Electricity & Magnetism

Gauss's Law

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Summary

The Electric Field is related to Coulomb's Force by

$$\mathbf{E} = \frac{\mathbf{F}}{Q_0}$$

Thus knowing the field we can calculate the force on a charge

$$\mathbf{F} = Q\mathbf{E}$$

The Electric Field is a vector field

Using superposition we thus find

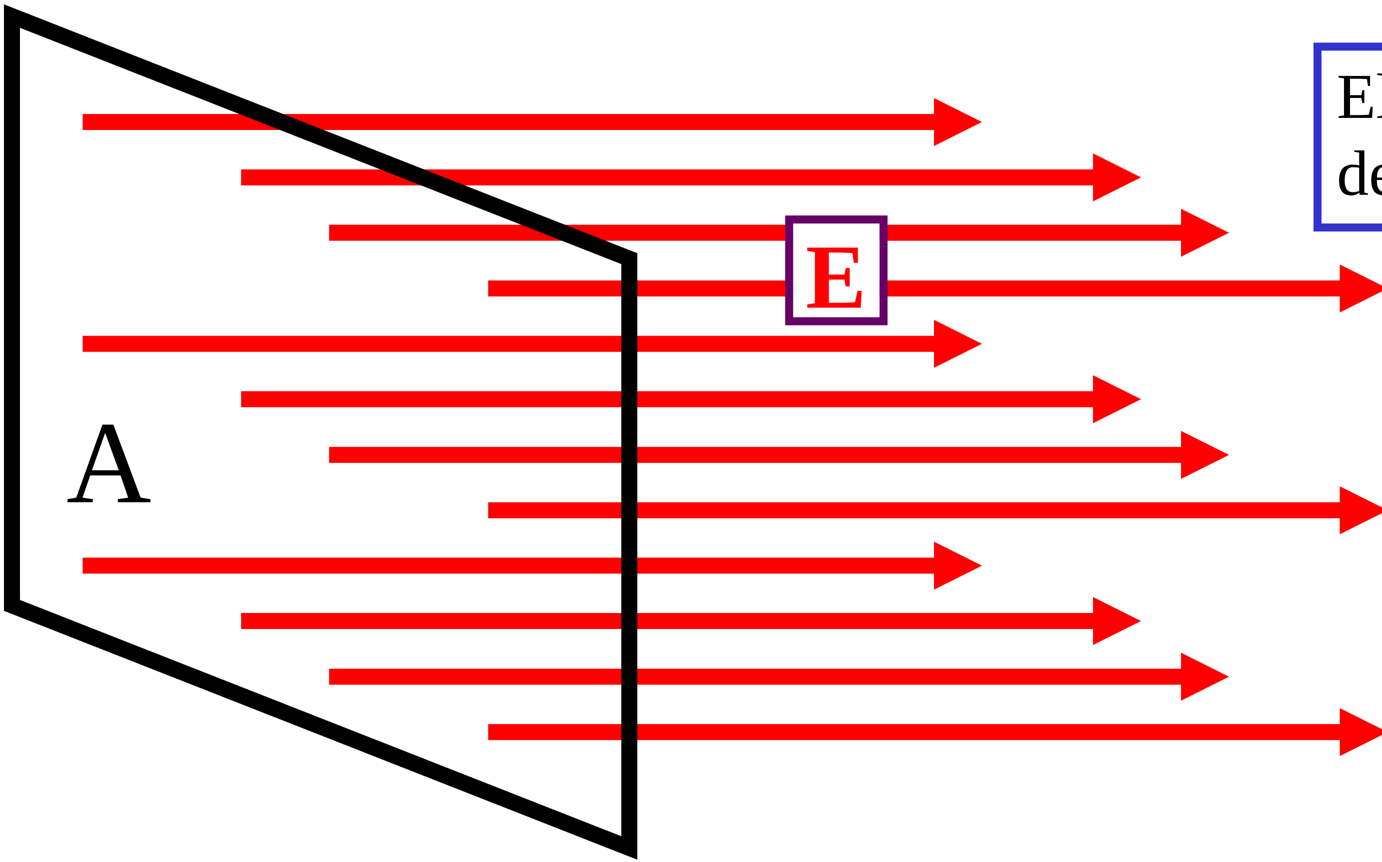
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{Q_i}{|\mathbf{r}_i|^2} \hat{\mathbf{r}}_i$$

Field lines illustrate the *strength* & *direction* of the Electric field

Today

- Electric Flux
- Gauss's Law
- Examples of using Gauss's Law
- Properties of Conductors

Electric Flux: Field Perpendicular



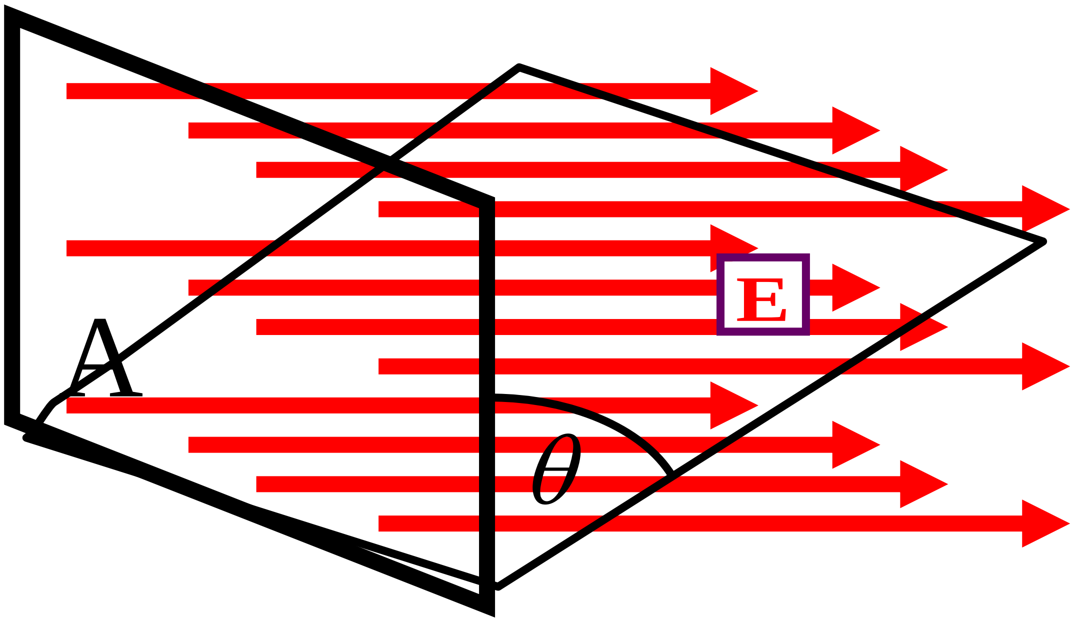
Electric Flux is defined as

$$\Phi = |\mathbf{E}|\, A$$

For a constant field perpendicular to a surface A

Electric Flux: Non perpendicular

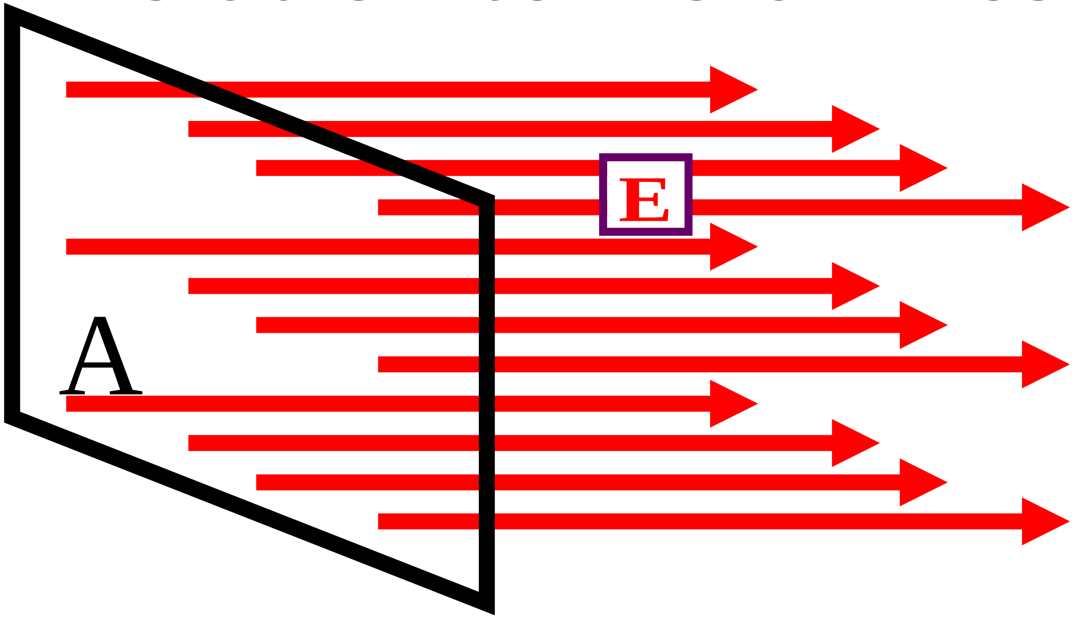
For a constant field NOT perpendicular to a surface A



Electric Flux is defined as

$$\Phi = |\mathbf{E}|\, A \cos\theta$$

Electric Flux:
Relation to field lines



$$\Phi = |\mathbf{E}|\, A$$

Field line density $\rho \propto |\mathbf{E}|$

Field line density × Area $\rho A \propto |\mathbf{E}|\, A$

FLUX

Number of flux lines $N \propto \Phi$

Quiz

What is the electric flux through a cylindrical surface? The electric field, E, is uniform and perpendicular to the surface. The cylinder has radius r and length L

A) $E\ 4/3\ \pi\ r^3\ L$
B) $E\ r\ L$
C) $E\ \pi\ r^2\ L$
D) $E\ 2\ \pi\ r\ L$
E) 0

Gauss's Law

Relates flux through a closed surface to charge within that surface

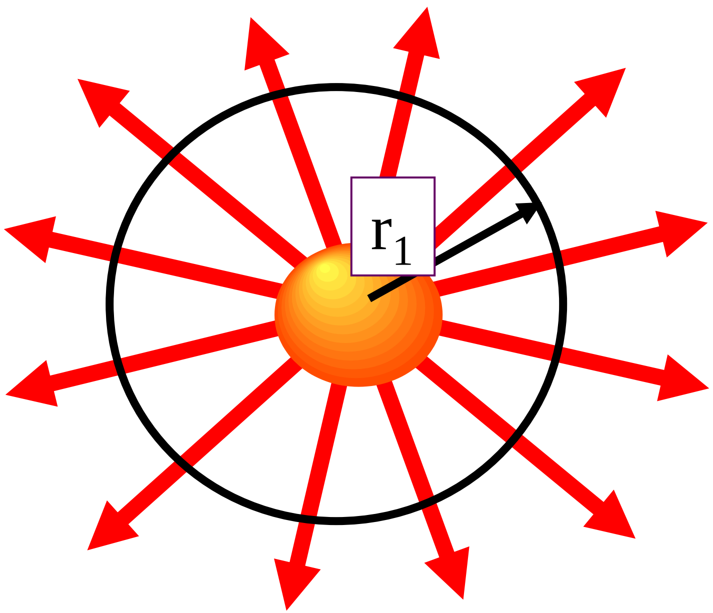
Flux through a sphere from a point charge

The electric field around a point charge

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2}$$

E

Area



Thus the flux on a sphere is E × Area

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2} \times 4\pi|\mathbf{r}_1|^2$$

Cancelling we get

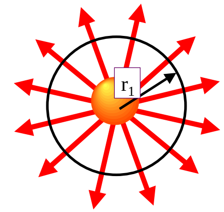
$$\Phi = \frac{Q}{\varepsilon_0}$$

Now we change the radius of sphere

Flux through a sphere from a point charge

The electric field around a point charge

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2}$$



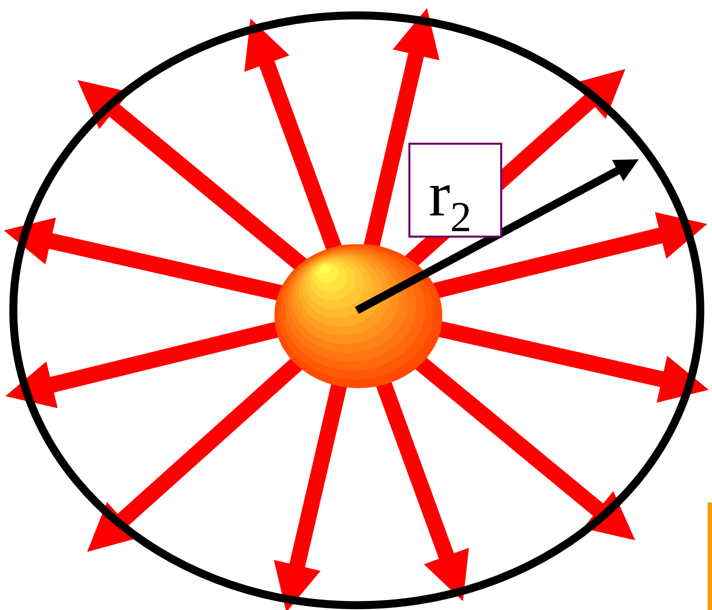
Thus the flux on a sphere is E × Area

E   Area

$$\Phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_1|^2} \times 4\pi|\mathbf{r}_1|^2$$

Cancelling we get

$$\Phi = \frac{Q}{\varepsilon_0}$$



$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_2|^2}$$

$$\Phi_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}_2|^2} \times 4\pi|\mathbf{r}_2|^2$$

$$\Phi_2 = \frac{Q}{\varepsilon_0}$$

The flux is the same as before

$$\Phi_2 = \Phi_1 = \frac{Q}{\varepsilon_0}$$

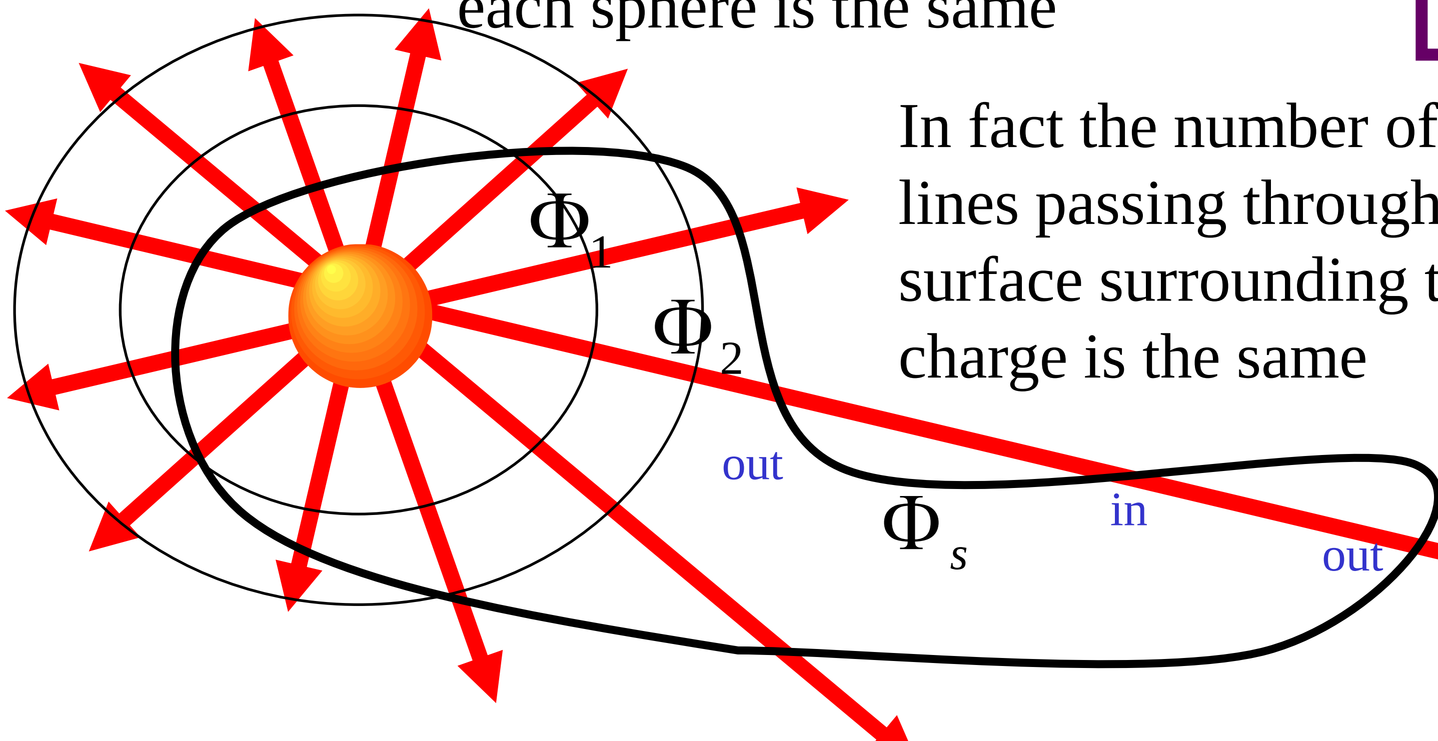
Flux lines & Flux

Just what we would expect because the number of field lines passing through each sphere is the same

$$N \propto \Phi \qquad \Phi \propto N$$

and number of lines passing through each sphere is the same

$$\Phi_S = \Phi_2 = \Phi_1 = \frac{Q}{\varepsilon_0}$$



In fact the number of flux lines passing through any surface surrounding this charge is the same

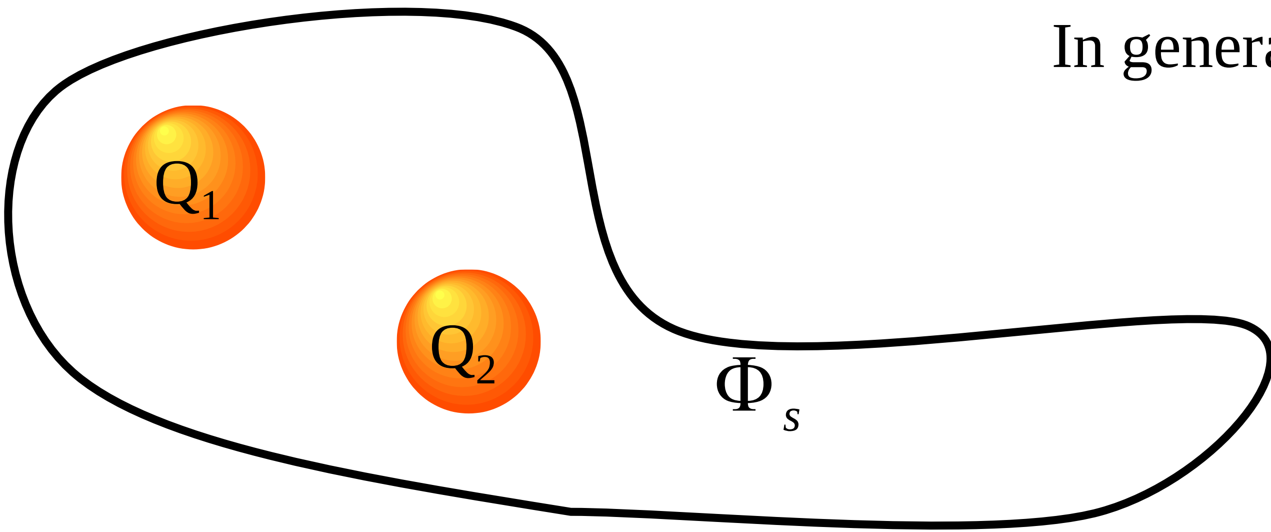
even when a line passes in and out of the surface it crosses out once more than in

Principle of superposition:

What is the flux from two charges?

Since the flux is related to the number of field lines passing through a surface the total flux is the total from each charge

$$\Phi_S = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0}$$



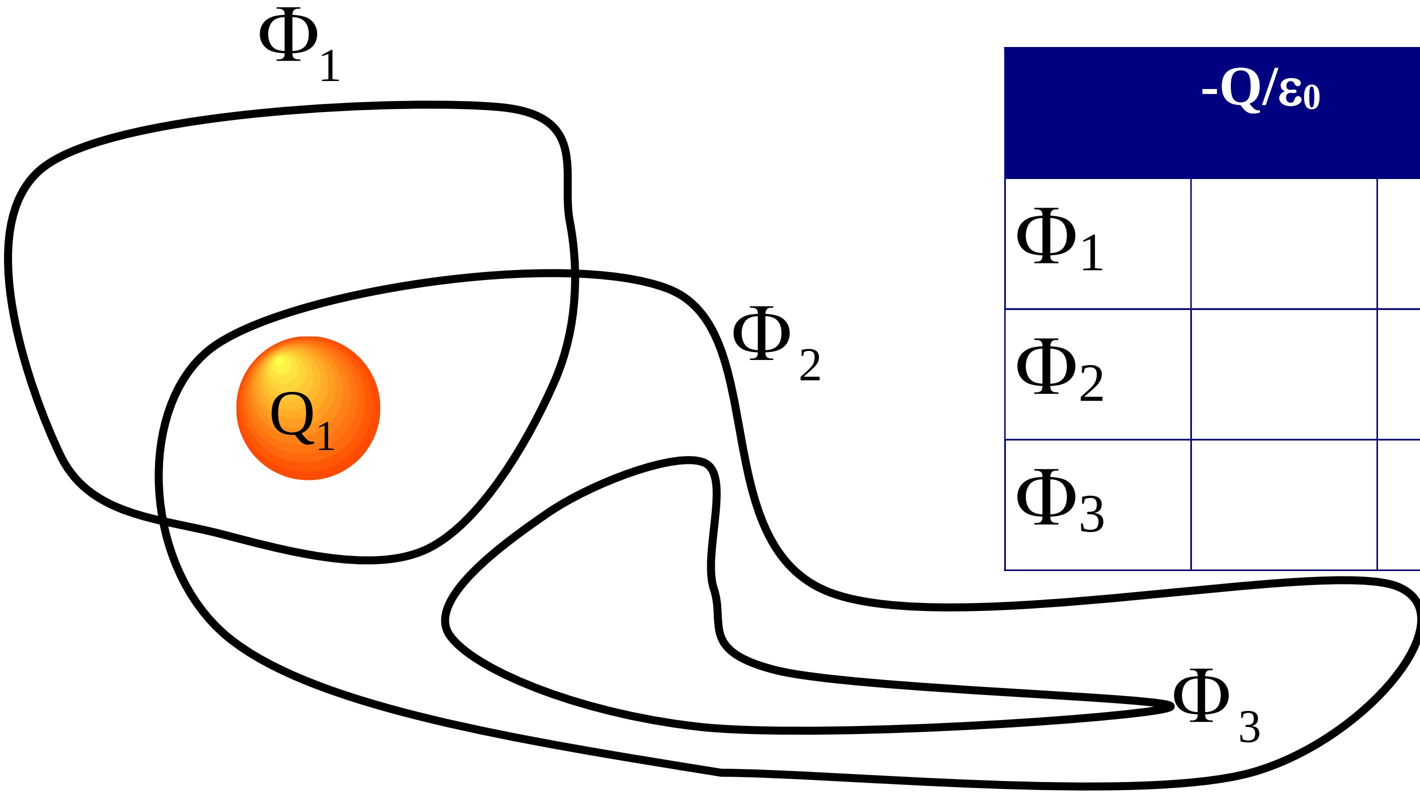
In general

$$\Phi_S = \sum \frac{Q_i}{\varepsilon_0}$$

For any surface

Gauss's Law

Quiz: **What flux is passing through each of these surfaces?**



	$-Q/\varepsilon_0$	0	$+Q/\varepsilon_0$	$+2Q/\varepsilon_0$
Φ_1				
Φ_2				
Φ_3				

What is Gauss's Law?

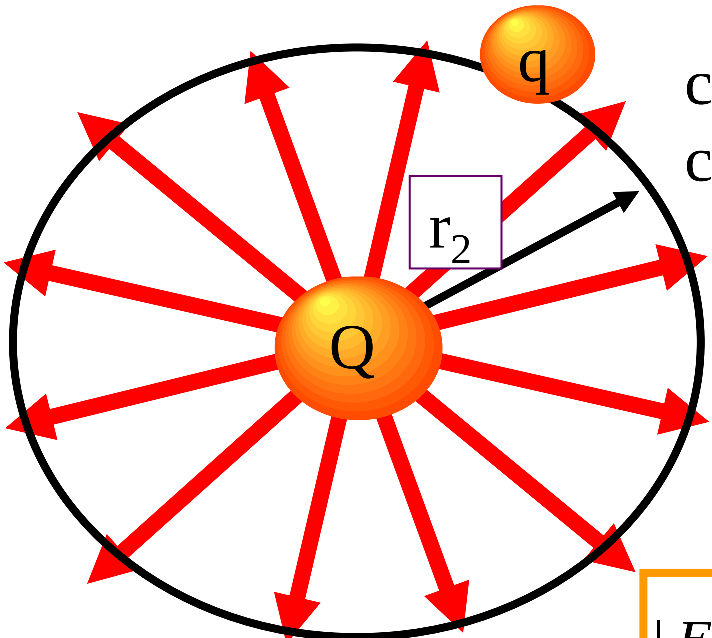
Gauss's Law does not tell us anything new, it is NOT a new law of physics, but another way of expressing Coulomb's Law

Gauss's Law is sometimes easier to use than Coulomb's Law, especially if there is lots of symmetry in the problem

Example of using Gauss's Law 1

oh no! I've just forgotten Coulomb's Law!

Not to worry I remember Gauss's Law



consider spherical surface centred on charge

$$\Phi = \frac{Q}{\varepsilon_0}$$

By symmetry **E** is ⊥ to surface

$$\Phi = |E|A = \frac{Q}{\varepsilon_0} \qquad = |E|4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$|E| = \frac{1}{4\pi r^2}\frac{Q}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

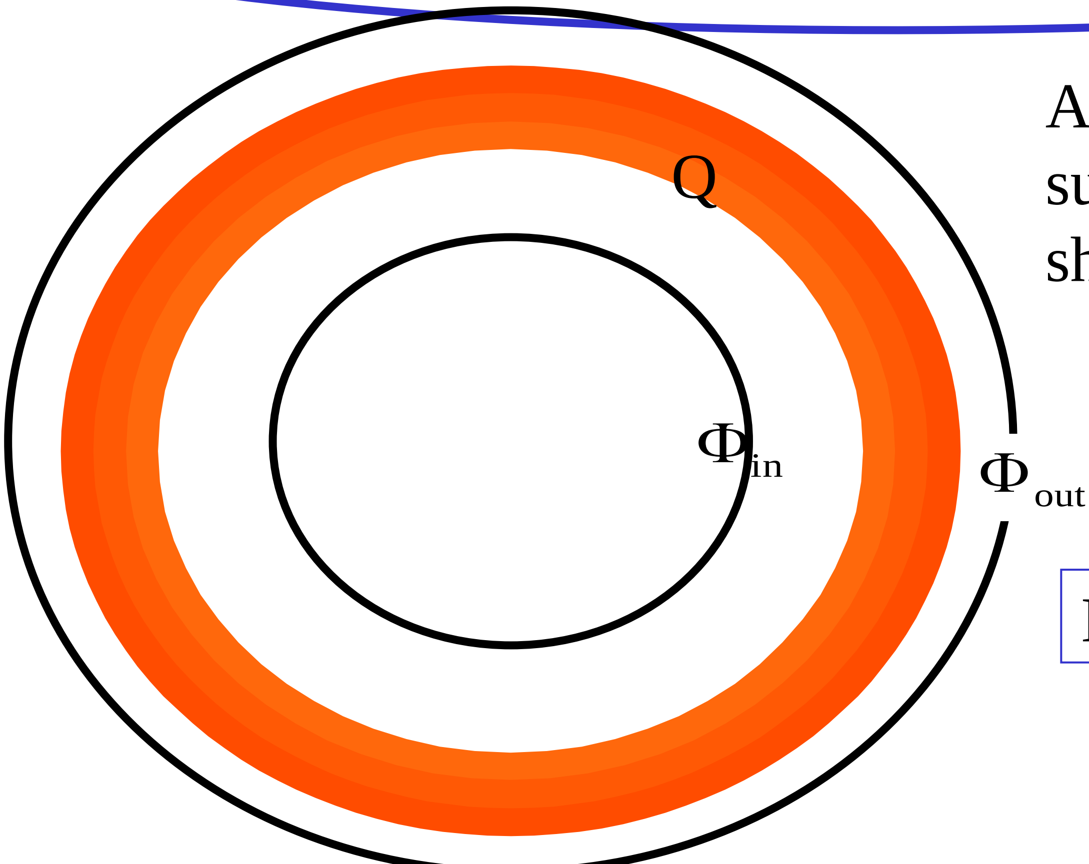
F=qE

$$F = \frac{1}{4\pi r^2}\frac{qQ}{\varepsilon_0}$$

Phew!

Example of using Gauss's Law 2

What's the field around a charged spherical shell?



Again consider spherical surface centred on charged shell

Outside

$$\Phi_{\text{out}} = \frac{Q}{\varepsilon_0}$$

So as e.g. 1

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Inside

charge within surface = 0

$$\Phi_{\text{in}} = 0$$

$$E = 0$$

Quiz

In a model of the atom the nucleus is a uniform ball of +ve charge of radius R. At what distance is the E field strongest?

A) $r = 0$

B) $r = R/2$

C) $r = R$

D) $r = 2\,R$

E) $r = 1.5\,R$

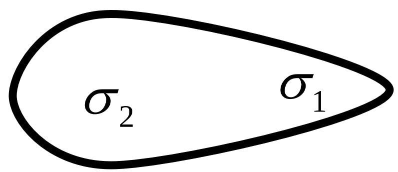
Properties of Conductors

Using Gauss's Law

Properties of Conductors

For a conductor in electrostatic equilibrium

1. E is zero within the conductor
2. Any net charge, Q, is distributed on surface (surface charge density σ=Q/A)
3. E immediately outside is $\perp$ to surface
4. σ is greatest where the radius of curvature is smaller



$\sigma_1 >> \sigma_{21}$

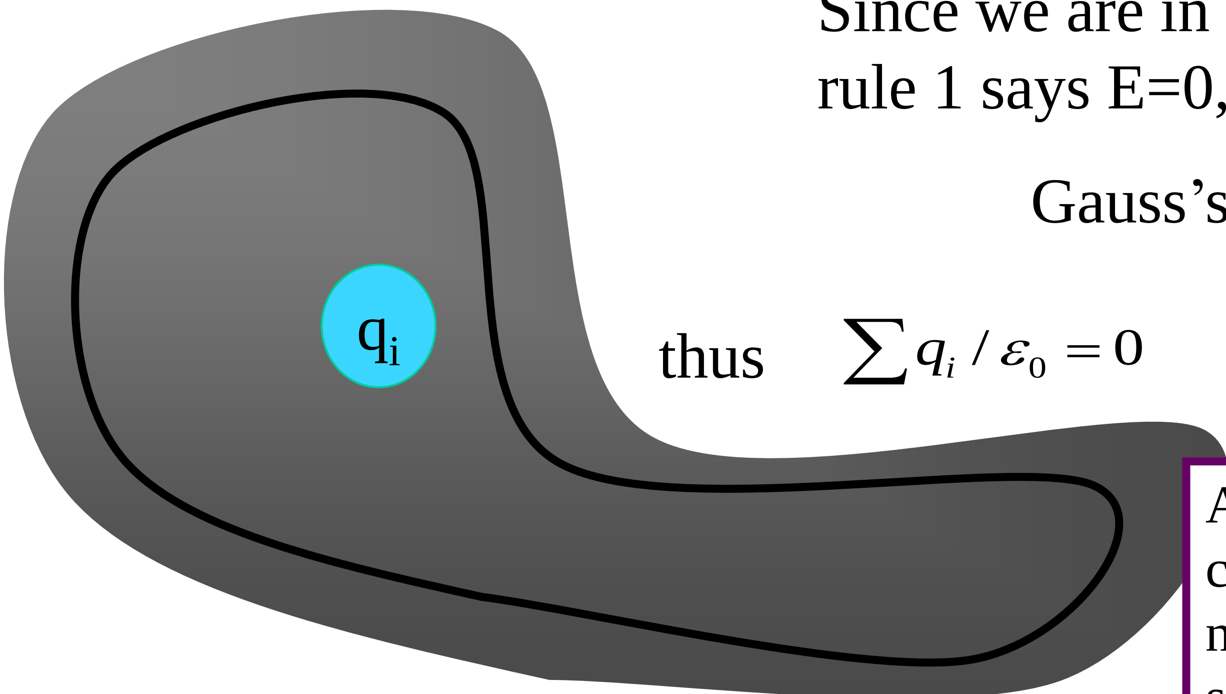
1. E is zero within conductor

If there is a field in the conductor, then the free electrons would feel a force and be accelerated. They would then move and since there are charges moving the conductor would not be in electrostatic equilibrium

Thus E=0

2. Any net charge, Q, is distributed on surface

Consider surface S below surface of conductor



Since we are in a conductor in equilibrium, rule 1 says E=0, thus Φ=0

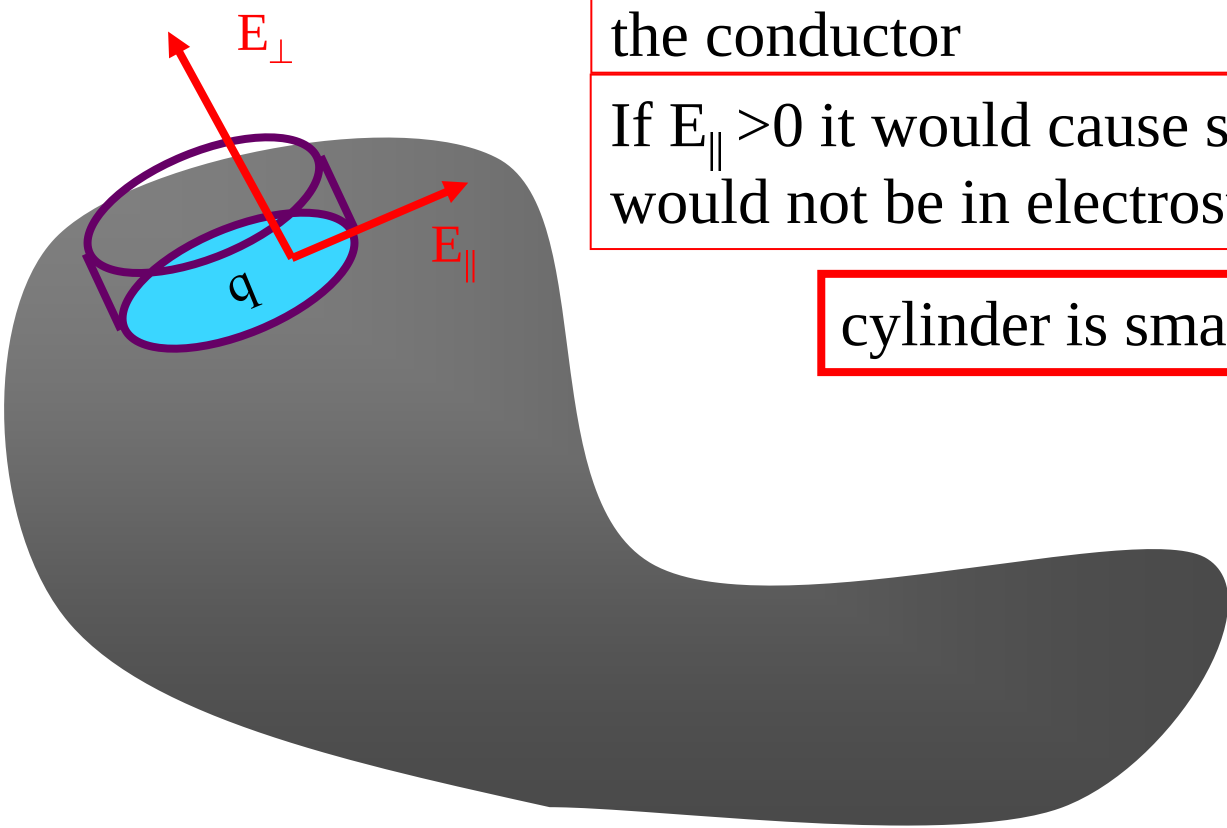
Gauss's Law $\Phi = EA = \sum q / \varepsilon_0$

thus $\sum q_i / \varepsilon_0 = 0$

So, net charge within the surface is zero

As surface can be drawn arbitrarily close to surface of conductor, all net charge must be distributed on surface

3. E immediately outside is ⊥ to surface



Consider a small cylindrical surface at the surface of the conductor

If $E_\parallel > 0$ it would cause surface charge q to move thus it would not be in electrostatic equilibrium, thus $E_\parallel = 0$

cylinder is small enough that E is constant

Gauss's Law $\Phi = EA = q/\varepsilon$

thus $E = q/A\varepsilon$

$$E_\perp = \sigma/\varepsilon$$

Summary

If there is a field in the conductor, then the free electrons would feel a force and be accelerated. They would then move and since there are charges moving the conductor would not be in electrostatic equilibrium. Thus E=0

$$\Phi = |\mathbf{E}| A \cos\theta$$

$$\Phi_S = \sum \frac{Q_i}{\varepsilon_0}$$

Properties of Conductors

- E is zero within the conductor
- Any net charge, Q, is distributed on surface (surface charge density $\sigma = Q/A$)
- E immediately outside is $\perp$ to surface
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