2.1 Average Rates of change and secant lines

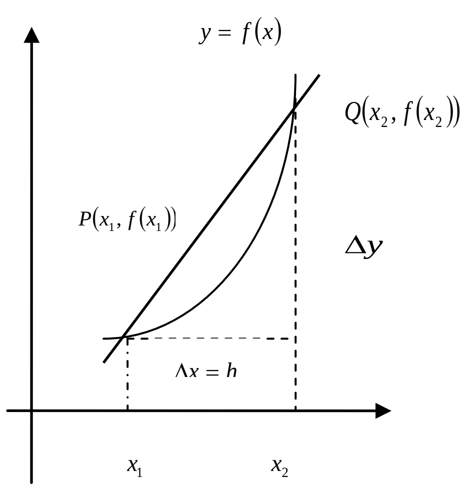
Given an arbitrary function $y = f(x)$, we calculate the average rate of change of y with respect to x over the interval $[x_1, x_2]$ by dividing the change in the value of y, $\Delta y = f(x_2) - f(x_1)$, by the length $\Delta x = x_2 - x_1 = h$ of the interval over which the change occurs (where h is Δx).

Definition

The Average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

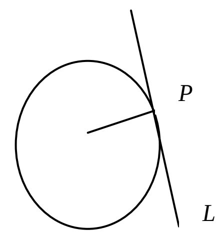
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h} \quad , \qquad h \neq 0$$

Geometrically, the rate of change of f over $[x_1, x_2]$ is the slope of the line through the points $P(x_1, f(x_1))$ and $Q(x_2, f(x_2))$, a line joining two points of a curve is a secant to the curve. The average rate of change of f from x_1 to x_2 is identical with the slope of secant PQ.



2.2 Defining the slope of curve

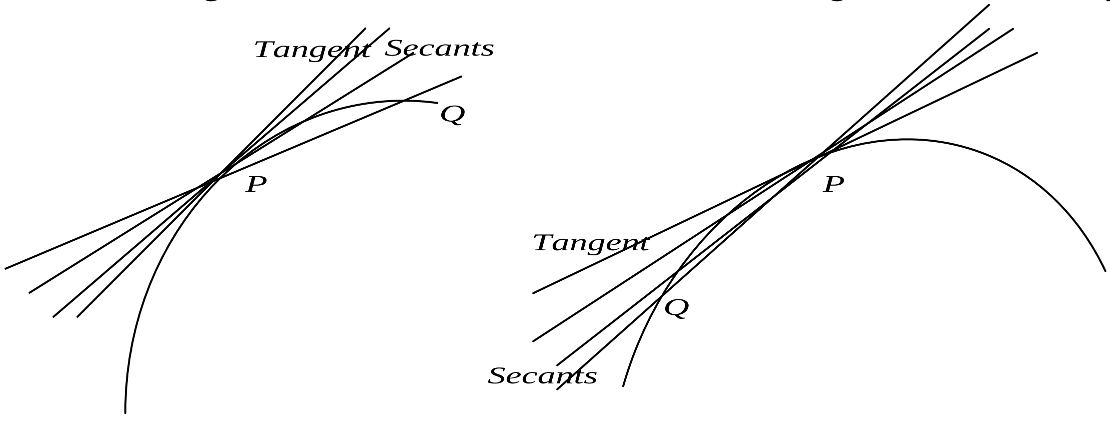
If there is a tangent line to the curve at p- a line that just touches the curve like the tangent to a circle- it would be reasonable to identify the slope of the tangent as the slope of the curve at P.



For circles, A line L is tangent to a circle at a point P if L passes through P perpendicular to the radius r at P.

To define tangency for general curves, we need an approach that takes into account the behavior of the secants through P and nearby points Q as Q moves toward P along the curve.

1. Start with what we can calculate, namely the slope of the secant PQ.
2. Investigate the limiting value of the secant slope as Q approaches P along the curve.
3. If the limit exists, take it to be the slope of the curve at P and define the tangent to the curve at P to be the line through P with this slope.



Example 1 :

Find the speed of the falling rock from the top of a tall cliff at t=1 and t=2 sec, where $y= 4.9\ t^2$.

Solution :

We calculate the average speed of the rock over a time interval (t_0, t_0+h), where $h=\Delta t$

$$\frac{\Delta y}{\Delta t}=\frac{4.9(t_0+h)^2-4.9(t_0)^2}{h}$$ average speed

We cannot use this formula to calculate speed at h=0, since $\frac{\Delta y}{\Delta t}=\infty$, we can use it to calculate average speeds $\left(\frac{\Delta y}{\Delta t}\right)$ for increasing short time $\Delta t=h=0.00001$ etc. $\Delta t\neq 0$ starting at $t_0=1$ and $t_0=2$ show as table below

Interval time (h)	Average speed $\left(\frac{\Delta y}{\Delta t}\right)$ starting at $t_0=1$	Average speed $\left(\frac{\Delta y}{\Delta t}\right)$ starting at $t_0=2$
1	14.7	24.5
0.1	10.29	20.09
0.01	9.849	19.649
0.001	9.8049	19.6049
0.0001	9.80049	19.60049
	$\frac{\Delta y}{\Delta t}=9.8+4.9h$	$\frac{\Delta y}{\Delta t}=19.6+4.9h$

Example 2 :

Find the slope of the parabola $y=x^2$ at the point P(2,4). Write an equation for the tangent to the parabola at this point.

Solution :

We begin with a secant line through P(2,4) and Q(2+h, $(2+h)^2$) nearby.

Secant slope = $\frac{\Delta y}{\Delta x}=\frac{(2+h)^2-(2)^2}{h}=\frac{h^2+4h+4-4}{h}=\frac{h^2+4h}{h}=h+4$

In the curve Q approaches P along the curve, h approaches zero and the secant slope (h+4) approaches 4 .

The tangent to the parabola at P is the line through P with slope 4 .

y= 4 + 4 (x-2)

y= 4 x – 4

2.3 Limit of function values

Suppose f(x) is defined on an open interval about x0 , except possibly at x0 , itself. If f(x) is arbitrarily close t0 L (as close to L as we like) for all x sufficiently close to x0 . we say that f approaches the limit L as x approaches x0 , and write.

$\lim_{x\to x_0} f(x)=L$, which is read " the limit of f(x) as approaches x0 is L"

Example 3 :

How does the function $f(x)=\frac{(x^2-1)}{(x-1)}$ behaves near x=1?

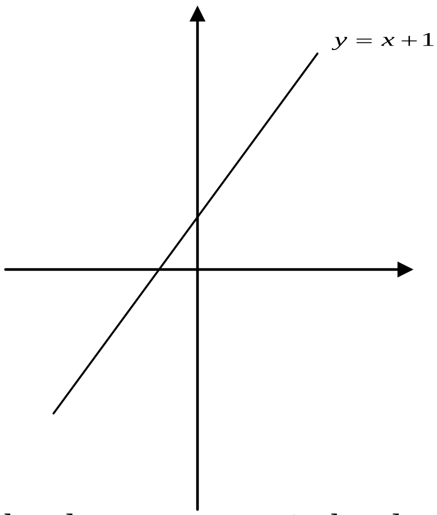
Solution :

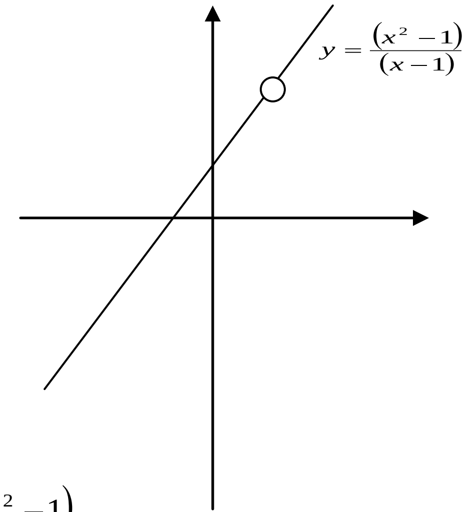
The given formula defines f for all real numbers x except x=1 (we cannot divide by zero). For any x ≠ 1, we can simplify the formula by factoring the numerator and canceling common factors :

$f(x)=\frac{(x^2-1)}{(x-1)}=x+1$ for x ≠ 1

The graph of f is the line y = x+1 with the point (1,2) removed, This removed point is shown as a " hole" in figure.

Even f(1) is not defined, it is clear that we can make the value of f(x) as close as we want to 2 by choosing x close enough to 1

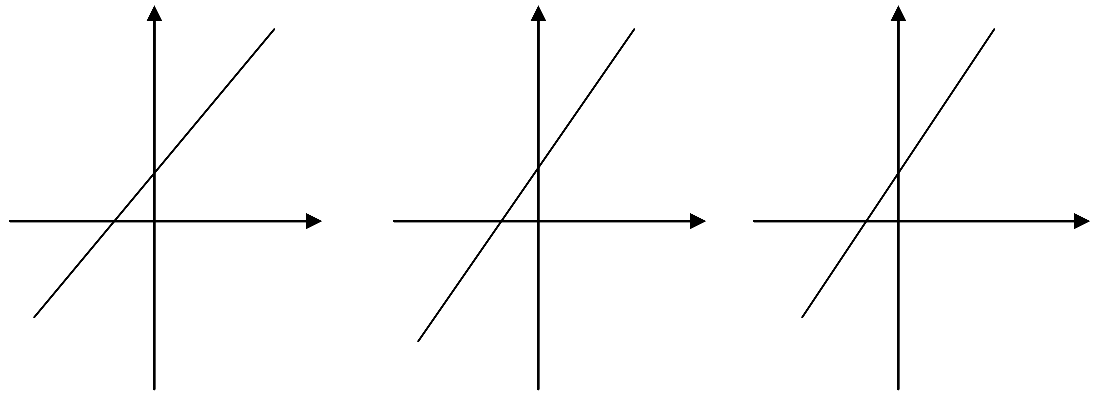




The closer x gets to 1, the closer $f(x) = \frac{(x^2-1)}{(x-1)}$ seems to get to 2

Value of x below and above 1	$f(x) = \frac{(x^2-1)}{(x-1)} = x+1$ for $x \neq 1$
0.9	1.9
1.1	2.1
0.99	1.99
1.01	2.01
0.999	1.999
1.001	2.001
0.999999	1.999999
1.000001	2.000001

Example 4 : this illustrates that the limit value of a function does not depend on how the function is defined at the point being approached. There are three function as below when f has limit as $x \to 1$ even though f is not defined at x=1



$$f(x) = \frac{(x^2-1)}{(x-1)} \qquad g(x) = \begin{cases} \dfrac{(x^2-1)}{(x-1)} & x \neq 1 \\ 1 & x = 1 \end{cases} \qquad h(x) = x+1$$

The limits of f(x), g(x) and h(x) all equal 2 as x approaches 1. only h(x) has the same function value as its limit at x=1.

<u>Example 5 :</u>

a- if f is the identity function f(x)= x, then for any value of x0

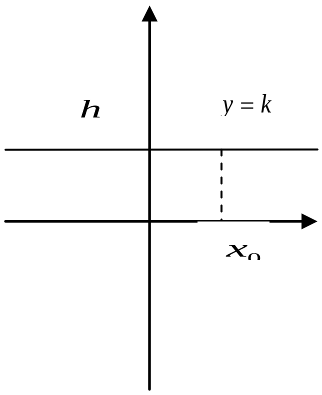
$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} x = x_0$$

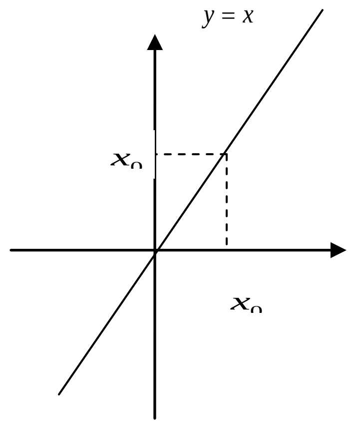
b- if f is the constant function f(x)=k (function with the constant value k), then for any value of x0

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} k = k$$

For instances of each of these rules we have

$\lim_{x \to 3} x = 3$ and $\lim_{x \to -7} 4 = \lim_{x \to 2} 4 = 4$





Constant function identity function

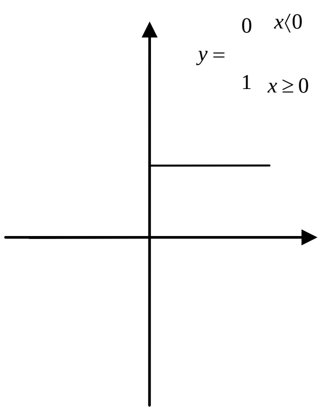
<u>Example 6 :</u>

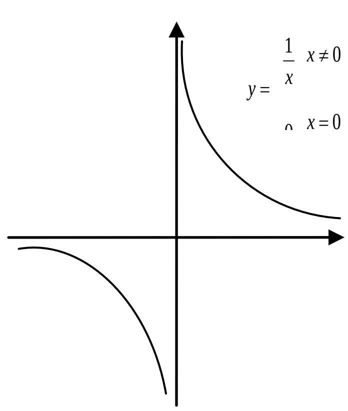
Discuss the behavior of the following functions as x→ 0 .

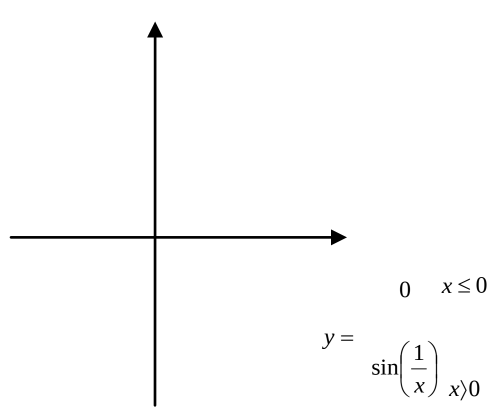
a- $U(x)=\begin{cases} 0 & x\langle 0 \\ 1 & x\geq 0 \end{cases}$

b- $g(x)=\begin{cases} \frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$

c- $f(x)=\begin{cases} 0 & x\leq 0 \\ \sin\left(\frac{1}{x}\right) & x\rangle 0 \end{cases}$







a- It jumps: the unit step function U(x) has no limit as $x \to 0$ because its values jump at x=0. for negative values of x arbitrarily close to zero, U(x)=0. for positive values of x arbitrarily close to zero, U(x) =1. there is no single value L approached by U(x) as $x \to 0$, see figure above.

b- It grows too " large " to have a limit : g(x) has no limit as $x \to 0$ because the values of g(x) grow arbitrarily large in absolute value as $x \to 0$ and do not stay close to any fixed real number, see figure above.

c- It oscillates too much to have a limit: f(x) has no limit as $x \to 0$ because the function's values oscillate between +1 and -1 in every open interval containing 0. the values do not stay close to any one number as $x \to 0$, see figure above.

2.4 The Limit Laws

To calculate limits of functions (when $x \to x0$, $x \to c$, $x \to a$, x0,c,a are constant values) that are arithmetic combinations of functions having known limits, we can use several easy rules.

Theorm 1 limit Laws if L, M, c and k are real numbers and

$\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then the properties of limits are :

1. Sum Rule : $\lim_{x \to c}(f(x) + g(x)) = L + M$
2. Difference Rule : $\lim_{x \to c}(f(x) - g(x)) = L - M$
3. Constant Multiple Rule : $\lim_{x \to c}(k \cdot f(x)) = k \cdot L$
4. Product Rule : $\lim_{x \to c}(f(x) \cdot g(x)) = L \cdot M$
5. Quotient Rule : $\lim_{x \to c}\left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}$, $M \neq 0$

6. Power Rule : $\lim\limits_{x\to c}[f(x)]^n = L^n$, n a positive integer

7. Root Rule : $\lim\limits_{x\to c}\sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(if n is even, we assume that $\lim\limits_{x\to c} f(x) = L \rangle 0$)

<u>Example 7 :</u>

Use the observations $\lim\limits_{x\to c} k = k$ and $\lim\limits_{x\to c} x = c$, and the properties of limits to find the following limits.

a- $\lim\limits_{x\to c}\left(x^3 + 4x^2 - 3\right)$ b- $\lim\limits_{x\to c}\dfrac{\left(x^4 + x^2 - 1\right)}{x^2 + 5}$ c- $\lim\limits_{x\to -2}\sqrt{\left(4x^2 - 3\right)}$

<u>Solution</u> :

a-
$$\begin{aligned}\lim_{x\to c}\left(x^3 + 4x^2 - 3\right) &= \lim_{x\to c}\left(x^3\right) + \lim_{x\to c}\left(4x^2\right) - \lim_{x\to c}(3)\\ &= c^3 + 4c^2 - 3\end{aligned}$$

b-
$$\begin{aligned}\lim_{x\to c}\frac{\left(x^4 + x^2 - 1\right)}{x^2 + 5} &= \frac{\lim\limits_{x\to c}\left(x^4 + x^2 - 1\right)}{\lim\limits_{x\to c}\left(x^2 + 5\right)}\\ &= \frac{\lim\limits_{x\to c}\left(x^4\right) + \lim\limits_{x\to c}\left(x^2\right) - \lim\limits_{x\to c}(1)}{\lim\limits_{x\to c}\left(x^2\right) + \lim\limits_{x\to c}(5)}\\ &= \frac{c^4 + c^2 - 1}{c^2 + 5}\end{aligned}$$

c-
$$\begin{aligned}\lim_{x\to -2}\sqrt{\left(4x^2 - 3\right)} &= \sqrt{\lim_{x\to -2}\left(4x^2 - 3\right)}\\ &= \sqrt{\lim_{x\to -2}\left(4x^2\right) - \lim_{x\to -2}(3)}\\ &= \sqrt{4(-2)^2 - 3}\\ &= \sqrt{16 - 3}\\ &= \sqrt{13}\end{aligned}$$

Theorm 2 limits of Polynomials

if $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0$ is the polynomial function, then

$$\lim_{x\to c} P(x) = P(c) = a_n c^n + a_{n-1}c^{n-1} + a_{n-2}c^{n-2} + ... + a_0$$

Theorm 3 limits of Rational functions

if $P(x)$ and $Q(x)$ are polynomial functions and $Q(c) \neq 0$, then

$$\lim_{x\to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$$

Example 8 :

The following calculation illustrates Theorems 2 and 3 :

$$\lim_{x\to -1} \frac{(x^3 + 4x^2 - 3)}{x^2 + 5} = \frac{(-1)^3 + 4(-1)^2 - 3}{(-1)^2 + 5} = \frac{0}{6} = 0$$

Example 9 :

Evaluate $\lim_{x\to -1} \frac{(x^2 + x - 2)}{x^2 - x}$

Solution :

We cannot substitute x=1 because it makes the denominator zero. We test the numerator to see if it, too, is zero at x=1. it is, so it has a factor of (x-1) in common with the denominator. Canceling the (x-1)'s given a simpler fraction with the same values as the original for x ≠ 1:

$$\frac{(x^2 + x - 2)}{x^2 - x} = \frac{(x-1)(x+2)}{x(x-1)} = \frac{(x+2)}{x} \quad , \quad \text{if } x \neq 1$$

Using the simpler fraction, we find the limit of these values as x→1 by substitution :

$$\lim_{x\to -1} \frac{(x^2 + x - 2)}{x^2 - x} = \lim_{x\to -1} \frac{(x+2)}{x} = \frac{1+2}{1} = 3$$

Example 10 :

Evaluate $\lim\limits_{x\to 0}\frac{\sqrt{(x^2+100)}-10}{x^2}$

Solution

We can create a common factor by multiplying both numerator and denominator by the conjugate radical expression $\left(\sqrt{(x^2+100)}+10\right)$

$$\frac{\sqrt{(x^2+100)}-10}{x^2}=\frac{\sqrt{(x^2+100)}-10}{x^2}\cdot\frac{\sqrt{(x^2+100)}+10}{\sqrt{(x^2+100)}+10}$$

$$=\frac{x^2+100-100}{x^2\left(\sqrt{(x^2+100)}+10\right)}=\frac{x^2}{x^2\left(\sqrt{(x^2+100)}+10\right)}$$

$$=\frac{1}{\left(\sqrt{(x^2+100)}+10\right)}$$

Therefore;

$$\lim_{x\to 0}\frac{\sqrt{(x^2+100)}-10}{x^2}=\lim_{x\to 0}\frac{1}{\left(\sqrt{(x^2+100)}+10\right)}$$

$$=\frac{1}{\left(\sqrt{(0^2+100)}+10\right)}$$

$$=\frac{1}{20}=0.05$$

Theorm 4 The Sandwich Theorem

Suppose that $g(x)\le f(x)\le h(x)$ for all x in some open interval containing c, except possibly at x =c itself. Suppose also that

$$\lim_{x\to c}g(x)=\lim_{x\to c}h(x)=L$$

Then $\lim\limits_{x\to c}f(x)=L$

The Sandwich Theorem is also called the Squeeze Theorem or Pinching Theorem.

Example 11 :

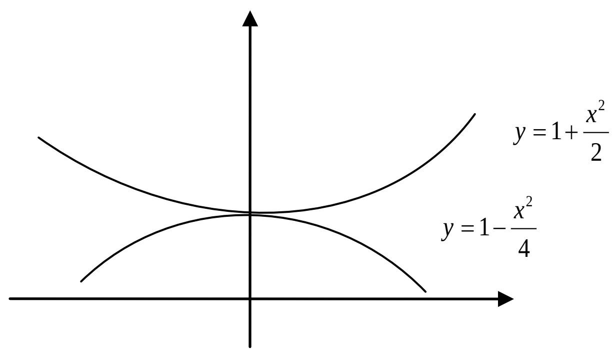
Given that $1-\frac{x^2}{4} \le u(x) \le 1+\frac{x^2}{2}$ for all x $\neq 0$

Find $\lim_{x\to 0} u(x)$, no matter how complicated u is.

Solution

Since $\lim_{x\to 0}\left(1-\frac{x^2}{4}\right)=1$ and $\lim_{x\to 0}\left(1+\frac{x^2}{2}\right)=1$

The Sandwich Theorem implies that $\lim_{x\to 0} u(x)=1$, see figure above



Example 12 :

The Sandwich Theorem help us establish several important limit rules:

(a) $\lim_{x\to \theta} \sin(\theta)=0$ (b) $\lim_{x\to \theta} \cos(\theta)=1$

(c) for any function f , $\lim_{x\to c}|f(x)|=0$ implies $\lim_{x\to c} f(x)=0$

Solution

(a) we established that $-|\theta| \le \sin\theta \le |\theta|$ for all θ (see fig.)

Since $\lim_{\theta\to 0}(-|\theta|)=\lim_{\theta\to 0}(|\theta|)=0$, we have $\lim_{\theta\to 0}\sin(\theta)=0$

(b) from $0 \le 1-\cos\theta \le |\theta|$ for all θ (see fig.), and we have

$\lim_{\theta\to 0}(1-\cos(\theta))=0$ or $\lim_{\theta\to 0}(\cos(\theta))=1$

(c) since $-|f(x)| \le f(x) \le |f(x)|$ and $-|f(x)|$ and $|f(x)|$ have limit 0 as x→c it follows that $\lim_{x\to c}|f(x)| = 0$.

Theorm 5 if $f(x) \le g(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself, and the limits of f and g both exist as x approaches c, then

$$\lim_{x\to c} f(x) \le \lim_{x\to c} g(x)$$

Example 13 : what values of x satisfy the inequality $|2x-8| \langle 2$?

Solution

To find out, we solve the inequality:

$$|2x-8| \langle 2$$
$$-2 \langle 2x-8 \langle 2$$
$$6 \langle 2x \langle 10$$
$$3 \langle x \langle 5$$

Theorm 6

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \quad (\theta \text{ in radians}) \qquad \text{(the proof page 70 in book)}$$

Example 14 : show that

a- $\lim_{h\to 0}\frac{\cos(h)-1}{h} = 0$ and b- $\lim_{x\to 0}\frac{\sin 2x}{5x} = \frac{2}{5}$

Solution

a- Using the half-angle formula $\cos(h) = 1 - 2\sin^2(h/2)$, we calculate

$$\lim_{h\to 0}\frac{\cos(h)-1}{h}=\lim_{h\to 0}-\frac{2\sin^2(h/2)}{h}$$
$$=-\lim_{h\to 0}\frac{\sin(\theta)}{\theta}\sin\theta \qquad let \quad \theta=h/2$$
$$=-(1)(0)=0$$

b- we produce it by multiplying numerator and denominator by (2/5):

$$\lim_{x\to 0}\frac{\sin(2x)}{5x}=\lim_{x\to 0}-\frac{\left(\frac{2}{5}\right)\cdot\sin(2x)}{\left(\frac{2}{5}\right)\cdot 5x}$$
$$=\left(\frac{2}{5}\right)\lim_{x\to 0}\frac{\sin(2x)}{2x} \qquad let \quad \theta=2x \qquad since \quad \lim_{x\to 0}\frac{\sin(2x)}{2x}=1$$
$$=\left(\frac{2}{5}\right)(1)=\frac{2}{5}$$

Example 15 : find $\lim_{t\to 0}\frac{\tan(t)\sec(2t)}{3t}$

Solution

$$\lim_{t\to 0}\frac{\tan(t)\sec(2t)}{3t}=\frac{1}{3}\lim_{t\to 0}\frac{\sin(t)}{t}\cdot\frac{1}{\cos(t)}\cdot\frac{1}{\cos(2t)} \qquad when \quad \tan(t)=\frac{\sin(t)}{\cos(t)}$$
$$=\frac{1}{3}(1)(1)(1)=\frac{1}{3} \qquad also \quad \sec(2t)=\frac{1}{\cos(2t)}$$

Theorem 7

All the limit laws in **Theorem1** are true when we replace $\lim_{x\to c}$ by $\lim_{x\to\infty}$ or $\lim_{x\to-\infty}$.

That is, the variable x may approach a finite number c or $\pm\infty$.

Example 16 : find

a- $\lim_{x\to\infty}\left(5+\frac{1}{x}\right)$ b- $\lim_{x\to-\infty}\left(\frac{\pi\sqrt{3}}{x^2}\right)$

Solution

a- $\lim_{x\to\infty}\left(5+\frac{1}{x}\right)=\lim_{x\to\infty}(5)+\lim_{x\to\infty}\left(\frac{1}{x}\right)=5+0=5$

b-

$$\lim_{x\to-\infty}\left(\frac{\pi\sqrt{3}}{x^2}\right)=\lim_{x\to-\infty}\left(\pi\sqrt{3}\cdot\frac{1}{x}\cdot\frac{1}{x}\right)$$
$$=\lim_{x\to-\infty}\left(\pi\sqrt{3}\right)\cdot\lim_{x\to-\infty}\left(\frac{1}{x}\right)\cdot\lim_{x\to-\infty}\left(\frac{1}{x}\right)$$
$$=\left(\pi\sqrt{3}\right)\cdot(0)\cdot(0)=0$$

Example 17 : find

a- $\lim_{x\to\infty}\left(\frac{5x^2+8x-3}{3x^2+2}\right)$ b- $\lim_{x\to-\infty}\left(\frac{11x+2}{2x^3-1}\right)$

Solution

a- (divide numerator and denominator by x^2)

$$\lim_{x\to\infty}\left(\frac{5x^2+8x-3}{3x^2+2}\right)=\lim_{x\to\infty}\left(\frac{5+(8/x)-(3/x^2)}{3+(2/x^2)}\right)$$
$$=\frac{5+0-0}{3+0}=\frac{5}{3}$$

b- (divide numerator and denominator by x^3)

$$\lim_{x\to-\infty}\left(\frac{11x+2}{2x^3-1}\right)=\lim_{x\to-\infty}\left(\frac{(11/x^2)+(2/x^3)}{2-(1/x^3)}\right)$$
$$=\frac{0+0}{2-0}=0$$

Example 18 : find a- $\lim_{x\to\infty}\sin\left(\frac{1}{x}\right)$ and b- $\lim_{x\to\pm\infty}x\cdot\sin\left(\frac{1}{x}\right)$

Solution

a- We introduce the new variable $t=\left(\frac{1}{x}\right)$, we know that $t\to0^+$ as $x\to\infty$ therefore,

$$\lim_{x\to\infty}\sin\left(\frac{1}{x}\right)=\lim_{t\to 0^+}\sin(t)=0$$

b- By the same way, we calculate the limits as $x\to\infty$ and $x\to-\infty$:

$$\lim_{x\to+\infty}x\cdot\sin\left(\frac{1}{x}\right)=\lim_{t\to 0^+}\left(\frac{\sin t}{t}\right)=1 \quad \text{and}$$

$$\lim_{x\to-\infty}x\cdot\sin\left(\frac{1}{x}\right)=\lim_{t\to 0^-}\left(\frac{\sin t}{t}\right)=1$$

Example 19 : find $\lim_{x\to\infty}\left(x-\sqrt{x^2+16}\right)$

Solution

$$\lim_{x\to\infty}\left(x-\sqrt{x^2+16}\right)=\lim_{x\to\infty}\left(x-\sqrt{x^2+16}\right)\frac{\left(x+\sqrt{x^2+16}\right)}{\left(x+\sqrt{x^2+16}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(x^2-\left(x^2+16\right)\right)}{\left(x+\sqrt{x^2+16}\right)}=\lim_{x\to\infty}\frac{-16}{\left(x+\sqrt{x^2+16}\right)}$$

As $x\to\infty$, the denominator becomes large, therefore dividing numerator and denominator by (x).

$$\lim_{x\to\infty}\frac{-16}{\left(x+\sqrt{x^2+16}\right)}=\lim_{x\to\infty}\frac{\frac{-16}{x}}{\left(1+\sqrt{\frac{x^2}{x^2}+\frac{16}{x^2}}\right)}=\frac{0}{\left(1+\sqrt{1+0}\right)}=0$$

Infinite limits

$$\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}\left(\frac{1}{x}\right)=\infty$$

$$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^-}\left(\frac{1}{x}\right)=-\infty$$

Example 20 : find $\lim_{x\to1^+}\frac{1}{x-1}$ and $\lim_{x\to1^-}\frac{1}{x-1}$

Solution

Think about the number $(x-1)$ and its reciprocal. As $x\to1^+$, we have $(x-1)\to0^+$ and $1/(x-1)\to\infty$. As $x\to1^-$, we have $(x-1)\to0^-$ and $1/(x-1)\to-\infty$. As shown below:

$$\lim_{x\to1^+}\frac{1}{x-1}=\infty$$

$$\lim_{x\to1^-}\frac{1}{x-1}=-\infty$$

Example 21 : find the limit of the rational functions a- $\lim_{x\to2}\frac{(x-2)^2}{(x^2-4)}$

b- $\lim_{x\to2}\frac{(x-2)}{(x^2-4)}$ c- $\lim_{x\to2^+}\frac{(x-3)}{(x^2-4)}$ d- $\lim_{x\to2^-}\frac{(x-3)}{(x^2-4)}$ e- $\lim_{x\to2}\frac{(x-3)}{(x^2-4)}$ f- $\lim_{x\to2}\frac{(2-x)}{(x-2)^3}$

Solution

a- $\lim_{x\to2}\frac{(x-2)^2}{(x^2-4)}=\lim_{x\to2}\frac{(x-2)(x-2)}{(x-2)(x+2)}=\lim_{x\to2}\frac{(x-2)}{(x+2)}=\frac{0}{4}=0$

b- $\lim_{x\to2}\frac{(x-2)}{(x^2-4)}=\lim_{x\to2}\frac{(x-2)}{(x-2)(x+2)}=\lim_{x\to2}\frac{1}{(x+2)}=\frac{1}{4}$

c- $\lim_{x\to2^+}\frac{(x-3)}{(x^2-4)}=\lim_{x\to2^+}\frac{(x-3)}{(x-2)(x+2)}=-\infty$ the value are negative for $x\rangle 2, x near 2$

d- $\lim_{x\to2^-}\frac{(x-3)}{(x^2-4)}=\lim_{x\to2^-}\frac{(x-3)}{(x-2)(x+2)}=\infty$ the value are negative for $x\langle 2, x near 2$

e- $\lim_{x\to2}\frac{(x-3)}{(x^2-4)}=\lim_{x\to2}\frac{(x-3)}{(x-2)(x+2)}$ does not exist

f- $\lim_{x\to2}\frac{(2-x)}{(x-2)^3}=\lim_{x\to2}\frac{-(x-2)}{(x-2)(x-2)(x-2)}=\lim_{x\to2}\frac{-1}{(x-2)^2}=-\infty$