كيفية إيجاد الحل الخاص (z2) :

من المعلوم بان الحل الخاص يعتمد على نوع الدالة (f (x , y) [الطرف الأيمن للمعادلة] و الذي يأخذ الحالات الآتية :

١ - الدالة الأسية ($f(x,y)=e^{ax+by}$.) ، **عندئذ يكون :**

$$Df(x,y)=ae^{ax+by}.\rightarrow D^2f(x,y)=a^2e^{ax+by}.,.....\rightarrow D^nf(x,y)=a^ne^{ax+by},$$

$$D'f(x,y)=be^{ax+by}.\rightarrow D'^2f(x,y)=b^2e^{ax+by}.,.....\rightarrow D'^nf(x,y)=b^ne^{ax+by}.$$

$$f(D,D')e^{ax+by}=f(a,b)e^{ax+by}.\rightarrow f(D,D')=f(a,b).$$

$$\because f(D,D')z=e^{ax+by}.\rightarrow z2=\frac{1}{f(D,D')}e^{ax+by}.,\rightarrow z2=\frac{1}{f(a,b)}e^{ax+by},suchthat,b\neq 0.\square$$

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مثال ١ : جد الحل العام للمعادلة : $(D^2-DD'-6D'^2)Z=e^{(2x-3y)}.$

$$\rightarrow m^2-m-6=0.,(subsdiralequation).$$

$$\rightarrow (m+2)(m-3)=0.,\rightarrow m1=-2,m3=3.$$

$$\rightarrow z1=\phi1(y-2x)+\phi2(y+3x).$$

$$\because a=2.,b=-3.\rightarrow f(2,-3)=2^2-2(-3)-6(-3)^2=4+6-54=-44\neq 0.$$

$$\because z2=\frac{1}{f(a,b)}e^{ax+by},suchthat,b\neq 0.,\rightarrow z2=\frac{-1}{44}e^{2x-3y}.$$

$$\because z=z1+z2.,\rightarrow z=\phi1(y-2x)+\phi2(y+3x)-\frac{1}{44}e^{2x-3y}.$$

مثال ٢ : جد الحل العام للمعادلة : $(D^3-3D^2D'+4D'^3)Z=e^{-x-y}.$

$\rightarrow m^3-3m^2+4=0.,(subsdiralequation).$

$\rightarrow (m+1)(m^2-4m+4)=0.,\rightarrow (m+1)(m-2)(m-2)=0.,\rightarrow m1=-1.,m2=m3=2.$

$\rightarrow z1=\phi 1(y-x)+\phi 2(y+2x)+\phi 2x(y+2x).$

$\because a=-1.,b=-1.\rightarrow f(-1,-1)=(-1)^3-3(-1)^2(-1)+4(-1)^3=-1+3-4=-2\neq 0.$

$\because z2=\frac{1}{f(a,b)}e^{ax+by},suchthat,b\neq 0.,\rightarrow z2=\frac{-1}{2}e^{-x-y}.$

$\because z=z1+z2.,\rightarrow z=\phi 1(y-x)+\phi 2(y+2x)+\phi 2x(y+2x)-\frac{1}{2}e^{-x-y}.$

تمارين ((جد الحل العام للمعادلات الآتية)) :

1	$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=e^{2x+y}.$	**5**	$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{x-y}.$
2	$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{x-y}$	**6**	$\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x2\partial y}-\frac{\partial^3 z}{\partial x\ \partial y^2}+2\frac{\partial^3 z}{\partial y^3}=e^{x+y}$
3	$(D^2-4D'^2)Z=e^{2x+y}.$	**7**	$(D^2-5DD'+4D'^2)Z=e^{x+y}.$
4	$(D^2-2DD'+D'^2)Z=e^{x+y}$	**8**	$(D^3-4D^2D'+4DD'^2)Z=e^{2x+y}.$