Electric Circuits

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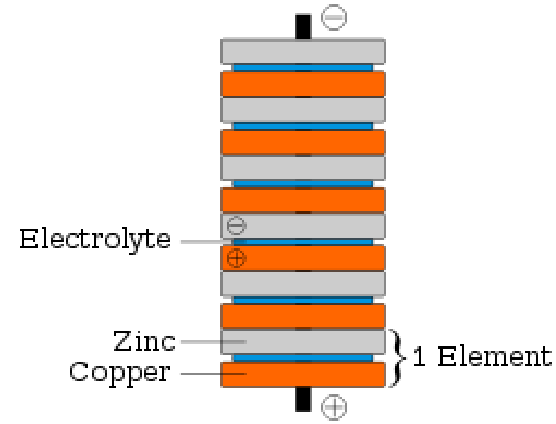
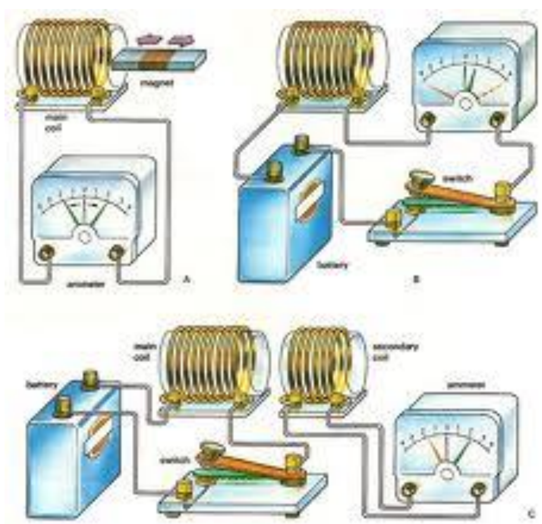
A brief history

1800 – voltaic pile developed by Alessandro Volta, a precursor to the **battery**



Voltaic pile

1831 – Michael Faraday discovers **electromagnetic induction**



Circuits containing inductors

1873 – *Electricity and Magnetism* published by James Maxwell, describing a **theory for electromagnetism**

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Maxwell's equations

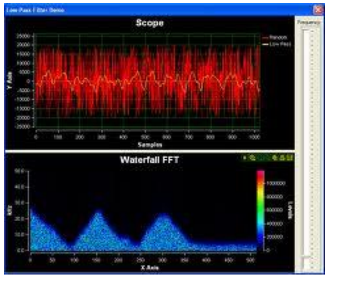
Fields of study

Power:
Creation, storage, and distribution of electricity

Control:
Design of dynamic systems and controllers for the systems

Electronics/Microelectronics:
Design of integrated circuits, microprocessors, etc.

Signal Processing: Analysis of signals



Fields of study

Telecommunications:
Design of transmission systems (voice, data)

Computer:
Design and development of computer systems

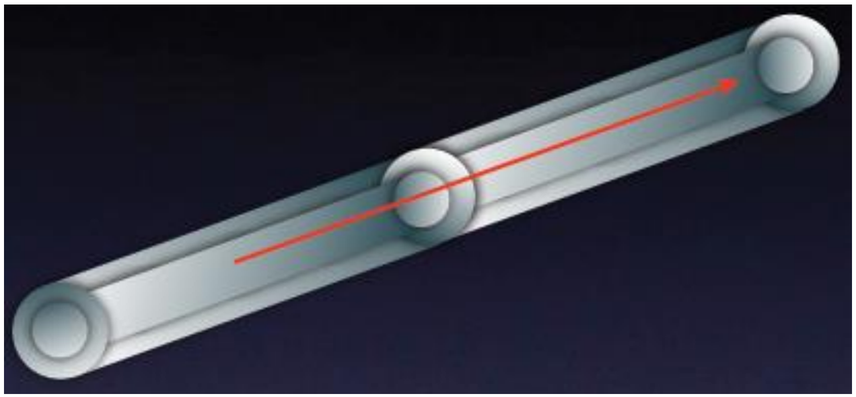
Instrumentation:
Design of sensors and data acquisition equipment

Basic concepts

Electricity: *Physical phenomenon arising from the existence and interactions of* ***electric charge***

- Charge
- Current
- Voltage
- Power and Energy

Electric current



$$I = \frac{\text{charge}}{\text{time}} = \frac{\text{coulombs}}{\text{seconds}}$$

$$I = \frac{Q}{t} \text{ amperes}$$

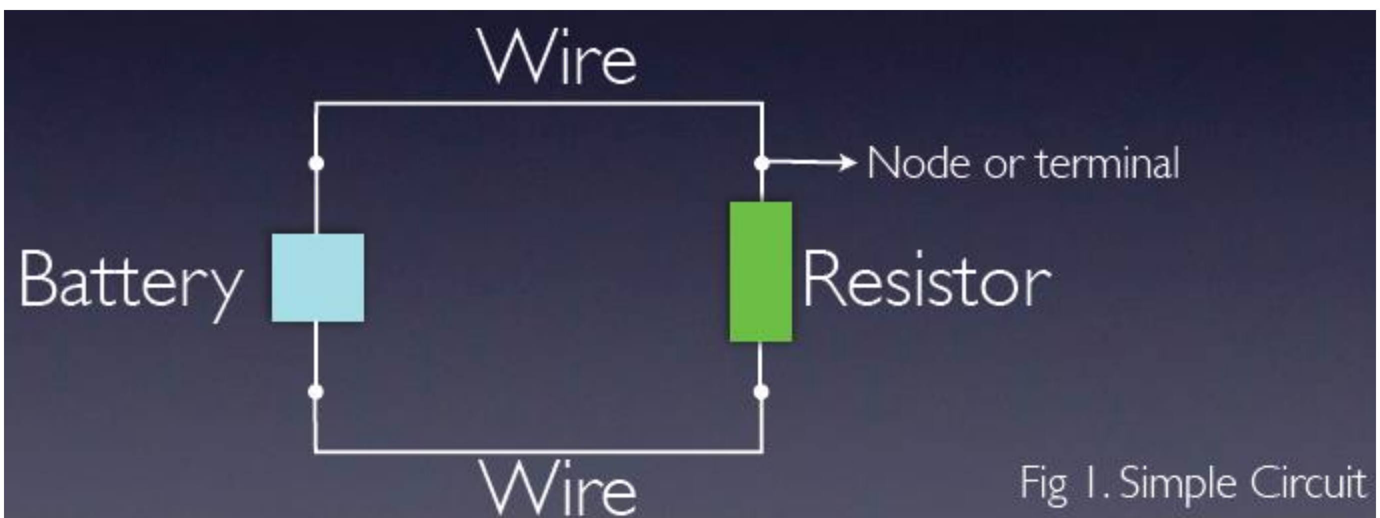
An **ampere (A)** is the number of electrons having a total charge of 1 C moving through a given cross section in 1 sec.

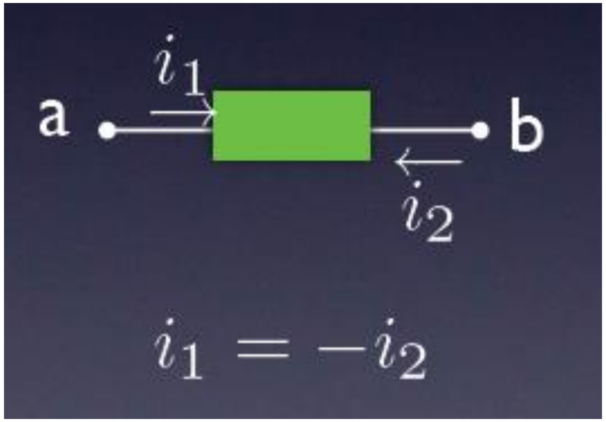
As defined, current flows in direction of **positive charge** flow

Electric circuit

An electric circuit *is an interconnection of* ***electrical elements*** *linked together in a* ***closed path*** *so that electric current may flow continuously*

Circuit diagrams are the standard for electrical engineers

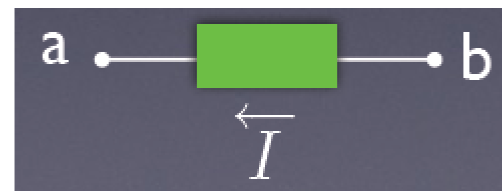




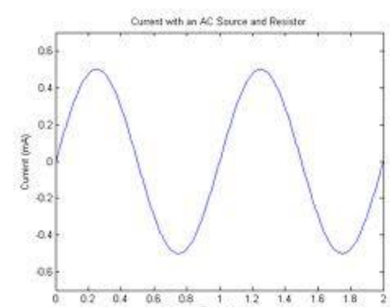
$\overrightarrow{i_1}$ Rate of flow of charge form **node a to node b**

$\overleftarrow{i_2}$ Rate of flow of charge form **node b to node a**

A **direct current (dc)** is a current of constant magnitude

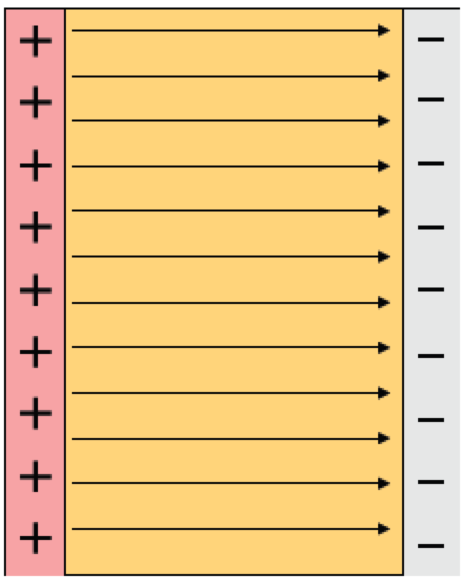


An **alternating current (ac)** is a current of varying magnitude and direction



Voltage

The voltage across an element is the work (energy) required to move a unit of positive charge from the “**−**” terminal to the “**+**” terminal



$$V = \frac{W}{Q} = \frac{\text{joules}}{\text{coulombs}} = \text{volts}$$

A **volt** is the potential difference (voltage) between two points when **1 joule of energy** is used to move **1 coulomb of charge** from one point to the other

Power

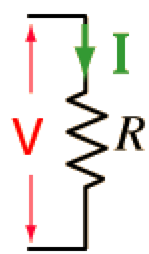
The rate at which energy is converted or work is performed

$$P = \frac{W}{t} = \frac{\text{joules}}{\text{second}} = \text{watt}$$

$$P = IV$$

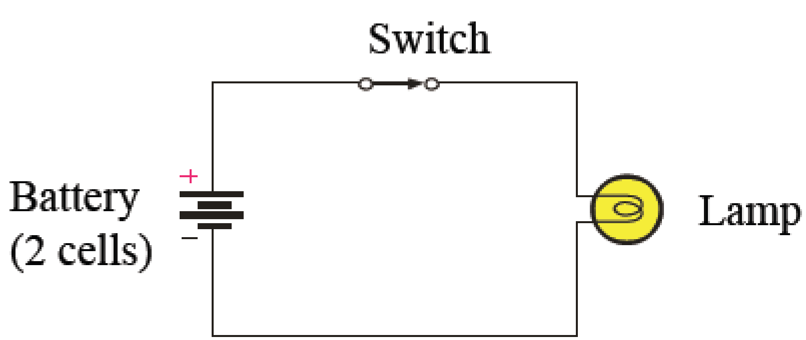
A **watt** results when **1 joule of energy** is converted or used in **1 second**

Power Dissipated in Resistor

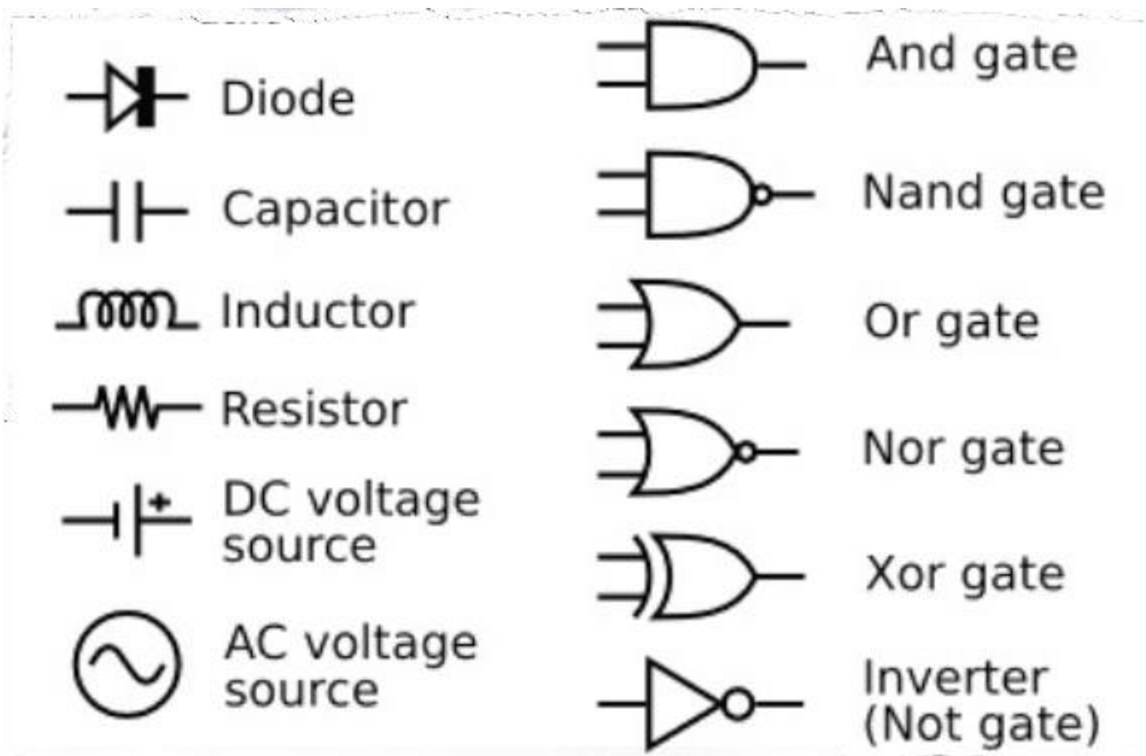


$$P = VI = \frac{V^2}{R} = I^2 R$$

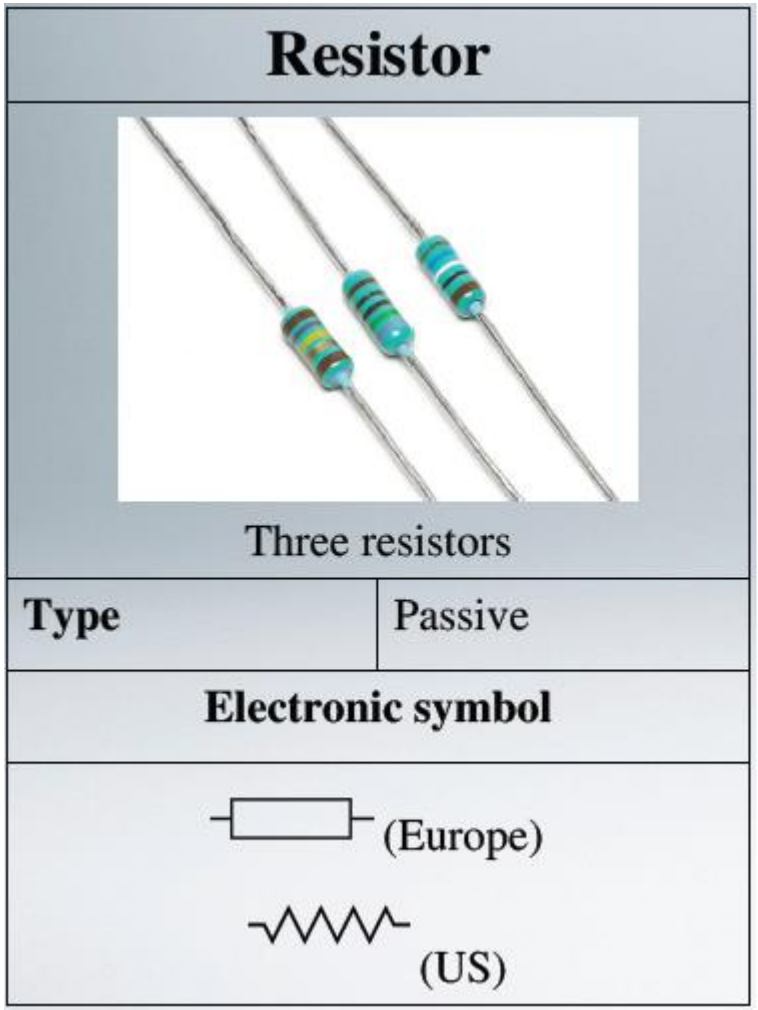
Circuit schematic example



Circuit elements



Resistors



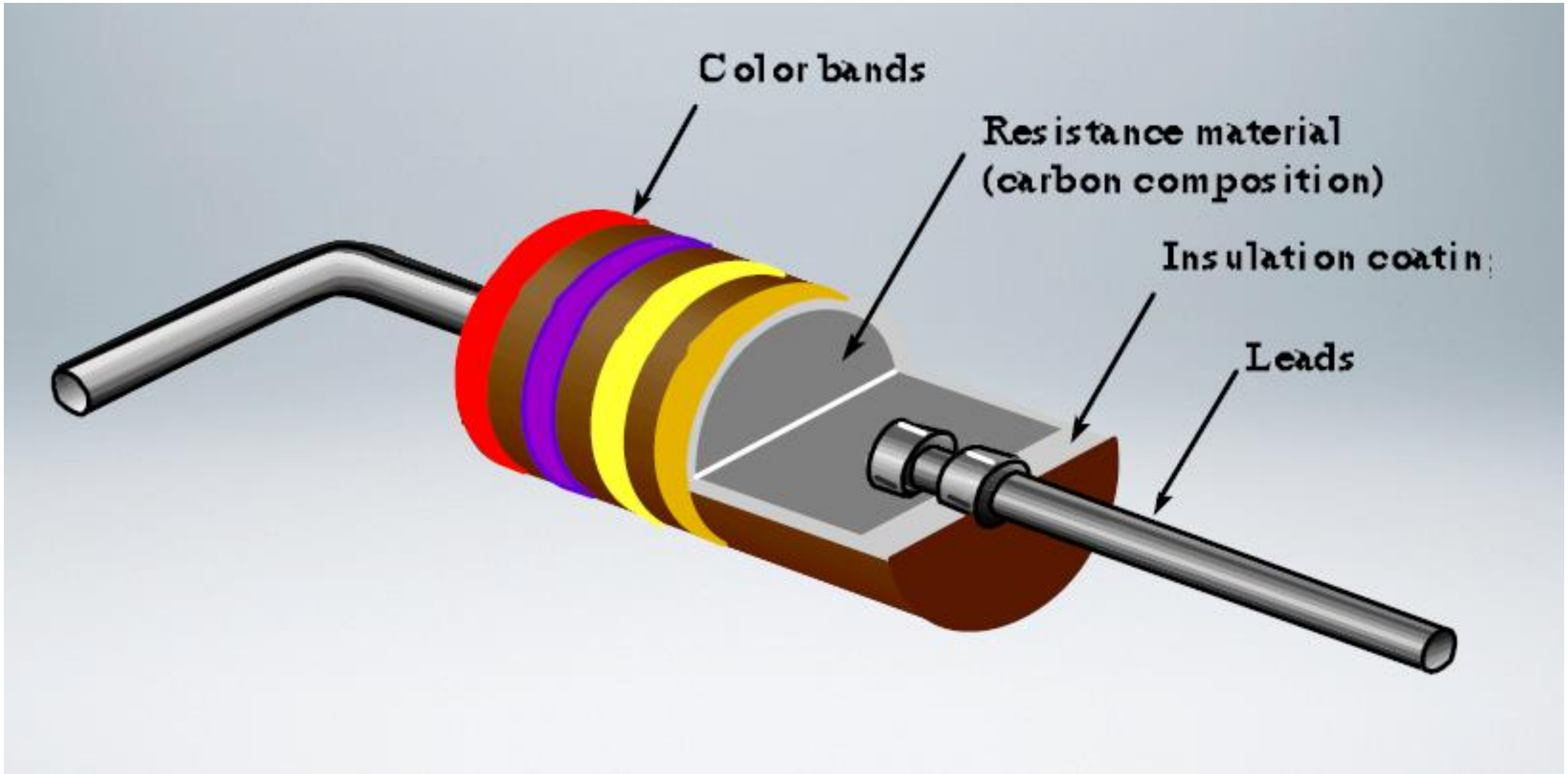
Resistance (R) is the physical property of an element that impedes the flow of current . The units of resistance are **Ohms (Ω)**

Resistivity (ρ) is the ability of a material to resist current flow. The units of resistivity are **Ohm-meters (Ω-m)**

Example:

Resistivity of copper	1.68×10^{-8} Ω·m
Resistivity of glass	10^{10} to 10^{14} Ω·m

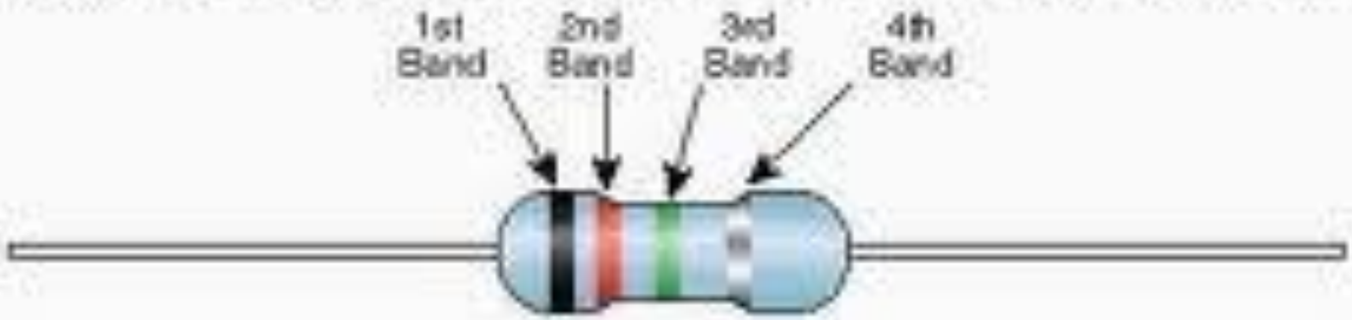
Resistors



Color bands
Resistance material
(carbon composition)
Insulation coatin
Leads

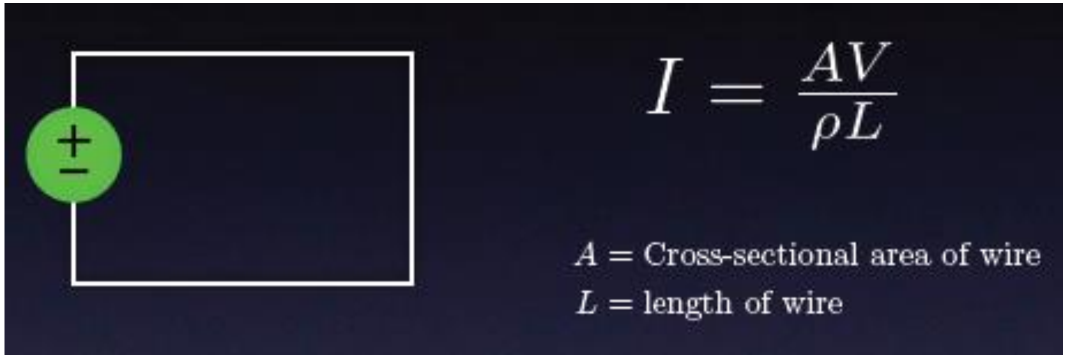
Resistors

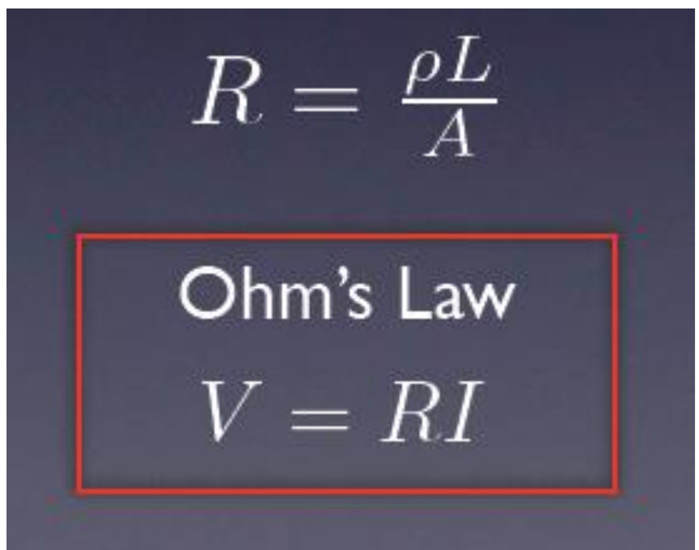
Standard EIA Color Code Table 4 Band: ±2%, ±5%, and ±10%



Color	1st Band (1st figure)	2nd Band (2nd figure)	3rd Band (multiplier)	4th Band (tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	±2%
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%

Ohm’s Law





(remember, R is in Ω and ρ is in Ω-m)

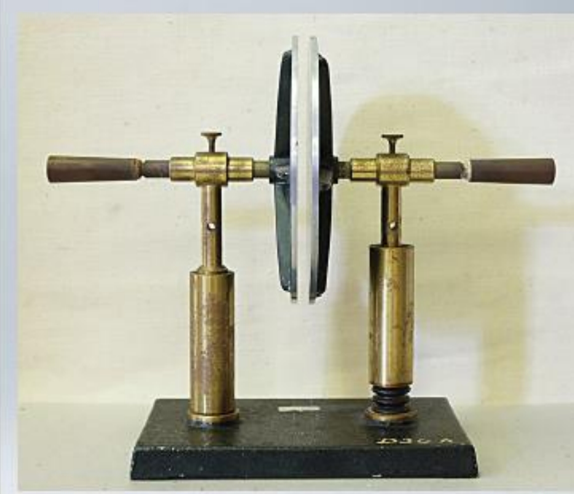
Capacitors

Capacitance (C) *is the ability of a material to store charge in the form of **separated charge or an electric field**. It is the ratio of charge stored to voltage difference between two plates.*

Capacitance is measured in **Farads (F)**

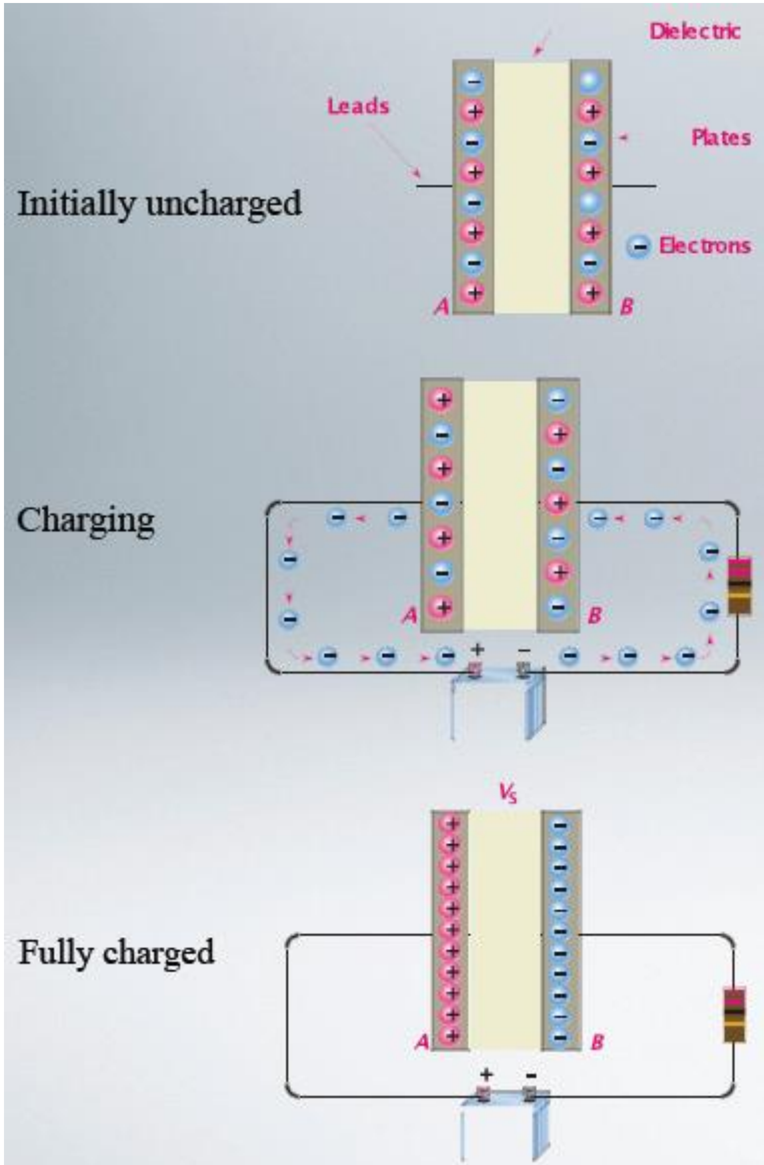
Capacitor	
 Modern capacitors, by a cm rule	
Type	Passive
Invented	Ewald Georg von Kleist (October 1745)
Electronic symbol	
	

$$C = \frac{Q}{V} = \frac{Coloumb}{Volt} = Farad$$



A simple demonstration of a parallel-plate capacitor

Capacitors



*A **capacitor** consists of a pair of conductors separated by a dielectric (insulator).*

$$C = \frac{\epsilon A}{d}$$

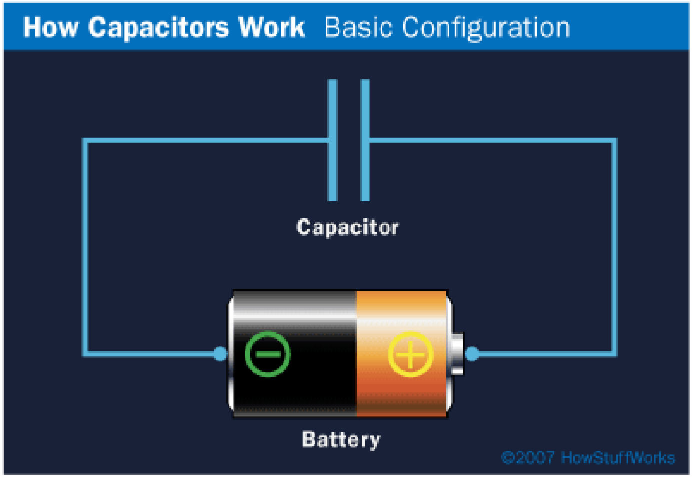
$\epsilon = permitivity$
$A = area$
$d = distance$

(ε indicates how penetrable a substance is to an electric field)

Electric charge is stored in the plates – a capacitor can become “charged”

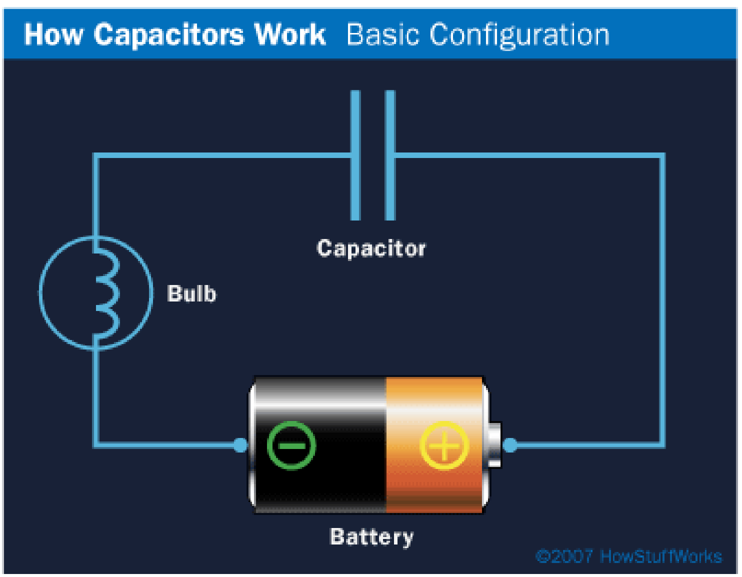
When a voltage exists across the conductors, it provides the energy to move the charge from the positive plate to the other plate.

Capacitors



The capacitor plate attached to the **negative terminal** accepts **electrons** from the battery.

The capacitor plate attached to the **positive terminal** accepts **protons** from the battery.



What happens when the light bulb is initially connected in the circuit?

What happens if you replace the battery with a piece of wire?

Energy storage

Work must be done by an **external influence** (e.g. a battery) to separate charge between the plates in a capacitor. The charge is stored in the capacitor until the external influence is removed and the separated charge is given a path to travel and dissipate.

Work exerted to charge a capacitor is given by the equation:

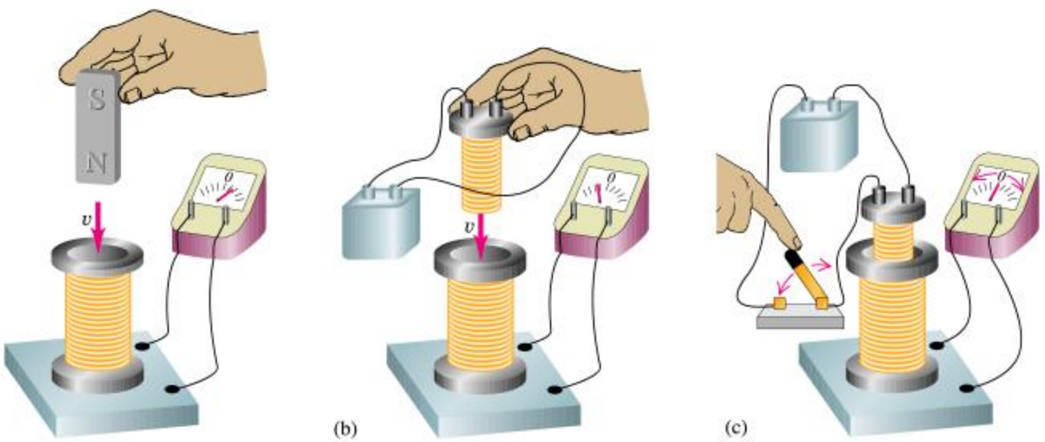
$$W = \frac{1}{2}CV^2$$

Inductors

The magnetic field from an inductor can generate an induced voltage, which can be used to drive current

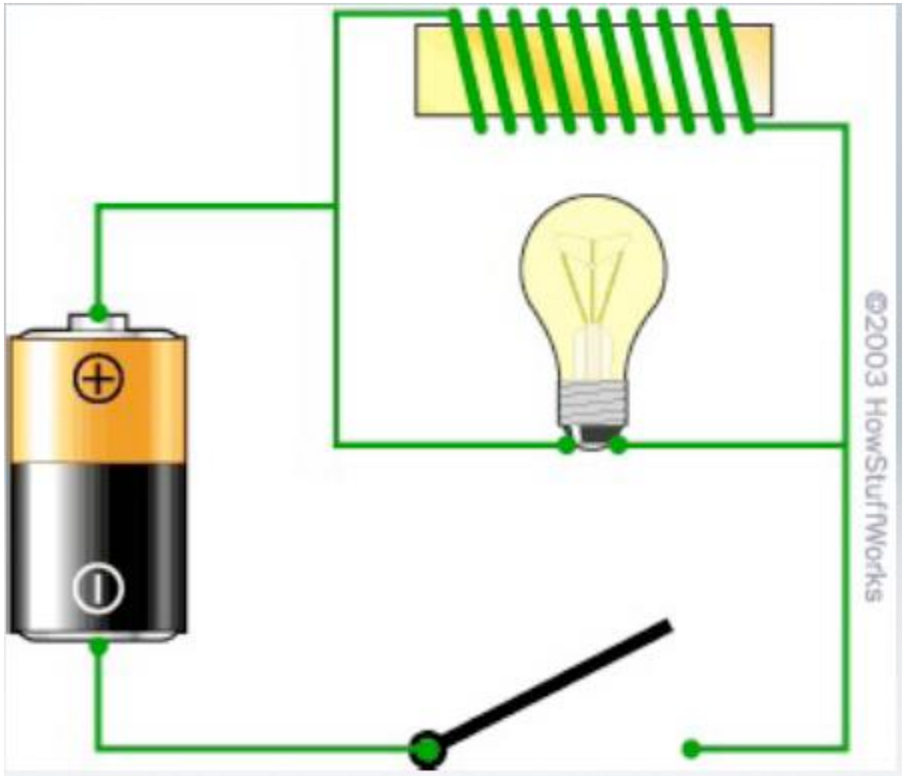
$$v = L\frac{di}{dt}$$

While building the magnetic field, the inductor **resists current flow**



(a) (b) (c)

Inductors



What happens to the light bulb when the switch is closed?

What happens to the light bulb when the switch is then opened?

Energy storage

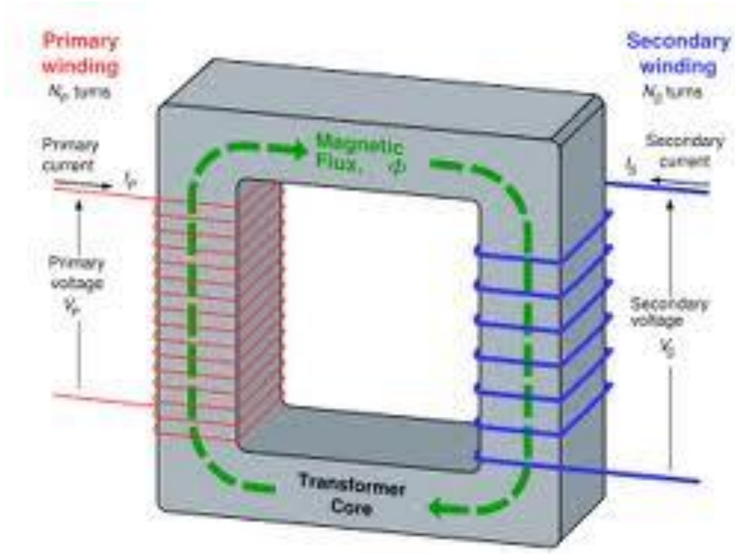
Inductors can store energy in the form of a magnetic field when a current is passed through them.

The work required to establish current through the coil, and therefore the magnetic field, is given by

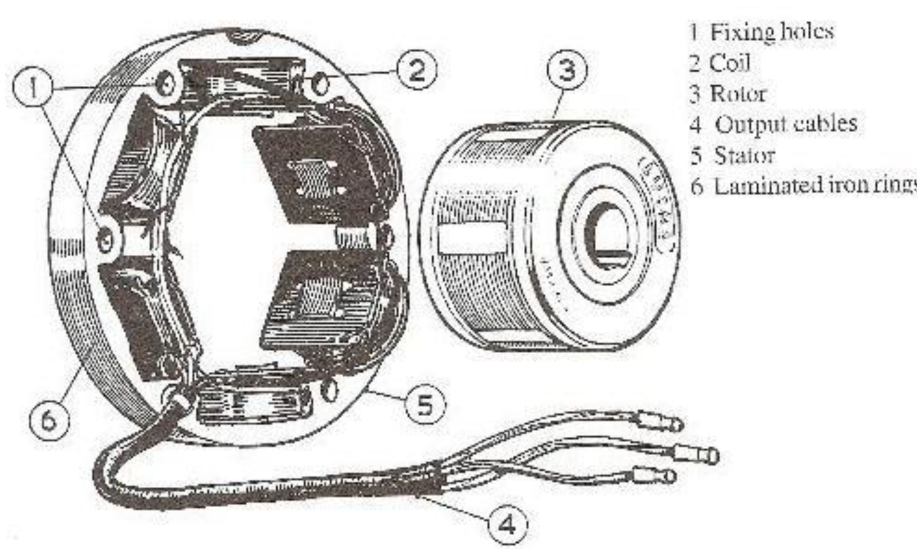
$$W = \frac{1}{2}LI^2$$

Transformers and alternators

Inductors are located in both **transformers** and **alternators**, allowing **voltage conversion** and **current generation**, respectively

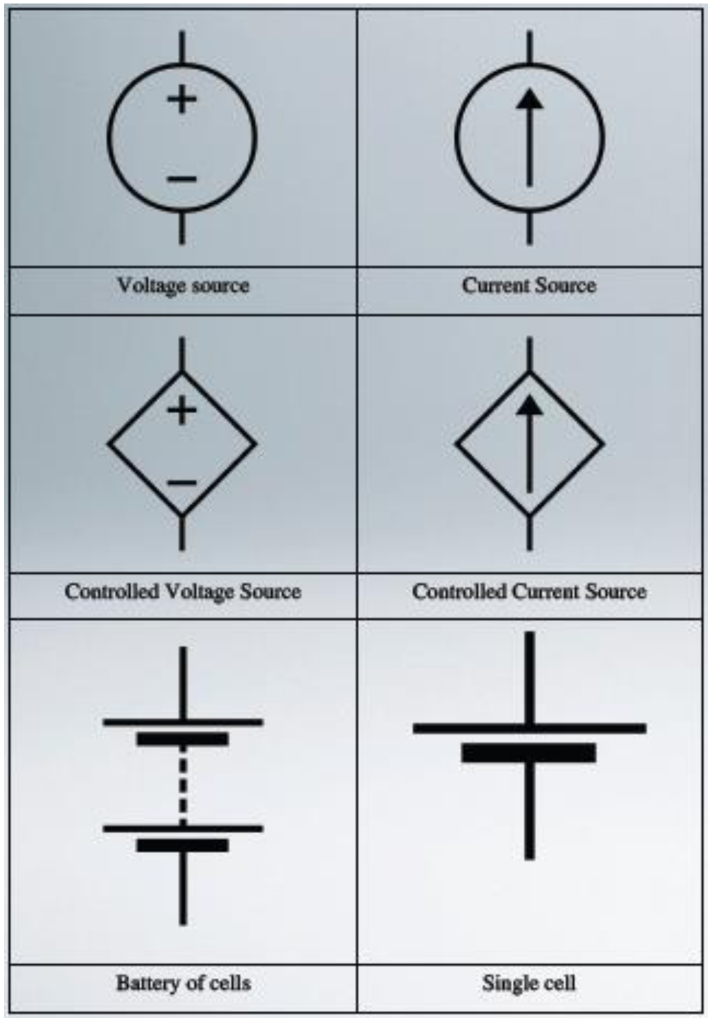


Transformer converts from one voltage to another



Alternator produces AC current

Electrical sources



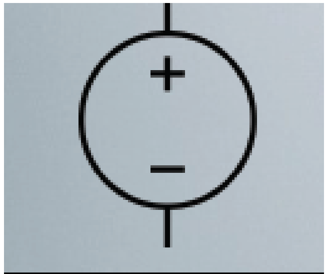
An **electrical source** is a **voltage** or **current generator** capable of supplying energy to a circuit

Examples:

- -AA batteries
- -12-Volt car battery
- -Wall plug

Ideal voltage source

An **ideal voltage source** is a circuit element where the **voltage across the source is independent of the current through it**.



Voltage source

Recall Ohm's Law: V=IR

The internal resistance of an ideal voltage source is zero.

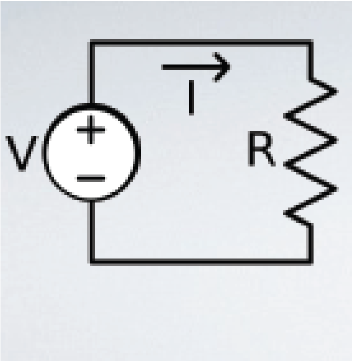
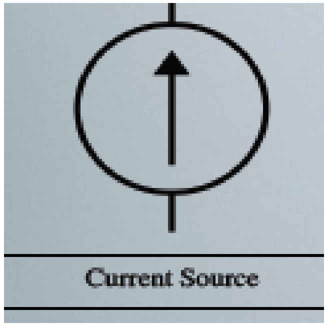


Figure 1: An ideal voltage source, *V*, driving a resistor, *R*, and creating a current *I*

If the current through an ideal voltage source is completely determined by the external circuit, it is considered an **independent voltage source**

Ideal current source

An **ideal current source** is a circuit element where the **current through the source is independent of the voltage across it**.



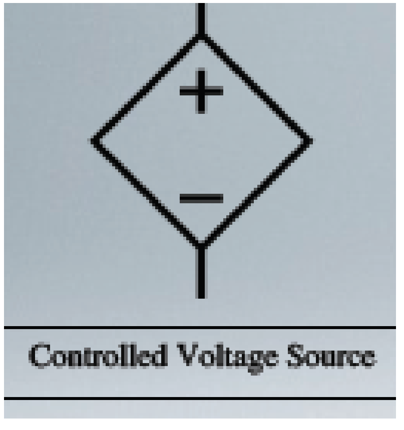
Recall Ohm's Law: I = V/R

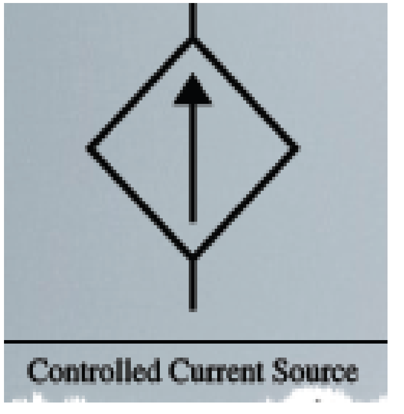
The internal resistance of an ideal current source is infinite.

If the voltage across an **ideal current source** is completely determined by the external circuit, it is considered an **independent current source**

Dependent Sources

A **dependent** or **controlled** source depends upon a different voltage or current in the circuit

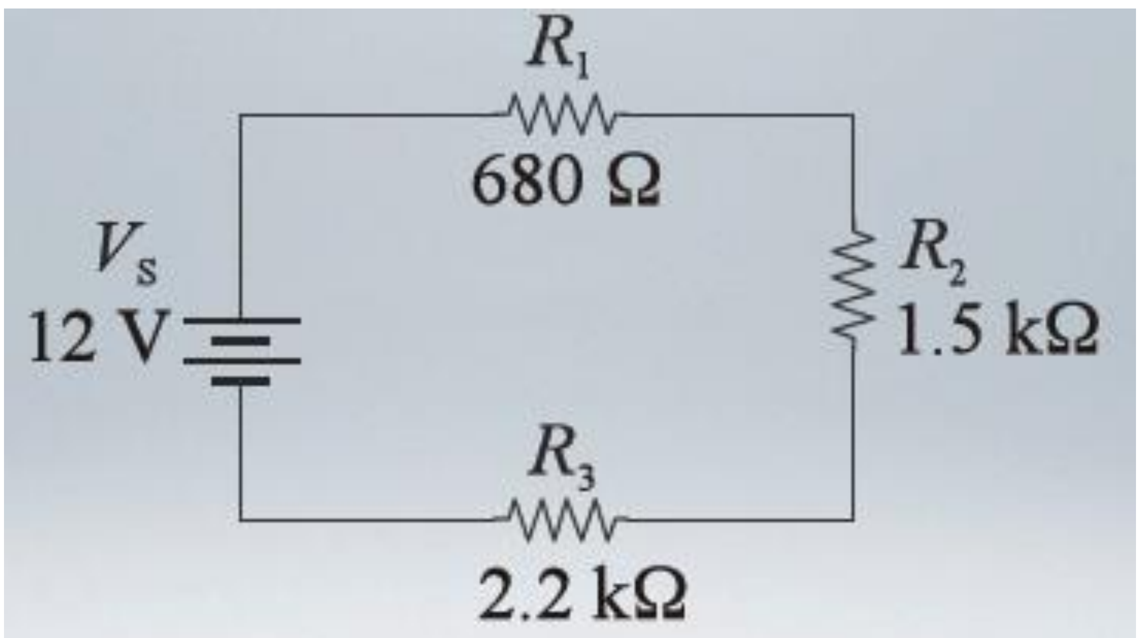




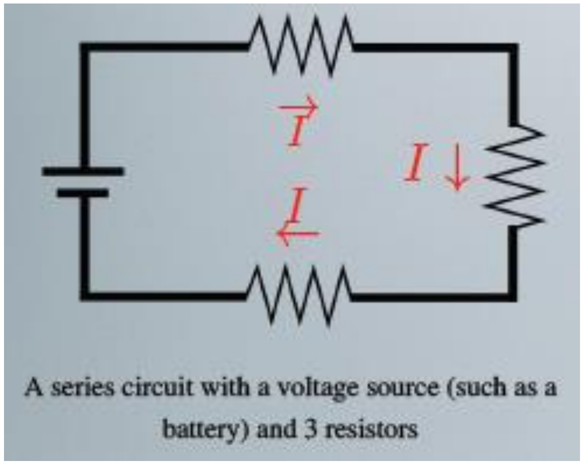
Electric Circuit Design Principles

Resistors in series

The resistors in a series circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?



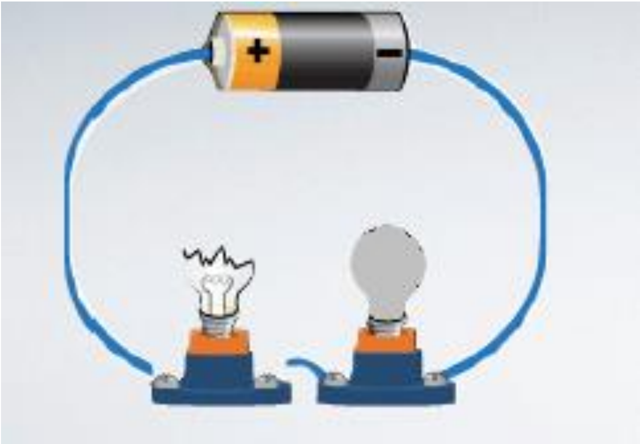
Series circuits



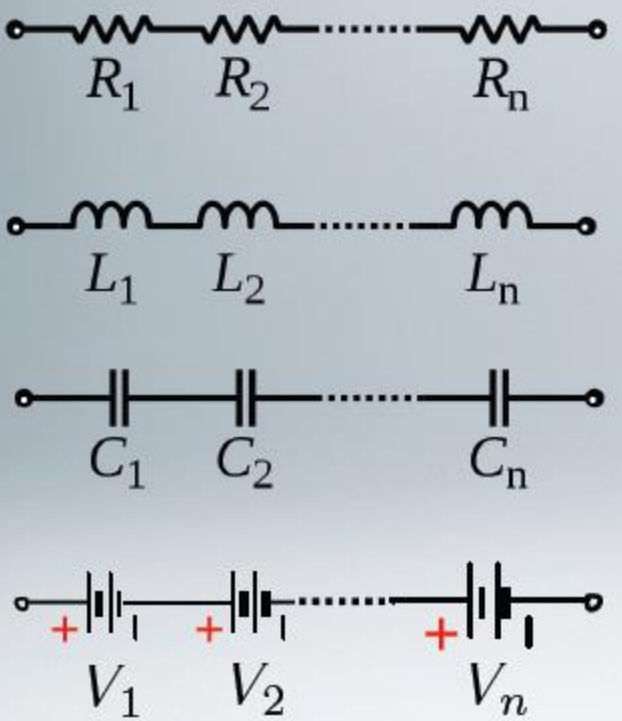
A **series circuit** has only **one current path**

Current through each component is the same

In a series circuit, all elements must function for the circuit to be complete



Multiple elements in a series circuit



$$R_{total} = R_1 + R_2 + \ldots + R_n$$

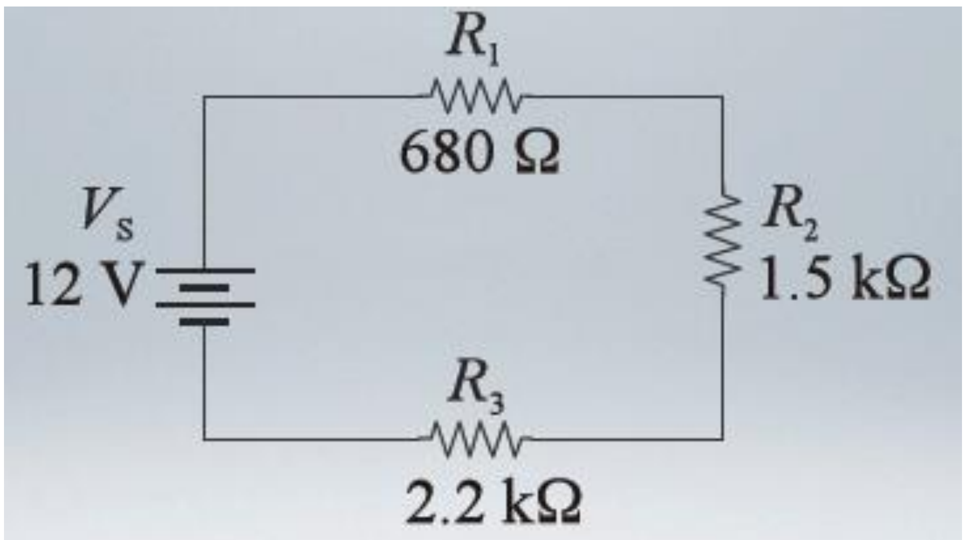
$$L_{total} = L_1 + L_2 + \ldots + L_n$$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

$$V_{total} = V_1 + V_2 + \ldots + V_n$$

Example: Resistors in series

The resistors in a series circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?



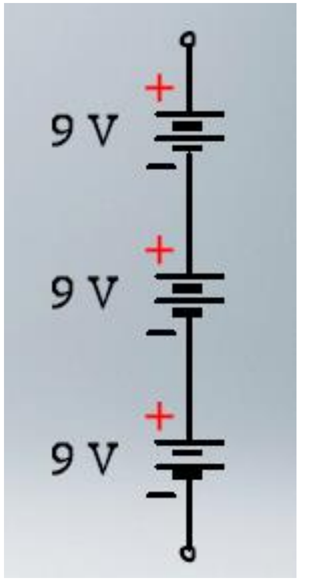
$$\begin{aligned} R_{total} &= R_1 + R_2 + R_3 \\ &= 680\Omega + 1500\Omega + 2200\Omega \\ &= 4380\Omega \\ &= 4.38k\Omega \end{aligned}$$

The current through each resistor?

$$I = \frac{V}{R_{total}} = \frac{12V}{4380\Omega} = 2.74mA$$

Example: Voltage sources in series

Find the total voltage of the sources shown

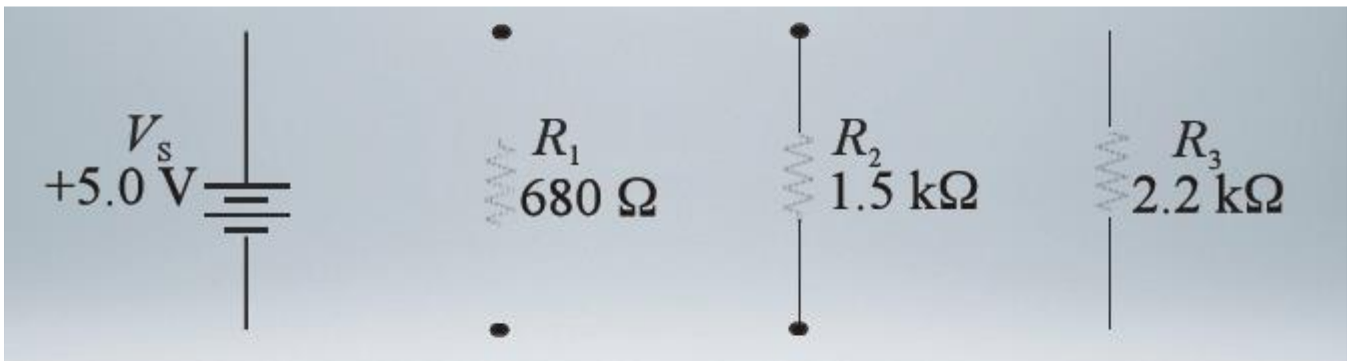


$$V_{total} = V_1 + V_2 + V_3 = 27V$$

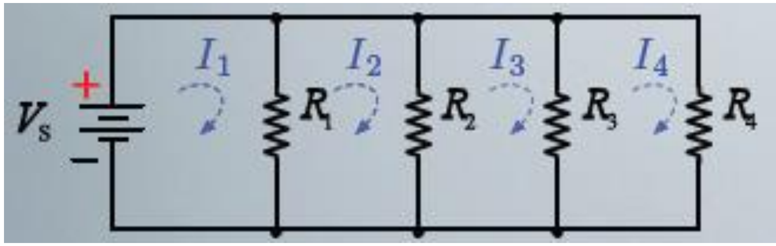
What happens if you reverse a battery?

Example: Resistors in parallel

The resistors in a parallel circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?

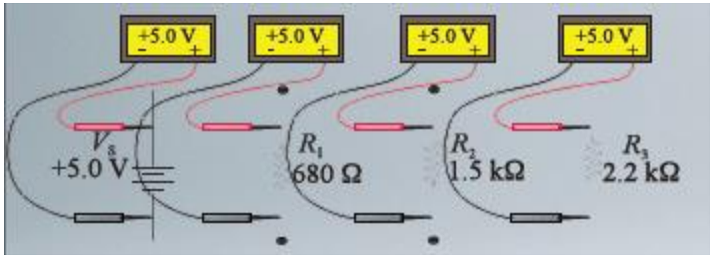


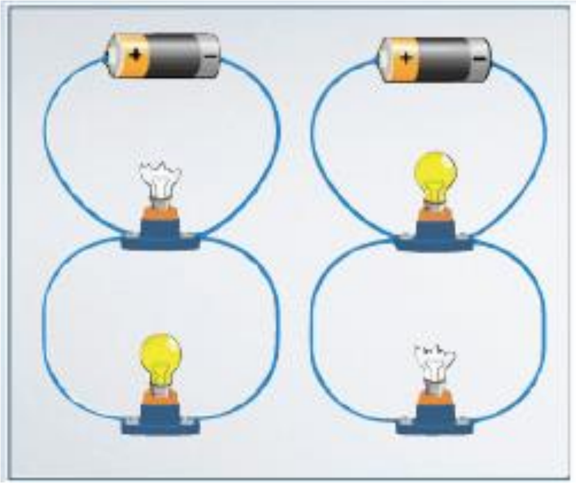
Parallel circuits



A **parallel circuit** has **more than one current path** branching from the energy source

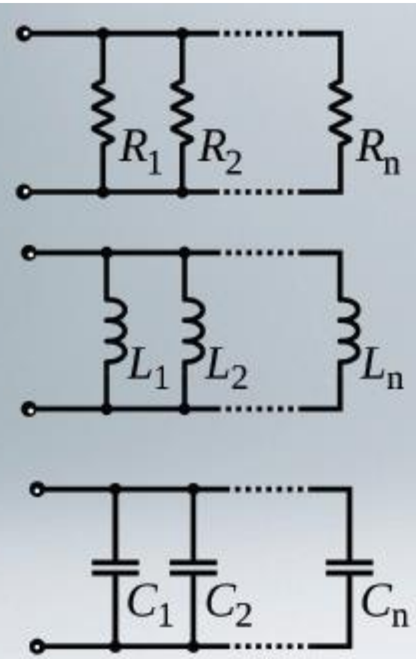
Voltage across each pathway is the same





In a parallel circuit, separate current paths function independently of one another

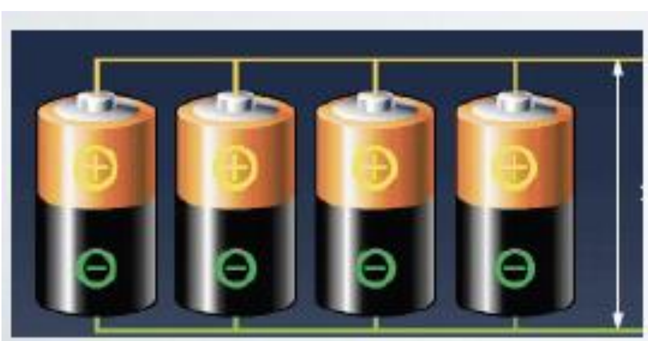
Multiple elements in a parallel circuit



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

$$\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$$

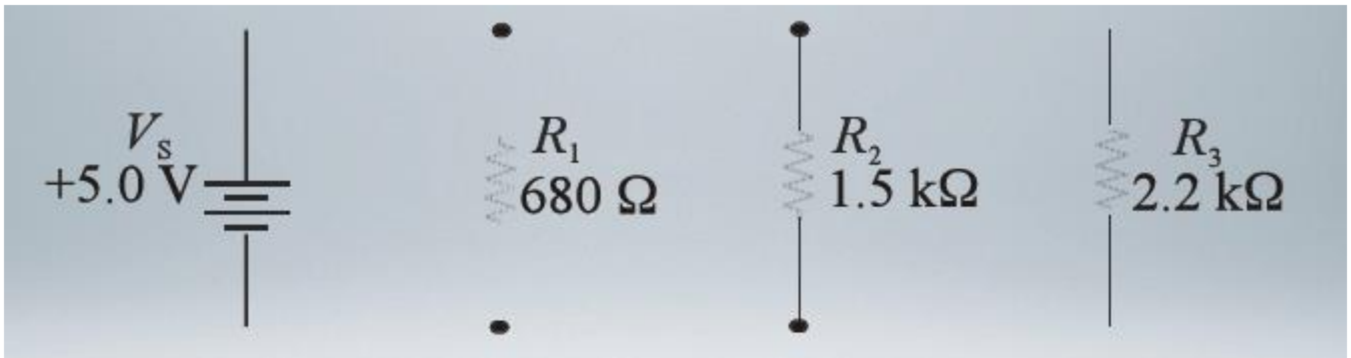
$$C_{total} = C_1 + C_2 + \ldots + C_n$$



For parallel voltage sources, the voltage is the same across all batteries, but the current supplied by each element is a fraction of the total current

Example: Resistors in parallel

The resistors in a parallel circuit are 680 Ω, 1.5 kΩ, and 2.2 kΩ. What is the total resistance?



$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$
$$= 386\Omega$$

Voltage across each resistor?
Dissipated power?

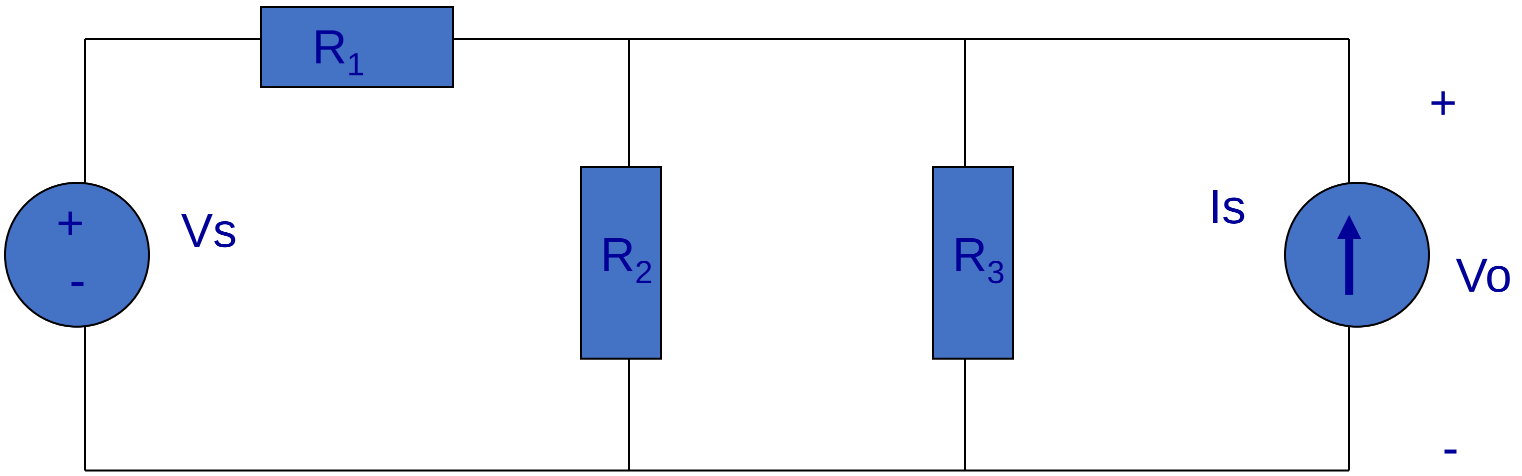
Current through each resistor?

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

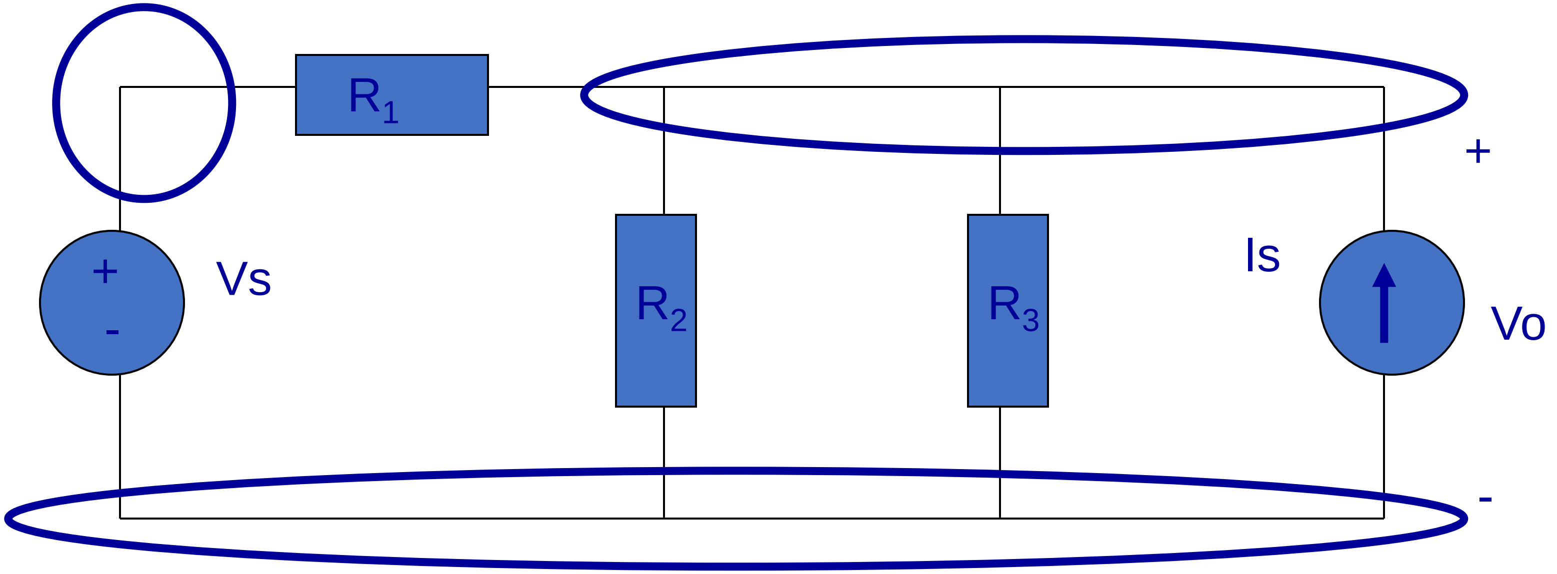
Example

- How many nodes, branches & loops?



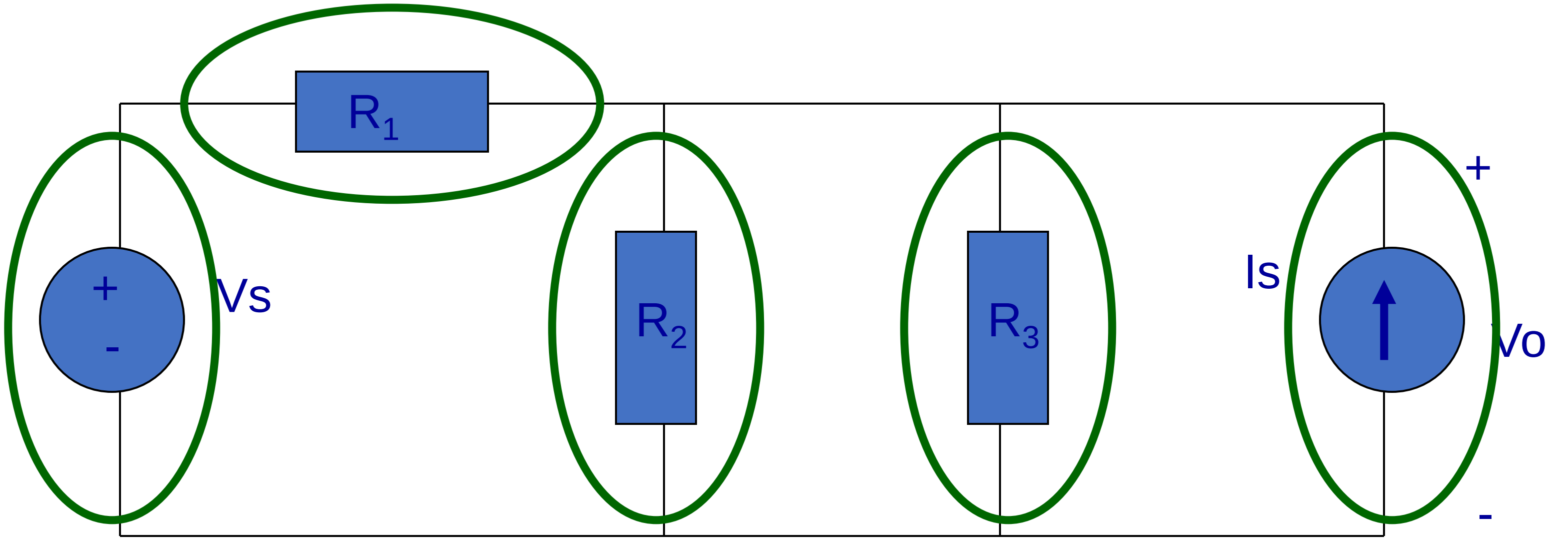
Example

- Three nodes



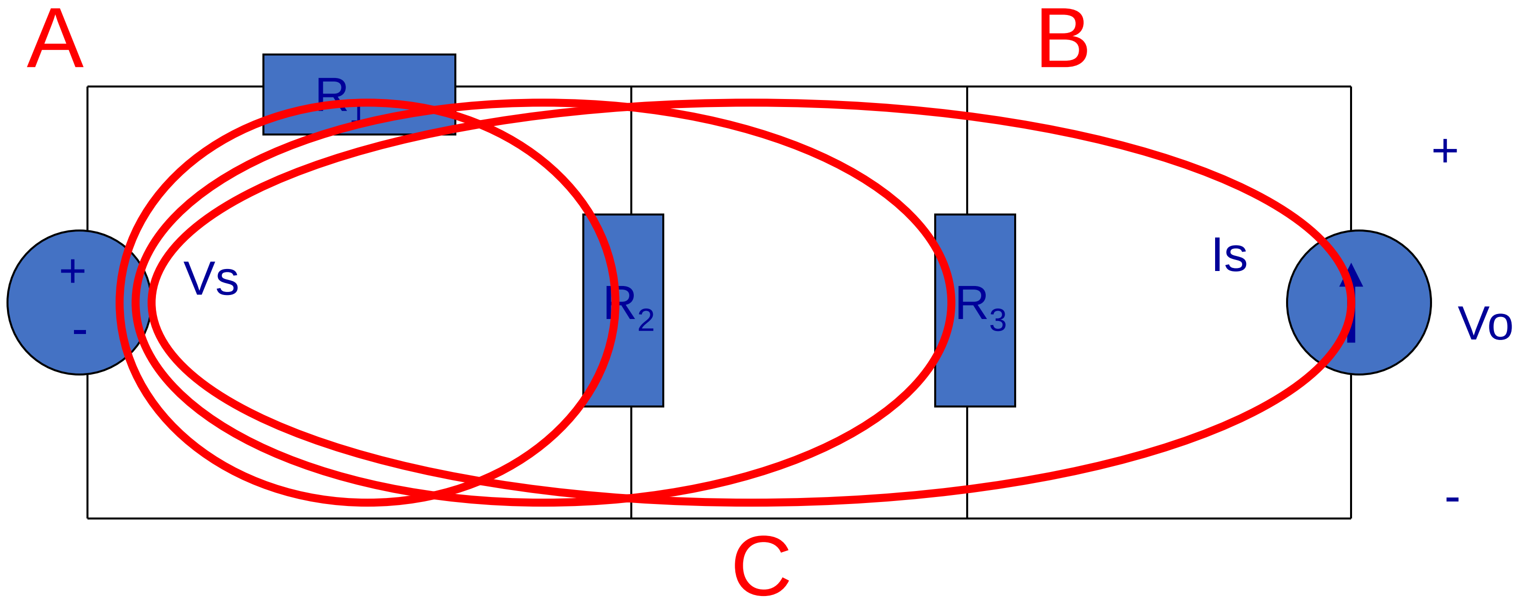
Example

- 5 Branches



Example

- Three Loops, if starting at node A

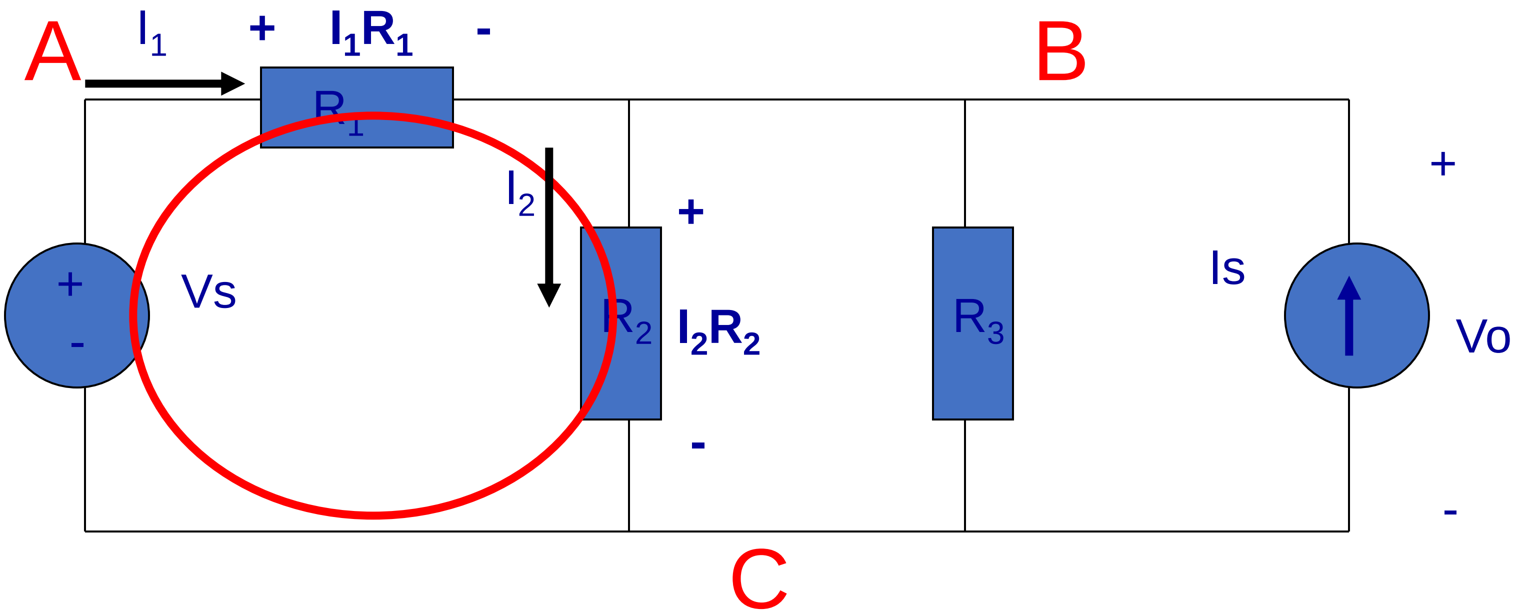


Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1st Loop



Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

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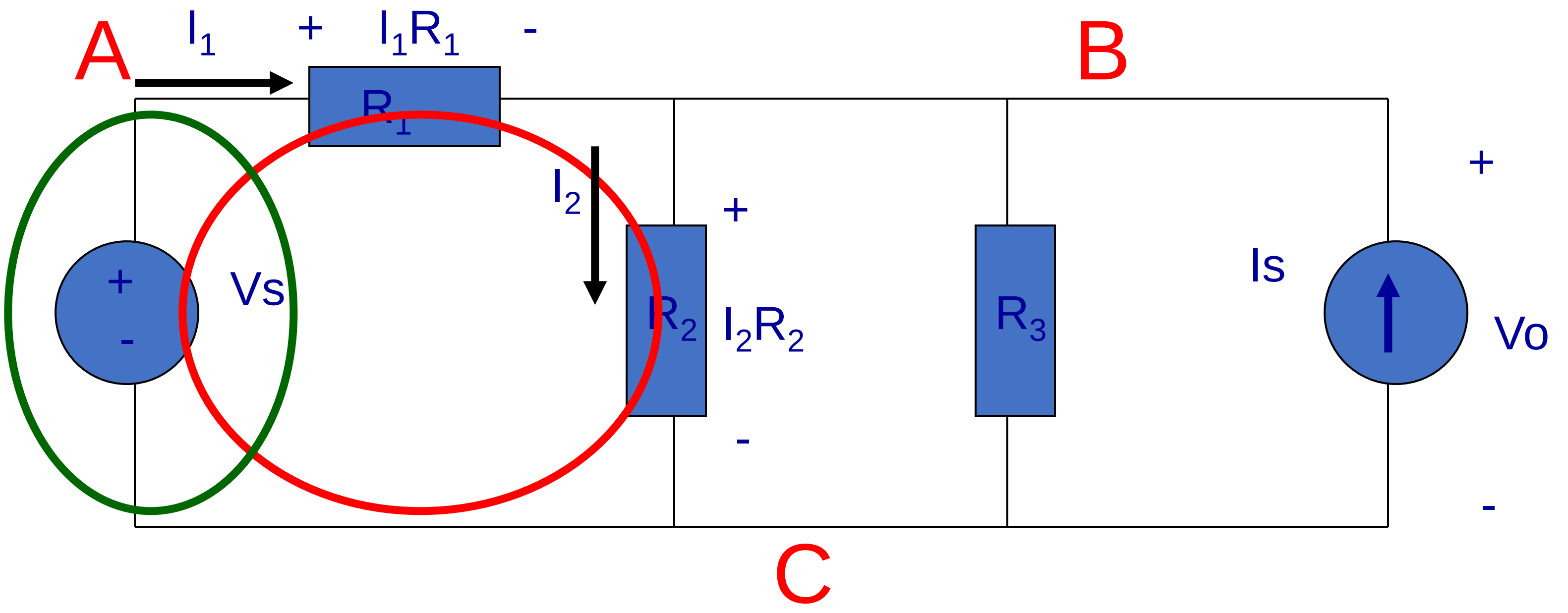
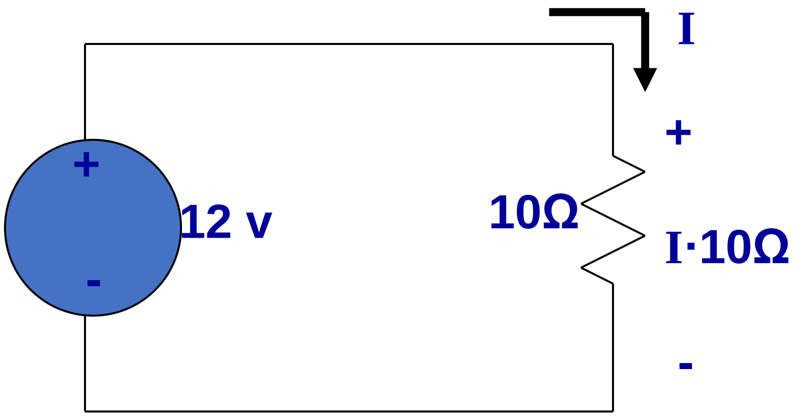


$$- I_1R_1 - I_2R_2 + Vs = 0$$

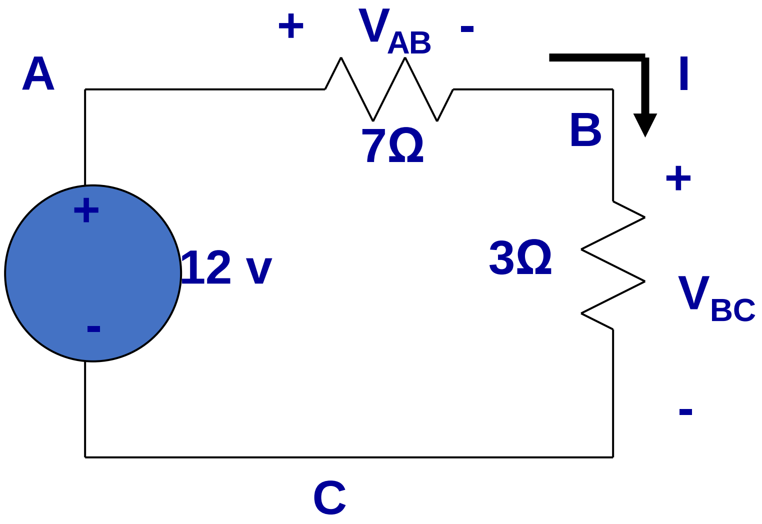
Series Resistors

- KVL: $+I \cdot 10\Omega - 12\text{ v} = 0$, So $I = 1.2$ A
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



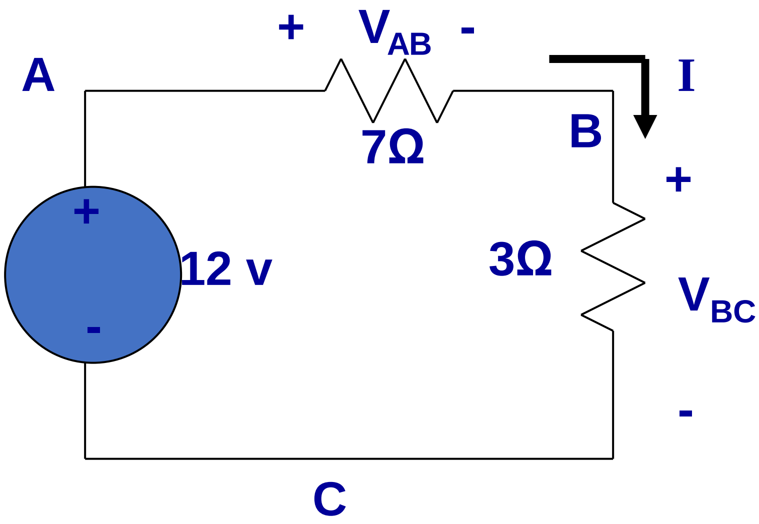
Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff's two laws and Ohm's law to determine all branch voltages and currents



Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$
- Solving: $I = 1.2\text{ A}$



Since $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$

And $I = 1.2\text{ A}$

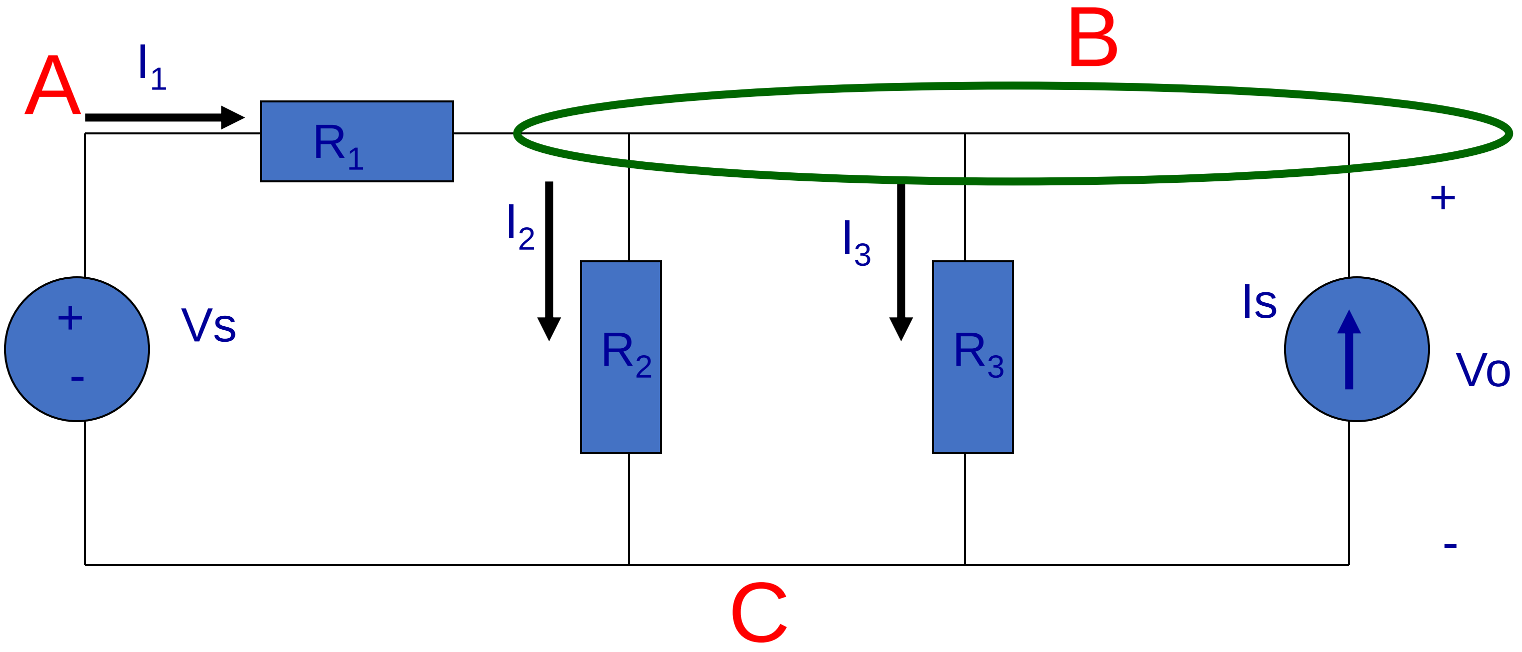
So $V_{AB} = 8.4\text{ v}$ and $V_{BC} = 3.6\text{ v}$

Kirchoff’s Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

Example

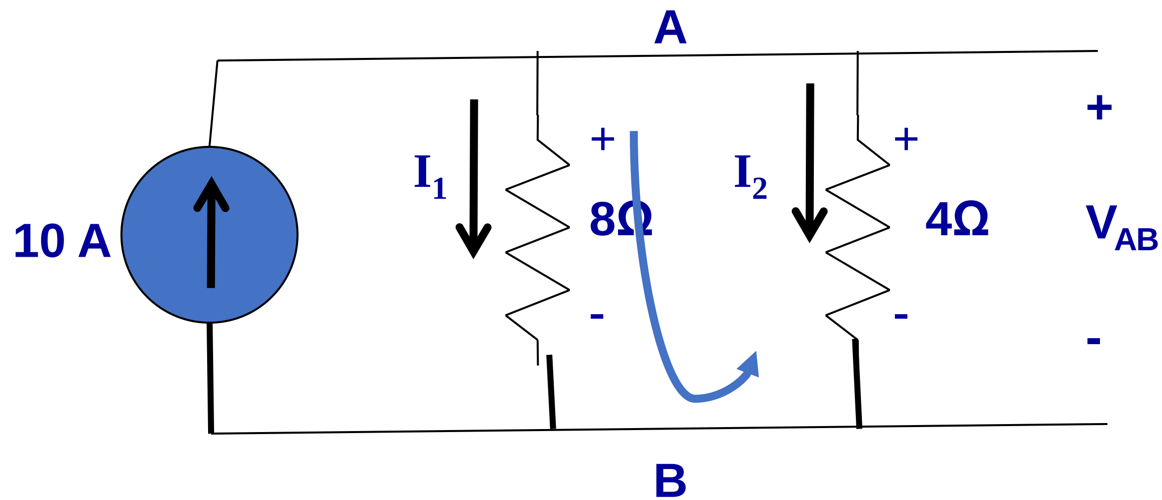
- Kirchoff's Current Law at B



Assign current variables and directions

Add currents in, subtract currents out: $I_1 - I_2 - I_3 + Is = 0$

Example: Find VAB for the Figure below



By KVL: $-I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$ → $I_2 = 2 \cdot I_1$

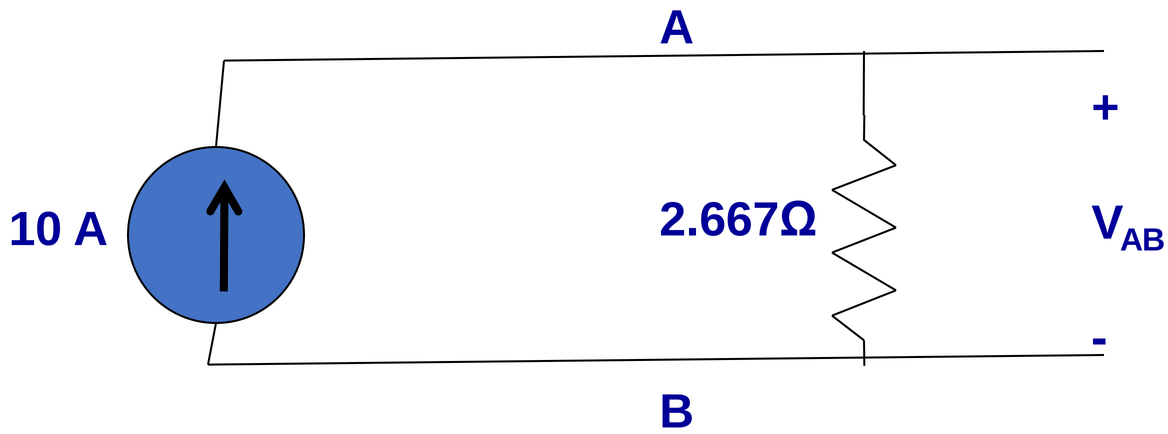
By KCL: $10A = I_1 + I_2$

Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$

So $I_1 = 3.33\ A$ → $I_2 = 6.67\ A$

And $V_{AB} = I_2 \cdot 4 = 26.33$ **volts**

Another Way



By Ohm's Law: V_{AB} = 10 A · 2.667 Ω

So V_{AB} = 26.67 volts

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

H.W) For the cct as shown in fig below.
Find V_o and the power dissipated in the resistance R_1