LECTURE NOTE

ON

PROBABILITY AND STATISTICS 2

BY

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LECTURE 13#

➢ Outline :-

✓ Continuous distributions

4- Student's t-distribution

Definition

Expected value Variance

Moment generating function

Solved exercises

Student's t-distribution

Definition :A continuous random variable X is said to have a t-distribution with ν degrees of freedom if its probability density function is of the form:

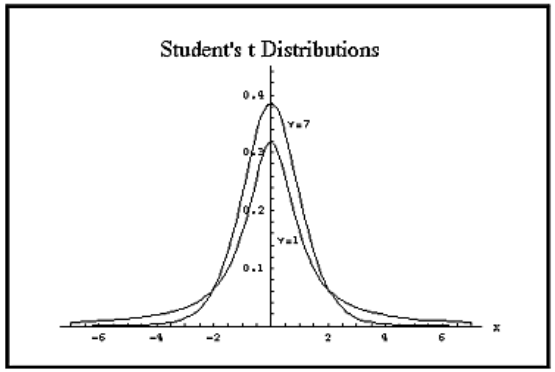
$$f(x;\,\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\,\nu}\;\Gamma\left(\frac{\nu}{2}\right)\;\left(1+\frac{x^2}{\nu}\right)^{\left(\frac{\nu+1}{2}\right)}}, \qquad -\infty < x < \infty$$

where ν > 0. If X has a t-distribution with ν degrees of freedom, then we denote it by writing X~t(ν). The t-distribution was discovered by W.S. Gosset (1876-1936) of England who published his work under the pseudonym of student. Therefore, this distribution is known as Student's t-distribution.

Note: if ν → ∞ , then

$$\lim_{\nu\to\infty} f(x;\,\nu) = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}x^2} \qquad -\infty < x < \infty,$$

which is the probability density function of the standard normal distribution.



Student's t-distribution

Theorem : If the random variable X has a t-distribution with ν degrees of freedom, then:

$$E[X] = \begin{cases} 0 & \text{if} \quad \nu \geq 2 \\ DNE & \text{if} \quad \nu = 1 \end{cases} \quad \text{and} \quad Var[X] = \begin{cases} \frac{\nu}{\nu-2} & \text{if} \quad \nu \geq 3 \\ DNE & \text{if} \quad \nu = 1, 2 \end{cases}$$

where DNE means does not exist.

Theorem: If Z ~ N(0, 1) and $U \sim \chi^2(\nu)$ and in addition, Z and U are independent, then the random variable W defined by :

$$W = \frac{Z}{\sqrt{\frac{U}{\nu}}}$$

has a t-distribution with ν degrees of freedom.

Note: A standard Student's t random variable X does not possess a moment generating function.

Student's t-distribution

Example: If T ~ t(10), then what is the probability that T is at least 2.228 ?

Solution:

$$\begin{aligned} P(T \geq 2.228) &= 1 - P(T < 2.228) \\ &= 1 - 0.975 \qquad (\text{from t} - \text{table}) \\ &= 0.025. \end{aligned}$$

Example: If T ~ t(19), then what is the value of the constant c such that

$P(|T| \leq c) = 0.95$?

Solution: (1-α) compared to the table

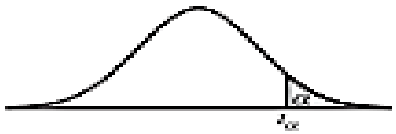
$$0.95 = P(|T| \leq c) = P(-c \leq T \leq c) = P(T \leq c) - 1 + P(T \leq c) = 2P(T \leq c) - 1.$$

Hence:

$$P(T \leq c) = 0.975.$$

$\boxed{0.025}$

Thus, using the t-table, we get for 19 degrees of freedom c = 2.093.



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

SEE YOU IN THE NEXT LECTURE