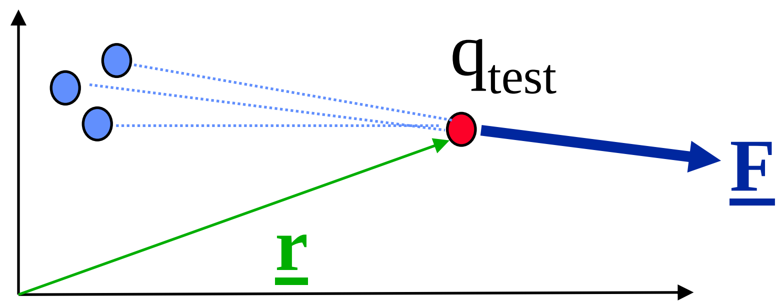
Electric and Magnetic Fields

Mohammed Qasim Taha

The Electric Field

- Group of fixed charges exert a force $\underline{F}$, given by Coulomb's law, on a test charge q_{test} at position $\underline{\mathbf{r}}$.

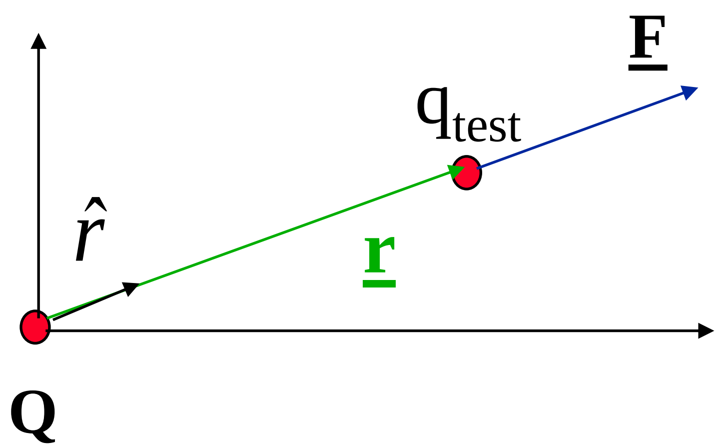


- The electric field $\underline{\mathbf{E}}$ (at a given point in space) is the **force per unit charge** that would be experienced by a test charge at that point.

$$\underline{\mathbf{E}} = \underline{\mathbf{F}} / q_{test}$$

This is a vector function of position.

Electric Field of a Point Charge



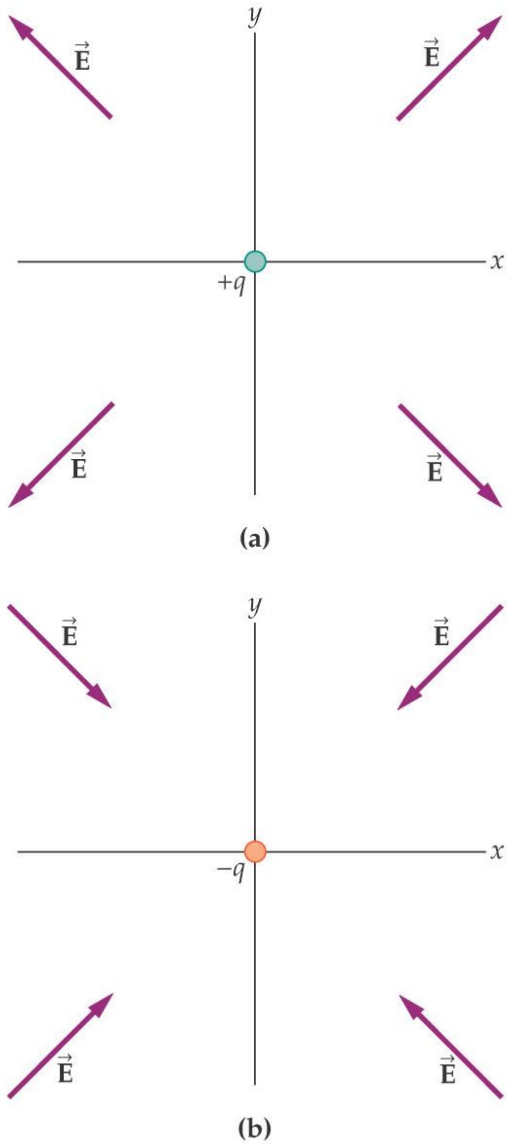
$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_{test}}{r^2}\,\hat{\mathbf{r}}$$

- Dividing out q_{test} gives the electric field at **r**:

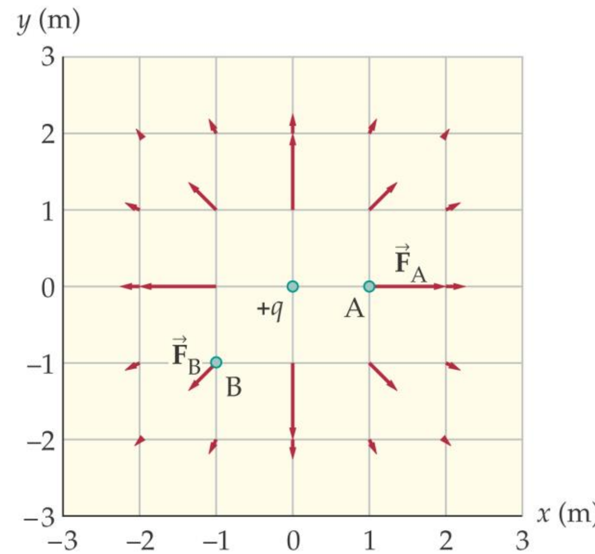
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\hat{\mathbf{r}}$$

Radially outward, falling off as $1/r^2$

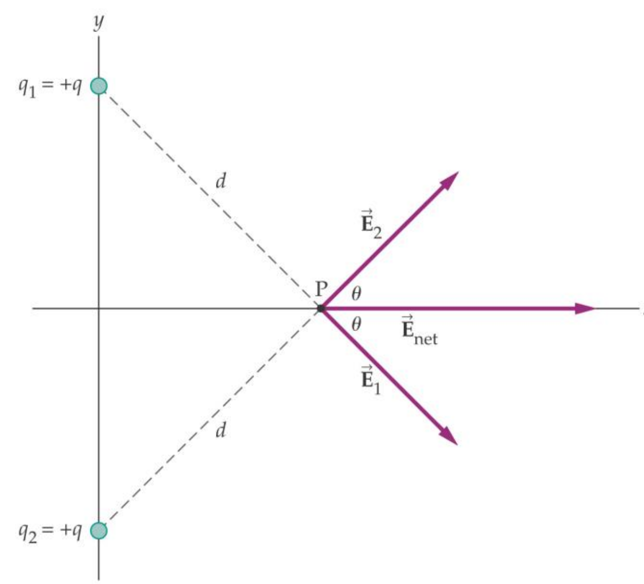
The Electric Field



Direction of the electric field



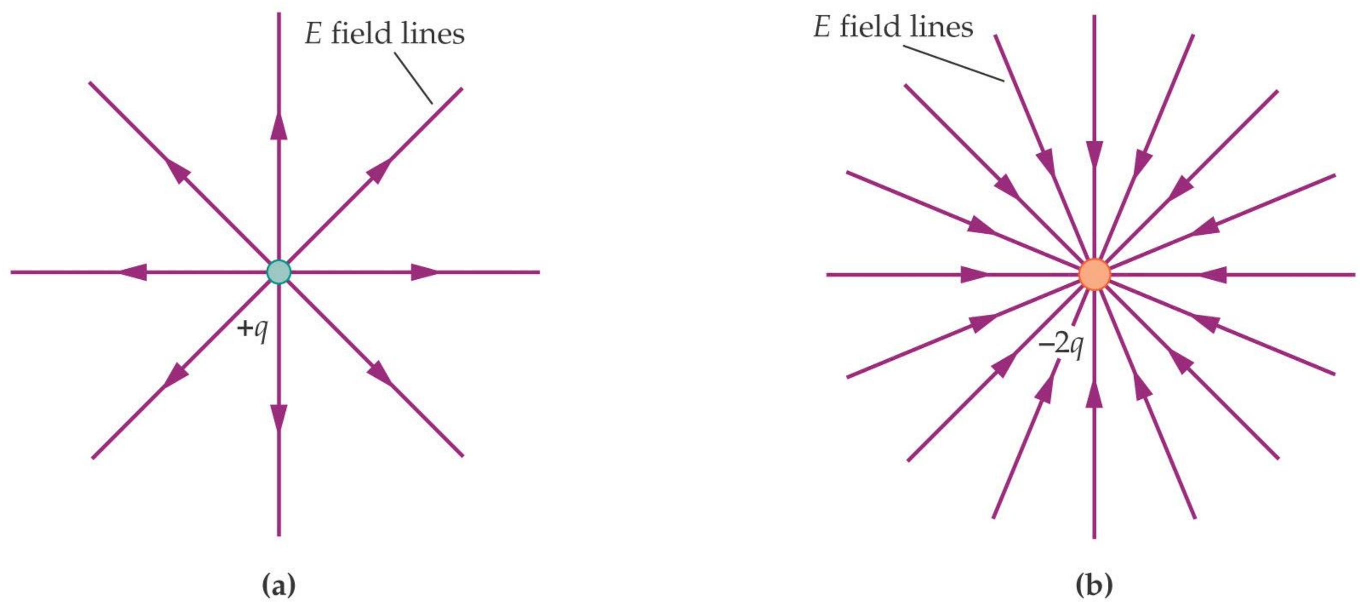
Strength of electric field



Superposition of electric field

Electric Field Lines

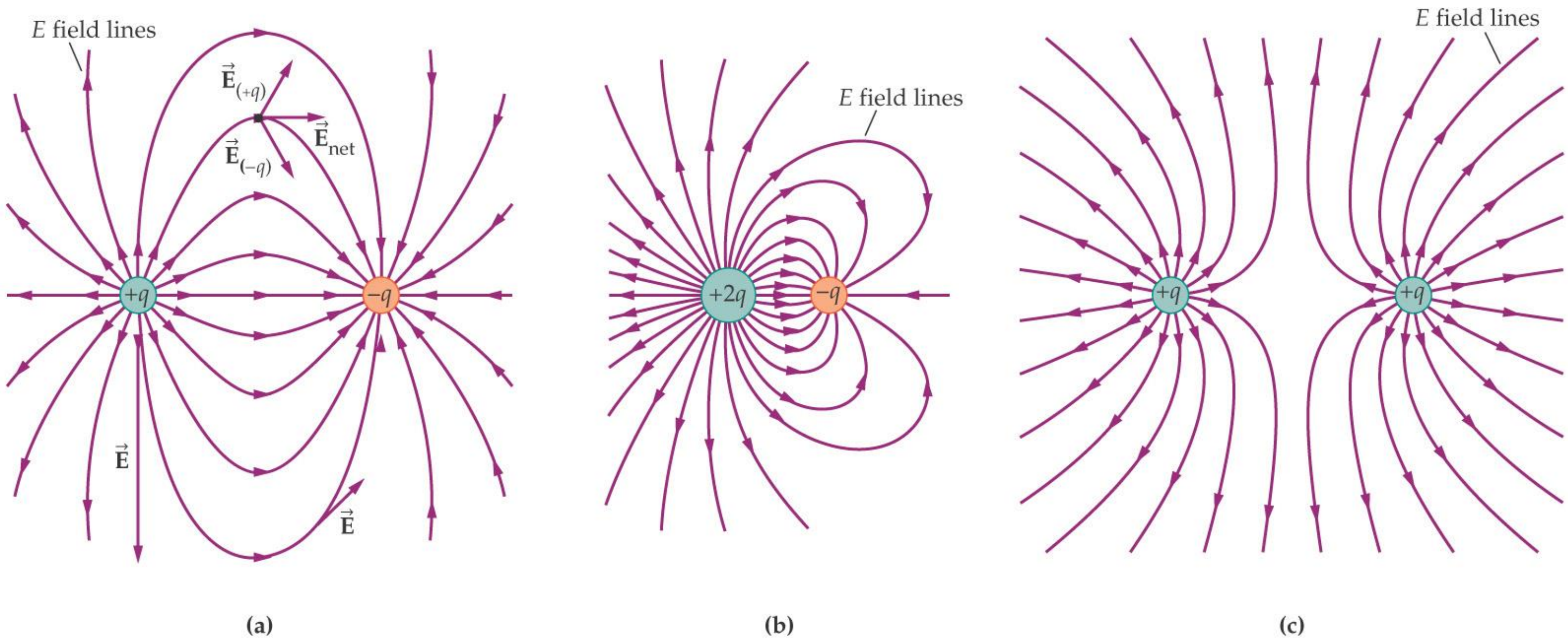
Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field



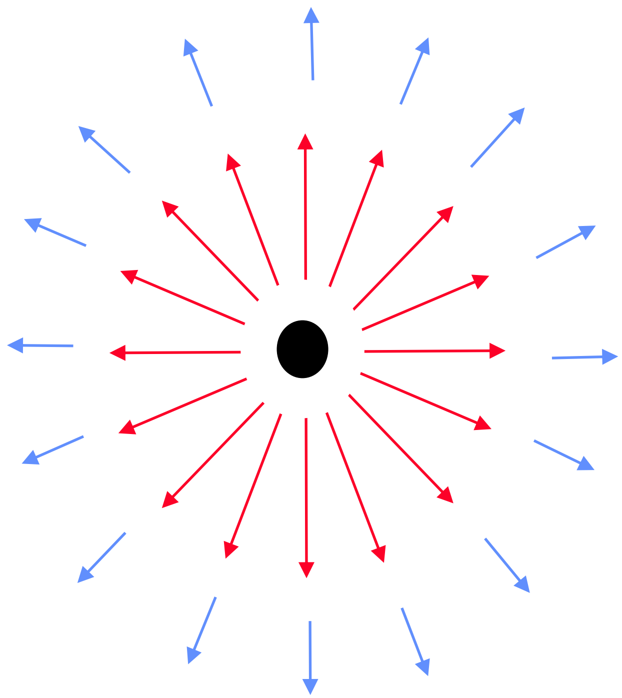
Electric field lines:

1) Point in the direction of the electric field E
2) Start at positive charges or at infinity
3) End at negative charges or at infinity
4) Are more dense where the field has greater magnitude

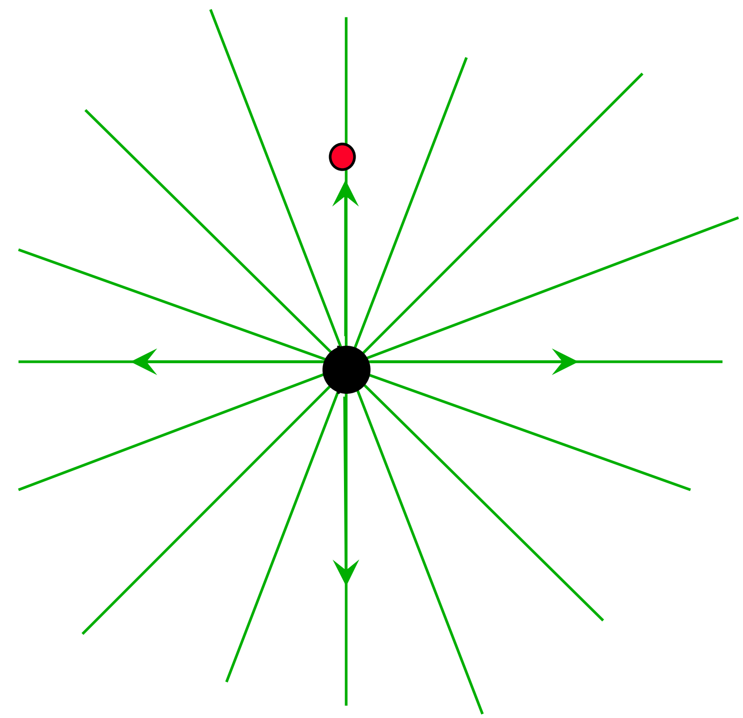
Electric Field Lines



Electric Field Lines (Point Charge)



Electric Field
(vector)



Field Lines
(Lines of force)

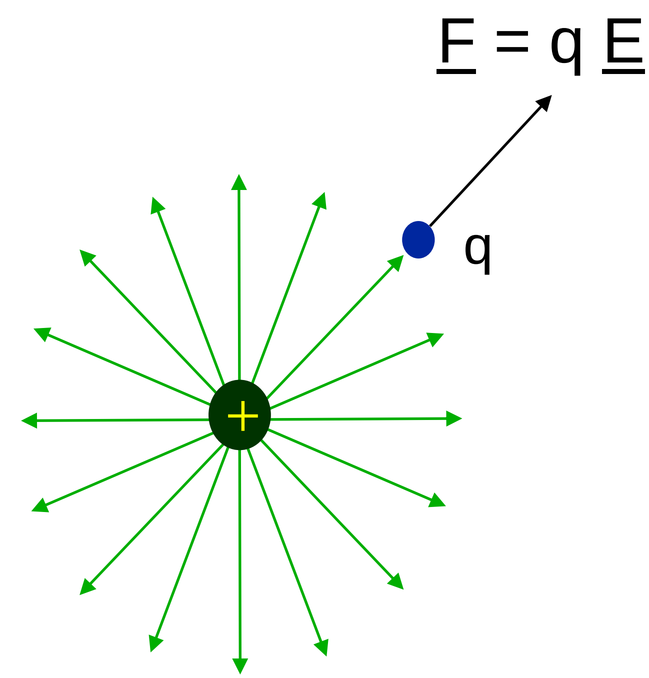
Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field

Force Due to an Electric Field

Just turn the definition of **E** around. If **E**(**r**) is known, the force **F** on a charge **q**, at point **r** is:

$$\underline{\mathbf{F}} = \mathbf{q}\,\underline{\mathbf{E}}(\underline{\mathbf{r}})$$

The electric field at **r** points in the direction that a positive charge placed at **r** would be pushed.

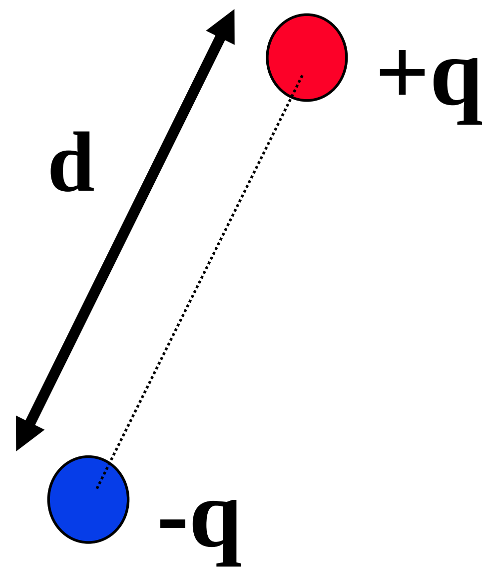


Electric field lines are bunched closer where the field is stronger.

Two point charges, + 2 μC each, are located on the x axis. One charge is at x = 1 m, and the other at x = - 1 m.

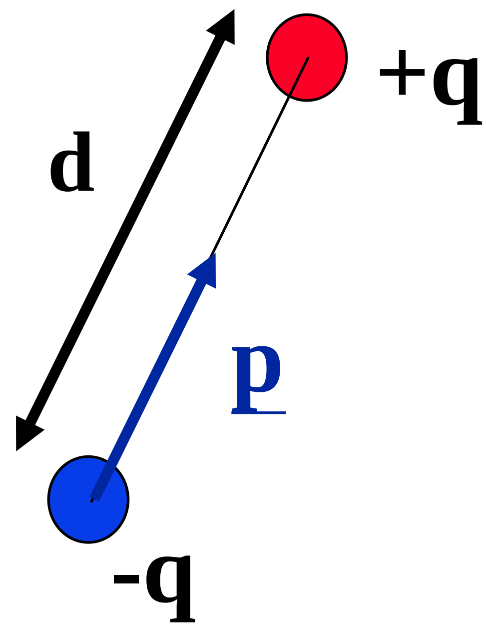
a) Calculate the electric field at the origin.
b) Calculate (and plot) the electric field along the + y axis.
c) Calculate the force exerted on a + 5 μC charge, located at an arbitrary location on the + y axis

The Electric Dipole



An electric dipole consists of two equal and opposite charges (**q** and **-q**) separated a distance **d.**

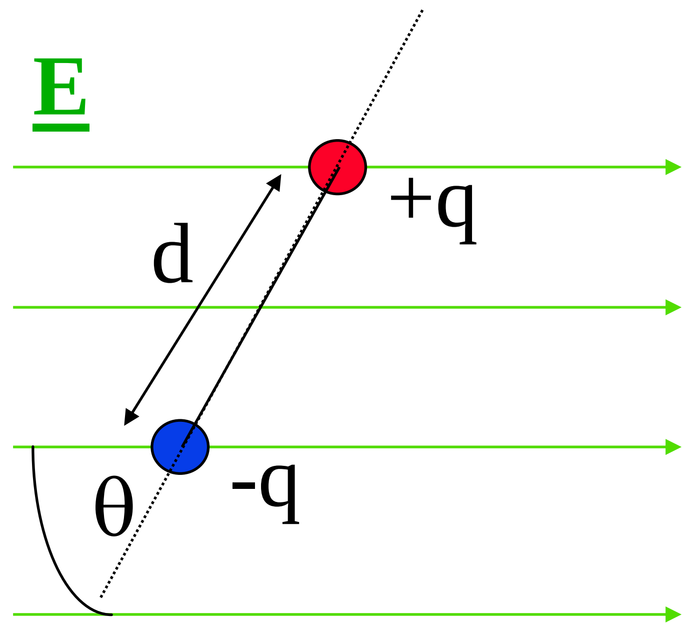
The Electric Dipole



We define the **Dipole Moment** **p**

p
- magnitude = **qd**,
- direction = from -q to +q

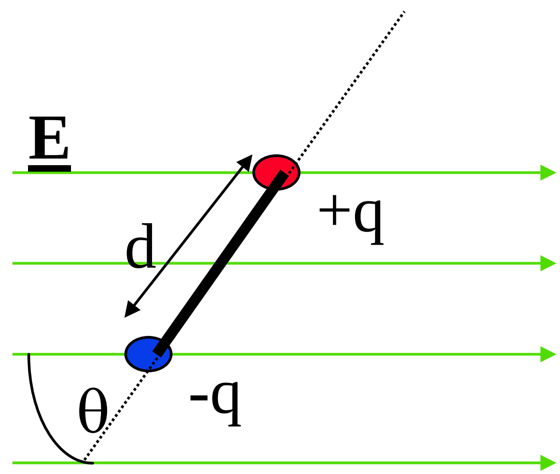
The Electric Dipole



Suppose the dipole is placed in a uniform electric field (i.e., **E** is the same everywhere in space).

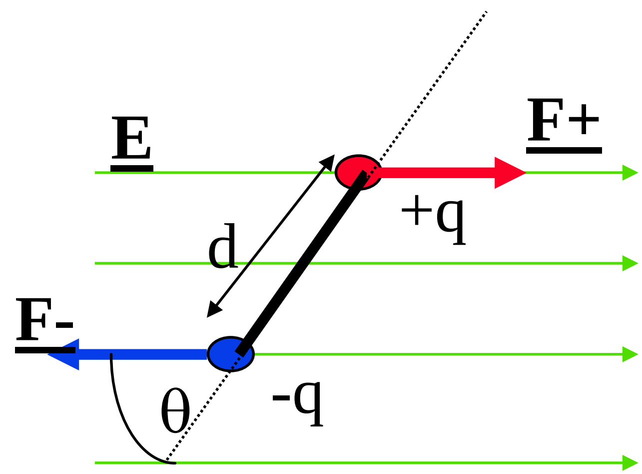
Will the dipole move ??

The Electric Dipole



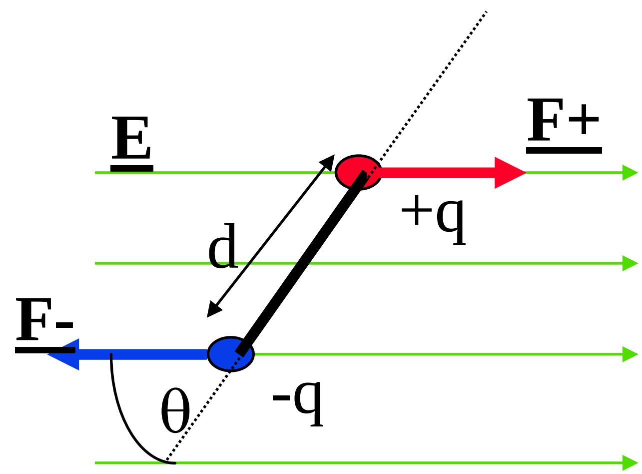
What is the total force acting on the dipole?

The Electric Dipole



What is the total force acting on the dipole?

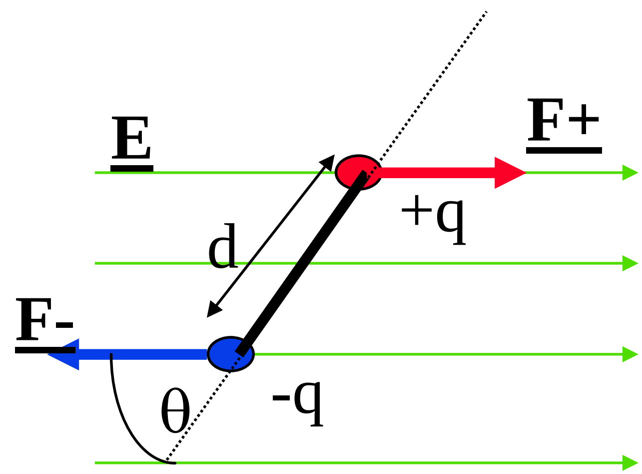
The Electric Dipole



What is the total force acting on the dipole?

Zero, because the force on the two charges cancel: both have magnitude qE. The center of mass does not accelerate.

The Electric Dipole



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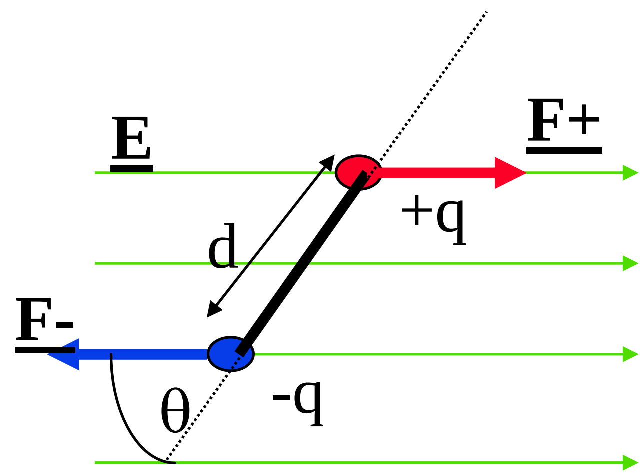
But the charges start to move (rotate). Why?

The Electric Dipole



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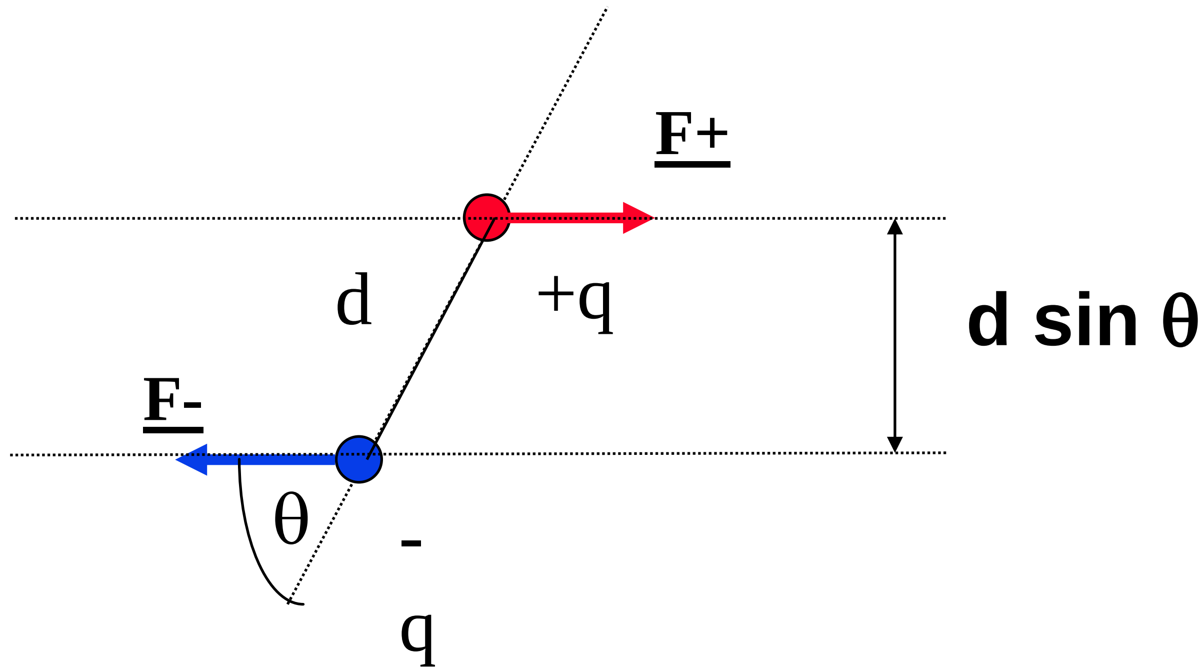
Zero, because the force on the two charges cancel: both have magnitude qE. The center of mass does not accelerate.

But the charges start to move (rotate). Why?

There's a torque because the forces aren't colinear.

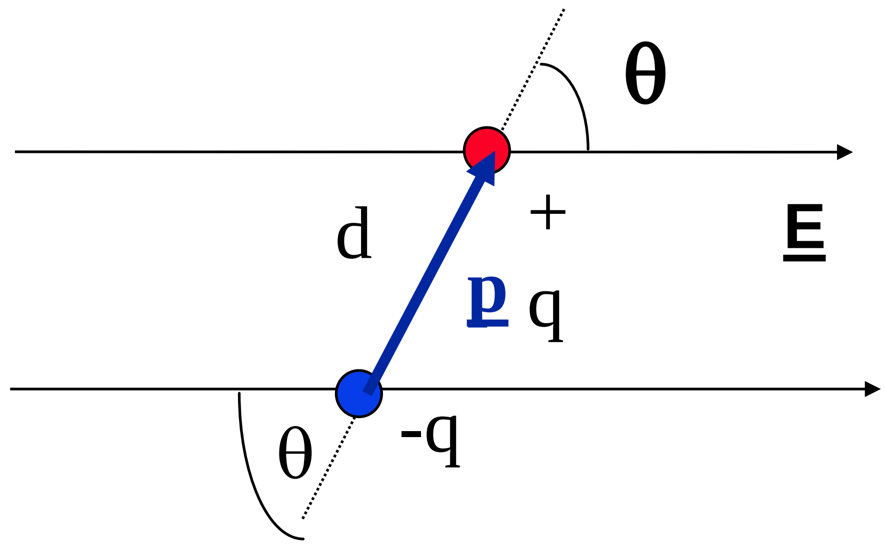


The torque is:

$\boldsymbol{\tau}$ = (magnitude of force) (moment arm)

$$\boldsymbol{\tau} = (qE)(d \sin \theta)$$

and the direction of $\underline{\boldsymbol{\tau}}$ is (in this case) into the page



but we have defined : **p = q d**

and the direction of **p** is from -q to +q

Then, the **torque** can be written as:

$$\underline{\tau} = \underline{\mathbf{p}} \times \underline{\mathbf{E}} \qquad \tau = p\, E \sin\theta$$

with an associated **potential energy**

$$U = -\,\underline{\mathbf{p}} \cdot \underline{\mathbf{E}}$$

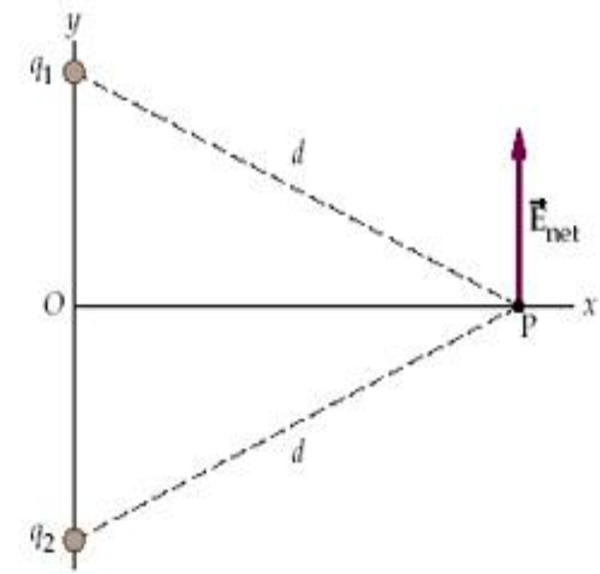
Electric fields due to various charge distributions

The electric field is a vector which obeys the superposition principle.

The electric field of a charge distribution is the sum of the fields produced by individual charges, or by differential elements of charge

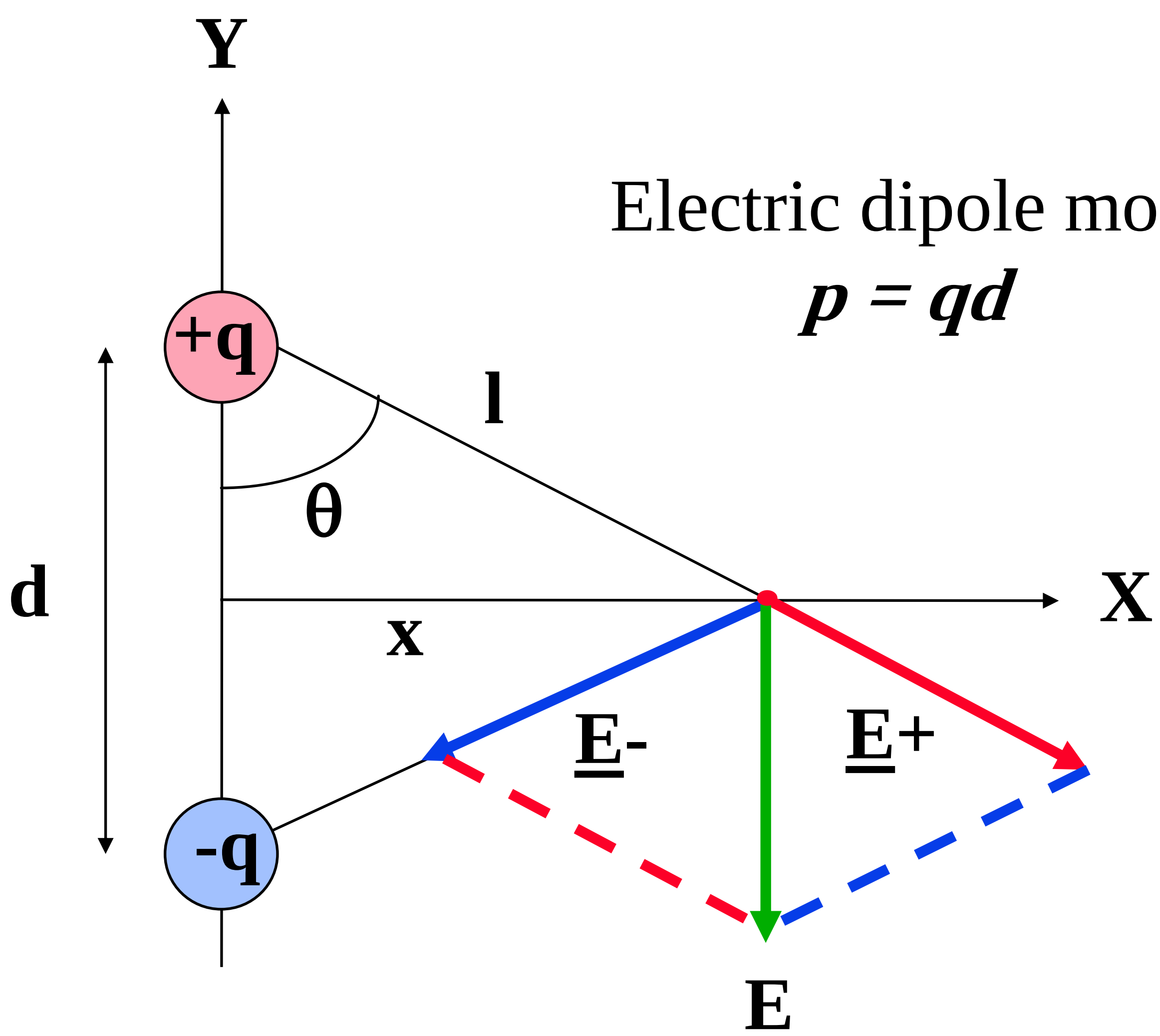
CONCEPTUAL CHECKPOINT 19–4

Two charges, q_1 and q_2, have equal magnitudes q and are placed as shown in the figure at the right. The net electric field at point P is vertically upward. We conclude that **(a)** q_1 is positive, q_2 is negative; **(b)** q_1 is negative, q_2 is positive; or **(c)** q_1 and q_2 have the same sign.



Field Due to an Electric Dipole

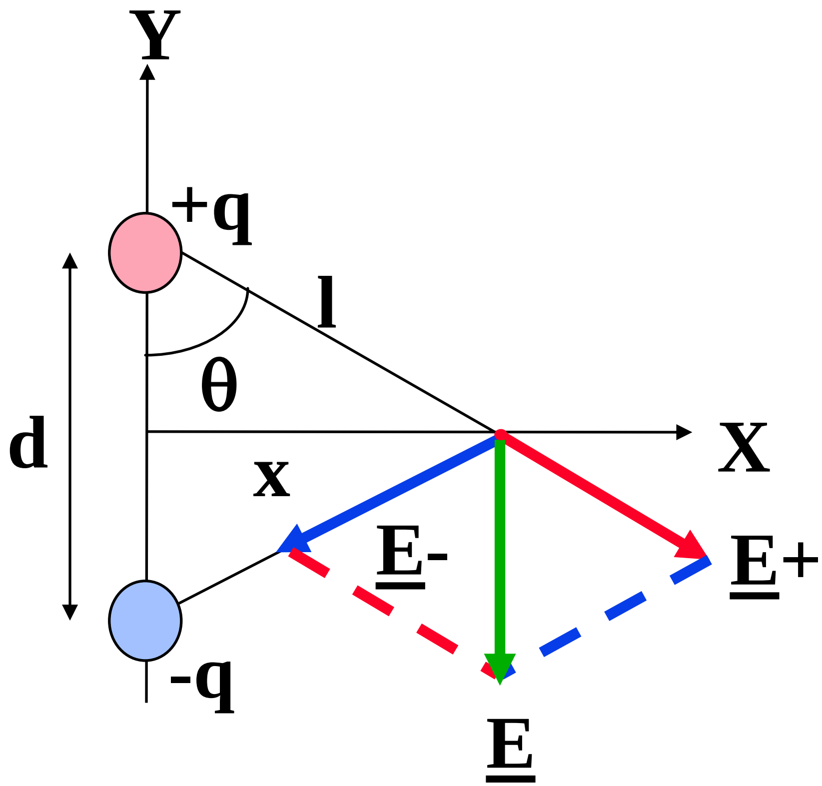
at a point x straight out from its midpoint



Electric dipole moment

$$p = qd$$

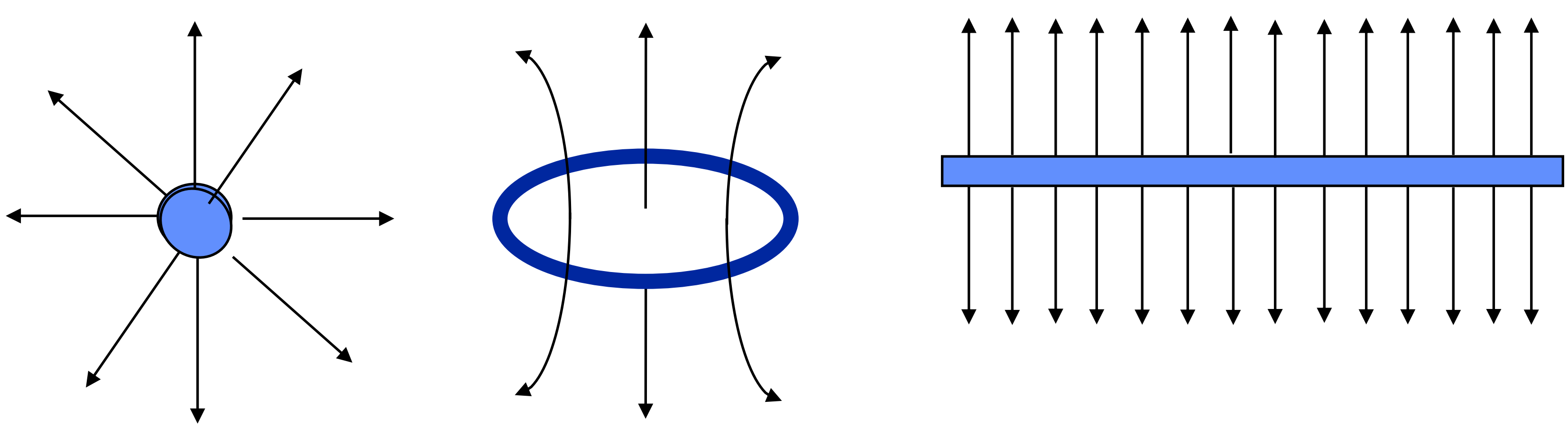
Calculate E as a function of p, x, and d, and approximate for x >> d


Y
+q
l
θ
d
X
x
E-
E+
-q
E

Electric Fields From Continuous Distributions of Charge

Up to now we have only considered the electric field of point charges.

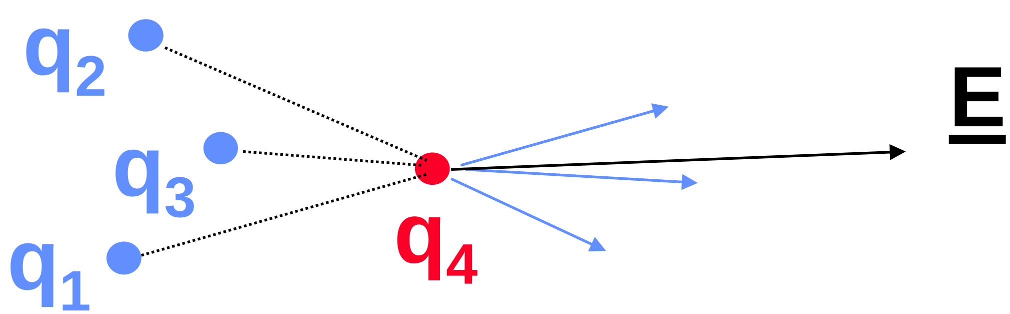
Now let's look at continuous distributions of charge
lines - surfaces - volumes of charge
and determine the resulting electric fields.



Sphere | Ring | Sheet

Electric Fields Produced by Continuous Distributions of Charge

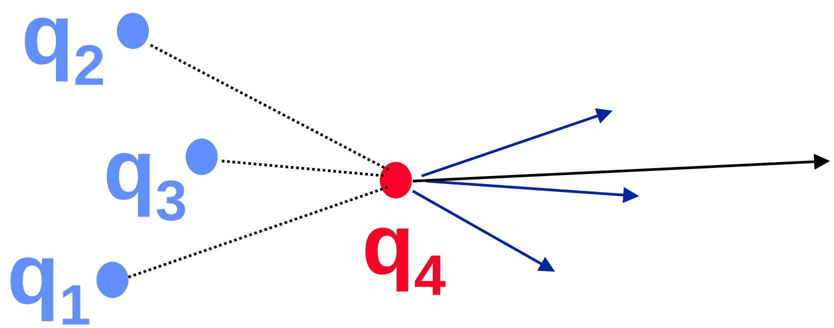
For discrete point charges, we can use the
Superposition Principle,
and sum the fields due to each point charge:



Electric field experienced by q_4

Electric Fields From Continuous Distributions

For discrete point charges, we can use the superposition principle and sum the fields due to each point charge:

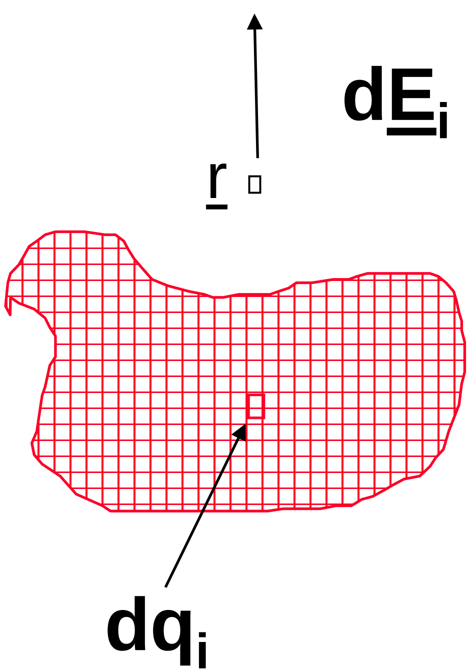


$$\vec{E}(\vec{r}) = \sum_i \vec{E}_i$$

What if we now have a *continuous* charge distribution?

Electric Field Produced by a Continuous Distribution of Charge

In the case of a continuous distribution of charge we first divide the distribution up into small pieces, and then we sum the contribution, to the field, from each piece:



The small piece of charge dq_i produces a small field $d\underline{E}_i$ at the point $\underline{r}$

Note: dq_i and $d\underline{E}_i$ are differentials

In the limit of very small pieces, the sum is an *integral*

Electric Field Produced by a Continuous Distribution of Charge

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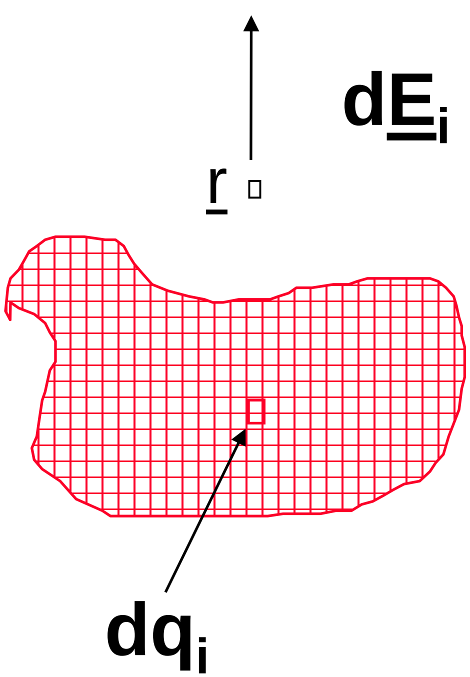
In the limit of very small pieces, the sum is an *integral*



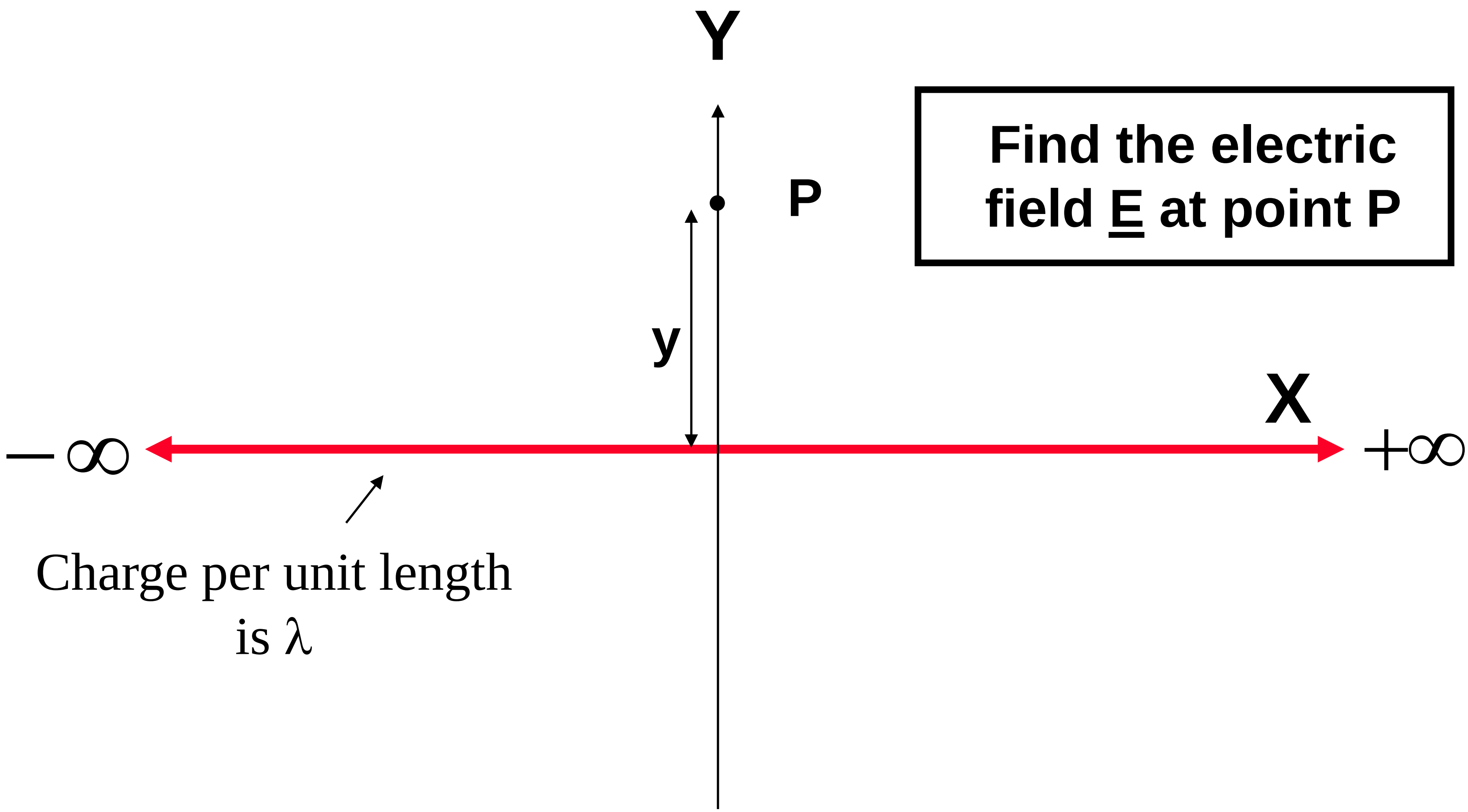
Each dq: $d\vec{E}(\vec{r}) = k\dfrac{dq}{r^2}\hat{r}$

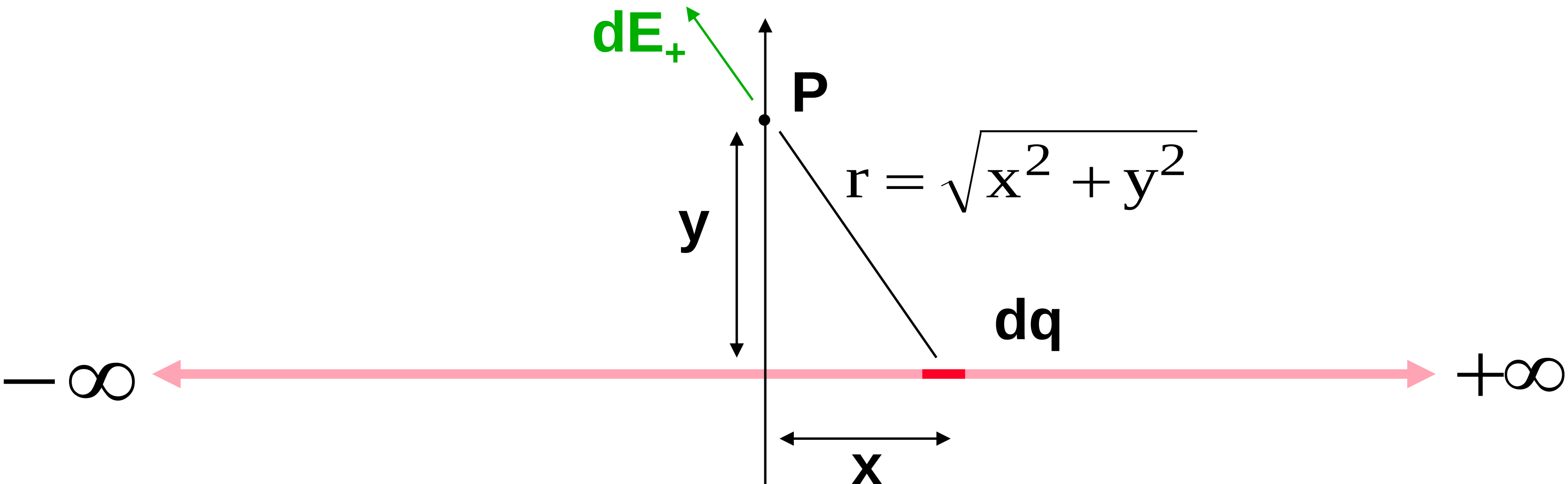
Then: $\underline{E} = \sum d\underline{E}_i$

For very small pieces: $\vec{E}(\vec{r}) = \int d\vec{E}$

$$\vec{E}(\vec{r}) = \int \frac{k\,dq}{r^2}\hat{r}$$

Example: An infinite thin line of charge



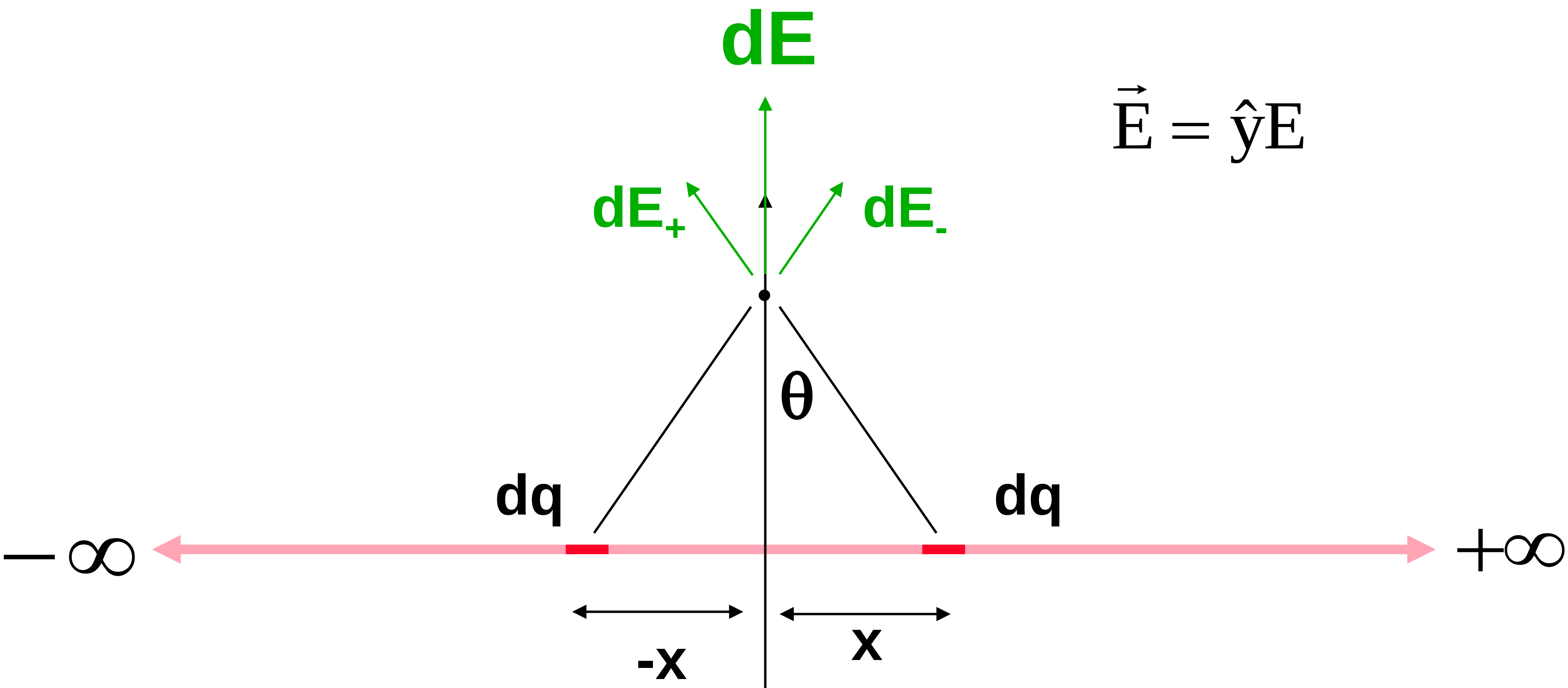


$$d\vec{E}_+ = k\frac{dq}{r^2}\hat{r}$$

- **Consider small element of charge, dq, at position x.**

- **dq is distance r from P.**

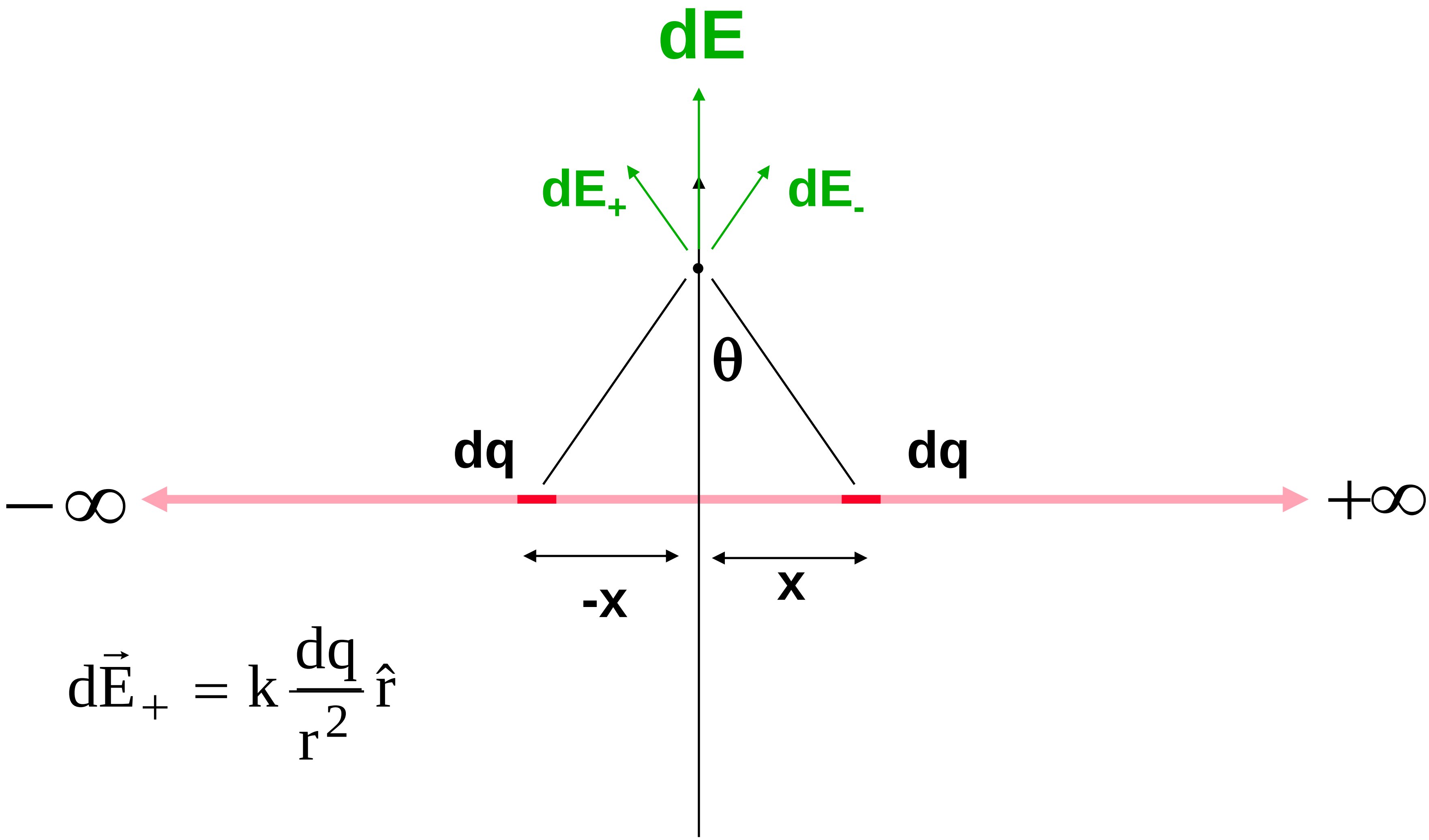
$$\vec{E} = \hat{y}E$$



$$d\vec{E}_{+} = k\frac{dq}{r^2}\hat{r}$$

$$dE_y = 2k\frac{dq}{r^2}\cos\theta$$

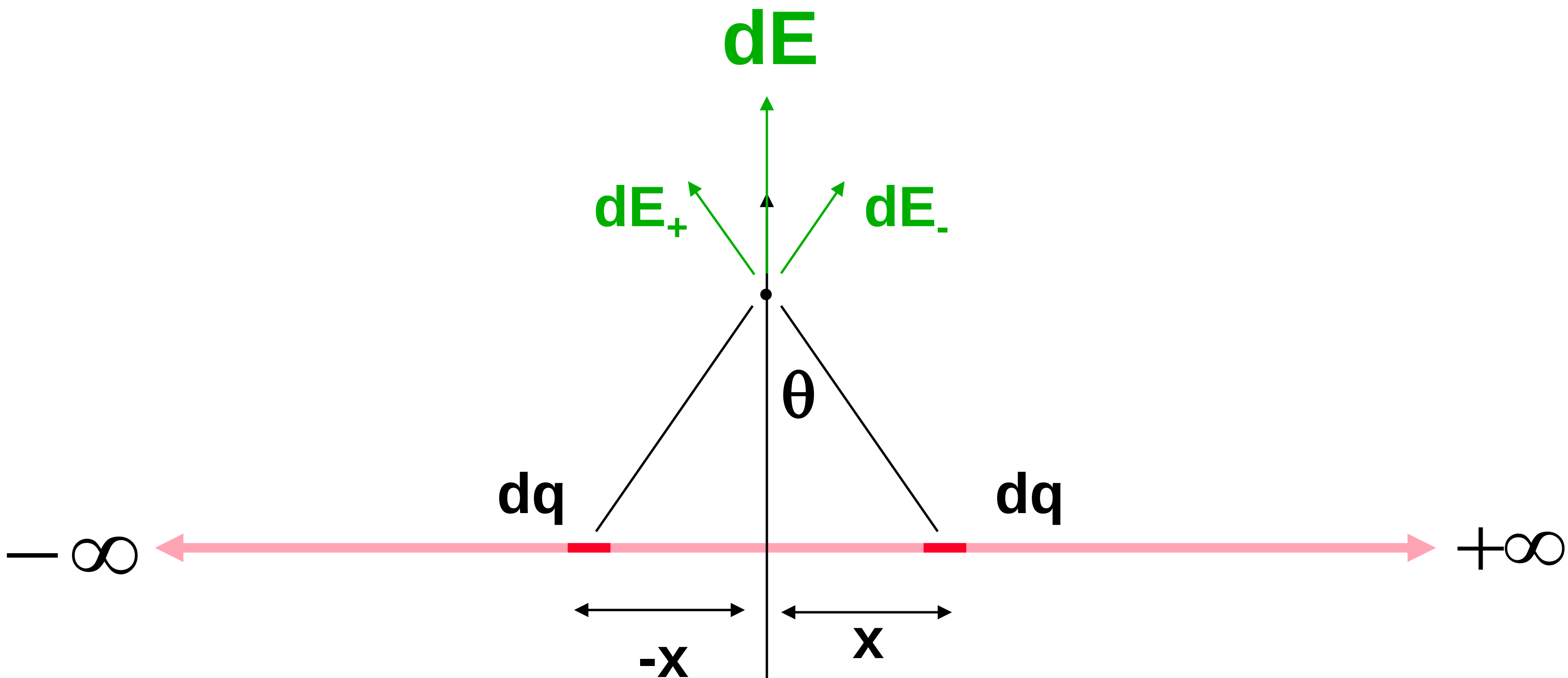
- **Consider small element of charge, dq, at position x.**
- **dq is distance r from P.**
- **For each dq at +x, there is a dq at -x.**



$$d\vec{E}_+ = k\frac{dq}{r^2}\hat{r}$$

$$\mathbf{dE_y} = \mathbf{2k\frac{dq}{r^2}\cos}\theta$$

$dq=\lambda\, dx$, $\cos\theta = y/r$



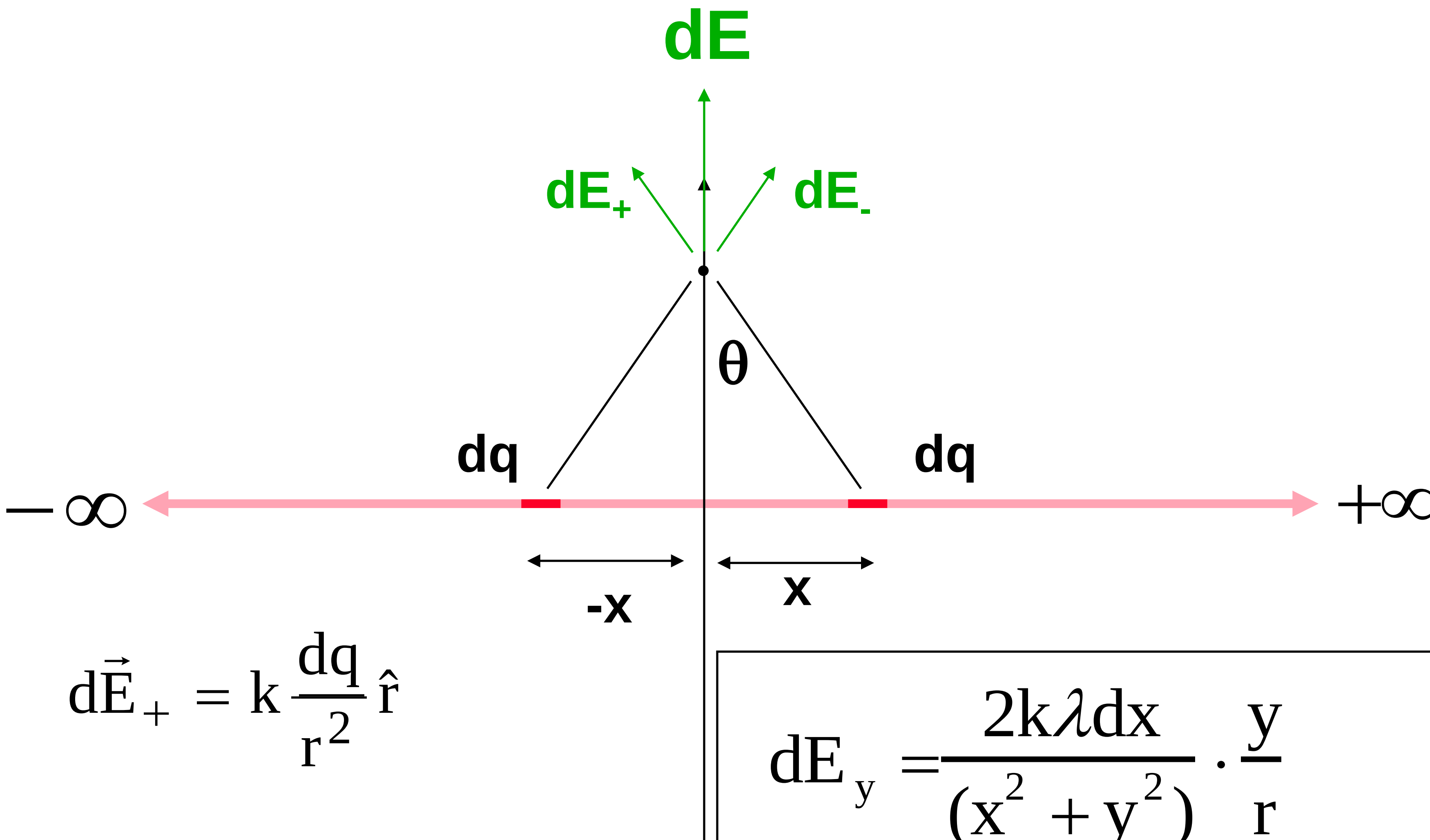
$$d\vec{E}_{+} = k\frac{dq}{r^2}\hat{r}$$

$$dE_y = 2k\frac{dq}{r^2}\cos\theta$$

dq=λ dx, cosθ=y/r

$$dE_y = \frac{2k\lambda dx}{(x^2+y^2)}\cdot\frac{y}{r}$$

$$E_y = \int_{x=0}^{x=\infty}\frac{2k\lambda y}{(x^2+y^2)^{3/2}}dx$$



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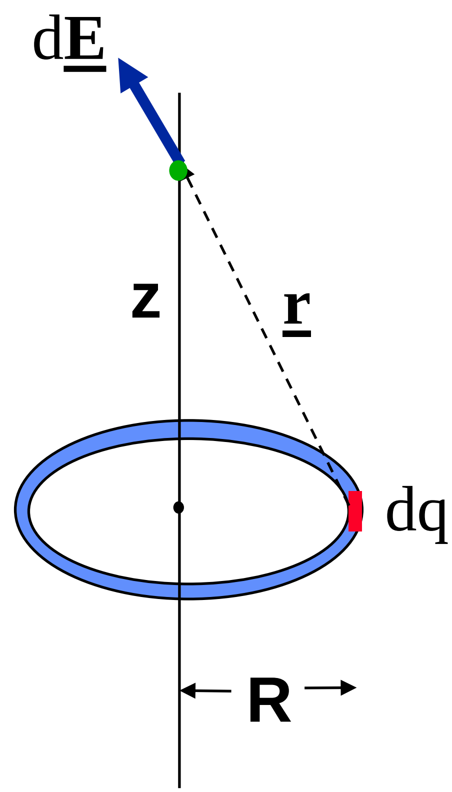
$$E_y = \int_{x=0}^{x=\infty}\frac{2k\lambda y}{(x^2+y^2)^{3/2}}dx = \frac{2k\lambda}{y}$$

Example of Continuous Distribution: Ring of Charge

Find the electric field at a point along the axis.
Hint: be sure to use the symmetry of the problem!



Break the charge up into little bits, and find the field due to each bit at the observation point. Then integrate.

Thin ring with total charge q
charge per unit length is
$\lambda = q/2\pi R$

Continuous Charge Distributions

	LINE	AREA	VOLUME
charge density	$\lambda = Q / L$	$\sigma = Q / A$	$\rho = Q / V$
units	C / m	C / m^2	C / m^3
differential	$dq = \lambda\, dL$	$dq = \sigma\, dA$	$dq = \rho\, dV$

Charge differential dq
to be used when finding
the electric field of a
continuous charge distribution

$$\vec{E}(\vec{r}) = \int \frac{k\,dq}{r^2}\hat{r}$$