Thus, by integration,

$$V(x) = \frac{q}{4\pi\epsilon_0\, x} + V_0, \tag{5.21}$$

where V_0 is an arbitrary constant. Finally, making use of the fact that $V = V(r)$, we obtain

$$V(\mathbf{r}) = \frac{q}{4\pi\epsilon_0\, r}. \tag{5.22}$$

Here, we have adopted the common convention that the potential at infinity is zero. A potential defined according to this convention is called an *absolute potential*.

Suppose that we have N point charges distributed in space. Let the ith charge q_i be located at position vector $\mathbf{r}_i$. Since electric potential is superposable, and is also a scalar quantity, the absolute potential at position vector $\mathbf{r}$ is simply the algebraic sum of the potentials generated by each charge taken in isolation:

$$V(\mathbf{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0\, |\mathbf{r} - \mathbf{r}_i|}. \tag{5.23}$$

The work W we would perform in taking a charge q from infinity and slowly moving it to point $\mathbf{r}$ is the same as the increase in electric potential energy of the charge during its journey [see Eq. (5.4)]. This, by definition, is equal to the product of the charge q and the increase in the electric potential. This, finally, is the same as q times the absolute potential at point $\mathbf{r}$: *i.e.*,

$$W = q\, V(\mathbf{r}). \tag{5.24}$$

5.5 Worked Examples

Example 5.1: Charge in a uniform electric field

Question: A charge of $q = +1.20\,\mu\mathrm{C}$ is placed in a uniform x-directed electric field of magnitude $E_x = 1.40 \times 10^3\,\mathrm{N\,C^{-1}}$. How much work must be performed in order to move the charge a distance $c = -3.50$ cm in the x-direction? What is the potential difference between the initial and final positions of the charge? If the electric field is produced by two oppositely charged parallel plates separated by a distance $d = 5.00$ cm, what is the potential difference between the plates?

Solution: Let us denote the initial and final positions of the charge A and B, respectively. The work which we must perform in order to move the charge from A to B is minus the product of the electrostatic force on the charge due to the electric field (since the force we exert on the charge is minus this force) and the distance that the charge moves in the direction of this force [see Eq. (5.1)]. Thus,

$$W = -q\,E_x\,c = -(1.2 \times 10^{-6})\,(1.40 \times 10^{3})\,(-3.50 \times 10^{-2}) = +5.88 \times 10^{-5}\,\text{J}.$$

Note that the work is positive. This makes sense, because we would have to do real work (*i.e.*, we would lose energy) in order to move a positive charge in the opposite direction to an electric field (*i.e.*, against the direction of the electrostatic force acting on the charge).

The work done on the charge goes to increase its electric potential energy, so $P_B - P_A = W$. By definition, this increase in potential energy is equal to the product of the potential difference $V_B - V_A$ between points B and A, and the magnitude of the charge q. Thus,

$$q\,(V_B - V_A) = P_B - P_A = W = -q\,E_x\,c,$$

giving

$$V_B - V_A = -E_x\,c = -(1.40 \times 10^{3})\,(-3.50 \times 10^{-2}) = 49.0\,\text{V}.$$

Note that the electric field is directed from point B to point A, and that the former point is at a higher potential than the latter.

It is clear, from the above formulae, that the magnitude of the potential difference between two points in a uniform electric field is simply the product of the electric field-strength and the distance between the two points (in the direction of the field). Thus, the potential difference between the two metal plates is

$$\Delta V = E_x\,d = (1.40 \times 10^{3})\,(5.00 \times 10^{-2}) = 70.0\,\text{V}.$$

If the electric field is directed from plate 1 (the positively charged plate) to plate 2 (the negatively charged plate) then the former plate is at the higher potential.

Example 5.2: Motion of an electron in an electric field

Question: An electron in a television set is accelerated from the cathode to the screen through a potential difference of +1000 V. The screen is 35 mm from the cathode. What is the net change in the potential energy of the electron during the acceleration process? How much work is done by the electric field in accelerating the electron? What is the speed of the electron when it strikes the screen?

Solution: Let call the cathode point A and the screen point B. We are told that the potential difference between points B and A is +1000 V, so

$$V_B - V_A = 1000\,\mathrm{V}.$$

By definition, the difference in electric potential energy of some charge q at points B and A is the product of the charge and the difference in electric potential between these points. Thus,

$$P_B - P_A = q\,(V_B - V_A) = (-1.6\times 10^{-19})\,(1000) = -1.6\times 10^{-16}\,\mathrm{J},$$

since $q = -1.6\times 10^{-19}\,\mathrm{C}$ for an electron. Note that the potential energy of the electron *decreases* as it is accelerated towards the screen. As we have seen, the electric potential energy of a charge is actually held in the surrounding electric field. Thus, a decrease in the potential energy of the charge corresponds to a reduction in the energy of the field. In this case, the energy of the field decreases because it does work W' on the charge. Clearly, the work done (*i.e.*, energy lost) by the field equals the decrease in potential energy of the charge,

$$W' = -\Delta P.$$

Thus,

$$W' = 1.6\times 10^{-16}\,\mathrm{J}.$$

The total energy E of the electron is made up of two components—the electric potential energy P, and the kinetic energy K. Thus,

$$E = P + K.$$

Of course,

$$K = \frac{1}{2} m v^2,$$

where $m = 9.11 \times 10^{-31}$ kg is the mass of the electron, and v its speed. By conservation of energy, E is a constant of the motion, so

$$K_B - K_A = \Delta K = -\Delta P.$$

In other words, the decrease in electric potential energy of the electron, as it is accelerated towards the screen, is offset by a corresponding increase in its kinetic energy. Assuming that the electron starts from rest (*i.e.* $v_A = 0$), it follows that

$$\frac{1}{2} m v_B^2 = -\Delta P,$$

or

$$v_B = \sqrt{\frac{-2\,\Delta P}{m}} = \sqrt{\frac{-2\,(-1.6 \times 10^{-16})}{9.11 \times 10^{-31}}} = 1.87 \times 10^7\,\mathrm{m\,s^{-1}}.$$

Note that the distance between the cathode and the screen is immaterial in this problem. The final speed of the electron is entirely determined by its charge, its initial velocity, and the potential difference through which it is accelerated.

Example 5.3: Electric potential due to point charges

Question: A particle of charge $q_1 = +6.0\,\mu C$ is located on the x-axis at the point $x_1 = 5.1$ cm. A second particle of charge $q_2 = -5.0\,\mu C$ is placed on the x-axis at $x_2 = -3.4$ cm. What is the absolute electric potential at the origin ($x = 0$)? How much work must we perform in order to slowly move a charge of $q_3 = -7.0\,\mu C$ from infinity to the origin, whilst keeping the other two charges fixed?

Solution: The absolute electric potential at the origin due to the first charge is

$$V_1 = k_e \frac{q_1}{x_1} = (8.988 \times 10^9) \frac{(6 \times 10^{-6})}{(5.1 \times 10^{-2})} = 1.06 \times 10^6\,\mathrm{V}.$$

Likewise, the absolute electric potential at the origin due to the second charge is

$$V_2 = k_e \frac{q_2}{|x_2|} = (8.988 \times 10^9) \frac{(-5 \times 10^{-6})}{(3.4 \times 10^{-2})} = -1.32 \times 10^6\,\mathrm{V}.$$

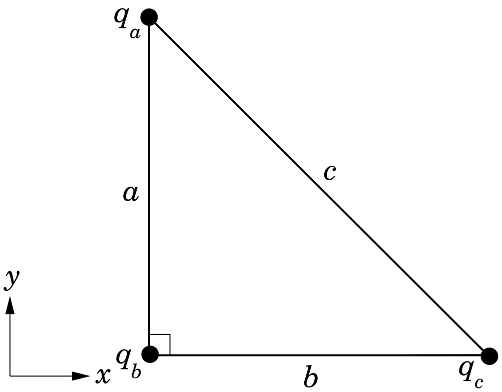
The net potential V at the origin is simply the algebraic sum of the potentials due to each charge taken in isolation. Thus,

$$V = V_1 + V_2 = -2.64 \times 10^5\,\text{V}.$$

The work W which we must perform in order to slowly moving a charge q_3 from infinity to the origin is simply the product of the charge and the potential difference V between the end and beginning points. Thus,

$$W = q_3\,V = (-7 \times 10^{-6})\,(-2.64 \times 10^5) = 1.85\,\text{J}.$$

Example 5.4: Electric potential due to point charges



Question: Suppose that three point charges, q_a, q_b, and q_c, are arranged at the vertices of a right-angled triangle, as shown in the diagram. What is the absolute electric potential of the third charge if $q_a = -6.0\,\mu\text{C}$, $q_b = +4.0\,\mu\text{C}$, $q_c = +2.0\,\mu\text{C}$, $a = 4.0\,\text{m}$, and $b = 3.0\,\text{m}$? Suppose that the third charge, which is initially at rest, is repelled to infinity by the combined electric field of the other two charges, which are held fixed. What is the final kinetic energy of the third charge?

Solution: The absolute electric potential of the third charge due to the presence of the first charge is

$$V_a = k_e\,\frac{q_a}{c} = (8.988 \times 10^9)\,\frac{(-6 \times 10^{-6})}{(\sqrt{4^2 + 3^2})} = -1.08 \times 10^4\,\text{V},$$

where use has been made of the Pythagorean theorem. Likewise, the absolute electric potential of the third charge due to the presence of the second charge is

$$V_b = k_e \frac{q_b}{b} = (8.988 \times 10^9) \frac{(4 \times 10^{-6})}{(3)} = 1.20 \times 10^4 \, \text{V}.$$

The net absolute potential of the third charge V_c is simply the algebraic sum of the potentials due to the other two charges taken in isolation. Thus,

$$V_c = V_a + V_b = 1.20 \times 10^3 \, \text{V}.$$

The change in electric potential energy of the third charge as it moves from its initial position to infinity is the product of the third charge, q_c, and the difference in electric potential ($-V_c$) between infinity and the initial position. It follows that

$$\Delta P = -q_c \, V_c = -(2 \times 10^{-6}) \, (1.2 \times 10^3) = -2.40 \times 10^{-3} \, \text{J}.$$

This decrease in the potential energy of the charge is offset by a corresponding increase $\Delta K = -\Delta P$ in its kinetic energy. Since the initial kinetic energy of the third charge is zero (because it is initially at rest), the final kinetic energy is simply

$$K = \Delta K = -\Delta P = 2.40 \times 10^{-3} \, \text{J}.$$