**

Polar Coordinates Technique →

- *Polar Coordinates*
- Common Polar Coordinate Graphs.
- *Tangents with Polar Coordinates.*

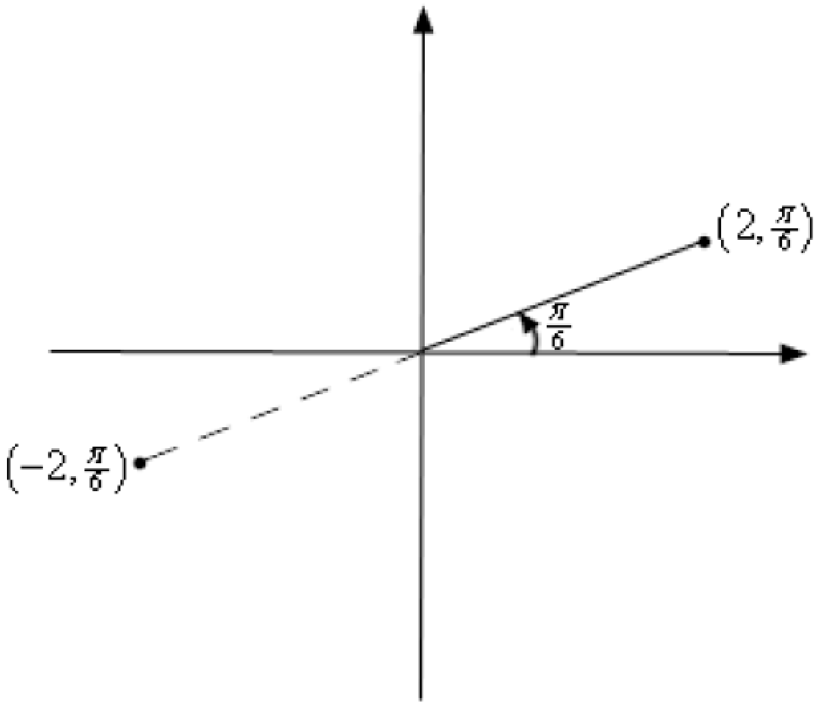
1- Polar Coordinates

This is not, however, the only way to define a point in two dimensional space. Instead of moving vertically and horizontally from the origin to get to the point we could instead go straight out of the origin until we hit the point and then determine the angle this line makes with the positive x-axis. We could then use the distance of the point from the origin and the amount we needed to rotate from the positive x-axis as the coordinates of the point. This is shown in the sketch below.



Coordinates in this form are called **polar coordinates**.

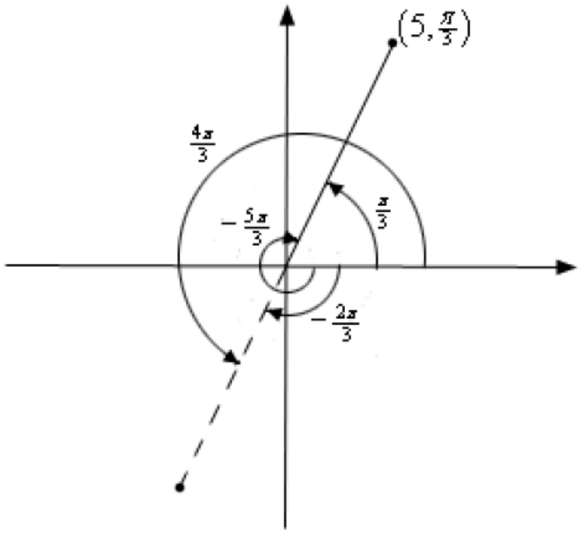
The above discussion may lead one to think that r must be a positive number. However, we also allow r to be negative. Below is a sketch of the two points $\left(2, \frac{\pi}{6}\right)$ and $\left(-2, \frac{\pi}{6}\right)$.



This leads to an important difference between Cartesian coordinates and polar coordinates. In Cartesian coordinates there is exactly one set of coordinates for any given point. With polar coordinates this isn't true. In polar coordinates there is literally an infinite number of coordinates for a given point. For instance, the following four points are all coordinates for the same point.

$$\left(5, \frac{\pi}{3}\right) = \left(5, -\frac{5\pi}{3}\right) = \left(-5, \frac{4\pi}{3}\right) = \left(-5, -\frac{2\pi}{3}\right)$$

Here is a sketch of the angles used in these four sets of coordinates.



These four points only represent the coordinates of the point without rotating around the system more than once. If we allow the angle to make as many complete rotations about the axis system as we want then there are an infinite number of coordinates for the same point. In fact the point (r, θ) can be represented by any of the following coordinate pairs.

$$(r, \theta + 2\pi n) \qquad (-r, \theta + (2n+1)\pi), \qquad \text{where } n \text{ is any integer.}$$

Polar to Cartesian Conversion Formulas

$x = r\cos\theta$	$y = r\sin\theta$

Converting from Cartesian is almost as easy. Let's first notice the following.

$$\begin{aligned} x^2 + y^2 &= (r\cos\theta)^2 + (r\sin\theta)^2 \\ &= r^2\cos^2\theta + r^2\sin^2\theta \\ &= r^2\left(\cos^2\theta + \sin^2\theta\right) = r^2 \end{aligned}$$

This is a very useful formula that we should remember, however we are after an equation for r so let's take the square root of both sides. This gives,

$$r = \sqrt{x^2 + y^2}$$

Note that technically we should have a plus or minus in front of the root since we know that r can be either positive or negative. We will run with the convention of positive r here.

Getting an equation for θ is almost as simple. We'll start with,

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

Taking the inverse tangent of both sides gives,

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

We will need to be careful with this because inverse tangents only return values in the range $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Recall that there is a second possible angle and that the second angle is given by $\theta + \pi$.

Summarizing then gives the following formulas for converting from Cartesian coordinates to polar coordinates.

Cartesian to Polar Conversion Formulas

$$r^2 = x^2 + y^2 \qquad\qquad r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Example 1 Convert each of the following points into the given coordinate system.

(a) $\left(-4, \frac{2\pi}{3}\right)$ into Cartesian coordinates. [Solution]

(b) (-1,-1) into polar coordinates. [Solution]

Solution

(a) Convert $\left(-4, \frac{2\pi}{3}\right)$ into Cartesian coordinates.

This conversion is easy enough. All we need to do is plug the points into the formulas.

$$x = -4\cos\left(\frac{2\pi}{3}\right) = -4\left(-\frac{1}{2}\right) = 2$$

$$y = -4\sin\left(\frac{2\pi}{3}\right) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

So, in Cartesian coordinates this point is $\left(2, -2\sqrt{3}\right)$.

(b) Convert (-1,-1) **into polar coordinates.**

Let's first get r.

$$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

Now, let's get θ.

$$\theta = \tan^{-1}\left(\frac{-1}{-1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

This is not the correct angle however. This value of θ is in the first quadrant and the point we've been given is in the third quadrant. As noted above we can get the correct angle by adding π onto this. Therefore, the actual angle is,

$$\theta = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

So, in polar coordinates the point is $\left(\sqrt{2}, \frac{5\pi}{4}\right)$. Note as well that we could have used the first θ that we got by using a negative r. In this case the point could also be written in polar coordinates as $\left(-\sqrt{2}, \frac{\pi}{4}\right)$.

Example 2 Convert each of the following into an equation in the given coordinate system.

(a) Convert $2x - 5x^3 = 1 + xy$ into polar coordinates. [Solution]

(b) Convert $r = -8\cos\theta$ into Cartesian coordinates. [Solution]

Solution

(a) Convert $2x - 5x^3 = 1 + xy$ **into polar coordinates.**

In this case there really isn't much to do other than plugging in the formulas for x and y (*i.e.* the Cartesian coordinates) in terms of r and θ (*i.e.* the polar coordinates).

$$2(r\cos\theta) - 5(r\cos\theta)^3 = 1 + (r\cos\theta)(r\sin\theta)$$

$$2r\cos\theta - 5r^3\cos^3\theta = 1 + r^2\cos\theta\sin\theta$$

(b) Convert $r = -8\cos\theta$ **into Cartesian coordinates.**

This one is a little trickier, but not by much. First notice that we could substitute straight for the r. However, there is no straight substitution for the cosine that will give us only Cartesian coordinates. If we had an r on the right along with the cosine then we could do a direct substitution. So, if an r on the right side would be convenient let's put one there, just don't forget to put one on the left side as well.

$$r^2 = -8r\cos\theta$$

We can now make some substitutions that will convert this into Cartesian coordinates.

$$x^2 + y^2 = -8x$$

2- Common Polar Coordinate Graphs.

Let's identify a few of the more common graphs in polar coordinates. We'll also take a look at a couple of special polar graphs.

Lines

Some lines have fairly simple equations in polar coordinates.

1. $\theta = \beta$.

We can see that this is a line by converting to Cartesian coordinates as follows

$$\theta = \beta$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \beta$$

$$\frac{y}{x} = \tan\beta$$

$$y = (\tan\beta)x$$

This is a line that goes through the origin and makes an angle of β with the positive x-axis. Or, in other words it is a line through the origin with slope of $\tan\beta$.

2. $r\cos\theta = a$

This is easy enough to convert to Cartesian coordinates to $x = a$. So, this is a vertical line.

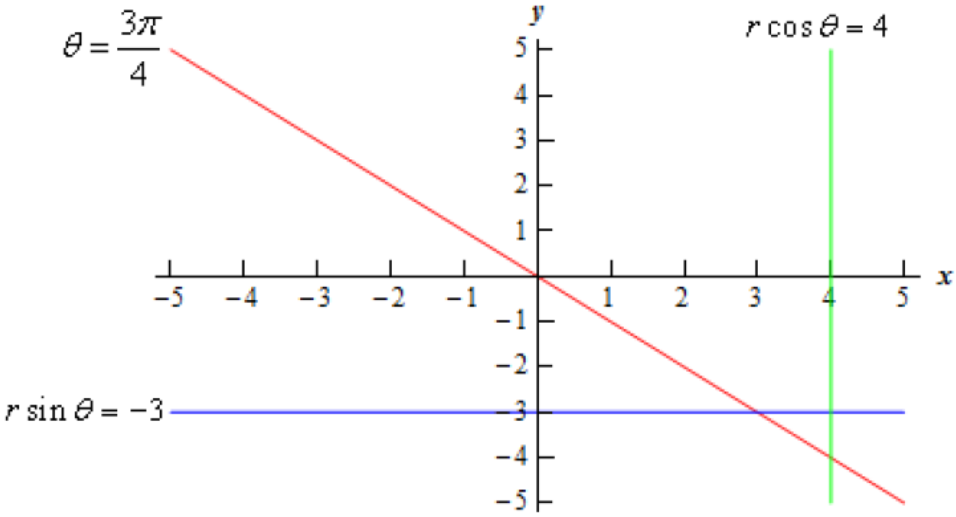
3. $r\sin\theta = b$

Likewise, this converts to $y = b$ and so is a horizontal line.

Example 3 Graph $\theta = \frac{3\pi}{4}$, $r\cos\theta = 4$ and $r\sin\theta = -3$ on the same axis system.

Solution

There really isn't too much to this one other than doing the graph so here it is.



Circles

Let's take a look at the equations of circles in polar coordinates.

1. $r = a$.

This equation is saying that no matter what angle we've got the distance from the origin must be a. If you think about it that is exactly the definition of a circle of radius a centered at the origin.

So, this is a circle of radius a centered at the origin. This is also one of the reasons why we might want to work in polar coordinates. The equation of a circle centered at the origin has a very nice equation, unlike the corresponding equation in Cartesian coordinates.

2. $r = 2a\cos\theta$.

We looked at a specific example of one of these when we were converting equations to Cartesian coordinates.

This is a circle of radius $|a|$ and center $(a,0)$. Note that a might be negative (as it was in our example above) and so the absolute value bars are required on the radius. They should not be used however on the center.

3. $r = 2b\sin\theta$.

This is similar to the previous one. It is a circle of radius $|b|$ and center $(0,b)$.

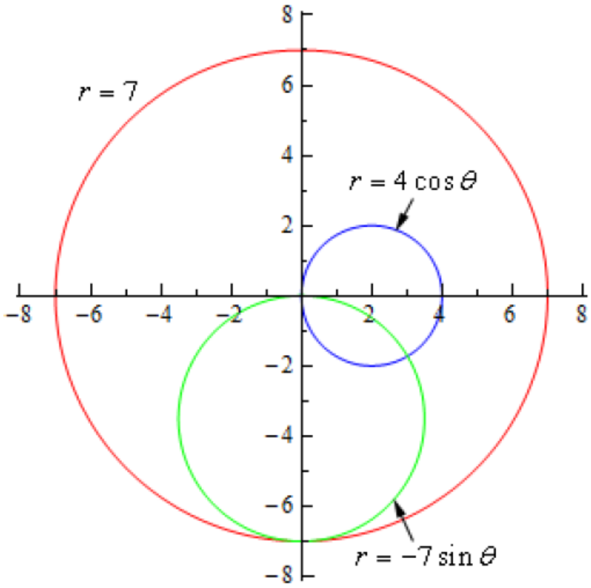
4. $r = 2a\cos\theta + 2b\sin\theta$.

This is a combination of the previous two and by completing the square twice it can be shown that this is a circle of radius $\sqrt{a^2+b^2}$ and center (a,b). In other words, this is the general equation of a circle that isn't centered at the origin.

Example 4 Graph $r=7$, $r=4\cos\theta$, and $r=-7\sin\theta$ on the same axis system.

Solution

The first one is a circle of radius 7 centered at the origin. The second is a circle of radius 2 centered at (2,0). The third is a circle of radius $\frac{7}{2}$ centered at $\left(0,-\frac{7}{2}\right)$. Here is the graph of the three equations.



Cardioids and Limacons

These can be broken up into the following three cases.

1. Cardioids : $r=a\pm a\cos\theta$ and $r=a\pm a\sin\theta$.
   These have a graph that is vaguely heart shaped and always contain the origin.

2. Limacons with an inner loop : $r=a\pm b\cos\theta$ and $r=a\pm b\sin\theta$ with $a<b$.
   These will have an inner loop and will always contain the origin.

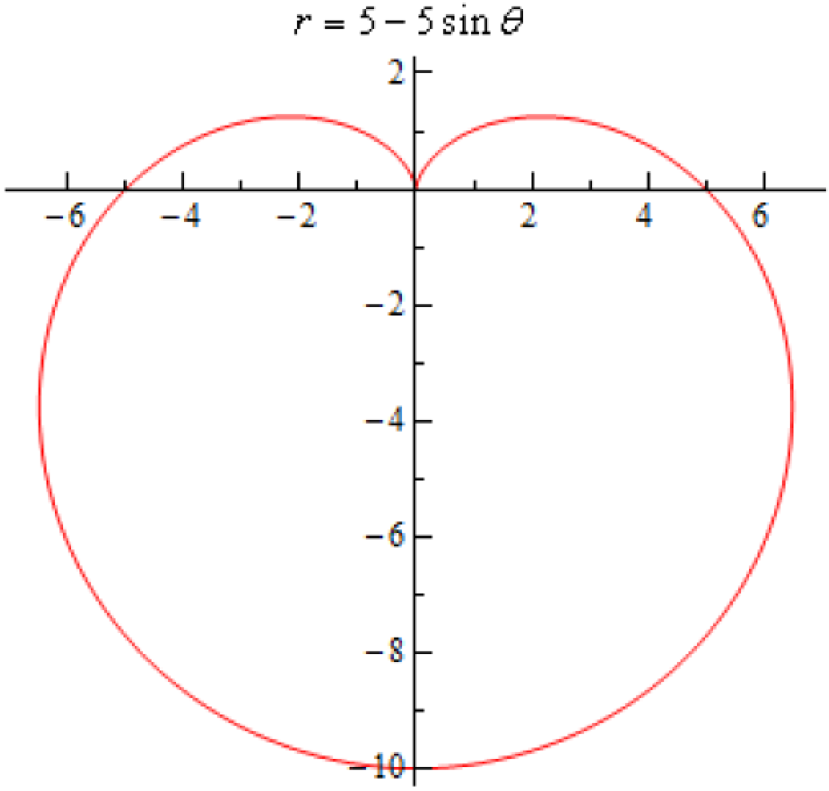
3. Limacons without an inner loop : $r=a\pm b\cos\theta$ and $r=a\pm b\sin\theta$ with $a>b$.
   These do not have an inner loop and do not contain the origin.

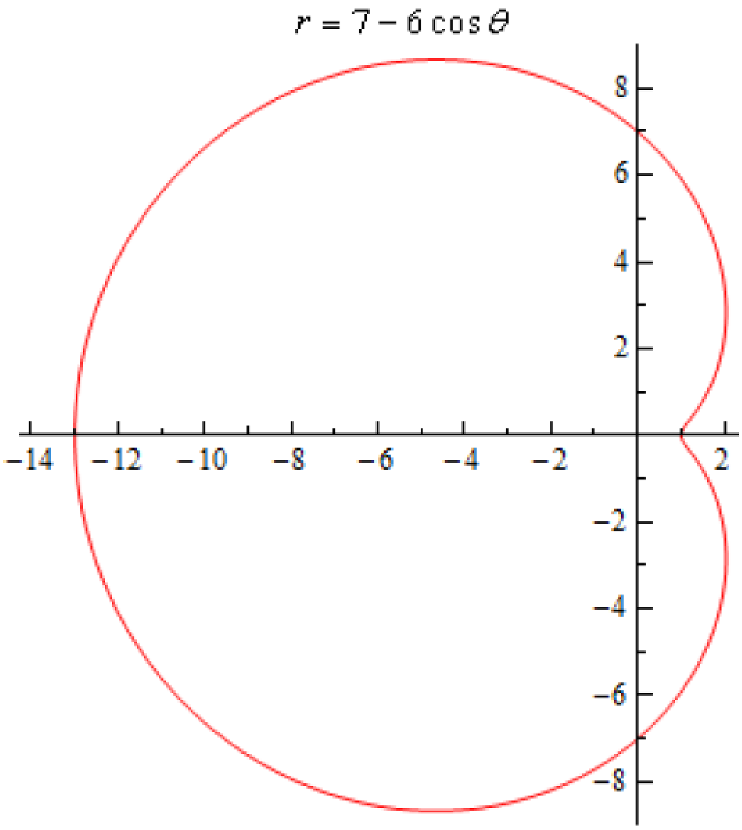
Example 5 Graph $r=5-5\sin\theta$, $r=7-6\cos\theta$, and $r=2+4\cos\theta$.

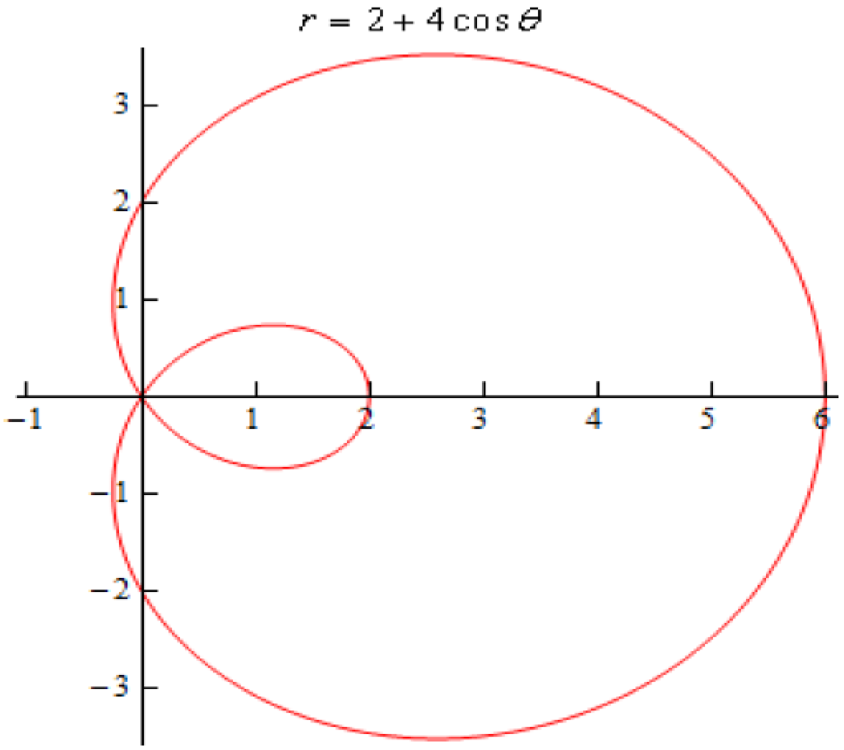
Solution

These will all graph out once in the range $0\le\theta\le2\pi$. Here is a table of values for each followed by graphs of each.

θ	$r=5-5\sin\theta$	$r=7-6\cos\theta$	$r=2+4\cos\theta$
0	5	1	6
$\frac{\pi}{2}$	0	7	2
π	5	13	-2
$\frac{3\pi}{2}$	10	7	2
2π	5	1	6


$r = 5 - 5\sin\theta$
2
−6
−4
−2
2
4
6
−2
−4
−6
−8
−10


$r = 7 - 6\cos\theta$
8
6
4
2
−14
−12
−10
−8
−6
−4
−2
2
−2
−4
−6
−8



3- *Tangents with Polar Coordinates.*

We now need to discuss some calculus topics in terms of polar coordinates.

We will start with finding tangent lines to polar curves. In this case we are going to assume that the equation is in the form $r = f(\theta)$. With the equation in this form we can actually use the equation for the derivative $\frac{dy}{dx}$ we derived when we looked at tangent lines with parametric equations. To do this however requires us to come up with a set of parametric equations to represent the curve. This is actually pretty easy to do.

From our work in the previous section we have the following set of conversion equations for going from polar coordinates to Cartesian coordinates.

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

Now, we'll use the fact that we're assuming that the equation is in the form $r = f(\theta)$. Substituting this into these equations gives the following set of parametric equations (with θ as the parameter) for the curve.

$$x = f(\theta)\cos\theta \qquad\qquad y = f(\theta)\sin\theta$$

Now, we will need the following derivatives.

$$\begin{aligned}\frac{dx}{d\theta} &= f'(\theta)\cos\theta - f(\theta)\sin\theta \\ &= \frac{dr}{d\theta}\cos\theta - r\sin\theta\end{aligned} \qquad\qquad \begin{aligned}\frac{dy}{d\theta} &= f'(\theta)\sin\theta + f(\theta)\cos\theta \\ &= \frac{dr}{d\theta}\sin\theta + r\cos\theta\end{aligned}$$

The derivative $\frac{dy}{dx}$ is then,

Derivative with Polar Coordinates

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Example 1 Determine the equation of the tangent line to $r = 3 + 8\sin\theta$ at $\theta = \frac{\pi}{6}$.

Solution
We'll first need the following derivative.

$$\frac{dr}{d\theta} = 8\cos\theta$$

The formula for the derivative $\frac{dy}{dx}$ becomes,

$$\frac{dy}{dx} = \frac{8\cos\theta\sin\theta + (3 + 8\sin\theta)\cos\theta}{8\cos^2\theta - (3 + 8\sin\theta)\sin\theta} = \frac{16\cos\theta\sin\theta + 3\cos\theta}{8\cos^2\theta - 3\sin\theta - 8\sin^2\theta}$$

The slope of the tangent line is,

$$m = \left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{6}} = \frac{4\sqrt{3} + \frac{3\sqrt{3}}{2}}{4 - \frac{3}{2}} = \frac{11\sqrt{3}}{5}$$

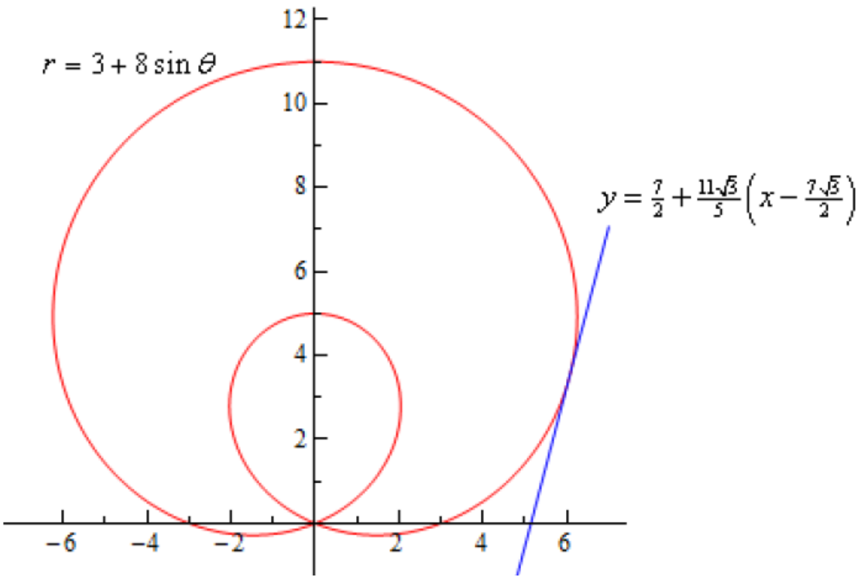
Now, at $\theta = \frac{\pi}{6}$ we have $r = 7$. We'll need to get the corresponding x-y coordinates so we can get the tangent line.

$$x = 7\cos\left(\frac{\pi}{6}\right) = \frac{7\sqrt{3}}{2} \qquad\qquad y = 7\sin\left(\frac{\pi}{6}\right) = \frac{7}{2}$$

The tangent line is then,

$$y = \frac{7}{2} + \frac{11\sqrt{3}}{5}\left(x - \frac{7\sqrt{3}}{2}\right)$$

For the sake of completeness here is a graph of the curve and the tangent line.



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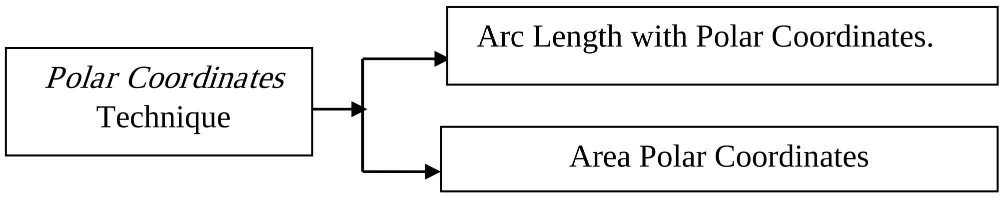
Lecture – Ten

***Polar Coordinates* Technique**



4- *Arc Length with Polar Coordinates.*

In this section we'll look at the arc length of the curve given by,

$$r = f(\theta) \qquad \alpha \le \theta \le \beta$$

where we also assume that the curve is traced out exactly once. Just as we did with the tangent lines in polar coordinates we'll first write the curve in terms of a set of parametric equations,

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= f(\theta)\cos\theta & &= f(\theta)\sin\theta \end{aligned}$$

and we can now use the parametric formula for finding the arc length.

We'll need the following derivatives for these computations.

$$\begin{aligned} \frac{dx}{d\theta} &= f'(\theta)\cos\theta - f(\theta)\sin\theta & \frac{dy}{d\theta} &= f'(\theta)\sin\theta + f(\theta)\cos\theta \\ &= \frac{dr}{d\theta}\cos\theta - r\sin\theta & &= \frac{dr}{d\theta}\sin\theta + r\cos\theta \end{aligned}$$

We'll need the following for our ds.

$$\begin{aligned} \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 &= \left(\frac{dr}{d\theta}\cos\theta - r\sin\theta\right)^2 + \left(\frac{dr}{d\theta}\sin\theta + r\cos\theta\right)^2 \\ &= \left(\frac{dr}{d\theta}\right)^2 \cos^2\theta - 2r\frac{dr}{d\theta}\cos\theta\sin\theta + r^2\sin^2\theta \\ &\qquad + \left(\frac{dr}{d\theta}\right)^2 \sin^2\theta + 2r\frac{dr}{d\theta}\cos\theta\sin\theta + r^2\cos^2\theta \\ &= \left(\frac{dr}{d\theta}\right)^2 \left(\cos^2\theta + \sin^2\theta\right) + r^2\left(\cos^2\theta + \sin^2\theta\right) \\ &= r^2 + \left(\frac{dr}{d\theta}\right)^2 \end{aligned}$$

The arc length formula for polar coordinates is then,

$$L = \int ds$$

where,

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Example 1 Determine the length of $r = \theta \quad 0 \le \theta \le 1$.

Solution

Okay, let's just jump straight into the formula since this is a fairly simple function.

$$L = \int_0^1 \sqrt{\theta^2 + 1}\, d\theta$$

We'll need to use a trig substitution here.

$$\theta = \tan x \qquad\qquad d\theta = \sec^2 x\, dx$$

$$\theta = 0 \qquad 0 = \tan x \qquad x = 0$$

$$\theta = 1 \qquad 1 = \tan x \qquad x = \frac{\pi}{4}$$

$$\sqrt{\theta^2 + 1} = \sqrt{\tan^2 x + 1} = \sqrt{\sec^2 x} = \left|\sec x\right| = \sec x$$

The arc length is then,

$$\begin{aligned} L &= \int_0^1 \sqrt{\theta^2 + 1}\, d\theta \\ &= \int_0^{\frac{\pi}{4}} \sec^3 x\, dx \\ &= \frac{1}{2}\left(\sec x \tan x + \ln\left|\sec x + \tan x\right|\right)\Big|_0^{\frac{\pi}{4}} \\ &= \frac{1}{2}\left(\sqrt{2} + \ln\left(1 + \sqrt{2}\right)\right) \end{aligned}$$

5- ***Area Polar Coordinates.***

The equation of a curve in polar coordinates is given by $r = f(\theta)$. To find the area bounded by the curve $r = f(\theta)$, the rays $\theta = \alpha$ and $\theta = \beta$, divide the angle $\beta - \alpha$ into n-parts by defining $\Delta\theta = \frac{\beta-\alpha}{n}$ and then defining the rays

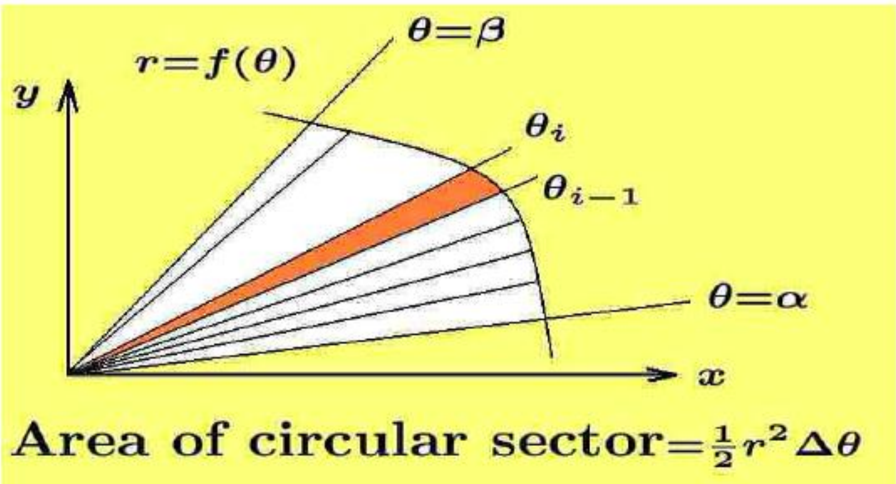
$$\theta_0 = \alpha,\ \theta_1 = \theta_0 + \Delta\theta, \ldots, \theta_i = \theta_{i-1} + \Delta\theta, \ldots, \theta_n = \theta_{n-1} + \Delta\theta = \beta$$

The area between the rays $\theta = \theta_{i-1}$, $\theta = \theta_i$ and the curve $r = f(\theta)$, illustrated in the figure below, is approximated by a circular sector with area element

$$dA_i = \frac{1}{2} r_i^2 \Delta\theta_i = \frac{1}{2} f^2(\theta_i)\,\Delta\theta_i$$

where $\Delta\theta_i = \theta_i - \theta_{i-1}$ and $r_i = f(\theta_i)$. A summation of these elements of area between the rays $\theta = \alpha$ and $\theta = \beta$ gives the approximate area

$$\sum_{i=1}^{n} dA_i = \sum_{i=1}^{n} \frac{1}{2} r_i^2 \Delta\theta_i = \sum_{i=1}^{n} \frac{1}{2} f^2(\theta_i)\,\Delta\theta_i$$



Approximation of area by summation of circular sectors.

This approximation gets better as $\Delta\theta_i$ gets smaller. Using the fundamental theorem of integral calculus, it can be shown that in the limit as $n \to \infty$, the equation defines the element of area $dA = \frac{1}{2} r^2\, d\theta$. A summation of these elements of area gives

$$\text{Polar Area} = \int_{\alpha}^{\beta} dA = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta = \frac{1}{2}\int_{\alpha}^{\beta} f^2(\theta)\, d\theta$$

Example 1. Find the area bounded by the polar curve

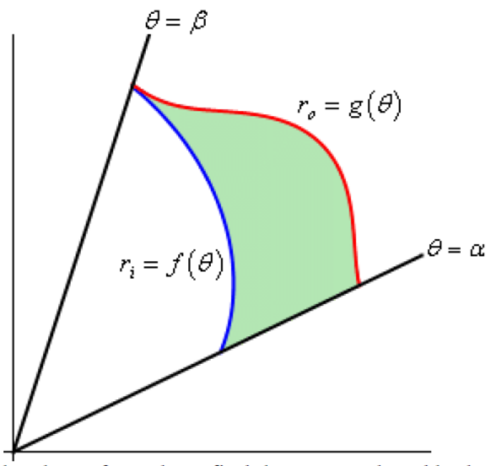
$$r = 2r_0 \cos\theta \quad \text{for } 0 \le \theta \le \pi.$$

Solution

One finds that the polar curve $r = 2r_0 \cos\theta$, for $0 \le \theta \le \pi$, is a circle of radius r_0 which has its center at the point $(r_0, 0)$ in polar coordinates. Using the area formula given by equation (3.121) one obtains

$$\text{Area} = \frac{1}{2}\int_0^\pi (2r_0\cos\theta)^2\, d\theta = 2r_0^2 \int_0^\pi \cos^2\theta\, d\theta = r_0^2 \int_0^\pi (\cos 2\theta + 1)\, d\theta = r_0^2 \left[\frac{\sin 2\theta}{2} + \theta\right]_0^\pi = \pi r_0^2$$

So, that's how we determine areas that are enclosed by a single curve, but what about situations like the following sketch were we want to find the area between two curves.



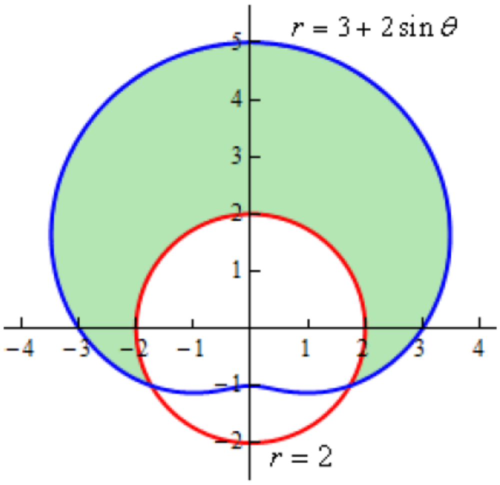
In this case we can use the above formula to find the area enclosed by both and then the actual area is the difference between the two. The formula for this is,

$$A = \int_\alpha^\beta \frac{1}{2}\left(r_o^2 - r_i^2\right) d\theta$$

Example 2 Determine the area that lies inside $r = 3 + 2\sin\theta$ and outside $r = 2$.

Solution

Here is a sketch of the region that we are after.

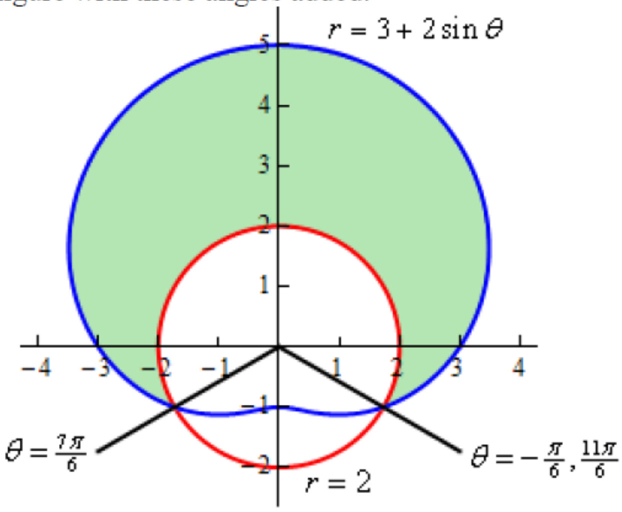


To determine this area we'll need to know the values of θ for which the two curves intersect. We can determine these points by setting the two equations and solving.

$$3+2\sin\theta=2$$

$$\sin\theta=-\frac{1}{2} \qquad \Rightarrow \qquad \theta=\frac{7\pi}{6},\frac{11\pi}{6}$$

Here is a sketch of the figure with these angles added.



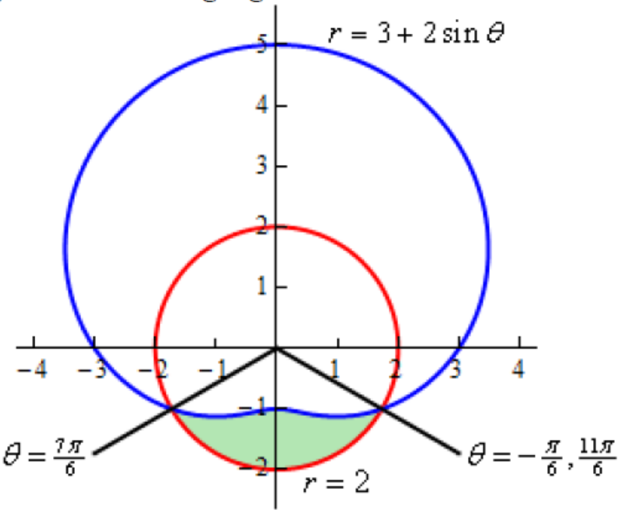
Note as well here that we also acknowledged that another representation for the angle $\frac{11\pi}{6}$ is $-\frac{\pi}{6}$. This is important for this problem. In order to use the formula above the area must be enclosed as we increase from the smaller to larger angle. So, if we use $\frac{7\pi}{6}$ to $\frac{11\pi}{6}$ we will not enclose the shaded area, instead we will enclose the bottom most of the three regions. However if we use the angles $-\frac{\pi}{6}$ to $\frac{7\pi}{6}$ we will enclose the area that we're after.

So, the area is then,

$$\begin{aligned}
A &= \int_{-\frac{\pi}{6}}^{\frac{7\pi}{6}} \frac{1}{2}\left((3+2\sin\theta)^2-(2)^2\right)d\theta \\
&= \int_{-\frac{\pi}{6}}^{\frac{7\pi}{6}} \frac{1}{2}\left(5+12\sin\theta+4\sin^2\theta\right)d\theta \\
&= \int_{-\frac{\pi}{6}}^{\frac{7\pi}{6}} \frac{1}{2}\left(7+12\sin\theta-2\cos(2\theta)\right)d\theta \\
&= \frac{1}{2}\left(7\theta-12\cos\theta-\sin(2\theta)\right)\Big|_{-\frac{\pi}{6}}^{\frac{7\pi}{6}} \\
&= \frac{11\sqrt{3}}{2}+\frac{14\pi}{3}=24.187
\end{aligned}$$

Example 3 Determine the area of the region outside $r = 3 + 2\sin\theta$ and inside $r = 2$.

Solution
This time we're looking for the following region.

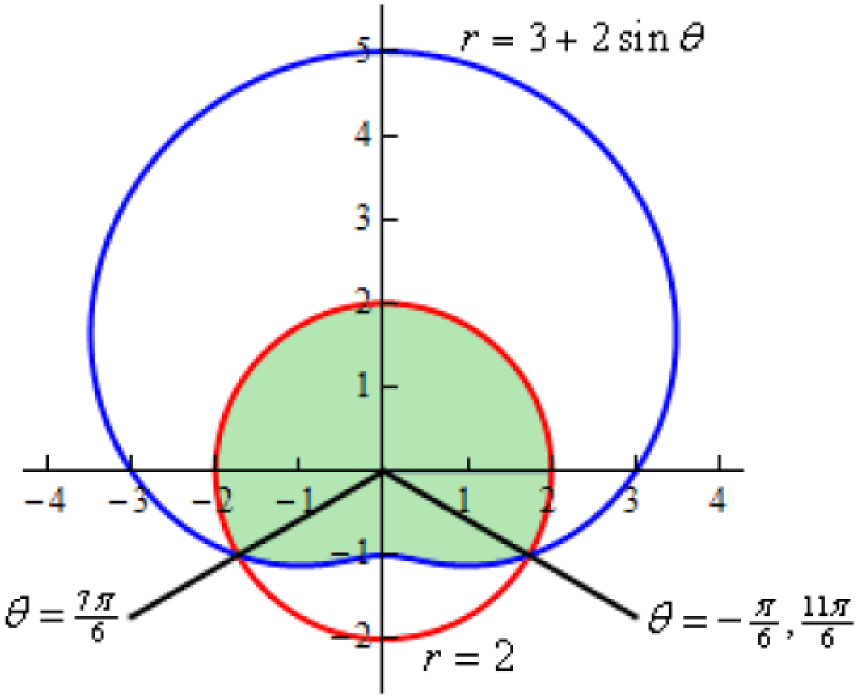


So, this is the region that we get by using the limits $\frac{7\pi}{6}$ to $\frac{11\pi}{6}$. The area for this region is,

$$
\begin{aligned}
A &= \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}\left((2)^2 - (3 + 2\sin\theta)^2\right) d\theta \\
&= \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}\left(-5 - 12\sin\theta - 4\sin^2\theta\right) d\theta \\
&= \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}\left(-7 - 12\sin\theta + 2\cos(2\theta)\right) d\theta \\
&= \frac{1}{2}\left(-7\theta + 12\cos\theta + \sin(2\theta)\right)\Big|_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \\
&= \frac{11\sqrt{3}}{2} - \frac{7\pi}{3} = 2.196
\end{aligned}
$$

Example 4 Determine the area that is inside both $r = 3 + 2\sin\theta$ and $r = 2$.

Solution
Here is the sketch for this example.



In this case however, that is not a major problem. There are two ways to do get the area in this problem. We'll take a look at both of them.

Solution 1
In this case let's notice that the circle is divided up into two portions and we're after the upper portion. Also notice that we found the area of the lower portion in Example 3. Therefore, the area is,

$$\begin{aligned}\text{Area} &= \text{Area of Circle} - \text{Area from Example 3}\\ &= \pi(2)^2 - 2.196\\ &= 10.370\end{aligned}$$

$$\begin{aligned}\text{Area} &= \text{Area of Limacon} - \text{Area from Example 2}\\ &= \int_0^{2\pi} \frac{1}{2}(3+2\sin\theta)^2\,d\theta - 24.187\\ &= \int_0^{2\pi} \frac{1}{2}\left(9+12\sin\theta+4\sin^2\theta\right)d\theta - 24.187\\ &= \int_0^{2\pi} \frac{1}{2}\left(11+12\sin\theta-2\cos(2\theta)\right)d\theta - 24.187\\ &= \frac{1}{2}\left(11\theta-12\cos(\theta)-\sin(2\theta)\right)\Big|_0^{2\pi} - 24.187\\ &= 11\pi - 24.187\\ &= 10.370\end{aligned}$$

First year/ 2nd Semester - 2018-2019- Chemical and Petroleum Engineering Department

By

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Lecture Eleven

Polar Coordinates Technique

Problems & Tutorials

Problems.

A- ***Polar Coordinates.***

1. For the point with polar coordinates $\left(2, \frac{\pi}{7}\right)$ determine three different sets of coordinates for the same point all of which have angles different from $\frac{\pi}{5}$ and are in the range $-2\pi \le \theta \le 2\pi$.

2. The polar coordinates of a point are $(-5, 0.23)$. Determine the Cartesian coordinates for the point.

3. The Cartesian coordinate of a point are $(2, -6)$. Determine a set of polar coordinates for the point.

4. The Cartesian coordinate of a point are $(-8, 1)$. Determine a set of polar coordinates for the point.

For problems 5 and 6 convert the given equation into an equation in terms of polar coordinates.

5. $\dfrac{4x}{3x^2 + 3y^2} = 6 - xy$

6. $x^2 = \dfrac{4x}{y} - 3y^2 + 2$

For problems 7 and 8 convert the given equation into an equation in terms of Cartesian coordinates.

7. $6r^3 \sin\theta = 4 - \cos\theta$

8. $\dfrac{2}{r} = \sin\theta - \sec\theta$

For problems 9 – 16 sketch the graph of the given polar equation.

9. $\cos\theta = \dfrac{6}{r}$

10. $\theta = -\dfrac{\pi}{3}$

11. $r = -14\cos\theta$

12. $r = 7$

13. $r = 9\sin\theta$

14. $r = 8 + 8\cos\theta$

15. $r = 5 - 2\sin\theta$

16. $r = 4 - 9\sin\theta$

B- ***Tangents with Polar Coordinates.***

1. Find the tangent line to $r = \sin(4\theta)\cos(\theta)$ at $\theta = \frac{\pi}{6}$.

2. Find the tangent line to $r = \theta - \cos(\theta)$ at $\theta = \frac{3\pi}{4}$.

C- ***Area with Polar Coordinates.***

1. Find the area inside the inner loop of $r = 3 - 8\cos\theta$.

2. Find the area inside the graph of $r = 7 + 3\cos\theta$ and to the left of the y-axis.

3. Find the area that is inside $r = 3 + 3\sin\theta$ and outside $r = 2$.

4. Find the area that is inside $r = 2$ and outside $r = 3 + 3\sin\theta$.

5. Find the area that is inside $r = 4 - 2\cos\theta$ and outside $r = 6 + 2\cos\theta$.

6. Find the area that is inside both $r = 1 - \sin\theta$ and $r = 2 + \sin\theta$.

D- ***Arc Length with Polar Coordinates.***

1. Determine the length of the following polar curve. You may assume that the curve traces out exactly once for the given range of θ.

$$r = -4\sin\theta, \ 0 \le \theta \le \pi$$

For problems 2 and 3 set up, but do not evaluate, an integral that gives the length of the given polar curve. For these problems you may assume that the curve traces out exactly once for the given range of θ.

2. $r = \theta\cos\theta, \ 0 \le \theta \le \pi$

3. $r = \cos(2\theta) + \sin(3\theta), \ 0 \le \theta \le 2\pi$