Electricity & Magnetism

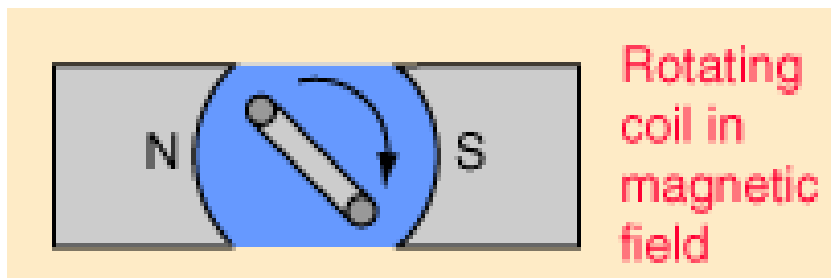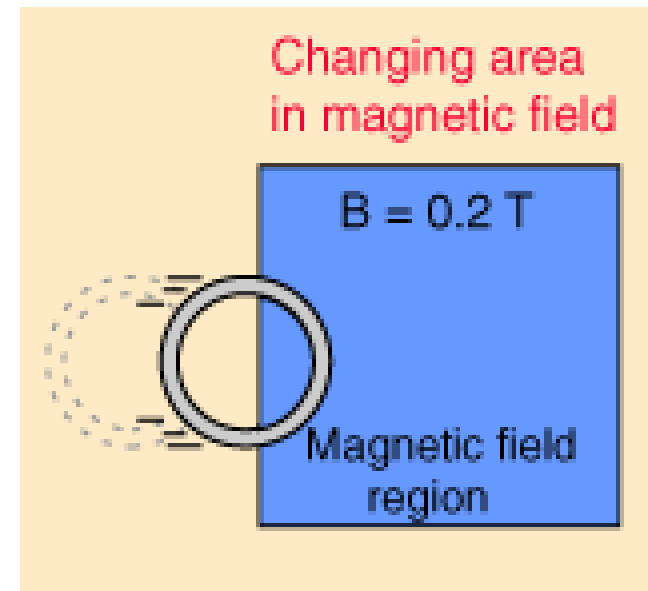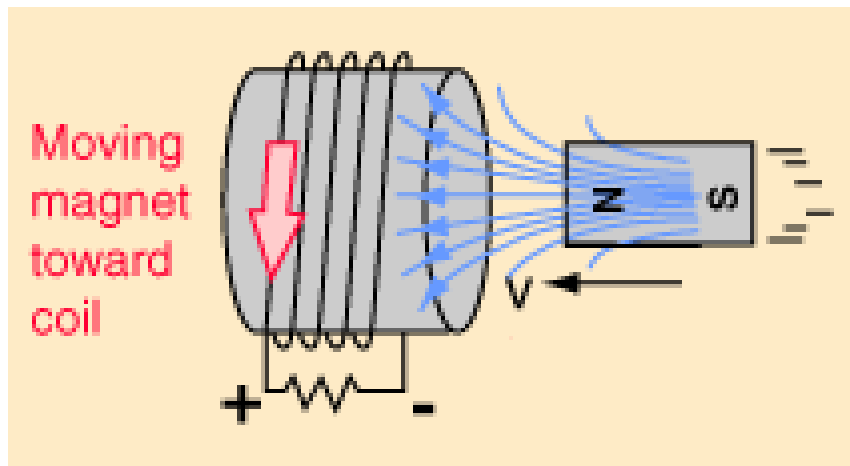## Induction and Inductor

Mohammed Q. Taha

Faraday's Law
AC Circuits

The electric current can give rise to a magnetic field, and that a magnetic field can exert a force on a moving charge.

I wonder if a magnetic field can somehow give rise to an electric current…

• A magnet may move through a loop of wire, or a loop of wire may be moved through a magnetic field (as suggested in the previous slide). These involve observable motion.

Moving magnet toward coil

Changing area in magnetic field

B = 0.2 T

Magnetic field region

Rotating coil in magnetic field

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/farlaw2.html#c1

Mathematically, magnetic flux ϕ_B through a surface of area A is defined by
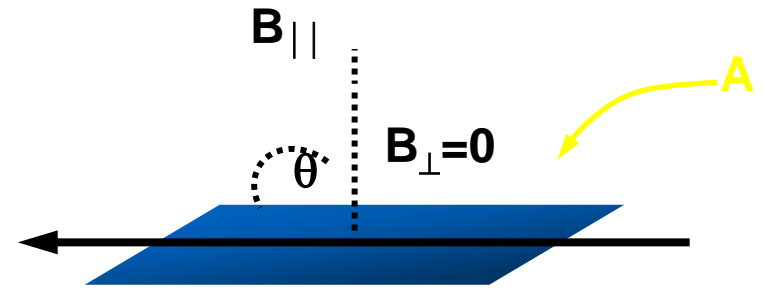
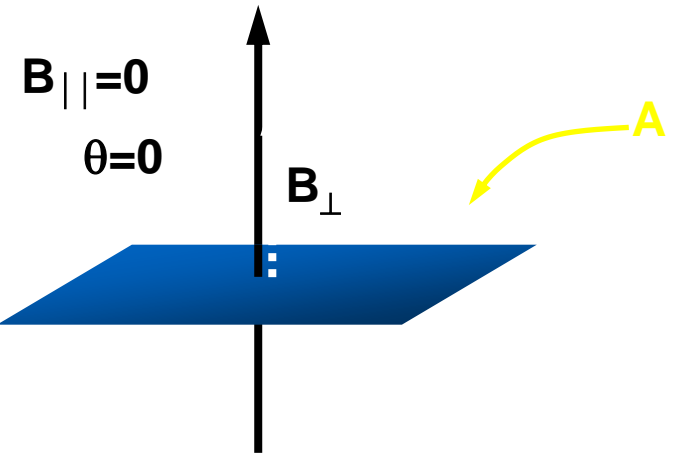$$\phi_B = B\ A\ \cos\theta$$

$$\phi_B = B_{\perp}A$$

where $B_{\perp}$ is the component of field perpendicular to the surface, A is the area of the surface, and θ is the angle between B and the normal to the surface.
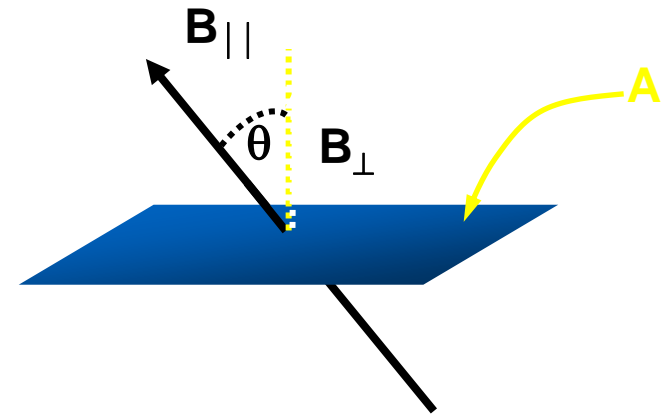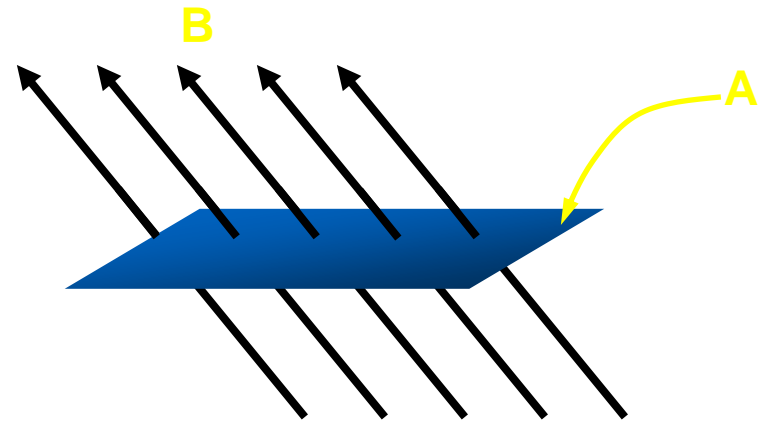
When B is parallel to the surface, $\theta=90°$ and $\phi_B = 0$.

$B_{||}$
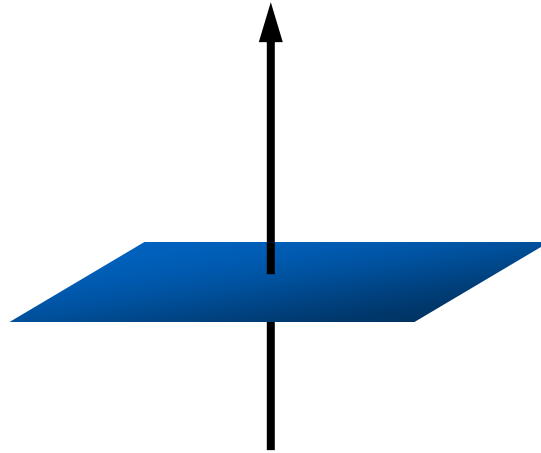
$B_\perp = 0$

A

θ

When B is perpendicular to the surface, $\theta=0°$ and $\phi_B = BA$.

$B_{||}=0$

$\theta=0$

$B_\perp$

A

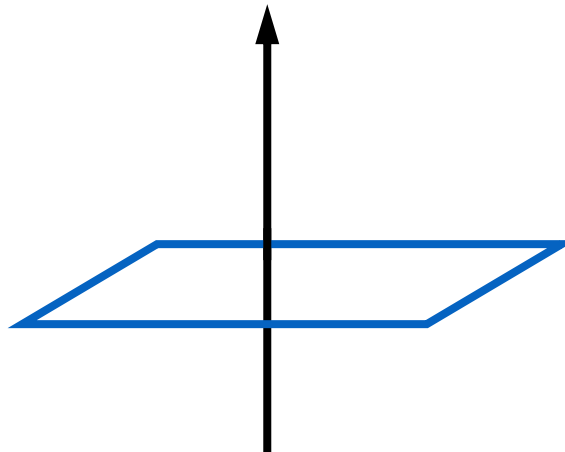The unit of magnetic flux is the T·m², called a weber:

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2 .$$

In the following discussion, we switch from talking about surfaces in a magnetic field…

…to talking about loops of wire in a magnetic field.

Now we can quantify the induced emf described qualitatively in the previous section

Experimentally, if the flux through N loops of wire changes by an amount $\Delta\phi_B$ in a time Δt, the induced emf is

$$\varepsilon = -N\frac{\Delta\phi_B}{\Delta t}.$$

Not an OSE—not quite yet.

This is called Faraday's law of induction. It is one of the fundamental laws of electricity and magnetism, and an important component of the theory that explains electricity and magnetism.

$$\varepsilon = -N \frac{\Delta \varphi_B}{\Delta t}.$$
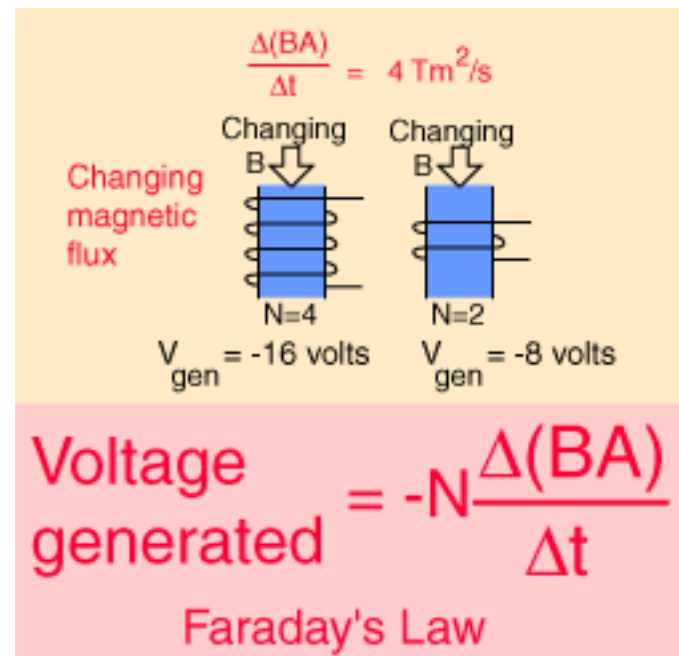
Voltage generated $= -N\frac{\Delta(BA)}{\Delta t}$

Faraday's Law

N = 5 turns   A = 0.002 m^2

Moving magnet toward coil

$\frac{\Delta B}{\Delta t}$ = 0.4 T/s

V_{gen} = -5 x 0.002 m^2 x 0.4 T/s
= -0.004 volts

Experimentally…

an induced emf always gives rise to a current whose magnetic field opposes the change in flux—Lenz's law.*
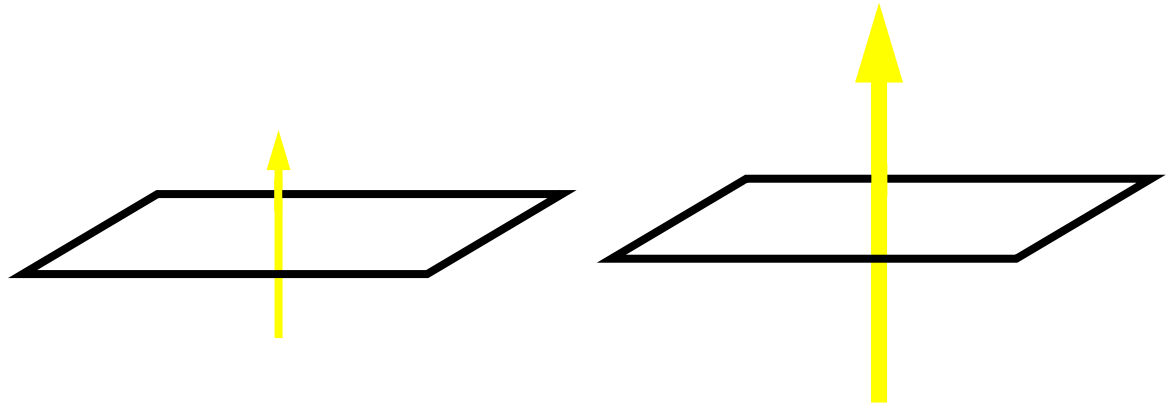
Think of the current resulting from the induced emf as "trying" to maintain the status quo—to prevent change.
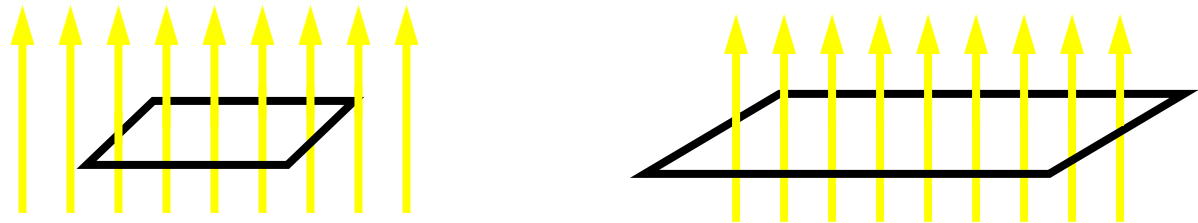
*We'll practice with this in a bit.

Ways to induce an emf:

$$\varepsilon = -N \frac{\Delta \phi_B}{\Delta t} . \qquad \phi_B = B_\perp A$$
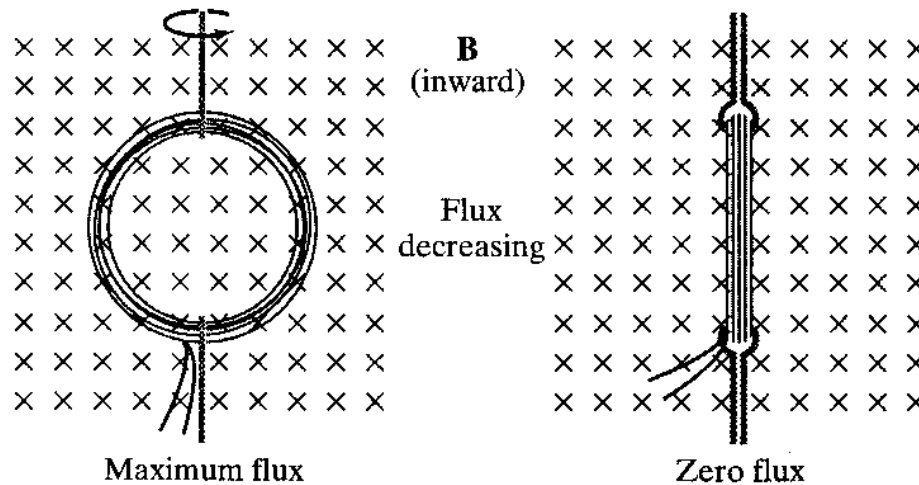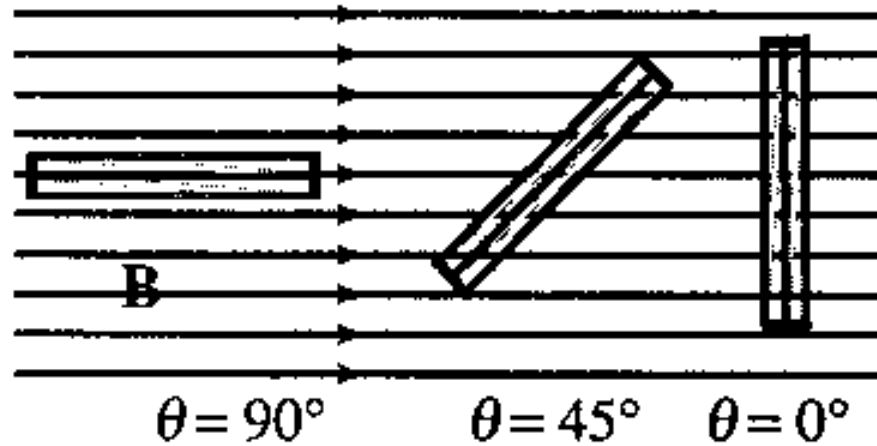
- change **B**

- change **A** the area of the loop in the field

Ways to induce an emf (continued):

• Changing The Angle by change the orientation of the loop in the field

$\theta = 90°$ $\theta = 45°$ $\theta = 0°$

Maximum flux   Zero flux

Conceptual example: Induction Stove

An ac current in a coil in the stove top produces a changing magnetic field at the bottom of a metal pan.

The changing magnetic field gives rise to a current in the bottom of the pan.

Because the pan has resistance, the current heats the pan. If the coil in the stove has low resistance it doesn't get hot but the pan does.
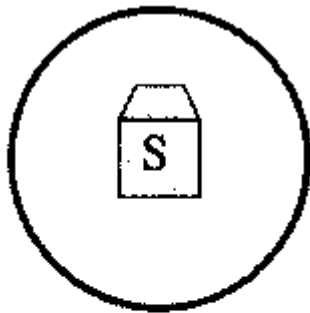
An insulator won't heat up on an induction stove.
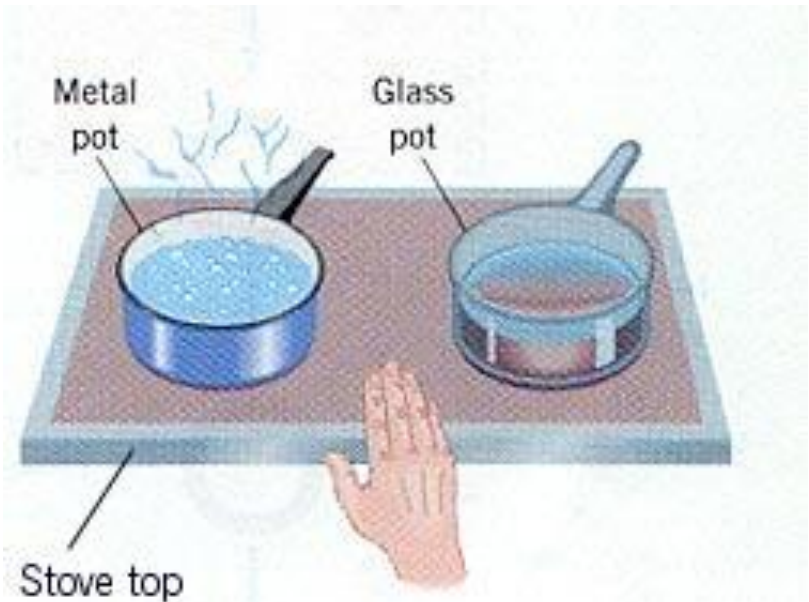
Remember the controversy about cancer from power lines a few years back? Careful studies showed no harmful effect. Nevertheless, some believe induction stoves are hazardous.
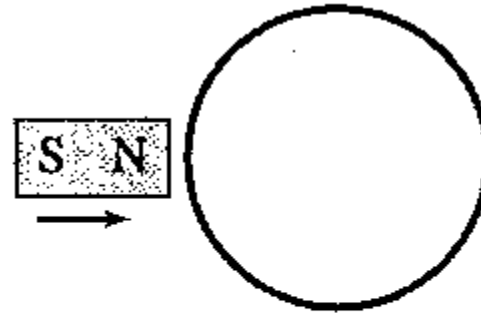
In which direction is the current induced in the coil for each situation shown?

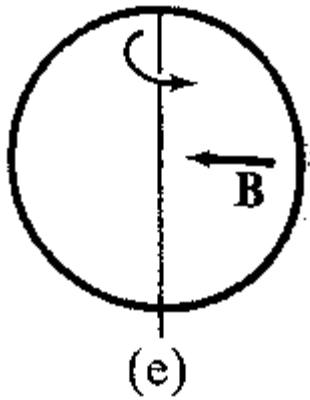
(a)
N magnetic pole
moving toward coil
from above the page

$$|\varepsilon| = \left| N \frac{\Delta \varphi_B}{\Delta t} \right|.$$

(b)
N magnetic pole moving
toward the coil in the plane
of the page

(no current)

(e)

(counterclockwise)

Rotating the coil about the vertical diameter by pulling the left side toward the reader and pushing the right side away from the reader in a magnetic field that points from right to left in the plane of the page.

Remember $\varepsilon = -N\dfrac{\Delta\varphi_B}{\Delta t}$?

Now that we are experts on the application of Lenz's law, lets make our induced emf equation official:

$$\left|\varepsilon\right| = \left|N\dfrac{\Delta\varphi_B}{\Delta t}\right|.$$

This means it is up to you to use Lenz's law to figure out the direction of the induced current (or the direction of whatever the problem wants.

Example 21-3 Pulling a Coil from a Magnetic Field

A square coil of side 5 cm contains 100 loops and is positioned perpendicular to a uniform 0.6 T magnetic field. It is quickly and uniformly pulled from the field (moving ⊥ to B) to a region where the field drops abruptly to zero. It takes 0.10 s to remove the coil, whose resistance is 100 Ω.

B = 0.6 T

5 cm

Initial: ϕ_{Bi} = BA .
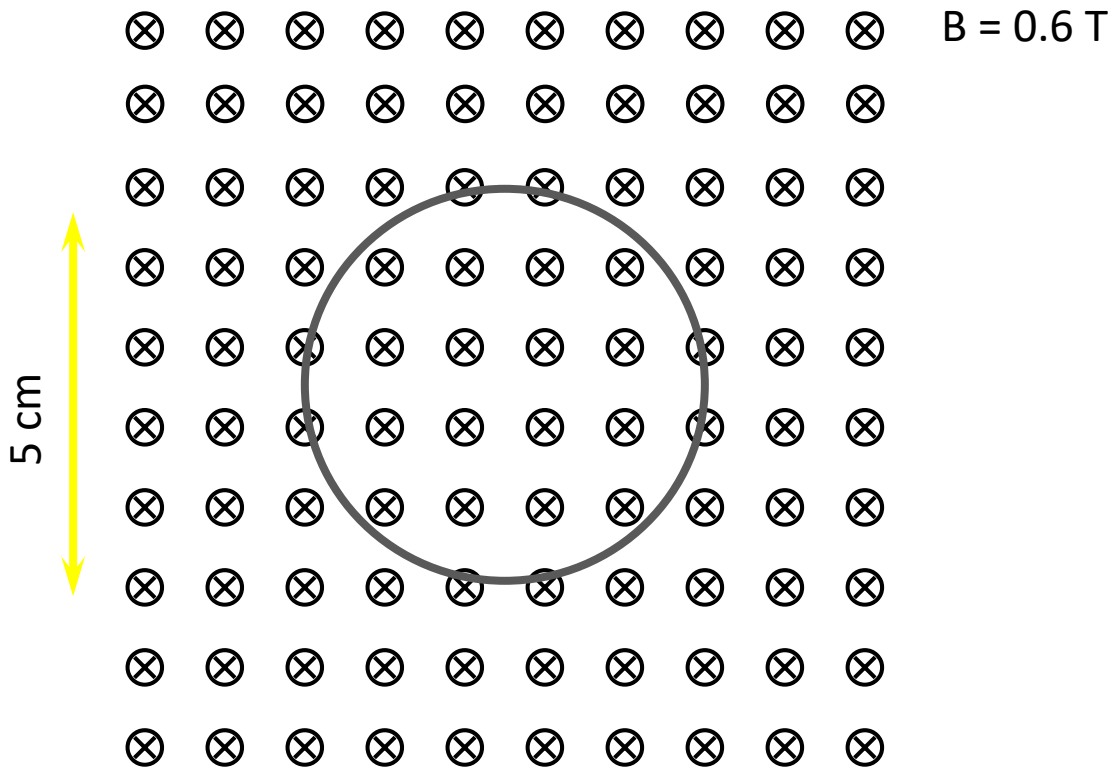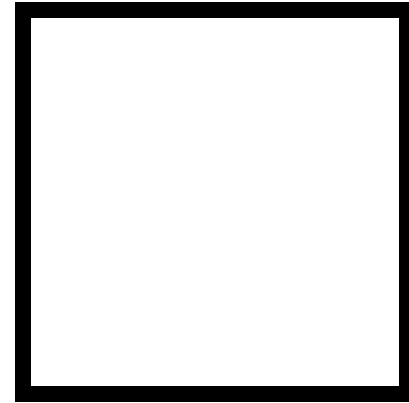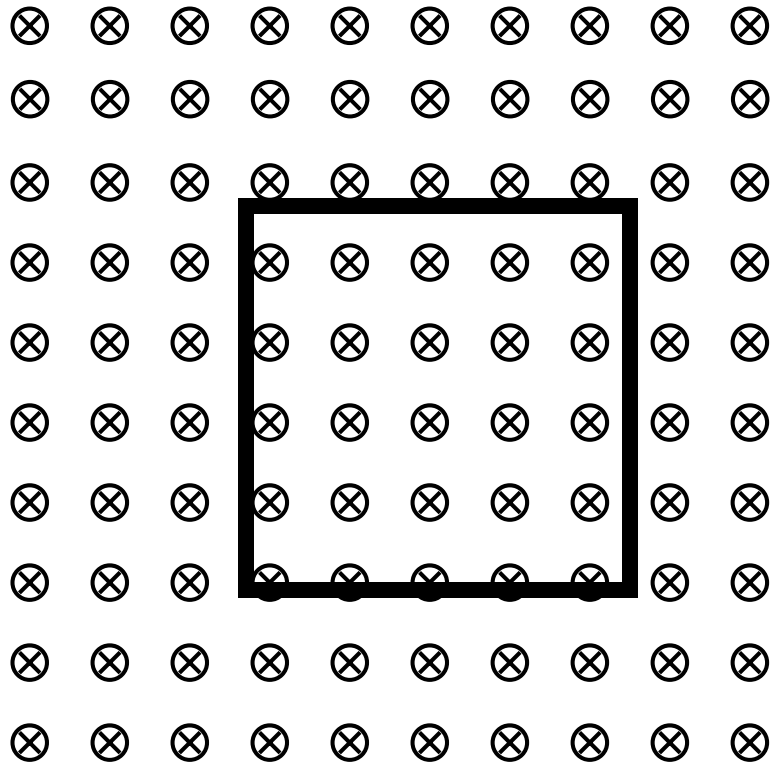
Final: ϕ_{Bf} = 0 .

$\Delta\phi_B = \phi_{Bf} - \phi_{Bi} = 0 - BA = -(0.6\ T)(0.05\ m)^2 = -1.5 \times 10^{-3}\ Wb$.

initial

final
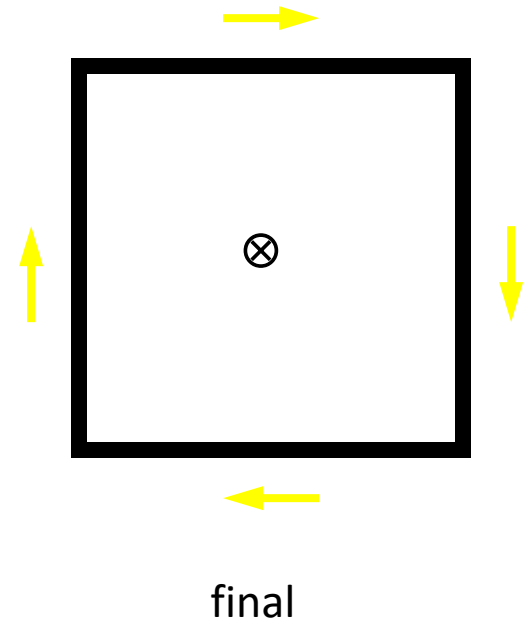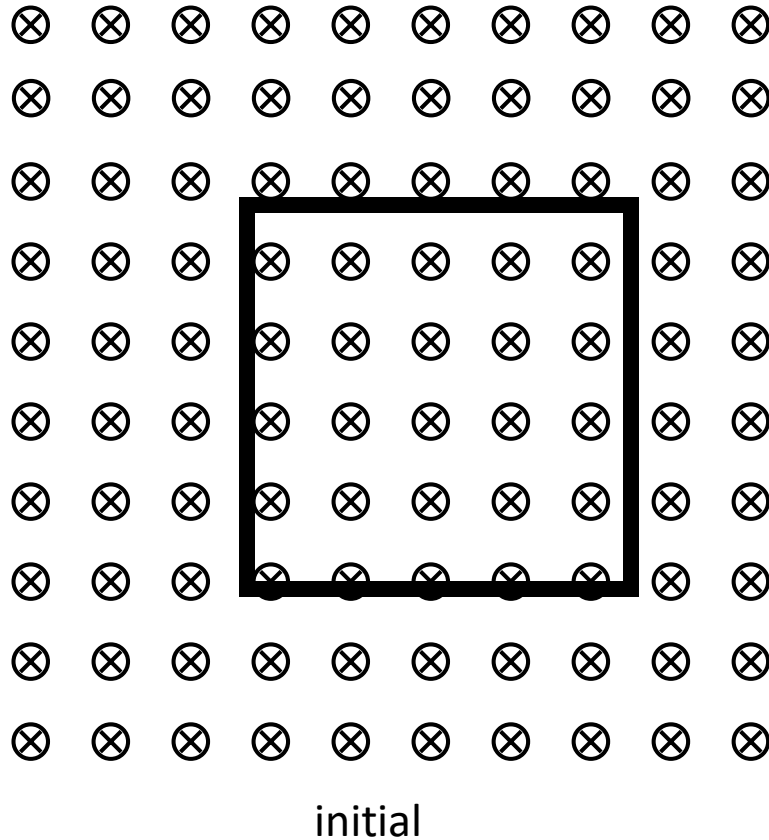
The current must flow counterclockwise to induce a downward magnetic field (which replaces the "removed" magnetic field).

The induced emf is

$$|\varepsilon| = \left| N \frac{\Delta \varphi_B}{\Delta t} \right| \implies |\varepsilon| = \left| (100) \frac{(-1.5 \times 10^{-3} \, \text{Wb})}{(0.1 \, \text{s})} \right| \implies \boxed{|\varepsilon| = 1.5 \, \text{V}}$$

The induced current is
$$\boxed{I = \frac{\varepsilon}{R} = \frac{1.5 \, \text{V}}{100 \, \Omega} = 15 \, \text{mA} \, .}$$

(c) How much energy is dissipated in the coil?

Current flows "only*" during the time flux changes.

E = Pt = I²Rt = (1.5x10⁻² A) (100 Ω) (0.1 s) = 2.3x10⁻³ J .

(d) What was the average force required?

The loop had to be "pulled" out of the magnetic field, so the pulling force did work

The flux change occurs only when the coil is in the process of leaving the region of magnetic field.
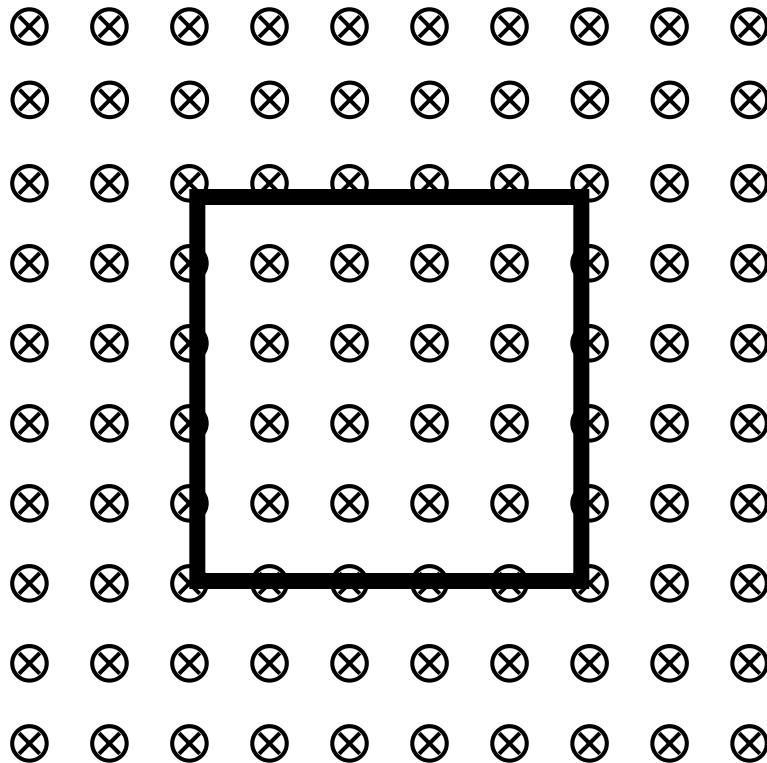
No flux change.
No emf.
No current.
No work (why?).

$F_{applied}$

D
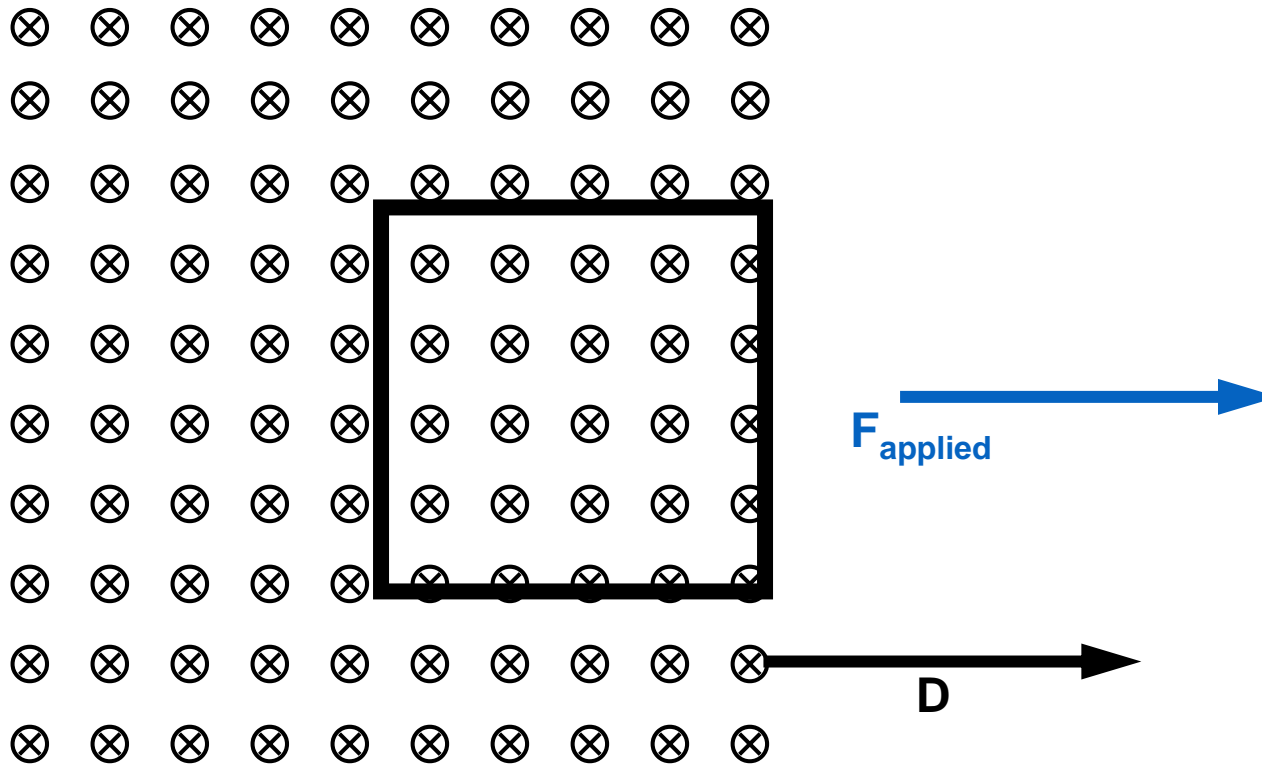
Flux changes. emf induced. Current flows. Work done.

⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
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⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗
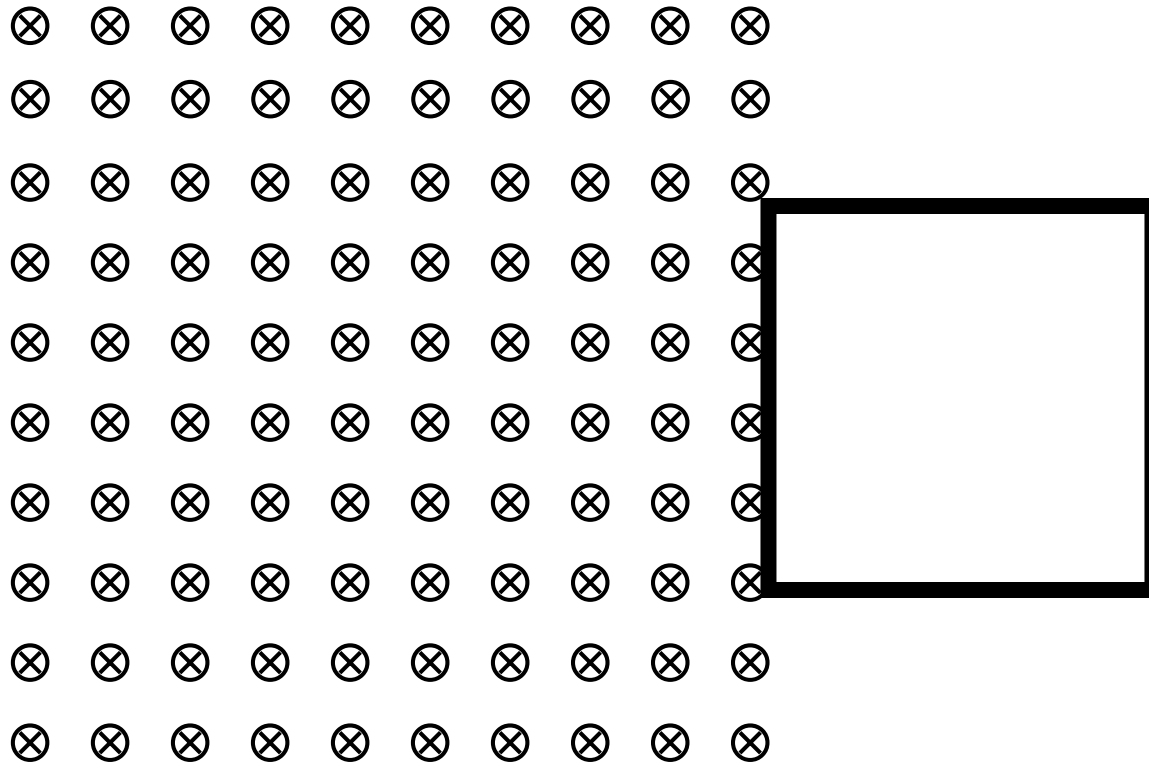
No flux change. No emf. No current. (No work.)

The energy calculated in part (c) is the energy dissipated in the coils while the current is flowing. The amount calculated in part (c) is also the mechanical energy put into the system by the force.

$$E_f - \cancel{E_i} = [\, W_{other}]_{I \to f}$$

$$E_f - E_i = F\, D$$

$$F = E_f / D$$

See 2 slides back for F and D.

$$F = (2.3 \times 10^{-3}\ J) / (0.05\ m)$$

$$F = 0.046\ N$$

Induced electric fields

To change the magnetic flux we can change:

1. the magnitude B of the magnetic field within the coil

2. the area of the coil, or the portion of that area that lies within the magnetic field (eg expanding the coil or moving it in or out of the field)

3. the angle between the direction of the field <u>B</u> and the area of the coil (eg by rotation of the coil)

Closed and open circuits

An induced emf does not necessarily mean an induced *current*

B_{in} increasing in strength

Field of induced current points out of page . . .

. . . so right-hand rule gives counter-clockwise current.

$\vec{B}_{in}$

I

$\vec{B}$

+

—
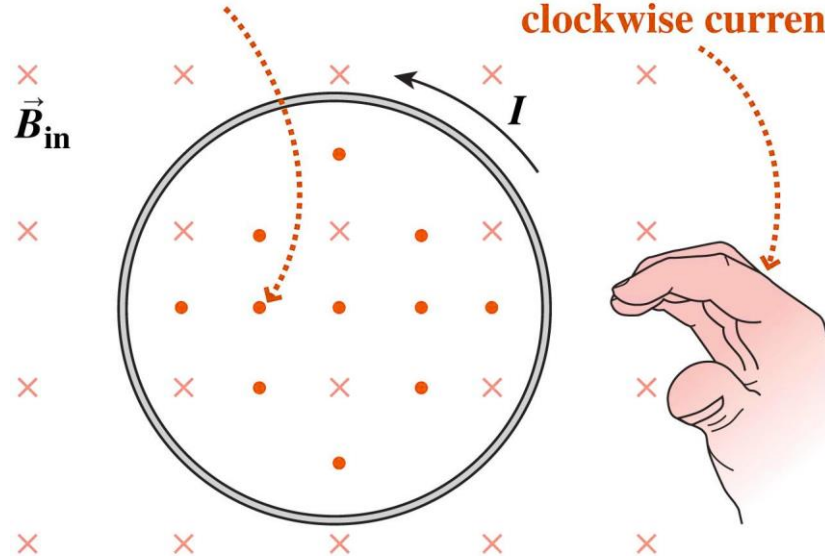
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Voltage across the gap
= induced emf

Increasing the strength of the magnetic field at a steady rate
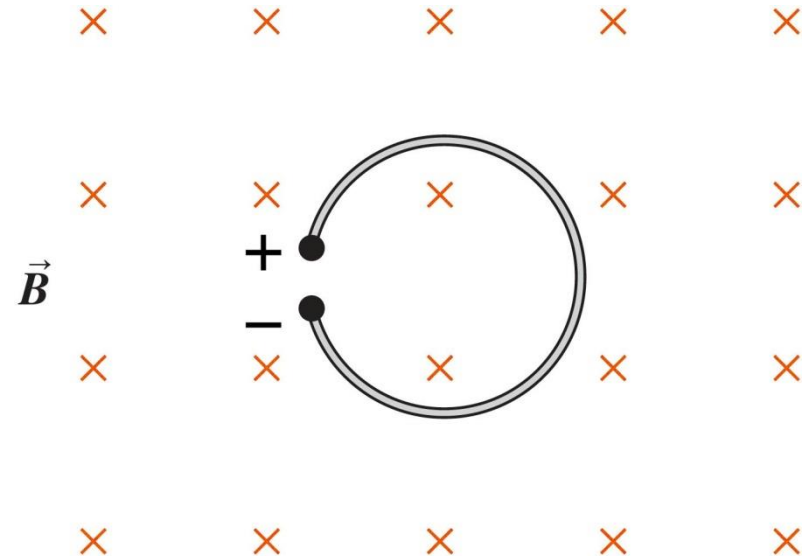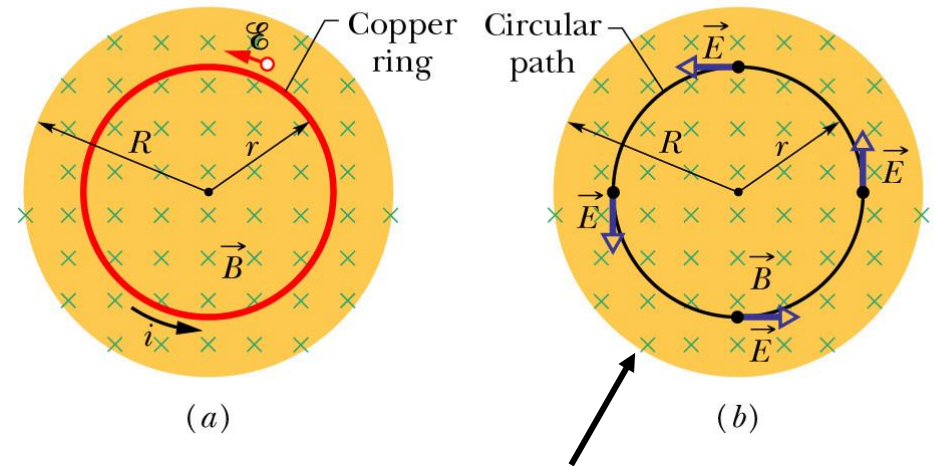
$\Rightarrow$ Faraday's law

Induced emf and induced current appear in ring

$\Rightarrow$ Lenz's law

Induced current is anti-clockwise

The electric field is induced even if there is no copper ring

Electric field lines

R

$\vec{B}$
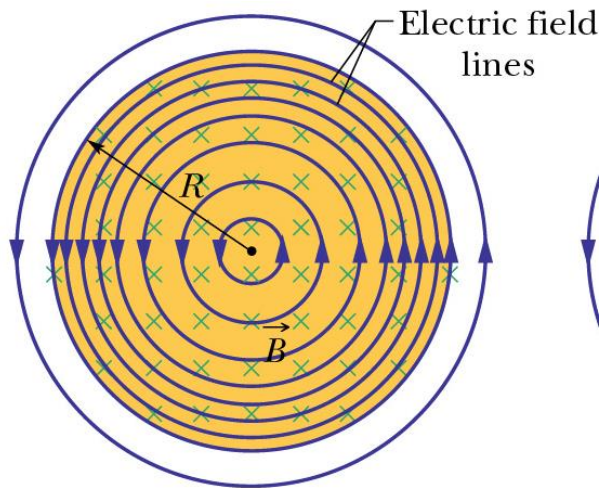
(c)

A changing magnetic field produces an electric field

As long as the magnetic field is increasing with time, the electric field will be present as circular lines (by symmetry).

If the magnetic field is constant with time, there will be no electric field lines.

If the magnetic field is decreasing with time, the electric field lines will be circles in the opposite direction

Another way of stating Faraday's Law

B changing
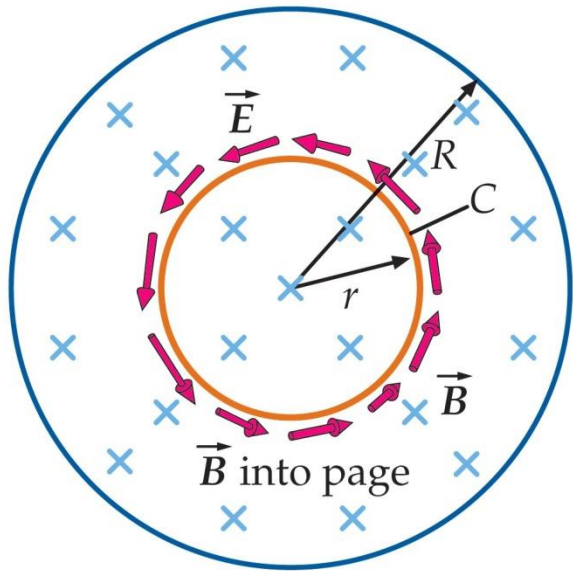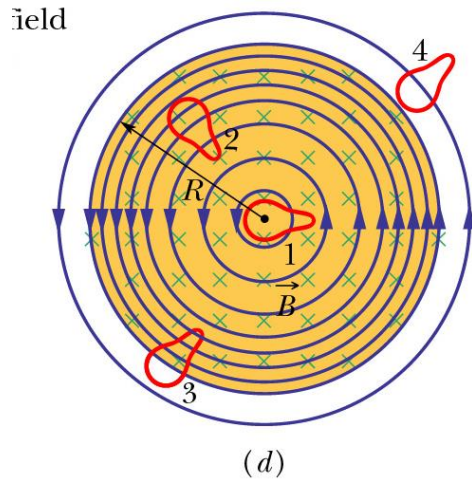
Consider a particle of charge q_0 moving around the circular path in figure. **The work done** in one revolution by the induced electric field is εq_0, where ε is the induced emf, ie the work done per unit charge in moving the test charge around the path.

$$\varepsilon = \oint \underline{E} \cdot d\underline{l}$$

Another way of stating Faraday's Law

We can now expand the meaning of induced emf.

$$\varepsilon = \oint \underline{E} \cdot d\underline{l}$$
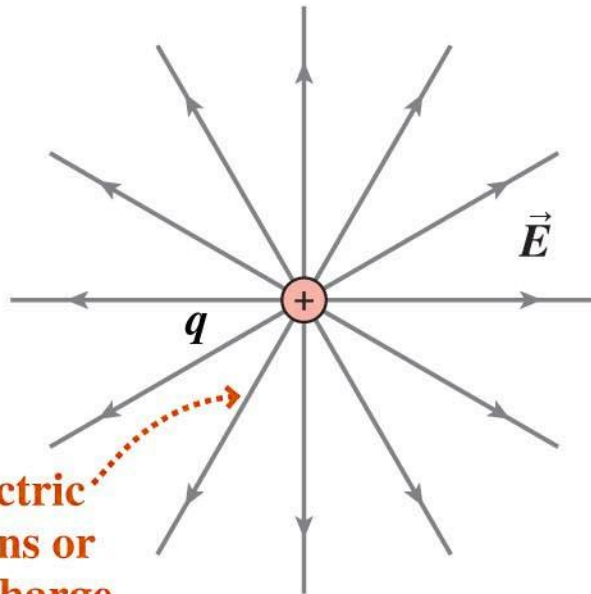
This means that an induced emf can exist without the need for a current or a particle; it is the sum (by integration) of $\underline{E}.\underline{dl}$ around a closed path.

Which of the red loops has the largest induced emf?

$$-\frac{d\Phi_B}{dt} = \oint \underline{E} \cdot d\underline{l}$$ Faraday's Law

Two sorts of electric fields

$\vec{E}$

q

Static electric field begins or ends on charge.

(a)

$\vec{E}_{\text{induced}}$

× × ×

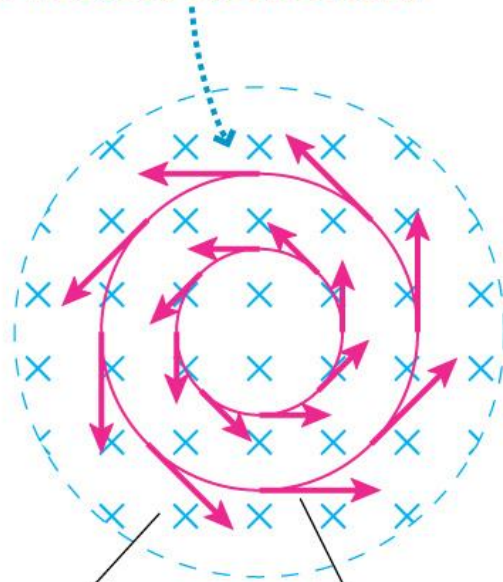× × × ×

× $\vec{B}$ × × ×

× × ×
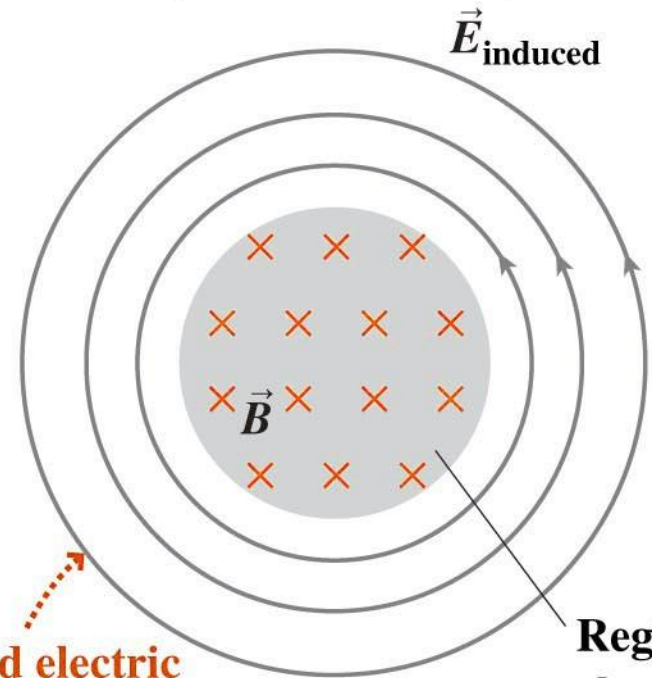
Induced electric field forms closed loops.

(b)

Region of changing magnetic field

Maxwell hypothesised the existence of induced *magnetic* fields

A changing magnetic field creates an induced electric field.

Region of increasing $\vec{B}$

Induced electric field E

A changing electric field creates an induced magnetic field.
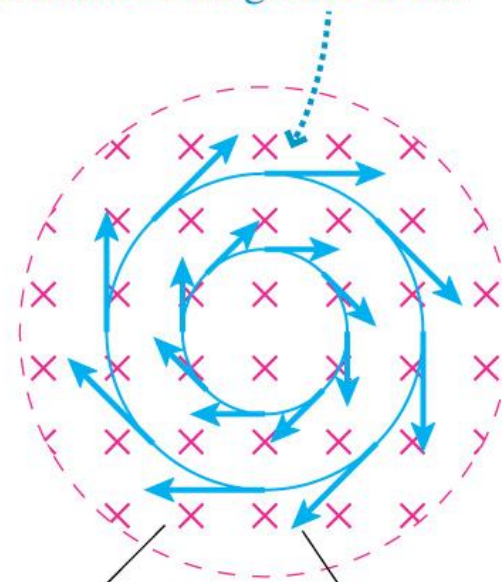
Region of increasing $\vec{E}$

Induced magnetic field $\vec{B}$

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A self-sustaining electromagnetic wave

Direction of propagation at speed $v_{\text{em wave}}$

$\vec{E}$
$\vec{B}$
$\vec{E}$
$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{E}$
$\vec{E}$
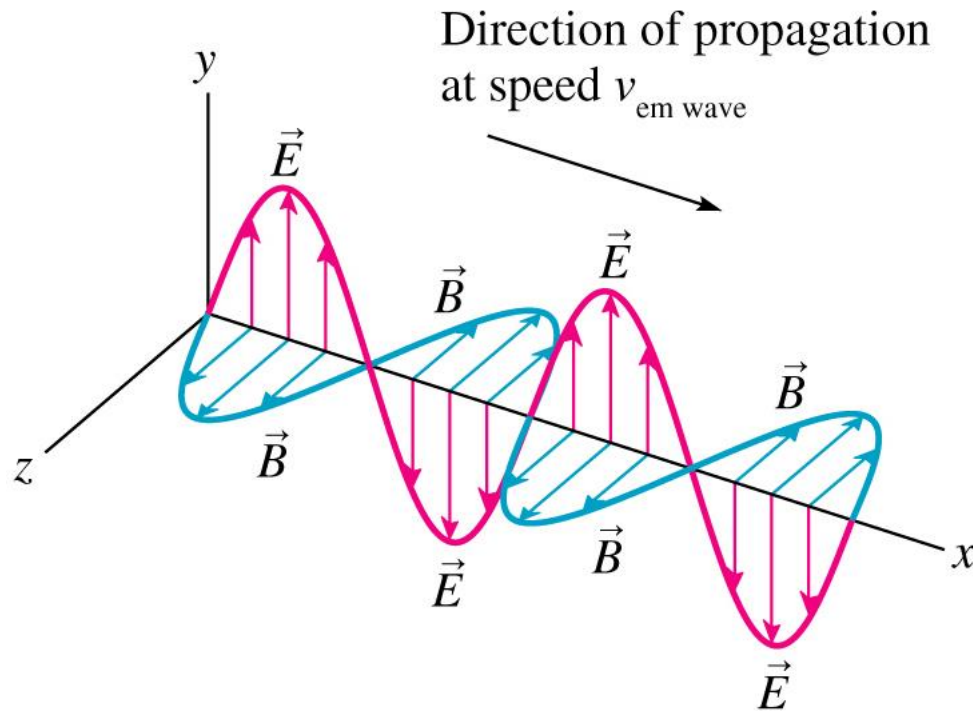
TABLE 29.2 Maxwell's Equations

| Law	Mathematical Statement	What It Says	Equation Number
Gauss for $\vec{E}$	$\oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\epsilon_0}$	How charges produce electric field; field lines begin and end on charges.	(29.2)
Gauss for $\vec{B}$	$\oint \vec{B} \cdot d\vec{A} = 0$	No magnetic charge; magnetic field lines don't begin or end.	(29.3)
Faraday	$\oint \vec{E} \cdot d\vec{r} = -\dfrac{d\Phi_B}{dt}$	Changing magnetic flux produces electric field.	(29.4)
Ampère	$\oint \vec{B} \cdot d\vec{r} = \mu_0 I \boxed{+ \mu_0 \epsilon_0 \dfrac{d\Phi_E}{dt}}$	Electric current and changing electric flux produce magnetic field.	(29.5)

Mutual inductance

- occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit.

  - Mutual inductance occurs because some of the magnetic flux produced by one circuit passes through the other circuit.
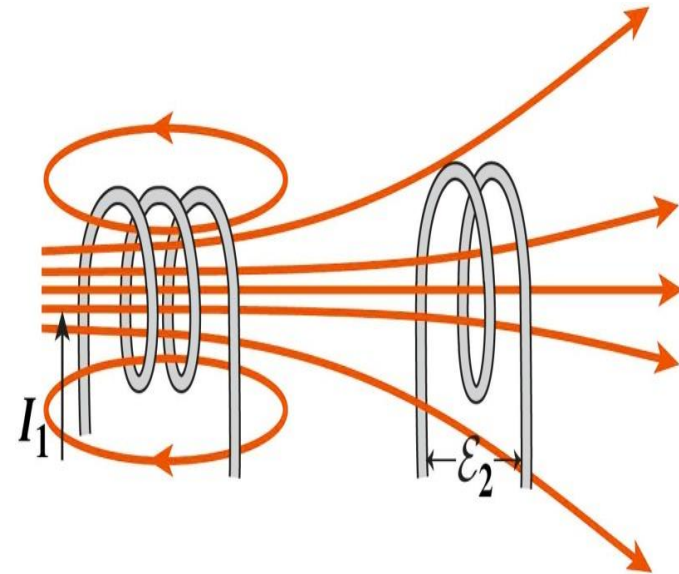
Transformers and power supplies

- A **transformer** is a pair of coils linked by mutual inductance.

- An AC current in the primary induces a current in the secondary.

- The secondary voltage differs from the primary voltage by the ratio of the number of turns.

- Both **step-up** and **step-down** transformers are possible.

- Transformers are used to produce low voltages for electronic equipment.

- Then they're combined with diodes that convert AC to DC and capacitors to smooth the DC voltage.

Primary   Secondary

(a)   (b)

(a)

Voltage across R

Voltage

Time

AC voltage from transformer

(b)
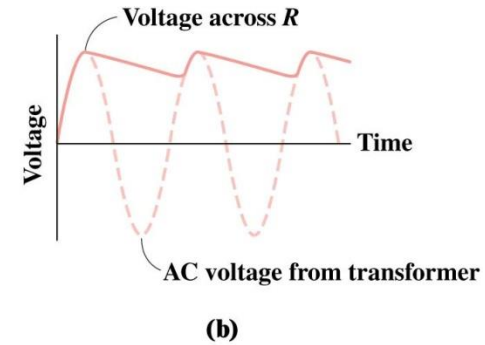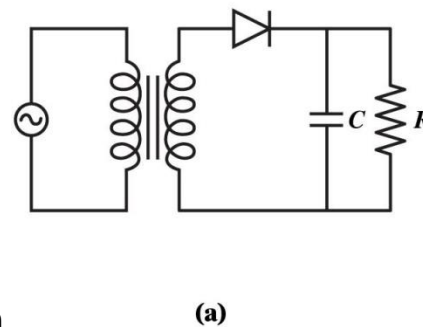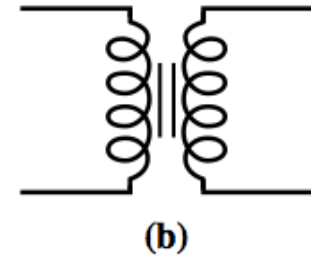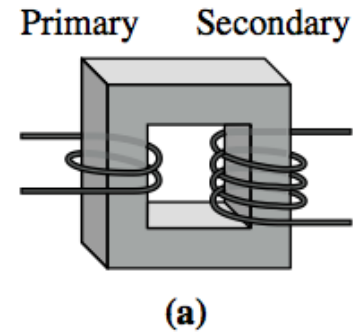
Transformers and power transmission

- Electric power is most efficiently transmitted at high voltages.
    - This reduces I^2R energy losses in the power lines.
    - But most end uses require lower voltages.
    - Transformers accomplish voltage changes throughout the power grid.
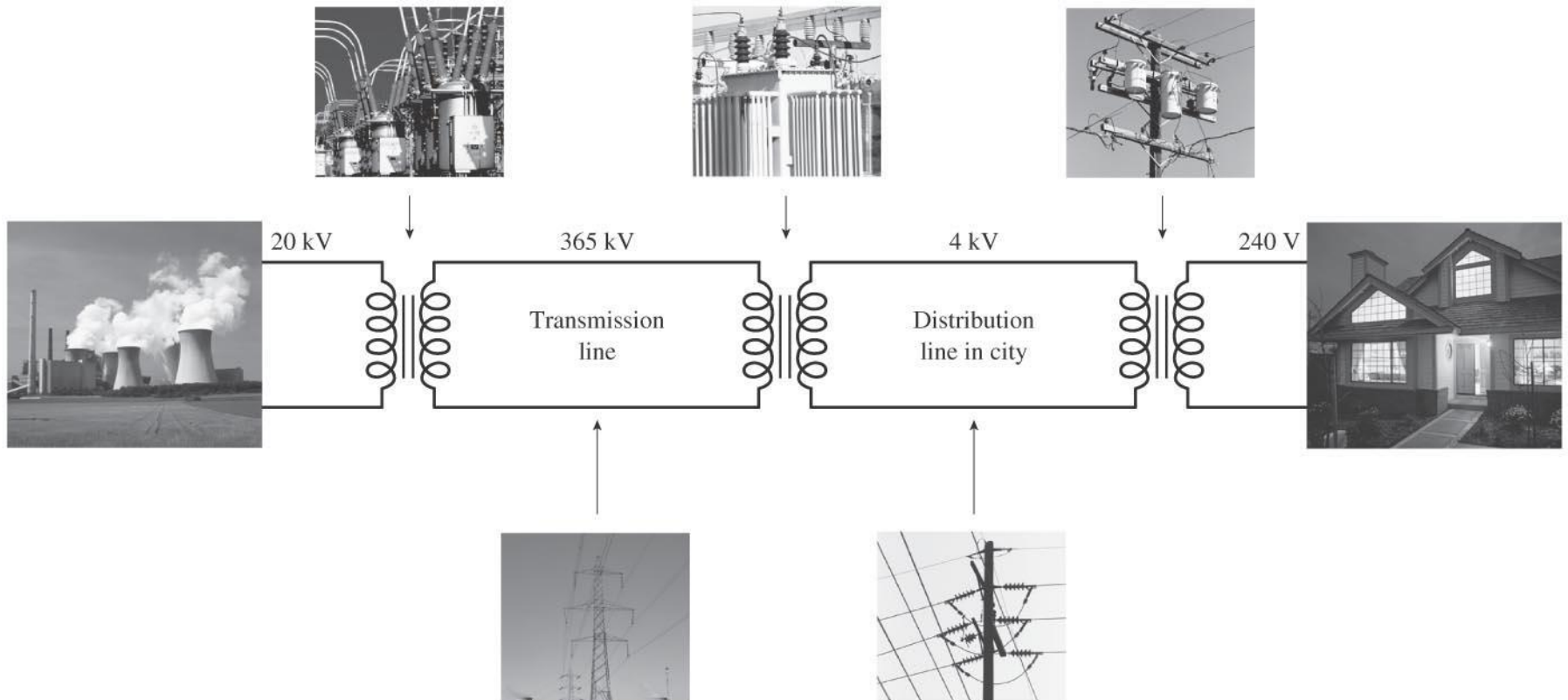
Inductance

The current i in the circuit causes a magnetic field $\underline{B}$ in the coil and hence a flux through the coil. When the current **changes**, the flux changes also and a self-induced emf appears.
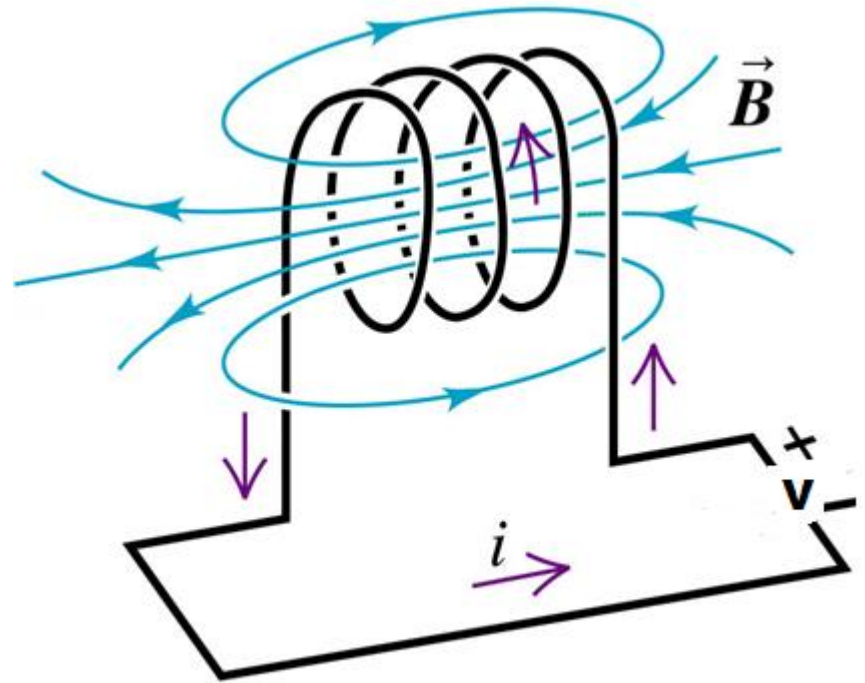
Inductance

The magnetic flux through a circuit is related to the current in that circuit and in other circuits nearby.

For a coil carrying current I, there is a magnetic field produced around it. The value of <u>B</u> at each point is proportional to the current. Therefore the magnetic flux through the coil is also proportional to I:

$$\phi_m = LI$$

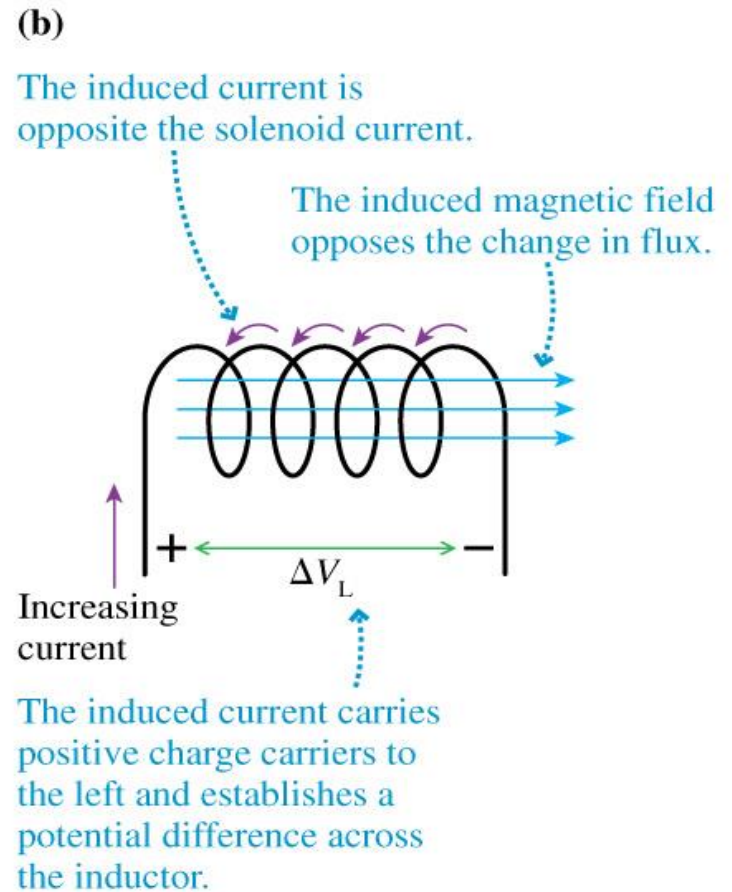DEFINITION—SELF-INDUCTANCE

Constant of proportionality

The potential difference across an Inductor
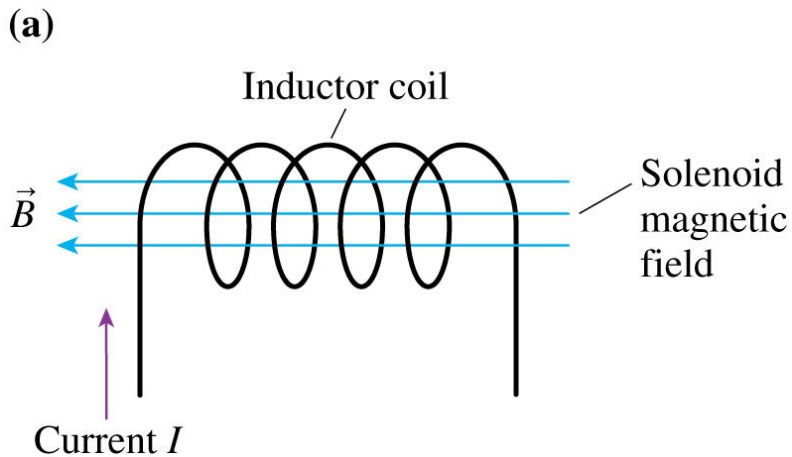
(a)

Inductor coil

$\vec{B}$

Solenoid magnetic field

Current I

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(b)

The induced current is opposite the solenoid current.

The induced magnetic field opposes the change in flux.

ΔV_L

Increasing current

The induced current carries positive charge carriers to the left and establishes a potential difference across the inductor.
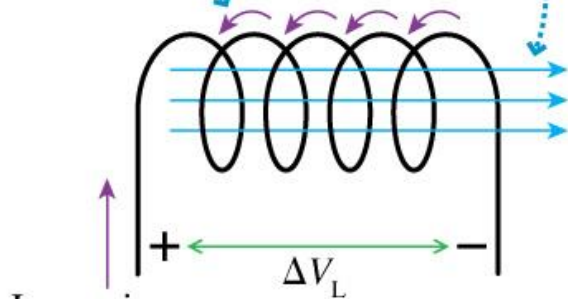
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(b)

The induced current is opposite the solenoid current.

The induced magnetic field opposes the change in flux.

ΔV_{L}

Increasing current

The induced current carries positive charge carriers to the left and establishes a potential difference across the inductor.

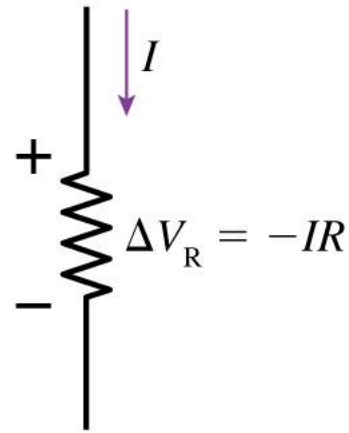Induced current   Induced field

ΔV_{L}
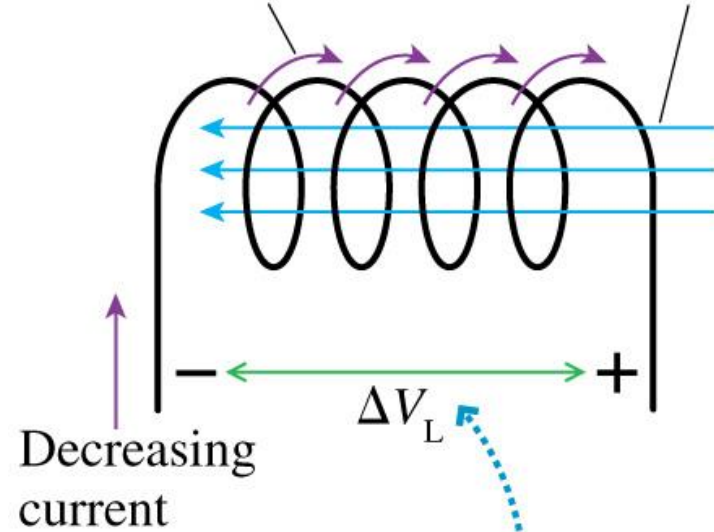
Decreasing current

The induced current carries positive charge carriers to the right. The potential difference is opposite that of Figure 33.39.
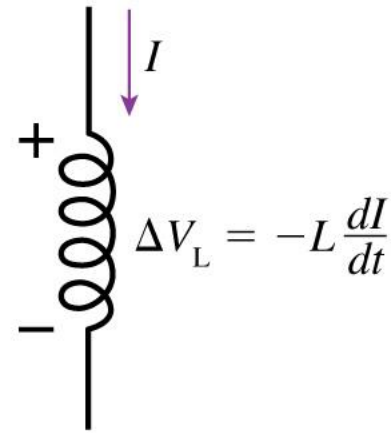
Inductance

Resistor

I

$+$

$\Delta V_R = -IR$

$-$

The potential always decreases.

Inductor
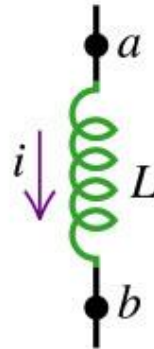
I

$+$

$\Delta V_L = -L\dfrac{dI}{dt}$

$-$

The potential decreases if the current is increasing.

The potential increases if the current is decreasing.

Magnetic Energy

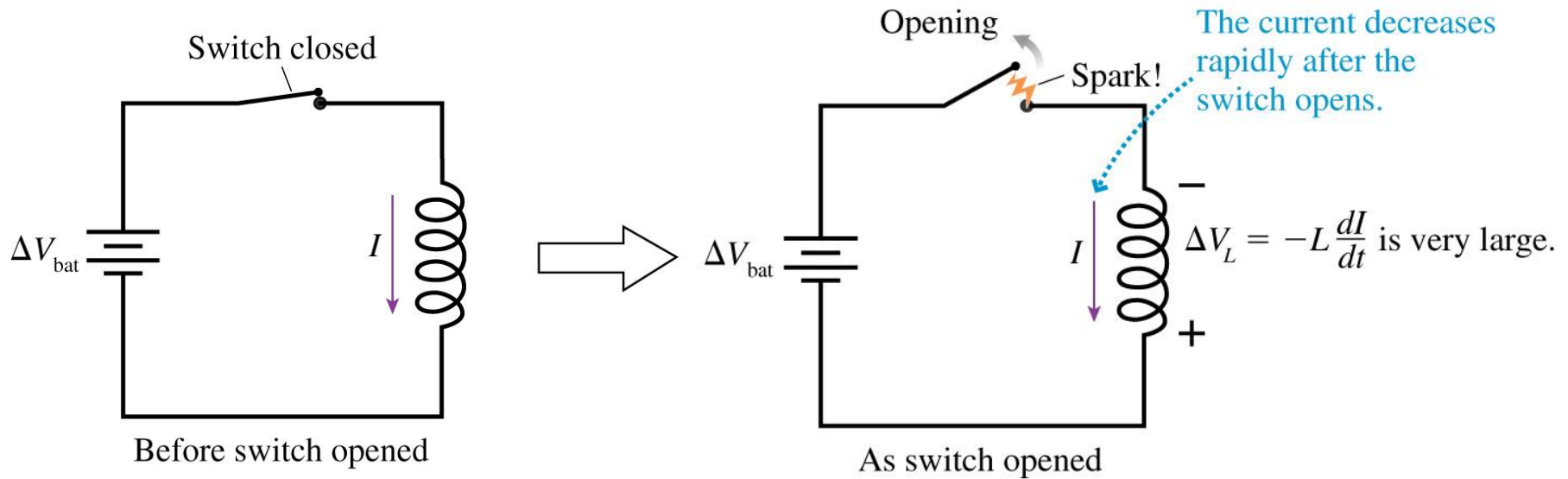
Resistor with current i:
energy is *dissipated*

Inductor with current i:
energy is *stored*

Magnetic Energy

$$U = L \int_0^I i \, di = \frac{1}{2} L I^2 \qquad \text{(energy stored in an inductor)} \qquad (30.9)$$

What is the energy **density** stored in an inductor?

Homework: Find this for a solenoid with N turns, area A, and length *l*.

Switch closed

ΔV_{bat}

I

Before switch opened

Opening

Spark!

The current decreases rapidly after the switch opens.

ΔV_{bat}

I

$-$

$\Delta V_L = -L\dfrac{dI}{dt}$ is very large.

$+$

As switch opened

This is how spark plugs work. The car's generator sends a large current through the coil (inductor). A switch in the distributor is suddenly opened, breaking the current. The induced voltage, usually a few 1000 V, appears across the terminals of the spark plug, creating the spark that ignites the fuel.