LECTURE NOTE

ON

PROBABILITY AND STATISTICS 2

BY

ASSIST. PRF. DR. MUSTAFA I. NAIF

DEPARTMENT OF MATHEMATICS
COLLEGE OF EDUCATION FOR PURE SCIENCE
UNIVERISTY OF ANBAR

LECTURE 14#

- Outline :-

Solved exercises for Normal distribution

Solved exercises

1- Let Z denote a normal random variable with mean 0 and standard deviation 1.

a Find $P(Z > 2)$.
b Find $P(-2 \leq Z \leq 2)$.
c Find $P(0 \leq Z \leq 1.73)$.

By SND table

Solution : a) Since μ = 0 and σ = 1, then , P (Z > 2) = 1-P(Z≤ 2) $= 1 - 0.9772 = .0228$.

b) $P(-2 \leq Z \leq 2) =$ 2P(Z≤ 2)-1=0.9544

c) $P(0 \leq Z \leq 1.73) =$ P(Z≤ 1.73) -P(Z≤ 0)= 0.9582-0.5=0.5482

2- If Z is a standard normal random variable, find the value z_0 such that:

a $P(Z > z_0) = .5$.
b $P(Z < z_0) = .8643$.
c $P(-z_0 < Z < z_0) = .90$.
d $P(-z_0 < Z < z_0) = .99$.

Solved exercises

Solution : a) P (Z > z_0) = 1-P(Z≤ z_0) = 0.5 $\Longrightarrow$ P(Z≤ z_0) = 0.5 $\longrightarrow$ $z_0 = 0$

Table III
Normal Distribution

The following table presents the standard normal distribution. The probabilities tabled are

$$P(X \le x) = \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw.$$

Note that only the probabilities for $x \ge 0$ are tabled. To obtain the probabilities for $x < 0$, use the identity $\Phi(-x) = 1 - \Phi(x)$.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Solved exercises

Solution : b) $P(Z< z_0) = 0.8643 \Longrightarrow z_0 = 1.10$ (by table)

c) $P(-z_0 < Z < z_0) = .90 \Longrightarrow 2P(Z< z_0) - 1 = 0.90 \Longrightarrow P(Z< z_0) = 0.95$

Thus, $z_0 = 1.645$

3) company that manufactures and bottles apple juice uses a machine that automatically fills 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed into the bottles that are filled. The amount dispensed has been observed to be approximately normally distributed with mean 16 ounces and standard deviation 1 ounce. Use Table of SND, to determine the proportion of bottles that will have more than 17 ounces dispensed into them.

Solution:

Note that the value 17 is $\underbrace{(17 - 16)/1 = 1}_{\text{Transform the value 17 to SND}}$ standard deviation above the mean.

So, $P(Z > 1) = .1587$.

SEE YOU IN THE NEXT LECTURE